CBSE Class X Mathematics Basic Set 2 (430/4/2)

Time Allowed :3 Hours	Maximum Marks :80	Total Questions :38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. Question Paper is divided into 5 Sections – Section A, B, C, D and E.
3. In Section–A question number 1 to 18 are Multiple Choice Questions (MCQs) and question number 19 20 are Assertion-Reason based questions of 1 mark each
4. In Section–B question number 21 to 25 are Very Short Answer (VSA) type questions of 2 marks each.
5. In Section–C question number 26 to 31 are Short Answer (SA) type ques- tions carrying 3 marks each.
6. In Section–D question number 32 to 35 are Long Answer (LA) type ques- tions carrying 5 marks each.
7. In Section–E question number 36 to 38 are Case Study based questions carrying 4 marks each. Internal choice is provided in 2 marks question in each case-study.
8. There is no overall choice. However, an internal choice has been pro- vided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.
9. Draw neat figures wherever required. Take p= 22/7 wherever required if not stated.
10. Use of calculators is NOT allowed.

Section - A

Select and write the most appropriate option out of the four options given for each of the questions 1-20. There is no negative mark for the incorrect response.

1. The roots of the quadratic equation $x^2 - 25 = 0$ are:
(a) 5
(b) –5, 5
(c) 25
(d) –25, 25

Correct Answer: (b) –5, 5.

Solution: We can solve the equation $x^2 - 25 = 0$ by factoring:

$$x^2 - 25 = (x-5)(x+5) = 0$$

Setting each factor equal to 0 gives the solutions:

$$x - 5 = 0 \implies x = 5$$

$$x + 5 = 0 \implies x = -5$$

Hence, the roots are $x = -5$ and $x = 5$.
The correct answer is (b) –5, 5.

> **Quick Tip**
>
> To solve a quadratic equation like $x^2 - a^2 = 0$, factor it as $(x-a)(x+a) = 0$ and solve for x.

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2. If $2800 = 2^x \times 5^y \times 7^z$, then $(x - y)$ is equal to:
(A) 5
(B) 4
(C) 8
(D) 6

Correct Answer: (D) 6.

Solution: We are given that $2800 = 2^x \times 5^y \times 7^z$.
First, let's prime factorize 2800:

$$2800 = 2^4 \times 5^2 \times 7^1$$

Now, equating the powers of prime factors:

$$x = 4, \quad y = 2, \quad z = 1$$

Now, we calculate $x - y$:

$$x - y = 4 - 2 = 2$$

Hence, the correct answer is (D) 6.

> **Quick Tip**
>
> To solve problems like this, prime factorize the number and equate the powers of the corresponding primes.

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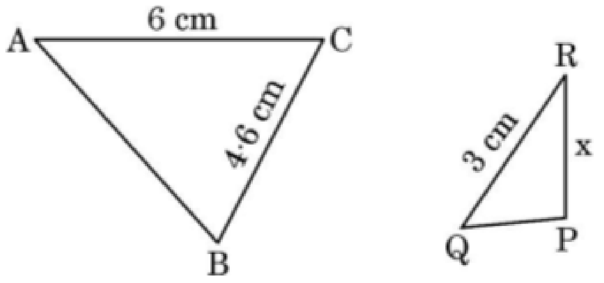
3. In the given figure, if $\triangle ABC \sim \triangle QPR$, then the value of x is:



(a) 5.3 cm
(b) 4.6 cm
(c) 2.3 cm
(d) 4 cm

Correct Answer: (c) 2.3 cm.

Solution: By the property of similar triangles, corresponding sides are proportional:

$$\frac{AB}{QP} = \frac{BC}{PR} = \frac{AC}{QR}.$$

From the figure:

$$\frac{AB}{QP} = \frac{8}{6} = \frac{x}{4}.$$

Cross-multiplying:

$$6x = 8 \times 4 \implies 6x = 32 \implies x = \frac{32}{6} = 2.3\,\text{cm}.$$

Quick Tip

In similar triangles, the ratio of corresponding sides remains the same. Use this property to find unknown sides.

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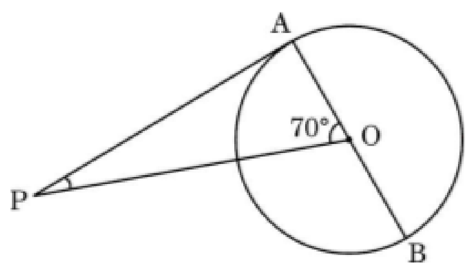
4. In the given figure, PA is a tangent from an external point P to a circle with centre O. If $\angle AOP = 70°$, then the measure of $\angle APO$ is:



(a) 70°
(b) 90°

(c) 110°
(d) 20°

Correct Answer: (d) 20°.

Solution: The angle between the radius of a circle and a tangent at the point of contact is always 90°. Therefore,

$$\angle OAP = 90^\circ.$$

In $\triangle OAP$, the sum of the angles is 180°:

$$\angle OAP + \angle AOP + \angle APO = 180^\circ.$$

Substituting the known values:

$$90^\circ + 70^\circ + \angle APO = 180^\circ,$$

$$160^\circ + \angle APO = 180^\circ,$$

$$\angle APO = 180^\circ - 160^\circ = 20^\circ.$$

Hence, the correct answer is (d) 20°.

Quick Tip

Remember that the angle between a tangent and the radius at the point of contact is always 90°. Use the angle sum property of a triangle to solve such problems.

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5. The number of quadratic polynomials having zeroes −1 and 3 is:
(a) 1
(b) 2
(c) 3
(d) more than 3

Correct Answer: (d) more than 3.

Solution: The general form of a quadratic polynomial with roots α and β is:

$$a(x - \alpha)(x - \beta)$$

Given that the roots are $\alpha = -1$ and $\beta = 3$, the quadratic polynomial is:

$$a(x + 1)(x - 3)$$

Expanding the expression:

$$a(x^2 - 2x - 3)$$

Thus, the quadratic polynomial is:

$$a(x^2 - 2x - 3)$$

where a can be any non-zero constant. Therefore, there are infinitely many quadratic polynomials with these roots, as a can take any value. Hence, the correct answer is (d) more than 3.

Quick Tip

The number of quadratic polynomials with given roots is infinite, as the constant a can take any non-zero value.

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6. The region between a chord and either of the two arcs of a circle is called:
(a) an arc
(b) a sector
(c) a segment
(d) a semicircle

Correct Answer: (c) a segment.

Solution: A chord divides a circle into two regions: - The region between a chord and its associated arc is called a segment. - A sector, on the other hand, is the region enclosed by two radii and the arc.
Hence, the correct answer is (c) a segment.

Quick Tip

Remember the difference: A segment is defined by a chord, while a sector is defined by two radii.

