UP Board Class 12 Physics 2026 Set 346 DY Question Paper with Solutions

ZOLLEGE

General Instructions

(i) This booklet contains 30 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 6 single-correct multiple-choice questions.

(iii) Attempt each question on your own before reviewing the given solution.

1. If the accelerating potential of a moving charged particle of λ de Broglie wavelength is increased by four times, then the de Broglie wavelength will be:

(A) $\lambda/4$

(B) $\lambda/2$

(C) 2λ

(D) 4λ

Correct Answer: (B) $\lambda/2$

SOLUTION:

Step 1: The work done by the accelerating field equals the kinetic energy: $qV = \frac{1}{2}mv^2$, so the speed is $v = \sqrt{2qV/m}$.

Step 2: Momentum is $p = mv = \sqrt{2mqV}$, and wavelength is $\lambda = h/p$. Since h, m and q are fixed, only V changes, giving $\lambda \propto V^{-1/2}$.

Step 3: Write the ratio directly. Quadrupling V multiplies $\sqrt{V}$ by $\sqrt{4}$ = 2, and because λ sits in the denominator, λ is divided by 2.

Step 4: Hence the wavelength becomes half of the original value.

$$\boxed{\lambda' = \frac{\lambda}{2}}$$

Quick Tip: Use $\lambda = h/\sqrt{2mqV}$; the wavelength varies inversely with the square root of the accelerating potential.

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2. In which of the following spherical mirrors is the magnification of the image always less than 1?

(A) Convex

(B) Plane

(C) Concave

(D) All of these

Correct Answer: (A) Convex

SOLUTION:

Step 1: Think about the image a mirror produces for a distant to near real object. The size of the image relative to the object decides the magnification.

Step 2: For a convex mirror the reflected rays diverge and appear to meet behind the mirror, producing an image that is always smaller than the object. This holds for every object distance, so $|m|$ never reaches 1.

Step 3: A plane mirror produces an image of exactly the same size, so $|m| = 1$ (not less than 1). A concave mirror behaves like a converging mirror and can enlarge the image, so $|m|$ can exceed 1.

Step 4: Since only the convex mirror keeps the image diminished in every case, it is the answer.

$$\boxed{\text{Convex mirror}}$$

Quick Tip: Which mirror always gives a virtual, erect and diminished image regardless of object position?

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3. Coulomb's repulsive force of F newton acts between two point charges of $+3\,\mu C$ and $+8\,\mu C$. If $-5\,\mu C$ charge is given to both of them, then the force and its nature between them will be:

(A) $F/4$ newton, attractive

(B) $F/4$ newton, repulsive

(C) $3F/13$ newton, repulsive

(D) $4F$ newton, attractive

Correct Answer: (A) $F/4$ newton, attractive

SOLUTION:

Step 1: Since the distance is unchanged, the force is proportional only to the product of the two charges: $F \propto q_1 q_2$.

Step 2: Original product $= (+3)(+8) = +24$ (in μC), a positive product meaning a repulsive force of value F.

Step 3: After sharing $-5\,\mu C$ to each plate, the charges become $-2\,\mu C$ and $+3\,\mu C$. Their product $= (-2)(+3) = -6$. The magnitude is 6 and the negative sign tells us the force is attractive.

Step 4: Ratio of magnitudes $= \dfrac{6}{24} = \dfrac{1}{4}$, so the new force is one quarter of F and attractive.

$$\boxed{F' = \frac{F}{4}\text{, attractive}}$$

Quick Tip: Update each charge by adding $-5\,\mu C$; force scales as the product of charges, and opposite signs mean attraction.

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4. In the given nuclear reaction, particle W is:

$${}^{4}_{2}\text{He} + {}^{A}_{Z}X \rightarrow {}^{A+3}_{Z+2}Y + W$$

(A) Electron

(B) Proton

(C) Positron

(D) Neutron

Correct Answer: (D) Neutron

SOLUTION:

Step 1: Write W as ${}^{a_W}_{z_W}W$ and demand that the subscripts (charge) and superscripts (mass number) match across the arrow.

Step 2: Superscript equation: incoming $4 + A$ must equal outgoing $(A + 3) + a_W$. Cancelling A gives $a_W = 1$.

Step 3: Subscript equation: incoming $2 + Z$ must equal outgoing $(Z + 2) + z_W$. Cancelling Z gives $z_W = 0$.

Step 4: A particle carrying one unit of mass and zero charge is the neutron ${}_0^1n$. Therefore W is a neutron.

$$\boxed{W = \text{neutron}}$$

Quick Tip: Conserve mass number and atomic number separately; the missing particle has mass number 1 and charge 0.

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5. A coil of one loop is formed by a wire of length L, and thereafter a coil of 3 loops is formed by the same wire. If the current remains the same in both cases, then the ratio of the magnetic fields at the centre will be:

(A) 1 : 3

(B) 3 : 1

(C) 1 : 9

(D) 9 : 1

Correct Answer: (C) 1 : 9

SOLUTION:

Step 1: For a coil made from a wire of fixed length, express the field only in terms of the number of turns N. If N turns share a wire of length L, each turn has circumference L/N, so the radius is $r = \dfrac{L}{2\pi N}$.

Step 2: Substitute into $B = \dfrac{\mu_0 NI}{2r}$: $B = \dfrac{\mu_0 NI}{2} \cdot \dfrac{2\pi N}{L} = \dfrac{\mu_0 \pi I N^2}{L}$. Hence $B \propto N^2$ for a fixed wire length and current.

Step 3: Compare $N_1 = 1$ and $N_2 = 3$: $\dfrac{B_1}{B_2} = \dfrac{1^2}{3^2} = \dfrac{1}{9}$.

Step 4: Therefore the fields are in the ratio $1 : 9$.

$$\boxed{B_1 : B_2 = 1 : 9}$$

Quick Tip: For a fixed wire length, $B \propto N^2$ because more turns shrink the radius; compare $N = 1$ with $N = 3$.

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6. Equivalent resistance R is obtained when n wires having the same length and same thickness of the same material are joined in parallel. The equivalent resistance on joining them in series will be:

(A) nR

(B) n^2R

(C) R/n

(D) R/n^2

Correct Answer: (B) n^2R

SOLUTION:

Step 1: Note a standard result: for n identical resistors, the series total is always n^2 times the parallel total. We verify it here.

Step 2: Let one wire have resistance r. Parallel combination of n equal resistors gives $R_p = r/n$, and this is the given R, so $r = nR$.

Step 3: Series combination of the same n wires gives $R_s = nr$. Their ratio is $\frac{R_s}{R_p} = \frac{nr}{r/n} = n^2$.

Step 4: Therefore $R_s = n^2 R_p = n^2 R$.

$$\boxed{R_{series} = n^2 R}$$

Quick Tip: Let each wire be r; parallel gives $r/n = R$ so $r = nR$, and series gives nr.

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7. Define and write the formula of mobility.

Correct Answer: —

SOLUTION:

Step 1: What the term means.

