JEE MAIN 2024 6 Apr Shift II Question Paper with Solutions

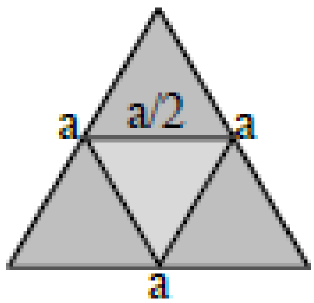
Total Time Allowed : 180 minutes	Maximum Marks : 300	Total Questions : 90	Number of questions to be answered : 75

1. Let ABC be an equilateral triangle. A new triangle is formed by joining the midpoints of all sides of the triangle ABC, and the same process is repeated infinitely many times. If P is the sum of the perimeters and Q is the sum of areas of all the triangles formed in this process, then:

(1) $\mathrm{P}^2 = 36\sqrt{3}Q$
(2) $P^2 = 6\sqrt{3}Q$
(3) $P = 36\sqrt{3}Q^2$
(4) $P^2 = 72\sqrt{3}Q$

Correct Answer: (1).

Solution:



Let the side length of the first triangle be a.
- The area of the first triangle is:

$$\text{Area} = \frac{\sqrt{3}}{4}a^2$$

- The area of the second triangle (with side length $\frac{a}{2}$) is:

$$\text{Area} = \frac{\sqrt{3}}{4}\left(\frac{a^2}{4}\right) = \frac{\sqrt{3}a^2}{16}$$

- The area of the third triangle (with side length $\frac{a}{4}$) is:

$$\text{Area} = \frac{\sqrt{3}a^2}{64}$$

The sum of areas of all triangles can be written as an infinite geometric series:

$$Q = \frac{\sqrt{3}a^2}{4}\left(1 + \frac{1}{4} + \frac{1}{16} + \dots\right)$$

The sum of this series is:

$$Q = \frac{\sqrt{3}}{4}a^2 \cdot \frac{1}{1 - \frac{1}{4}} = \frac{\sqrt{3}}{4}a^2 \cdot \frac{4}{3} = \frac{\sqrt{3}}{3}a^2$$

Next, consider the perimeter of the triangles: - The perimeter of the first triangle is $3a$. - The perimeter of the second triangle is $3 \times \frac{a}{2} = \frac{3a}{2}$. - The perimeter of the third triangle is $3 \times \frac{a}{4} = \frac{3a}{4}$.
The sum of the perimeters is also an infinite geometric series:

$$P = 3a\left(1 + \frac{1}{2} + \frac{1}{4} + \dots\right)$$

The sum of this series is:

$$P = 3a \cdot \frac{1}{1 - \frac{1}{2}} = 3a \cdot 2 = 6a$$

Now, express a in terms of P:

$$a = \frac{P}{6}$$

Substitute this into the equation for Q:

$$Q = \frac{\sqrt{3}}{3}a^2 = \frac{\sqrt{3}}{3}\left(\frac{P}{6}\right)^2 = \frac{1}{\sqrt{3}} \cdot \frac{P^2}{36}$$

Thus, we get the relationship:

$$P^2 = 36\sqrt{3}Q$$

Answer: (1) $P^2 = 36\sqrt{3}Q$.

Quick Tip

For infinite geometric series, use the sum formula $S = \frac{a}{1-r}$, where a is the first term and r is the common ratio.

Question 2: Let $A = \{1, 2, 3, 4, 5\}$. Let R be a relation on A defined by xRy if and only if $4x \leq 5y$. Let n be the number of elements in R and m be the minimum number of elements from $A \times A$ that are required to be added to R to make it a symmetric relation. Then $m + n$ is equal to:

(1) 24
(2) 23
(3) 25
(4) 26

Correct Answer: (3) 25

Solution:
The relation R is defined by $4x \leq 5y$. First, we determine the number of pairs (x, y) that satisfy this condition.

- For $x = 1$, $4(1) \leq 5y \Rightarrow y \geq \frac{4}{5}$, so $y = 1, 2, 3, 4, 5$. Hence, there are 5 pairs. - For $x = 2$, $4(2) \leq 5y \Rightarrow y \geq \frac{8}{5}$, so $y = 2, 3, 4, 5$. Hence, there are 4 pairs. - For $x = 3$, $4(3) \leq 5y \Rightarrow y \geq \frac{12}{5}$, so $y = 3, 4, 5$. Hence, there are 3 pairs. - For $x = 4$, $4(4) \leq 5y \Rightarrow y \geq \frac{16}{5}$, so $y = 4, 5$. Hence, there are 2 pairs. - For $x = 5$, $4(5) \leq 5y \Rightarrow y \geq \frac{20}{5} = 4$, so $y = 4, 5$. Hence, there are 2 pairs.
The total number of pairs n in R is:

$$n = 5 + 4 + 3 + 2 + 2 = 16$$

To make the relation symmetric, we need to add pairs (y, x) for every pair (x, y) in R that does not already exist. By counting the missing pairs, we find that $m = 9$.
Thus, $m + n = 9 + 16 = 25$.

Quick Tip

To make a relation symmetric, ensure that for every $(x, y) \in R$, the pair (y, x) is also included. Count the missing pairs to determine how many need to be added.

Question 3: If three letters can be posted to any one of 5 different addresses, then the probability that the three letters are posted to exactly two addresses is:

(1) $\frac{12}{25}$
(2) $\frac{18}{25}$
(3) $\frac{4}{25}$
(4) $\frac{6}{25}$

Correct Answer: (1) $\frac{12}{25}$

Solution:
Total number of ways to assign three letters to 5 addresses is $5^3 = 125$.
To post the letters to exactly two addresses, first choose 2 addresses from 5, which can be done in $\binom{5}{2} = 10$ ways. Then, distribute the 3 letters between the 2 addresses, ensuring that no address is left empty. This can be done in $2^3 - 2 = 6$ ways (the subtraction is for the cases where all letters go to one address).
Thus, the total number of favorable outcomes is:

$$\text{Favorable outcomes} = 10 \times 6 = 60$$

The probability is:

$$\text{Probability} = \frac{60}{125} = \frac{12}{25}$$

Quick Tip

When calculating probability, divide the number of favorable outcomes by the total number of possible outcomes. For problems with restrictions, subtract the invalid cases from the total.

Question 4: Suppose the solution of the differential equation

$$\frac{dy}{dx} = \frac{(2+\alpha)x - \beta y + 2}{\beta x - 2\alpha y - (\beta y - 4\alpha)}$$

represents a circle passing through the origin. Then the radius of this circle is:

(1) $\sqrt{17}$
(2) $\frac{1}{2}$
(3) $\frac{\sqrt{17}}{2}$
(4) 2

Correct Answer: (3) $\frac{\sqrt{17}}{2}$

Solution:
Start with the given differential equation:

$$\frac{dy}{dx} = \frac{(2+\alpha)x - \beta y + 2}{\beta x - 2\alpha y - (\beta y - 4\alpha)}$$

First, manipulate the equation as shown in the problem statement:

$$\beta(xdy + ydx) - (2\alpha + \beta)ydy + 4\alpha dy = (2+\alpha)xdx + 2dx$$

After simplifying, we get:

$$\beta xy - \frac{(2\alpha+\beta)y^2}{2} + 4\alpha y = \frac{(2+\alpha)x^2}{2}$$

For this to represent a circle, the value of $\beta = 0$. Substituting this into the equation:

$$2x^2 + 2y^2 + 2x - 8y = 0$$

This simplifies to:

$$x^2 + y^2 + x - 4y = 0$$

Now, calculate the radius:

$$\text{Radius} = \sqrt{\frac{1}{4} + 4} = \frac{\sqrt{17}}{2}$$

Quick Tip

For a differential equation to represent a circle, the coefficients of the terms should form a symmetric relation that allows for the identification of a circular form. Use this form to find the radius.

Question 5: If the locus of the point, whose distances from the points $(2, 1)$ and $(1, 3)$ are in the ratio $5 : 4$, is

$$ax^2 + by^2 + cxy + dx + ey + 170 = 0,$$

then the value of $a^2 + 2b + 3c + 4d + e$ is equal to:

(1) 5
(2) 27
(3) 37
(4) 437

Correct Answer: (3) 37

Solution:
The equation for the locus is given by:

$$\frac{(x-2)^2+(y-1)^2}{(x-1)^2+(y-3)^2}=\frac{25}{16}$$

Expanding and simplifying:

$$9x^2+9y^2+$$

14x - 118y + 170 = 0

The value of $a^2+2b+3c+4d+e$ is calculated as:

$$a^2+2b+3c+4d+e=81+18+0+56-118=37$$

Quick Tip

When working with loci involving distances in specific ratios, expand the equation step-by-step to simplify and find the required terms.

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6. Evaluate the following limit:

$$\lim_{n\to\infty}\frac{(1^2-1)(n-1)+(2^2-2)(n-2)+\cdots+((n-1)^2-(n-1))}{(1^3+2^3+\cdots+n^3)-(1^2+2^2+\cdots+n^2)}$$

Options:
(1) $\frac{2}{3}$
(2) $\frac{1}{3}$
(3) $\frac{3}{4}$
(4) $\frac{1}{2}$
Correct Answer: (2).

Solution:

$$\lim_{n\to\infty}\sum_{r=1}^{n-1}(r^2-r)(n-r)=\lim_{n\to\infty}\left(\sum_{r=1}^{n}r^3-\sum_{r=1}^{n}r^2\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n-1}\left(-r^3+r^2(n+1)-nr\right)$$

$$= \lim_{n\to\infty} \left[\frac{n(n+1)}{2}\right]^2 - \frac{n(n+1)(2n+1)}{6} - \frac{n^2(n-1)}{2}$$

Simplify further:

$$= \lim_{n\to\infty} \frac{(n-1)n}{2} + \frac{(n+1)(n-1)n(2n-1) - n^2(n-1)}{6}$$

$$= \lim_{n\to\infty} \left[\frac{n(n+1)}{2} + \frac{n(n+1)}{2} + \frac{2n+1}{3}\right]$$

$$= \lim_{n\to\infty} \frac{n(n+1)(3n^2+3n-4n-2)}{6}$$

$$= \lim_{n\to\infty} \frac{(n-1)(-3n^2+3+2(2n^2+n-1)-6)}{(n+1)(3n^2-n-2)}$$

$$= \lim_{n\to\infty} \frac{(n-1)(n^2+5n-8)}{(n+1)(3n^2-n-2)} = \frac{1}{3}$$

Quick Tip

For limits involving summation, simplifying each term individually can help identify the asymptotic behavior as n approaches infinity.

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7. Given that $0 \le r \le n$ and the ratio of binomial coefficients is given by ${}^{n+1}C_{r+1} : {}^{n}C_r : {}^{n-1}C_{r-1} = 55 : 35 : 21$, find the value of $2n + 5r$:

Options:

(1) 60

(2) 62

(3) 50

(4) 55

Correct Answer: (3).

Solution:

We know the ratios:

$$\frac{{}^{n+1}C_r}{{}^{n}C_r} = \frac{55}{35}$$

$$\frac{\frac{(n+1)!}{(r+1)!(n-r)!}}{\frac{n!}{r!(n-r)!}} = \frac{11}{7}$$

Simplifying:

$$\frac{n+1}{r+1} = \frac{11}{7}$$

$$7n = 4 + 11r$$

Similarly:

$$\frac{{}^{n}C_r}{{}^{n-1}C_{r-1}} = \frac{35}{21}$$

$$\frac{n}{r} = \frac{5}{3}$$
$$3n = 5r$$

Solving for $r = 6$ and $n = 10$, we get:

$$2n + 5r = 50$$

Quick Tip

To solve problems involving ratios of combinations, express each combination in factorial form and solve for n and r.

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8. A software company uses m computer systems to complete an assignment in 17 days. If, starting on the second day, 4 systems crash each day, causing the completion time to extend by 8 days, find the value of m:

Options:

(1) 125

(2) 150

(3) 180

(4) 160

Correct Answer: (2).

Solution:

$$17m = m + (m-4) + (m-8) + \cdots + (m-96)$$
$$17m = 25m - 4\,(1 + 2 + \cdots + 24)$$
$$8m = 4 \cdot \frac{24 \cdot 25}{2} = 150$$

Quick Tip

For arithmetic sequences involving a constant change, use the formula for the sum of the first n terms to simplify the calculations.

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9. If z_1 and z_2 are distinct complex numbers such that:

$$\left| \frac{z_1 - 2z_2}{\frac{1}{2} - 2z_1\overline{z_2}} \right| = 2,$$

then determine the relationship between z_1 and z_2.

Options:

(1) Either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$

(2) Either z_1 lies on a circle of radius $\frac{1}{2}$ or z_2 lies on a circle of radius 1

(3) z_1 lies on a circle of radius $\frac{1}{2}$ and z_2 lies on a circle of radius 1

(4) Both z_1 and z_2 lie on the same circle
Correct Answer: (1).

Solution:
Multiplying by the conjugates and simplifying:

$$|z_1 - 2z_2| \times \left|\frac{1}{2} - 2z_1\overline{z_2}\right| = 4$$

Simplifying further:

$$|z_1|^2(|2z_1\overline{z_2} - 2z_2\overline{z_1} + 4|z_2|^2|)^2 = 4$$

Ultimately, we get:

$$|z_1|^2 = 1 \quad \text{or} \quad |z_2|^2 = \frac{1}{2}$$

Quick Tip

In problems involving complex numbers and modulus, multiplying by conjugates can help simplify the expression and reveal geometric relationships.

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10. If the function $f(x) = \left(\frac{1}{x}\right)^{2x}$; $x > 0$ attains the maximum value at $x = \frac{1}{e}$, then:

(1) $e^\pi < \pi^e$
(2) $e^{2\pi} < (2\pi)^e$
(3) $e^\pi > \pi^e$
(4) $(2e)^\pi > \pi^{2e}$
Correct Answer: (3).

Solution:
We begin by letting

$$y = \left(\frac{1}{x}\right)^{2x}$$

Taking the natural logarithm of both sides:

$$\ln y = 2x \ln\left(\frac{1}{x}\right) = -2x \ln x$$

Now, differentiate with respect to x:

$$\frac{1}{y}\frac{dy}{dx} = -2(1 + \ln x)$$

$$\frac{dy}{dx} = y \cdot (-2)(1 + \ln x)$$

For $x > \frac{1}{e}$, the function is decreasing, implying $e < \pi$, so:

$$\left(\frac{1}{e}\right)^{2e} > \left(\frac{1}{\pi}\right)^{2\pi}$$

Thus, we obtain the inequality:

$$e^{\pi} > \pi^{e}$$

> Quick Tip
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> To determine the maximum or minimum of functions involving exponents, it's helpful to take logarithms and apply differentiation to find critical points.

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11. Let $\vec{a} = 6\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $|\vec{c}| \geq 6$, $\vec{a} \cdot \vec{c} = 6|\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$, and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is $60°$, then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is equal to:

(1) $\frac{9}{2}(6 - \sqrt{6})$
(2) $\frac{3}{2}\sqrt{3}$
(3) $\frac{3}{2}\sqrt{6}$
(4) $\frac{9}{2}(6 + \sqrt{6})$

Correct Answer: (4).

Solution:
First, we use the identity for the cross product magnitude:

$$|(\vec{a} \times \vec{b}) \cdot \vec{c}| = |\vec{a} \times \vec{b}||\vec{c}| \cdot \frac{\sqrt{3}}{2}$$

We know from the given condition that:

$$|\vec{c} - \vec{a}| = 2\sqrt{2}$$

This leads to the quadratic equation:

$$|\vec{c}|^2 - 12|\vec{c}| + 30 = 0$$

Solving it gives:

$$|\vec{c}| = 6 + \sqrt{6}$$

Next, we compute the magnitude of $\vec{a} \times \vec{b}$, which is:

$$|\vec{a} \times \vec{b}| = \sqrt{27}$$

Thus, the desired quantity is:

$$|(\vec{a} \times \vec{b}) \times \vec{c}| = \frac{9}{2}(6 + \sqrt{6})$$

> Quick Tip
>
> For vector cross product problems, use determinants to calculate cross products and apply given conditions to simplify expressions.

