CBSE Class X Mathematics (Standard) Set 3 (30/5/3)

Time Allowed :3 Hours	Maximum Marks :80	Total Questions :38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question Paper is divided into **FIVE Sections – Section A, B, C, D, and E**.
3. In Section–A, questions number 1 to 18 are **Multiple Choice Questions (MCQs)** and questions number 19 & 20 are **Assertion-Reason based questions**, carrying **1 mark each**.
4. In Section–B, questions number 21 to 25 are **Very Short-Answer (VSA)** type questions, carrying **2 marks each**.
5. In Section–C, questions number 26 to 31 are **Short Answer (SA)** type questions, carrying **3 marks each**.
6. In Section–D, questions number 32 to 35 are **Long Answer (LA)** type questions, carrying **5 marks each**.
7. In Section–E, questions number 36 to 38 are **Case Study based questions** carrying **4 marks each**. *Internal choice is provided in each case-study.*
8. There is **no overall choice.** However, *an internal choice has been provided in 2 questions in Section–B, 2 questions in Section–C, 2 questions in Section–D, and 3 questions in Section–E.*
9. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is **not allowed.**

Section - A

This section comprises Multiple Choice Questions (MCQs) of 1 mark each.

Question 1: The LCM of the smallest prime number and the smallest odd composite number is:
(A) 10
(B) 6
(C) 9
(D) 18

Correct Answer: (D) 18

Solution:
The smallest prime number is 2, and the smallest odd composite number is 9. The LCM of two numbers is calculated by finding the smallest number divisible by both.

$$\text{Prime factorization of } 2 : 2.$$

$$\text{Prime factorization of } 9 : 3 \times 3.$$

The LCM is the product of the highest powers of all prime factors:

$$\text{LCM} = 2 \times 3^2 = 18.$$

—

Conclusion:
The LCM of 2 and 9 is 18.

> Quick Tip
>
> For finding the LCM of two numbers, use the highest power of each prime factor involved.

Question 2: If the mean of the first n natural numbers is $\frac{5n}{9}$, then the value of n is:
(A) 5
(B) 4
(C) 9
(D) 10

Correct Answer: (C) 9

Solution:
The formula for the mean of the first n natural numbers is:

$$\text{Mean} = \frac{\text{Sum of the first } n \text{ natural numbers}}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}.$$

Equating this to $\frac{5n}{9}$:

$$\frac{n+1}{2} = \frac{5n}{9}.$$

Cross-multiply:

$$9(n+1) = 10n.$$

Simplify:

$$9n + 9 = 10n.$$

$$n = 9.$$

—

Conclusion:
The value of n is 9.

💡 Quick Tip

For mean calculations involving natural numbers, always simplify the equation step-by-step for accuracy.

Question 3: If $5\tan\theta - 12 = 0$, then the value of $\sin\theta$ is:
(A) $\frac{5}{12}$
(B) $\frac{12}{13}$
(C) $\frac{5}{13}$
(D) $\frac{12}{5}$

Correct Answer: (B) $\frac{12}{13}$

Solution:
From the given equation:

$$5\tan\theta - 12 = 0 \implies \tan\theta = \frac{12}{5}.$$

Using the Pythagorean identity:

$$\sec^2\theta = 1 + \tan^2\theta.$$

Substitute $\tan\theta = \frac{12}{5}$:

$$\sec^2\theta = 1 + \left(\frac{12}{5}\right)^2 = 1 + \frac{144}{25} = \frac{169}{25}.$$

$$\sec\theta = \frac{13}{5}.$$

The reciprocal gives:

$$\cos\theta = \frac{5}{13}.$$

Using $\sin^2\theta + \cos^2\theta = 1$:

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{144}{169}.$$

$$\sin\theta = \sqrt{\frac{144}{169}} = \frac{12}{13}.$$

—

Conclusion:
The value of $\sin\theta$ is $\frac{12}{13}$.

💡 Quick Tip

Remember, $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$, and use the Pythagorean theorem for finding $\sin\theta$ and $\cos\theta$.

Question 4: The next (4^{th}) term of the A.P. $\sqrt{18}$, $\sqrt{50}$, $\sqrt{98}$, ... is:

(A) $\sqrt{128}$
(B) $\sqrt{140}$
(C) $\sqrt{162}$
(D) $\sqrt{200}$

Correct Answer: (C) $\sqrt{162}$

Solution:
The given sequence $\sqrt{18}, \sqrt{50}, \sqrt{98}, \ldots$ is in arithmetic progression (A.P.) since the difference between consecutive terms is constant.
Step 1: Calculate the common difference (d):

$$d = \sqrt{50} - \sqrt{18}.$$

Simplify each term:

$$\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}, \quad \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}.$$

Thus:

$$d = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}.$$

Step 2: Find the 4^{th} term:
The general term of an A.P. is given by:

$$a_n = a + (n-1)d,$$

where $a = \sqrt{18} = 3\sqrt{2}$, $n = 4$, and $d = 2\sqrt{2}$. Substituting:

$$a_4 = 3\sqrt{2} + (4-1)(2\sqrt{2}) = 3\sqrt{2} + 6\sqrt{2} = 9\sqrt{2}.$$

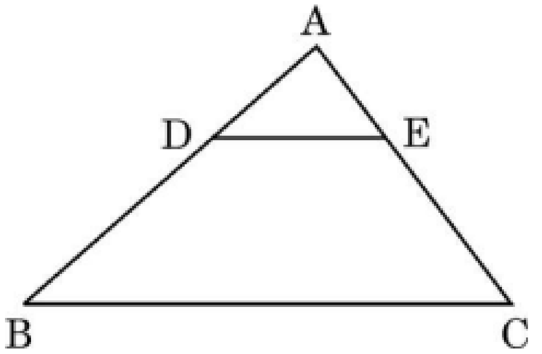
Simplify:

$$a_4 = \sqrt{81 \cdot 2} = \sqrt{162}.$$

Conclusion:
The 4^{th} term is $\sqrt{162}$.

Quick Tip
To find the next term in an A.P., calculate the common difference and apply the formula for the n-th term.

Question 5: In the given figure, in $\triangle ABC$, $DE \parallel BC$. If $AD = 2.4\,\text{cm}$, $DB = 4\,\text{cm}$, and $AE = 2\,\text{cm}$, then the length of AC is:



(A) $\frac{10}{3}$ cm
(B) $\frac{3}{10}$ cm
(C) $\frac{16}{3}$ cm
(D) 1.2 cm

Correct Answer: (C) $\frac{16}{3}$ cm

Solution:
Given $DE \parallel BC$, by the Basic Proportionality Theorem (Thales' theorem), we have:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

Step 1: Express EC in terms of AE, AD, and DB:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

Substitute $AD = 2.4\,\text{cm}, DB = 4\,\text{cm}, AE = 2\,\text{cm}$:

$$\frac{2.4}{4} = \frac{2}{EC}.$$

Step 2: Solve for EC:

$$EC = \frac{2 \times 4}{2.4} = \frac{8}{2.4} = \frac{10}{3}\,\text{cm}.$$

Step 3: Find AC:

$$AC = AE + EC = 2 + \frac{10}{3} = \frac{6}{3} + \frac{10}{3} = \frac{16}{3}\,\text{cm}.$$

—

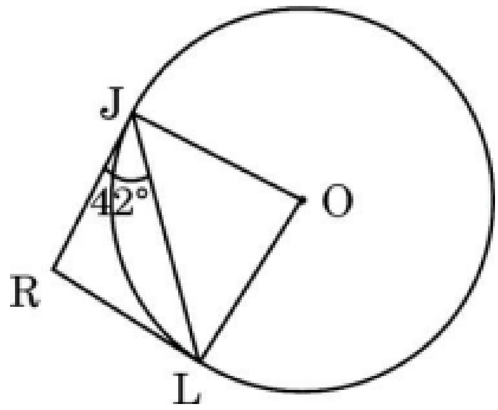
Conclusion:
The length of AC is:

$$AC = \frac{16}{3}\,\text{cm}.$$

Quick Tip

Use the Basic Proportionality Theorem ($\frac{AD}{DB} = \frac{AE}{EC}$) to solve problems involving parallel lines in triangles.