Mobility (μ) measures the responsiveness of a charge carrier to an electric field. A carrier with high mobility drifts fast even in a weak field; a carrier with low mobility drifts slowly. It is a property of the material and the type of carrier.

Step 2: Write it as a ratio.

By definition it is the drift speed produced per unit field strength:

$$\mu = \frac{\text{drift velocity}}{\text{electric field}} = \frac{v_d}{E}$$

Step 3: Connect to current density (a different framing).

The current density is $J = nev_d$, and also $J = \sigma E$ where σ is conductivity. Comparing, $\sigma = ne\,(v_d/E) = ne\mu$. Hence mobility can equally be viewed through $\sigma = ne\mu$, giving $\mu = \dfrac{\sigma}{ne}$, where n is carrier number density and e the carrier charge.

Step 4: Unit.

From $\mu = v_d/E$, the unit is $\dfrac{\mathrm{m\,s^{-1}}}{\mathrm{V\,m^{-1}}} = \mathrm{m^2\,V^{-1}\,s^{-1}}$.

$$\boxed{\mu = \frac{v_d}{E}}$$

Quick Tip: Mobility is drift velocity per unit electric field; recall $\mu = v_d/E$ $= e\tau/m$.

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8. What are the uses of microwaves?

Correct Answer: —

SOLUTION:

Step 1: Place microwaves in the spectrum.

Microwaves sit between radio waves and infrared radiation, with wavelengths of the order of a centimetre. This short wavelength is the reason for most of their applications.

Step 2: List applications with the reason for each.

• **RADAR and navigation:** being short-wavelength and sharply beamable, they reflect off distant objects, letting us locate aircraft, ships and measure vehicle speeds.

• **Cooking (microwave ovens):** the microwave frequency matches the rotational response of water molecules, so energy is absorbed and food heats from within.

• **Communication:** satellite links, mobile/cellular networks and point-to-point microwave relays use them to carry large amounts of information.

• **Analysis of molecular structure** (microwave spectroscopy) is a further scientific use.

$$\boxed{\text{Radar, cooking ovens, satellite and mobile communication}}$$

Quick Tip: Think of the very short wavelength: it makes microwaves ideal for RADAR, ovens and satellite communication.

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9. What is the difference between paramagnetic and diamagnetic substances?

Correct Answer: —

SOLUTION:

Step 1: Compare their behaviour in a field.
Put a rod of each material in a magnetic field. A **paramagnetic** rod turns to align *along* the field and moves toward the stronger-field region (weak attraction). A **diamagnetic** rod sets itself *across* (perpendicular to) the field and moves toward the weaker-field region (weak repulsion).

Step 2: Compare susceptibility and permeability.
For paramagnetics the susceptibility is small and positive ($0 < \chi \ll 1$) and relative permeability $\mu_r > 1$. For diamagnetics the susceptibility is small and negative ($-1 < \chi < 0$) and $\mu_r < 1$.

Step 3: Compare the atomic origin.
Paramagnetism arises because each atom already has a permanent magnetic dipole moment that partly aligns with the field. Diamagnetism arises in atoms with zero net moment; the applied field induces a tiny opposing moment (Lenz-law style). Paramagnetism also weakens with rising temperature, while diamagnetism is essentially temperature independent.

$$\boxed{\text{Para} = \text{permanent moments, attracted; Dia} = \text{no moment, repelled}}$$

Quick Tip: Compare the sign of susceptibility and whether the atoms carry a permanent magnetic moment.

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10. What is meant by the optical center of a lens?

Correct Answer: —

SOLUTION:

Step 1: Where it lies.

Every thin lens has one special point on its principal axis, its optical center O, which for a symmetric lens coincides with the geometric center.

Step 2: The defining behaviour.

Consider a ray aimed at O. At the point where it meets the two lens surfaces, those surfaces are effectively parallel (like the two faces of a thin glass slab). A ray crossing a parallel-sided slab emerges parallel to its original path; for a thin lens the slab thickness is negligible, so the ray comes out along the *same straight line*, that is, undeviated.

Step 3: Why it is useful.

This undeviated ray is one of the three principal rays used to locate images in geometrical optics, which makes the optical center an important reference point.

$$\boxed{\text{Central axial point giving zero deviation to a passing ray}}$$

Quick Tip: It is the point on the principal axis through which a ray passes without any deviation.

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11. What are the charge carriers in p-type of semiconductors?

Correct Answer: —

SOLUTION:

Step 1: Identify the dopant.

A p-type material comes from adding a group-13 (trivalent) acceptor impurity to silicon or germanium. Because it accepts an electron to complete its bonding, it is called an acceptor.

Step 2: Reason out the carrier.

The acceptor atom is short of one electron for the fourth covalent bond. This missing electron behaves as a positively charged vacancy, a *hole*. When a neighbouring electron jumps in to fill it, the hole effectively moves in the opposite direction, so holes carry the current.

Step 3: State majority and minority.

Hence the majority charge carriers in a p-type semiconductor are **holes**, and the thermally generated electrons form only a small minority.

$$\boxed{\text{Holes are the majority charge carriers in p-type}}$$

Quick Tip: Trivalent (acceptor) doping creates vacancies; recall which positive carrier dominates in p-type.

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12. What is an equipotential surface?

Correct Answer: —

SOLUTION:

Step 1: Picture it.

Imagine joining together all the points in a field region that happen to have exactly the same electric potential. The surface formed by these points is an equipotential surface, for instance concentric spheres around a single point charge or parallel planes for a uniform field.

Step 2: Deduce the work property.

Moving a test charge between two points on such a surface involves a potential difference of zero, so $W = q\Delta V = 0$. No work is spent moving a charge along the surface.

Step 3: Deduce the field direction.

If the field had a component along the surface it would do work as the charge moved, contradicting $W = 0$. Therefore the electric field must be entirely **normal (perpendicular)** to the equipotential surface everywhere.

$$\boxed{\text{Constant-potential surface; field is normal to it, } W = 0}$$

Quick Tip: Constant potential everywhere means zero work along it and the electric field is perpendicular to the surface.

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13. Work function of a metal surface is 3.2 eV. Find out the kinetic energy of the emitted photoelectrons in Joule, when a photon of 4.0 eV energy is incident on its surface.

Correct Answer: —

SOLUTION:

Step 1: Convert every energy into Joule first.

Since $1 \text{ eV} = 1.6 \times 10^{-19}$ J:

Photon energy $E = 4.0 \times 1.6 \times 10^{-19} = 6.4 \times 10^{-19}$ J.

Work function $W_0 = 3.2 \times 1.6 \times 10^{-19} = 5.12 \times 10^{-19}$ J.

Step 2: Apply conservation of energy at the surface.

