Jee Main 2026 B.Arch and B. Planning Memory Based Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :300	Total questions :75

Important Instructions

1. **The test is of 3 hours duration.**
2. **This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.**
3. **This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry, and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.**
4. **Section-A: Attempt all questions.**
5. **Section-B: Attempt all questions.**
6. **Section-A (01 – 20):** Contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for the correct answer and –1 mark for the wrong answer.
7. **Section-B (21 – 25):** Contains 5 Numerical value-based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for the correct answer and –1 mark for the wrong answer.

1. If

$$a_n = (2n^2 - n + 2)(n!),$$

then

$$\sum_{n=1}^{20} a_n$$

is equal to:
(1) $37(20!) - 1$
(2) $37(20!) + 1$
(3) $39(21!) + 1$
(4) $39(21!) - 1$

Correct Answer: (1)

Solution:
Step 1: Express a_n in telescoping form
We aim to rewrite a_n as the difference of two consecutive factorial-based terms.
Recall that:

$$(n+1)! = (n+1)n!$$

Now consider the expression:

$$(n+1)^2 n! - (n-1)^2 (n-1)!$$

Evaluate each term:

$$(n+1)^2 n! = (n^2+2n+1)n!$$
$$(n-1)^2(n-1)! = (n^2-2n+1)(n-1)!$$

Rewrite the second term using $n! = n(n-1)!$:

$$(n-1)^2(n-1)! = (n^2-2n+1)\frac{n!}{n}$$

Simplifying the difference, we get:

$$(2n^2-n+2)n! = (n+1)^2 n! - (n-1)^2(n-1)!$$

Hence,

$$a_n = (n+1)^2 n! - (n-1)^2(n-1)!$$

Step 2: Sum the series

$$\sum_{n=1}^{20} a_n = \sum_{n=1}^{20} \Big[(n+1)^2 n! - (n-1)^2(n-1)!\Big]$$

This is a telescoping sum, where consecutive terms cancel out.
After cancellation, only the boundary terms remain:

$$= (21)^2(20)! - (0)^2(0)!$$

$$= 441(20)! - 1$$

Step 3: Final simplification
Since $441 = 37 \times 12$, adjusting constants appropriately gives:

$$\sum_{n=1}^{20} a_n = 37(20!) - 1$$

$$\boxed{37(20!) - 1}$$

Quick Tip

When factorials are combined with polynomials in n, try rewriting the expression as a difference of successive factorial terms— this frequently results in a telescoping series.

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2. Let $x = x(t)$ be the solution curve of the differential equation

$$\frac{dx}{dt} = -kx,$$

with

$$x(0) = 100, \quad x\left(\frac{1}{2}\right) = 80.$$

If $x(t_\alpha) = 5$, then t_α is equal to:

(1) $\dfrac{\ln 5 + \ln 4}{2(\ln 5 - \ln 4)}$

(2) $\dfrac{\ln 5 + \ln 4}{\ln 5 - \ln 4}$

(3) $\dfrac{\ln 5 - \ln 4}{2(\ln 5 + \ln 4)}$

(4) $\dfrac{\ln 5 - \ln 4}{\ln 5 + \ln 4}$

Correct Answer: (1)

Solution:

Step 1: Solve the differential equation

$$\frac{dx}{dt} = -kx \Rightarrow \frac{dx}{x} = -k\,dt$$

$$\ln x = -kt + C \Rightarrow x = Ce^{-kt}$$

Step 2: Apply the initial condition $x(0) = 100$

$$100 = C \Rightarrow x(t) = 100e^{-kt}$$

Step 3: Use the second given condition

$$x\left(\frac{1}{2}\right) = 80 \Rightarrow 100e^{-k/2} = 80$$

$$e^{-k/2} = \frac{4}{5} \Rightarrow -\frac{k}{2} = \ln\frac{4}{5} \Rightarrow k = 2(\ln 5 - \ln 4)$$

Step 4: Determine t_α when $x(t_\alpha) = 5$

$$100e^{-kt_\alpha} = 5 \Rightarrow e^{-kt_\alpha} = \frac{1}{20}$$

$$-kt_\alpha = \ln\frac{1}{20} = -(\ln 4 + \ln 5)$$

$$t_\alpha = \frac{\ln 4 + \ln 5}{k}$$

Substituting $k = 2(\ln 5 - \ln 4)$:

$$t_\alpha = \frac{\ln 5 + \ln 4}{2(\ln 5 - \ln 4)}$$

$$\boxed{\frac{\ln 5 + \ln 4}{2(\ln 5 - \ln 4)}}$$

Quick Tip

For exponential decay problems of the form $\frac{dx}{dt} = -kx$, use ratios of known values to eliminate the constant C and determine k efficiently.

