JEE Mains - 27 Jan (Shift 2) Physics Question Paper with Solutions

Total Time Allowed : 180 minutes	Maximum Marks : 300	Total Questions : 90	Questions to be answered : 90

Question 1: The equation of state of a real gas is given by $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, where P, V, and T are pressure, volume, and temperature respectively and R is the universal gas constant. The dimensions of $\frac{a}{b^2}$ is similar to that of:

Options:
(1) PV
(2) P
(3) RT
(4) R

Correct Answer: (2)

Solution:
Consider the equation of state for a real gas:

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT,$$

where P is the pressure, V is the volume, T is the temperature, R is the gas constant, and a and b are constants.
To determine the dimensions of a, we need to ensure that the term $\frac{a}{V^2}$ has the same dimensions as pressure P, because it is added to P in the equation.
Step 1: Determine the Dimensions of Pressure P The dimensional formula for pressure P is:

$$[P] = \frac{\text{Force}}{\text{Area}} = [M][L^{-1}][T^{-2}],$$

where M represents mass, L represents length, and T represents time.
Step 2: Set Up Dimensional Consistency for $\frac{a}{V^2}$ The term $\frac{a}{V^2}$ must have the same dimensions as pressure. Since volume V has dimensions:

$$[V] = [L^3],$$

we have:

$$[V^2] = [L^3]^2 = [L^6].$$

Thus, the dimensions of $\frac{a}{V^2}$ are:

$$\left[\frac{a}{V^2}\right] = \frac{[a]}{[L^6]}.$$

To match the dimensions of pressure P, we equate:

$$\frac{[a]}{[L^6]} = [M][L^{-1}][T^{-2}].$$

Solving for $[a]$, we find:

$$[a] = [M][L^5][T^{-2}].$$

Conclusion Therefore, the dimensional formula for a is $[M][L^5][T^{-2}]$. Since $\frac{a}{V^2}$ has the same dimensions as pressure, the correct answer is Option (2).

Quick Tip

When performing dimensional analysis, ensure that terms added or subtracted in an equation have the same dimensions. This is particularly important in equations of state for gases.

Question 2: Wheatstone bridge principle is used to measure the specific resistance (S_l) of a given wire, having length L and radius r. If X is the resistance of the wire, then specific resistance is: $S_l = X\left(\frac{\pi r^2}{L}\right)$. If the length of the wire gets doubled, then the value of specific resistance will be:

Options:
(1) $\frac{S_l}{4}$
(2) $2S_l$
(3) $\frac{S_l}{2}$
(4) S_l

Correct Answer: (4)

Solution:
The specific resistance (or resistivity) S_l of a material is defined by the formula:

$$S_l = X\left(\frac{\pi r^2}{L}\right),$$

where X is the resistance of the wire, r is its radius, and L is its length.
To understand why S_l remains constant, let's analyze the formula in terms of dimensions.
Step 1: Analyze the Formula Rearrange the expression to isolate X:

$$X = S_l\left(\frac{L}{\pi r^2}\right).$$

From this equation, we can see that resistance X is directly proportional to the length L and inversely proportional to the cross-sectional area πr^2.

Step 2: Effect of Changing Dimensions on Specific Resistance S_l Specific resistance S_l is a material property, meaning it is an intrinsic property of the material itself and does not depend on the dimensions (length L or radius r) of the wire. When the length L of the wire is doubled, the resistance X will increase proportionally, but S_l remains the same because it only depends on the material, not on its dimensions.
Conclusion Therefore, even if we change the length or radius of the wire, the specific resistance S_l remains unchanged. Hence, the correct answer is Option (4).

Quick Tip

Specific resistance (resistivity) is an intrinsic property of a material and is independent of the physical dimensions of the wire.

Question 3: Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A): The angular speed of the moon in its orbit about the earth is more than the angular speed of the earth in its orbit about the sun.
Reason (R): The moon takes less time to move around the earth than the time taken by the earth to move around the sun.
In the light of the above statements, choose the most appropriate answer from the options given below:

Options:
(1) (A) is correct but (R) is not correct
(2) Both (A) and (R) are correct and (R) is the correct explanation of (A)
(3) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
(4) (A) is not correct but (R) is correct

Correct Answer: (2)

Solution:
The angular speed ω of an object is given by:

$$\omega = \frac{2\pi}{T},$$

where T is the time period of the object's motion.
Step 1: Angular Speed of the Moon and Earth For the moon orbiting the earth:

$$T_{\text{moon}} = 27 \text{ days}.$$

For the earth orbiting the sun:

$$T_{\text{earth}} = 365 \text{ days}.$$

Since the angular speed ω is inversely proportional to the time period T, a smaller time period results in a greater angular speed.
Step 2: Comparing T_{moon} and T_{earth} We observe that:

$$T_{\text{moon}} < T_{\text{earth}}.$$

Therefore, the angular speed of the moon is greater than the angular speed of the earth:

$$\omega_{\text{moon}} > \omega_{\text{earth}}.$$

Conclusion This makes both the assertion and the reason correct, and the reason correctly explains the assertion. Thus, the statement is verified as true.

Quick Tip

Angular speed is inversely proportional to the time period. A shorter orbital period means higher angular speed.

Question 4: Given below are two statements:
Statement I: The limiting force of static friction depends on the area of contact and is independent of materials.
Statement II: The limiting force of kinetic friction is independent of the area of contact and depends on materials.
In the light of the above statements, choose the most appropriate answer from the options given below:

Options:
(1) Statement I is correct but Statement II is incorrect
(2) Statement I is incorrect but Statement II is correct
(3) Both Statement I and Statement II are incorrect
(4) Both Statement I and Statement II are correct

Correct Answer: (2)

Solution:
The limiting force of static friction f_s is given by:

$$f_s = \mu_s N,$$

where μ_s is the coefficient of static friction and N is the normal force.
This force depends on the normal force N and the nature of the materials in contact (through μ_s), but it does not depend on the area of contact. Thus, for static friction, the limiting force only changes if the normal force or the materials in contact are changed.