The photon energy that is not used up in liberating the electron appears as its kinetic energy:

$$KE_{max} = E - W_0$$

Step 3: Subtract the two Joule values.

$$KE_{max} = 6.4 \times 10^{-19} - 5.12 \times 10^{-19}$$

$$KE_{max} = 1.28 \times 10^{-19} \text{ J}$$

So the fastest photoelectrons leave the surface with a kinetic energy of about 1.28×10^{-19} J (equivalently 0.8 eV).

$$\boxed{KE_{max} = 1.28 \times 10^{-19} \text{ J}}$$

Quick Tip: Use Einstein's photoelectric equation $KE_{max} = E - W_0$, then convert the answer from eV to Joule using $1 \text{ eV} = 1.6 \times 10^{-19}$ J.

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14. What will be the effect on the drift velocity of electrons, when the current in a copper wire is passed in a wire, twice of its radius (the same current being maintained)?

Correct Answer: —

SOLUTION:

Step 1: Start from the microscopic current formula.

For a wire carrying current I, the drift speed of the conduction electrons is

$$v_d = \frac{I}{neA}$$

Step 2: Identify what stays fixed and what changes.

The wire is still copper, so n and e do not change, and the problem keeps the current I the same. Only the cross-sectional area $A = \pi r^2$ changes because the radius changes.

Step 3: Take a ratio of the two situations.

Let the original radius be r and the new radius be $2r$. Writing the ratio of drift speeds:

$$\frac{v_d'}{v_d} = \frac{A}{A'} = \frac{\pi r^2}{\pi (2r)^2} = \frac{r^2}{4r^2} = \frac{1}{4}$$

Step 4: Interpret the result.

Because the same number of electrons per second (same current) now has four times the area to spread across, each electron needs to move only a quarter as fast.

$$v_d' = \frac{v_d}{4}$$

$$\boxed{v_d' = \frac{1}{4}\, v_d}$$

Quick Tip: Use $v_d = I/(neA)$ with $A = \pi r^2$; at constant current $v_d \propto 1/r^2$, so doubling the radius quarters the drift velocity.

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15. What is the phenomenon of radioactivity? How many types are there of radioactive decay?

Correct Answer: —

SOLUTION:

Step 1: Explain the phenomenon in plain terms.
Some naturally occurring heavy elements (such as uranium, thorium and radium) have nuclei that carry too much energy to remain stable. Such a nucleus, without any external cause, breaks up and throws out invisible penetrating radiation. This self-driven emission of radiation from an unstable nucleus is called *radioactivity*.

Step 2: Note who and how it was found.
Becquerel first noticed it with uranium salts, and the Curies later isolated other radioactive elements. The activity depends only on the nucleus, so heating, cooling or forming compounds cannot switch it off.

Step 3: Classify the emitted radiations, which fixes the number of decay types.
When the radiation is passed through an electric or magnetic field it

splits into three parts, showing there are three kinds of decay:
(i) α-decay, giving positively charged helium nuclei;
(ii) β-decay, giving negatively charged electrons;
(iii) γ-decay, giving uncharged electromagnetic radiation of very short wavelength.
Hence radioactive decay is of three types.

$$\boxed{\alpha,\ \beta \text{ and } \gamma \text{ decay (three types)}}$$

Quick Tip: Radioactivity is spontaneous emission of radiation from an unstable nucleus; the decay is of three kinds, alpha, beta and gamma.

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16. Explain the difference between conductors and semiconductors on the basis of energy bands in solids.

Correct Answer: —

SOLUTION:

Step 1: Set up the comparison using the forbidden gap.
Whether a solid conducts is decided by how easily electrons can reach the conduction band from the valence band, which depends on the width of the forbidden energy gap E_g that lies between these two bands.

Step 2: Describe a conductor.
In metals the conduction band and valence band are not separated at all, they overlap. There is effectively no forbidden gap ($E_g = 0$), so electrons are already free to move and even a tiny voltage produces a large current. This is why the resistance of a metal is low and rises

slightly with temperature.

Step 3: Describe a semiconductor.

In a semiconductor the two bands are separate but the gap is narrow, of the order of 1 eV. At absolute zero no electron is in the conduction band, so it is an insulator; as the temperature rises, thermal energy lifts a few electrons across the gap, leaving holes behind, and both the electrons and holes carry current. Its conductivity therefore increases as temperature increases.

Step 4: Summarise with numbers.

Conductor: bands overlap, $E_g \approx 0$, many free electrons, high conductivity. Semiconductor: small gap $E_g \approx 0.7$ to 1.1 eV, few free carriers at room temperature, moderate and temperature-sensitive conductivity.

$$\boxed{\text{Overlapping bands} \Rightarrow \text{conductor; small gap} \Rightarrow \text{semiconductor}}$$

Quick Tip: Compare the forbidden energy gap: conductors have overlapping valence and conduction bands (no gap), semiconductors have a small gap of about 1 eV.

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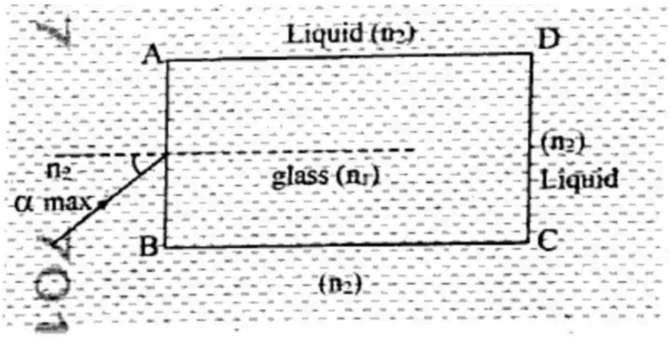
17.



A rectangular glass slab ABCD of refractive index n_1 is placed in a liquid of refractive index n_2 $(n_1 > n_2)$. A ray of light is incident at the surface AB of the slab as shown in the figure. Find out the maximum value of angle α, so that the ray comes out only from the other surface CD.

Correct Answer: —

SOLUTION:

Step 1: Fix the idea of the limiting ray.
As the incidence angle α on AB is increased, the internal ray bends more and strikes the neighbouring side face more steeply (closer to grazing). Refraction out of that side face stops the instant the internal angle of incidence there reaches the critical angle θ_c. The value of α at that instant is α_{max}.

Step 2: Relate the two internal angles by geometry.
Inside a rectangular slab the normal to AB and the normal to the adjoining side face are at 90°. If the refraction angle at AB is r, then the angle of incidence at the side face is its complement, $\left(90^\circ - r\right)$.

Step 3: Impose TIR at the side face.
Total internal reflection just sets in when this equals the critical angle:

$$90° - r = \theta_c, \qquad \sin\theta_c = \frac{n_2}{n_1}.$$

Taking sine of both sides gives $\cos r = \dfrac{n_2}{n_1}$, hence

$\tan\theta_c = \dfrac{\sin\theta_c}{\cos\theta_c}$ is not needed; instead use $\sin r = \sqrt{1 - \dfrac{n_2^2}{n_1^2}} = \dfrac{\sqrt{n_1^2 - n_2^2}}{n_1}$.

Step 4: Refract back out through AB using Snell's law.

At AB, light passes from liquid to glass, so $n_2 \sin\alpha_{max} = n_1 \sin r$.