Question 6: In the given figure, RJ and RL are two tangents to the circle. If $\angle RJL = 42°$, then the measure of $\angle JOL$ is:



(A) $42°$
(B) $84°$
(C) $96°$
(D) $138°$

Correct Answer: (B) $84°$

Solution:
Given:

$$\angle RJL = 42°, \quad \text{RJ and RL are tangents.}$$

In a circle, the angle formed by the tangents at the external point ($\angle RJL$) is half the angle subtended by the chord at the center ($\angle JOL$):

$$\angle JOL = 2 \cdot \angle RJL.$$

Substitute the given value:

$$\angle JOL = 2 \cdot 42° = 84°.$$

—

Conclusion:
The measure of $\angle JOL$ is $84°$.

Quick Tip

For tangents and angles in circles, the angle subtended at the center is twice the angle between the tangents.

Question 7: The perimeter of the sector of a circle of radius 21 cm which subtends an angle of $60°$ at the center of the circle is:
(A) 22 cm
(B) 43 cm
(C) 64 cm
(D) 462 cm

Correct Answer: (C) 64 cm

Solution:
The perimeter of the sector is given by:

$$\text{Perimeter} = 2r + \text{Arc length}.$$

The arc length is:

$$\text{Arc length} = \frac{\theta}{360^\circ} \cdot 2\pi r.$$

Substitute $\theta = 60^\circ$, $r = 21$, and $\pi = \frac{22}{7}$:

$$\text{Arc length} = \frac{60}{360} \cdot 2 \cdot \frac{22}{7} \cdot 21 = \frac{1}{6} \cdot 2 \cdot \frac{22}{7} \cdot 21 = 22\,\text{cm}.$$

The perimeter is:

$$\text{Perimeter} = 2(21) + 22 = 42 + 22 = 64\,\text{cm}.$$

—

Conclusion:
The perimeter is 64 cm.

Quick Tip

For sector problems, always add the arc length and twice the radius to find the total perimeter.

Question 8: The ratio of the sum and product of the roots of the quadratic equation $5x^2 - 6x + 21 = 0$ is:
(A) 5 : 21
(B) 2 : 7
(C) 21 : 5
(D) 7 : 2

Correct Answer: (B) 2 : 7

Solution:
For a quadratic equation of the form:

$$ax^2 + bx + c = 0,$$

the sum of the roots is:

$$-\frac{b}{a}.$$

The product of the roots is:

$$\frac{c}{a}.$$

Substitute the values $a = 5$, $b = -6$, and $c = 21$:

$$\text{Sum of the roots:} \quad -\frac{-6}{5} = \frac{6}{5}.$$

$$\text{Product of the roots: } \frac{21}{5}.$$

Find the ratio of the sum to the product:

$$\text{Ratio} = \frac{\frac{6}{5}}{\frac{21}{5}} = \frac{6}{21} = \frac{2}{7}.$$

—

Conclusion:
The ratio of the sum and product of the roots is $2 : 7$.

> Quick Tip
>
> For quadratic equations, use the relationships between coefficients and roots:
> Sum of roots $= -\frac{b}{a}$, Product of roots $= \frac{c}{a}$.

Question 9: The 14th term from the end of the A.P. $-11, -8, -5, \ldots, 49$ is:
(A) 7
(B) 10
(C) 13
(D) 28

Correct Answer: (B) 10

Solution:
The nth term of an arithmetic progression (A.P.) is given by:

$$a_n = a + (n - 1)d.$$

For terms from the end, the sequence reverses, and the new first term (a') becomes 49, with a common difference of -3 (reverse of the original $d = 3$).
The 14th term from the end is:

$$a_{14} = 49 + (14 - 1)(-3).$$

Simplify:

$$a_{14} = 49 - 39 = 10.$$

Conclusion:
The 14th term from the end is 10.

> Quick Tip
>
> To find terms from the end of an A.P., reverse the sequence and recalculate the term using the modified common difference.

Question 10: The length of the shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. The angle of elevation of the Sun is:

(A) 30°
(B) 45°
(C) 60°
(D) 90°

Correct Answer: (A) 30°

Solution:
The relationship between the height of the tower (h), the length of the shadow (l), and the angle of elevation (θ) is given by:

$$\tan\theta = \frac{\text{Height of the tower}}{\text{Length of the shadow}} = \frac{h}{l}.$$

Given:

$$l = \sqrt{3}h \implies \tan\theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}.$$

The angle θ such that $\tan\theta = \frac{1}{\sqrt{3}}$ is:

$$\theta = 30^\circ.$$

Conclusion:
The angle of elevation of the Sun is 30°.

Quick Tip

For shadow problems, remember standard trigonometric values: $\tan 30^\circ = \frac{1}{\sqrt{3}}$, $\tan 45^\circ = 1$, and $\tan 60^\circ = \sqrt{3}$.

Question 11: What is the probability that a number selected randomly from the numbers $1, 2, 3, \ldots, 15$ **is a multiple of 4?**
(A) $\frac{4}{15}$
(B) $\frac{6}{15}$
(C) $\frac{3}{15}$
(D) $\frac{5}{15}$

Correct Answer: (C) $\frac{3}{15}$

Solution:
The multiples of 4 between 1 and 15 are:

$$4, 8, 12.$$

Thus, there are 3 favorable outcomes.
The total number of outcomes is:

$$15 \,(\text{numbers from 1 to 15}).$$

The probability is given by:

$$\text{Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{15}.$$

Conclusion:
The probability of selecting a multiple of 4 is $\frac{3}{15}$.

> 💡 Quick Tip
>
> To calculate probability, always divide the count of favorable outcomes by the total number of possible outcomes.

Question 12: If $\frac{x}{3} = 2\sin A$ and $\frac{y}{3} = 2\cos A$, then the value of $x^2 + y^2$ is:
(A) 36
(B) 9
(C) 6
(D) 18

Correct Answer: (A) 36

Solution:
From the given equations:

$$\frac{x}{3} = 2\sin A \implies x = 6\sin A,$$
$$\frac{y}{3} = 2\cos A \implies y = 6\cos A.$$

We need to find:

$$x^2 + y^2.$$

Substitute the values of x and y:

$$x^2 + y^2 = (6\sin A)^2 + (6\cos A)^2.$$

Simplify:

$$x^2 + y^2 = 36(\sin^2 A + \cos^2 A).$$

Using the trigonometric identity $\sin^2 A + \cos^2 A = 1$:

$$x^2 + y^2 = 36 \cdot 1 = 36.$$

Conclusion:
The value of $x^2 + y^2$ is 36.

> 💡 Quick Tip
>
> Always remember the trigonometric identity: $\sin^2 A + \cos^2 A = 1$.

Question 13: If α and β are the zeroes of the polynomial $p(x) = kx^2 - 30x + 45k$ and $\alpha + \beta = \alpha\beta$, then the value of k is:
(A) $-\frac{2}{3}$
(B) $-\frac{3}{2}$

(C) $\frac{3}{2}$
(D) $\frac{2}{3}$

Correct Answer: (D) $\frac{2}{3}$

Solution:
For a quadratic polynomial $p(x) = ax^2 + bx + c$, the sum and product of the roots are given by:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}.$$

Given:

$$p(x) = kx^2 - 30x + 45k \implies a = k,\ b = -30,\ c = 45k.$$

Substitute into the formulas:

$$\alpha + \beta = -\frac{-30}{k} = \frac{30}{k},$$

$$\alpha\beta = \frac{45k}{k} = 45.$$

It is also given that:

$$\alpha + \beta = \alpha\beta \implies \frac{30}{k} = 45.$$

Solve for k:

$$30 = 45k \implies k = \frac{30}{45} = \frac{2}{3}.$$

Conclusion:
The value of k is $\frac{2}{3}$.

Quick Tip

For polynomials, always use the sum and product of roots formulas: $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$.