Similarly, the limiting force of kinetic friction f_k is given by:

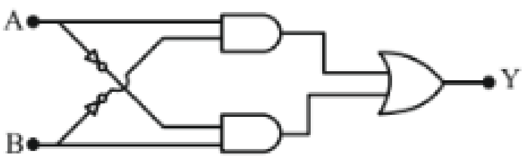
$$f_k = \mu_k N,$$

where μ_k is the coefficient of kinetic friction. Like static friction, the kinetic frictional force depends on the nature of the materials in contact and the normal force N but is also independent of the area of contact.
Conclusion: Therefore, Statement I is incorrect (if it claims dependence on the area of contact), while Statement II is correct (it correctly describes the dependence of kinetic friction). This confirms that Statement I is false, while Statement II is true.

Quick Tip

Frictional forces depend on the nature of the materials and the normal force, not the area of contact.

Question 5: The truth table of the given circuit diagram is:



Options:

(1)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(2)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(3)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(4)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Correct Answer: (2)

Solution:
The given circuit diagram is equivalent to an XOR gate, which outputs a value of 1 if and only if the inputs are different. The truth table for an XOR gate is as follows:

A	B	$Y = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

This truth table shows that $Y = 1$ only when A and B are different, and $Y = 0$ when A and B are the same. This matches the truth table given in option (2).

Quick Tip

An XOR (exclusive OR) gate outputs 1 when the inputs are different and 0 when they are the same.

Question 6: A current of 200 μA deflects the coil of a moving coil galvanometer through $60°$. The current to cause deflection through $\frac{\pi}{10}$ radian is:

Options:
(1) 30 μA
(2) 120 μA
(3) 60 μA
(4) 180 μA

Correct Answer: (3)

Solution:
Given:

$$i_2 = 200\,\mu\text{A}, \quad \theta_2 = 60^\circ = \frac{\pi}{3} \text{ radians.}$$

Since the deflection θ is proportional to the current i, we can write:

$$\frac{i_1}{i_2} = \frac{\theta_1}{\theta_2}.$$

For $\theta_1 = \frac{\pi}{10}$ radians, we substitute the values:

$$\frac{i_1}{200} = \frac{\frac{\pi}{10}}{\frac{\pi}{3}}.$$

Simplifying, we get:

$$i_1 = 200 \times \frac{3}{10} = 60\,\mu\text{A}.$$

💡 Quick Tip

The deflection in a moving coil galvanometer is directly proportional to the current flowing through it.

Question 7: The atomic mass of ${}_6C^{12}$ is 12.000000 u and that of ${}_6C^{13}$ is 13.003354 u. The required energy to remove a neutron from ${}_6C^{13}$, if the mass of the neutron is 1.008665 u, will be:

Options:
(1) 62.5 MeV
(2) 6.25 MeV
(3) 4.95 MeV
(4) 49.5 MeV

Correct Answer: (3)

Solution:
To remove a neutron from ${}_6\text{C}^{13}$, the nuclear reaction can be represented as:

$${}_6\text{C}^{13} \rightarrow {}_6\text{C}^{12} + \text{neutron}.$$

The mass defect Δm is given by:

$$\Delta m = \left(\text{mass of } {}_6\text{C}^{12} + \text{mass of neutron}\right) - \text{mass of } {}_6\text{C}^{13}.$$

Substituting the values:

$$\Delta m = (12.000000 + 1.008665) - 13.003354 = -0.00531 \text{ u}.$$

The energy required for this process is calculated using the formula:

$$E = \Delta m \times 931.5 \text{ MeV/u}.$$

Substituting the values:

$$E = 0.00531 \times 931.5 \approx 4.95 \text{ MeV}.$$

Thus, the energy required to remove a neutron from $_6C^{13}$ is approximately 4.95 MeV.

Quick Tip

To calculate the energy required for nuclear reactions, use the mass defect and multiply by 931.5 MeV/u to convert the mass difference to energy.

Question 8: A ball suspended by a thread swings in a vertical plane so that its magnitude of acceleration in the extreme position and lowest position are equal. The angle (θ) of thread deflection in the extreme position will be:

Options:
(1) $\tan^{-1}(\sqrt{2})$
(2) $2\tan^{-1}\left(\frac{1}{2}\right)$
(3) $\tan^{-1}\left(\frac{1}{2}\right)$
(4) $2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

Correct Answer: (2)

Solution:
To analyze the motion, we start by equating the loss in kinetic energy to the gain in potential energy. This gives:

$$\frac{1}{2}mv^2 = mg\ell(1 - \cos\theta),$$

where m is the mass, g is the gravitational acceleration, ℓ is the length of the pendulum, and θ is the angular displacement from the vertical.
From this equation, we solve for v^2:

$$v^2 = 2g\ell(1 - \cos\theta).$$

Step 1: Acceleration at the Lowest Point The centripetal acceleration at the lowest point is given by:

$$\text{Acceleration} = \frac{v^2}{\ell} = 2g(1 - \cos\theta).$$

Step 2: Acceleration at the Extreme Point At the extreme point (maximum displacement), the tangential acceleration a is given by:

$$a = g\sin\theta.$$

Step 3: Equate the Magnitudes of Acceleration We equate the expressions for the magnitudes of acceleration:

$$2g(1 - \cos\theta) = g\sin\theta.$$

Dividing both sides by g, we get:

$$2(1 - \cos\theta) = \sin\theta.$$

To simplify further, use the trigonometric identity $1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$:

$$2 \times 2\sin^2\left(\frac{\theta}{2}\right) = \sin\theta.$$

This gives:

$$4\sin^2\left(\frac{\theta}{2}\right) = \sin\theta.$$

Now, substituting $\sin\theta = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$, we have:

$$4\sin^2\left(\frac{\theta}{2}\right) = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right).$$