Substituting $\sin r$:

$$n_2 \sin\alpha_{max} = n_1 \cdot \frac{\sqrt{n_1^2 - n_2^2}}{n_1} = \sqrt{n_1^2 - n_2^2}.$$

Step 5: Solve for the maximum incidence angle.

$$\sin\alpha_{max} = \frac{\sqrt{n_1^2 - n_2^2}}{n_2}$$

$$\alpha_{max} = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_2}\right).$$

(If $n_1 \geq \sqrt{2}n_2$ this fraction reaches 1, meaning every incidence angle up to grazing works.)

$$\boxed{\alpha_{max} = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_2}\right)}$$

Quick Tip: Apply Snell's law at AB and impose total internal reflection at the side face, where the internal angle is $(90° - r)$ and $\sin\theta_c = n_2/n_1$.

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18. Find out the formula for the electric field at a point outside a uniformly charged spherical shell by using Gauss's theorem.

Correct Answer: —

SOLUTION:

Step 1: Write down what Gauss's law provides.
Gauss's law links the outward electric flux of a closed surface to the charge it traps:

$$\Phi_E = \oint \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\varepsilon_0}.$$

Step 2: Exploit the symmetry to pick the surface.
A uniformly charged shell of radius R and charge q looks the same from every direction. So the field at an outside point must be radial and its size can depend only on the distance r from the centre. The natural Gaussian surface is a sphere of radius r $(r > R)$ centred on the shell.

Step 3: Simplify the surface integral.
On this sphere $\vec{E}$ is constant in magnitude and everywhere

perpendicular to the surface, so the dot product becomes a plain product and E comes out of the integral:

$$\oint \vec{E} \cdot d\vec{S} = E \oint dS = E \cdot 4\pi r^2.$$

Step 4: Insert the trapped charge.

Because $r > R$, the sphere encloses the complete charge of the shell, $q_{enc} = q$. Substituting into Gauss's law:

$$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}.$$

Step 5: Isolate the field.

$$E = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{k\,q}{r^2}, \qquad k = \frac{1}{4\pi\varepsilon_0}.$$

This is an inverse-square field, identical to that of a point charge q sitting at the centre of the shell.

$$\boxed{E = \frac{q}{4\pi\varepsilon_0 r^2} \quad (r > R)}$$

Quick Tip: Take a concentric spherical Gaussian surface of radius r > R; flux $= E(4\pi r^2) = q/\varepsilon_0$ gives $E = q/(4\pi\varepsilon_0 r^2)$.

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19. How a galvanometer is converted into an ammeter? The resistance of the coil of a galvanometer is 15 Ω and it gives full scale deflection by a current of 4 mA. How it can be converted into an ammeter in order to measure a current upto 6 A?

Correct Answer: —

SOLUTION:

Step 1 (Idea of range multiplication): An ammeter is made by wiring a low resistance shunt across a galvanometer. Its range is expressed through the multiplying factor $n = I/I_g$, which tells how many times larger the measurable current is compared with the coil's full-scale current.

Step 2 (Find the factor): With $I = 6$ A and $I_g = 4 \times 10^{-3}$ A,

$$n = \frac{I}{I_g} = \frac{6}{4 \times 10^{-3}} = 1500$$

Step 3 (Shunt in terms of the factor): Starting from $I_g G = (I - I_g)S$ and dividing throughout by I_g, the shunt becomes

$$S = \frac{G}{n-1}$$

Step 4 (Substitute):

$$S = \frac{15}{1500 - 1} = \frac{15}{1499}$$

Step 5 (Compute):

$$S = 0.01001\ \Omega \approx 0.01\ \Omega$$

Result: A parallel shunt of roughly $0.01\ \Omega$ diverts 1499 parts of the current while 1 part flows through the coil, so the meter now reads up to 6 A.

$$\boxed{S = \frac{G}{n-1} \approx 0.01\ \Omega}$$

Quick Tip: Connect a small shunt in parallel so the coil voltage equals the shunt voltage: $I_g G = (I - I_g)S$. Solve for S.

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20. The de Broglie wavelengths of a photon and a proton are λ_1 and λ_2 respectively. Both have same energy E. If mass of proton is m and speed of light is C, then prove that $\frac{\lambda_1}{\lambda_2} = C\sqrt{\frac{2m}{E}}$.

Correct Answer: —

SOLUTION:

Step 1 (Common tool): Both wavelengths come from $\lambda = h/p$. The whole proof is really about expressing each momentum p in terms of the shared energy E.

Step 2 (Momentum of the photon): Light carries momentum $p_1 = E/C$ because a photon obeys $E = p_1 C$. So its wavelength is $\lambda_1 = h/p_1 = hC/E$.

Step 3 (Momentum of the proton): A massive particle with kinetic energy E satisfies $p_2 = \sqrt{2mE}$, obtained from $E = \frac{1}{2}mv^2$ combined with $p_2 = mv$. So $\lambda_2 = h/\sqrt{2mE}$.

Step 4 (Divide by cancelling h): Instead of substituting fractions, divide the wavelengths directly:

$$\frac{\lambda_1}{\lambda_2} = \frac{p_2}{p_1} = \frac{\sqrt{2mE}}{E/C} = \frac{C\sqrt{2mE}}{E}$$

Notice the ratio of wavelengths equals the reversed ratio of momenta, which is a neat cross-check.

Step 5 (Reduce the surd): Since $\dfrac{\sqrt{2mE}}{E} = \sqrt{\dfrac{2m}{E}}$,

$$\frac{\lambda_1}{\lambda_2} = C\sqrt{\frac{2m}{E}}$$

which is exactly what had to be shown.

$$\boxed{\frac{\lambda_1}{\lambda_2} = C\sqrt{\frac{2m}{E}}}$$

Quick Tip: Use $\lambda = h/p$. For the photon $p = E/C$; for the proton $p = \sqrt{2mE}$. Then divide.

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21. What is meant by displacement current? What is the difference between displacement current and conduction current? Write down their formulae.

Correct Answer: —

SOLUTION:

Step 1 (Why the idea was needed): Consider charging a capacitor. Current flows in the connecting wires, but between the plates no charge crosses. If we apply Ampere's circuital law to two surfaces bounded by the same loop, one pierced by the wire and one passing through the gap, we get contradictory results. To remove this inconsistency Maxwell introduced a second kind of current in the gap.

Step 2 (Definition): The **displacement current** is the current-like quantity associated with a time-varying electric field between the plates. Its size equals the rate of change of electric flux times ε_0:

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

Step 3 (Conduction current): The familiar current in wires is the **conduction current**, arising from drifting free electrons. Its magnitude is the charge passing per second:

$$I_c = \frac{dq}{dt}$$

Step 4 (Contrast in a table of ideas):
• Origin: I_c from moving charges, I_d from a changing $\vec{E}$.
• Medium: I_c flows through conductors, I_d appears even in vacuum or a dielectric gap.
• Continuity: across a charging capacitor I_d in the gap exactly equals I_c in the wire, so the total current is continuous.