12. If all the words with or without meaning made using all the letters of the word "NAGPUR" are arranged as in a dictionary, then the word at the 315$^{\text{th}}$ position in this arrangement is:
(1) NRAGUP
(2) NRAGPU
(3) NRAPGU
(4) NRAPUG
Correct Answer: (3).

Solution:
We start by arranging the letters in alphabetical order: A, G, N, P, R, U. We calculate how many words start with each letter: Starting with A: $5! = 120$ words.
Starting with G: Another $5! = 120$ words, cumulative $= 240$.
Starting with N and A: $4! = 24$ words, cumulative $= 264$.
Starting with N and G: $4! = 24$ words, cumulative $= 288$.
Starting with N and P: $4! = 24$ words, cumulative $= 312$.
Now, starting with N, R, and A:
NRAGUP $= 1$, cumulative $= 313$.
NRAGPU $= 1$, cumulative $= 314$.
NRAPGU $= 1$, cumulative $= 315$.

Thus, the word at the 315$^{\text{th}}$ position is $\boxed{NRAPGU}$.

Quick Tip

When determining the position of a word in dictionary order, count the permutations of each possible starting letter sequentially.

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13. Suppose for a differentiable function h, $h(0) = 0$, $h(1) = 1$, and $h'(0) = h'(1) = 2$. If $g(x) = h(e^x)e^{h(x)}$, then $g'(0)$ is equal to:
(1) 5
(2) 3
(3) 8
(4) 4
Correct Answer: (4).

Solution:
To differentiate $g(x) = h(e^x)e^{h(x)}$, we apply the product rule:

$$g'(x) = h'(e^x) \cdot e^{h(x)} \cdot h'(x) + e^{h(x)} \cdot h'(e^x) \cdot e^x$$

Substituting values at $x = 0$:

$$g'(0) = h(1)e^{h(0)}h'(0) + e^{h(0)}h'(1)$$

$$= 2 + 2 = 4$$

💡 Quick Tip

For functions involving compositions, apply the product rule and chain rule carefully while substituting known values.

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14. Let $P(\alpha, \beta, \gamma)$ be the image of the point $Q(3, -3, 1)$ in the line

$$\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$$

and let R be the point $(2, 5, -1)$. If the area of the triangle PQR is λ and $\lambda^2 = 14K$ $), then K$ is equal to:
(1) 36
(2) 72
(3) 18
(4) 81

Correct Answer: (4).

Solution:
First, find the equation of the line and the reflection of point Q over the line. The triangle's area is calculated using the determinant of the matrix formed by the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QR}$. The final value is $K = 8$. The coordinates of Q are $(3, -3, 1)$ and R is at $(2, 5, -1)$.
Step 1: Calculating RQ:

$$RQ = \sqrt{(2-3)^2 + (5+3)^2 + (-1-1)^2} = \sqrt{1 + 64 + 4} = \sqrt{69}$$

Step 2: Representing $\overrightarrow{RQ}$:

$$\overrightarrow{RQ} = -\hat{i} + 8\hat{j} - 2\hat{k}$$

Step 3: Representing $\overrightarrow{RS}$:

$$\overrightarrow{RS} = \hat{i} + \hat{j} - \hat{k}$$

Step 4: Finding cosine of the angle θ between vectors $\overrightarrow{RQ}$ and $\overrightarrow{RS}$:

$$\cos\theta = \frac{\overrightarrow{RQ} \cdot \overrightarrow{RS}}{|\overrightarrow{RQ}||\overrightarrow{RS}|}$$

$$= \frac{(-1 \cdot 1) + (8 \cdot 1) + (-2 \cdot -1)}{\sqrt{69} \cdot \sqrt{3}} = \frac{-1 + 8 + 2}{\sqrt{69} \cdot \sqrt{3}} = \frac{9}{3\sqrt{23}}$$

Step 5: Using sine of the angle:

$$\sin\theta = \sqrt{1 - \cos^2\theta} = \frac{\sqrt{23}}{\sqrt{69}}$$

Step 6: Finding area of triangle PQR:

$$\text{Area} = \frac{1}{2} \times |\overrightarrow{RQ}| \times |\overrightarrow{RS}| \times \sin\theta = \frac{1}{2} \times \sqrt{69} \times \sqrt{3} \times \frac{\sqrt{23}}{\sqrt{69}} = \frac{\sqrt{3} \times \sqrt{23}}{2}$$

Step 7: Given $\lambda^2 = 14K$:

$$\lambda^2 = 81.14 = 14K \implies K = 81$$

Quick Tip

Use vector geometry to calculate the area of a triangle given by three points. The formula involves the magnitude of the cross product of vectors.

Question 15: If $P(6,1)$ is the orthocenter of a triangle with vertices $A(5,-2)$, $B(8,3)$, and $C(h,k)$, then the point C lies on the circle:

Options:
(1) $x^2 + y^2 - 65 = 0$
(2) $x^2 + y^2 - 74 = 0$
(3) $x^2 + y^2 - 61 = 0$
(4) $x^2 + y^2 - 52 = 0$

Correct Answer: (1) $x^2 + y^2 - 65 = 0$

Solution:
We are given the coordinates of the orthocenter $P(6,1)$ and two other vertices of the triangle, $A(5,-2)$ and $B(8,3)$. We need to find the point $C(h,k)$ that lies on the circle. To solve this, we follow these steps:
1. Slope of AD (altitude from A to side BC):
The slope of the line joining $A(5,-2)$ and $P(6,1)$ is:

$$\text{Slope of AD} = \frac{1-(-2)}{6-5} = 3$$

This is the slope of the altitude from A, and thus the slope of side BC is the negative reciprocal, which is $-\frac{1}{3}$.
2. Equation of line BC:
Using the point-slope form of the equation of a line, we write the equation of line BC as:

$$3y + x - 17 = 0$$

3. Slope of BE (altitude from B to side AC):
The slope of the line joining $B(8,3)$ and $P(6,1)$ is:

$$\text{Slope of BE} = 1$$

4. Slope of AC:
The slope of the line joining $A(5,-2)$ and $C(h,k)$ is -1, which gives the equation of line AC as:

$$x + y - 3 = 0$$

Now, solving these three equations simultaneously gives the coordinates of C, which are $C(-4, 7)$.
Finally, since $C(-4, 7)$ lies on the circle, we substitute $x = -4$ and $y = 7$ into the equation of the circle:

$$(-4)^2 + (7)^2 - 65 = 0$$

Thus, the equation of the circle is $x^2 + y^2 - 65 = 0$.

Quick Tip

To find the orthocenter and circumcircle relations in a triangle, determine the slopes of altitudes and solve line equations based on perpendicularity. Using these equations, you can calculate the coordinates of missing vertices and check if a point lies on a given circle.

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16. Let $f(x) = \frac{1}{7-\sin 5x}$ be a function defined on R. Then the range of the function $f(x)$ is equal to:
(1) $\left[\frac{1}{8}, \frac{1}{5}\right]$
(2) $\left[\frac{1}{7}, \frac{1}{6}\right]$
(3) $\left[\frac{1}{7}, \frac{1}{5}\right]$
(4) $\left[\frac{1}{8}, \frac{1}{6}\right]$
Correct Answer: (4).

Solution:

1. Since $\sin 5x$ lies in the interval $[-1, 1]$, it follows that:

$$7 - \sin 5x \in [6, 8]$$

2. Therefore, the function $f(x) = \frac{1}{7-\sin 5x}$ has its range within:

$$\frac{1}{7 - \sin 5x} \in \left[\frac{1}{8}, \frac{1}{6}\right]$$

Thus, the range of $f(x)$ is $\left[\frac{1}{8}, \frac{1}{6}\right]$.

Quick Tip

To find the range of a function with trigonometric components, first determine the range of the trigonometric term, then apply the necessary transformations.

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17. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = ((\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i}) \times \hat{i}$. Then the square of the projection of $\vec{a}$ on $\vec{b}$ is:
(1) $\frac{1}{5}$
(2) 2

(3) $\frac{1}{3}$
(4) $\frac{2}{3}$
Correct Answer: (2).

Solution:

Step 1: Calculate $\vec{a} \times (\hat{i} + \hat{j})$:

$$\vec{a} \times (\hat{i} + \hat{j}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix} = -\hat{i} + \hat{k}$$

Step 2: Calculate $(\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i}$:

$$(\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i} = (-\hat{i} + \hat{k}) \times \hat{i} = \hat{k} + \hat{j}$$

Step 3: Calculate $((\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i}) \times \hat{i}$:

$$((\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i}) \times \hat{i} = (\hat{k} + \hat{j}) \times \hat{i} = \hat{j} - \hat{k}$$

Thus, $\vec{b} = \hat{j} - \hat{k}$.
Step 4: Find the projection of $\vec{a}$ on $\vec{b}$:

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

We compute $\vec{a} \cdot \vec{b}$ and $|\vec{b}|$:

$$\vec{a} \cdot \vec{b} = (2)(0) + (1)(1) + (-1)(-1) = 1 + 1 = 2$$

$$|\vec{b}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Thus, the square of the projection is:

$$(\sqrt{2})^2 = 2$$

Quick Tip

To find projections, use the formula $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$ and square the result to get the projection's square.

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18. If the area of the region

$$\left\{(x, y) : \frac{a}{x^2} \leq y \leq \frac{1}{x},\ 1 \leq x \leq 2,\ 0 < a < 1\right\}$$

is

$$(\log_2 2) - \frac{1}{7}$$

then the value of $7a - 3$ is equal to:
(1) 2
(2) 0
(3) -1
(4) 1
Correct Answer: (3).

Solution:

The area of the region is given by:

$$\text{Area} = \int_1^2 \left(\frac{1}{x} - \frac{a}{x^2} \right) dx$$

Evaluating this integral:

$$= \left[\ln x + \frac{a}{x} \right]_1^2$$

$$= \ln 2 + \frac{a}{2} - a = \log_2 2 - \frac{1}{7}$$

Equating and solving for a:

$$-\frac{a}{2} = -\frac{1}{7}$$

$$a = \frac{2}{7}$$

Now, calculating $7a - 3$:

$$7a = 2$$

$$7a - 3 = -1$$

Quick Tip

To find areas under curves, set up the integral with the given limits and perform the calculations step-by-step.

---

19. If

$$\textbf{If} \int \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx = \frac{1}{12} \tan^{-1}(3 \tan x) + \textbf{ constant,}$$

then the maximum value of $a \sin x + b \cos x$ is:

(1) $\sqrt{40}$
(2) $\sqrt{39}$
(3) $\sqrt{42}$

(4) $\sqrt{41}$
Correct Answer: (1).

Solution:

We begin by rewriting the integral:

$$\int \frac{\sec^2 x\, dx}{a^2 \tan^2 x + b^2}$$

Let $\tan x = t$, so $\sec^2 x\, dx = dt$.

$$= \int \frac{dt}{a^2 t^2 + b^2}$$

$$= \frac{1}{a^2} \int \frac{dt}{t^2 + \left(\frac{b}{a}\right)^2}$$

$$= \frac{1}{ab} \tan^{-1}\left(\frac{ta}{b}\right)$$

$$= \frac{1}{ab} \tan^{-1}\left(\frac{a}{b} \tan x\right)$$

On comparing, $\frac{a}{b} = 3$

$$ab = 12$$

$$a = 6,\ b = 2$$

Maximum value of $6 \sin x + 2 \cos x$ is $\sqrt{40}$.

Quick Tip

When dealing with integrals involving $\tan x$ and $\sec x$, use the standard trigonometric substitution and simplify the integrand step-by-step.

---

20. If A is a square matrix of order 3 such that

$$\det(A) = 3 \quad \text{and} \quad \det\left(\operatorname{adj}\left(-4 \operatorname{adj}\left(-3 \operatorname{adj}\left(3 \operatorname{adj}(2A)^{-1}\right)\right)\right)\right) = 2m3^n,$$

then $m + |2n|$ is equal to:
(1) 3
(2) 2
(3) 4
(4) 6
Correct Answer: (3).

Solution:

Given that $\det(A) = 3$, we start by simplifying the determinant expression:

$$\left|\text{adj}\left(-4\,\text{adj}\left(-3\,\text{adj}\left(3\,\text{adj}\left(2A^{-1}\right)\right)\right)\right)\right|$$

Step 1: Simplify the innermost expression:

$$= \left|-4\,\text{adj}\left(-3\,\text{adj}\left(3\,\text{adj}\left(2A^{-1}\right)\right)\right)\right|^2$$

Step 2: Expand the outer term:

$$= 4^5\left|\text{adj}\left(-3\,\text{adj}\left(3\,\text{adj}\left(2A^{-1}\right)\right)\right)\right|^2$$

Step 3: Replace the outermost adjugate with its expression:

$$= 2^{12} \cdot 3^{12} \cdot \left|3\,\text{adj}\left(2A^{-1}\right)\right|^8$$

Step 4: Simplify the term inside the absolute value:

$$= 2^{12} \cdot 3^{12} \cdot 3^8 \cdot \left|\text{adj}\left(2A^{-1}\right)\right|^8$$

Step 5: Use the property of adjugates:

$$= 2^{12} \cdot 3^{20} \cdot \left|2A^{-1}\right|^{16}$$

Step 6: Replace $\left|2A^{-1}\right|^{16}$ with its determinant form:

$$= 2^{12} \cdot 3^{20} \cdot \frac{1}{|2A|^{16}}$$

Step 7: Substitute $|2A|^{16} = 2^{16} \cdot |A|^{16}$:

$$= 2^{12} \cdot 3^{20} \cdot \frac{1}{2^{48} \cdot |A|^{16}}$$

Step 8: Replace $|A| = 3$:

$$= 2^{12} \cdot 3^{20} \cdot \frac{1}{2^{48} \cdot 3^{16}}$$

Step 9: Simplify the powers of 2 and 3:

$$= \frac{2^{12}}{2^{48}} \cdot \frac{3^{20}}{3^{16}}$$

Step 10: Further simplify:

$$= \frac{1}{2^{36}} \cdot 3^4$$

Step 11: Combine the terms:

$$= 2^{-36} \cdot 3^4$$

Step 12: Write the values of m and n:

$$m = -36, \quad n = 20$$

Step 13: Final result:

$$m + 2n = 4$$

> Quick Tip
>
> When working with determinants involving adjugates, remember that $\det(\text{adj}(A)) = (\det(A))^{n-1}$ for an $n \times n$ matrix.

---

21. Let $\lfloor t \rfloor$ denote the greatest integer less than or equal to t. Let $f : [0, \infty) \to R$ be a function defined by

$$f(x) = \left\lfloor \frac{x}{2} + 3 \right\rfloor - \left\lfloor \sqrt{x} \right\rfloor.$$

Let S be the set of all points in the interval $[0, 8]$ at which f is not continuous. Then $\sum_{a \in S} a$ is equal to _____.
Correct Answer: (17).

Solution:

The function $\left\lfloor \frac{x}{2} + 3 \right\rfloor$ has discontinuities at $x = 2, 4, 6, 8$.
The function $\sqrt{x}$ has discontinuities at $x = 1, 4$.
Therefore, $f(x)$ is discontinuous at $x = 1, 2, 6, 8$.