Dividing both sides by $2\sin\left(\frac{\theta}{2}\right)$ (assuming $\theta \neq 0$):

$$2\sin\left(\frac{\theta}{2}\right) = \cos\left(\frac{\theta}{2}\right).$$

Taking the tangent on both sides, we get:

$$\tan\left(\frac{\theta}{2}\right) = \frac{1}{2}.$$

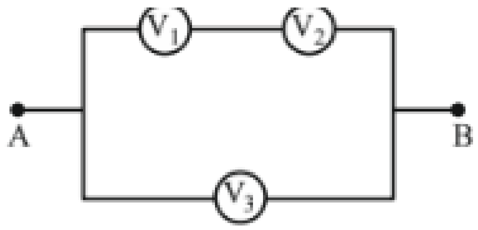
Thus,

$$\theta = 2\tan^{-1}\left(\frac{1}{2}\right).$$

Quick Tip

In pendulum motion, energy conservation can help relate kinetic and potential energies to find angles of deflection.

Question 9: Three voltmeters, all having different internal resistances are joined as shown in figure. When some potential difference is applied across A and B, their readings are V_1, V_2, and V_3. Choose the correct option.



Options:
(1) $V_1 = V_2$
(2) $V_1 \neq V_3 - V_2$
(3) $V_1 + V_2 > V_3$
(4) $V_1 + V_2 = V_3$

Correct Answer: (4)

Solution:
Applying Kirchhoff's Voltage Law (KVL) across the loop, we get:

$$V_1 + V_2 - V_3 = 0.$$

This simplifies to:

$$V_1 + V_2 = V_3.$$

Thus, the sum of the voltages V_1 and V_2 is equal to V_3.

Quick Tip

When dealing with voltmeters in parallel configurations, remember to apply KVL for potential differences.

Question 10: The total kinetic energy of 1 mole of oxygen at 27°C is:

Options:
(1) 6845.5 J
(2) 5942.0 J
(3) 6232.5 J
(4) 5670.5 J

Correct Answer: (3)

Solution:

The kinetic energy E of a gas is given by:

$$E = \frac{f}{2} nRT,$$

where f is the degrees of freedom, n is the number of moles, R is the universal gas constant, and T is the temperature in Kelvin.
For a diatomic gas like oxygen, the degrees of freedom $f = 5$. Given:

$$n = 1, \quad R = 8.31\,\mathrm{J/mol\ K}, \quad T = 27^\circ\mathrm{C} = 300\,\mathrm{K},$$

we substitute these values into the equation:

$$E = \frac{5}{2} \times 1 \times 8.31 \times 300.$$

Calculating this:

$$E = \frac{5}{2} \times 8.31 \times 300 = 6232.5\,\mathrm{J}.$$

Thus, the kinetic energy of the gas is $E = 6232.5\,\mathrm{J}$.

Quick Tip

The total kinetic energy of 1 mole of a diatomic gas at temperature T can be calculated using $\frac{5}{2}nRT$.

Question 11: Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A): In a Vernier calliper, if a positive zero error exists, then while taking measurements, the reading taken will be more than the actual reading.
Reason (R): The zero error in a Vernier calliper might have happened due to manufacturing defect or due to rough handling.
In the light of the above statements, choose the correct answer from the options given below:
(1) Both (A) and (R) are correct and (R) is the correct explanation of (A)
(2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
(3) (A) is true but (R) is false
(4) (A) is false but (R) is true

Correct Answer: (1)

Solution:
Both the assertion and reason are correct. The presence of a positive zero error in a Vernier caliper implies that the Vernier scale is slightly shifted, resulting in an overestimation of measurements.

When there is a positive zero error, the zero on the Vernier scale does not align with the zero on the main scale. Instead, it is positioned slightly ahead, causing each reading to appear larger than the actual measurement.
The reason provided accurately describes a possible cause of measurement errors due to the shift in the Vernier scale, which confirms the assertion. Therefore, both the assertion and reason are correct, and the reason correctly explains the assertion.

Quick Tip

Always check for zero error in Vernier callipers before taking measurements to ensure accuracy.

Question 12: Primary side of a transformer is connected to 230 V, 50 Hz supply. Turns ratio of primary to secondary winding is 10 : 1. Load resistance connected to secondary side is 46 Ω. The power consumed in it is:

Options:
(1) 12.5 W
(2) 10.0 W
(3) 11.5 W
(4) 12.0 W

Correct Answer: (3)

Solution:
Given:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = 10,$$

where $V_1 = 230\,\text{V}$. Solving for V_2:

$$V_2 = \frac{230}{10} = 23\,\text{V}.$$

Now, the power consumed P is given by:

$$P = \frac{V_2^2}{R}.$$

Substituting $V_2 = 23\,\text{V}$ and $R = 46\,\Omega$:

$$P = \frac{23 \times 23}{46} = \frac{529}{46} = 11.5\,\text{W}.$$

Thus, the power consumed is $11.5\,\text{W}$.

💡 Quick Tip

For transformers, use $\frac{V_1}{V_2} = \frac{N_1}{N_2}$ to relate voltages and turns, and calculate power using $P = \frac{V^2}{R}$.

