

## JEE Mains - 27 Jan (Shift 2) Mathematics Question Paper

|                                     |                        |                      |                                  |
|-------------------------------------|------------------------|----------------------|----------------------------------|
| Total Time Allowed<br>: 180 minutes | Maximum Marks :<br>300 | Total Questions : 90 | Questions to be<br>answered : 90 |
|-------------------------------------|------------------------|----------------------|----------------------------------|

**Question 1:** Considering only the principal values of inverse trigonometric functions, the number of positive real values of  $x$  satisfying  $\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{4}$  is:

**Options:**

- (1) More than 2
  - (2) 1
  - (3) 2
  - (4) 0
- 

**Question 2:** Consider the function  $f : (0, 2) \rightarrow R$  defined by  $f(x) = \frac{x}{2} + \frac{2}{x}$  and the function  $g(x)$  defined by

$$g(x) = \begin{cases} \min\{f(t)\} & \text{for } 0 < t \leq x \text{ and } 0 < x \leq 1 \\ \frac{3}{2} + x & \text{for } 1 < x < 2 \end{cases}.$$

**Then:**

**Options:**

- (1)  $g$  is continuous but not differentiable at  $x = 1$
  - (2)  $g$  is not continuous for all  $x \in (0, 2)$
  - (3)  $g$  is neither continuous nor differentiable at  $x = 1$
  - (4)  $g$  is continuous and differentiable for all  $x \in (0, 2)$
- 

**Question 3:** Let the image of the point  $(1, 0, 7)$  in the line  $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$  be the point (

**Options:**

- (1)  $(1, -2, 1 + \sqrt{2})$
  - (2)  $(1, 2, 1 - \sqrt{2})$
  - (3)  $(3, 4, 3 - 2\sqrt{2})$
  - (4)  $(3, -4, 3 + 2\sqrt{2})$
- 

**Question 4:** Let  $R$  be the interior region between the lines  $3x - y + 1 = 0$  and  $x + 2y - 5 = 0$  containing the origin. The set of all values of  $a$ , for which the points  $(a^2, a + 1)$  lie in  $R$ , is:

**Options:**

- (1)  $(-3, -1) \cup (-\frac{1}{3}, 1)$
  - (2)  $(-3, 0) \cup (-\frac{1}{3}, 1)$
  - (3)  $(-3, 0) \cup (\frac{2}{3}, 1)$
  - (4)  $(-3, -1) \cup (-\frac{1}{3}, 1)$
- 

**Question 5:** The 20th term from the end of the progression  $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots, -129\frac{1}{4}$  is:

**Options:**

- (1)  $-118$
  - (2)  $-110$
  - (3)  $-115$
  - (4)  $-100$
- 

**Question 6:** Let  $f : R \setminus \{-\frac{1}{2}\} \rightarrow R$  and  $g : R \setminus \{-\frac{5}{2}\} \rightarrow R$  be defined as  $f(x) = \frac{2x+3}{2x+1}$  and  $g(x) = \frac{|x|+1}{2x+5}$ . Then the domain of the function  $f \circ g$  is:

**Options:**

- (1)  $R \setminus \{-\frac{5}{2}\}$
  - (2)  $R$
  - (3)  $R \setminus \{-\frac{7}{4}\}$
  - (4)  $R \setminus \{-\frac{5}{2}, -\frac{7}{4}\}$
- 

**Question 7:** For  $0 < a < 1$ , the value of the integral  $\int_0^\pi \frac{dx}{1-2a \cos x + a^2}$  is:

**Options:**

- (1)  $\frac{\pi^2}{\pi+a^2}$
  - (2)  $\frac{\pi^2}{\pi-a^2}$
  - (3)  $\frac{\pi}{1-a^2}$
  - (4)  $\frac{\pi}{1+a^2}$
- 

**Question 8:** Let  $g(x) = 3f(\frac{x}{3}) + f(3-x)$  and  $f''(x) > 0$  for all  $x \in (0, 3)$ . If  $g$  is decreasing in  $(0, \alpha)$  and increasing in  $(\alpha, 3)$ , then  $8\alpha$  is:

**Options:**

- (1)  $24$
- (2)  $0$

- (3) 18  
(4) 20
- 

**Question 9:** If  $\lim_{x \rightarrow 0} \frac{3 + \alpha \sin x + \beta \cos x + \log_e(1-x)}{3 \tan^2 x} = \frac{1}{3}$ , then  $2\alpha - \beta$  is equal to:

**Options:**

- (1) 2  
(2) 7  
(3) 5  
(4) 1
- 

**Question 10:** If  $\alpha, \beta$  are the roots of the equation  $x^2 - x - 1 = 0$  and  $S_n = 2023\alpha^n + 2024\beta^n$ , then:

**Options:**

- (1)  $2S_{12} = S_{11} + S_{10}$   
(2)  $S_{12} = S_{11} + S_{10}$   
(3)  $2S_{11} = S_{12} + S_{10}$   
(4)  $S_{11} = S_{10} + S_{12}$
- 

**Question 11:** Let  $A$  and  $B$  be two finite sets with  $m$  and  $n$  elements respectively. The total number of subsets of the set  $A$  is 56 more than the total number of subsets of  $B$ . Then the distance of the point  $P(m, n)$  from the point  $Q(-2, -3)$  is:

**Options:**

- (1) 10  
(2) 6  
(3) 4  
(4) 8
- 

**Question 12:** The values of  $\alpha$  for which

$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$$

lie in the interval:

**Options:**

- (1)  $(-2, 1)$

- (2)  $(-3, 0)$
  - (3)  $(-\frac{3}{2}, \frac{3}{2})$
  - (4)  $(0, 3)$
- 

**Question 13:** An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability that the first draw gives all white balls and the second draw gives all black balls is:

**Options:**

- (1)  $\frac{5}{256}$
  - (2)  $\frac{5}{715}$
  - (3)  $\frac{3}{715}$
  - (4)  $\frac{3}{256}$
- 

**Question 14:** The integral  $\int \frac{(x^8 - x^2)dx}{(x^{12} + 3x^6 + 1) \tan^{-1}(x^3 + \frac{1}{x^3})}$  is equal to:

**Options:**

- (1)  $\log_e \left[ \tan^{-1} \left( x^3 + \frac{1}{x^3} \right) \right]^{1/3} + C$
  - (2)  $\log_e \left[ \tan^{-1} \left( x^3 + \frac{1}{x^3} \right) \right]^{1/2} + C$
  - (3)  $\log_e \left[ \tan^{-1} \left( x^3 + \frac{1}{x^3} \right) \right] + C$
  - (4)  $\log_e \left[ \tan^{-1} \left( x^3 + \frac{1}{x^3} \right) \right]^3 + C$
- 

**Question 15:** If  $2 \tan^2 \theta - 5 \sec \theta = 1$  has exactly 7 solutions in the interval  $[0, \frac{n\pi}{2}]$ , for the least value of  $n \in N$ , then  $\sum_{k=1}^n \frac{k}{2^k}$  is equal to:

**Options:**

- (1)  $\frac{1}{2^{15}}(2^{14} - 14)$
  - (2)  $\frac{1}{2^{14}}(2^{15} - 15)$
  - (3)  $1 - \frac{15}{2^{13}}$
  - (4)  $\frac{1}{2^{13}}(2^{14} - 15)$
- 

**Question 16:** The position vectors of the vertices  $A, B$ , and  $C$  of a triangle are  $\vec{A} = 2\vec{i} - 3\vec{j} + 3\vec{k}$ ,  $\vec{B} = 2\vec{i} + 2\vec{j} + 3\vec{k}$ , and  $\vec{C} = -\vec{i} + \vec{j} + 3\vec{k}$  respectively. Let  $\ell$  denote the length of the angle bisector  $AD$  of  $\angle BAC$  where  $D$  is on the line segment  $BC$ . Then  $2\ell^2$  equals:

**Options:**

- (1) 49

- (2) 42
  - (3) 50
  - (4) 45
- 

**Question 17:** If  $y = y(x)$  is the solution curve of the differential equation  $(x^2 - 4)dy - (y^2 - 3y)dx = 0$ ,  $x > 2$ ,  $y(4) = \frac{3}{2}$ , and the slope of the curve is never zero, then the value of  $y(10)$  equals:

**Options:**

- (1)  $\frac{3}{1+(8)^{1/4}}$
  - (2)  $\frac{3}{1+2\sqrt{2}}$
  - (3)  $\frac{3}{1-2\sqrt{2}}$
  - (4)  $\frac{3}{1-(8)^{1/4}}$
- 

**Question 18:** Let  $e_1$  be the eccentricity of the hyperbola  $\frac{x^2}{16} - \frac{y^2}{9} = 1$  and  $e_2$  be the eccentricity of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $a > b$ , which passes through the foci of the hyperbola. If  $e_1 e_2 = 1$ , then the length of the chord of the ellipse parallel to the  $x$ -axis and passing through  $(0, 2)$  is:

**Options:**

- (1)  $4\sqrt{5}$
  - (2)  $\frac{8\sqrt{5}}{3}$
  - (3)  $\frac{10\sqrt{5}}{3}$
  - (4)  $3\sqrt{5}$
- 

**Question 19:** Let  $\alpha = \frac{(4!)!}{(4!)^3!}$  and  $\beta = \frac{(5!)!}{(5!)^4!}$ . Then:

**Options:**

- (1)  $\alpha \in N$  and  $\beta \notin N$
  - (2)  $\alpha \notin N$  and  $\beta \in N$
  - (3)  $\alpha \in N$  and  $\beta \in N$
  - (4)  $\alpha \notin N$  and  $\beta \notin N$
- 

**Question 20:** Let the position vectors of the vertices  $A, B$  and  $C$  of a triangle be  $2\vec{i} + 2\vec{j} + \vec{k}$ ,  $\vec{i} + 2\vec{j} + 2\vec{k}$ , and  $2\vec{i} + \vec{j} + 2\vec{k}$  respectively. Let  $l_1, l_2$  and  $l_3$  be the lengths of perpendiculars drawn from the orthocenter of the triangle on the

sides  $AB, BC$  and  $CA$  respectively, then  $l_1^2 + l_2^2 + l_3^2$  equals:

Options:

- (1)  $\frac{1}{5}$
- (2)  $\frac{1}{2}$
- (3)  $\frac{1}{4}$
- (4)  $\frac{1}{3}$

---

**Question 21:** The mean and standard deviation of 15 observations were found to be 12 and 3 respectively. On rechecking, it was found that an observation was read as 10 in place of 12. If  $\mu$  and  $\sigma^2$  denote the mean and variance of the correct observations respectively, then  $15(\mu + \mu^2 + \sigma^2)$  is equal to:

---

**Question 22:** If the area of the region  $\{(x, y) : 0 \leq y \leq \min(2x, 6x - x^2)\}$  is  $A$ , then  $12A$  is equal to:

---

**Question 24:** If the sum of squares of all real values of  $\alpha$ , for which the lines  $2x - y + 3 = 0$ ,  $6x + 3y + 1 = 0$ , and  $ax + 2y - 2 = 0$  do not form a triangle is  $p$ , then the greatest integer less than or equal to  $p$  is:

---

**Question 25:** The coefficient of  $x^{2012}$  in the expansion of  $(1 - x)^{2008}(1 + x + x^2)^{2007}$  is equal to:

---

**Question 26:** If the solution curve of the differential equation  $\frac{dy}{dx} = \frac{x+y-2}{x-y}$  passing through the point  $(2, 1)$  is

$$\tan^{-1} \left( \frac{y-1}{x-1} \right) - \frac{1}{\beta} \log_e \left( \alpha + \left( \frac{y-1}{x-1} \right)^2 \right) = \log_e |x-1|,$$

then  $5\beta + \alpha$  is equal to:

---

**Question 27:** Let  $f(x) = \int_0^x g(t) \log_e \left( \frac{1-t}{1+t} \right) dt$ , where  $g$  is a continuous odd function. If

$$I = \int_{-\pi/2}^{\pi/2} (f(x) \cdot \frac{x^2 \cos x}{1+e^x}) dx = \left( \frac{\pi}{\alpha} \right)^2 - \alpha,$$

then  $\alpha$  is equal to:

---

**Question 28:** Consider a circle  $(x - \alpha)^2 + (y - \beta)^2 = 50$ , where  $\alpha, \beta > 0$ . If the

circle touches the line  $y + x = 0$  at the point  $P$ , whose distance from the origin is  $4\sqrt{2}$ , then  $(\alpha + \beta)^2$  is equal to:

---

**Question 29:** The lines  $\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z-7}{8}$  and  $\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1}$  intersect at the point  $P$ . If the distance of  $P$  from the line  $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z-1}{1}$  is  $l$ , then  $14l^2$  is equal to:

---

**Question 30:** Let the complex numbers  $\alpha$  and  $\frac{1}{\alpha}$  lie on the circles  $|z - z_0|^2 = 4$  and  $|z - z_0|^2 = 16$  respectively, where  $z_0 = 1 + i$ . Then, the value of  $100|\alpha|^2$  is:

---