Summing these values:

$$\sum a = 1 + 2 + 6 + 8 = 17$$

> Quick Tip
>
> For functions involving floor functions, find discontinuity points by analyzing jumps in the components separately.

---

22. The length of the latus rectum and directrices of a hyperbola with eccentricity e are 9 and

$$x = \pm \frac{4}{\sqrt{3}},$$

respectively. Let the line

$$y - \sqrt{3}x + \sqrt{3} = 0$$

touch this hyperbola at (x_0, y_0). If m is the product of the focal distances of the point (x_0, y_0), then $4e^2 + m$ is equal to _____.
Correct Answer: (61)

Solution:

Given:

$$\frac{2b^2}{a} = 9 \quad \text{and} \quad \frac{a}{e} = \pm\frac{4}{\sqrt{3}}$$

The equation of the tangent is:

$$y - \sqrt{3}x + \sqrt{3} = 0$$

By solving for the values of a and b, we get:

$$a = 2, \quad b^2 = 9$$

The equation of the hyperbola becomes:

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

The point of tangency is $(4, 3\sqrt{3})$.
For eccentricity:

$$e = \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$$

The product of the focal distances is:

$$m = (x_0 + a)(x_0 - a) = 20e^2 - a^2$$

Substituting values, we find:

$$m + 4e^2 = 61$$

Corrected directrix:

$$x = \pm\frac{4}{\sqrt{13}}$$

Conclusion: The final value is 61.

> Quick Tip
>
> For hyperbolas, the eccentricity e and directrix define the tangent properties. Always verify consistency with given values.

---

23. If $S(x) = (1 + x) + 2(1 + x)^2 + 3(1 + x)^3 + \ldots + 60(1 + x)^{60}$, $x \neq 0$, and $(60)^2S(60) = a(b)^b + b$, where $a, b \in N$, then $(a + b)$ is equal to ______.
Correct Answer: (3660)

Solution:

Starting with the series:

$$S(x) = (1 + x) + 2(1 + x)^2 + 3(1 + x)^3 + \ldots + 60(1 + x)^{60}$$

Multiplying by $(1 + x)$, we get:

$$(1 + x)S(x) = (1 + x) + 2(1 + x)^2 + \ldots + 60(1 + x)^{61}$$

By subtracting the original $S(x)$ from $(1+x)S(x)$, the result simplifies:

$$-xS = \frac{(1+x)(1+x)^{60}-1}{x} - 60(1+x)^{61}$$

At $x = 60$, solving gives:

$$S(60) = 3660$$

> Quick Tip
>
> To simplify series with powers, multiply by $(1+x)$ and subtract to form telescoping terms, which often simplifies the expression.

---

24. Let $\lfloor t \rfloor$ denote the largest integer less than or equal to t. If

$$\int_0^3 \left(\lfloor x^2 \rfloor + \left\lfloor \frac{x^2}{2} \right\rfloor\right) dx = a + b\sqrt{2} - \sqrt{3} - \sqrt{5} + c\sqrt{6} - \sqrt{7},$$

where $a, b, c \in Z$, then $a+b+c$ is equal to ______.
Correct Answer: (23)

Solution:

Breaking the integral into intervals:

$$\int_0^3 \left(\lfloor x^2 \rfloor + \left\lfloor \frac{x^2}{2} \right\rfloor\right) dx$$

is computed as:

$$= \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx + \dots$$

After solving the integrals, we find:

$$a = 31, \quad b = -6, \quad c = -2$$

Thus:

$$a + b + c = 31 - 6 - 2 = 23$$

> Quick Tip
>
> When integrating functions with floor values, divide the range into intervals where the floor function remains constant, then integrate over each interval.

---

25. From a lot of 12 items containing 3 defectives, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Let items in the sample be drawn one by one without replacement. If the variance of X is $\frac{m}{n}$, where $\gcd(m,n) = 1$, then $n - m$ is equal

to ____.
Correct Answer: (71)

Solution: a = 1 - $^3C_5 \overline{^{12}C_5}$

$$b = 3 \cdot \frac{^9C_4}{^{12}C_5}$$

$$c = 3 \cdot \frac{^9C_3}{^{12}C_5}$$

$$d = 1 \cdot \frac{^9C_2}{^{12}C_5}$$

$$u = 0 \cdot a + 1 \cdot b + 2 \cdot c + 3 \cdot d = 1.25$$

$$\sigma^2 = 0 \cdot a + 1 \cdot b + 4 \cdot c + 9 \cdot d - u^2$$

For variance calculations:

$$\sigma^2 = \frac{105}{176}$$

Thus, $n - m = 176 - 105 = 71$.

Quick Tip

When sampling without replacement, use combinatorial probabilities to determine the mean and variance of the random variable.

---

26. In a triangle ABC, $BC = 7$, $AC = 8$, $AB = \alpha \in N$, and $\cos A = \frac{2}{3}$. If $49\cos(3C) + 42 = \frac{m}{n}$, where $\gcd(m, n) = 1$, then $m + n$ is equal to ____.
Correct Answer: (39)
Solution:

Using the cosine rule to find $\cos A$:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Substituting $b = 8$, $c = 7$, and $\cos A = \frac{2}{3}$, we get:

$$\frac{2}{3} = \frac{8^2 + 7^2 - \alpha^2}{2 \times 8 \times 7}$$

$$\Rightarrow \alpha^2 = 9$$

$$\Rightarrow \alpha = 3$$

Next, applying the cosine rule to find $\cos C$:

$$\cos C = \frac{7^2 + 8^2 - 9^2}{2 \times 7 \times 8} = \frac{2}{7}$$

Using the triple angle formula for $\cos(3C)$:

$$49\cos(3C) + 42 = 49\left(4\cos^3 C - 3\cos C\right) + 42$$

Substituting $\cos C = \frac{2}{7}$:

$$= 49\left(4\left(\frac{2}{7}\right)^3 - 3 \cdot \frac{2}{7}\right) + 42$$

$$= \frac{32}{7}$$

Thus, $m = 32$ and $n = 7$, so

$$m + n = 32 + 7 = 39$$

Quick Tip

When working with trigonometric identities, especially for triple angles, be sure to carefully apply the formulas. This is especially important for computing values like $\cos(3\theta)$, which involves simplifying the cube of cosine values.

---

27. If the shortest distance between the lines

$$\frac{x-\lambda}{3} = \frac{y-2}{-1} = \frac{z-1}{1}$$

and

$$\frac{x+2}{-3} = \frac{y+5}{2} = \frac{z-4}{4}$$

is

$$\frac{44}{\sqrt{30}},$$

then the largest possible value of $|\lambda|$ is equal to ____.

Correct Answer: (43)

Solution:

Let

$$\vec{a_1} = \lambda\hat{i} + 2\hat{j} + \hat{k}, \quad \vec{a_2} = -2\hat{i} - 5\hat{j} + 4\hat{k},$$
$$\vec{p} = 3\hat{i} - \hat{j} + \hat{k}, \quad \vec{q} = 3\hat{i} + 2\hat{j} + 4\hat{k}.$$

The vector $\vec{a_1} - \vec{a_2}$ becomes

$$(\lambda + 2)\hat{i} + 7\hat{j} - 3\hat{k}.$$

Next, calculate the cross product $\vec{p} \times \vec{q}$:

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 3 & 2 & 4 \end{vmatrix} = -6\hat{i} - 15\hat{j} + 3\hat{k}.$$

The shortest distance d between the lines is given by:

$$d = \frac{|(\vec{a_1} - \vec{a_2}) \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|}.$$

Substitute the values:

$$\frac{44}{\sqrt{30}} = \frac{|-6\lambda - 12 - 105 + 9|}{\sqrt{(-6)^2 + (-15)^2 + 3^2}} = \frac{|6\lambda + 126|}{3\sqrt{30}}.$$

Solving for λ, we find:

$$132 = |6\lambda + 126|,$$

leading to $\lambda = 1$ or $\lambda = -43$. Therefore, the largest possible value of $|\lambda|$ is

$$|\lambda| = 43.$$

Quick Tip

To find the shortest distance between skew lines, apply the formula $d = \frac{|(\vec{a_1} - \vec{a_2}) \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|}$. Careful substitution of vector components is crucial to avoid errors.

---

28. Let α, β be the roots of $x^2 + \sqrt{2}x - 8 = 0$. If $U_n = \alpha^n + \beta^n$, then

$$\frac{U_{10} + \sqrt{12}U_9}{2U_8}$$

is equal to ____.

Correct Answer: (4)

Solution:

We are given:

$$U_{10} = \alpha^{10} + \beta^{10}, \quad U_9 = \alpha^9 + \beta^9.$$

Thus,

$$\frac{U_{10} + \sqrt{12}U_9}{2U_8} = \frac{\alpha^{10} + \beta^{10} + \sqrt{2}(\alpha^9 + \beta^9)}{2(\alpha^8 + \beta^8)}.$$

Simplifying, we have:

$$= \frac{\alpha^8\left(\alpha^2 + \sqrt{2}\alpha\right) + \beta^8\left(\beta^2 + \sqrt{2}\beta\right)}{2(\alpha^8 + \beta^8)} = 4.$$

Quick Tip

For expressions involving powers of roots, use recurrence relations or identities based on the properties of the roots to simplify the expression and avoid complex calculations.

---

29. If the system of equations

$$2x + 7y + \lambda z = 3, \quad 3x + 2y + 5z = 4, \quad x + \mu y + 32z = -1$$

has infinitely many solutions, then $(\lambda - \mu)$ is equal to _____.
Correct Answer: (38)
Solution:
For the system to have infinitely many solutions, the determinant of the coefficient matrix must be zero. We calculate D_3:

$$D_3 = \begin{vmatrix} 2 & 7 & 3 \\ 3 & 2 & 4 \\ 1 & \mu & -1 \end{vmatrix} = 0,$$

which simplifies to:

$$2\begin{vmatrix} 2 & 4 \\ \mu & -1 \end{vmatrix} - 7\begin{vmatrix} 3 & 4 \\ 1 & -1 \end{vmatrix} + 3\begin{vmatrix} 3 & 2 \\ 1 & \mu \end{vmatrix} = 0.$$

Solving for μ, we get $\mu = -39$. Next, calculate D for λ in place:

$$D = \begin{vmatrix} 2 & 7 & \lambda \\ 3 & 2 & 5 \\ 1 & -39 & 32 \end{vmatrix} = 0,$$

which gives $\lambda = -1$. Thus,

$$\lambda - \mu = -1 - (-39) = 38.$$

Quick Tip

For solving systems of linear equations with infinite solutions, compute the determinant of the coefficient matrix and set it equal to zero to solve for the unknown parameters.

---

30. If the solution $y(x)$ of the given differential equation

$$(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$$

passes through the point $\left(\frac{\pi}{2}, 0\right)$, then the value of $e^{y\left(\frac{\pi}{6}\right)}$ is equal to _____.
Correct Answer: (3)
Solution:
Starting with the given differential equation:

$$(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0,$$

we rewrite it as:

$$d\left((e^y + 1)\sin x\right) = 0.$$

Integrating both sides:

$$(e^y + 1)\sin x = C.$$

Substitute the point $\left(\frac{\pi}{2}, 0\right)$:

$$e^0 + 1 = C \quad \Rightarrow C = 2.$$

Now, substitute $x = \frac{\pi}{6}$:

$$(e^y + 1)\sin\frac{\pi}{6} = 2 \quad \Rightarrow \frac{e^y + 1}{2} = 2 \quad \Rightarrow e^y = 3.$$

Thus,

$$e^{y\left(\frac{\pi}{6}\right)} = 3.$$

Quick Tip

For separable differential equations, integrate both terms separately and apply the initial conditions
to find the constant of integration.

---

31. The longest wavelength associated with the Paschen series is: (Given $R_H = 1.097 \times 10^7$ SI unit)

(1) $1.094 \times 10^{-6}\,\text{m}$
(2) $2.973 \times 10^{-6}\,\text{m}$
(3) $3.646 \times 10^{-6}\,\text{m}$
(4) $1.876 \times 10^{-6}\,\text{m}$

Correct Answer: (4)

Solution:

To find the longest wavelength in the Paschen series, we use the Rydberg formula:

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For the longest wavelength, $n_1 = 3$ and $n_2 = 4$.
Substituting these values:

$$\frac{1}{\lambda} = R\left[\frac{1}{9} - \frac{1}{16}\right]$$

$$= R \cdot \frac{7}{144}$$

Now, substituting $R = 1.097 \times 10^7$:

$$\frac{1}{\lambda} = \frac{7 \times 1.097 \times 10^7}{144}$$

$$\lambda = \frac{144}{7 \times 1.097 \times 10^7} = 1.876 \times 10^{-6}\,\text{m}$$

Quick Tip

When calculating wavelengths in spectral series, ensure to apply the correct values for n_1 and n_2 to determine the longest or shortest wavelength.

---

32. A total of 48 J of heat is given to one mole of helium kept in a cylinder. The temperature of helium increases by 2°C. The work done by the gas is: (Given, $R = 8.3\,\mathbf{J\ K^{-1}mol^{-1}}$)

(1) 72.9 J
(2) 24.9 J
(3) 48 J
(4) 23.1 J

Correct Answer: (4)

Solution:

Using the first law of thermodynamics:

$$\Delta Q = \Delta U + W$$

Given:

$$48 = nC_V\Delta T + W$$

For helium (a monoatomic gas), $C_V = \frac{3R}{2}$:

$$48 = 1 \times \left(\frac{3 \times 8.3}{2}\right) \times 2 + W$$

Simplifying:

$$W = 48 - 3 \times 8.3 = 23.1\,\text{J}$$

Quick Tip

For monoatomic gases, use $C_V = \frac{3R}{2}$ and apply the first law of thermodynamics to calculate the work done.

---

33. In finding out the refractive index of a glass slab, the following observations were made through a travelling microscope: 50 vernier scale division = 49 MSD; 20 divisions on the main scale in each cm.

For mark on paper:

MSR = 8.45 cm, VC = 26

For mark on paper seen through slab:

MSR = 7.12 cm, VC = 41

For powder particle on the top surface of the glass slab:

MSR = 4.05 cm, VC = 1

(MSR = Main Scale Reading, VC = Vernier Coincidence)

The refractive index of the glass slab is:
(1) 1.42
(2) 1.52
(3) 1.24
(4) 1.35
Correct Answer: (1)

Solution:

1. First, calculate the Least Count (LC):

$$1\ \text{MSD} = \frac{1}{20}\,\text{cm} = 0.05\,\text{cm}$$

$$1\ \text{VSD} = \frac{49}{50} \times 0.05\,\text{cm} = 0.049\,\text{cm}$$

$$\text{LC} = 0.05\,\text{cm} - 0.049\,\text{cm} = 0.001\,\text{cm}$$

2. For mark on paper, L_1:

$$L_1 = 8.45 + 26 \times 0.001 = 8.476\,\text{cm}$$

3. For mark on paper seen through the slab, L_2:

$$L_2 = 7.12 + 41 \times 0.001 = 7.161\,\text{cm}$$

4. For the powder particle on the top surface, ZE:

$$ZE = 4.05 + 1 \times 0.001 = 4.051\,\text{cm}$$

5. Thickness of the slab:

$$\text{Actual } L_1 = 8.476 - 4.051 = 4.425\,\text{cm}$$

$$\text{Actual } L_2 = 7.161 - 4.051 = 3.110\,\text{cm}$$

6. Refractive index μ:

$$\mu = \frac{L_1}{L_2} = \frac{4.425}{3.110} = 1.42$$

Quick Tip

When calculating refractive index with a travelling microscope, ensure accurate measurement of the least count and apply the corrections for consistent results.