Step 5 (Unified law): Maxwell's corrected Ampere law reads $\oint \vec{B} \cdot d\vec{l} = \mu_0(I_c + I_d)$, showing both currents create magnetic fields on equal footing.

$$\boxed{I_d = \varepsilon_0 \frac{d\Phi_E}{dt}, \quad I_c = \frac{dq}{dt}}$$

Quick Tip: Displacement current comes from a changing electric flux, not moving charge: $I_d = \varepsilon_0 \, d\Phi_E/dt$, while $I_c = dq/dt$.

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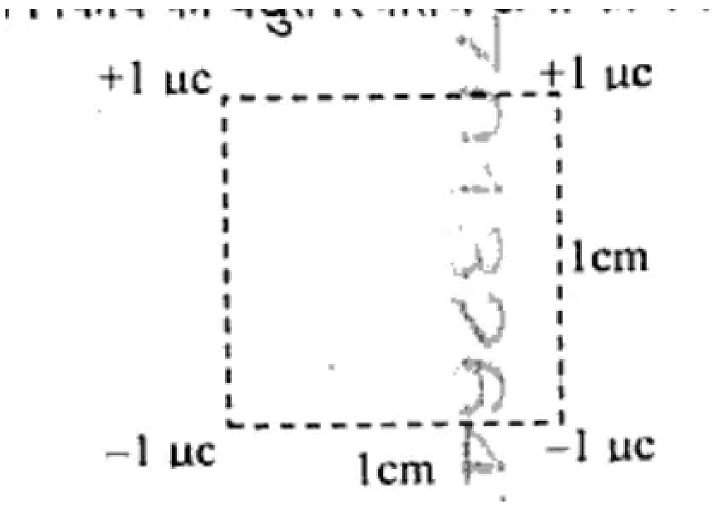
22.



Compute the electric potential energy of the system with the help of the figure given above. The square has side 1 cm with charges $+1\ \mu C$ and $+1\ \mu C$ on the two top corners and $-1\ \mu C$ and $-1\ \mu C$ on the two bottom corners.

OR

The capacitance of a parallel plate capacitor is 20 pF and distance between the plates is 4 mm. It is charged by a battery of 100 volt and then battery is removed. Now a dielectric slab (K = 4) and 2 mm thickness is placed in between the plates. Find out: (i) the final charge on each plate (ii) the potential difference between the plates.

Correct Answer: —

SOLUTION:

Option 1 (square array by grouping symmetry):

Step 1 (Bond-counting): Picture the four charges as forming bonds: 4 edge bonds of length a and 2 diagonal bonds of length $a\sqrt{2}$. Each bond stores energy kq_iq_j/r. Sum all bonds.

Step 2 (Edges cancel by pairing): The top edge $(+,+)$ is repulsive $(+kq^2/a)$ and the bottom edge $(-,-)$ is also repulsive $(+kq^2/a)$; the two vertical edges are each $(+,-)$ attractive $(-kq^2/a)$. Adding, $+kq^2/a + kq^2/a - kq^2/a - kq^2/a = 0$. The edge energy vanishes.

Step 3 (Only diagonals survive): Both diagonals connect unlike charges, so each stores $-kq^2/(a\sqrt{2})$. Two of them give

$$U = -\frac{2kq^2}{a\sqrt{2}} = -\frac{\sqrt{2}\,kq^2}{a}$$

Step 4 (Plug numbers): With $k = 9 \times 10^9$, $q^2 = 10^{-12}$, $a = 0.01$: $kq^2/a = 0.9\ \text{J}$, so $U = -1.414 \times 0.9 = -1.27\ \text{J}$.

$$\boxed{U \approx -1.27\ \text{J}}$$

Option 2 (capacitor via field/thickness reasoning):

Step 1 (Fix the charge first): Charged to $V = 100$ V on $C_0 = 20$ pF, the plates hold $Q = C_0V = 2\ \text{nC}$. Once the battery is pulled off, the isolated plates keep this charge no matter what is inserted. Part (i): $Q = 2\ \text{nC}$.

Step 2 (Split the gap into two regions): The 4 mm gap becomes a 2 mm air part in series with a 2 mm dielectric part. The equivalent capacitance of series regions gives

$$C = \frac{\varepsilon_0 A}{(d-t) + t/K}.$$

Step 3 (Get $\varepsilon_0 A$): From $C_0 = \varepsilon_0 A/d$, $\varepsilon_0 A = 20\text{ pF} \times 4\text{ mm}$. Effective thickness $= (4-2) + 2/4 = 2.5\text{ mm}$, i.e. $\frac{4}{2.5}$ smaller denominator, so $C = 20 \times \frac{4}{2.5} = 32$ pF.

Step 4 (Voltage from $Q = CV$):

$$V' = \frac{Q}{C} = \frac{2 \times 10^{-9}}{32 \times 10^{-12}} = 62.5\text{ V}$$

The voltage drops from 100 V to 62.5 V because inserting the dielectric raises the capacitance while charge is fixed.

$$\boxed{Q = 2\text{ nC}, \quad V' = 62.5\text{ V}}$$

Quick Tip: Option 1: sum kq_iq_j/r over 6 pairs; the four edges cancel, only the two unlike diagonals remain. Option 2: charge is fixed after the battery is removed; use $C = \varepsilon_0 A/(d - t + t/K)$ then $V = Q/C$.

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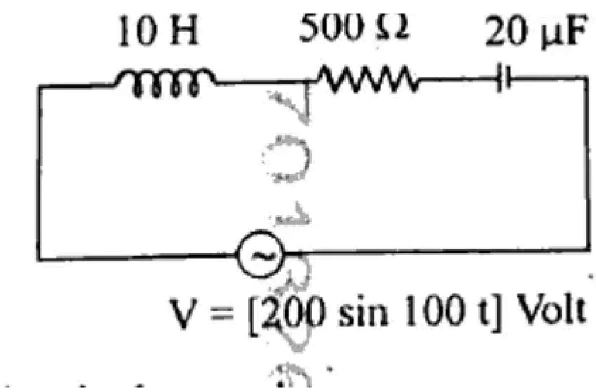
23.



Find out from the given circuit shown above: (i) Impedance (ii) power factor (iii) phase difference between current and voltage. The series circuit has $L = 10$ H, $R = 500\ \Omega$ and $C = 20\ \mu F$, driven by an AC source $V = [200 \sin 100t]$ Volt.

Correct Answer: —

SOLUTION:

Step 1 (Identify ω): The applied EMF $V = 200 \sin 100t$ tells us the drive frequency directly: $\omega = 100$ rad/s (the coefficient of t).

Step 2 (Two opposing reactances): The coil and capacitor push the phase in opposite directions. Compute each: $X_L = \omega L = 100(10) = 1000\ \Omega$ and $X_C = 1/(\omega C) = 1/(100 \cdot 20 \times 10^{-6}) = 500\ \Omega$.

Step 3 (Reactance triangle): Represent R along the horizontal and the net reactance $(X_L - X_C) = 500\ \Omega$ along the vertical. Because $R = 500\ \Omega$ and the net reactance is also $500\ \Omega$, this is an isosceles right triangle.

