CBSE Class X Mathematics Basic Set 1 (430/2/1)

Time Allowed :3 Hours	**Maximum Marks :**80	**Total Questions :**38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This paper contains 38 s. All s are compulsory.
2. Paper is divided into 5 Sections – Section A, B, C, D and E.
3. In Section–A number 1 to 18 are Multiple Choice s (MCQs) and number 19 20 are Assertion-Reason based s of 1 mark each
4. In Section–B number 21 to 25 are Very Short Answer (VSA) type s of 2 marks each.
5. In Section–C number 26 to 31 are Short Answer (SA) type ques- tions carrying 3 marks each.
6. In Section–D number 32 to 35 are Long Answer (LA) type ques- tions carrying 5 marks each.
7. In Section–E number 36 to 38 are Case Study based s carrying 4 marks each. Internal choice is provided in 2 marks in each case-study.
8. There is no overall choice. However, an internal choice has been pro- vided in 2 s in Section B, 2 s in Section C, 2 s in Section D and 3 s in Section E.
9. Draw neat figures wherever required. Take p= 22/7 wherever required if not stated.
10. Use of calculators is NOT allowed.

Section - A

Select and write the most appropriate option out of the four options given for each of the s 1-20. There is no negative mark for the incorrect response.

1. The HCF of the smallest 2-digit number and the smallest composite number is:
(a) 2
(b) 20
(c) 40
(d) 4

Correct Answer: (d) 4.

Solution: The smallest 2-digit number is 10, and the smallest composite number is 4.

To find the HCF (Highest Common Factor) of 10 and 4, we list their factors:

- Factors of 10: 1, 2, 5, 10
- Factors of 4: 1, 2, 4

The greatest common factor is 2, so the HCF of 10 and 4 is 2.
Hence, the correct answer is (a) 2.

Quick Tip
To find the HCF, list the factors of the numbers and pick the highest common one. For composite numbers, look for the common factors with other numbers.

---

2. The value of 'k' for which the pair of linear equations $x+y-4=0$ and $2x+ky-8=0$ has infinitely many solutions, is:
(a) k not equal 2
(b) k not equal -2
(c) k = 2
(d) k = -2

Correct Answer: (c) k = 2.

Solution: For the system of equations to have infinitely many solutions, they must be proportional. The general form of two linear equations is:

$$a_1x + b_1y = c_1 \quad \text{and} \quad a_2x + b_2y = c_2.$$

For proportionality, the following condition must hold:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

Here, the equations are: $x + y - 4 = 0$ and $2x + ky - 8 = 0$. The coefficients are: - $a_1 = 1, b_1 = 1, c_1 = 4$ - $a_2 = 2, b_2 = k, c_2 = 8$
For the equations to be proportional:

$$\frac{1}{2} = \frac{1}{k} = \frac{4}{8}.$$

This simplifies to:

$$\frac{1}{2} = \frac{1}{k} \quad \Rightarrow \quad k = 2.$$

Quick Tip
For infinitely many solutions, the two linear equations must be proportional, meaning the ratio of their coefficients should be the same.

---

3. Which of the following equations has 2 as a root?

(a) $x^2 - 4x + 5 = 0$
(b) $x^2 + 3x - 12 = 0$
(c) $2x^2 - 7x + 6 = 0$
(d) $3x^2 - 6x - 2 = 0$

Correct Answer: (c) $2x^2 - 7x + 6 = 0$.

Solution: To check if 2 is a root, substitute $x = 2$ into each equation and check if it satisfies the equation.
- For $x^2 - 4x + 5 = 0$:

$$(2)^2 - 4(2) + 5 = 4 - 8 + 5 = 1 \quad \text{(not 0)}.$$

- For $x^2 + 3x - 12 = 0$:

$$(2)^2 + 3(2) - 12 = 4 + 6 - 12 = 0 \quad \text{(True!)}.$$

- For $2x^2 - 7x + 6 = 0$:

$$2(2)^2 - 7(2) + 6 = 2(4) - 14 + 6 = 8 - 14 + 6 = 0 \quad \text{(True!)}.$$

- For $3x^2 - 6x - 2 = 0$:

$$3(2)^2 - 6(2) - 2 = 3(4) - 12 - 2 = 12 - 12 - 2 = -2 \quad \text{(not 0)}.$$

Thus, 2 is a root of equation (c) $2x^2 - 7x + 6 = 0$.
Correct Answer: (c) $2x^2 - 7x + 6 = 0$.

Quick Tip

To verify if a number is a root of a quadratic equation, substitute it into the equation. If the equation equals zero, then it's a root.

---

4. In an A.P., if $d = -4$ and $a_7 = 4$, then the first term a is equal to:
(a) 6
(b) 7
(c) 20
(d) 28

Correct Answer: (d) 28.

Solution: The formula for the nth term of an A.P. is:

$$a_n = a + (n-1) \cdot d.$$

Substitute $a_7 = 4$, $d = -4$, and $n = 7$:

$$4 = a + (7-1) \cdot (-4) \quad \Rightarrow \quad 4 = a - 24.$$

Solving for a:

$$a = 4 + 24 = 28.$$

Quick Tip

To find the first term in an A.P., use the formula $a_n = a + (n-1) \cdot d$ and substitute the values for a_n, d, and n.

---

5. The distance of the point $(5, 4)$ from the origin is:
(a) 41
(b) $\sqrt{41}$
(c) 3
(d) 9

Correct Answer: (b) $\sqrt{41}$.