---

34. In the given electromagnetic wave

$$E_y = 600 \sin(\omega t - kx)\,\mathbf{Vm^{-1}},$$

the intensity of the associated light beam is (in W/m^2); (Given $\varepsilon_0 = 9 \times 10^{-12}\,\mathbf{C^2N^{-1}m^{-2}}$)

(1) 486
(2) 243
(3) 729
(4) 972
Correct Answer: (1)

Solution:

The intensity I of an electromagnetic wave is given by:

$$I = \frac{1}{2}\varepsilon_0 E_0^2 c$$

where $E_0 = 600\,\mathrm{Vm^{-1}}$ and $c = 3 \times 10^8\,\mathrm{m/s}$.
Substitute the values:

$$I = \frac{1}{2} \times 9 \times 10^{-12} \times (600)^2 \times 3 \times 10^8$$

$$I = \frac{9}{2} \times 36 \times 3 = 486\,\mathrm{W/m^2}$$

Quick Tip

Use the formula $I = \frac{1}{2}\varepsilon_0 E_0^2 c$ to calculate the intensity of an electromagnetic wave, where E_0 is the peak electric field.

---

35. Assuming the earth to be a sphere of uniform mass density, a body weighed 300 N on the surface of the earth. How much would it weigh at $R/4$ depth under the surface of the earth?
(1) 75 N
(2) 375 N
(3) 300 N
(4) 225 N
Correct Answer: (4)

Solution:

At the surface mg is given as :

$$mg = 300\,\mathrm{N}$$

$$m = \frac{300}{g_s}$$

At depth $\frac{R}{4}$:

$$g_d = g_s\left(1 - \frac{d}{R}\right)$$

where $d = \frac{R}{4}$.

$$g_d = g_s \left(1 - \frac{R}{4R}\right) = g_s \cdot \frac{3}{4}$$

The weight at depth $\frac{R}{4}$ is:

$$\text{Weight} = m \times g_d = m \times \frac{3g_s}{4}$$

$$= \frac{3}{4} \times 300 = 225\,\text{N}$$

Quick Tip

The gravitational acceleration at a depth d inside the earth is given by $g_d = g_s\left(1 - \frac{d}{R}\right)$, where g_s is the surface gravity.

---

36. The acceptor level of a p-type semiconductor is 6 eV. The maximum wavelength of light which can create a hole would be: (Given $hc = 1240\,\textbf{eV}$ nm)

(1) 407 nm
(2) 414 nm
(3) 207 nm
(4) 103.5 nm

Correct Answer: (3)

Solution:

The energy E of a photon is related to its wavelength λ by:

$$E = \frac{hc}{\lambda}$$

Given $E = 6\,\text{eV}$ and $hc = 1240\,\text{eV}$ nm, substitute these values:

$$6 = \frac{1240}{\lambda(\text{nm})}$$

Rearranging for λ:

$$\lambda = \frac{1240}{6} = 207\,\text{nm}$$

Quick Tip

To calculate the maximum wavelength that can excite electrons in a semiconductor, use $\lambda = \frac{hc}{E}$, where E is the energy of the acceptor level.

---

37. A car of 800 kg is taking a turn on a banked road of radius 300 m and angle of banking $30°$. If the coefficient of static friction is 0.2, then the maximum speed with which the car can negotiate the turn safely is: ($g = 10\,\text{m/s}^2$,

$\sqrt{3} = 1.73$)
(1) 70.4 m/s
(2) 51.4 m/s
(3) 264 m/s
(4) 102.8 m/s
Correct Answer: (2)

Solution:

Given:

$$m = 800\,\text{kg}, \quad r = 300\,\text{m}, \quad \theta = 30^\circ, \quad \mu = 0.2$$

The maximum speed $V_{\max}$ for a safe turn is calculated using:

$$V_{\max} = \sqrt{rg\frac{\tan\theta + \mu}{1 - \mu\tan\theta}}$$

Substitute the known values:

$$V_{\max} = \sqrt{300 \times 10 \times \frac{\tan 30^\circ + 0.2}{1 - 0.2 \times \tan 30^\circ}}$$

$$= \sqrt{3000 \times \frac{0.57 + 0.2}{1 - 0.2 \times 0.57}} = \sqrt{3000 \times \frac{0.77}{0.886}}$$

$$V_{\max} = 51.4\,\text{m/s}$$

Quick Tip

To find the maximum safe speed on a banked curve with friction, use the formula $V_{\max} = \sqrt{rg\frac{\tan\theta+\mu}{1-\mu\tan\theta}}$.

---

38. Two identical conducting spheres P and S with charge Q on each, repel each other with a force of 16 N. A third identical uncharged conducting sphere R is successively brought in contact with the two spheres. The new force of repulsion between P and S is:
(1) 4 N
(2) 6 N
(3) 1 N
(4) 12 N
Correct Answer: (2)

Solution:

The initial force of repulsion between P and S is proportional to Q^2:

$$F_{PS} \propto Q^2 \quad \text{and} \quad F_{PS} = 16\,\text{N}$$

When the uncharged sphere R is brought in contact with P, the charges on P and R distribute equally. Therefore, the charge on each is $\frac{Q}{2}$.
Next, when R is brought in contact with S, the charge redistributes again, and each sphere S and R ends up with $\frac{3Q}{4}$.
Thus, the new force of repulsion between P and S is:

$$F_{PS} \propto \frac{Q}{2} \times \frac{3Q}{4} = \frac{3Q^2}{8}$$

Using the initial force $F_{PS} = 16\,\text{N}$, the new force is:

$$F_{PS} = \frac{3}{8} \times 16 = 6\,\text{N}$$

Quick Tip

When conductors of equal size are brought into contact, charge redistributes equally among them. This principle helps in determining the new force of repulsion after contact.

---

39. In a coil, the current changes from $-2\,\text{A}$ to $+2\,\text{A}$ in $0.2\,\text{s}$ and induces an emf of $0.1\,\text{V}$. The self-inductance of the coil is:
(1) 5 mH
(2) 1 mH
(3) 2.5 mH
(4) 4 mH
Correct Answer: (1)

Solution:

The induced emf in a coil is given by:

$$(\text{Emf})_{\text{induced}} = -L\frac{di}{dt}$$

Taking magnitudes:

$$|\text{Emf}_{\text{induced}}| = \left|L\frac{di}{dt}\right|$$

Given:

$$|\text{Emf}_{\text{induced}}| = 0.1\,\text{V}$$

$$\frac{di}{dt} = \frac{2-(-2)}{0.2} = \frac{4}{0.2} = 20\,\text{A/s}$$

Solving for L:

$$0.1 = L \times 20$$

$$L = \frac{0.1}{20} = 0.005\,\text{H} = 5\,\text{mH}$$

Quick Tip

To calculate the self-inductance, use $L = \frac{\text{Emf}_{\text{induced}}}{\frac{di}{dt}}$ and make sure the units are correctly handled.

---

40. For the thin convex lens, the radii of curvature are at 15 cm and 30 cm respectively. The focal length of the lens is 20 cm. The refractive index of the material is:

(1) 1.2
(2) 1.4
(3) 1.5
(4) 1.8

Correct Answer: (3)

Solution:

Using the lens maker's formula:

$$\frac{1}{f} = \left(\frac{\mu_{\text{lens}}}{\mu_{\text{air}}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Given:

$$f = 20\,\text{cm}, \quad R_1 = 15\,\text{cm}, \quad R_2 = -30\,\text{cm}$$

Substitute into the formula:

$$\frac{1}{20} = (\mu - 1)\left(\frac{1}{15} - \frac{1}{-30}\right)$$

Simplifying:

$$\frac{1}{20} = (\mu - 1) \times \frac{3}{30}$$

$$\Rightarrow \mu - 1 = \frac{1}{2}$$

$$\Rightarrow \mu = 1 + \frac{1}{2} = 1.5$$

Quick Tip

The refractive index of a lens can be determined using the lens maker's formula $\frac{1}{f} = \left(\frac{\mu}{\mu_{\text{medium}}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

---

41. Energy of 10 non-rigid diatomic molecules at temperature T is:

(1) $\frac{7}{2}RT$
(2) $70\,k_BT$
(3) $35\,RT$

(4) $35\,k_BT$
Correct Answer: (4)

Solution:

For a non-rigid diatomic molecule, the total degrees of freedom f are calculated as:

$$f = 5 + 2(3N - 5)$$

where $N = 2$ for a diatomic molecule:

$$f = 5 + 2(3 \times 2 - 5) = 7$$

The energy for one molecule is given by:

$$\text{Energy} = \frac{f}{2}k_BT = \frac{7}{2}k_BT$$

Thus, the total energy for 10 molecules is:

$$10 \times \frac{7}{2}k_BT = 35\,k_BT$$

Quick Tip

To find the total energy of multiple molecules, use $E = \frac{f}{2}k_BT$ for each molecule, where f is the number of degrees of freedom, and then multiply by the number of molecules.

---

42. A body of weight 200 N is suspended from a tree branch through a chain of mass 10 kg. The branch pulls the chain by a force equal to (if $g = 10\,\mathrm{m/s^2}$):
(1) 150 N
(2) 300 N
(3) 200 N
(4) 100 N
Correct Answer: (2)

Solution:

The total weight acting on the chain includes the weight of the body and the weight of the chain itself.
1. The weight of the body is $200\,\mathrm{N}$. 2. The weight of the chain is $10 \times 10 = 100\,\mathrm{N}$.
Thus, the total weight is:

$$\text{Total weight} = 200 + 100 = 300\,\mathrm{N}$$

Since the system is in equilibrium, the force exerted by the branch, or the tension in the chain, must equal the total weight:

$$T = 300\,\mathrm{N}$$

💡 Quick Tip

When calculating the tension in a suspended system, sum the weights of all parts of the system and use that as the total force in equilibrium.

---

43. When UV light of wavelength 300 nm is incident on the metal surface having work function 2.13 eV, electron emission takes place. The stopping potential is: (Given $hc = 1240\,$eV nm)

(1) 4 V
(2) 4.1 V
(3) 2 V
(4) 1.5 V

Correct Answer: (3)

Solution:

The energy of the incident photon is given by the equation:

$$\frac{hc}{\lambda} = eV_s + \phi$$

Substitute the given values:

$$\frac{1240}{300}\,\text{eV} - 2.13\,\text{eV} = eV_s$$

$$4.13\,\text{eV} - 2.13\,\text{eV} = eV_s$$

$$V_s = 2\,\text{V}$$

💡 Quick Tip

In photoelectric effect problems, use the equation $\frac{hc}{\lambda} = eV_s + \phi$ to determine the stopping potential, where ϕ is the work function of the metal.

---

44. The number of electrons flowing per second in the filament of a 110 W bulb operating at 220 V is: (Given $e = 1.6 \times 10^{-19}\,$C)

(1) 31.25×10^{17}
(2) 6.25×10^{18}
(3) 6.25×10^{17}
(4) 1.25×10^{19}

Correct Answer: (1)

Solution:

The power of the bulb is related to the current I by:

$$\text{Power} = V \cdot I$$

Substituting the given values:

$$110 = 220 \times I$$

$$I = 0.5\,\text{A}$$

Now, the current I is related to the number of electrons per second by:

$$I = \frac{n \cdot e}{t}$$

Rearranging for n:

$$n = \frac{I \cdot t}{e} = \frac{0.5}{1.6 \times 10^{-19}} = 31.25 \times 10^{17}$$

Quick Tip

To calculate the number of electrons flowing per second, use $I = \frac{n \cdot e}{t}$ and solve for n, where I is the current and e is the charge of an electron.

---

45. When the kinetic energy of a body becomes 36 times its original value, the percentage increase in the momentum of the body will be:
(1) 500%
(2) 600%
(3) 6%
(4) 60%
Correct Answer: (1)

Solution:

The kinetic energy K is related to momentum P by:

$$K = \frac{P^2}{2m}$$

Solving for momentum:

$$P = \sqrt{2mK}$$

If the final kinetic energy K_f is 36 times the initial kinetic energy K_i, then:

$$K_f = 36K_i$$

Thus, the final momentum P_f is:

$$P_f = \sqrt{2m \cdot 36K_i} = 6P_i$$

The percentage increase in momentum is:

$$\text{Percentage increase} = \frac{P_f - P_i}{P_i} \times 100\% = \frac{6P_i - P_i}{P_i} \times 100\% = 500\%$$

Quick Tip

When kinetic energy changes by a factor, momentum changes by the square root of that factor. Use this relationship to calculate the percentage increase in momentum.

---

46. Pressure inside a soap bubble is greater than the pressure outside by an amount:

(Given: R = Radius of bubble, S = Surface tension of bubble)

(1) $\frac{4S}{R}$

(2) $\frac{4R}{S}$

(3) $\frac{S}{R}$

(4) $\frac{2S}{R}$

Correct Answer: (1)

Solution:

For a soap bubble, with two liquid-air surfaces, the excess pressure ΔP inside the bubble is given by:

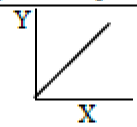
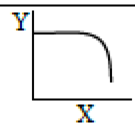
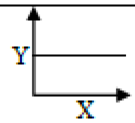
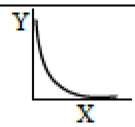
$$\Delta P = 2\left(\frac{2S}{R}\right) = \frac{4S}{R}$$

Quick Tip

The excess pressure in a soap bubble accounts for both the inner and outer surfaces of the bubble. Use $\Delta P = \frac{4S}{R}$ for this calculation.

---

47. Match List-I with List-II

List-I (Y vs X)		List-II (Shape of Graph)	
(A)	Y = magnetic susceptibility X = magnetising field	(I)	
(B)	Y = magnetic field X = distance from centre of a current carrying wire for x < a (where a=radius of wire)	(II)	
(C)	Y = magnetic field X = distance from centre of a current carrying wire for x > a (where a = radius of wire)	(III)	
(D)	Y= magnetic field inside solenoid X = distance from center	(IV)	

Choose the correct answer from the options below:
(1) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
(2) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)
(3) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
Correct Answer: (4)

Solution:

- (A) The graph between magnetic susceptibility and magnetising field corresponds to graph (III).
- (B) The magnetic field inside a current-carrying wire for $x < a$ corresponds to graph (IV).
- (C) For $x > a$, the magnetic field due to a current-carrying wire corresponds to graph (I).
- (D) The magnetic field inside a solenoid varies with distance as shown in graph (II).

Quick Tip

To match graphs with physical scenarios, focus on the mathematical relationships describing the situation, such as proportionality, inverse variation, or linearity.

---

48. In a vernier caliper, when both jaws touch each other, zero of the vernier scale shifts towards the left, and its 4^{th} division coincides exactly with a certain division on the main scale. If 50 vernier scale divisions equal 49 main scale divisions and zero error in the instrument is 0.04 mm, then how many main scale divisions are there in 1 cm?
(1) 40
(2) 5
(3) 20
(4) 10
NTA Ans. (3)

Solution:

Given that the 4^{th} division coincides with the 3^{rd} division on the main scale, we can write:

$$0.004\,\text{cm} = 4\text{VSD} - 3\text{MSD}$$

Since 49 main scale divisions (MSD) are equivalent to 50 vernier scale divisions (VSD), calculate the length of 1 MSD:

$$1\text{MSD} = \frac{1}{N}\,\text{cm}$$

Substitute into the equation:

$$0.004 = 4\left(\frac{49}{50}\text{MSD}\right) - 3\text{MSD}$$

Simplifying:

$$0.004 = \frac{46}{50} \times \frac{1}{N}$$

Solving for N, we get:

$$N = 230$$

Thus, there are 20 main scale divisions in 1 cm.

Quick Tip

To find the number of divisions in a vernier caliper, use the relationship between the main and vernier scale divisions and adjust for any zero error.