Step 4 (Impedance = hypotenuse): For an isosceles right triangle the hypotenuse is $\sqrt{2}$ times a leg:

$$Z = R\sqrt{2} = 500\sqrt{2} \approx 707\ \Omega$$

Step 5 (Angle of the triangle): Equal legs mean a 45° angle, so the phase difference is $\phi = 45°$, with voltage ahead of current (inductive side larger).

Step 6 (Power factor = adjacent/hypotenuse):

$$\cos\phi = \cos 45^\circ = \frac{1}{\sqrt{2}} \approx 0.707$$

All three answers follow from one right triangle.

$$\boxed{Z \approx 707\ \Omega, \quad \cos\phi = 0.707, \quad \phi = 45^\circ}$$

Quick Tip: Read $\omega = 100$ from $200\sin 100t$. Find $X_L = \omega L$, $X_C = 1/\omega C$, then $Z = \sqrt{R^2 + (X_L - X_C)^2}$, $\cos\phi = R/Z$, $\tan\phi = (X_L - X_C)/R$.

• • •

24. What is the law of Malus? A plane polarized light of intensity 16 watt/m^2, is incident on a polaroid in such a way that the vibration of its electric vector makes an angle of 30° from the transmission axis of the polaroid. Find out the intensity of the transmitted light from the polaroid. Write down any one use of polaroid.

Correct Answer: —

SOLUTION:

Step 1 (State the rule): Malus's law describes how a polaroid dims polarized light. If the electric vector of the incoming polarized beam makes angle θ with the polaroid axis, only the component along the axis is passed. Since intensity goes as amplitude squared, and the passed amplitude is $E_0 \cos \theta$, the transmitted intensity is

$$I = I_0 \cos^2 \theta.$$

Step 2 (List knowns): Incident intensity $I_0 = 16 \text{ W/m}^2$; angle between vibration and axis $\theta = 30°$.

Step 3 (Insert the angle): Here $\cos 30° = 0.866$, hence $\cos^2 30°$ $= (0.866)^2 = 0.75 = \frac{3}{4}$.

Step 4 (Final intensity):

$$I = 16 \times 0.75 = 12 \text{ W/m}^2$$

So one quarter of the intensity is blocked and three quarters passes through.

$$\boxed{I = 12 \text{ W/m}^2}$$

Step 5 (A use): Polaroids are used as analysers to detect and measure the polarization of light; everyday examples include anti-glare sunglasses and camera polarizing filters.

Quick Tip: Malus's law: $I = I_0 \cos^2\theta$. Put $I_0 = 16$ and $\theta = 30°$ with $\cos^2 30°$ = 3/4.

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25. Explain Ampere's circuital law. Obtain the expression for the magnetic field inside a current carrying solenoid with its help.

Correct Answer: —

SOLUTION:

Step 1: Meaning of the law.

Ampere's circuital law connects a magnetic field with the current that produces it. For any imagined closed path, adding up $\vec{B} \cdot d\vec{l}$ all the way round gives μ_0 multiplied by whatever net current pierces the path: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$. It plays the same role for magnetism that Gauss's law plays for electricity and is most useful when the geometry is symmetric.

Step 2: Symmetry of a solenoid.

Picture a very long tightly wound coil with n turns in every metre carrying steady current I. By symmetry the interior field is straight, axial and of one constant magnitude B; just outside a long solenoid the returning field is spread over a huge region and is taken as zero.

Step 3: Build a rectangular path.

Draw a rectangle whose long arm (length ℓ) runs through the interior parallel to the axis and whose opposite long arm lies well outside. The two short arms cross from inside to outside.

Step 4: Split the circulation.

Interior long arm gives $B\ell$. On the short arms $\vec{B}$ is at right angles to the path (and zero once outside), giving nothing. The outer long arm sits where $B = 0$, giving nothing. Hence the whole circulation is simply $B\ell$.

Step 5: Count the linked current.

The rectangle of length ℓ encloses $n\ell$ turns, so the current linked is $I_{enc} = n\ell I$.

Step 6: Equate and cancel.

$B\ell = \mu_0 n\ell I$, and ℓ cancels to leave $B = \mu_0 nI$. The field is independent of the loop length and of position inside, confirming uniformity.

$$\boxed{B = \mu_0 nI}$$

Quick Tip: Use $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ with a rectangular loop having one long side inside the solenoid; only that side contributes, and the enclosed current is nLI.

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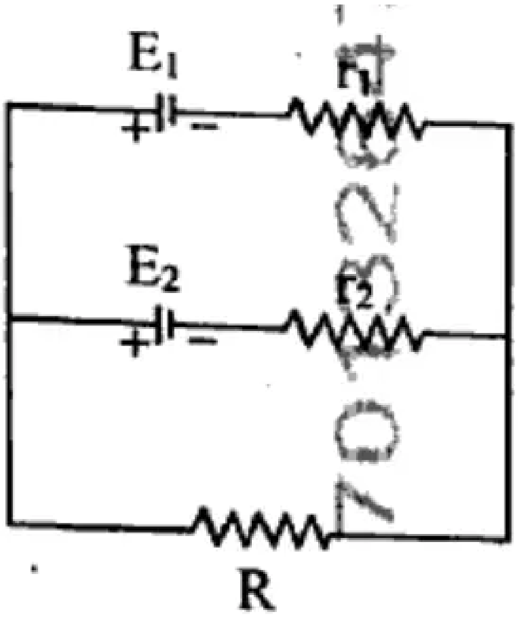
26.



Obtain the formula for the current flowing in the resistor R, with the help of the circuit given above. (Two cells of e.m.f. E_1 and E_2 with internal resistances r_1 and r_2 are connected in parallel and feed a common external resistor R.)

Correct Answer: —

SOLUTION:

Step 1: Replace the two cells by one equivalent cell.

Two sources in parallel behave like a single cell of effective e.m.f. E_{eq} and effective internal resistance r_{eq}. This is the fastest route to the current in R.

Step 2: Equivalent internal resistance.

The internal resistances r_1 and r_2 are in parallel:

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

Step 3: Equivalent e.m.f.

For parallel cells the equivalent e.m.f. is the conductance-weighted average:

$$E_{eq} = \frac{\dfrac{E_1}{r_1} + \dfrac{E_2}{r_2}}{\dfrac{1}{r_1} + \dfrac{1}{r_2}} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

Step 4: Apply Ohm's law to the single-loop circuit.

The equivalent cell drives current through r_{eq} in series with R:

$$I = \frac{E_{eq}}{R + r_{eq}}$$

Step 5: Substitute.

$$I = \frac{\dfrac{E_1 r_2 + E_2 r_1}{r_1 + r_2}}{R + \dfrac{r_1 r_2}{r_1 + r_2}}$$

Step 6: Clear the fractions.

