CBSE Class X Mathematics Basic Set 1 (430/3/1)

Time Allowed :3 Hours	**Maximum Marks** :80	**Total Questions** :38

General Instructions

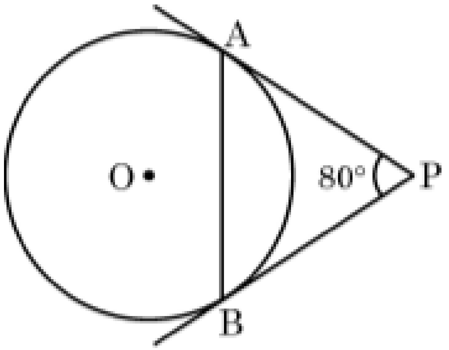
Read the following instructions very carefully and strictly follow them:

1. This paper contains 38 s. All s are compulsory.
2. Paper is divided into 5 Sections – Section A, B, C, D and E.
3. In Section–A number 1 to 18 are Multiple Choice s (MCQs) and number 19 20 are Assertion-Reason based s of 1 mark each
4. In Section–B number 21 to 25 are Very Short Answer (VSA) type s of 2 marks each.
5. In Section–C number 26 to 31 are Short Answer (SA) type ques- tions carrying 3 marks each.
6. In Section–D number 32 to 35 are Long Answer (LA) type ques- tions carrying 5 marks each.
7. In Section–E number 36 to 38 are Case Study based s carrying 4 marks each. Internal choice is provided in 2 marks in each case-study.
8. There is no overall choice. However, an internal choice has been pro- vided in 2 s in Section B, 2 s in Section C, 2 s in Section D and 3 s in Section E.
9. Draw neat figures wherever required. Take p= 22/7 wherever required if not stated.
10. Use of calculators is NOT allowed.

Section - A

Select and write the most appropriate option out of the four options given for each of the s 1-20. There is no negative mark for the incorrect response.

1. In the given figure, tangents PA and PB drawn from P to circle are inclined to each other at an angle of 80°. The measure of ∠PAB is:
(a) 80°
(b) 60°
(c) 50°
(d) 40°



Correct Answer: (d) 40°.

Solution: The angle between two tangents drawn from an external point to a circle is half of the angle subtended by the chord at the center. Given that the angle between the tangents is 80°, the measure of PAB is:

$$\angle PAB = \frac{1}{2} \times 80^{\circ} = 40^{\circ}$$

Hence, the correct answer is (d) 40°.

Quick Tip

When two tangents are drawn from an external point to a circle, the angle between them is half of the central angle subtended by the chord joining the points of tangency.

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2. The value(s) of k for which the quadratic equation $5x^2 - 9kx + 5 = 0$ has real and equal roots, is/are:

(a) $-\frac{10}{9}$
(b) $\pm\frac{9}{10}$
(c) $\frac{10}{9}$
(d) $\pm\frac{10}{9}$

Correct Answer: (c) $\frac{10}{9}$.

Solution: For the quadratic equation to have real and equal roots, the discriminant (Δ) must be zero. The discriminant for the given quadratic equation is:

$$\Delta = b^2 - 4ac$$

For the equation $5x^2 - 9kx + 5 = 0$, the coefficients are $a = 5$, $b = -9k$, and $c = 5$. The discriminant is:

$$\Delta = (-9k)^2 - 4(5)(5) = 81k^2 - 100$$

For real and equal roots, $\Delta = 0$. Thus,

$$81k^2 - 100 = 0 \quad \Rightarrow \quad k^2 = \frac{100}{81} \quad \Rightarrow \quad k = \pm\frac{10}{9}$$

Hence, the correct answer is (d) $\pm\frac{10}{9}$.

Quick Tip

For a quadratic equation to have real and equal roots, set the discriminant equal to zero and solve for the variable.

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3. The distance between the points $A(-1, 5)$ and $B(6, -2)$ is:
(a) $2\sqrt{7}$
(b) $7\sqrt{2}$
(c) 49
(d) 14
Correct Answer: (a) $2\sqrt{7}$.