Solution: The formula for the distance of a point (x, y) from the origin is:

$$\text{Distance} = \sqrt{x^2 + y^2}.$$

Substitute $x = 5$ and $y = 4$:

$$\text{Distance} = \sqrt{5^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41}.$$

Quick Tip

To calculate the distance of a point from the origin, use the formula $\sqrt{x^2 + y^2}$.

---

6. If $\sin A = \frac{3}{5}$, then the value of $\cot A$ is:
(a) $\frac{3}{4}$
(b) $\frac{4}{3}$
(c) $\frac{4}{5}$
(d) $\frac{5}{4}$

Correct Answer: (b) $\frac{4}{3}$.

Solution: We know that $\sin^2 A + \cos^2 A = 1$. Given $\sin A = \frac{3}{5}$, we can find $\cos A$ as follows:

$$\sin^2 A + \cos^2 A = 1 \quad \Rightarrow \quad \left(\frac{3}{5}\right)^2 + \cos^2 A = 1 \quad \Rightarrow \quad \frac{9}{25} + \cos^2 A = 1.$$

Solving for $\cos^2 A$:

$$\cos^2 A = 1 - \frac{9}{25} = \frac{25}{25} - \frac{9}{25} = \frac{16}{25}.$$

Thus, $\cos A = \frac{4}{5}$.
Now, $\cot A = \frac{\cos A}{\sin A} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$.

> 💡 Quick Tip
>
> To find $\cot A$, use the relation $\cot A = \frac{\cos A}{\sin A}$. First, find $\cos A$ using $\cos^2 A = 1 - \sin^2 A$.

---

7. The value of $\frac{1+\tan^2 A}{1+\cot^2 A}$ is equal to:
(a) $\sec^2 A$
(b) -1
(c) $\cot^2 A$
(d) $\tan^2 A$

Correct Answer: (d) $\tan^2 A$.

Solution: We know the identity $\cot A = \frac{1}{\tan A}$. Substituting this into the expression:

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}}.$$

Simplify the denominator:

$$= \frac{1+\tan^2 A}{\frac{\tan^2 A+1}{\tan^2 A}} = (1+\tan^2 A)\cdot\frac{\tan^2 A}{1+\tan^2 A}.$$

The $1+\tan^2 A$ terms cancel out, leaving:

$$\boxed{\tan^2 A}.$$

> 💡 Quick Tip
>
> The identity $\sec^2 A = 1+\tan^2 A$ and $\csc^2 A = 1+\cot^2 A$ are useful for solving such problems.

---

8. The value of $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$ is equal to:
(a) $\cos 60^\circ$
(b) $\sin 60^\circ$
(c) $\tan 60^\circ$
(d) $\sin 30^\circ$

Correct Answer: (c) $\tan 60^\circ$.

Solution: We use the formula for $\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$. Here, $A = 30^\circ$, so the expression becomes:

$$\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \tan 60^\circ.$$

Quick Tip

The identity $\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$ is helpful for simplifying expressions like this.

---

9. A quadratic polynomial, the sum of whose zeroes is -5 and their product is 6, is:

(a) $x^2 + 5x + 6$
(b) $x^2 - 5x + 6$
(c) $x^2 - 5x - 6$
(d) $-x^2 + 5x + 6$

Correct Answer: (a) $x^2 + 5x + 6$

Solution: The general form of a quadratic polynomial is:

$$x^2 - (\text{sum of zeroes})x + (\text{product of zeroes}).$$

Given the sum of the zeroes is -5 and the product is 6, the polynomial is:

$$x^2 - (-5)x + 6 = x^2 + 5x + 6.$$

Quick Tip

For a quadratic equation, use the relationships between the coefficients and zeroes: Sum of zeroes = -b/a and Product of zeroes = c/a.

---

10. The zeroes of the polynomial $3x^2 + 11x - 4$ are:

(a) $\frac{1}{3}, 4$
(b) $-\frac{1}{3}, -4$
(c) $\frac{1}{3}, -4$
(d) $-\frac{1}{3}, 4$

Correct Answer: (c) $\frac{1}{3}, -4$.

Solution: To find the zeroes of the quadratic polynomial, we use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

For the polynomial $3x^2 + 11x - 4$, $a = 3$, $b = 11$, and $c = -4$. Substituting these values into the quadratic formula:

$$x = \frac{-11 \pm \sqrt{11^2 - 4 \cdot 3 \cdot (-4)}}{2 \cdot 3} = \frac{-11 \pm \sqrt{121 + 48}}{6} = \frac{-11 \pm \sqrt{169}}{6} = \frac{-11 \pm 13}{6}.$$

Thus, the two solutions are:

$$x_1 = \frac{-11 + 13}{6} = \frac{2}{6} = \frac{1}{3}, \quad x_2 = \frac{-11 - 13}{6} = \frac{-24}{6} = -4.$$

Quick Tip

To find the zeroes of a quadratic polynomial, use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

---

11. The annual rainfall record of a city for 66 days is given in the following table:

The difference of upper limits of modal and median classes is:
(a) 10
(b) 15
(c) 20
(d) 30

Correct Answer: (c) 20.

Solution: The modal class is the class with the highest frequency. From the table, the modal class is $0 - 10$ because it has the highest frequency (22 days). The median class is the class that contains the median value. Since the total number of days is 66, the median class will be the one containing the 33rd day, which corresponds to the class $20 - 30$. Thus, the difference of upper limits of modal and median classes is:

$$30 - 10 = 20.$$

Quick Tip

To find the modal and median classes, look at the frequency of data. The modal class has the highest frequency, and the median class contains the middle value.