Multiply top and bottom by $(r_1 + r_2)$:

$$I = \frac{E_1 r_2 + E_2 r_1}{R(r_1 + r_2) + r_1 r_2}$$

which matches the direct Kirchhoff result.

$$\boxed{I = \frac{E_1 r_2 + E_2 r_1}{r_1 r_2 + R(r_1 + r_2)}}$$

Quick Tip: Take a common terminal voltage V across R, write each cell current as $(E - V)/r$, add them for the R current, then put $V = IR$; or reduce the two parallel cells to one equivalent cell E_{eq}, r_{eq}.

• • •

27. Draw a circuit diagram of a p-n junction diode in reverse-biased condition and show a graph between the variation of current and voltage related with it. What is meant by avalanche breakdown?
OR
What is the difference between p-type and n-type semiconductors? A silicon diode and a load resistance of 500 Ω are joined in series with an alternating voltage source of 20 V. The forward resistance of the diode is 10 Ω and the barrier voltage is 0.7 V. Find out: (i) the peak current through the diode (ii) the peak voltage across the load.

Correct Answer: —

SOLUTION:

Option 1

Step 1: How to bias in reverse.
A junction is reverse biased when the external battery aids, rather than opposes, the built-in junction field. So join battery + to the n-material and battery − to the p-material, and put a sensitive current meter in

the loop to read the small current.

Step 2: Depletion region response.
Because the external voltage pulls the mobile carriers back from the boundary, the carrier-free depletion zone grows thicker and the barrier height rises. Diffusion of majority carriers is now blocked, and the circuit carries only the thermally generated minority-carrier current, which is nearly independent of the applied voltage.

Step 3: Shape of the characteristic.
On an I-V plot the reverse branch lies in the third quadrant hugging the voltage axis (current a few μA). It remains flat as the reverse voltage is raised, until the breakdown voltage where it plunges steeply downward, showing a large reverse current for a nearly fixed voltage.

Step 4: Meaning of avalanche breakdown.
At large reverse voltage across a lightly doped junction the wide depletion region has a strong electric field. A minority carrier crossing it is accelerated so hard that on hitting a lattice atom it ionises the atom, releasing an electron-hole pair. Those new carriers are accelerated too and ionise further atoms, causing a chain (avalanche) multiplication and a sudden large reverse current. That is avalanche breakdown.

$$\boxed{\text{Chain impact-ionisation at } V_{br} \Rightarrow \text{steep rise of reverse current}}$$

Option 2

Step 1: Contrasting the two doped materials.

Doping silicon with a Group-13 (trivalent) atom leaves a shortage of one bonding electron, i.e. a hole; such p-type silicon conducts mainly through holes. Doping with a Group-15 (pentavalent) atom leaves one spare electron; such n-type silicon conducts mainly through electrons. Majority carriers: holes in p-type, electrons in n-type. Minority carriers are the opposite kind, and each sample stays neutral overall.

Step 2: Series loop reasoning.
When the diode conducts, going round the loop the source peak V_0 must supply the fixed barrier drop V_b plus the ohmic drops across the diode's own resistance R_f and the load R_L (they carry the same current). Hence $V_0 = V_b + I_0(R_f + R_L)$.

Step 3: Peak current.
Rearrange: $I_0 = \dfrac{V_0 - V_b}{R_f + R_L} = \dfrac{20 - 0.7}{10 + 500} = \dfrac{19.3}{510}$.
Dividing, $I_0 = 0.03784\text{ A} = 37.8\text{ mA}$.

Step 4: Load peak voltage.
Only the load resistance matters here: $V_{L,0} = I_0 R_L = 0.03784 \times 500 = 18.92\text{ V} \approx 18.9\text{ V}$.

$$\boxed{I_0 \approx 37.8\text{ mA},\quad V_{L,0} \approx 18.9\text{ V}}$$

Quick Tip: Reverse bias: p to battery negative, n to positive; reverse current is tiny until breakdown, then rises sharply by avalanche (impact-ionisation chain). For the diode: peak current $= (V_0 - V_b)/(R_f + R_L)$, then load peak voltage $= I_0 R_L$.

• • •

28. Derive the formula for the focal length of a thin lens.

OR

Explain the difference between diffraction and interference of light waves. Write down the formula for the fringe width in Young's double slit experiment. What will be the effect on the fringe width, when the separation between the slits is made one third and the distance between the slits and the screen is doubled? What will be the effect on the fringe width if the whole experiment is placed in water?

Correct Answer: —

SOLUTION:

Option 1: Focal length of a thin lens

Step 1: Idea of the derivation.

A lens is just two refracting spherical surfaces placed back to back. If we track the image formed by the first surface and feed it as the object to the second surface, the two single-surface refraction relations add up to the lens formula. The single-surface relation used is $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$.

Step 2: First surface (air to glass, radius R_1).

With $n_1 = 1,\ n_2 = n$, the intermediate image sits at v_1:

$$\frac{n}{v_1} - \frac{1}{u} = \frac{n-1}{R_1}$$

Step 3: Second surface (glass to air, radius R_2).

Now $n_1 = n,\ n_2 = 1$, and because the lens is thin the object distance is still v_1:

$$\frac{1}{v} - \frac{n}{v_1} = \frac{1-n}{R_2}$$

Step 4: Superpose.

Adding the two equations removes v_1 and gives $\frac{1}{v} - \frac{1}{u} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

Step 5: Identify $1/f$.

Since $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ by the thin-lens definition, we read off

$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

which fixes the focal length from the material and the two curvatures.

Option 2: Diffraction, interference and fringe width

Step 1: Contrast in one line each.

Interference needs two independent coherent beams that overlap; diffraction arises from one wavefront bending round an edge or through a slit. Interference bands are uniform in width and brightness with truly dark gaps; diffraction bands shrink and dim away from a dominant central band and its minima are only partly dark.

Step 2: Standard fringe width.

For Young's experiment $\beta = \frac{\lambda D}{d}$, linear in λ and D and inverse in slit spacing d.

Step 3: Scale the two lengths.

Replace $d \to d/3$ (denominator shrinks, ×3) and $D \to 2D$ (numerator grows, ×2). Net factor $2 \times 3 = 6$, so $\beta \to 6\beta$: fringes get six times wider.

Step 4: Effect of water.

Inside water of index n_w light slows and its wavelength shrinks to λ/n_w, so every fringe narrows by the same factor: $\beta \to \beta/n_w$. Taking $n_w \approx 1.33$ gives roughly 0.75β, the pattern tightening up.

$$\boxed{\beta_{new} = 6\beta, \qquad \beta_{water} = \beta/n_w \approx 0.75\beta}$$

Quick Tip: For the lens: apply the single spherical-surface refraction formula twice (air-glass then glass-air) and add to get $1/f = (n-1)(1/R_1 - 1/R_2)$. For YDSE: $\beta = \lambda D/d$; with $d \to d/3$, $D \to 2D$ the width becomes 6β, and in water it becomes β/n_w.

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29. Enunciate the postulates of Bohr\'s model for the hydrogen atom. Show the spectral lines in the (i) emission and (ii) the absorption spectrum corresponding to the transition of hydrogen atom from $n = 1$ to $n = 3$ energy states.