Question 15: The length of an arc of a circle with radius 12 cm is 10π cm. The angle subtended by the arc at the center of the circle is:
(A) $120°$
(B) $6°$
(C) $75°$
(D) $150°$

Correct Answer: (D) $150°$

Solution:
The length of an arc is given by:

$$\text{Arc length} = \frac{\theta}{360°} \cdot 2\pi r.$$

Substitute Arc length $= 10\pi$, $r = 12$, and solve for θ:

$$10\pi = \frac{\theta}{360} \cdot 2\pi \cdot 12.$$

Simplify:

$$10 = \frac{\theta}{360} \cdot 24 \implies \frac{\theta}{360} = \frac{10}{24}.$$

$$\theta = \frac{10}{24} \cdot 360 = 150^\circ.$$

—

Conclusion:
The angle subtended by the arc is 150°.

> Quick Tip
>
> For arc length problems, substitute known values into the formula and solve for the unknown step-by-step.

Question 15: The LCM of three numbers $28, 44, 132$ is:
(A) 258
(B) 231
(C) 462
(D) 924

Correct Answer: (D) 924

Solution:
To find the LCM of $28, 44, 132$: 1. Perform prime factorization:

$$28 = 2^2 \cdot 7, \quad 44 = 2^2 \cdot 11, \quad 132 = 2^2 \cdot 3 \cdot 11.$$

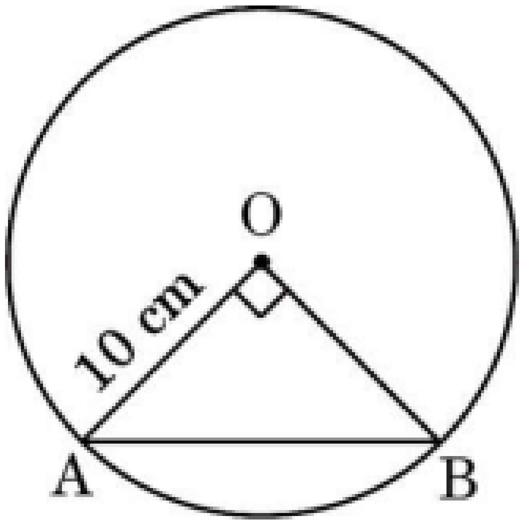
2. Take the highest powers of all prime factors:

$$\text{LCM} = 2^2 \cdot 3 \cdot 7 \cdot 11 = 924.$$

Conclusion:
The LCM of $28, 44, 132$ is 924.

> Quick Tip
>
> For LCM, take the highest powers of all prime factors common or unique to the given numbers.

Question 16: A chord of a circle of radius 10 cm subtends a right angle at its center. The length of the chord (in cm) is:



(A) $5\sqrt{2}$
(B) $10\sqrt{2}$
(C) $\frac{5}{\sqrt{2}}$
(D) 5

Correct Answer: (B) $10\sqrt{2}$

Solution:
Given: - Radius $r = 10\,\text{cm}$. - The chord subtends a 90° angle at the center.
In $\triangle OAB$, O is the center, and $\angle AOB = 90^\circ$. Using the Pythagoras theorem:

$$\text{Chord length (AB)} = \sqrt{OA^2 + OB^2}.$$

Substitute $OA = OB = r = 10$:

$$\text{Chord length (AB)} = \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}.$$

Conclusion:
The length of the chord is $10\sqrt{2}$.

> Quick Tip
>
> For chords subtending 90° at the center, use the Pythagoras theorem to calculate the chord length.

Question 17: Which out of the following types of straight lines will be represented by the system of equations $3x + 4y = 5$ and $6x + 8y = 7$?
(A) Parallel
(B) Intersecting
(C) Coincident
(D) Perpendicular to each other

Correct Answer: (A) Parallel

Solution:
The given equations are:

$$1.\ 3x + 4y = 5 \quad \text{and} \quad 2.\ 6x + 8y = 7$$

Rewrite Equation 2 to check if it is a multiple of Equation 1:

$$6x + 8y = 7 \quad \Longrightarrow \quad 2(3x + 4y) = 7$$

Since the constant terms (5 and 7) are not in the same ratio as the coefficients, the lines are not coincident. The coefficients of x and y are in the same ratio:

$$\frac{3}{6} = \frac{4}{8}.$$

This implies the lines are parallel.
Conclusion:
The lines represented by the equations are parallel.

> Quick Tip
>
> For a system of linear equations, lines are parallel if the ratios of the coefficients of x and y are equal but differ in constant terms.

Question 18: The greatest number which divides 281 and 1249, leaving remainders 5 and 7 respectively, is:
(A) 23
(B) 276
(C) 138
(D) 69

Correct Answer: (C) 138

Solution:
If a number N divides 281 leaving a remainder of 5 and 1249 leaving a remainder of 7, then:

$$281 - 5 = 276 \quad \text{and} \quad 1249 - 7 = 1242$$

must be divisible by N.
Thus, N is the greatest common divisor (GCD) of 276 and 1242.
Step 1: Find the GCD of 276 and 1242. Using the Euclidean algorithm:

$$1242 = 276 \cdot 4 + 138$$

$$276 = 138 \cdot 2 + 0$$

The GCD is 138.
—
Conclusion:
The greatest number which divides 281 and 1249, leaving the specified remainders, is 138.

Quick Tip

For such problems, subtract the remainders from the given numbers and find the GCD of the resulting values.

Questions 19 and 20 are Assertion and Reason-based questions.
Two statements are given, one labelled as Assertion (A) and the other as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C), and (D) as given below:
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Question 19:
Assertion (A): The degree of a zero polynomial is not defined.
Reason (R): The degree of a non-zero constant polynomial is 0.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of Assertion (A).

Solution:

Both Assertion (A) and Reason (R) are true. However, Reason (R) is not the correct explanation for Assertion (A). The degree of a zero polynomial is undefined because it does not have any terms, while the degree of a non-zero constant polynomial is defined as 0.

Quick Tip

Remember: The degree of a zero polynomial is undefined, while the degree of any non-zero constant polynomial is 0.

Question 20:
Assertion (A): ABCD is a trapezium with $DC \parallel AB$. E and F are points on AD and BC, respectively, such that $EF \parallel AB$. Then:

$$\frac{AE}{ED} = \frac{BF}{FC}.$$

Reason (R): Any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

Solution:

Both Assertion (A) and Reason (R) are true. The line EF, being parallel to the parallel sides of the trapezium (AB and DC), divides the non-parallel sides proportionally. The Reason (R) provides the correct explanation for Assertion (A).

> Quick Tip
>
> Use the property of proportionality in trapeziums: A line parallel to the parallel sides divides the non-parallel sides proportionally.

Section - B

This section comprises Very Short Answer (VSA) type questions of 2 marks each.

Question 21: The king, queen, and ace of clubs and diamonds are removed from a deck of 52 playing cards, and the remaining cards are shuffled. A card is randomly drawn from the remaining cards. Find the probability of getting:
(i) **A card of clubs.**
(ii) **A red-colored card.**

Solution:
Total cards left after removing the king, queen, and ace of clubs and diamonds:

$$52 - 3 - 3 = 46.$$

(i) Probability of getting a card of clubs:
The total number of club cards is originally 13. After removing 3 (king, queen, and ace), the remaining number of club cards is:

$$13 - 3 = 10.$$

The probability is:

$$P(\text{card of clubs}) = \frac{\text{Number of club cards left}}{\text{Total number of cards left}} = \frac{10}{46} = \frac{5}{23}.$$

(ii) Probability of getting a red-colored card:
The total number of red cards (hearts and diamonds) is originally 26. After removing 3 red cards (king, queen, and ace of diamonds), the remaining number of red cards is:

$$26 - 3 = 23.$$

The probability is:

$$P(\text{red-colored card}) = \frac{\text{Number of red cards left}}{\text{Total number of cards left}} = \frac{23}{46} = \frac{1}{2}.$$

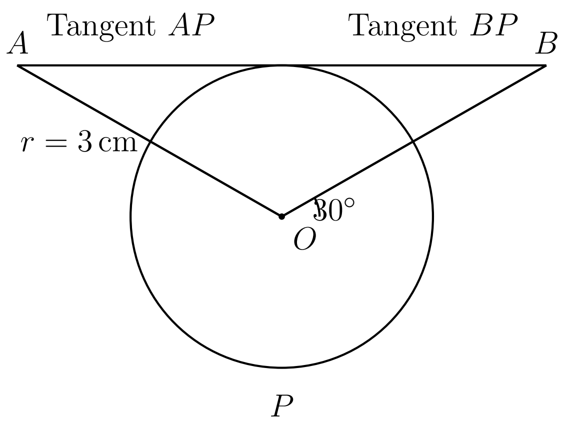
—

Conclusion:
(i) The probability of getting a card of clubs is $\frac{5}{23}$.
(ii) The probability of getting a red-colored card is $\frac{1}{2}$.