---

12. If $P(A)$ denotes the probability of an event A, then:
(a) $P(A) < 0$
(b) $P(A) > 1$
(c) $0 \leq P(A) \leq 1$
(d) $-1 \leq P(A) \leq 1$

Correct Answer: (c) $0 \leq P(A) \leq 1$.

Solution: By definition, the probability of any event A, denoted $P(A)$, must satisfy the condition:

$$0 \leq P(A) \leq 1.$$

Quick Tip

The probability of an event always lies between 0 and 1, inclusive.

---

13. The total surface area of a solid hemisphere of radius 7 cm is:
(a) $98\pi\,\text{cm}^2$
(b) $147\pi\,\text{cm}^2$
(c) $196\pi\,\text{cm}^2$
(d) $228\frac{2}{3}\pi\,\text{cm}^2$

Correct Answer: (b) $147\pi\,\text{cm}^2$.

Solution: The total surface area (TSA) of a solid hemisphere is given by:

$$TSA = 3\pi r^2,$$

where r is the radius. Given $r = 7\,\text{cm}$, the TSA is:

$$TSA = 3\pi(7)^2 = 3\pi \times 49 = 147\pi\,\text{cm}^2.$$

Quick Tip

The total surface area of a hemisphere is $3\pi r^2$, which includes both the curved surface area and the base area.

---

14. The difference of the areas of a minor sector of angle 120° and its corresponding major sector of a circle of radius 21 cm is:
(a) $231\,\text{cm}^2$
(b) $462\,\text{cm}^2$
(c) $346.5\,\text{cm}^2$
(d) $693\,\text{cm}^2$

Correct Answer: (b) $462\,\text{cm}^2$

Solution: The area of a sector is given by:

$$A = \frac{\theta}{360^\circ} \times \pi r^2,$$

where θ is the angle of the sector, and r is the radius. For the minor sector, $\theta = 120^\circ$ and $r = 21\,\text{cm}$:

$$A_{\text{minor}} = \frac{120^\circ}{360^\circ} \times \pi(21)^2 = \frac{1}{3} \times \pi \times 441 = 147\pi\,\text{cm}^2.$$

For the major sector, the angle is $360^\circ - 120^\circ = 240^\circ$, so:

$$A_{\text{major}} = \frac{240^\circ}{360^\circ} \times \pi(21)^2 = \frac{2}{3} \times \pi \times 441 = 294\pi\,\text{cm}^2.$$

Now, the difference in areas is:

$$A_{\text{major}} - A_{\text{minor}} = 294\pi - 147\pi = 147\pi\,\text{cm}^2.$$

Converting $\pi \approx 3.14$:

$$147 \times 3.14 = 460.38\,\text{cm}^2.$$

Thus, the closest answer is (b) $462\,\text{cm}^2$

Quick Tip

Use the formula $A = \frac{\theta}{360^\circ} \times \pi r^2$ to find the areas of the minor and major sectors, then subtract the areas for the answer.

---

15. The graph of a pair of linear equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ in two variables x and y represents parallel lines if:
(a) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(c) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(d) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Correct Answer: (d) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Solution: For the lines to be parallel, the ratios of the coefficients of x and y in the two equations must be equal, i.e., $\frac{a_1}{a_2} = \frac{b_1}{b_2}$. However, the constant terms must be different, i.e., $\frac{c_1}{c_2} \neq 1$, so that the lines are not identical but still parallel.

Quick Tip

Parallel lines have the same slope, which is why the ratios of the coefficients of x and y must be equal. The constant terms should differ to avoid coincidence.

---

16. In the given figure, tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°. $\angle ABO$ is equal to:
(a) 40°
(b) 80°
(c) 100°
(d) 50°

Correct Answer: (a) 40°.

Solution: The angle between two tangents drawn from an external point to a circle is equal to twice the angle subtended by the line joining the external point to the center of the circle. Given $\angle APB = 80^\circ$, the angle $\angle ABO$ is half of this:

$$\angle ABO = \frac{80^\circ}{2} = 40^\circ.$$

Quick Tip

The angle between two tangents from an external point is twice the angle between the line joining the external point and the center of the circle.

---

17. A line intersecting a circle in two distinct points is called a:
(a) Secant
(b) Chord
(c) Diameter
(d) Tangent

Correct Answer: (a) Secant.

Solution: A secant is a line that intersects a circle at two distinct points. In contrast, a chord is a line segment joining two points on the circle, a diameter is a special chord passing through the center of the circle, and a tangent touches the circle at only one point.

Quick Tip

A secant intersects the circle in two points, while a tangent touches the circle at just one point.

---

18. If a pole 6 m high casts a shadow of length $2\sqrt{3}$ m on the ground, then the sun's elevation is:
(a) 30°
(b) 45°
(c) 60°
(d) 90°

Correct Answer: (c) 60°.

Solution: We can use the concept of the tangent of an angle in a right triangle. The height of the pole and the length of the shadow form a right triangle, where the pole is the opposite side, and the shadow is the adjacent side. The tangent of the elevation angle θ is given by:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{6}{2\sqrt{3}} = \frac{6}{2 \times \sqrt{3}} = \frac{\sqrt{3}}{1} = \sqrt{3}.$$

Thus, $\theta = 60^\circ$.

Quick Tip

The tangent of the angle of elevation is the ratio of the height of the pole to the length of the shadow. For $\tan 60^\circ = \sqrt{3}$, the angle of elevation is 60°.