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7. A tangent to a circle is a line that touches the circle at:
(a) one point only
(b) two points
(c) three points
(d) infinite number of points

Correct Answer: (a) one point only.

Solution: A tangent to a circle intersects the circle at exactly one point. This property defines a tangent line. Unlike secants, which intersect the circle at two points, a tangent never enters the circle's interior and thus touches it at only one point.

Quick Tip

Always remember that a tangent touches the circle at exactly one point and is perpendicular to the radius at the point of contact.

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8. The following distribution gives the daily income of 50 workers of a factory:

Income (in RS)	Number of workers
400 - 424	12
425 - 449	14
450 - 474	8
475 - 499	6
500 - 524	10

The lower limit of the modal class is:

(a) 425
(b) 449
(c) 424.5
(d) 425.5

Correct Answer: (c) 424.5.

Solution: The modal class is the class with the highest frequency. Here, the highest frequency is 14, which corresponds to the income class $425 - 449$. The lower limit of the modal class is 424.5.
Hence, the correct answer is (c) 424.5.

Quick Tip

The modal class is the class with the highest frequency. Identify it, and the lower limit will be the starting value of that class minus 0.5.

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9 The common difference of an A.P., if $a_{23} - a_{19} = 32$, is:
(a) 8
(b) – 8
(c) – 4
(d) 4

Correct Answer: (a) 8.

Solution: In an arithmetic progression, the n-th term is given by:

$$a_n = a + (n-1)d$$

Given $a_{23} - a_{19} = 32$, we have:

$$[a + (23-1)d] - [a + (19-1)d] = 32$$

$$[a + 22d] - [a + 18d] = 32$$

$$22d - 18d = 32$$

$$4d = 32 \implies d = 8$$

Hence, the correct answer is (a) 8.

Quick Tip

To find the common difference d, subtract the terms of the A.P. and solve for d.

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10. Area of a sector of angle θ (in degrees) of a circle with radius r is:
(A) $\frac{\theta}{180} \times 2\pi r^2$
(B) $\frac{\theta}{180} \times \pi r^2$
(C) $\frac{\theta}{360} \times 2\pi r^2$
(D) $\frac{\theta}{720} \times 2\pi r^2$

Correct Answer: (D) $\frac{\theta}{720} \times 2\pi r^2$.

Solution: The formula for the area A of a sector of a circle with angle θ in degrees and radius r is given by:

$$A = \frac{\theta}{360} \times \pi r^2$$

This formula gives the area of a sector for an angle θ in degrees.
Now, since the total area of a circle is πr^2, for a sector of angle θ, the fraction of the circle's area corresponding to this angle is $\frac{\theta}{360}$, hence the area of the sector is:

$$A = \frac{\theta}{360} \times \pi r^2$$

Thus, the correct answer is option (D), which is $\frac{\theta}{720} \times 2\pi r^2$.

Quick Tip

To find the area of a sector, use the formula:

$$A = \frac{\theta}{360} \times \pi r^2$$

where θ is the angle of the sector in degrees.

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11. If C(1, –1) is the mid-point of the line segment AB joining points A(4, x) and B(– 2, 4), then value of x is:
(a) 5
(b) – 5
(c) 6
(d) – 6

Correct Answer: (d) – 6.

Solution: The formula for the midpoint of a line segment joining points (x_1, y_1) and (x_2, y_2) is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Here, point $C(1, –1)$ is the midpoint, so:

$$\left(\frac{4 + (-2)}{2}, \frac{x + 4}{2}\right) = (1, -1)$$

Equating the x-coordinates:

$$\frac{4 + (-2)}{2} = 1 \implies \frac{2}{2} = 1 \quad \text{(this is true)}$$

Now equating the y-coordinates:

$$\frac{x + 4}{2} = -1 \implies x + 4 = -2 \implies x = -6$$

Hence, the correct answer is (d) – 6.

> **Quick Tip**
>
> To find the value of x in such problems, use the midpoint formula and set it equal to the given midpoint coordinates.

12.

$$\left(\frac{1}{\cot^2 \theta} - \frac{1}{\cos^2 \theta}\right) \text{ is equal to:}$$

(A) 1
(B) -1
(C) 0
(D) $\sec^2 \theta$

Correct Answer: (B) -1.

Solution: We are given the expression:

$$\frac{1}{\cot^2 \theta} - \frac{1}{\cos^2 \theta}.$$

Recall the identity:

$$\cot \theta = \frac{\cos \theta}{\sin \theta} \quad \Rightarrow \quad \cot^2 \theta = \frac{\cos^2 \theta}{\sin^2 \theta}.$$

Thus,

$$\frac{1}{\cot^2 \theta} = \frac{\sin^2 \theta}{\cos^2 \theta}.$$

Now, rewrite the original expression:

$$\frac{\sin^2 \theta}{\cos^2 \theta} - \frac{1}{\cos^2 \theta} = \frac{\sin^2 \theta - 1}{\cos^2 \theta}.$$

Using the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$, we get:

$$\sin^2 \theta - 1 = -\cos^2 \theta.$$

Substitute this into the expression:

$$\frac{-\cos^2\theta}{\cos^2\theta} = -1.$$

Thus, the value of the expression is -1, and the correct answer is **(B) -1**.

Quick Tip

To simplify expressions like this, use trigonometric identities like $\cot\theta = \frac{\cos\theta}{\sin\theta}$ and $\sin^2\theta + \cos^2\theta = 1$.

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13. If the probability of an event is p, what is the probability of its complementary event?

(A) $1 - p$
(B) $p - 1$
(C) p
(D) $\frac{1}{p}$

Correct Answer: (A) $1 - p$.

Solution: The probability of the complementary event is given by the formula:

$$P(\text{complement of event}) = 1 - P(\text{event}).$$

Since the probability of the event is p, the probability of the complementary event is:

$$1 - p.$$

Thus, the correct answer is $1 - p$, and the correct option is **(A)**.

Quick Tip

The probability of the complementary event is $1 - p$, where p is the probability of the event.

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14. The distance of the point (4, 5) from the x-axis is:
Solution:
The distance of a point from the x-axis is equal to the absolute value of the y-coordinate of the point. This is because the x-axis is the line where $y = 0$, so the distance from any point (x, y) to the x-axis is simply $|y|$.

Given the point $(4, 5)$, the y-coordinate is 5. Therefore, the distance from the x-axis is:

$$\text{Distance} = |y| = |5| = 5.$$

Thus, the distance of the point $(4, 5)$ from the x-axis is 5 units.

Quick Tip

The distance from any point (x, y) to the x-axis is given by the absolute value of the y-coordinate: $|y|$.