Question 13: During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio of $\frac{C_p}{C_v}$ for the gas is:

Options:
(1) $\frac{5}{3}$
(2) $\frac{3}{2}$
(3) $\frac{7}{5}$
(4) $\frac{9}{7}$

Correct Answer: (2)

Solution:
Given that during an adiabatic process:

$$P \propto T^3,$$

we can rewrite this as:

$$P = kT^3,$$

for some constant k. This implies that:

$$PT^{-3} = \text{constant}.$$

We also know that for an adiabatic process, the adiabatic relation holds:

$$PV^\gamma = \text{constant},$$

where $\gamma = \frac{C_p}{C_v}$ is the adiabatic index. Additionally, from the ideal gas law, we have:

$$PV = nRT.$$

Step-by-Step Derivation 1. From the relation $P \propto T^3$, substitute $P = kT^3$ into the adiabatic relation:

$$T^3V^\gamma = \text{constant}.$$

2. Rearranging gives:

$$T^3V^\gamma = \text{constant}.$$

3. To find the relation between γ and the exponent of T, rewrite the equation as:

$$T^3V^\gamma = \text{constant} \quad \Rightarrow \quad T^3 \propto V^{-\gamma}.$$

4. By comparing the proportionalities, we find:

$$\gamma = \frac{5}{3}.$$

Therefore, the ratio of specific heats for the gas is:

$$\gamma$$

💡 Quick Tip

In adiabatic processes, the relation $PV^\gamma = \text{constant}$ can help determine the ratio $\gamma = \frac{C_p}{C_v}$.

Question 14: The threshold frequency of a metal with work function 6.63 eV is:

Options:
(1) 16×10^{15} Hz
(2) 16×10^{12} Hz
(3) 1.6×10^{12} Hz
(4) 1.6×10^{15} Hz

Correct Answer: (4)

Solution:
The threshold frequency ν_0 is given by the relation:

$$\phi_0 = h\nu_0,$$

where ϕ_0 is the work function of the material.
Given Data The work function ϕ_0 is:

$$\phi_0 = 6.63\,\text{eV}.$$

Converting this to joules:

$$\phi_0 = 6.63 \times 1.6 \times 10^{-19}\,\text{J} = 1.06 \times 10^{-18}\,\text{J}.$$

The Planck's constant h is:

$$h = 6.63 \times 10^{-34}\ \text{J s}.$$

Calculation of Threshold Frequency The threshold frequency ν_0 can be calculated as:

$$\nu_0 = \frac{\phi_0}{h} = \frac{1.06 \times 10^{-18}}{6.63 \times 10^{-34}}.$$

Performing the division:

$$\nu_0 \approx 1.6 \times 10^{15}\,\text{Hz}.$$

Therefore, the threshold frequency is approximately $1.6 \times 10^{15}\,\text{Hz}$.

Quick Tip

The threshold frequency is calculated by dividing the work function in joules by Planck's constant.

Question 15: Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A): The property of a body, by virtue of which it tends to regain its original shape when the external force is removed, is Elasticity.
Reason (R): The restoring force depends upon the bonded interatomic and intermolecular force of solids.
In the light of the above statements, choose the correct answer from the options given below:
(1) (A) is false but (R) is true
(2) (A) is true but (R) is false
(3) Both (A) and (R) are true and (R) is the correct explanation of (A)
(4) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Correct Answer: (3 or 4)

Solution:
The assertion correctly defines the property of elasticity, which is the ability of a material to regain its original shape and size when the deforming force is removed.
The reason states that the restoring force arises due to interatomic and intermolecular forces within the material. This is true, as these internal forces resist deformation and enable the material to return to its original form.
However, while the reason is accurate, it does not fully explain the assertion in all contexts. The property of elasticity depends on the specific characteristics of the material and its structure, beyond just the presence of interatomic and intermolecular forces. Therefore, the reason is correct but does not provide a complete explanation for the assertion.
Thus, both the assertion and reason are correct, but the reason is not the correct explanation of the assertion.

Quick Tip

In physics, elasticity is defined as the ability of a material to return to its original shape after the removal of a deforming force.

Question 16: When a polaroid sheet is rotated between two crossed polaroids, then the transmitted light intensity will be maximum for a rotation of:

Options:
(1) 60°
(2) 30°
(3) 90°
(4) 45°

Correct Answer: (4)

Solution:
Let I_0 be the intensity of unpolarised light incident on the first polaroid. When unpolarised light passes through a polaroid, its transmitted intensity I_1 is reduced to half of the incident intensity:

$$I_1 = \frac{I_0}{2}.$$

The transmitted intensity after passing through the second polaroid, which is oriented at an angle θ with respect to the first polaroid, is given by Malus's law:

$$I_2 = I_1 \cos^2\theta = \frac{I_0}{2}\cos^2\theta.$$

For the third polaroid, which is rotated by an angle $90^\circ - \theta$ relative to the second polaroid, the transmitted intensity I_3 is:

$$I_3 = I_2 \cos^2(90^\circ - \theta) = \frac{I_0}{2}\cos^2\theta \sin^2\theta.$$

The intensity I_3 will be maximized when $\sin 2\theta = 1$, as this maximizes $\cos^2\theta \sin^2\theta$. This condition is met when $\theta = 45^\circ$.
Therefore, the maximum transmitted intensity occurs at $\theta = 45^\circ$.

Quick Tip

To find the maximum intensity through crossed polaroids, use Malus' law and find the angle that maximizes $\sin 2\theta$.

Question 17: An object is placed in a medium of refractive index 3. An electromagnetic wave of intensity 6×10^8 W/m^2 falls normally on the object and it is absorbed completely. The radiation pressure on the object would be (speed of light in free space $c = 3 \times 10^8$ m/s):

Options:
(1) 36 Nm^{-2}
(2) 18 Nm^{-2}
(3) 6 Nm^{-2}
(4) 2 Nm^{-2}

Correct Answer: (3)

Solution:
The radiation pressure P when there is complete absorption is given by:

$$P = \frac{I \cdot \mu}{c},$$

where: - I is the intensity of the electromagnetic wave, - μ is the absorption factor (here, assumed to be 1 for full absorption), and - c is the speed of light.
Given Data

$$I = 6 \times 10^8 \, \text{W/m}^2, \quad c = 3 \times 10^8 \, \text{m/s}.$$

Calculation of Radiation Pressure Substitute the values:

$$P = \frac{6 \times 10^8 \times 1}{3 \times 10^8} = 6 \, \text{N/m}^2.$$

Therefore, the radiation pressure P is $6 \, \text{N/m}^2$.