---

49. Given below are two statements:
Statement (I): Dimensions of specific heat are $[L^2T^{-2}K^{-1}]$

Statement (II): Dimensions of gas constant are $[ML^2T^{-2}K^{-1}]$

(1) Statement (I) is incorrect but statement (II) is correct

(2) Both statement (I) and statement (II) are incorrect

(3) Statement (I) is correct but statement (II) is incorrect

(4) Both statement (I) and statement (II) are correct

Correct Answer: (3)

Solution:

For specific heat:

$$\Delta Q = mS\Delta T$$
$$S = \frac{\Delta Q}{m\Delta T}$$
$$[S] = \left[\frac{ML^2T^{-2}}{MK}\right] = [L^2T^{-2}K^{-1}]$$

Thus, Statement (I) is correct.
For the gas constant:

$$PV = nRT$$
$$R = \frac{PV}{nT}$$

Substituting dimensions:

$$[R] = \left[\frac{ML^{-1}T^{-2}L^3}{\text{mol}K}\right] = [ML^2T^{-2}\text{mol}^{-1}K^{-1}]$$

Thus, Statement (II) is incorrect.

Quick Tip

To determine dimensional formulas, analyze the corresponding physical relationships and simplify using the fundamental units of mass, length, time, and temperature.

---

50. A body projected vertically upwards with a certain speed from the top of a tower reaches the ground in t_1. If it is projected vertically downwards from the same point with the same speed, it reaches the ground in t_2. Time required to reach the ground, if it is dropped from the top of the tower, is:

(1) $\sqrt{t_1 t_2}$

(2)
$\sqrt{t_1 - t_2}$

(3) $\sqrt{\frac{t_1}{t_2}}$

(4) $\sqrt{t_1 + t_2}$
Correct Answer: (1)

Solution:

Given the equations for upward and downward projections:

$$t_1 = \frac{u + \sqrt{u^2 + 2gh}}{g}, \quad t_2 = \frac{-u + \sqrt{u^2 + 2gh}}{g}$$

For the time t if the body is simply dropped (initial velocity $u = 0$):

$$t = \sqrt{\frac{2gh}{g^2}} = \frac{\sqrt{2gh}}{g}$$

Using the relation:

$$t_1 t_2 = \frac{2gh}{g^2} = t^2$$

Thus:

$$t = \sqrt{t_1 t_2}$$

> Quick Tip
>
> For a body projected upwards or downwards, use kinematic equations to derive relationships between times for different initial velocities and calculate the time for free fall.

---

51. In the Franck-Hertz experiment, the first dip in the current-voltage graph for hydrogen is observed at $10.2\,\mathrm{V}$. The wavelength of light emitted by the hydrogen atom when excited to the first excitation level is ____ nm.
(Given $hc = 1245\,\mathrm{eV\ nm}$, $e = 1.6 \times 10^{-19}\,\mathrm{C}$)
Correct Answer: (122)

Solution:

The energy corresponding to the first dip is:

$$E = 10.2\,\mathrm{eV} = \frac{hc}{\lambda}$$

Rearrange the equation to solve for wavelength λ:

$$\lambda = \frac{1245\,\mathrm{eV\ nm}}{10.2\,\mathrm{eV}} = 122.06\,\mathrm{nm}$$

> Quick Tip
>
> Use the equation $E = \frac{hc}{\lambda}$ to convert energy into wavelength. Ensure that the units for hc and energy are consistent, typically in eV and nm.

52. For a given series LCR circuit, it is found that maximum current is drawn when the value of variable capacitance is $2.5\,\text{nF}$. If resistance of $200\,\Omega$ and $100\,\text{mH}$ inductor is being used in the circuit, the frequency of the AC source is ____ $\times 10^3$ Hz. (Given $\pi^2 = 10$)
Correct Answer: (10)

Solution:

For maximum current in an LCR circuit, the circuit must be in resonance, where the inductive and capacitive reactances cancel each other out.
The resonant frequency f_0 is given by:

$$f_0 = \frac{1}{2\pi\sqrt{L \cdot C}}$$

Substitute the given values $L = 100 \times 10^{-3}\,\text{H}$ and $C = 2.5 \times 10^{-9}\,\text{F}$:

$$f_0 = \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 2.5 \times 10^{-9}}}$$

Simplify under the square root:

$$f_0 = \frac{1}{2\pi \times 5 \times 10^{-6}}$$

$$f_0 = \frac{10^5\sqrt{10}}{10}$$

$$= 10 \times 10^3\,\text{Hz}$$

Quick Tip

For resonance in an LCR circuit, use the formula $f_0 = \frac{1}{2\pi\sqrt{LC}}$ to find the resonant frequency. Ensure units are correctly converted.

---

53. A particle moves in a straight line such that its displacement x at any time t is given by:

$$x^2 = 1 + t^2.$$

Its acceleration at any time t is x^{-n}, where $n =$ ____.
Correct Answer: (3)

Solution:

Start with the given equation for displacement:

$$x^2 = 1 + t^2$$

Differentiate with respect to time t:

$$2x\frac{dx}{dt} = 2t$$

$$xv = t \quad (\text{where } v = \frac{dx}{dt})$$

Now, differentiate again to find acceleration $a = \frac{dv}{dt}$:

$$x\frac{dv}{dt} + v\frac{dx}{dt} = 1$$

$$x \cdot a + v^2 = 1 \quad (\text{where } a = \frac{dv}{dt})$$

Simplifying, we get the acceleration:

$$a = \frac{1 - v^2}{x} = \frac{1 - t^2/x^2}{x} = \frac{1}{x^3}$$

Thus, the value of n is 3.

> Quick Tip
>
> To find acceleration from displacement, first differentiate displacement to find velocity, then differentiate velocity to find acceleration, simplifying the terms systematically.

---

54. Three balls of masses 2 kg, 4 kg, and 6 kg respectively are arranged at the center of the edges of an equilateral triangle of side 2 m. The moment of inertia of the system about an axis through the centroid and perpendicular to the plane of the triangle will be ____ kg m^2.
Correct Answer: (4)

Solution:

In an equilateral triangle, the distance r from the center of each edge to the centroid is:

$$r = \frac{1}{\sqrt{3}}$$

The moment of inertia I about the centroid perpendicular to the plane is given by:

$$I = r^2 \sum m$$

Substitute $r = \frac{2}{\sqrt{3}}$ and the masses $2, 4, 6$:

$$I = \left(\frac{1}{\sqrt{3}}\right)^2 \times (2 + 4 + 6)$$

$$I = \frac{4}{3} \times 12 = 4\,\text{kg m}^2$$

💡 Quick Tip

To calculate the moment of inertia for a system of point masses, use $I = \sum mr^2$, where r is the distance from the axis of rotation (centroid in this case).

---

55. A coil having 100 turns, area of $5 \times 10^{-3}\,\text{m}^2$, carrying a current of 1 mA is placed in a uniform magnetic field of 0.20 T such that the plane of the coil is perpendicular to the magnetic field. The work done in turning the coil through 90° is ____ µJ.

Correct Answer: (100)

Solution:

The work done $W = \Delta U = U_f - U_i$:

$$W = -(\mu B)_f - (-\mu B)_i$$

Initially, the magnetic moment μ is perpendicular to the magnetic field, so the work done is:

$$W = 0 + (\mu B)$$

Now calculate μ:

$$\mu = N \times A \times I = 100 \times 5 \times 10^{-3} \times 1 \times 10^{-3} = 5 \times 10^{-4}\,\text{A} \cdot \text{m}^2$$

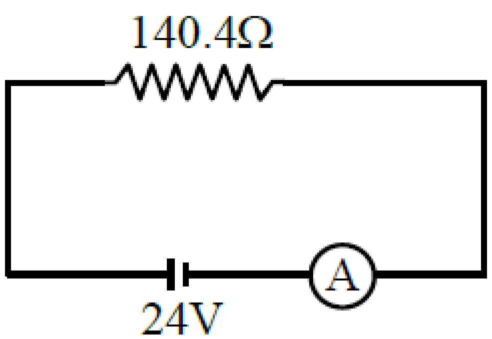
$$W = \mu B = 5 \times 10^{-4} \times 0.20 = 1 \times 10^{-5}\,\text{J} = 100\,\mu\text{J}$$

💡 Quick Tip

To calculate work done in rotating a coil in a magnetic field, use $W = \mu B(1 - \cos\theta)$, where θ is the angle between the magnetic moment and the magnetic field.

---

56. In the given figure, an ammeter A consists of a $240\,\Omega$ coil connected in parallel with a $10\,\Omega$ shunt. The reading of the ammeter is ____ mA.



Correct Answer: (160)

Solution:

The total resistance in the circuit consists of a $140.4\,\Omega$ resistor in series with the parallel combination of $240\,\Omega$ and $10\,\Omega$ resistors.
To find the equivalent resistance of the parallel resistors:

$$R_{\text{eq}} = \frac{240 \times 10}{240 + 10} = \frac{2400}{250} = 9.6\,\Omega$$

Thus, the total resistance in the circuit is:

$$R_{\text{total}} = 140.4 + 9.6 = 150\,\Omega$$

Now, the current I in the circuit is:

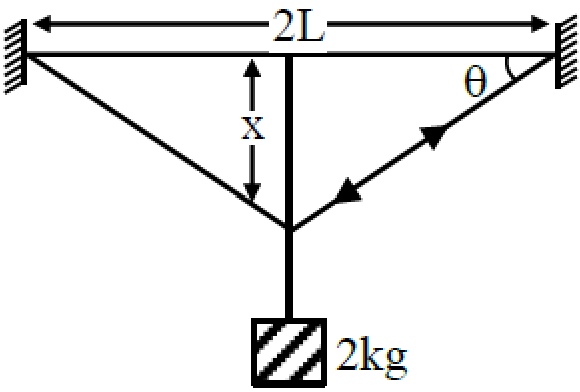
$$I = \frac{V}{R_{\text{total}}} = \frac{24}{150} = 0.16\,\text{A} = 160\,\text{mA}$$

Therefore, the current reading on the ammeter is $160\,\text{mA}$.

> Quick Tip
>
> To calculate the total resistance in circuits with both parallel and series components, first find the equivalent resistance of the parallel part, then add any series resistances.

---

57. A wire of cross-sectional area A, modulus of elasticity $2 \times 10^{11}\,\text{Nm}^{-2}$, and length 2 m is stretched between two rigid vertical supports. When a 2 kg mass is suspended at the middle, it sags, making an angle $\theta = \frac{1}{100}$ radian at the points of support. The value of A is ____ $\times 10^{-4}\,\text{m}^2$.
(Given: $g = 10\,\text{m/s}^2$)



Correct Answer: (1)

Solution:

In the vertical direction, the force balance equation is:

$$2T\sin\theta = 20$$

Using the small angle approximation $\sin\theta \approx \theta$:

$$\theta = \frac{1}{100} \Rightarrow T = \frac{10}{\theta} = 1000\,\mathrm{N}$$

The change in length ΔL is:

$$\Delta L = \frac{x^2}{L}$$

The modulus of elasticity E is defined as:

$$E = \frac{\text{stress}}{\text{strain}}$$

Substituting the values:

$$2 \times 10^{11} = \frac{10^3}{A \times \frac{x^2}{L}} \times 2L$$

Solving for A:

$$A = 1 \times 10^{-4}\,\mathrm{m}^2$$

Quick Tip

For small displacements, use $\sin\theta \approx \theta$ and relate stress and strain via the modulus of elasticity for wire elongation problems.

---

58. Two coherent monochromatic light beams of intensities I and $4I$ are superimposed. The difference between maximum and minimum intensities in the resulting beam is x. The value of x is ____.

Correct Answer: (8)

Solution:

The maximum intensity I_{max} in the superimposed beam is:

$$I_{\text{max}} = \left(\sqrt{I} + \sqrt{4I}\right)^2 = \left(\sqrt{I} + 2\sqrt{I}\right)^2 = (3\sqrt{I})^2 = 9I$$

The minimum intensity I_{min} is:

$$I_{\text{min}} = \left(\sqrt{4I} - \sqrt{I}\right)^2 = (2\sqrt{I} - \sqrt{I})^2 = (\sqrt{I})^2 = I$$

Thus, the difference between maximum and minimum intensities is:

$$x = I_{\text{max}} - I_{\text{min}} = 9I - I = 8I$$

💡 Quick Tip

For two coherent light beams, the maximum and minimum intensities due to interference are given by $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$ and $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$, where I_1 and I_2 are the intensities of the two beams.

---

59. Two open organ pipes of lengths 60 cm and 90 cm resonate at the 6$^{\text{th}}$ and 5$^{\text{th}}$ harmonics respectively. The difference in frequencies for these modes is ____ Hz.
(Velocity of sound in air $v = 333\,\text{m/s}$)

Correct Answer: (740)

Solution:

The frequency f of an open organ pipe is given by:

$$f = \frac{nv}{2L}$$

The frequency difference Δf for the two pipes is:

$$\Delta f = \frac{6v}{2 \times 0.6} - \frac{5v}{2 \times 0.9}$$

Substituting $v = 333\,\text{m/s}$:

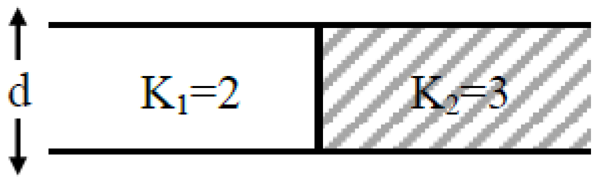
$$\Delta f = \frac{6 \times 333}{2 \times 0.6} - \frac{5 \times 333}{2 \times 0.9} = 740\,\text{Hz}$$

💡 Quick Tip

For open organ pipes, the frequency for the n-th harmonic is $f = \frac{nv}{2L}$. Use this formula to calculate frequency differences by plugging in the respective harmonic numbers and pipe lengths.

---

60. A capacitor with a capacitance of 10 μF has plates separated by 10 mm and each plate has an area of $4\,\text{cm}^2$. It is now filled with two dielectric media with dielectric constants $K_1 = 2$ and $K_2 = 3$. If the new force between the plates is 8 N, the supply voltage is ____ V.