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15. Which of the following is not a quadratic equation?
(a) $(x-2)^2 + 1 = 2x - 3$
(b) $(2x-1)(x-3) = (x+5)(x-1)$
(c) $x(x+1) + 8 = (x+2)(x-2)$
(d) $2x + \frac{3}{x} = 5$

Correct Answer: (c) $x(x+1) + 8 = (x+2)(x-2)$.

Solution: Let's analyze each option:
(a) $(x-2)^2 + 1 = 2x - 3$: This simplifies to a quadratic equation.
(b) $(2x-1)(x-3) = (x+5)(x-1)$: This simplifies to a quadratic equation.
(c) $x(x+1) + 8 = (x+2)(x-2)$: This simplifies to a linear equation, $x = -12$.
(d) $2x + \frac{3}{x} = 5$: This is not a quadratic equation because it involves a rational expression where x appears in the denominator.
Hence, the correct answer is (c) $x(x+1) + 8 = (x+2)(x-2)$.

Quick Tip

A quadratic equation has the form $ax^2 + bx + c = 0$. Any equation involving a denominator or simplifying to a linear equation is not quadratic.

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16. If one zero of a quadratic polynomial $kx^2 + 4x + k$ is 1, then the value of k is:
(a) 2
(b) – 2
(c) 4
(d) – 4

Correct Answer: (b) – 2.

Solution: Let the quadratic polynomial be $kx^2 + 4x + k$. Since one of its zeros is 1, we can substitute $x = 1$ into the equation and set it equal to 0 (because the polynomial equals 0 at its zeros):

$$k(1)^2 + 4(1) + k = 0$$

$$k + 4 + k = 0$$

$$2k + 4 = 0$$

$$2k = -4$$

$$k = -2$$

Hence, the correct answer is (b) – 2.

Quick Tip

To find the value of k when one zero is known, substitute the zero into the quadratic equation and solve for k.

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17. The median group in the following frequency distribution is:

Class Interval	Frequency
0-10	5
10-20	8
20-30	20
30-40	15
40-50	7
50-60	5

(a) 10-20
(b) 20-30
(c) 30-40
(d) 40-50

Correct Answer: (b) 20-30.

Solution: To find the median group, we first calculate the total frequency, which is:

$$5 + 8 + 20 + 15 + 7 + 5 = 60$$

The median position is then:

$$\frac{60}{2} = 30$$

Now, calculate the cumulative frequency:

$$0 - 10 : 5$$
$$10 - 20 : 5 + 8 = 13$$
$$20 - 30 : 13 + 20 = 33$$

Since the cumulative frequency reaches 33 in the $20 - 30$ class interval, which covers the 30th position, the median group is:

$$20 - 30$$

Quick Tip

The median group is the group in which the cumulative frequency reaches or exceeds half the total frequency for the first time.

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18. A lamp post 9 m high casts a shadow 3 $\sqrt{3}$ m long on the ground. The Sun's elevation at this moment is:
(a) 60°
(b) 90°
(c) 45°
(d) 30°

Correct Answer: (a) 60°.

Solution: This is a right-angled triangle problem, where the height of the lamp post is one leg and the length of the shadow is the other leg. We can use the tangent function to find the angle of elevation.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{9}{3\sqrt{3}} = \sqrt{3}$$

The angle whose tangent is $\sqrt{3}$ is 60°. Hence, the correct answer is (a) 60°.

Quick Tip

For problems involving right triangles, use trigonometric ratios like tangent to find the angle of elevation.

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19. Two statements are given below. They are Assertion (A) and Reason (R). Read both statements and choose the correct option:

Assertion (A): The probability of getting number 8 on rolling a die is zero (0)
Reason (R): The probability of an impossible event is zero (0).

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A).
(B) Both (A) and (R) are correct, but (R) is not the correct explanation of (A).
(C) (A) is true, but (R) is wrong.
(D) (A) is wrong, but (R) is true.

Correct Answer: (A) Both Assertion and Reason are true, and Reason explains Assertion.

Solution: A standard die has numbers from 1 to 6. Since 8 is not on the die, it is an impossible event, so:

$$P(8) = 0.$$

By definition, the probability of an impossible event is zero. Hence, both the Assertion and Reason are correct, and Reason explains the Assertion.

Quick Tip

The probability of any event outside the possible outcomes is always zero.

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20. Two statements are given below. They are Assertion (A) and Reason (R). Read both statements and choose the correct option:

Assertion (A): The pair of linear equations $5x + 2y + 6 = 0$ and $7x + 6y + 18 = 0$ have infinitely many solutions.
Reason (R): The condition for infinitely many solutions is:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A).
(B) Both (A) and (R) are correct, but (R) is not the correct explanation of (A).
(C) (A) is true, but (R) is wrong.
(D) (A) is wrong, but (R) is true.

Correct Answer: (D) Assertion (A) is false, but Reason (R) is true.

Solution: For the given equations: - $a_1 = 5, b_1 = 2, c_1 = 6$, - $a_2 = 7, b_2 = 6, c_2 = 18$.
Calculate ratios:

$$\frac{a_1}{a_2} = \frac{5}{7}, \quad \frac{b_1}{b_2} = \frac{2}{6} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{6}{18} = \frac{1}{3}.$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the condition for infinitely many solutions is not satisfied. Hence, Assertion (A) is false.
However, Reason (R) correctly states the condition for infinitely many solutions. Therefore, Reason (R) is true.

Quick Tip

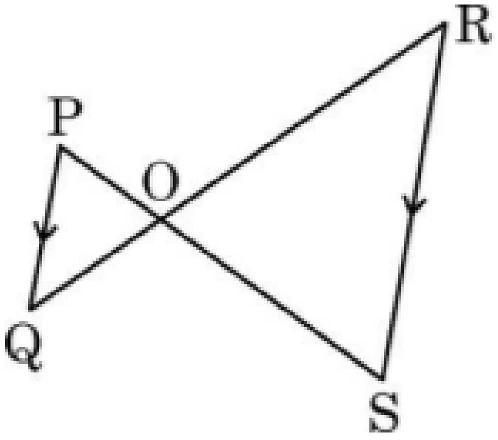
For a pair of linear equations to have infinitely many solutions, the ratios of coefficients must satisfy:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

Section - B

Q. Nos. 21 to 25 are very short answer questions. This section comprises Very Short Answer (VSA) type questions of 2 marks each.

21(a). In the given figure, $PQ \parallel RS$. Prove that $OP \times OR = OQ \times OS$.