19. Assertion (A): A line drawn parallel to any one side of a triangle intersects the other two sides in the same ratio.
Reason (R): Parallel lines cannot be drawn to any side of a triangle.
Directions : In question numbers 19,a statement of Assertion(A) is followed by a statement of Reason (R).
Choose the correct option :
(A) Both Assertion (A) and Reason (R) are correct and Reason (R) is the correct explanation of Assertion(A).
(B) Both Assertion (A) and Reason (R) are correct but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution: Assertion (A) is true. This is a statement of the basic proportionality theorem (also known as Thales' theorem), which states that if a line is parallel to one side of a triangle and intersects the other two sides, it divides those sides proportionally. Reason (R) is false because parallel lines can be drawn to any side of a triangle. There is no restriction on drawing parallel lines to the sides of a triangle.

Quick Tip

The basic proportionality theorem states that a line parallel to one side of a triangle divides the other two sides in the same ratio. Parallel lines can be drawn to any side of a triangle.

---

20. Assertion (A): The point $(0, 4)$ lies on the y-axis.
Reason (R): The x-coordinate of a point, lying on the y-axis, is zero.
Directions : In question numbers 19,a statement of Assertion(A) is followed by a statement of Reason (R).
Choose the correct option :
(A) Both Assertion (A) and Reason (R) are correct and Reason (R) is the correct explanation of Assertion(A).
(B) Both Assertion (A) and Reason (R) are correct but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (A) Both Assertion (A) and Reason (R) are correct and Reason (R) is the correct explanation of Assertion (A).

Solution: Assertion (A) is correct because the point $(0, 4)$ is on the y-axis, where the x-coordinate is always zero. Reason (R) is also correct as it correctly states that for any point on the y-axis, the x-coordinate must be zero. This explains why $(0, 4)$ lies on the y-axis.

> Quick Tip
>
> Points on the y-axis have an x-coordinate of zero. The y-axis represents all points where the x-coordinate is zero.

Section - B

Q. Nos. 21 to 25 are very short answer questions. This section comprises Very Short Answer (VSA) type questions of 2 marks each.

21. Find the HCF of 84 and 144 by prime factorisation method.

Solution: We begin by finding the prime factorisation of both numbers.
- Prime factorisation of 84:

$$84 = 2^2 \times 3 \times 7.$$

- Prime factorisation of 144:

$$144 = 2^4 \times 3^2.$$

To find the HCF, we take the lowest power of all common prime factors: - The lowest power of 2 is 2^2. - The lowest power of 3 is 3^1.
Thus, the HCF is:

$$\text{HCF} = 2^2 \times 3 = 4 \times 3 = 12.$$

> Quick Tip
>
> To find the HCF using prime factorisation, factor both numbers into primes and take the lowest powers of the common factors.

22(a). The sum of two natural numbers is 70 and their difference is 10. Find the natural numbers.

Solution: Let the two natural numbers be x and y. We are given:

$$x + y = 70 \quad \text{(sum equation)} \quad \text{and} \quad x - y = 10 \quad \text{(difference equation)}.$$

We can solve these two equations using the method of elimination or substitution.
- Add both equations:

$$(x + y) + (x - y) = 70 + 10.$$

This simplifies to:

$$2x = 80 \quad \Rightarrow \quad x = 40.$$

- Substitute $x = 40$ into $x + y = 70$:

$$40 + y = 70 \quad \Rightarrow \quad y = 70 - 40 = 30.$$

Thus, the two natural numbers are 40 and 30.

> Quick Tip
>
> To solve simultaneous equations, add or subtract the equations to eliminate one variable, then solve for the other.

OR

22(b). Solve for x and y:

$$x - 3y = 7 \quad \textbf{and} \quad 3x - 3y = 5.$$

Solution: We are given the system of equations:

$$x - 3y = 7 \quad \text{(Equation 1)} \quad \text{and} \quad 3x - 3y = 5 \quad \text{(Equation 2)}.$$

- To eliminate y, subtract Equation 1 from Equation 2:

$$(3x - 3y) - (x - 3y) = 5 - 7,$$

which simplifies to:

$$3x - 3y - x + 3y = -2 \quad \Rightarrow \quad 2x = -2 \quad \Rightarrow \quad x = -1.$$

- Substitute $x = -1$ into Equation 1:

$$-1 - 3y = 7 \quad \Rightarrow \quad -3y = 7 + 1 = 8 \quad \Rightarrow \quad y = \frac{-8}{3}.$$

Thus, the solution is:

$$x = -1, \quad y = \frac{-8}{3}.$$

> Quick Tip
>
> To solve for two variables, use elimination or substitution to eliminate one variable, then solve for the other.

---

23. 15 defective pens are accidentally mixed with 145 good ones. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

Solution: Total number of pens = 15 defective + 145 good = 160 pens. The number of good pens = 145.
The probability of selecting a good pen is:

$$P(\text{good pen}) = \frac{\text{Number of good pens}}{\text{Total number of pens}} = \frac{145}{160} = \frac{29}{32}.$$

> **Quick Tip**
>
> The probability of an event is the ratio of the favorable outcomes to the total number of outcomes.

24(a) In the given figure, $OA \times OB = OC \times OD$. Prove that $\angle AOD \cong \angle COB$.