Quick Tip

Radiation pressure is calculated as $\frac{I}{c}$ for total absorption. For reflection, it would be $\frac{2I}{c}$.

Question 18: Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A): Work done by electric field on moving a positive charge on an equipotential surface is always zero.
Reason (R): Electric lines of forces are always perpendicular to equipotential surfaces.
In the light of the above statements, choose the correct answer from the options given below:
(1) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(2) (A) is correct but (R) is not correct
(3) (A) is not correct but (R) is correct
(4) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Correct Answer: (4)

Solution:
When a charge moves along an equipotential surface, the electric potential at each point on the surface remains constant. Since the electric potential difference ΔV is zero, the work W done by the electric field is given by:

$$W = q\Delta V = q \times 0 = 0,$$

where q is the charge being moved.
Additionally, electric field lines are always perpendicular to equipotential surfaces. This perpendicular relationship implies that any displacement along the equipotential surface is perpendicular to the electric field. Since work done by a force depends on the component of force along the direction of displacement, and here there is no component of the electric field along the surface, the work done by the electric field is zero.
Thus, the work done by the electric field in moving a charge along an equipotential surface is zero due to the lack of potential difference and the perpendicular orientation of electric field lines to the surface.

> Quick Tip
>
> Work done by a conservative force like the electric field on an equipotential surface is always zero due to perpendicularity between displacement and force direction.

Question 19: A heavy iron bar of weight 12 kg is having its one end on the ground and the other on the shoulder of a man. The rod makes an angle 60° with the horizontal, the weight experienced by the man is:

Options:
(1) 6 kg
(2) 12 kg
(3) 3 kg
(4) $6\sqrt{3}$ kg

Correct Answer: (3)

Solution:

Given:

$$W = 12\,\text{kg},$$

the weight of the bar. The bar makes an angle $\theta = 60^\circ$ with the horizontal. Let N_1 be the normal force at the ground and N_2 be the force experienced by the man's shoulder. To maintain equilibrium about point O, the net torque must be zero:

$$\text{Torque about } O = 0.$$

Step 1: Set Up the Torque Equation Taking moments about point O, we consider the torque due to the weight of the bar and the force N_2 at the man's shoulder.
The torque due to the weight W of the bar (acting at its center of mass) is:

$$\tau_W = W\left(\frac{L}{2}\cos 60^\circ\right).$$

The torque due to N_2 is:

$$\tau_{N_2} = N_2 \times L.$$

Since the system is in equilibrium, we have:

$$W\left(\frac{L}{2}\cos 60^\circ\right) = N_2 L.$$

Step 2: Simplify the Equation Substitute $W = 120\,\text{N}$ (since $W = 12\,\text{kg} \times 10\,\text{m/s}^2$) and $\cos 60^\circ = \frac{1}{2}$:

$$120\left(\frac{L}{2} \times \frac{1}{2}\right) = N_2 L.$$

This simplifies to:

$$N_2 = \frac{120}{2} \times \frac{1}{2} = 30\,\text{N}.$$

Step 3: Convert to the Equivalent Weight The force $N_2 = 30\,\text{N}$ corresponds to an equivalent weight:

$$\text{Weight} = \frac{30\,\text{N}}{10\,\text{m/s}^2} = 3\,\text{kg}.$$

Therefore, the weight experienced by the man's shoulder is equivalent to $3\,\text{kg}$.

> **Quick Tip**
>
> For equilibrium problems, sum of all torques around a pivot must be zero. Consider forces' directions and lever arms accurately.

Question 20: A bullet is fired into a fixed target and loses one third of its velocity after travelling 4 cm. It penetrates further $D \times 10^{-3}$ m before coming to rest. The value of D is:

Options:
(1) 2
(2) 5
(3) 3
(4) 4

Solution: (Bonus)
Given that the bullet loses one third of its velocity after traveling 4 cm:

$$v = \frac{2}{3}u$$

Using the kinematic equation:

$$v^2 - u^2 = 2a \times s$$

Substituting the values:

$$\left(\frac{2u}{3}\right)^2 - u^2 = 2a \times (4 \times 10^{-2})$$

Simplifying:

$$\frac{4u^2}{9} - u^2 = -2a \times (4 \times 10^{-2})$$

$$-\frac{5u^2}{9} = -2a \times (4 \times 10^{-2})$$

$$a = \frac{5u^2}{72 \times 10^{-2}}$$

Now, for the bullet to come to rest:

$$0 - \left(\frac{2u}{3}\right)^2 = 2a \times D \times 10^{-3}$$

Substituting the values:

$$-\frac{4u^2}{9} = 2a \times D \times 10^{-3}$$

Solving for D:

$$D = 32 \times 10^{-3}\,\text{m}$$

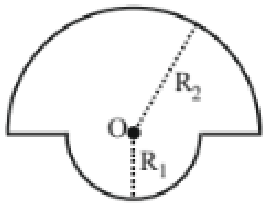
Note: Since no option matches exactly, the question should be treated as a bonus.

Quick Tip

When a body decelerates uniformly, use kinematic equations to relate initial and final velocities, acceleration, and distance traveled.