> Quick Tip
>
> Always adjust the total number of cards based on what has been removed, and use the ratio of favorable outcomes to total outcomes for probability calculations.

Question 22(a): If two tangents inclined at an angle of $60°$ are drawn to a circle of radius 3 cm, then find the length of each tangent.

Solution:
Given: - Radius of the circle: $r = 3\,\text{cm}$, - Angle between the tangents: $\angle APB = 60°$.



Step 1: Analyze the geometry.
The tangents form two right triangles with the center of the circle. In $\triangle APO$, where O is the center of the circle:

$$\angle APO = \frac{\angle APB}{2} = \frac{60°}{2} = 30°.$$

Step 2: Use trigonometric ratios.
From $\triangle APO$, using $\tan 30°$:

$$\tan 30° = \frac{\text{Opposite side (radius)}}{\text{Adjacent side (tangent length)}}.$$

Substitute the values:

$$\tan 30° = \frac{1}{\sqrt{3}}, \quad \frac{1}{\sqrt{3}} = \frac{3}{AP}.$$

Solve for AP:

$$AP = 3\sqrt{3}\,\text{cm}.$$

Step 3: Finalize the result.
Since the tangents are symmetrical, the length of each tangent is:

$$AP = 3\sqrt{3}\,\text{cm}.$$

—

Conclusion:
The length of each tangent is $3\sqrt{3}\,\text{cm}$.

> 💡 Quick Tip
>
> When two tangents are drawn to a circle, use trigonometric ratios in the formed right triangles to find tangent lengths.

Question 22(b): Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Solution:
Let AB be the diameter of a circle with center O, and let P and Q be the tangents at A and B, respectively.
Step 1: Analyze the geometry.
The tangent to a circle is perpendicular to the radius at the point of tangency. Therefore:

$$\angle OAY = 90^\circ \quad \text{and} \quad \angle OBP = 90^\circ.$$

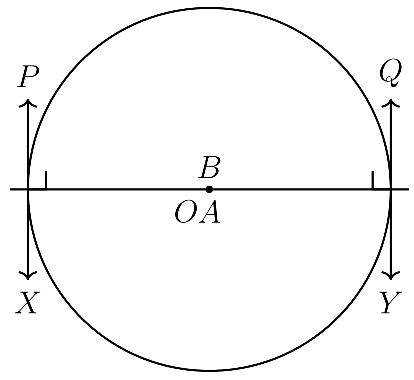
Step 2: Prove parallelism.
The angles $\angle OAY$ and $\angle OBP$ are equal and form alternate interior angles between the lines PQ and XY. Hence, by the property of alternate interior angles:

$$PQ \parallel XY.$$

—

Conclusion:
The tangents drawn at the ends of a diameter of a circle are parallel.

Diagram:



> 💡 Quick Tip
>
> The tangents at the ends of the diameter are always parallel because they form equal alternate interior angles with the line joining the ends of the diameter.

Question 23(a): Find the ratio in which the point $P(-4, 6)$ divides the line segment joining the points $A(-6, 10)$ and $B(3, -8)$.

Solution:
Let the ratio in which P divides the line AB be $k : 1$. Using the section formula, the coordinates of the dividing point $P(x, y)$ are:

$$x = \frac{kx_2 + x_1}{k+1}, \quad y = \frac{ky_2 + y_1}{k+1}.$$

Substitute the coordinates of $P(-4, 6)$, $A(-6, 10)$, and $B(3, -8)$:

$$-4 = \frac{3k-6}{k+1}, \quad 6 = \frac{-8k+10}{k+1}.$$

Solve the first equation:

$$-4(k+1) = 3k-6 \implies -4k-4 = 3k-6 \implies -7k = -2 \implies k = \frac{2}{7}.$$

Thus, the ratio is $k : 1 = 2 : 7$.

—

Conclusion:
The required ratio in which $P(-4, 6)$ divides AB is:

$$2 : 7.$$

Quick Tip

For points dividing a line segment, use the section formula:

$$x = \frac{kx_2 + x_1}{k+1}, \quad y = \frac{ky_2 + y_1}{k+1}.$$

Question 23(b): Prove that the points $(3, 0)$, $(6, 4)$, and $(-1, 3)$ are the vertices of an isosceles triangle.

Solution:
Let the points be $A(3, 0)$, $B(6, 4)$, and $C(-1, 3)$. Calculate the lengths of the sides using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

1. Length of AB:

$$AB = \sqrt{(6-3)^2 + (4-0)^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5.$$

2. Length of BC:

$$BC = \sqrt{(6-(-1))^2 + (4-3)^2} = \sqrt{(6+1)^2 + 1^2} = \sqrt{7^2 + 1^2} = \sqrt{49+1} = \sqrt{50}.$$

3. Length of CA:

$$CA = \sqrt{(3-(-1))^2 + (0-3)^2} = \sqrt{(3+1)^2 + (-3)^2} = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5.$$

Since $AB = CA$, the triangle is isosceles.
—
Conclusion:
The points $A(3, 0)$, $B(6, 4)$, and $C(-1, 3)$ form an isosceles triangle as:

$$AB = CA = 5.$$

Quick Tip

To prove a triangle is isosceles, calculate the lengths of all sides using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Check if any two sides are equal.

Question 24: If α, β are zeroes of the polynomial $p(x) = 5x^2 - 6x + 1$, then find the value of $\alpha + \beta + \alpha\beta$.

Solution:
For a polynomial $ax^2 + bx + c$, the sum and product of the roots are:

$$\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{-6}{5} = \frac{6}{5}.$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{1}{5}.$$

Add $\alpha + \beta$ and $\alpha\beta$:

$$\alpha + \beta + \alpha\beta = \frac{6}{5} + \frac{1}{5} = \frac{7}{5}.$$

—
Conclusion:
The value of $\alpha + \beta + \alpha\beta$ is:

$$\frac{7}{5}.$$

Quick Tip

For zeroes of a polynomial, use the relationships:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}.$$

Question 25: Evaluate:

$$\frac{2\tan 30^\circ \cdot \sec 60^\circ \cdot \tan 45^\circ}{1 - \sin^2 60^\circ}.$$

Solution:
Substitute the trigonometric values:

$$\tan 30^\circ = \frac{1}{\sqrt{3}}, \quad \sec 60^\circ = 2, \quad \tan 45^\circ = 1, \quad \sin^2 60^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}.$$

Simplify the numerator:

$$2 \cdot \frac{1}{\sqrt{3}} \cdot 2 \cdot 1 = \frac{4}{\sqrt{3}}.$$

Simplify the denominator:

$$1 - \sin^2 60^\circ = 1 - \frac{3}{4} = \frac{1}{4}.$$

The entire expression becomes:

$$\frac{\frac{4}{\sqrt{3}}}{\frac{1}{4}} = \frac{4}{\sqrt{3}} \cdot 4 = \frac{16}{\sqrt{3}}.$$

Rationalize the denominator:

$$\frac{16}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{16\sqrt{3}}{3}.$$

—

Conclusion:
The value of the expression is:

$$\frac{16\sqrt{3}}{3}.$$

Quick Tip

Always simplify trigonometric functions step-by-step and rationalize the denominator when necessary.

Section - C

This section comprises Short Answer (SA) type questions of 3 marks each.