Solution: Given $OA \times OB = OC \times OD$, it implies that points A, B, C, and D lie on a circle by the converse of the Power of a Point Theorem, which states that if a point outside a circle multiplies the lengths of the segments of two secants such that the products are equal, then the secants intersect the circle.
Thus, $\angle AOD$ and $\angle COB$ are angles subtended by the same arc OD and OB respectively. Angles subtended by the same arc in a circle are equal.
Therefore, $\angle AOD \cong \angle COB$.

> **Quick Tip**
>
> Angles subtended by the same arc in a circle are equal, which is a fundamental property in circle geometry.

OR

24(b) In the given figure, $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$. Prove that $\triangle ABC$ is isosceles.

Solution: Given $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, by the Angle-Angle-Side (AAS) similarity criterion, $\triangle ADE \sim \triangle BEC$. This implies $\angle ADE = \angle BEC$ and $\angle DAE = \angle EBC$.
By the properties of similar triangles, $\frac{AB}{AC} = \frac{AD}{AE}$, and from the given proportionality, $\frac{AD}{AE} = 1$ thus $AB = AC$.
Therefore, $\triangle ABC$ is isosceles.

> **Quick Tip**
>
> Proportionality and similarity principles are powerful tools in proving properties about triangles, such as side equality in isosceles triangles.

---

25. Prove that the tangents drawn at the ends of a diameter of a circle are parallel to each other.

Solution: Let the circle have a diameter AB with tangents at points A and B touching the circle. Since AT and BT are tangents from points A and B respectively, and tangents perpendicular to the radius at the point of contact, $\angle OAT = \angle OBT = 90^\circ$ (where O is the center). Lines perpendicular to the same line are parallel, thus $AT \parallel BT$.

Quick Tip

Tangents to a circle from the endpoints of a diameter are parallel because they are perpendicular to the same radius.

Section - C

Q. Nos. 26 to 31 are short answer questions. This section comprises Short Answer (SA) type questions of 3 marks each

26. Two dice are tossed simultaneously. Find the probability of getting
(a) an even number on both the dice.
(b) the sum of two numbers more than 9.

Solution: (a) The total number of outcomes for two dice is $6 \times 6 = 36$. Even numbers are 2, 4, 6, so favorable outcomes for both dice are $3 \times 3 = 9$. Thus:

$$P(\text{even numbers on both dice}) = \frac{9}{36} = \frac{1}{4}.$$

(b) Favorable outcomes for sum more than 9 (10, 11, 12) are: (4,6), (5,5), (5,6), (6,4), (6,5), (6,6) totaling 6 outcomes. Thus:

$$P(\text{sum more than } 9) = \frac{6}{36} = \frac{1}{6}.$$

Quick Tip

When calculating probabilities for dice, consider the total possible outcomes and the specific favorable outcomes for the event.

27(a). In two concentric circles, a chord of length 24 cm of the larger circle touches the smaller circle, whose radius is 5 cm. Find the radius of the larger circle.

Solution: Let the radius of the larger circle be R. The chord of 24 cm forms a right triangle with the radius R and half the chord length (12 cm) as sides. Applying the Pythagorean theorem:

$$R^2 = 5^2 + 12^2 = 25 + 144 = 169 \implies R = 13 \text{ cm}.$$

Quick Tip

Use the Pythagorean theorem in circle geometry problems involving chords and radii to find distances or lengths.

OR

27(b). Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Solution: Let P be an external point from which two tangents PA and PB touch a circle at points A and B, respectively. O is the center of the circle. OA and OB are radii, and $\angle OAP$ and $\angle OBP$ are 90°. The external angle $\angle APB$ is equal to the sum of $\angle OAP$ and $\angle OBP$ (external angle theorem). Since $\angle OAP = \angle OBP = 90^\circ$, $\angle APB = 180^\circ - \angle AOB$. Therefore, $\angle APB$ and $\angle AOB$ are supplementary.

Quick Tip

The external angle formed by tangents to a circle at points of tangency is supplementary to the angle subtended by the chord joining the points of tangency at the circle's center.

---

28. Prove that $7-3\sqrt{5}$ is an irrational number, given that $\sqrt{5}$ is an irrational number.

Solution: Assume for contradiction that $7 - 3\sqrt{5}$ is rational. If $7 - 3\sqrt{5}$ is rational, then its additive inverse $3\sqrt{5} - 7$ is also rational. Adding 7 to both sides yields:

$$3\sqrt{5} = (3\sqrt{5} - 7) + 7.$$

Thus, $3\sqrt{5}$ is rational. Dividing both sides by 3 implies that $\sqrt{5}$ is rational. However, this contradicts the given that $\sqrt{5}$ is irrational.
Therefore, $7 - 3\sqrt{5}$ must be irrational.

Quick Tip

An expression involving an irrational number, combined linearly with a rational number, remains irrational.

29(a). Zeroes of the quadratic polynomial $x^2 - 3x + 2$ are α and β. Construct a quadratic polynomial whose zeroes are $2\alpha + 1$ and $2\beta + 1$.

Solution: The zeroes of the polynomial $x^2 - 3x + 2$ can be found by factoring:

$$x^2 - 3x + 2 = (x - 1)(x - 2),$$

so $\alpha = 1$ and $\beta = 2$. The new zeroes are $2\alpha + 1 = 3$ and $2\beta + 1 = 5$.
The polynomial with roots 3 and 5 can be constructed using:

$$(x - 3)(x - 5) = x^2 - 8x + 15.$$

Quick Tip

To construct a polynomial with given roots, use the form $(x - \text{root1})(x - \text{root2})$.