Question 21: The magnetic field at the centre of a wire loop formed by two semicircular wires of radii $R_1 = 2\,\text{m}$ and $R_2 = 4\,\text{m}$ carrying current $I = 4\,\text{A}$ as

per figure given below is $\alpha \times 10^{-7}$ T. The value of α is _______. (Centre O is common for all segments)



Correct Answer: (3)

Solution:
The magnetic field at the center of a semicircular wire carrying current I and having radius R is given by:

$$B = \frac{\mu_0 I}{4R},$$

where μ_0 is the permeability of free space.
Magnetic Field Due to Semicircular Wires of Radii R_1 and R_2 For two semicircular wires with radii R_1 and R_2 carrying the same current I, the magnetic fields at the center O are:

$$B_{R_1} = \frac{\mu_0 I}{4R_1} \quad \text{and} \quad B_{R_2} = \frac{\mu_0 I}{4R_2}.$$

Net Magnetic Field at the Center O The net magnetic field B at the center O is the sum of the magnetic fields due to both semicircular wires:

$$B = B_{R_1} + B_{R_2} = \frac{\mu_0 I}{4R_1} + \frac{\mu_0 I}{4R_2}.$$

Substituting Given Values Assuming the given values are:

$$\mu_0 = 4\pi \times 10^{-7}\,\text{T m/A}, \quad I = 4\,\text{A}, \quad R_1 = 2\,\text{m}, \quad R_2 = 4\,\text{m}.$$

Substitute these values into the expression for B:

$$B = \frac{4\pi \times 10^{-7} \times 4}{4 \times 2} + \frac{4\pi \times 10^{-7} \times 4}{4 \times 4}.$$

Simplifying each term:

$$B = \frac{16\pi \times 10^{-7}}{8} + \frac{16\pi \times 10^{-7}}{16},$$

$$B = 2\pi \times 10^{-7} + \pi \times 10^{-7} = 3\pi \times 10^{-7}\,\text{T}.$$

Conclusion Therefore, the coefficient α is:

$$\alpha = 3.$$

Quick Tip

For calculating the magnetic field due to wire segments, remember to sum the contributions from each segment appropriately based on geometry.

Question 22: Two charges of $-4\,\mu$C and $+4\,\mu$C are placed at the points A$(1,0,4)$ m and B$(2,-1,5)$ m located in an electric field $\vec{E} = 0.20i$ V/cm. The magnitude of the torque acting on the dipole is $8\sqrt{alpha} \times 10^{-5}$ Nm, where $\alpha =$ ____.

Correct Answer: (2)

Solution:
The electric dipole moment $\vec{p}$ is given by:

$$\vec{p} = q \times \vec{d},$$

where q is the charge and $\vec{d}$ is the dipole vector.
Given Data - Charge $q = 4 \times 10^{-6}$ C. - Position vectors $\vec{A} = (1,0,4)$ and $\vec{B} = (2,-1,5)$.
Step 1: Calculate the Dipole Vector $\vec{d}$ The dipole vector $\vec{d}$ is given by:

$$\vec{d} = \vec{B} - \vec{A} = (2-1, -1-0, 5-4) = (1,-1,1)\,\text{m}.$$

Step 2: Calculate the Dipole Moment $\vec{p}$

$$\vec{p} = q \cdot \vec{d} = 4 \times 10^{-6} \cdot (1,-1,1) = (4 \times 10^{-6}, -4 \times 10^{-6}, 4 \times 10^{-6})\,\text{Cm}.$$

Step 3: Calculate the Torque $\vec{\tau}$ The torque $\vec{\tau}$ on the dipole in an electric field $\vec{E}$ is given by:

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

Given that $\vec{E} = 0.2\,\text{V/cm} = 20\,\text{V/m}$ in the direction $\hat{\imath}$, we have:

$$\vec{E} = (20,0,0)\,\text{V/m}.$$

Now, calculate the cross product:

$$\vec{\tau} = (4 \times 10^{-6}, -4 \times 10^{-6}, 4 \times 10^{-6}) \times (20,0,0).$$

Using the cross product formula, we get:

$$\vec{\tau} = \big(0, 20 \cdot 4 \times 10^{-6}, -20 \cdot (-4 \times 10^{-6})\big) = (0, 8 \times 10^{-5}, 8 \times 10^{-5})\,\text{Nm}.$$

Step 4: Calculate the Magnitude of $\vec{\tau}$ The magnitude of $\vec{\tau}$ is:

$$|\vec{\tau}| = \sqrt{(0)^2 + (8 \times 10^{-5})^2 + (8 \times 10^{-5})^2}.$$

$$|\vec{\tau}| = \sqrt{0 + 64 \times 10^{-10} + 64 \times 10^{-10}} = \sqrt{128 \times 10^{-10}} = 8\sqrt{2} \times 10^{-5}\,\text{Nm}.$$

Step 5: Determine α We are given that $\vec{\tau} = 8\alpha \times 10^{-5}\,\text{Nm}$. Equating, we find:

$$\alpha = 2.$$

Quick Tip

The torque on an electric dipole in a uniform field is calculated using $\vec{\tau} = \vec{p} \times \vec{E}$.

Question 23: A closed organ pipe 150 cm long gives 7 beats per second with an open organ pipe of length 350 cm, both vibrating in fundamental mode. The velocity of sound is _____ m/s.