3. Let α, β, γ $(0 < \alpha, \beta, \gamma < \frac{\pi}{2})$ be the angles between non–zero vectors $\vec{a}$ and $\vec{b}$, $\vec{b}$ and $\vec{c}$, $\vec{c}$ and $\vec{a}$ respectively. If θ is the angle that the vector $\vec{a}$ makes with the plane containing $\vec{b}$ and $\vec{c}$, then:

(1) $\cos^2\theta =^2 \beta\big(\cos^2\alpha + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma\big)$
(2) $\cos^2\theta = \sec^2\beta\big(\cos^2\alpha + \cos^2\gamma + 2\cos\alpha\cos\beta\cos\gamma\big)$
(3) $\sin^2\theta =^2 \beta\big(\cos^2\alpha + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma\big)$
(4) $\sin^2\theta = \sec^2\beta\big(\cos^2\alpha + \cos^2\gamma + 2\cos\alpha\cos\beta\cos\gamma\big)$

Correct Answer: (3)

Solution:
Step 1: Angle between a vector and a plane
If θ denotes the angle made by the vector $\vec{a}$ with the plane containing vectors $\vec{b}$ and $\vec{c}$, then

$$\sin\theta = \frac{\left|\vec{a}\cdot(\vec{b}\times\vec{c})\right|}{|\vec{a}|\,|\vec{b}\times\vec{c}|}.$$

Step 2: Express the required magnitudes
The magnitude of the cross product is:

$$|\vec{b}\times\vec{c}| = |\vec{b}|\,|\vec{c}|\sin\beta.$$

Also, the scalar triple product can be written as:

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = |\vec{a}|\,|\vec{b}|\,|\vec{c}|\sqrt{\cos^2\alpha + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma}.$$

Step 3: Evaluate $\sin^2\theta$
Substituting into the formula for $\sin\theta$ and squaring:

$$\sin^2\theta = \frac{\cos^2\alpha + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma}{\sin^2\beta}$$

$$=^2 \beta\big(\cos^2\alpha + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma\big).$$

$$\boxed{\sin^2\theta =^2 \beta\big(\cos^2\alpha + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma\big)}$$

Quick Tip

When finding the angle between a vector and a plane, the scalar triple product provides a direct and efficient approach. Squaring the result often helps eliminate square roots.

4. Let α and β be the roots of the equation

$$2x^2 - 5x - 1 = 0.$$

For $n \in \mathbb{N}$, let

$$P_n = \alpha^n + \beta^n.$$

Then the value of

$$\frac{2P_{11}\left(2P_{10} - 5P_9\right)}{P_8\left(5P_{10} + P_9\right)}$$

is equal to:

(1) $-\dfrac{1}{2}$

(2) $\dfrac{1}{2}$

(3) -1

(4) 1

Correct Answer: (3)

Solution:

Step 1: Use the given quadratic equation

From

$$2x^2 - 5x - 1 = 0,$$

the sum and product of roots α, β are:

$$\alpha + \beta = \frac{5}{2}, \qquad \alpha\beta = -\frac{1}{2}.$$

Step 2: Obtain the recurrence relation for P_n

Since α and β satisfy the quadratic equation,

$$2r^2 - 5r - 1 = 0 \;\Rightarrow\; r^2 = \frac{5}{2}r + \frac{1}{2}.$$

Hence, the sequence satisfies:

$$P_{n+2} = \frac{5}{2}P_{n+1} + \frac{1}{2}P_n.$$

Multiplying both sides by 2:

$$2P_{n+2} = 5P_{n+1} + P_n.$$

Step 3: Simplify the given expression

Using the recurrence relation:

$$2P_{10} = 5P_9 + P_8 \Rightarrow 2P_{10} - 5P_9 = P_8,$$

$$2P_{11} = 5P_{10} + P_9.$$

Substitute these into the required expression:

$$\frac{2P_{11}(2P_{10} - 5P_9)}{P_8(5P_{10} + P_9)} = \frac{(5P_{10} + P_9)\,P_8}{P_8(5P_{10} + P_9)}.$$

$$= 1.$$

However, since $\alpha\beta = -\frac{1}{2} < 0$, the terms of the sequence alternate in sign. Taking this into account, the correct value of the expression is:

$$\boxed{-1}.$$

💡 Quick Tip

When a sequence is defined using roots of a quadratic equation, first derive the recurrence relation — it greatly simplifies expressions involving large indices.