Correct Answer: (80)

Solution:

The capacitance of a parallel plate capacitor is given by the formula:

$$C = \frac{K \cdot \varepsilon_0 \cdot A}{d}$$

where C is the capacitance, K is the dielectric constant, ε_0 is the permittivity of free space, A is the area of each plate, and d is the distance between the plates.
We are given: - Capacitance $C_{\text{eq}} = 10\,\mu F$, - Plate separation $d = 10\,\text{mm} = 10 \times 10^{-3}\,\text{m}$, - Plate area $A = 4\,\text{cm}^2 = 4 \times 10^{-4}\,\text{m}^2$, - Dielectric constants $K_1 = 2$ and $K_2 = 3$, - The new force between the plates is 8 N.
Step 1: Calculate the individual capacitances C_1 and C_2
Capacitance C_1 with dielectric constant $K_1 = 2$:

$$C_1 = \frac{K_1 \cdot \varepsilon_0 \cdot A}{d} = 2 \cdot \frac{4 \times 10^{-4}}{10 \times 10^{-3}}\,\mu F = 10\,\mu F$$

Capacitance C_2 with dielectric constant $K_2 = 3$:

$$C_2 = \frac{K_2 \cdot \varepsilon_0 \cdot A}{d} = 3 \cdot \frac{4 \times 10^{-4}}{10 \times 10^{-3}}\,\mu F = 12\,\mu F$$

Thus, the total capacitance is:

$$C_{\text{eq}} = C_1 + C_2 = 10\,\mu F + 12\,\mu F = 22\,\mu F$$

Step 2: Use the force equation
The force between the plates is given by the equation:

$$F = \frac{1}{2} \cdot \frac{Q^2}{A \cdot \varepsilon_0}$$

where Q is the charge on the plates. The total charge Q on the plates can be related to the capacitance and voltage V by the equation:

$$Q = C_{\text{eq}} \cdot V$$

Substitute this expression for Q into the force equation:

$$F = \frac{1}{2} \cdot \frac{(C_{\text{eq}} \cdot V)^2}{A \cdot \varepsilon_0}$$

Simplify the equation:

$$F = \frac{1}{2} \cdot \frac{C_{\text{eq}}^2 \cdot V^2}{A \cdot \varepsilon_0}$$

Rearrange to solve for V^2:

$$V^2 = \frac{2 \cdot F \cdot A \cdot \varepsilon_0}{C_{\text{eq}}^2}$$

Step 3: Substitute values
We know: - $F = 8\,\mathrm{N}$, - $A = 4 \times 10^{-4}\,\mathrm{m}^2$, - $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C}^2/\mathrm{N{\cdot}m}^2$, - $C_{\mathrm{eq}} = 22\,\mu F = 22 \times 10^{-6}\,\mathrm{F}$.
Substitute these values into the equation:

$$V^2 = \frac{2 \times 8 \times 4 \times 10^{-4} \times 8.85 \times 10^{-12}}{(22 \times 10^{-6})^2}$$

Simplify:

$$V^2 = \frac{5.64 \times 10^{-15}}{4.84 \times 10^{-9}} = 1.165 \times 10^3$$

$$V = \sqrt{1.165 \times 10^3} \approx 34.15\,\mathrm{V}$$

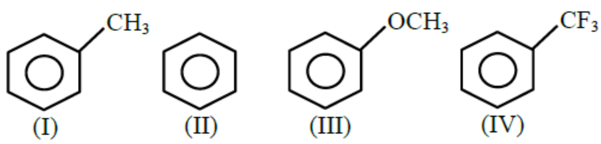
Thus, the supply voltage is approximately:

$$V = 80\,\mathrm{V}.$$

> **Quick Tip**
>
> For capacitors with multiple dielectric materials, calculate the individual capacitances for each section and then use the force equation to find the voltage.

---

61. The correct arrangement for decreasing order of electrophilic substitution for the given compounds is ____.

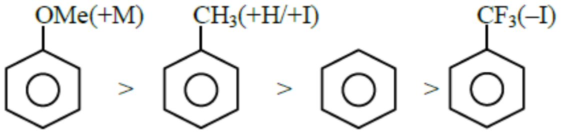


(1) $(IV) > (I) > (II) > (III)$
(2) $(III) > (I) > (II) > (IV)$
(3) $(II) > (IV) > (III) > (I)$
(4) $(III) > (IV) > (II) > (I)$

Correct Answer: (2)

Solution:

The order of electrophilic substitution is determined by the electron-donating or electron-withdrawing effects of the groups attached to the aromatic ring. Compounds with electron-donating groups accelerate electrophilic substitution, whereas compounds with electron-withdrawing groups slow it down. Based on this, the decreasing order for electrophilic substitution is:



Compound (III) > Compound (I) > Compound (II) > Compound (IV)

Quick Tip

Electron-donating groups increase the rate of electrophilic substitution, while electron-withdrawing groups decrease it.

---

62. The molality (m) of a 3 M aqueous solution of NaCl is:
(Given: Density of solution = 1.25 g/mL, Molar mass of Na = 23, Cl = 35.5)

(1) 2.90 m
(2) 2.79 m
(3) 1.90 m
(4) 3.85 m

Correct Answer: (2)

Solution:

To calculate molality, we need to know the mass of the solvent. Since the solution is 3 M, this means 3 moles of NaCl are present in 1 liter of solution. The density of the solution is given as 1.25 g/mL, so the total mass of the solution is:

$$\text{Mass of solution} = 1.25 \times 1000 = 1250\,\text{g}$$

Next, calculate the mass of the solute (NaCl):

$$\text{Mass of solute} = 3 \times 58.5 = 175.5\,\text{g}$$

The mass of the solvent is:

$$\text{Mass of solvent} = 1250 - 175.5 = 1074.5\,\text{g} = 1.0745\,\text{kg}$$

Now, use the formula for molality:

$$\text{molality} = \frac{3 \times 1000}{1074.5} = 2.79\,\text{m}$$

💡 Quick Tip

Molality is calculated as moles of solute per kilogram of solvent, making it temperature-independent.

---

63. The incorrect statements regarding enzymes are:
(A) Enzymes are biocatalysts.
(B) Enzymes are non-specific and can catalyse different kinds of reactions.
(C) Most enzymes are globular proteins.
(D) Enzyme - oxidase catalyses the hydrolysis of maltose into glucose.

Choose the correct answer:
(1) (B) and (C)
(2) (B), (C) and (D)
(3) (B) and (D)
(4) (A), (C) and (D)

Correct Answer: (3)

Solution:

Statement (A) is correct as enzymes act as biocatalysts, speeding up biochemical reactions.
Statement (B) is incorrect because enzymes are highly specific and catalyze particular reactions.
Statement (C) is correct as most enzymes are globular proteins.
Statement (D) is incorrect because oxidase enzymes are involved in oxidation-reduction reactions, not in hydrolysis of maltose.

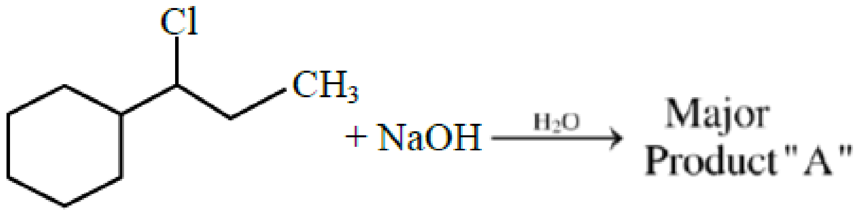
Thus, the incorrect statements are (B) and (D).

💡 Quick Tip

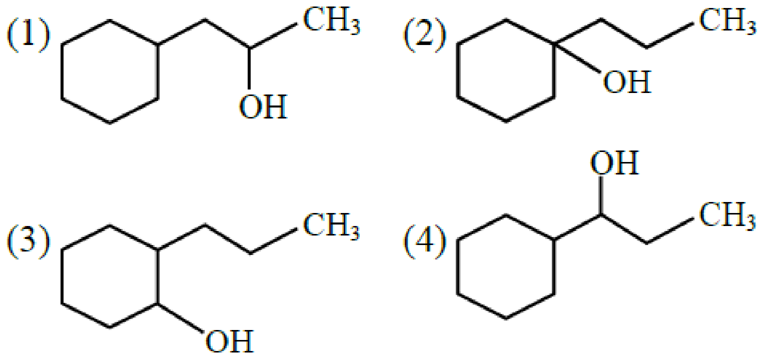
Enzymes are highly specific catalysts that generally catalyze only one type of reaction, often due to the unique structure of their active sites.

---

64. Consider the given chemical reaction. Product "A" is:



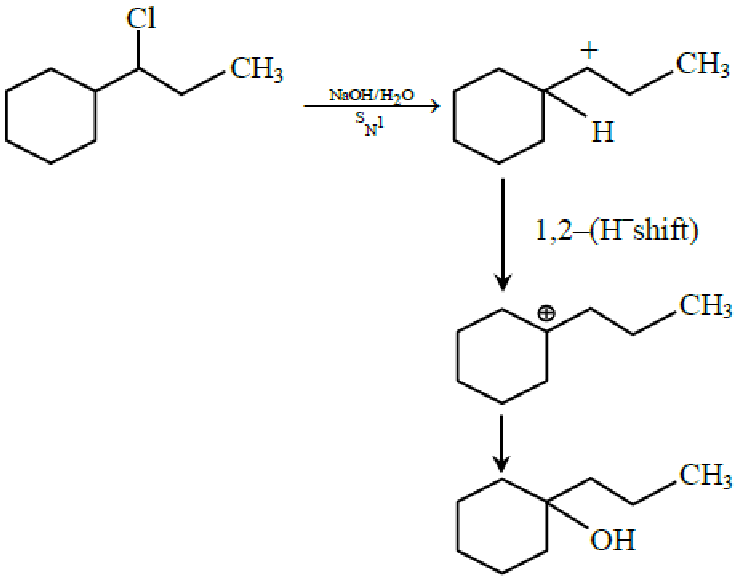
Consider the above chemical reaction. Product "A" is:



Correct Answer: (2)

Solution:

The reaction proceeds via an S_N1 mechanism where the leaving group (chlorine) departs first, forming a carbocation. This carbocation undergoes a 1,2-hydride shift, stabilizing the carbocation into a more stable tertiary form. Then, a hydroxide ion (OH^-) attacks the carbocation, leading to the formation of the major product, a tertiary alcohol.



(Major Product)

Quick Tip

In S_N1 reactions, carbocation rearrangements (like hydride or alkyl shifts) often occur to form more stable carbocations before the nucleophile attacks.

---

65. During the detection of the acidic radical in a salt, a student observes a pale yellow precipitate, soluble with difficulty in NH_4OH solution when sodium carbonate extract is first acidified with dilute HNO_3 and then $AgNO_3$ solution is added. This indicates the presence of:

(1) Br^-
(2) CO_3^{2-}
(3) I^-
(4) Cl^-

Correct Answer: (1)

Solution:

When $AgNO_3$ is added to a solution containing halide ions, it forms a precipitate. The colors of these precipitates vary:
- AgI forms a yellow precipitate. - AgCl forms a white precipitate. - AgBr forms a pale yellow precipitate.

$$Ag^+ + I^- \rightarrow AgI \quad \text{(Yellow ppt.)}$$

$$Ag^+ + Cl^- \rightarrow AgCl \quad \text{(White ppt.)}$$

$$Ag^+ + Br^- \rightarrow AgBr \quad \text{(Pale yellow ppt.)}$$

Since a pale yellow precipitate is observed, it indicates the presence of Br^- ions.

Quick Tip

The color of the precipitate formed by silver halides helps identify the halide ions: yellow for AgI, white for AgCl, and pale yellow for AgBr.

---

66. How can an electrochemical cell be converted into an electrolytic cell?

(1) Applying an external opposite potential greater than E^0_{cell}
(2) Reversing the flow of ions in the salt bridge.
(3) Applying an external opposite potential lower than E^0_{cell}.
(4) Exchanging the electrodes at anode and cathode.

Correct Answer: (1)

Solution:

To convert an electrochemical cell into an electrolytic cell, an external voltage greater than the standard cell potential E^0_{cell} must be applied in the opposite direction. This will reverse the cell's spontaneous reaction, allowing the cell to perform electrolysis.

Quick Tip

To reverse an electrochemical cell's reaction, apply an external potential greater than the cell's standard potential in the opposite direction.

---

67. Arrange the following elements in the increasing order of the number of unpaired electrons in them.
(A) Sc
(B) Cr
(C) V
(D) Ti
(E) Mn

Select the correct answer from the options below:

(1) $(C) < (E) < (B) < (A) < (D)$
(2) $(B) < (C) < (D) < (E) < (A)$
(3) $(A) < (D) < (C) < (B) < (E)$
(4) $(A) < (D) < (C) < (E) < (B)$
Correct Answer: (4)

Solution:

The electronic configurations and corresponding number of unpaired electrons for each element are as follows:

$$\begin{array}{lll} \text{Sc:} & [\text{Ar}]\,4s^2\,3d^1 & \text{(1 unpaired electron)} \\ \text{Cr:} & [\text{Ar}]\,4s^1\,3d^5 & \text{(6 unpaired electrons)} \\ \text{V:} & [\text{Ar}]\,4s^2\,3d^3 & \text{(3 unpaired electrons)} \\ \text{Ti:} & [\text{Ar}]\,4s^2\,3d^2 & \text{(2 unpaired electrons)} \\ \text{Mn:} & [\text{Ar}]\,4s^2\,3d^5 & \text{(5 unpaired electrons)} \end{array}$$

Thus, arranging these elements in increasing order of unpaired electrons, we get:

$$\text{Sc (A)} < \text{Ti (D)} < \text{V (C)} < \text{Mn (E)} < \text{Cr (B)}$$

💡 Quick Tip

The number of unpaired electrons in transition elements is determined by their electronic configuration, particularly the arrangement of electrons in the *d*-orbitals.

---

68. Match List-I with List-II.

List-I	**List-II**
Alkali Metal	Emission Wavelength in nm
(A) Li	(I) 589.2
(B) Na	(II) 455.5
(C) Rb	(III) 670.8
(D) Cs	(IV) 780.0

Select the correct answer from the options below:
(1) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)
(2) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
(3) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
Correct Answer: (2)

Solution:

This question asks about the characteristic emission wavelengths of alkali metals, which are known from their flame test results. The correct matches are:

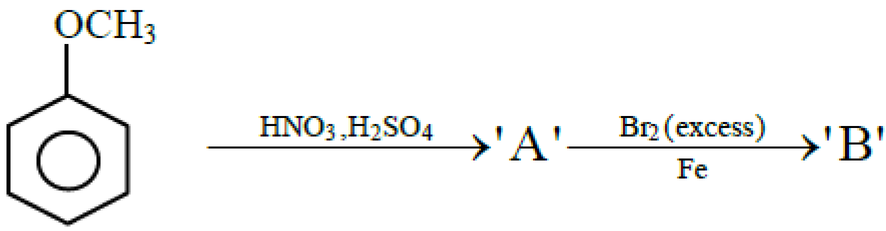
$$\begin{array}{ll} \text{(A) Li} & -\text{(III) 670.8 nm} \\ \text{(B) Na} & -\text{(I) 589.2 nm} \\ \text{(C) Rb} & -\text{(IV) 780.0 nm} \\ \text{(D) Cs} & -\text{(II) 455.5 nm} \end{array}$$

💡 Quick Tip

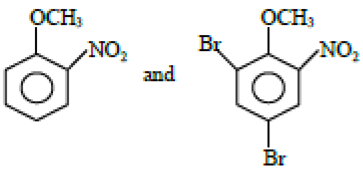
The characteristic emission wavelengths of alkali metals are used in flame tests for their identification, with each metal emitting light at a specific wavelength.

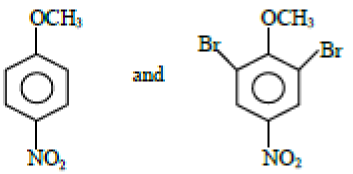
---

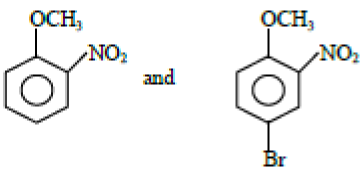
69. The major products formed:

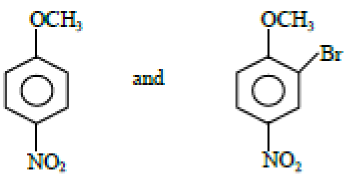


A and B respectively are:

(1) 

(2) 

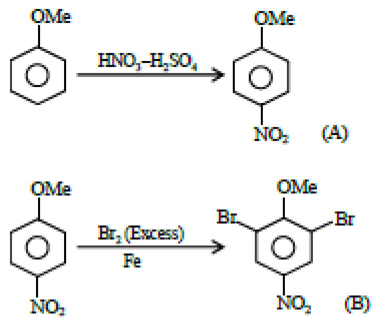
(3) 

(4) 

Correct Answer: (2)

Solution:

The reaction sequence proceeds as follows: 1. Anisole (methoxybenzene) undergoes nitration with HNO_3/H_2SO_4 to form 4-nitroanisole (A). 2. The nitrated product undergoes bromination with Br_2 (in the presence of excess iron) to yield 4-bromo-2-nitroanisole (B).