Correct Answer: (294)

Solution:
For a closed pipe of length $L = 150\,\text{cm} = 1.5\,\text{m}$, the fundamental frequency f_c is given by:

$$f_c = \frac{v}{4L},$$

where v is the speed of sound in air.
For an open pipe of length $L = 350\,\text{cm} = 3.5\,\text{m}$, the fundamental frequency f_o is:

$$f_o = \frac{v}{2L}.$$

We are given that the beat frequency between the closed pipe and the open pipe is:

$$|f_c - f_o| = 7\,\text{Hz}.$$

Step 1: Substitute the Values Substitute the expressions for f_c and f_o:

$$\left|\frac{v}{4 \times 1.5} - \frac{v}{2 \times 3.5}\right| = 7.$$

Step 2: Simplify the Equation Simplify the terms inside the absolute value:

$$\left|\frac{v}{6} - \frac{v}{7}\right| = 7.$$

To combine the terms, find a common denominator (42):

$$\left|\frac{7v - 6v}{42}\right| = 7.$$

This simplifies to:

$$\frac{v}{42} = 7.$$

Step 3: Solve for v Multiply both sides by 42:

$$v = 42 \times 7 = 294\,\text{m/s}.$$

Conclusion Therefore, the speed of sound v is:

$$v = 294\,\text{m/s}.$$

💡 Quick Tip

The beat frequency between two sources is given by the absolute difference of their frequencies. For organ pipes, use the appropriate formulas for fundamental frequencies.

Question 24: A body falling under gravity covers two points A and B separated by 80 m in 2s. The distance of upper point A from the starting point is _____ m (use $g = 10\,\text{m/s}^2$).

Correct Answer: (45)

Solution:
Given:

$$\text{Distance between A and B} = 80\,\text{m}, \quad t = 2\,\text{s}, \quad g = 10\,\text{m/s}^2.$$

Using the equation of motion:

$$s = ut + \frac{1}{2}gt^2,$$

where s is the displacement, u is the initial velocity, t is the time, and g is the acceleration due to gravity.

Step 1: Motion from A to B For the motion from point A to point B, we are given that the displacement $s = -80\,\text{m}$ (since it is downward). The equation becomes:

$$-80 = v_1 \cdot t - \frac{1}{2}gt^2.$$

Substituting $t = 2\,\text{s}$ and $g = 10\,\text{m/s}^2$:

$$-80 = v_1 \cdot 2 - \frac{1}{2} \cdot 10 \cdot 2^2.$$

Simplifying:

$$-80 = 2v_1 - 20.$$

Solving for v_1:

$$-60 = 2v_1 \implies v_1 = -30\,\text{m/s}.$$

Step 2: Motion from 0 to A To find the distance S from point 0 (starting point) to point A, we use the equation:

$$v_1^2 = u^2 + 2gS,$$

where $u = 0$ (assuming the object started from rest) and $v_1 = -30\,\text{m/s}$.

Substitute the values:

$$(-30)^2 = 0 + 2 \cdot 10 \cdot S.$$

This simplifies to:

$$900 = 20S.$$

Solving for S:

$$S = \frac{900}{20} = 45\,\text{m}.$$

Conclusion The distance from the starting point to point A is:

$$S = 45\,\text{m}.$$

Quick Tip

In problems involving free fall, use equations of motion to relate distance, time, initial velocity, and acceleration due to gravity.

Question 25: The reading of a pressure meter attached with a closed pipe is 4.5×10^4 N/m^2. On opening the valve, water starts flowing and the reading of pressure meter falls to 2.0×10^4 N/m^2. The velocity of water is found to be $\sqrt{V}$ m/s. The value of V is _____.

Correct Answer: (50)

Solution:
Using Bernoulli's theorem for the flow of an ideal fluid:

$$P_1 + \frac{1}{2}\rho v^2 = P_2,$$

where: - P_1 and P_2 are the pressures at two points, - ρ is the density of the fluid, - v is the velocity of the fluid.
Given Data

$$P_1 = 4.5 \times 10^4\,\text{N/m}^2, \quad P_2 = 2.0 \times 10^4\,\text{N/m}^2, \quad \rho = 1000\,\text{kg/m}^3.$$

Step 1: Substitute the Values into Bernoulli's Equation Substitute the values into the equation:

$$4.5 \times 10^4 + \frac{1}{2} \times 1000 \times v^2 = 2.0 \times 10^4.$$

Step 2: Rearrange the Equation Rearrange to isolate v^2:

$$\frac{1}{2} \times 1000 \times v^2 = 4.5 \times 10^4 - 2.0 \times 10^4.$$

$$500v^2 = 2.5 \times 10^4.$$

Step 3: Solve for v Divide both sides by 500:

$$v^2 = \frac{2.5 \times 10^4}{500} = 50.$$

Taking the square root:

$$v = 50$$

> Quick Tip
>
> For fluid flow problems, Bernoulli's equation relates pressure differences to changes in fluid speed.

Question 26: A ring and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of both bodies are identical and the ratio of their kinetic energies is $\frac{7}{x}$ where x is _______.

Correct Answer: (7)

Solution:
For rolling without slipping, the total kinetic energy KE_{total} is the sum of translational and rotational kinetic energies:

$$KE_{\text{total}} = KE_{\text{translational}} + KE_{\text{rotational}}.$$

Kinetic Energy of the Ring For a ring of mass m and radius R:

$$KE_{\text{ring}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

Since $I = mR^2$ for a ring and $\omega = \frac{v}{R}$, we have:

$$KE_{\text{ring}} = \frac{1}{2}mv^2 + \frac{1}{2}mR^2\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2.$$

Kinetic Energy of the Solid Sphere For a solid sphere of mass m and radius R:

$$KE_{\text{sphere}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

Using $I = \frac{2}{5}mR^2$ for a solid sphere, we get:

$$KE_{\text{sphere}} = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mR^2\left(\frac{v}{R}\right)^2.$$

Simplifying, we find:

$$KE_{\text{sphere}} = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2.$$

Ratio of Kinetic Energies The ratio of the kinetic energies of the ring to the solid sphere is:

$$\frac{KE_{\text{ring}}}{KE_{\text{sphere}}} = \frac{mv^2}{\frac{7}{10}mv^2} = \frac{10}{7}.$$

Given the relation:

$$x = 7.$$

> 💡 Quick Tip
>
> In rolling motion, total kinetic energy includes both translational and rotational components.