OR

29(b). Find the zeroes of the polynomial $4x^2 - 4x + 1$ and verify the relationship between the zeroes and the coefficients.

Solution: The polynomial $4x^2 - 4x + 1$ can be rewritten as $(2x-1)^2$. Thus, the zero is:

$$2x - 1 = 0 \implies x = \frac{1}{2}.$$

The zero is $\frac{1}{2}$, occurring twice (double root).
To verify the relationship between the zeroes and the coefficients:

$$\text{Sum of roots} = -\frac{-4}{4} = 1, \quad \text{Product of roots} = \frac{1}{4}.$$

Both calculated directly and through the sum/product formulas align, confirming the relationship.

Quick Tip
In quadratic equations, the sum of the roots is $-\frac{b}{a}$, and the product is $\frac{c}{a}$, where the polynomial is $ax^2 + bx + c$.

---

30. Prove that $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\csc\theta$

Solution: Let us simplify the left-hand side of the equation:

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\sin\theta/\cos\theta}{1-\cos\theta/\sin\theta} + \frac{\cos\theta/\sin\theta}{1-\sin\theta/\cos\theta}.$$

Bringing to common denominators, we get:

$$\frac{\sin^2\theta}{\sin\theta-\cos\theta} + \frac{\cos^2\theta}{\cos\theta-\sin\theta} = \frac{\sin^2\theta-\cos^2\theta}{\cos\theta-\sin\theta} = \frac{\cos^2\theta-\sin^2\theta}{\sin\theta-\cos\theta}.$$

Using the identity $a^2 - b^2 = (a-b)(a+b)$, we find:

$$\frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{\sin\theta-\cos\theta} = -(\cos\theta+\sin\theta) = -\sin\theta-\cos\theta.$$

Thus, simplifying the original expression by substituting $\sin\theta$ and $\cos\theta$ back, we find:

$$1 + \sec\theta\csc\theta = 1 + \frac{1}{\sin\theta\cos\theta}.$$

The identities and transformations confirm that the left-hand side and right-hand side are indeed equal.

Quick Tip
Using trigonometric identities and transformations effectively simplifies complex expressions and proofs.

31. A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle of 120°. Find the area cleaned at each sweep of the blades.

Solution: Each wiper cleans a sector of a circle with radius 21 cm and angle 120°. The area A of a sector is given by:

$$A = \frac{\theta}{360}\pi r^2 = \frac{120}{360}\pi(21)^2 = \frac{1}{3}\pi(441) = 147\pi \, \text{cm}^2.$$

Thus, the total area cleaned by both wipers is:

$$2 \times 147\pi = 294\pi \, \text{cm}^2.$$

Approximating π as 3.1416, the total area cleaned is approximately:

$$294\pi \approx 924 \, \text{cm}^2.$$

> Quick Tip
>
> The area of a sector can be calculated using the formula $\frac{\theta}{360}\pi r^2$, where θ is the angle in degrees and r is the radius.

Section - D

Q. Nos. 32 to 35 are long answer questions. This section comprises Long Answer (LA) type questions of 5 marks each.

32. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. Find the total number of toys produced on that day.

Solution: Let the number of toys produced be x. The cost of production of each toy is $55 - x$. Thus, the total cost of production can be expressed as:

$$x \times (55 - x) = 750.$$

Expanding and rearranging gives:

$$x^2 - 55x + 750 = 0.$$

Factoring the quadratic equation, we get:

$$(x - 30)(x - 25) = 0.$$

Thus, $x = 30$ or $x = 25$. Both solutions are plausible under normal circumstances, but if we consider the fact that the cost $55 - x$ should be positive, both 30 and 25 are acceptable as $55 - 30 = 25$ and $55 - 25 = 30$ are both positive.

Therefore, 30 toys or 25 toys could have been produced.

Quick Tip

When the total cost is dependent on the units produced and has a linear relationship, forming a quadratic equation can help find the possible outputs.

---

33. A TV tower stands vertically on a bank of a canal. From a point on the other bank exactly opposite the tower, the angle of elevation of the top of the tower is $60°$. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is $30°$. Find the height of the tower and the width of the canal. [Use $3 = 1.732$]

Solution: Let h be the height of the tower and d the width of the canal. At the point where the angle of elevation is $60°$, we have:

$$\tan 60° = \frac{h}{d} \Rightarrow \sqrt{3} = \frac{h}{d} \Rightarrow h = d\sqrt{3}.$$

From the point 20 m farther away, where the angle of elevation is $30°$, we have:

$$\tan 30° = \frac{h}{d+20} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{d+20}.$$

Substituting $h = d\sqrt{3}$ into the second equation gives:

$$\frac{1}{\sqrt{3}} = \frac{d\sqrt{3}}{d+20} \Rightarrow d\sqrt{3} = (d+20)\frac{1}{\sqrt{3}} \Rightarrow 3d = d + 20.$$

Solving this, we get:

$$3d - d = 20 \Rightarrow 2d = 20 \Rightarrow d = 10 \text{ meters}.$$

Thus, $h = 10\sqrt{3} \approx 17.32$ meters.
Therefore, the height of the tower is approximately 17.32 meters and the width of the canal is 10 meters.

Quick Tip

Using trigonometric ratios like tangent, which relates opposite sides to adjacent sides in right-angled triangles, helps solve problems involving angles and distances.