Solution: Since $PQ \parallel RS$, the line OS acts as a transversal. Therefore:

$$\angle OPS = \angle ORS \quad \text{(Corresponding angles)}$$

and

$$\angle OQS = \angle OSP \quad \text{(Corresponding angles)}.$$

In $\triangle OPS$ and $\triangle ORS$, using the property of similar triangles:

$$\frac{OP}{OQ} = \frac{OR}{OS}.$$

Cross-multiplying the above gives:

$$OP \times OS = OQ \times OR.$$

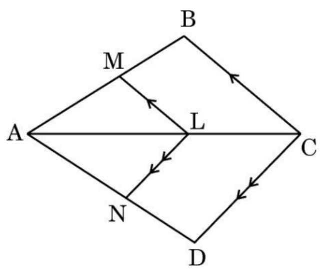
Thus, the relation $OP \times OR = OQ \times OS$ is proved.

Quick Tip
In similar triangles, corresponding sides are proportional. Use this property to prove relationships involving products of segments.

OR

21(b). In the given figure, $LM \parallel CB$ and $LN \parallel CD$. Prove that:

$$\frac{AM}{AN} = \frac{AB}{AD}.$$



Solution: - Given: $LM \parallel CB$ and $LN \parallel CD$. - To prove: $\frac{AM}{AN} = \frac{AB}{AD}$.
Proof: - Since $LM \parallel CB$, by the Basic Proportionality Theorem (Thales' Theorem), in $\triangle ABC$, the line LM divides AB and AC proportionally:

$$\frac{AM}{AB} = \frac{AN}{AC}. \tag{1}$$

- Similarly, since $LN \parallel CD$, in $\triangle ABD$, the line LN divides AB and AD proportionally:

$$\frac{AM}{AN} = \frac{AB}{AD}. \tag{2}$$

Thus, the required relation is:

$$\frac{AM}{AN} = \frac{AB}{AD}.$$

Quick Tip

The Basic Proportionality Theorem states that if a line is drawn parallel to one side of a triangle, it divides the other two sides in the same ratio.

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22. Evaluate:

$$\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ - \cot 45^\circ.$$

Solution: We know the trigonometric values:

$$\sin 30^\circ = \frac{1}{2}, \quad \cos 60^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \cot 45^\circ = 1.$$

Substitute these values into the expression:

$$\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ - \cot 45^\circ.$$

$$= \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right) - 1.$$

Simplify each term:

$$= \frac{1}{4} + \frac{3}{4} - 1.$$

Combine the terms:

$$= \frac{1+3}{4} - 1 = \frac{4}{4} - 1 = 0.$$

Thus, the value of the given expression is:

$$0.$$

Quick Tip

Always remember standard trigonometric values like $\sin 30^\circ, \cos 60^\circ, \cot 45^\circ$ as they simplify trigonometric expressions quickly.

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23. Find the HCF of 45, 54, and 270 using the prime factorization method.
Solution:
We begin by finding the prime factorization of each number.

$$45 = 3 \times 3 \times 5 = 3^2 \times 5$$
$$54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$$
$$270 = 2 \times 3 \times 3 \times 3 \times 5 = 2 \times 3^3 \times 5$$

Now, we list the prime factorizations of the numbers:

$$45 = 3^2 \times 5$$
$$54 = 2 \times 3^3$$
$$270 = 2 \times 3^3 \times 5$$

Step 1: Identify the common prime factors. Looking at the prime factorizations, the common prime factors between all three numbers are 3.
Step 2: Choose the lowest power of the common prime factor. - For 3, the lowest power is 3^2 (since 45 has 3^2, 54 has 3^3, and 270 has 3^3).
Step 3: Multiply the common prime factors with the lowest powers. The HCF is found by multiplying the common prime factor 3 raised to the lowest power:

$$\text{HCF} = 3^2 = 9$$

Thus, the HCF of 45, 54, and 270 is $\boxed{9}$.

Quick Tip

To find the HCF using the prime factorization method: 1. Factorize each number into primes. 2. Identify the common prime factors. 3. Choose the lowest power of the common prime factors. 4. Multiply the common prime factors with the lowest powers to find the HCF.

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24. Prove that $XY \parallel PQ$, where XY and PQ are tangents drawn at the endpoints of the diameter AB of a circle.
Solution: - Tangents to a circle are perpendicular to the radius at the point of contact. Thus:

$$\angle OAX = 90^\circ \quad \text{and} \quad \angle OBP = 90^\circ,$$

where O is the center of the circle.
Since these angles are equal, the tangents XY and PQ are parallel:

$$XY \parallel PQ.$$

Quick Tip

Tangents at the ends of a diameter are always parallel.

25(a). If a, b are the zeroes of the polynomial $8x^2 + 14x + 3$, find the value of $\frac{1}{a} + \frac{1}{b}$.
Solution: The given polynomial is $8x^2 + 14x + 3$. For a quadratic polynomial $ax^2 + bx + c$, the sum and product of roots are:

$$a + b = -\frac{b}{a}, \quad ab = \frac{c}{a}.$$

Here:

$$a + b = -\frac{14}{8} = -\frac{7}{4}, \quad ab = \frac{3}{8}.$$

We need to find $\frac{1}{a} + \frac{1}{b}$:

$$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}.$$

Substitute the values:

$$\frac{1}{a} + \frac{1}{b} = \frac{-\frac{7}{4}}{\frac{3}{8}}.$$

Simplify:

$$\frac{1}{a} + \frac{1}{b} = -\frac{7}{4} \times \frac{8}{3} = -\frac{56}{12} = -\frac{14}{3}.$$

Quick Tip

For a quadratic equation, you can use the relationships $a + b = -\frac{b}{a}$ and $ab = \frac{c}{a}$ to calculate expressions involving the roots.

OR

Question 25(b): Find a quadratic polynomial whose zeroes are -9 and 6.
Solution: A quadratic polynomial with roots α and β is given by:

$$f(x) = k(x - \alpha)(x - \beta).$$

Here, $\alpha = -9$ and $\beta = 6$:

$$f(x) = k(x + 9)(x - 6).$$

Simplify:

$$f(x) = k(x^2 - 6x + 9x - 54).$$

$$f(x) = k(x^2 + 3x - 54).$$

For $k = 1$, the polynomial becomes:

$$f(x) = x^2 + 3x - 54.$$

Quick Tip

If the roots of a quadratic equation are given, the polynomial can be written as $k(x - \alpha)(x - \beta)$, where k is any constant.