Question 26: Prove that:

$$\frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta} = \sec^2\theta - \csc^2\theta.$$

Solution:
Start with the left-hand side (LHS):

$$\text{LHS} = \frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta}.$$

Substitute $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$:

$$\text{LHS} = \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}}{\sin\theta\cos\theta}.$$

Simplify the numerator:

$$\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}.$$

Now substitute back into the LHS:

$$\text{LHS} = \frac{\frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}}{\sin\theta\cos\theta}.$$

Simplify further:

$$\text{LHS} = \frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta\cos^2\theta}.$$

Split the terms:

$$\text{LHS} = \frac{1}{\cos^2\theta} - \frac{1}{\sin^2\theta}.$$

Using the identities $\sec^2\theta = \frac{1}{\cos^2\theta}$ and $\csc^2\theta = \frac{1}{\sin^2\theta}$:

$$\text{LHS} = \sec^2\theta - \csc^2\theta.$$

Thus:

$$\text{LHS} = \text{RHS}.$$

—

Conclusion:
The given identity is proved:

$$\frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta} = \sec^2\theta - \csc^2\theta.$$

Quick Tip

For proving trigonometric identities, always convert terms to their sine and cosine equivalents to simplify.

Question 27: A sector is cut from a circle of radius 21 cm. The central angle of the sector is 150°. Find the length of the arc of this sector and the area of the sector.

Solution:
The length of the arc of a sector is given by:

$$\text{Arc length} = 2\pi r \cdot \frac{\theta}{360^\circ}.$$

Substitute $r = 21$, $\theta = 150^\circ$, and $\pi = \frac{22}{7}$:

$$\text{Arc length} = 2 \cdot \frac{22}{7} \cdot 21 \cdot \frac{150}{360}.$$

Simplify:

$$\text{Arc length} = \frac{2 \cdot 22 \cdot 21 \cdot 150}{7 \cdot 360} = 55\,\text{cm}.$$

The area of the sector is given by:

$$\text{Area of sector} = \pi r^2 \cdot \frac{\theta}{360^\circ}.$$

Substitute $r = 21$, $\theta = 150^\circ$, and $\pi = \frac{22}{7}$:

$$\text{Area of sector} = \frac{22}{7} \cdot 21 \cdot 21 \cdot \frac{150}{360}.$$

Simplify:

$$\text{Area of sector} = \frac{22 \cdot 21 \cdot 21 \cdot 150}{7 \cdot 360} = 577.5\,\text{cm}^2.$$

—

Conclusion:
The length of the arc is $55\,\text{cm}$, and the area of the sector is $577.5\,\text{cm}^2$.

Quick Tip

For sector calculations, use: 1. Arc length $= 2\pi r \cdot \frac{\theta}{360^\circ}$.
2. Area of sector $= \pi r^2 \cdot \frac{\theta}{360^\circ}$.

Question 28(a): Prove that $\sqrt{3}$ is an irrational number.

Solution:
Let us assume, for the sake of contradiction, that $\sqrt{3}$ is a rational number. Then it can be expressed as:

$$\sqrt{3} = \frac{p}{q},$$

where p and q are integers, $q \neq 0$, and p and q are coprime (have no common factors other than 1).

Step 1: Square both sides.

$$3 = \frac{p^2}{q^2} \quad \implies \quad p^2 = 3q^2.$$

Step 2: Analyze divisibility of p.
Since p^2 is divisible by 3, it follows that p must also be divisible by 3 (property of prime numbers). Let:

$$p = 3a, \quad \text{where } a \text{ is an integer.}$$

Step 3: Substitute $p = 3a$ into the equation.

$$p^2 = 3q^2 \quad \implies \quad (3a)^2 = 3q^2 \quad \implies \quad 9a^2 = 3q^2 \quad \implies \quad q^2 = 3a^2.$$

Step 4: Analyze divisibility of q.
Since q^2 is divisible by 3, it follows that q must also be divisible by 3.
Step 5: Contradiction.
If both p and q are divisible by 3, then p and q are not coprime, which contradicts our assumption that $\frac{p}{q}$ is in its simplest form.

—

Conclusion:
The assumption that $\sqrt{3}$ is a rational number leads to a contradiction.
Therefore, $\sqrt{3}$ is an irrational number.

> Quick Tip
>
> To prove a number is irrational, assume it is rational and derive a contradiction using properties of divisibility.

Question 28(b): Prove that $(\sqrt{2}+\sqrt{3})^2$ is an irrational number, given that $\sqrt{6}$ is an irrational number.

Solution:
Expand $(\sqrt{2}+\sqrt{3})^2$:

$$(\sqrt{2}+\sqrt{3})^2 = 2+3+2\sqrt{6} = 5+2\sqrt{6}.$$

Step 1: Assume, for contradiction, that $5+2\sqrt{6}$ is a rational number.
Let:

$$5+2\sqrt{6} = \frac{a}{b},$$

where a, b are integers, and $b \neq 0$.
Step 2: Rearrange to isolate $\sqrt{6}$.

$$2\sqrt{6} = \frac{a}{b} - 5 \quad \implies \quad \sqrt{6} = \frac{a-5b}{2b}.$$

Step 3: Analyze rationality.
Since a and b are integers, $\frac{a-5b}{2b}$ is a rational number. However, it is given that $\sqrt{6}$ is an irrational number. This leads to a contradiction.
Step 4: Conclude irrationality.
The assumption that $5+2\sqrt{6}$ is rational is incorrect. Therefore:

$$5+2\sqrt{6} = (\sqrt{2}+\sqrt{3})^2$$

is an irrational number.
Conclusion:
$(\sqrt{2}+\sqrt{3})^2$ is an irrational number.

> Quick Tip
>
> When proving irrationality, assume rationality, isolate the square root term, and demonstrate that it contradicts the given property of irrationality.

Question 29: Three unbiased coins are tossed simultaneously. Find the probability of getting:

1. At least one head.

2. Exactly one tail.
3. Two heads and one tail.

Solution:
Step 1: Total number of outcomes.
When three coins are tossed, the possible outcomes are:

$$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}.$$

Thus, the total number of outcomes is:

$$n(S) = 8.$$

Step 2: Calculate probabilities.

1. **At least one head:**
   Outcomes with at least one head:
   $$\{HHH, HHT, HTH, HTT, THH, THT, TTH\}.$$
   Number of favorable outcomes:
   $$n(E) = 7.$$
   Probability:
   $$P(\text{At least one head}) = \frac{n(E)}{n(S)} = \frac{7}{8}.$$
2. **Exactly one tail:**
   Outcomes with exactly one tail:
   $$\{HHT, HTH, THH\}.$$
   Number of favorable outcomes:
   $$n(E) = 3.$$
   Probability:
   $$P(\text{Exactly one tail}) = \frac{n(E)}{n(S)} = \frac{3}{8}.$$
3. **Two heads and one tail:**
   Outcomes with two heads and one tail:
   $$\{HHT, HTH, THH\}.$$
   Number of favorable outcomes:
   $$n(E) = 3.$$
   Probability:
   $$P(\text{Two heads and one tail}) = \frac{n(E)}{n(S)} = \frac{3}{8}.$$

—
Conclusion:

1. Probability of at least one head: $\frac{7}{8}$.
2. Probability of exactly one tail: $\frac{3}{8}$.
3. Probability of two heads and one tail: $\frac{3}{8}$.

Quick Tip

List all possible outcomes for clarity when calculating probabilities involving multiple events.

Question 30: Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:
Let the parallelogram $ABCD$ circumscribe a circle, touching the sides AB, BC, CD, and DA at points P, Q, R, and S, respectively.
Step 1: Use the property of tangents.
The lengths of tangents drawn from an external point to a circle are equal. Therefore:

$$AP = AS, \quad BP = BQ, \quad CR = CQ, \quad DR = DS.$$

Step 2: Add the tangent pairs.
Adding all the equal tangents:

$$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ).$$

Simplify:

$$AB + CD = AD + BC.$$

Step 3: Use the properties of a parallelogram.
In a parallelogram, opposite sides are equal:

$$AB = CD \quad \text{and} \quad AD = BC.$$

Substitute these values:

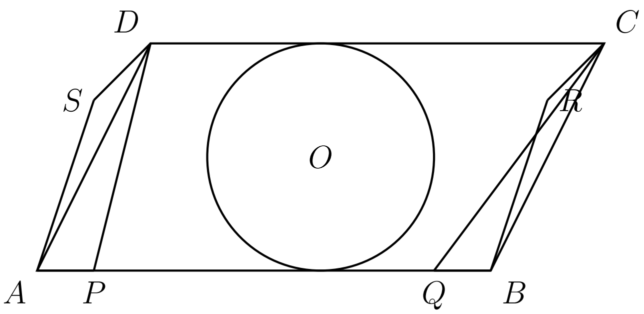
$$AB + AB = AB + AB \quad \implies \quad 2AB = 2BC.$$

Divide by 2:

$$AB = BC.$$

Step 4: Conclude that the parallelogram is a rhombus.
Since all sides of the parallelogram are equal ($AB = BC = CD = DA$), $ABCD$ is a rhombus.
—
Conclusion:
The parallelogram circumscribing a circle is a rhombus.