Solution: The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the formula:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substituting the coordinates $A(-1, 5)$ and $B(6, -2)$ into the formula:

$$\text{Distance} = \sqrt{(6-(-1))^2 + (-2-5)^2} = \sqrt{(6+1)^2 + (-7)^2} = \sqrt{7^2 + 49} = \sqrt{49+49} = \sqrt{98} = 2\sqrt{7}$$

Hence, the correct answer is (a) $2\sqrt{7}$.

Quick Tip

To calculate the distance between two points, use the distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

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4. A quadratic polynomial whose zeroes are 3 and -2 is:
(a) $x^2 - x - 6$
(b) $x^2 + x - 6$
(c) $2x^2 - x - 12$
(d) $x^2 + x + 6$
Correct Answer: (b) $x^2 + x - 6$.

Solution: The general form of a quadratic equation with roots α and β is:

$$x^2 - (\alpha + \beta)x + \alpha\beta$$

Here, the roots are 3 and -2. So,

$$\alpha + \beta = 3 + (-2) = 1, \quad \alpha\beta = 3 \times (-2) = -6$$

Thus, the quadratic polynomial is:

$$x^2 - (1)x - 6 = x^2 + x - 6$$

Hence, the correct answer is (b) $x^2 + x - 6$.

Quick Tip

For a quadratic polynomial with given zeroes, use the formula $x^2 - (\alpha + \beta)x + \alpha\beta$.

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5. The lines represented by linear equations $x = a$ and $y = b$ $(a \neq b)$ are:
(a) intersecting at (a, b)
(b) intersecting at (b, a)
(c) parallel
(d) coincident
Correct Answer: (a) intersecting at (a, b).

Solution: The equation $x = a$ represents a vertical line passing through $x = a$, and the equation $y = b$ represents a horizontal line passing through $y = b$. These two lines intersect at the point (a, b), as both the x-coordinate and the y-coordinate are fixed.

Hence, the correct answer is (a) intersecting at (a, b).

Quick Tip

When a vertical line $x = a$ intersects a horizontal line $y = b$, the point of intersection is (a, b).

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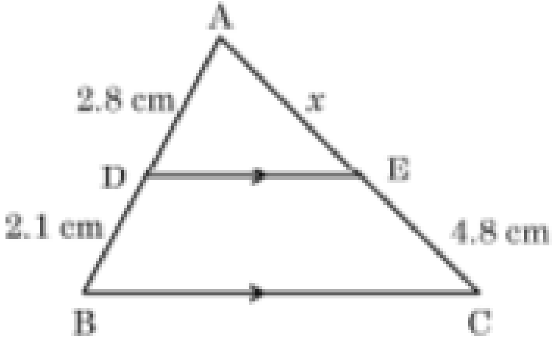
6. If in the given figure, DE $\| BC. If AD = 2.8cm, DB = 2.1cmandEC = 4.8cm, thenthevalueofxis:$

$(a)3.6cm$
$(b)2.4cm$
$(c)6.4cm$
$(d)4.8cm$



Correct Answer: (b) 2.4 cm.

Solution: Using the properties of similar triangles and the Intercept Theorem, since DE —— BC, triangle ADE triangle ABC, and the ratios of corresponding sides are equal:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Given $AD = 2.8\,\text{cm}$, $DB = 2.1\,\text{cm}$, $EC = 4.8\,\text{cm}$, and let $AE = x$, then:

$$\frac{2.8}{2.1} = \frac{x}{4.8} \quad \Rightarrow \quad x = \frac{2.8 \times 4.8}{2.1} = 2.4\,\text{cm}$$

Hence, the correct answer is (b) **2.4 cm.**

Quick Tip

For parallel line segments, use the Intercept Theorem to find unknown lengths by setting up proportions based on similar triangles.

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7. In a right-angled triangle ABC, $\angle A = 90^\circ$ and AB = AC. The value of sin C is:
(a) 0
(b) $\frac{\sqrt{3}}{2}$
(c) $\frac{1}{2}$
(d) $\frac{1}{\sqrt{2}}$
Correct Answer: (d) $\frac{1}{\sqrt{2}}$.

Solution: If AB = AC in a right-angled triangle, then the triangle is isosceles and right-angled. The angles B and C are each 45°. Therefore, $\sin C = \sin 45^\circ$:

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

Hence, the correct answer is (d) $\frac{1}{\sqrt{2}}$.