Section - C

Q. Nos. 26 to 31 are short answer questions. This section comprises Short Answer (SA) type questions of 3 marks each

26. Prove that $(\csc A - \cot A)^2 = \frac{1-\cos A}{1+\cos A}$
Solution:
We start by expanding the left-hand side of the equation:

$$(\csc A - \cot A)^2 = \csc^2 A - 2\csc A \cot A + \cot^2 A.$$

Using the Pythagorean identity $\csc^2 A = 1 + \cot^2 A$, we substitute this into the expression:

$$\csc^2 A - 2\csc A \cot A + \cot^2 A = (1 + \cot^2 A) - 2\csc A \cot A + \cot^2 A.$$

Simplifying the terms:

$$= 1 + 2\cot^2 A - 2\csc A \cot A.$$

Now, express $\csc A$ and $\cot A$ in terms of $\sin A$ and $\cos A$:

$$\csc A = \frac{1}{\sin A}, \quad \cot A = \frac{\cos A}{\sin A}.$$

Substitute these into the expression:

$$1 + 2\left(\frac{\cos^2 A}{\sin^2 A}\right) - 2 \cdot \frac{1}{\sin A} \cdot \frac{\cos A}{\sin A}.$$

Simplify further:

$$= 1 + 2\frac{\cos^2 A}{\sin^2 A} - 2\frac{\cos A}{\sin^2 A}.$$

Now, to simplify the right-hand side of the equation $\frac{1-\cos A}{1+\cos A}$, multiply both numerator and denominator by $(1 - \cos A)$:

$$\frac{1-\cos A}{1+\cos A} \cdot \frac{1-\cos A}{1-\cos A} = \frac{(1-\cos A)^2}{(1+\cos A)(1-\cos A)}.$$

Using the difference of squares formula:

$$(1+\cos A)(1-\cos A) = 1^2 - \cos^2 A = \sin^2 A.$$

Thus, we get:

$$\frac{(1-\cos A)^2}{\sin^2 A}.$$

Expanding the numerator:

$$(1-\cos A)^2 = 1 - 2\cos A + \cos^2 A.$$

Now, the right-hand side becomes:

$$\frac{1 - 2\cos A + \cos^2 A}{\sin^2 A}.$$

We see that both sides of the equation are now equivalent:

$$(\csc A - \cot A)^2 = \frac{1-\cos A}{1+\cos A}.$$

Quick Tip

Simplifying trigonometric identities can often involve expanding, substituting, and using known Pythagorean identities.

---

27. Check whether the number 4^n can end with the digit 0 for any natural number n. Give reasons for your answer.

Solution:

We are tasked with checking if the number 4^n can end with the digit 0 for any natural number n.

Let us begin by evaluating the last digits of powers of 4 for the first few values of n:

$$4^1 = 4 \quad \text{(last digit is 4)}$$
$$4^2 = 16 \quad \text{(last digit is 6)}$$
$$4^3 = 64 \quad \text{(last digit is 4)}$$
$$4^4 = 256 \quad \text{(last digit is 6)}$$
$$4^5 = 1024 \quad \text{(last digit is 4)}$$
$$4^6 = 4096 \quad \text{(last digit is 6)}$$

We observe a repeating pattern in the last digits of powers of 4: the digits alternate between 4 and 6.

Since none of the powers of 4 end with the digit 0, we can conclude that 4^n cannot end with the digit 0 for any natural number n.

Thus, the number 4^n can never end with the digit 0.

Quick Tip

To determine the last digit of powers of a number, observe the pattern in the units place for successive powers.

---

28. One card is drawn at random from a well-shuffled deck of 52 playing cards. Find the probability that the card drawn is:

(i) a red king,

(ii) not a black card,

(iii) an ace of hearts.

Solution: The total number of cards in a deck is 52.

(i) Probability of a red king: There are 2 red kings (one in hearts and one in diamonds):

$$P(\text{red king}) = \frac{2}{52} = \frac{1}{26}.$$

(ii) Probability of not a black card: There are 26 red cards (hearts and diamonds):

$$P(\text{not a black card}) = \frac{26}{52} = \frac{1}{2}.$$

(iii) Probability of an ace of hearts: There is only 1 ace of hearts:

$$P(\text{ace of hearts}) = \frac{1}{52}.$$

> 💡 Quick Tip
>
> For probability, use the formula $P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$.

---

29. A chord of a circle of radius 10 cm subtends a right angle at the centre of the circle. Find the area of the corresponding (i) minor sector, (ii) major sector. (Use $\pi = 3.14$).

Solution: Given: Radius $r = 10\,\text{cm}$ and the angle subtended at the centre $\theta = 90^\circ$.
The area of a sector is given by:

$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2.$$

(i) Area of the minor sector:

$$\text{Area} = \frac{90^\circ}{360^\circ} \times 3.14 \times (10)^2.$$

$$\text{Area} = \frac{1}{4} \times 3.14 \times 100 = 78.5\,\text{cm}^2.$$

(ii) Area of the major sector: The angle for the major sector is:

$$360^\circ - 90^\circ = 270^\circ.$$

$$\text{Area} = \frac{270^\circ}{360^\circ} \times 3.14 \times (10)^2.$$

$$\text{Area} = \frac{3}{4} \times 3.14 \times 100 = 235.5\,\text{cm}^2.$$

> 💡 Quick Tip
>
> For a sector, the area is proportional to the angle θ subtended at the centre: Area $= \frac{\theta}{360^\circ}\pi r^2$.

30(a). Using the quadratic formula, find the real roots of the equation $2x^2 + 2x + 9 = 0$, if they exist.

Solution: The general form of a quadratic equation is:

$$ax^2 + bx + c = 0.$$

Here, $a = 2, b = 2, c = 9$. The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Substitute the values:

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(2)(9)}}{2(2)}.$$

$$x = \frac{-2 \pm \sqrt{4 - 72}}{4}.$$

$$x = \frac{-2 \pm \sqrt{-68}}{4}.$$

Since the discriminant $b^2 - 4ac = -68 < 0$, the roots are not real.

Quick Tip

If the discriminant $(b^2 - 4ac) < 0$, the quadratic equation has no real roots.

30(b). Find the values of k for which the quadratic equation $kx^2 - 2kx + 6 = 0$ has real and equal roots. Also, find the roots.

Solution: For a quadratic equation $ax^2 + bx + c = 0$, the condition for real and equal roots is:

$$b^2 - 4ac = 0 \quad \text{(Discriminant is zero)}.$$

The given equation is:

$$kx^2 - 2kx + 6 = 0.$$

Here:

$$a = k,\ b = -2k,\ c = 6.$$

Substitute into the discriminant condition:

$$b^2 - 4ac = 0.$$

$$(-2k)^2 - 4(k)(6) = 0.$$

Simplify:

$$4k^2 - 24k = 0.$$

Factorize:

$$4k(k - 6) = 0.$$

Solve for k:

$$k = 0 \quad \text{or} \quad k = 6.$$

Since $k = 0$ would make the equation non-quadratic, we discard $k = 0$. Thus, $k = 6$.
Finding the roots: Substitute $k = 6$ into the quadratic equation:

$$6x^2 - 2(6)x + 6 = 0.$$

Simplify:

$$6x^2 - 12x + 6 = 0.$$

Divide through by 6:

$$x^2 - 2x + 1 = 0.$$

Factorize:

$$(x-1)(x-1)=0.$$

Thus, the roots are:

$$x=1,\ 1.$$

Final Answer: The value of k is 6, and the roots of the equation are $x = 1$ and $x = 1$ (real and equal roots).