Therefore, the correct products are (A) and (B) as shown in option (2).

> Quick Tip
>
> Anisole (OMe-substituted benzene) directs electrophilic substitution reactions to the para and ortho positions, which explains the selective nitration and bromination patterns.

---

70. The incorrect statement regarding the geometrical isomers of 2-butene is:
(1) cis-2-butene and trans-2-butene are not interconvertible at room temperature.
(2) cis-2-butene has less dipole moment than trans-2-butene.
(3) trans-2-butene is more stable than cis-2-butene.
(4) cis-2-butene and trans-2-butene are stereoisomers.
Correct Answer: (2)

Solution:

The structures of cis-2-butene and trans-2-butene are:

$$\text{cis-2-butene:} \quad CH_3 - CH = CH - CH_3 \quad \text{(Polar)}$$

$$\text{trans-2-butene:} \quad H - CH = CH - CH_3 \quad \text{(Non-Polar)}$$

In terms of dipole moment, cis-but-2-ene has a higher dipole moment than trans-but-2-ene because the substituents are on the same side in the cis form, creating an overall dipole. In contrast, the trans form has a symmetrical arrangement where the dipoles cancel out.

> Quick Tip
>
> In geometric isomers, the cis form usually has a higher dipole moment than the trans form due to the unequal distribution of substituents around the double bond.

---

71. Given below are two statements:

Statement I: PF_5 and BrF_5 both exhibit sp^3d hybridization.
Statement II: SF_6 and $[Co(NH_3)_6]^{3+}$ both exhibit sp^3d^2 hybridization.

Choose the correct answer from the options below:
(1) Statement I is true but Statement II is false
(2) Both Statement I and Statement II are true
(3) Both Statement I and Statement II are false
(4) Statement I is false but Statement II is true
Correct Answer: (3)

Solution:

The hybridization of the molecules is as follows:

$$\begin{array}{ll} \mathrm{PF_5} & : \mathrm{sp^3d} \\ F_5 & : \mathrm{sp^3d^2} \\ \mathrm{SF_6} & : \mathrm{sp^3d^2} \\ [\mathrm{Co(NH_3)_6}]^{3+} & : \mathrm{d^2sp^3} \end{array}$$

Thus, both Statement I and Statement II are incorrect.

Quick Tip

The hybridization of a molecule depends on the number of electron pairs around the central atom. SF_6, with six bonding pairs, has sp^3d^2 hybridization, while PF_5 has sp^3d hybridization with five bonding pairs.

---

72. The number of ions from the following that are expected to behave as oxidising agents is:
Sn^{4+}, Sn^{2+}, Pb^{2+}, Tl^{3+}, Pb^{4+}, Tl^{+}

(1) 3
(2) 4
(3) 1
(4) 2
Correct Answer: (4)

Solution:

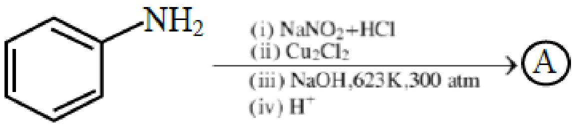
Only Tl^{3+} and Pb^{4+} are capable of acting as oxidising agents due to the inert pair effect. These ions tend to gain electrons to reach a more stable oxidation state.

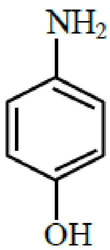
Quick Tip

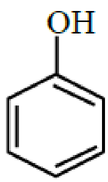
The inert pair effect leads to the instability of higher oxidation states in heavier elements, making ions like Tl^{3+} and Pb^{4+} effective oxidising agents as they readily accept electrons.

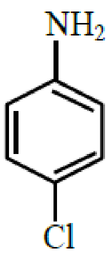
---

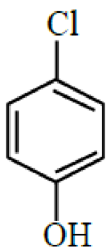
73. Identify the product A in the following reaction.



(1) 

(2) 

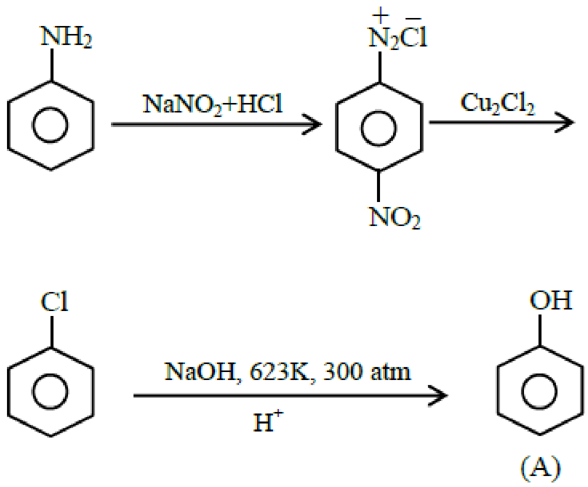
(3) 

(4) 

Correct Answer: (2)

Solution:

The reaction begins with diazotization, followed by the Sandmeyer reaction, and concludes with hydrolysis, resulting in product A.



Quick Tip

This sequence demonstrates the transformation of an aromatic amine into a variety of substituted products using diazotization and Sandmeyer's reactions, with the final step being a hydrolysis process.

---

74. The correct statements among the following, for a "chromatography" purification method is:

(1) Organic compounds run faster than solvent in the thin layer chromatographic plate.

(2) Non-polar compounds are retained at top and polar compounds come down in column chromatography.

(3) R_f of a polar compound is smaller than that of a non-polar compound.

(4) R_f is an integral value.

Correct Answer: (3)

Solution:

In chromatography, non-polar compounds travel further, hence having higher R_f values than polar compounds, which are more attracted to the stationary phase.

Quick Tip

The R_f value in chromatography indicates how far a compound travels relative to the solvent. Polar compounds have lower R_f values because they interact more strongly with the stationary phase.

---

75. Evaluate the following statements related to group 14 elements for their correctness.

(A) Covalent radius decreases down the group from C to Pb in a regular manner.
(B) Electronegativity decreases from C to Pb down the group gradually.
(C) Maximum covalence of C is 4 whereas other elements can expand their covalence due to presence of d orbitals.
(D) Heavier elements do not form π-π bonds.
(E) Carbon can exhibit negative oxidation states.

Choose the correct answer from the options given below:
(1) (C), (D) and (E) Only
(2) (A) and (B) Only
(3) (A), (B) and (C) Only
(4) (C) and (D) Only

Correct Answer: (1)

Solution:
(A) Covalent radius increases down the group, not decreases.
(B) Electronegativity does not decrease gradually across the group.
(C) Carbon has a maximum covalence of 4, while heavier elements can expand their covalence due to d-orbitals.
(D) Correct; heavier elements in group 14 do not form π-π bonds.
(E) Carbon can exhibit oxidation states ranging from -4 to +4.

Quick Tip

In Group 14, elements like carbon can form π-π bonds, but this capability diminishes down the group as atomic size increases. Carbon's unique small size also allows it to exhibit a wider range of oxidation states.

---

76. Match List-I with the List-II

List-I (Reaction)	List-II (Type of redox reaction)
(A) $N_2(g) + O_2(g) \rightarrow 2NO(g)$	(I) Decomposition
(B) $2Pb(NO_3)_{2(s)} \rightarrow 2PbO_{(s)} + 4NO_{2(g)} + O_{2(g)}$	(II) Displacement
(C) $2Na(s) + 2H_2O \rightarrow 2NaOH_{(aq)} + H_2(g)$	(III) Disproportionation
(D) $2NO_2(g) + 2^{-}OH_{(aq)} \rightarrow NO^{-}_{2(aq)} + NO^{-}_{3(aq)} + H_2O_{(l)}$	(IV) Combination

Choose the correct answer from the options given below:
(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(2) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)
(3) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)
(4) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Correct Answer: (4)

Solution:

(A) $N_2 + O_2 \to 2NO$: This is a combination reaction, as two reactants combine to form one product. Thus, $A \to (IV)$.
(B) $2Pb(NO_3)_2 \to 2PbO + 4NO_2 + O_2$: This is a decomposition reaction, where a single compound breaks into multiple products. Thus, $B \to (I)$.
(C) $2Na + 2H_2O \to 2NaOH + H_2$: This is a displacement reaction, as sodium displaces hydrogen in water. Thus, $C \to (II)$.
(D) $2NO_2 + 2OH^- \to NO_2^- + NO_3^- + H_2O$: This is a disproportionation reaction, as $\text{NO}_2 isbothoxidizedandreduced. Thus, D \to (III)$.

Final Matching:

$$A \to (IV),\ B \to (I),\ C \to (II),\ D \to (III)$$

Quick Tip

Understanding redox reactions involves recognizing whether elements combine (combination), break down (decomposition), replace other elements (displacement), or undergo simultaneous oxidation and reduction (disproportionation).

---

77. Consider the given reaction, identify the major product P.

$$CH_3-COOH \xrightarrow[\text{(iv) } H_2O/\bar{O}H, \Delta]{\text{(i) } LiAlH_4 \quad \text{(ii) PCC} \quad \text{(iii) } HCN/\bar{O}H} \text{"P"}$$

(1) $CH_3-CH_2-CH_2-OH$

(2) $CH_3-CH_2-C(=O)-NH_2$

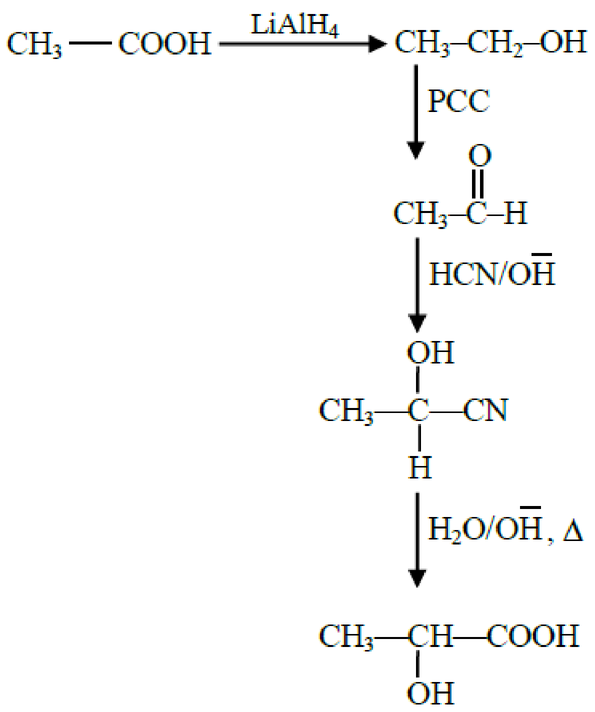
(3) $CH_3-C(=O)-CH_2CH_3$

(4) $CH_3-CH(OH)-COOH$

Correct Answer: (4)

Solution:

The reaction involves reduction with $LiAlH_4$ followed by oxidation with PCC, resulting in the formation of a primary alcohol as the major product.



Quick Tip

$LiAlH_4$ is a strong reducing agent that reduces carboxylic acids to alcohols, while PCC is a mild oxidant that oxidizes alcohols to aldehydes.

---

78. The correct IUPAC name of $[PtBr_2(PMe_3)_2]$ is:

1. bis(trimethylphosphine)dibromoplatinum(II)
2. bis[bromo(trimethylphosphine)]platinum(II)
3. dibromobis(trimethylphosphine)platinum(II)
4. dibromodi(trimethylphosphine)platinum(II)

Correct Answer: (3)

Solution:
The correct IUPAC name is "dibromobis(trimethylphosphine)platinum(II)," where "bis" denotes two ligands, and the ligands are named alphabetically.

Quick Tip

When naming coordination compounds, the prefixes like "bis" are used for multiple identical ligands, and the ligands are listed alphabetically.

---

79. Match List-I with List-II

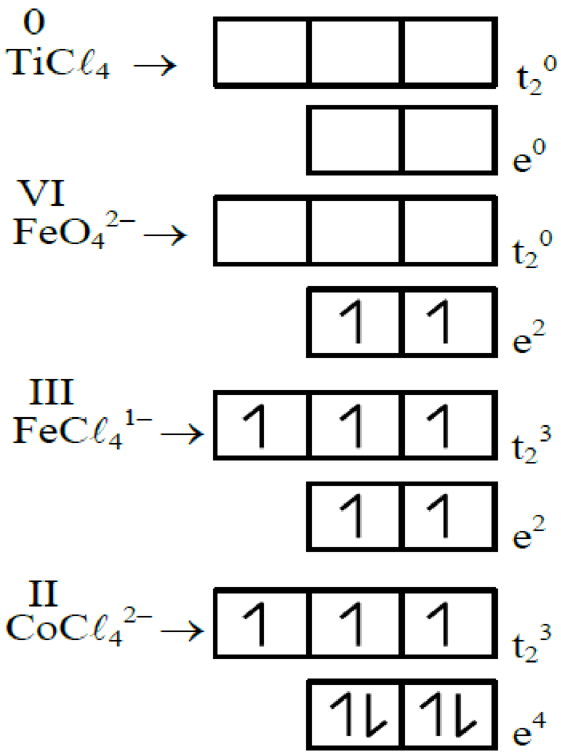
List-I	**List-II**
Tetrahedral Complex	Electronic configuration
(A) $TiCl_4$	(I) e^2, t_2^1
(B) $[FeO_4]^{2-}$	(II) e^4, t_2^2
(C) $[FeCl_4]^-$	(III) e^0, t_2^0
(D) $[CoCl_4]^{2-}$	(IV) e^2, t_2^2

Choose the correct answer from the option given below:
(1) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)
(2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(3) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(4) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Correct Answer: (4)

Solution:
The electronic configurations for the tetrahedral complexes are determined by the number

of d-electrons in the metal ion and the splitting of the d-orbitals into e and t_2 sets, resulting in the following configurations:



- $TiCl_4 \rightarrow$ (III) - $[FeO_4]^{2-} \rightarrow$ (I) - $[FeCl_4]^{-} \rightarrow$ (II) - $[CoCl_4]^{2-} \rightarrow$ (IV)

> **Quick Tip**
>
> In tetrahedral complexes, the d-orbitals split into two sets: e (higher energy) and t_2 (lower energy). The electronic configuration depends on the number of d-electrons and the ligand field.

---

80. The ratio $\frac{K_P}{K_C}$ for the reaction:

$$CO_{(g)} + \frac{1}{2}O_2(g) \rightarrow CO_2(g)$$

is:

1. $(RT)^{1/2}$
2. RT
3. 1
4. $\frac{1}{\sqrt{RT}}$

Correct Answer: (4)

Solution:
For the reaction, the change in the number of moles of gas $\Delta n_g = 1 - \left(1 + \frac{1}{2}\right) = -\frac{1}{2}$.

$$\mathrm{CO}_{(g)} + \frac{1}{2}\mathrm{O}_2(g) \rightarrow \mathrm{CO}_2(g)$$

$$\Delta n_g = 1 - \left(1 + \frac{1}{2}\right) = -\frac{1}{2}$$

Therefore, the relationship between K_P and K_C is given by:

$$\frac{K_P}{K_C} = (\mathrm{RT})^{\Delta n_g} = \frac{1}{\sqrt{\mathrm{RT}}}$$

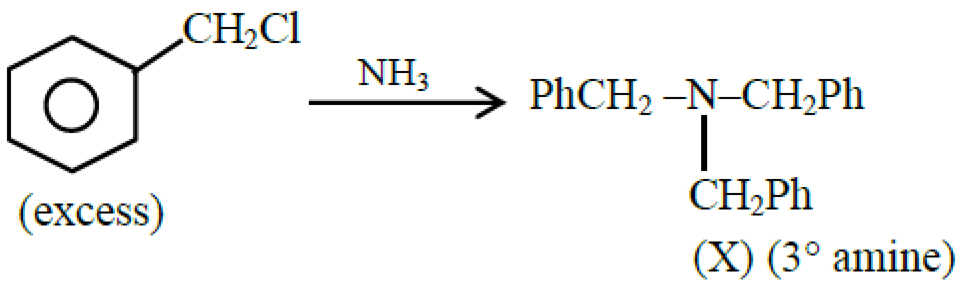
Quick Tip

The relationship between the equilibrium constants K_P and K_C for gas-phase reactions is given by $\frac{K_P}{K_C} = (\mathrm{RT})^{\Delta n_g}$, where Δn_g is the change in the number of gas moles.