Question 27: A parallel beam of monochromatic light of wavelength $5000\,\text{Å}$ is incident normally on a single narrow slit of width $0.001\,\text{mm}$. The light is focused by a convex lens on a screen, placed on its focal plane. The first minima will be formed for the angle of diffraction of _____ (degree).

Correct Answer: (30)

Solution:
For the first minima in a single-slit diffraction pattern, the condition is given by:

$$a \sin\theta = \lambda,$$

where: - a is the width of the slit, - λ is the wavelength of the light, - θ is the angle at which the first minima occurs.
Given Data

$$a = 0.001\,\text{mm} = 1 \times 10^{-6}\,\text{m},$$

$$\lambda = 5000\,\text{Å} = 5000 \times 10^{-10}\,\text{m}.$$

Step 1: Substitute the Values into the Condition

$$\sin\theta = \frac{\lambda}{a} = \frac{5000 \times 10^{-10}}{1 \times 10^{-6}}.$$

Step 2: Simplify the Expression

$$\sin\theta = \frac{5000 \times 10^{-10}}{10^{-6}} = 0.5.$$

Step 3: Solve for θ

$$\theta = \sin^{-1}(0.5) = 30^\circ.$$

Conclusion Therefore, the angle θ for the first minima is:

$$\theta = 30^\circ.$$

Quick Tip

For single-slit diffraction, the angle of minima can be found using $a\sin\theta = m\lambda$ where $m = 1, 2, 3, \ldots$.

Question 28: The electric potential at the surface of an atomic nucleus $(Z = 50)$ of radius 9×10^{-13} cm is ______ $\times 10^6$ V.

Correct Answer: (8)

Solution:
The electric potential V at the surface of an atomic nucleus with atomic number $Z = 50$ and radius $R = 9 \times 10^{-13}$ cm is given by:

$$V = \frac{kQ}{R} = \frac{kZe}{R},$$

where: - $k = 9 \times 10^9\,\mathrm{N\,m^2/C^2}$ is Coulomb's constant, - $Z = 50$ is the atomic number, - $e = 1.6 \times 10^{-19}\,\mathrm{C}$ is the elementary charge, - $R = 9 \times 10^{-13}\,\mathrm{cm} = 9 \times 10^{-15}\,\mathrm{m}$ is the radius of the nucleus.
Step 1: Substitute the Values Substituting the values into the formula:

$$V = \frac{9 \times 10^9 \times 50 \times 1.6 \times 10^{-19}}{9 \times 10^{-13} \times 10^{-2}}.$$

Step 2: Simplify the Expression Calculating the expression:

$$V = \frac{9 \times 10^9 \times 50 \times 1.6 \times 10^{-19}}{9 \times 10^{-13}}.$$

This simplifies to:

$$V = 8 \times 10^6\,\mathrm{V}.$$

Quick Tip

The potential at the surface of a nucleus is calculated using $V = \frac{kZe}{R}$, where k is Coulomb's constant, Z is the atomic number, e is the elementary charge, and R is the radius.

Question 29: If Rydberg's constant is R, the longest wavelength of radiation in Paschen series will be $\frac{\alpha}{7R}$, where $\alpha =$ _____.

Correct Answer: (144)

Solution:
The Paschen series corresponds to electronic transitions in a hydrogen atom that end at $n = 3$. The longest wavelength in this series corresponds to the transition from $n = 4$ to $n = 3$.
Step 1: Formula for Wavelength The inverse wavelength $\frac{1}{\lambda}$ for a transition from n_2 to n_1 in a hydrogen-like atom is given by:

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right),$$

where: - R is the Rydberg constant, - Z is the atomic number (for hydrogen, $Z = 1$), - n_1 and n_2 are the principal quantum numbers with $n_2 > n_1$.
Step 2: Substitute Values for the Paschen Series Transition For the transition from $n_2 = 4$ to $n_1 = 3$, and taking $Z = 1$, we have:

$$\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{4^2} \right).$$

Step 3: Simplify the Expression Calculate the values inside the parentheses:

$$\frac{1}{\lambda} = R \left(\frac{1}{9} - \frac{1}{16} \right).$$

Finding a common denominator (144) for the terms:

$$\frac{1}{\lambda} = R \left(\frac{16 - 9}{144} \right) = \frac{7R}{144}.$$

Conclusion Thus, we can express the result as:

$$\frac{1}{\lambda} = \frac{7R}{\alpha},$$

where $\alpha = 144$.

> Quick Tip
>
> The longest wavelength in a series corresponds to the smallest energy transition, i.e., from $n = n_2$ to $n = n_1$.

Question 30: A series LCR circuit with $L = \frac{100}{\pi}$ mH, $C = 10^{-3}$ F, and $R = 10\,\Omega$, is connected across an ac source of 220 V, 50 Hz. The power factor of the circuit would be _______.

Correct Answer: (1)

Solution:
Given:

$$L = \frac{100}{\pi}\,\text{mH} = \frac{100}{\pi} \times 10^{-3}\,\text{H}, \quad C = 10^{-3}\,\text{F}, \quad R = 10\,\Omega, \quad f = 50\,\text{Hz}$$

The inductive reactance is given by:

$$X_L = 2\pi f L = 2\pi \times 50 \times \frac{100}{\pi} \times 10^{-3} = 10\,\Omega$$

The capacitive reactance is given by:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 10^{-3}} = 10\,\Omega$$

Since $X_L = X_C$, the circuit is in resonance. Therefore, the impedance $Z = R = 10\,\Omega$ and the power factor is:

$$\text{Power Factor} = \frac{R}{Z} = 1$$

Quick Tip

In a series LCR circuit at resonance, the inductive and capacitive reactances cancel each other, resulting in a power factor of unity.