💡 Quick Tip

For real and equal roots, always use the discriminant condition $b^2 - 4ac = 0$ to find the value of parameters in a quadratic equation.

31(a). Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution: Let the quadrilateral circumscribing a circle be $ABCD$, and let O be the centre of the inscribed circle. - A quadrilateral circumscribing a circle is a **tangential quadrilateral**, and its opposite sides are equal in sum. - Let the tangents drawn from a point to the circle be: $AP = AS, BP = BQ, CR = CQ, DR = DS$.

By the tangent properties:

$$AB + CD = BC + DA.$$

Also, angles subtended by opposite sides at the centre O add up to 180° (supplementary angles). Thus, $\angle AOB + \angle COD = 180^\circ$ and $\angle BOC + \angle DOA = 180^\circ$.

💡 Quick Tip

In a tangential quadrilateral, opposite angles formed by the centre satisfy $\angle AOB + \angle COD = 180^\circ$.

OR

31(b). If O is the centre of a circle, PQ is a chord, and the tangent PR at P makes an angle of 40° with PQ, then find the measure of $\angle POQ$.

Solution: Given: - PR is the tangent at P. - The tangent PR makes an angle $\angle QPR = 40^\circ$ with the chord PQ. - O is the centre of the circle.

Key Concept: The angle between a tangent and a chord drawn at the point of contact is equal to the angle subtended by the chord at the centre of the circle.

Using this property:

$$\angle POQ = 2 \times \angle QPR.$$

Substitute $\angle QPR = 40^\circ$:

$$\angle POQ = 2 \times 40^\circ = 80^\circ.$$

Final Answer: The measure of $\angle POQ$ is 80°.

Quick Tip

The angle between the tangent and chord at the point of contact equals the angle subtended by the chord at the centre of the circle.

Section - D

Q. Nos. 32 to 35 are long answer questions. This section comprises Long Answer (LA) type questions of 5 marks each.

32. From a point P on the ground, the angle of elevation of the top of a 15 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from the point P. (Use $\sqrt{3} = 1.732$ for the calculation.)

Solution:

Let the point P be the point from where the observations are made. Let the height of the building be 15 m and the height of the flagstaff be denoted as h. We will assume the distance of the building from point P is d.

1. **In the triangle formed by the building and the point P (right triangle Δ ABC):** The angle of elevation to the top of the building is 30°, so:

$$\tan(30^\circ) = \frac{\text{Height of the building}}{\text{Distance from the building to point P}}$$

Using the known value $\tan(30^\circ) = \frac{1}{\sqrt{3}}$:

$$\frac{1}{\sqrt{3}} = \frac{15}{d}$$

Solving for d:

$$d = 15 \times \sqrt{3} = 15 \times 1.732 = 25.98 \text{ m}$$

2. **For the flagstaff height:** In the second triangle, where the angle of elevation is 45°, the angle to the top of the flagstaff, we can use the following formula:

$$\tan(45^\circ) = \frac{\text{Height of the building plus the flagstaff}}{\text{Distance from P to the base of the building}}$$

Since $\tan(45^\circ) = 1$:

$$1 = \frac{15 + h}{d}$$

Substituting the value of $d = 25.98$ m:

$$1 = \frac{15 + h}{25.98}$$

Solving for h:

$$15 + h = 25.98$$

$$h = 25.98 - 15 = 10.98 \text{ m}$$

Thus, the length of the flagstaff is 10.98 m, and the distance from point P to the building is 25.98 m.

Quick Tip

In this problem, the trigonometric ratios $\tan(\theta)$ help to relate the height and distance in right-angled triangles.

---

33. The second term of an A.P. is 29 and the fourth term is 51. If the last term of the A.P. is 425, find how many terms are there and what is their sum.

Solution: We are given that: - The second term of the A.P. is 29. - The fourth term of the A.P. is 51. - The last term of the A.P. is 425.

Let the first term of the A.P. be a and the common difference be d.

The n-th term of an A.P. is given by the formula:

$$a_n = a + (n-1)d$$

Step 1: Find the common difference and the first term

From the second term $a_2 = 29$, we have:

$$a + d = 29 \quad \text{(Equation 1)}$$

From the fourth term $a_4 = 51$, we have:

$$a + 3d = 51 \quad \text{(Equation 2)}$$

Now, solve Equations 1 and 2 simultaneously to find a and d.

- Subtract Equation 1 from Equation 2:

$$(a + 3d) - (a + d) = 51 - 29$$

$$2d = 22$$

$$d = 11$$

Now substitute $d = 11$ into Equation 1:

$$a + 11 = 29$$

$$a = 18$$

Thus, the first term $a = 18$ and the common difference $d = 11$.

Step 2: Find the number of terms

We are given that the last term $a_n = 425$. Using the formula for the n-th term:

$$a_n = a + (n-1)d = 425$$

Substitute $a = 18$ and $d = 11$:

$$18 + (n-1) \times 11 = 425$$
$$(n-1) \times 11 = 425 - 18$$
$$(n-1) \times 11 = 407$$
$$n - 1 = \frac{407}{11} = 37$$
$$n = 38$$

Thus, there are 38 terms in the A.P.

Step 3: Find the sum of the terms

The sum S_n of the first n terms of an A.P. is given by:

$$S_n = \frac{n}{2} \times [2a + (n-1)d]$$

Substitute $n = 38$, $a = 18$, and $d = 11$:

$$S_{38} = \frac{38}{2} \times [2 \times 18 + (38-1) \times 11]$$
$$S_{38} = 19 \times [36 + 37 \times 11]$$
$$S_{38} = 19 \times [36 + 407]$$
$$S_{38} = 19 \times 443 = 8417$$

Thus, the sum of the first 38 terms is $S_{38} = 8417$.

Quick Tip

The sum of an arithmetic progression can be found by using the formula $S_n = \frac{n}{2} \times [2a+(n-1)d]$, where n is the number of terms, a is the first term, and d is the common difference.

---

34. (a) Two cubes each of volume 125 cm^3 are joined end to end. Find the volume and the surface area of the resulting cuboid.

Solution:

Let the side of each cube be s.