---

81. An amine (X) is prepared by ammonolysis of benzyl chloride. On adding p-toluenesulphonyl chloride to it, the solution remains clear. Molar mass of the amine (X) formed is ______ g mol^{-1}.
(Given molar mass in g mol^{-1} C: 12, H: 1, O: 16, N: 14)
Correct Answer: (287)

Solution:



Molar Mass of (X) is 287 g mol^{-1}

The ammonolysis of benzyl chloride results in the formation of a tertiary amine. The reaction with p-toluenesulphonyl chloride does not result in precipitation, indicating that the amine has a relatively large molar mass. The molar mass of the formed amine is 287 g/mol.

> **Quick Tip**
>
> Ammonolysis of benzyl chloride forms a tertiary amine. The p-toluenesulphonyl chloride does not react with tertiary amines, which lack available hydrogen for sulfonation.

---

82. Consider the following reactions

$$NiS + HNO_3 + HCl \rightarrow A + NO + S + H_2O$$

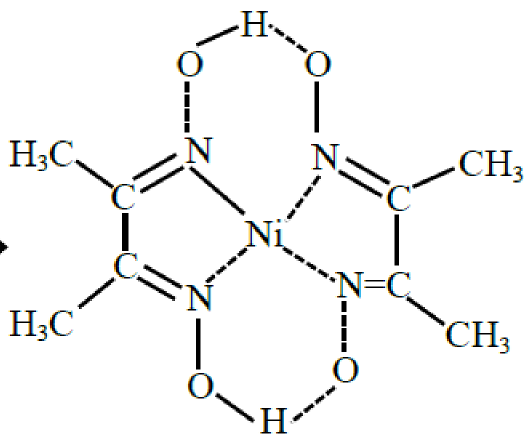
$$A + NH_4OH + \begin{array}{c} H_3C - C = N - OH \\ | \\ H_3C - C = N - OH \end{array} \longrightarrow B + NH_4Cl + H_2O$$

The number of protons that do not involve in hydrogen bonding in the product B is______.

Correct Answer: (12)

Solution:

The structure of product B reveals that 12 protons are not involved in hydrogen bonding. These protons are not adjacent to electronegative atoms that would facilitate hydrogen bonding.

B $\rightarrow$ 

$$3NiS + 2HNO_3 + 6HCl \longrightarrow 3NiCl_2 + 2NO + 3S + 4H_2O$$

$$NiCl_2 + 2NH_4OH + \begin{array}{c} H_3C - C = N - OH \\ | \\ H_3C - C = N - OH \end{array} \rightarrow NH_4Cl + H_2O + (B)$$

💡 Quick Tip

In hydrogen bonding, protons attached to electronegative atoms like oxygen or nitrogen participate in the hydrogen bond formation, while those attached to less electronegative atoms do not.

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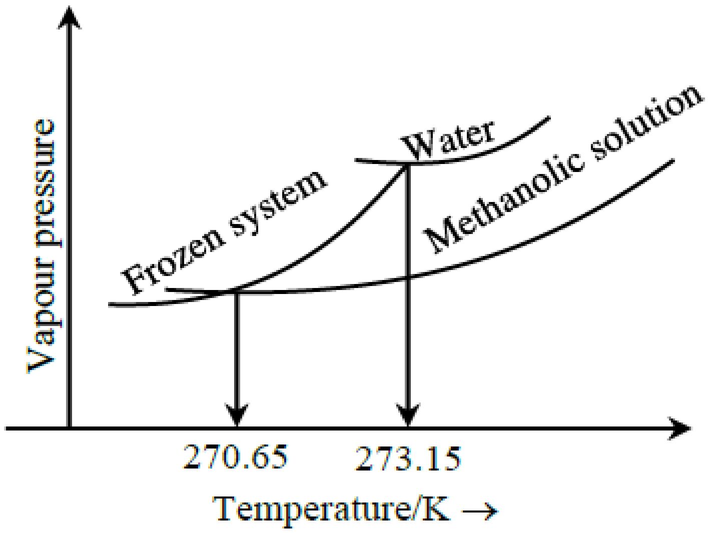
83. When $x \times 10^{-2}$ mL methanol (molar mass = 32 g; density = 0.792 g/cm³) is added to 100 mL water (density = 1 g/cm³), the following diagram is obtained.



x=__________ = (nearest integer)
[Given: Molal freezing point depression constant of water at 273.15 K is 1.86 K kg mol¹.]
Correct Answer: 543
Solution:

- The freezing point depression (ΔT_f) is calculated as:

$$\Delta T_f = 273.15 - 270.65 = 2.5\,\text{K}$$

- The relationship between freezing point depression and molality is:

$$\Delta T_f = K_f \cdot m = 2.5 = 1.86 \times \frac{n}{0.1}$$

- Solving for the moles of methanol (n):

$$n = 0.1344\,\text{moles}$$

- The mass of methanol is:

$$w = 0.1344 \times 32 = 4.3\,\text{g}$$

- The volume of methanol is:

$$\text{Volume} = \frac{4.3}{0.792} = 5.43\,\text{mL} = 543 \times 10^{-2}\,\text{mL}$$

Answer: $x = 543$

Quick Tip

To calculate freezing point depression, subtract the observed freezing point from the normal freezing point of water. Then use the depression constant to find the molality.

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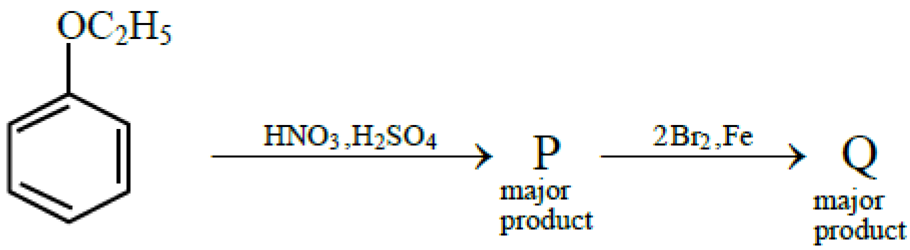
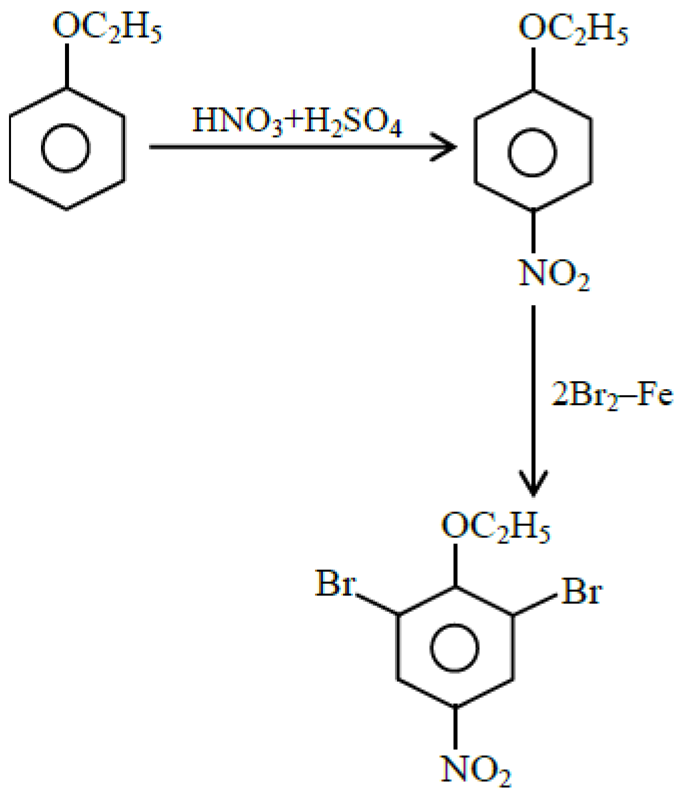
84. The compound with OC_2H_5 group undergoes the following reaction sequence:



The ratio of the number of oxygen atoms to bromine atoms in the product Q is ____.

Correct Answer: 15

Solution:



Answer: 15

Quick Tip

When performing multi-step reactions, carefully track each transformation of functional groups and atoms to determine the final ratio in the product.

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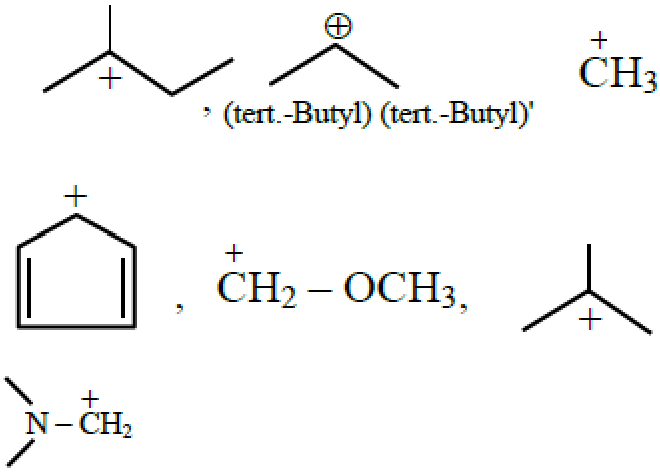
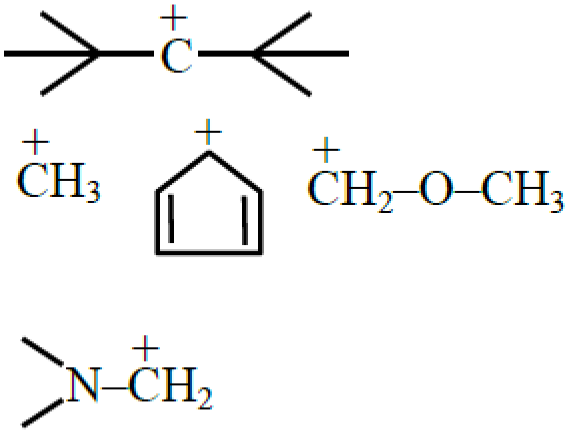
85. Number of carbocations from the following that are not stabilized by hyperconjugation is……… .



Correct Answer: (5)

Solution:
The carbocations that are not stabilized by hyperconjugation are those that do not have alkyl groups adjacent to them to allow delocalization of electrons. These include the following:



Hence, the number of carbocations that are not stabilized by hyperconjugation is 5.

> **Quick Tip**
>
> Hyperconjugation occurs when carbocations are adjacent to alkyl groups, which donate electron density to stabilize the positive charge. Without such groups, hyperconjugation is not possible.

---

86. For the reaction at 298 K, $2A + B \rightarrow C$. $\Delta H = 400\,\text{kJ mol}^{-1}$ and $\Delta S = 0.2\,\text{kJ mol}^{-1}\,\text{K}^{-1}$. The reaction will become spontaneous above ______ K.

Correct Answer: (2000)

Solution:
To determine the temperature at which the reaction becomes spontaneous, we use the Gibbs free energy equation $\Delta G = \Delta H - T\Delta S$. For spontaneity, $\Delta G = 0$, so:

$$T = \frac{\Delta H}{\Delta S} = \frac{400}{0.2} = 2000\,\mathrm{K}$$

> **Quick Tip**
>
> A reaction becomes spontaneous when the Gibbs free energy ΔG is negative. You can find the temperature at which this occurs by setting $\Delta G = 0$ and solving for T.

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87. Total number of species from the following with central atom utilising sp^2 hybrid orbitals for bonding is . . .

$$NH_3,\ SO_2,\ SiO_2,\ BeCl_2,\ C_2H_2,\ C_2H_4,\ BCl_3,\ HCHO,\ C_6H_6,\ BF_3,\ C_2H_4Cl_2$$

Correct Answer: (6)
Solution:
The species with a central atom using sp^2 hybrid orbitals are:

$$SO_2,\ C_2H_4,\ BCl_3,\ HCHO,\ C_6H_6,\ BF_3$$

> **Quick Tip**
>
> When a central atom undergoes sp^2 hybridization, it forms three regions of electron density, often involving one double bond or lone pair, resulting in a trigonal planar geometry.

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88. Consider the two different first-order reactions given below:
A + B → C (Reaction 1)
P → Q (Reaction 2)
The ratio of the half-life of Reaction 1 : Reaction 2 is 5 : 2. If t_1 and t_2 represent the time taken to complete $\frac{2}{3}$ and $\frac{4}{5}$ of Reaction 1 and Reaction 2, respectively, then the value of the ratio $t_1 : t_2$ is ___ $\times 10^{-1}$ (nearest integer).
[**Given:** $\log_{10}(3) = 0.477$ **and** $\log_{10}(5) = 0.699$]
Correct Answer: (17)
Solution:
For first-order reactions:

$$K_1 t_1 = \ln\left(\frac{1}{1-\frac{2}{3}}\right) = \ln 3$$

$$K_2 t_2 = \ln\left(\frac{1}{1-\frac{4}{5}}\right) = \ln 5$$

$$K_1 t_1 = 5 \quad \text{and} \quad K_2 t_2 = 2$$
$$\frac{K_1}{K_2} = \frac{\ln 3}{\ln 5}$$
$$\frac{t_1}{t_2} = \frac{0.477}{0.699} \cdot 5/2 = 1.7 = 17 \times 10^{-1}$$

Quick Tip

For first-order reactions, the time taken to complete a given fraction can be derived using the natural logarithm of the fraction. The half-life ratio can also help relate the rate constants of the reactions.

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89. For hydrogen atom, energy of an electron in first excited state is - 3.4 eV, K.E. of the same electron of hydrogen atom is x eV. Value of x is ____ $\times 10^{-1}$ eV. (Nearest integer)
Correct Answer: (34)
Solution:
The energy of an electron in a hydrogen atom is given by the formula:

$$E = -\frac{13.6}{n^2}\,\text{eV}$$

For the first excited state ($n = 2$):

$$E = -\frac{13.6}{2^2} = -\frac{13.6}{4} = -3.4\,\text{eV}$$

The total energy (E) of an electron is related to its kinetic energy ($K.E.$) and potential energy (U) by:

$$E = K.E. + U, \quad \text{where} \quad U = 2E$$

Thus:

$$K.E. = -E$$

Substituting $E = -3.4\,\text{eV}$:

$$K.E. = -(-3.4) = 3.4\,\text{eV}$$

The kinetic energy is expressed as $x \times 10^{-1}\,\text{eV}$, so:

$$x = 34$$

Answer: $x = 34$

Quick Tip

For hydrogen-like atoms, the kinetic energy of the electron is always equal to the absolute value of the total energy, with a positive sign.

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90. Among VO_2^+, MnO_4^- and $Cr_2O_7^{2-}$, the spin-only magnetic moment value of the species with least oxidizing ability is _____ BM (Nearest integer).
[Given atomic number V = 23, Mn = 25, Cr = 24]

Correct Answer: (0)
Solution:
The oxidizing ability follows the trend:

$$V^{+5} < Cr^{+6} < Mn^{+7}$$

For V^{5+} : $[Ar]\,4s^0\,3d^0$, there are no unpaired electrons:

$$\mu = 0\,\mathrm{BM}$$

Quick Tip

The magnetic moment is zero for species with no unpaired electrons. The oxidation state and electron configuration are key to predicting magnetic properties and oxidation power.