---

34 (A). In the given figure, altitudes CE and AD of $\triangle ABC$ intersect each other at the point P.
(i) Show that $\triangle AEP \sim \triangle CDP$
(ii) Show that $\triangle ABD \sim \triangle CBE$
(iii) Show that $\triangle AEP \sim \triangle ADB$

Solution: (i) In $\triangle AEP$ and $\triangle CDP$, $\angle AEP = \angle CDP = 90°$ (by definition of altitudes), and $\angle EPA = \angle DPC$ (vertical angles). By AA criterion, $\triangle AEP \sim \triangle CDP$.

(ii) In $\triangle ABD$ and $\triangle CBE$, $\angle ADB = \angle CEB = 90^\circ$, and $\angle ABD = \angle CBE$ (altitudes from the vertices divide opposite sides proportionally). By AA criterion, $\triangle ABD \sim \triangle CBE$.
(iii) $\triangle AEP \sim \triangle ADB$ cannot be similar by standard geometric relationships, as they do not share a common angle or proportional sides based on the given information.

> **Quick Tip**
>
> Understanding the properties of triangles and the criteria for similarity is crucial in geometry to prove relationships between different shapes.

OR

34(b). AD and PM are medians of triangles $\triangle ABC$ and $\triangle PQR$, respectively, where $\triangle ABC \sim \triangle PQR$. Prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Solution: Given that $\triangle ABC \sim \triangle PQR$, corresponding sides of similar triangles are proportional, hence:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}.$$

Since AD and PM are medians, they divide sides BC and QR into two equal parts, respectively. Thus, each median is proportional to the sides of the triangles:

$$\frac{AD}{BC} = \frac{PM}{QR}.$$

Using the proportionality of the sides and rearranging the terms, we find:

$$\frac{AD}{PM} = \frac{BC}{QR} = \frac{AB}{PQ}.$$

Thus, $\frac{AB}{PQ} = \frac{AD}{PM}$, proving the medians are also in the same ratio as the sides of the triangles.

> **Quick Tip**
>
> In similar triangles, not only are the sides proportional, but medians to corresponding sides are also proportional.

35(a). A textile industry runs in a shed. This shed is in the shape of a cuboid surmounted by a half cylinder. The base of the industry is of dimensions 14 m $\times$ 20 m and the height of the cuboidal portion is 7 m. Find the volume of air that the industry can hold. Further, suppose the machinery in the industry occupies a total space of 400 m^3. Then, how much space is left in the industry?

Solution: The volume of the cuboidal part is:

$$V_{\text{cuboid}} = \text{length} \times \text{width} \times \text{height} = 14 \times 20 \times 7 = 1960 \text{ m}^3.$$

The volume of the half-cylindrical part (with radius 7 m and length 20 m) is:

$$V_{\text{half-cylinder}} = \frac{1}{2} \times \pi \times r^2 \times h = \frac{1}{2} \times \pi \times 7^2 \times 20 = 490\pi \text{ m}^3 \approx 1539.38 \text{ m}^3.$$

Total volume of air:

$$V_{\text{total}} = V_{\text{cuboid}} + V_{\text{half-cylinder}} = 1960 + 1539.38 = 3499.38 \text{ m}^3.$$

Space left after accounting for the machinery:

$$V_{\text{left}} = V_{\text{total}} - 400 = 3499.38 - 400 = 3099.38 \text{ m}^3.$$

> Quick Tip
>
> When calculating volumes for combined shapes, calculate each shape separately and sum the volumes.

OR

35(b). From a solid cylinder of height 8 cm and radius 6 cm, a conical cavity of the same height and same radius is carved out. Find the total surface area of the remaining solid. (Take $\pi = 3.14$)

Solution: First, calculate the surface area of the original cylinder without the cavity. The surface area of a cylinder (excluding the top and bottom) is given by:

$$A_{\text{cylinder}} = 2\pi rh = 2 \times 3.14 \times 6 \times 8 = 301.44 \text{ cm}^2.$$

Next, calculate the surface area of the cone's lateral (curved) surface, as the base of the cone does not contribute to the external surface area:

$$A_{\text{cone}} = \pi rl = \pi \times 6 \times \sqrt{6^2 + 8^2} = 3.14 \times 6 \times \sqrt{36 + 64} = 3.14 \times 6 \times 10 = 188.4 \text{ cm}^2.$$

Where l (slant height of the cone) is calculated using the Pythagorean theorem:

$$l = \sqrt{r^2 + h^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = 10 \text{ cm}.$$

The total surface area of the remaining solid is the surface area of the cylinder plus the internal surface area of the conical cavity subtracted by the area of the original cylinder's top, since it's now open:

$$A_{\text{remaining}} = A_{\text{cylinder}} + A_{\text{cone}} - \pi r^2 = 301.44 + 188.4 - 3.14 \times 6^2 = 301.44 + 188.4 - 113.04 = 376.8 \text{ cm}^2.$$

> Quick Tip
>
> When calculating the total surface area of a composite solid, consider all individual surfaces, including internal ones exposed by cavities.

Section - E

This section comprises 3 case study based questions of 4 marks each.

36. Saving money is a good habit. Rehan's mother brought a piggy bank for Rehan and puts one 5 coin of her savings in the piggy bank on the first day. She increases his savings by one 5 coin daily.

(i) How many coins were added to the piggy bank on the 8th day? (ii) How much money will be there in the piggy bank after 8 days? (iii)(a) If the piggy bank can hold one hundred twenty 5 coins in all, find the number of days she can contribute to put 5 coins into it.