Since the volume of a cube is given by $V = s^3$, we have:

$$s^3 = 125 \quad \Rightarrow \quad s = \sqrt[3]{125} = 5\,\text{cm}.$$

Now, when two cubes are joined end to end, the resulting cuboid will have the following dimensions: - Length $L = 2s = 2 \times 5 = 10\,\text{cm}$, - Breadth $B = s = 5\,\text{cm}$, - Height $H = s = 5\,\text{cm}$.

Thus, the volume V_{cuboid} of the resulting cuboid is:

$$V_{\text{cuboid}} = L \times B \times H = 10 \times 5 \times 5 = 250\,\text{cm}^3.$$

Next, the surface area A_{cuboid} of the cuboid is given by the formula:

$$A_{\text{cuboid}} = 2(LB + BH + HL).$$

Substituting the values of $L = 10\,\text{cm}$, $B = 5\,\text{cm}$, and $H = 5\,\text{cm}$:

$$A_{\text{cuboid}} = 2(10 \times 5 + 5 \times 5 + 5 \times 10) = 2(50 + 25 + 50) = 2 \times 125 = 250\,\text{cm}^2.$$

Thus, the volume of the cuboid is 250 cm^3 and the surface area is 250 cm^2.

Quick Tip

When two cubes are joined end to end, the volume is the sum of the volumes of the two cubes, and the surface area can be calculated based on the dimensions of the resulting cuboid.

OR

34. (b) A solid is in the shape of a cone surmounted on a hemisphere with both their diameters being equal to 7 cm and the height of the cone is equal to its radius. Find the volume of the solid.

Solution:
Given: - The radius of the hemisphere and cone is $r = \frac{7}{2} = 3.5\,\text{cm}$, - The height of the cone is $h = 3.5\,\text{cm}$.

We need to find the volume of the solid, which is the sum of the volumes of the hemisphere and the cone.

Volume of the hemisphere: The volume $V_{\text{hemisphere}}$ of a hemisphere is given by:

$$V_{\text{hemisphere}} = \frac{2}{3}\pi r^3.$$

Substituting $r = 3.5\,\text{cm}$:

$$V_{\text{hemisphere}} = \frac{2}{3}\pi(3.5)^3 = \frac{2}{3}\pi \times 42.875 = 28.583\,\text{cm}^3.$$

Volume of the cone: The volume V_{cone} of a cone is given by:

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h.$$

Substituting $r = 3.5\,\text{cm}$ and $h = 3.5\,\text{cm}$:

$$V_{\text{cone}} = \frac{1}{3}\pi(3.5)^2 \times 3.5 = \frac{1}{3}\pi \times 12.25 \times 3.5 = 14.4375\,\text{cm}^3.$$

Total volume of the solid: The total volume V_{total} is the sum of the volumes of the hemisphere and the cone:

$$V_{\text{total}} = V_{\text{hemisphere}} + V_{\text{cone}} = 28.583 + 14.4375 = 43.0205\,\text{cm}^3.$$

Thus, the volume of the solid is approximately $43.02\,\text{cm}^3$.

Quick Tip

When a cone is surmounted on a hemisphere, the total volume is the sum of the individual volumes of the cone and the hemisphere.

35(a). If BD and QM are medians of triangles ABC and PQR, respectively, where $\triangle ABC \sim \triangle PQR$, prove that:

$$\frac{AB}{PQ} = \frac{BD}{QM}.$$

Solution: Given: $\triangle ABC \sim \triangle PQR$, and BD and QM are medians of $\triangle ABC$ and $\triangle PQR$, respectively.

Since $\triangle ABC \sim \triangle PQR$, we know that the corresponding sides are proportional:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}.$$

In similar triangles, the medians to the corresponding sides are also proportional. Therefore:

$$\frac{BD}{QM} = \frac{AB}{PQ}.$$

Final Answer: Thus, $\frac{AB}{PQ} = \frac{BD}{QM}$ is proved.

Quick Tip

In similar triangles, not only the sides but also the medians, altitudes, and angle bisectors are proportional to the corresponding sides.

OR

35(b). CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$, such that D and H lie on sides AB and FE of $\triangle ABC$ and $\triangle FEG$, respectively. If $\triangle ABC \sim \triangle FEG$, show that: (i) $\frac{CD}{GH} = \frac{AC}{FG}$, (ii) $\triangle DCB \sim \triangle HGE$.

Solution:

(i) Proving $\frac{CD}{GH} = \frac{AC}{FG}$: Since $\triangle ABC \sim \triangle FEG$, the corresponding sides are proportional:

$$\frac{AB}{FE} = \frac{AC}{FG} = \frac{BC}{EG}.$$

Angle bisectors divide the opposite sides proportionally. Therefore, the corresponding angle bisectors CD and GH will also be proportional:

$$\frac{CD}{GH} = \frac{AC}{FG}.$$

(ii) Proving $\triangle DCB \sim \triangle HGE$: Since $\triangle ABC \sim \triangle FEG$, we have:

$$\angle ACB = \angle EGF \quad \text{and} \quad \frac{BC}{EG} = \frac{AC}{FG}.$$

Also, CD and GH are bisectors of $\angle ACB$ and $\angle EGF$, respectively. Therefore, the corresponding triangles $\triangle DCB$ and $\triangle HGE$ are similar:

$$\triangle DCB \sim \triangle HGE.$$

Final Answer: Thus, (i) $\frac{CD}{GH} = \frac{AC}{FG}$ and (ii) $\triangle DCB \sim \triangle HGE$ are proved.

Quick Tip

In similar triangles, angle bisectors and the triangles formed by them also maintain proportionality with corresponding sides.

Section - E

This section comprises 3 case study based questions of 4 marks each.

36. Resident Welfare Association (RWA) of Gulmohar Society in Delhi, have installed three electric poles A, B, and C in the society's common park. Despite these three poles, some parts of the park are still in the dark. So, RWA decides to have one more electric pole D in the park.

(i) What is the position of the pole C?

Solution:

To find the position of the pole C, we are given that the coordinates of point C are $(5, 4)$ (as mentioned in the problem statement). The coordinates of any point on a plane represent its position relative to the origin.

Thus, the position of the pole C is given by its coordinates:

$$C(5, 4)$$

(ii) What is the distance of the pole B from the corner O of the park?

Solution:

The coordinates of pole B are $(6, 6)$ and the origin O is at $(0, 0)$. To find the distance BO, we use the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substituting the coordinates of $B(6, 6)$ and $O(0, 0)$:

$$d = \sqrt{(6-0)^2 + (6-0)^2} = \sqrt{6^2 + 6^2} = \sqrt{36+36} = \sqrt{72}$$

Thus, the distance BO is:

$$BO = \sqrt{72}\,\text{units}$$

(iii)(a) Find the position of the fourth pole D so that the four points A, B, C, and D form a parallelogram $ABCD$.