Diagram:



Quick Tip

The key to solving this problem is using the property that the sum of tangents from opposite sides of a circumscribed circle is equal.

Question 31(a): If the sum of the first 14 terms of an A.P. is 1050 and the first term is 10, then find the 20th term and the n-th term.

Solution:
The sum of the first n terms of an A.P. is given by:

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right].$$

Here:

$$S_{14} = 1050, \quad a = 10, \quad n = 14.$$

Substitute the values:

$$\frac{14}{2}\left[2(10) + 13d\right] = 1050.$$

Simplify:

$$7\left[20 + 13d\right] = 1050 \implies 20 + 13d = 150 \implies d = 10.$$

Find the 20th term (a_{20}):
The n-th term of an A.P. is given by:

$$a_n = a + (n-1)d.$$

For $n = 20$:

$$a_{20} = 10 + (20-1)10 = 10 + 190 = 200.$$

Find the general n-th term (a_n):
Substitute $a = 10$ and $d = 10$:

$$a_n = 10 + (n-1)10 = 10n.$$

—

Conclusion:
The 20th term is 200 and the n-th term is $10n$.

Quick Tip

Use the formula $S_n = \frac{n}{2}[2a + (n-1)d]$ to calculate the sum of n terms and $a_n = a + (n-1)d$ for specific terms.

Question 31(b): The first term of an A.P. is 5, the last term is 45, and the sum of all the terms is 400. Find the number of terms and the common difference of the A.P.

Solution:
The sum of the first n terms is given by:

$$S_n = \frac{n}{2}(a + l),$$

where $a = 5$, $l = 45$, and $S_n = 400$.
Substitute the values:

$$\frac{n}{2}(5 + 45) = 400.$$

Simplify:

$$\frac{n}{2}(50) = 400 \implies 25n = 400 \implies n = 16.$$

Find the common difference (d):
The last term of an A.P. is given by:

$$a_n = a + (n-1)d.$$

Substitute $a_n = 45$, $a = 5$, and $n = 16$:

$$45 = 5 + (16-1)d \implies 45 = 5 + 15d \implies 15d = 40 \implies d = \frac{40}{15} = \frac{8}{3}.$$

—

Conclusion:
The number of terms is 16 and the common difference is $\frac{8}{3}$.

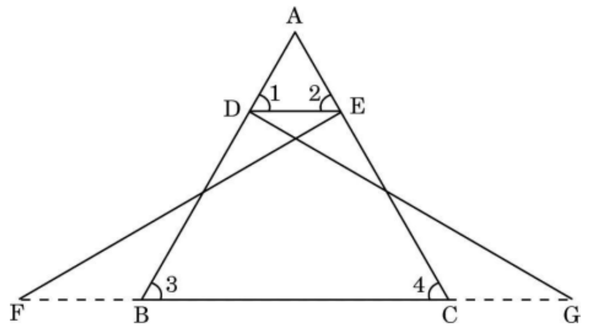
Quick Tip

For problems involving the sum of an A.P., use $S_n = \frac{n}{2}(a + l)$ when the first and last terms are given.

Section - D

This section comprises Long Answer (LA) type questions of 5 marks each.

Question 32(a): In the given figure, $\triangle FEC \cong \triangle GDB$ and $\angle 1 = \angle 2$. Prove that $\triangle ADE \sim \triangle ABC$.



Solution:
From the given information:

$$\triangle FEC \cong \triangle GDB.$$

This implies:

$$\angle 3 = \angle 4.$$

In $\triangle ABC$:

$$\angle 3 = \angle 4 \quad \text{(as given)}.$$

Thus:

$$AB = AC \quad \text{(i)}.$$

In $\triangle ADE$:

$$\angle 1 = \angle 2 \quad \text{(as given)}.$$

Thus:

$$AD = AE \quad \text{(ii)}.$$

Now, divide equation (ii) by equation (i):

$$\frac{AD}{AB} = \frac{AE}{AC}.$$

This implies:

$$DE \parallel BC \quad \text{(By Basic Proportionality Theorem)}.$$

Since:

$$\angle 1 = \angle 2 \quad \text{and} \quad \angle 3 = \angle 4,$$

it follows that:

$$\triangle ADE \sim \triangle ABC \quad \text{(by the AA similarity criterion)}.$$

—

Conclusion:

$$\triangle ADE \sim \triangle ABC.$$

Quick Tip
To prove similarity between triangles, look for proportional sides and equal angles. The AA similarity criterion is one of the simplest methods.

Question 32(b): Sides AB and AC and median AD of $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of another $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.

Solution:
To prove $\triangle ABC \sim \triangle PQR$, extend AD to E such that $AD = DE$, and join EC. Similarly, extend PM to L such that $PM = ML$, and join LR.
1. Since $\triangle ABD \cong \triangle ECD$ (by construction), we have:

$$AB = EC.$$

2. Similarly, in $\triangle PQR$, extend PM, and by symmetry, we have:

$$PQ = LR.$$

Now, by the given proportionality condition:

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}.$$

Since $AD = DE$ and $PM = ML$, the proportionality extends:

$$\frac{EC}{LR} = \frac{AC}{PR} = \frac{AD}{PM}.$$

Thus:

$$\triangle AEC \sim \triangle PLR \quad \text{(by the proportionality criterion).}$$

3. Since $\triangle AEC \sim \triangle PLR$, we have:

$$\angle BAC = \angle QPR.$$

4. Similarly, $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$, proving $\triangle ABC \sim \triangle PQR$ by the AA similarity criterion.
—
Conclusion:

$$\triangle ABC \sim \triangle PQR.$$

Quick Tip
To prove triangle similarity using medians: • Extend the median symmetrically and use congruence or similarity of smaller triangles. • Check proportionality and equal angles to apply the AA similarity criterion.

Question 33(a): Find the value of k for which the quadratic equation $(k+1)x^2 - 6(k+1)x + 3(k+9) = 0, k \neq -1$ has real and equal roots.

Solution:
For real and equal roots, the discriminant (D) of the quadratic equation must be zero:

$$D = b^2 - 4ac = 0.$$

Here, $a = (k+1)$, $b = -6(k+1)$, and $c = 3(k+9)$.
Substitute these values into the discriminant:

$$D = [-6(k+1)]^2 - 4(k+1)[3(k+9)].$$

Simplify:

$$D = 36(k+1)^2 - 12(k+1)(k+9).$$

Factorize:

$$D = 12(k+1)[3(k+1) - (k+9)].$$

Simplify further:

$$D = 12(k+1)[3k+3-k-9].$$

$$D = 12(k+1)(2k-6).$$

Set $D = 0$:

$$12(k+1)(2k-6) = 0.$$

Solve for k:

$$k+1 = 0 \implies k = -1 \quad \text{(not allowed, as } k \neq -1\text{)}.$$

$$2k-6 = 0 \implies k = 3.$$

—

Conclusion:
The value of k is $k = 3$.

Quick Tip

For real and equal roots of a quadratic equation, always use the condition $D = b^2 - 4ac = 0$, and solve systematically for unknown parameters.

Question 33(b): The age of a man is twice the square of the age of his son. Eight years hence, the age of the man will be 4 years more than three times the age of his son. Find their present ages.