OR

What is mass-energy conservation law? A neutron disintegrates into a proton, a β-particle and an antineutrino. The disintegration process is:

$$_0n^1 \to {}_1H^1 + {}_{-1}\beta^0 + \bar{\nu}$$

Find out the energy produced in this process in MeV. (Mass of neutron $= 1.6747 \times 10^{-27}$ kg, Mass of proton $= 1.6725 \times 10^{-27}$ kg, Mass of electron $= 9.1 \times 10^{-31}$ kg.)

Correct Answer: —

SOLUTION:

Option 1: Bohr atom explained through orbit relations and photon wavelengths

Step 1: Bohr assumed that the hydrogen electron can occupy only discrete non-radiating circular orbits (stationary states). The Coulomb pull equals the centripetal requirement, $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r}$, which fixes the speed on each orbit.

Step 2: He further restricted the orbits by quantising angular momentum, $mvr = nh/2\pi$. Combining this with Step 1 gives quantised radii $r_n \propto n^2$ and quantised energies $E_n = -13.6/n^2$ eV, so the levels are $E_1 = -13.6$, $E_2 = -3.4$, $E_3 = -1.51$ eV.

Step 3: Radiation appears only during a jump, and the photon carries exactly the energy gap, $h\nu = |E_i - E_f|$, equivalently $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$. This third postulate is what turns discrete levels into sharp spectral lines.

Step 4 (Counting the lines): Working between the three levels n = 1, 2, 3, the number of distinct photon energies equals the number of level pairs, $\binom{3}{2} = 3$. So the emission spectrum shows three lines produced by the downward jumps $3 \to 2$ (energy 1.89 eV, visible), $3 \to 1$ (12.09 eV, UV) and $2 \to 1$ (10.2 eV, UV).

Step 5: A cold hydrogen gas sits in the ground state n = 1, so it can only soak up photons that lift it upward. The two allowed upward jumps $1 \to 3$ and $1 \to 2$ remove exactly those two photon energies from a white beam, leaving two dark absorption lines. Thus emission = 3 lines, absorption = 2 lines.

Step 6 (Sketch): Ladder of levels with n = 1 at the bottom; downward arrows for the three emission lines and upward arrows from n = 1 for the two absorption lines.

Option 2: Neutron decay energy using the atomic-mass-unit method

Step 1: Einstein\'s relation $E = mc^2$ says any loss of rest mass in a reaction is liberated as kinetic energy of the products; the total mass-energy before and after the decay must balance. Here the neutron is slightly heavier than proton plus electron, and that surplus mass is released.
Step 2: Treat the antineutrino as massless. The lost mass is $\Delta m = m_n - m_p - m_e$. Plugging the data, $\Delta m = 1.6747 \times 10^{-27} - 1.6725 \times 10^{-27} - 9.1 \times 10^{-31}$.
Step 3: $\Delta m = 2.2 \times 10^{-30} - 0.91 \times 10^{-30} = 1.29 \times 10^{-30}$ kg.
Step 4 (Switch to amu): Using $1\ \text{u} = 1.66 \times 10^{-27}$ kg, $\Delta m = \dfrac{1.29 \times 10^{-30}}{1.66 \times 10^{-27}} = 7.77 \times 10^{-4}$ u.
Step 5 (Use 1 u = 931 MeV): Energy released $E = \Delta m \times 931$ MeV $= 7.77 \times 10^{-4} \times 931$.
Step 6: $E \approx 0.72$ MeV, which agrees with the $E = \Delta mc^2$ value of 0.726 MeV found by the direct method.

$$\boxed{E \approx 0.72\ \text{MeV}}$$

Quick Tip: Use Bohr\'s three postulates - non-radiating orbits, $mvr = nh/2\pi$, and $h\nu = E_i - E_f$ - and count 3 emission vs 2 absorption lines. For the OR part use $E = \Delta m c^2$ with $\Delta m = m_n - (m_p + m_e)$.

• • •

30. What are the Faraday\'s laws of electromagnetic induction? The magnetic flux linked with a coil of resistance 2 Ω is

$$\phi = (5t^3 - 100t + 300) \text{ Weber}$$

Find out: (i) induced e.m.f. produced in the coil at $t = 2$ seconds (ii) induced current in the coil.

OR

Explain the conditions required for the resonance in series LCR alternating circuit by the diagram. On which factors does the resonant frequency depend?

Correct Answer: —

SOLUTION:

Option 1: Induced emf treated as the instantaneous slope of the flux curve

Step 1: Faraday summarised induction in two statements - a changing flux linkage drives an e.m.f., and the strength of that e.m.f. is set by how fast the linkage changes. Symbolically the instantaneous e.m.f. is the negative slope of the flux-time graph, $e(t) = -\frac{d\phi}{dt}$, the sign being fixed by Lenz to oppose the change.
Step 2: The flux is a cubic in time, $\phi(t) = 5t^3 - 100t + 300$. Differentiating each term separately: the derivative of $5t^3$ is $15t^2$, of $-100t$ is -100, and of the constant 300 is 0.
Step 3: Hence the instantaneous e.m.f. is $e(t) = -(15t^2 - 100) = 100 - 15t^2$ volt, a formula valid at every instant.
Step 4: Substituting the required instant $t = 2$ s: $e = 100 - 15(4) = 40$ V. (Note the e.m.f. is still positive here because $t = 2$ s is before the turning instant $t = \sqrt{100/15} \approx 2.58$ s where it would reverse sign.)

Step 5: The coil resistance is the only element in the loop, so the induced current is simply $I = e/R = 40/2 = 20$ A, flowing in the sense demanded by Lenz to oppose the rising flux.

$$\boxed{e = 40\text{ V},\qquad I = 20\text{ A}}$$

Option 2: Resonance seen from the phasor / voltage-magnification viewpoint

Step 1: In a series LCR loop the same current flows through all elements, but the voltage across L leads the current by 90° while the voltage across C lags by 90°; these two voltage phasors point in exactly opposite directions.

Step 2: As the source frequency is raised, $X_L = \omega L$ grows and $X_C = 1/\omega C$ shrinks. At one special frequency the two opposing voltages become equal in size and cancel, leaving the whole source voltage across R alone.

Step 3 (Resonance condition): That cancellation is the resonance condition $X_L = X_C$. Here the impedance collapses to its least value $Z = R$, the circuit behaves as purely resistive, current peaks and the phase angle between current and voltage is zero.

Step 4: Setting $\omega L = 1/(\omega C)$ and solving gives $\omega_0 = 1/\sqrt{LC}$, i.e. $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$. The resonance curve of current versus frequency is a sharp peak centred on f_0.

Step 5: Because f_0 contains only L and C, the resonant frequency is decided solely by the inductance and capacitance and does not depend on the resistance R - the resistance only broadens or narrows the peak.

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

Quick Tip: Induced e.m.f. is $\varepsilon = -d\phi/dt = 100 - 15t^2$; put $t = 2$ then $I = \varepsilon/R$. For the OR part, resonance needs $X_L = X_C$, giving $f_0 = 1/(2\pi\sqrt{LC})$, which depends on L and C only.