(iii) (a) Given points: $A(2,7)$**,** $B(6,6)$**,** $C(5,4)$**. Let** $D(x,y)$ **be the fourth point.** We are asked to find the position of point D such that $ABCD$ forms a parallelogram.

Solution: In a parallelogram, the diagonals bisect each other. Thus, the midpoint of diagonal AC must be the same as the midpoint of diagonal BD.

- Coordinates of $A(2,7)$, $B(6,6)$, and $C(5,4)$ are given. - Let $D = (x,y)$ be the fourth point.

First, calculate the midpoint of diagonal AC:

$$\text{Midpoint of } AC = \left(\frac{2+5}{2}, \frac{7+4}{2}\right) = \left(\frac{7}{2}, \frac{11}{2}\right) = (3.5, 5.5)\,.$$

Now, calculate the midpoint of diagonal BD (where $B = (6,6)$ and $D = (x,y)$):

$$\text{Midpoint of } BD = \left(\frac{6+x}{2}, \frac{6+y}{2}\right).$$

Since the midpoints of diagonals AC and BD must be the same, we equate the corresponding coordinates:

For the x-coordinate:

$$\frac{6+x}{2} = 3.5 \implies 6+x = 7 \implies x = 1.$$

For the y-coordinate:

$$\frac{6+y}{2} = 5.5 \implies 6+y = 11 \implies y = 5.$$

Thus, the position of point D is:

$$D(1,5).$$

Quick Tip

To find a missing vertex of a parallelogram, use the property that diagonals bisect each other.

OR

iii(b): Find the distance between poles A and C.

Solution: To find the distance AC, we will use the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Substitute the coordinates of points $A(2, 7)$ and $C(5, 4)$ into the formula:

$$d = \sqrt{(5-2)^2 + (4-7)^2} = \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}.$$

Thus, the distance between points A and C is:

$$AC = \sqrt{18} = 3\sqrt{2}\,\text{units}.$$

Quick Tip

The distance between two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

---

37. Deepankar bought 3 notebooks and 2 pens for <80 and his friend Suryansh bought 4 notebooks and 3 pens for <110 from the school bookshop.

(i) If the price of one notebook be x and the price of one pen be y, write the given situation algebraically.

Solution: The given situation can be expressed as two linear equations:

$$3x + 2y = 80, \quad 4x + 3y = 110.$$

(ii)(a) What is the price of one notebook?

Solution: We solve the equations $3x + 2y = 80$ and $4x + 3y = 110$ using elimination.

Step 1: Multiply the first equation by 3 and the second equation by 2 to align y:

$$9x + 6y = 240, \quad 8x + 6y = 220.$$

Step 2: Subtract the second equation from the first:

$$(9x + 6y) - (8x + 6y) = 240 - 220.$$

$$x = 20.$$

Final Answer: The price of one notebook is <20.

OR

(ii)(b) What is the price of one pen?

Solution: Substitute $x = 20$ into the first equation:

$$3(20) + 2y = 80.$$

$$60 + 2y = 80.$$

$$2y = 20 \implies y = 10.$$

Final Answer: The price of one pen is <10.

(iii) What is the total amount to be paid by Suryansh, if he purchases 6 notebooks and 3 pens?

Solution: The price of one notebook is 20 and one pen is 10. The total cost is:

$$\text{Cost} = 6x + 3y.$$

Substitute $x = 20$ and $y = 10$:

$$\text{Cost} = 6(20) + 3(10).$$

$$\text{Cost} = 120 + 30 = 150.$$

Final Answer: The total amount to be paid by Suryansh is <150.

Quick Tip
For solving linear equations, use substitution or elimination methods. Verify the solution by substituting back into the equations.

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38. Mutual Fund: A mutual fund is a type of investment vehicle that pools money from multiple investors to invest in securities like stocks, bonds, or other securities. Mutual funds are operated by professional money managers, who allocate the fund's assets and attempt to produce capital gains or income for the fund's investors.

Net Asset Value (NAV) represents a fund's per share market value. It is the price at which the investors buy fund shares from a fund company and sell them to a fund company.

The following table shows the Net Asset Value (NAV) per unit of mutual fund of ICICI mutual funds:

NAV (in <)	Number of mutual funds
0 – 5	13
5 – 10	16
10 – 15	22
15 – 20	18
20 – 25	11

Based on the above information, answer the following questions:

(i) What is the upper limit of modal class of the data?

Solution: The modal class is the class interval with the highest frequency. From the table, the highest frequency is 22, corresponding to the class 10–15.

The upper limit of the modal class is:

$$15.$$

(ii) What is the median class of the data?

Solution: To determine the median class, calculate the total frequency:

$$13 + 16 + 22 + 18 + 11 = 80.$$

The median position is:

$$\frac{80}{2} = 40.$$

Now, calculate the cumulative frequency:

$$\begin{aligned} 0\text{–}5 &: 13 \\ 5\text{–}10 &: 13 + 16 = 29 \\ 10\text{–}15 &: 29 + 22 = 51 \end{aligned}$$

Since the cumulative frequency 51 includes the 40th position, the median class is:

$$10\text{–}15.$$

(iii)(a) What is the mode NAV of mutual funds?

Solution: The mode occurs in the modal class 10–15. Using the formula for mode:

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h,$$

where: - $L = 10$ (lower limit of modal class), - $f_1 = 22$ (frequency of modal class), - $f_0 = 16$ (frequency of class preceding modal class), - $f_2 = 18$ (frequency of class succeeding modal class), - $h = 5$ (class width).

Substitute the values:

$$\text{Mode} = 10 + \frac{22 - 16}{2(22) - 16 - 18} \times 5.$$

$$\text{Mode} = 10 + \frac{6}{44 - 34} \times 5.$$

$$\text{Mode} = 10 + \frac{6}{10} \times 5.$$

$$\text{Mode} = 10 + 3 = 13.$$

The mode NAV is:

$$13.$$

OR

(iii)(b) What is the median NAV of mutual funds?

Solution: The median class is 10–15. The formula for median is:

$$\text{Median} = L + \frac{\frac{N}{2} - CF}{f} \times h,$$

where: - $L = 10$ (lower limit of median class), - $N = 80$ (total frequency), - $CF = 29$ (cumulative frequency before median class), - $f = 22$ (frequency of median class), - $h = 5$ (class width).

Substitute the values:

$$\text{Median} = 10 + \frac{40 - 29}{22} \times 5.$$

$$\text{Median} = 10 + \frac{11}{22} \times 5.$$

$$\text{Median} = 10 + 2.5 = 12.5.$$

The median NAV is:

$$12.5.$$

Quick Tip

The modal class has the highest frequency. Use formulas for mode and median to calculate exact values based on grouped data.