Solution:
Let the present age of the son be x years. Then the present age of the man is $2x^2$ years.
Step 1: Form an equation based on the given condition.
After 8 years, the age of the man will be $2x^2+8$, and the age of the son will be $x+8$. According to the problem:

$$2x^2 + 8 = 4 + 3(x+8).$$

Simplify:

$$2x^2 + 8 = 4 + 3x + 24.$$

$$2x^2 + 8 = 3x + 28.$$

$$2x^2 - 3x - 20 = 0.$$

Step 2: Solve the quadratic equation.
Factorize:

$$2x^2 - 8x + 5x - 20 = 0.$$

$$(2x + 5)(x - 4) = 0.$$

Solve for x:

$$x = -\frac{5}{2} \quad \text{(not possible, as age cannot be negative)},$$

$$x = 4.$$

Step 3: Calculate the man's age.
Substitute $x = 4$ into $2x^2$:

$$\text{Man's age} = 2(4^2) = 2 \times 16 = 32\,\text{years}.$$

—

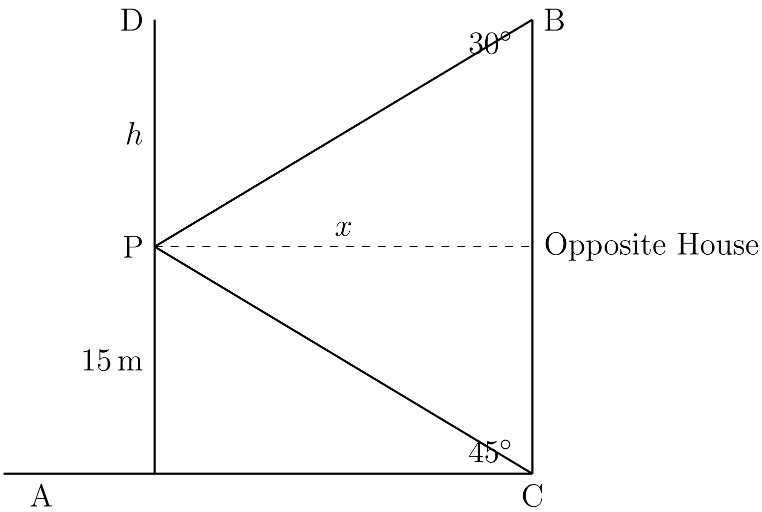
Conclusion:
The son's age is 4 years, and the man's age is 32 years.

Quick Tip

When solving age-related problems, carefully form equations based on given conditions and ensure the solutions are realistic (e.g., positive ages).

Question 34: From a window 15 metres high above the ground in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are 30° and 45°, respectively. Find the height of the opposite house. (Use $\sqrt{3} = 1.732$).

Solution:
Let the height of the opposite house be $CD = h + 15$, where 15 m is the height of the window above the ground. Let the horizontal distance between the house and the window be x.
Diagram:



Step 1: Calculate x using $\tan 45°$:
In $\triangle BPC$:

$$\tan 45° = \frac{\text{Height}}{\text{Base}} = \frac{15}{x}.$$

Since $\tan 45^\circ = 1$, we have:

$$x = 15.$$

—

Step 2: Calculate h using $\tan 30^\circ$:
In $\triangle BPD$:

$$\tan 30^\circ = \frac{\text{Height above window (h)}}{\text{Base (x)}}.$$

Substitute $\tan 30^\circ = \frac{1}{\sqrt{3}}$ and $x = 15$:

$$\frac{1}{\sqrt{3}} = \frac{h}{15}.$$

Solve for h:

$$h = \frac{15}{\sqrt{3}} = 5\sqrt{3}.$$

Using $\sqrt{3} = 1.732$:

$$h = 5 \cdot 1.732 = 8.66\,\text{m}.$$

Step 3: Total height of the opposite house:

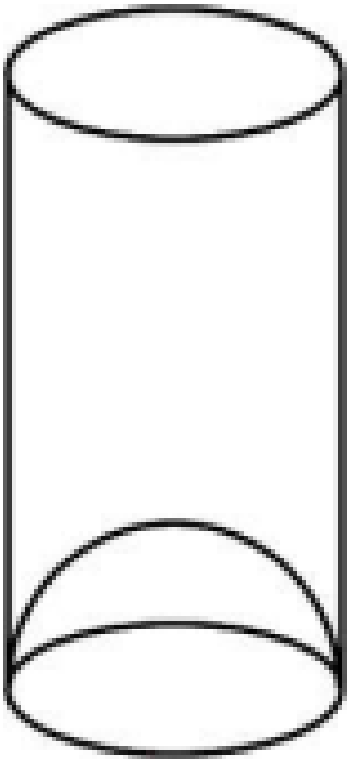
$$CD = h + 15 = 8.66 + 15 = 23.66\,\text{m}.$$

Conclusion:
The height of the opposite house is $23.66\,\text{m}$.

Quick Tip

For problems involving angles of elevation and depression, always break the problem into right-angled triangles and use trigonometric ratios step-by-step.

Question 35: A juice seller was serving his customers using glasses as shown in the figure. The inner diameter of the cylindrical glass was $5.6\,\text{cm}$, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of the glass was $10\,\text{cm}$, find the apparent capacity and the actual capacity of the glass.



Solution:
The radius of the cylindrical glass is:

$$r = \frac{\text{Diameter}}{2} = \frac{5.6}{2} = 2.8\,\text{cm}.$$

—

Step 1: Calculate the apparent capacity of the cylindrical glass:
The volume of the cylinder is given by:

$$\text{Volume of cylinder} = \pi r^2 h.$$

Substitute $\pi = \frac{22}{7}$, $r = 2.8$, and $h = 10$:

$$\text{Apparent capacity} = \frac{22}{7} \cdot 2.8 \cdot 2.8 \cdot 10.$$

Simplify:

$$\text{Apparent capacity} = 246.4\,\text{cm}^3.$$

—

Step 2: Calculate the volume of the hemispherical part:
The volume of a hemisphere is given by:

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3.$$

Substitute $\pi = \frac{22}{7}$ and $r = 2.8$:

$$\text{Volume of hemisphere} = \frac{2}{3} \cdot \frac{22}{7} \cdot 2.8 \cdot 2.8 \cdot 2.8.$$

Simplify:

$$\text{Volume of hemisphere} = 45.9\,\text{cm}^3.$$

—

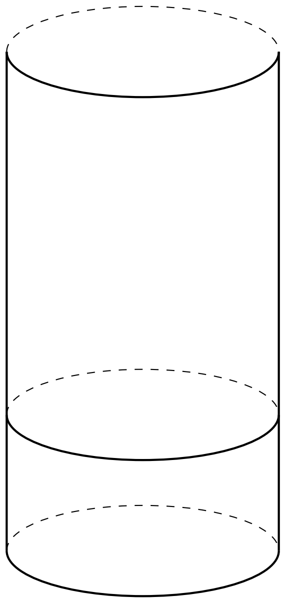
Step 3: Calculate the actual capacity of the glass:
The actual capacity is the apparent capacity minus the volume of the hemispherical part:

$$\text{Actual capacity} = 246.4 - 45.9 = 200.5\,\text{cm}^3.$$

Convert to milliliters ($1\ \text{cm}^3 = 1\,\text{ml}$):

$$\text{Actual capacity} = 200.5\,\text{ml}.$$

Conclusion:
The apparent capacity of the glass is $246.4\,\text{cm}^3$, and the actual capacity of the glass is $200.5\,\text{cm}^3$ or $200.5\,\text{ml}$.
Diagram:



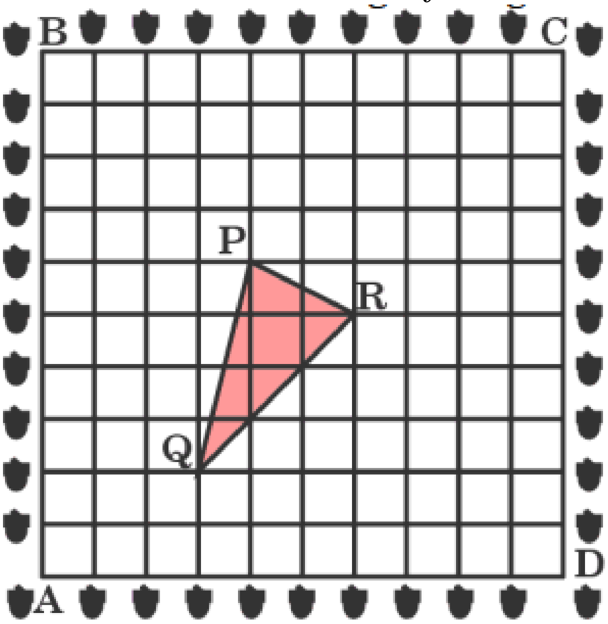
> Quick Tip
>
> To calculate volumes of combined shapes, always handle each shape separately and adjust the final result accordingly.