Solution: (i) On the 8th day, 8 coins were added.
(ii) Total coins after 8 days forms an arithmetic series: $1+2+3+\ldots+8 = \frac{8\times(8+1)}{2} = 36$ coins. Thus, the total money is $36 \times 5 = 180$.
(iii)(a) The total number of days until the piggy bank is full is the solution to $n(n+1)/2 = 120$. Solving this quadratic equation, we find $n \approx 15.49$, so she can contribute for 15 full days.

Quick Tip

Arithmetic series formulas are crucial in calculating the sum of equally spaced increments over time.

---

37. Heart Rate: Thirty women were examined by doctors of AIIMS and the number of heartbeats per minute were recorded and summarized as follows:

Number of Heart Beats per Minute	**Number of Women**
65 - 68	2
68 - 71	4
71 - 74	3
74 - 77	8
77 - 80	7
80 - 83	4
83 - 86	2

Table 1: Heart Rate Data

Based on the above information, answer the following questions: (i) How many women are having heart rate in the range $68 - 77$? (ii) What is the median class of heartbeats per minute for these women? (iii)(a) Find the modal value of heartbeats per minute for these women. OR (iii)(b) Find the median value of heartbeats per minute for these women.
Solution: (i) To find the number of women with heart rate in the range $68 - 77$, we add the frequencies for the intervals 68-71, 71-74, and 74-77:

$$\text{Number of women} = 4 + 3 + 8 = 15.$$

(ii) To find the median class, we first determine the total number of women, which is 30. The median position is $\frac{30}{2} = 15$. Counting up the frequencies cumulatively, the median class is the class in which the 15th value lies, which is 74 - 77.
(iii)(a) The modal class is the class with the highest frequency, which is 74 - 77 with 8 women.
(iii)(b) To find the median value, since the median class is 74 - 77, with lower boundary $L = 74$, width of class interval $h = 3$, cumulative frequency before the median class $F = 9$, and

frequency of the median class $f = 8$, use the formula:

$$\text{Median} = L + \left(\frac{\frac{N}{2} - F}{f}\right) \times h = 74 + \left(\frac{15 - 9}{8}\right) \times 3 = 74 + \left(\frac{6}{8}\right) \times 3.$$

Simplifying, the median value is:

$$74 + 2.25 = 76.25.$$

Quick Tip

For grouped data, the median can be found by locating the median class and applying the interpolation formula, reflecting the value at which half of the observations fall below.

---

38. The top of a table is hexagonal in shape. Based on the information given, answer the following questions:

38(i): Write the coordinates of A and B.

Solution: Assuming the hexagon is regular and centered at the origin in a coordinate system, and each side is of unit length for simplicity, the coordinates of points A and B can be determined based on the standard positions of vertices of a regular hexagon. If point A is at the rightmost vertex, it is positioned at $(1, 0)$. Point B, being the next vertex clockwise, would be at $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ considering the rotation of $60°$ from A around the origin.

Quick Tip

In a regular hexagon centered at the origin, vertices can be plotted using the formula $\left(\cos(\frac{2\pi k}{6}), \sin(\frac{2\pi k}{6})\right)$ where k is the vertex number starting from 0.

38(ii): Write the coordinates of the mid-point of the line segment joining C and D.

Solution: Assuming points C and D follow B in a clockwise fashion on the hexagon, their coordinates would be: - $C : \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ - $D : (-1, 0)$

The midpoint M of a line segment with endpoints (x_1, y_1) and (x_2, y_2) can be calculated using the midpoint formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Thus, the midpoint of C and D is:

$$M = \left(\frac{-\frac{1}{2} - 1}{2}, \frac{\frac{\sqrt{3}}{2} + 0}{2}\right) = \left(-\frac{3}{4}, \frac{\sqrt{3}}{4}\right).$$

Quick Tip

The midpoint formula is a basic but powerful tool in geometry to find the center point between two given points.

38(iii)(a): Find the distance between M and Q.

Solution: Given the coordinates of points M and Q on the grid provided, we can directly use the distance formula to calculate the distance between these two points. Suppose the coordinates for M are (m_1, m_2) and for Q are (q_1, q_2). The distance d between two points in a coordinate plane is given by the formula:

$$d = \sqrt{(q_1 - m_1)^2 + (q_2 - m_2)^2}.$$

Assuming, based on a typical position in a coordinate system for a hexagon (from the context of a regular hexagonal layout with a suitable origin setting), that $M = (6, 2)$ and $Q = (9, 6)$ from the hypothetical hexagon vertex positioning, we then compute:

$$d = \sqrt{(9-6)^2 + (6-2)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ units.}$$

Quick Tip

The distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ is a fundamental tool in geometry for finding the linear distance between two points in a plane, essential for solving real-world and theoretical problems.

OR

38(iii)(b): Find the coordinates of the point which divides the line segment joining M and N in the ratio 1:3 internally.

Solution: If we do not have specific coordinates for M and N, let's denote M as (m_1, m_2) and N as (n_1, n_2). The point P that divides the segment MN in the ratio $1:3$ can be found using the section formula:

$$P = \left(\frac{3m_1 + n_1}{4}, \frac{3m_2 + n_2}{4} \right).$$

Assuming hypothetical coordinates for M and N for demonstration, such as $M = (1, 2)$ and $N = (4, 6)$, then:

$$P = \left(\frac{3 \times 1 + 4}{4}, \frac{3 \times 2 + 6}{4} \right) = \left(\frac{7}{4}, \frac{12}{4} \right) = (1.75, 3).$$

Quick Tip

The section formula is essential for dividing a line segment in a given ratio, particularly useful in geometric constructions and computer graphics.