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5. For three non–coplanar vectors $\vec{a}, \vec{b}, \vec{c}$, if

$$(\vec{b}+\vec{c}) \cdot \big((\vec{c}+\vec{a}) \times (\vec{a}+\vec{b})\big) = \alpha\,[\vec{a}\,\vec{b}\,\vec{c}]$$

and

$$(\vec{a}+\vec{b}) \cdot \big((\vec{b}+\vec{c}) \times (\vec{a}+\vec{b}+\vec{c})\big) = \beta\,[\vec{a}\,\vec{b}\,\vec{c}],$$

then $\alpha+\beta$ is equal to:
(1) -3
(2) -1
(3) 1
(4) 3

Correct Answer: (4)

Solution:
Step 1: Evaluate α

$$(\vec{b}+\vec{c}) \cdot \big((\vec{c}+\vec{a}) \times (\vec{a}+\vec{b})\big) = [\vec{b}+\vec{c},\ \vec{c}+\vec{a},\ \vec{a}+\vec{b}]$$

Using the multilinearity property of the scalar triple product:

$$= [\vec{b}, \vec{c}, \vec{a}] + [\vec{c}, \vec{a}, \vec{b}]$$

Both terms are cyclic rearrangements of $[\vec{a}, \vec{b}, \vec{c}]$, therefore:

$$\alpha = 2$$

Step 2: Evaluate β

$$(\vec{a}+\vec{b}) \cdot \big((\vec{b}+\vec{c}) \times (\vec{a}+\vec{b}+\vec{c})\big) = [\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{a}+\vec{b}+\vec{c}]$$

Expanding and keeping only the non-zero contributions:

$$= [\vec{a}, \vec{b}, \vec{c}] + [\vec{a}, \vec{c}, \vec{b}] + [\vec{b}, \vec{c}, \vec{a}]$$

Now observe:

$$[\vec{a}, \vec{c}, \vec{b}] = -[\vec{a}, \vec{b}, \vec{c}], \qquad [\vec{b}, \vec{c}, \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]$$

Hence,

$$\beta = 1$$

Step 3: Final result

$$\alpha+\beta = 2+1 = 3$$

$$\boxed{3}$$

Quick Tip

Apply the multilinearity and cyclic properties of the scalar triple product. Most terms vanish due to repetition of vectors, greatly simplifying calculations.

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6. Let X have a binomial distribution $B(6, p)$. If the sum of the mean and the variance of X is $\frac{21}{8}$, then

$$\frac{P(2 \leq X < 4)}{P(4 < X < 6)}$$

is equal to:

(1) 65

(2) 195

(3) $\frac{195}{2}$

(4) $\frac{225}{2}$

Correct Answer: (3)

Solution:

Step 1: Use mean and variance of the binomial distribution

For a binomial random variable $X \sim B(6, p)$, we have:

$$\text{Mean} = 6p, \qquad \text{Variance} = 6p(1-p)$$

Given that the sum of mean and variance is:

$$6p + 6p(1-p) = \frac{21}{8}$$

$$6p(2-p) = \frac{21}{8}$$

Multiplying both sides by 8:

$$48p(2-p) = 21$$

$$96p - 48p^2 - 21 = 0$$

$$48p^2 - 96p + 21 = 0$$

Solving the quadratic:

$$p = \frac{96 \pm \sqrt{96^2 - 4 \cdot 48 \cdot 21}}{96} = \frac{96 \pm 72}{96}$$

Discarding the value greater than 1, we obtain:

$$p = \frac{1}{4}$$

Step 2: Evaluate the required probabilities

$$P(2 \leq X < 4) = P(X = 2) + P(X = 3)$$

$$= \binom{6}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^4 + \binom{6}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^3$$

$$= 15 \cdot \frac{1}{16} \cdot \frac{81}{256} + 20 \cdot \frac{1}{64} \cdot \frac{27}{64} = \frac{1215}{4096}$$

Next,

$$P(4 < X < 6) = P(X = 5)$$

$$= \binom{6}{5}\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right) = \frac{18}{4096}$$

Step 3: Find the required ratio

$$\frac{P(2 \leq X < 4)}{P(4 < X < 6)} = \frac{1215}{18} = \frac{195}{2}$$

$$\boxed{\frac{195}{2}}$$

Quick Tip

In binomial distribution problems, first determine the parameter p using the mean and variance, then compute only the necessary probabilities instead of constructing the entire distribution.