Section - E

This section consists of 3 Case-Study Based Questions of 4 marks each.

Question 36: Case Study – 1
A garden is in the shape of a square. The gardener grew saplings of Ashoka tree on the boundary of the garden at the distance of 1 m from each other. He wants

to decorate the garden with rose plants. He chose a triangular region inside the garden to grow rose plants. In the above situation, the gardener took help from the students of class 10. They made a chart for it which looks like the given figure.



Based on the above, answer the following questions:

1. If A is taken as origin, what are the coordinates of the vertices of ΔPQR?

2. (a) Find distances PQ and QR.
   (b) Find the coordinates of the point which divides the line segment joining points P and R in the ratio $2 : 1$ internally.

3. Find out if ΔPQR is an isosceles triangle.

Solution:
(i) Coordinates of the vertices of ΔPQR:
From the figure, the coordinates are:
$P(4, 6),\ Q(3, 2),\ R(6, 5)$.
(ii) Find distances and coordinates:
(a) Distance between P and Q:

$$PQ = \sqrt{(4-3)^2 + (6-2)^2} = \sqrt{1^2 + 4^2} = \sqrt{1+16} = \sqrt{17}.$$

Distance between Q and R:

$$QR = \sqrt{(3-6)^2 + (2-5)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}.$$

(b) The coordinates of the point dividing PR in the ratio $2 : 1$:

$$\text{Using section formula: } \left(\frac{2x_2 + 1x_1}{2+1}, \frac{2y_2 + 1y_1}{2+1}\right).$$

Substitute $P(4,6)$ and $R(6,5)$:

$$\left(\frac{2\times 6+1\times 4}{3}, \frac{2\times 5+1\times 6}{3}\right) = \left(\frac{12+4}{3}, \frac{10+6}{3}\right) = \left(\frac{16}{3}, \frac{16}{3}\right).$$

(iii) Check if ΔPQR is an isosceles triangle:
Distance PR:

$$PR = \sqrt{(4-6)^2+(6-5)^2} = \sqrt{(-2)^2+1^2} = \sqrt{4+1} = \sqrt{5}.$$

Since $PQ \neq QR \neq PR$, ΔPQR is not an isosceles triangle.

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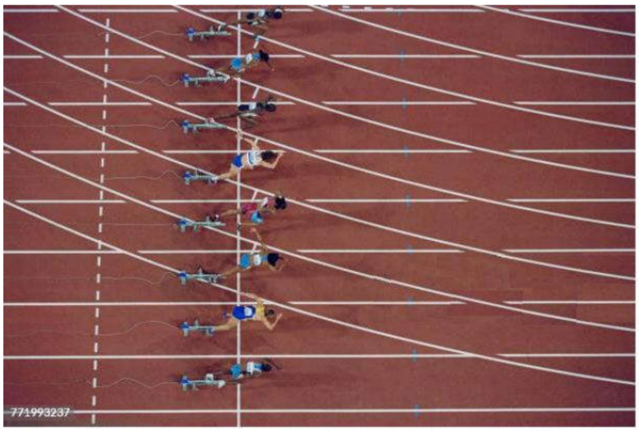
Conclusion:
(i) Coordinates of the vertices: $P(4,6), Q(3,2), R(6,5)$.
(ii) (a) $PQ = \sqrt{17}, QR = \sqrt{18}$.
(b) The coordinates of the point dividing PR are $\left(\frac{16}{3}, \frac{16}{3}\right)$.
(iii) ΔPQR is not isosceles.

> **Quick Tip**
>
> To determine the nature of a triangle, compute the lengths of all sides using the distance formula and compare them.

Question 37: Case Study – 2
Activities like running or cycling reduce stress and the risk of mental disorders like depression. Running helps build endurance. Children develop stronger bones and muscles and are less prone to gain weight. The physical education teacher of a school has decided to conduct an inter-school running tournament on his school premises. The time taken by a group of students to run 100 m was noted as follows:



Time (in seconds)	Number of students (f)
0 – 20	8
20 – 40	10
40 – 60	13
60 – 80	6
80 – 100	3

Based on the above, answer the following questions:

1. What is the median class of the above-given data?

2. (a) Find the mean time taken by the students to finish the race.
   (b) Find the mode of the above-given data.

3. How many students took time less than 60 seconds?

Solution:
(i) Median Class:
The cumulative frequency is calculated as follows:

Time (in seconds)	Number of students (f)	Cumulative Frequency (cf)
$0 - 20$	8	8
$20 - 40$	10	18
$40 - 60$	13	31
$60 - 80$	6	37
$80 - 100$	3	40

The total number of students is 40. Since $N/2 = 20$, the median class is $40 - 60$.
(ii) Mean Time:
The table for x_i (midpoints) and $f_i x_i$ is as follows:

Time (in seconds)	Midpoint(x_i)	Number of students (f)	$f_i x_i$
$0 - 20$	10	8	80
$20 - 40$	30	10	300
$40 - 60$	50	13	650
$60 - 80$	70	6	420
$80 - 100$	90	3	270

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1720}{40} = 43.$$

(ii) Mode:
The modal class is $40 - 60$. Using the formula:

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

where $L = 40,\ f_1 = 13,\ f_0 = 10,\ f_2 = 6,\ h = 20$:

$$\text{Mode} = 40 + \left(\frac{13 - 10}{2(13) - 10 - 6}\right) \times 20 = 40 + \left(\frac{3}{26 - 16}\right) \times 20 = 40 + 6 = 46.$$

(iii) Students Taking Less than 60 Seconds:
From the cumulative frequency table, the number of students taking less than 60 seconds is 31.
—

Conclusion:
(i) Median class: $40-60$.
(ii) Mean time: 43, Mode: 46.
(iii) Number of students: 31.

Quick Tip
To calculate the mean, use midpoints and summations. The modal class is identified as the class with the highest frequency.

Question 38: Case Study – 3
Essel World is one of India's largest amusement parks that offers a diverse range of thrilling rides, water attractions, and entertainment options for visitors of all ages. The park is known for its iconic "Water Kingdom" section, making it a popular destination for family outings and fun-filled adventure. The ticket charges for the park are 150 per child and 250 per adult.

On a day, the cashier of the park found that 300 tickets were sold, and an amount of 55,000 was collected.
Based on the above, answer the following questions:

1. If the number of children visited be x and the number of adults visited be y, then write the given situation algebraically.

2. (a) How many children visited the amusement park that day?
   (b) How many adults visited the amusement park that day?

3. How much amount will be collected if 250 children and 100 adults visit the amusement park?

Solution:
(i) Formulate the equations:
Let the number of children be x and the number of adults be y. The given conditions can be

written as:

$$x + y = 300 \quad \cdots (i)$$

$$150x + 250y = 55000 \quad \cdots (ii)$$

(ii) Solve for the number of children and adults:
(a) From equations (i) and (ii), solve for x:
Substitute $y = 300 - x$ into equation (ii):

$$150x + 250(300 - x) = 55000.$$

Simplify:

$$150x + 75000 - 250x = 55000.$$

$$-100x + 75000 = 55000 \quad \Longrightarrow \quad -100x = -20000 \quad \Longrightarrow \quad x = 200.$$

Therefore, the number of children is $x = 200$.

(b) Substituting $x = 200$ into equation (i):

$$y = 300 - 200 = 100.$$

Therefore, the number of adults is $y = 100$.

(iii) Calculate the amount collected if 250 children and 100 adults visit the park:

$$\text{Amount collected} = 150 \times 250 + 250 \times 100.$$

$$\text{Amount collected} = 37500 + 25000 = 62500.$$

—

Conclusion:
(i) The algebraic equations are $x + y = 300$ and $150x + 250y = 55000$.
(ii) Number of children: 200, Number of adults: 100.
(iii) Total amount collected: 62500.

Quick Tip

Use substitution or elimination methods for solving linear equations systematically. Ensure accurate substitution and simplification in word problems.