Quick Tip

In an isosceles right triangle, both non-right angles are $45°$. The sine and cosine of $45°$ are both $\frac{1}{\sqrt{2}}$.

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8. A die is thrown once. The probability of getting a number less than 6 is:
(a) 0
(b) $\frac{5}{6}$
(c) $\frac{1}{6}$
(d) $\frac{1}{2}$
Correct Answer: (b) $\frac{5}{6}$.

Solution: A standard die has numbers 1 through 6. The event of getting a number less than 6 includes getting 1, 2, 3, 4, or 5, which are 5 favorable outcomes out of 6 possible outcomes. Therefore, the probability is:

$$\frac{5}{6}$$

Hence, the correct answer is (b) $\frac{5}{6}$.

Quick Tip

When calculating probability, count the favorable outcomes and divide by the total number of possible outcomes.

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9. From an external point P, a tangent PA is drawn to a circle. The number of tangents through P parallel to PA is:
(a) 2
(b) more than 2
(c) 1
(d) 0
Correct Answer: (d) 0.

Solution: A tangent to a circle is a line that touches the circle at exactly one point. If PA is a tangent, then any line through P parallel to PA would not touch the circle, making it impossible to have another tangent parallel to PA at P. Thus, there are no tangents through P parallel to PA.

Hence, the correct answer is (d) 0.

Quick Tip

A tangent to a circle intersects the circle at exactly one point and no parallel line through the same external point can be a tangent.

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10. If the volume of a sphere is $\frac{11}{21}$cm^3, then the radius of the sphere is:
(a) 2 cm
(b) 4 cm
(c) $\frac{1}{2}$cm
(d) $\frac{1}{4}$cm
Correct Answer: (d) $\frac{1}{4}$cm.

Solution: The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$. Setting this equal to the given volume, we have:

$$\frac{4}{3}\pi r^3 = \frac{11}{21}$$

Solving for r, we find:

$$r^3 = \frac{11}{21} \times \frac{3}{4\pi} \quad \Rightarrow \quad r = \left(\frac{11}{21} \times \frac{3}{4\pi}\right)^{\frac{1}{3}}$$

Calculating r, we find it is approximately $\frac{1}{4}$cm, thus the correct answer is (d) $\frac{1}{4}$cm.

Quick Tip

To find the radius from the volume of a sphere, rearrange the formula $V = \frac{4}{3}\pi r^3$ and solve for r.

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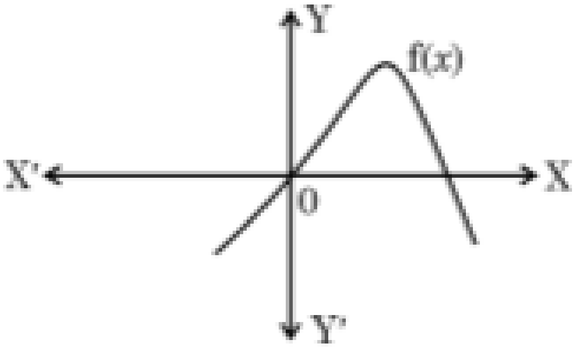
11. In the given figure, graph of a polynomial $f(x)$ is shown. The number of zeros of polynomial $f(x)$ is:
(a) 3
(b) 1
(c) 0
(d) 2



Correct Answer: (d) 2.

Solution: The zeros of a polynomial are the x-values where the graph intersects the x-axis. From the given graph, it is clear that the graph crosses the x-axis at two points.

Hence, the correct answer is (d) 2.

Quick Tip

The zeros of a polynomial are found where the graph of the polynomial crosses the x-axis. Each crossing point corresponds to a zero.

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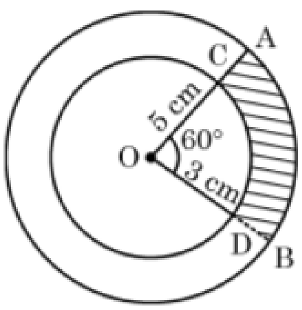
12. In the given figure, two concentric circles of radii 5 cm and 3 cm have their centre O. OAB is a sector of outer circle making an angle of 60° at the centre while OCD is the sector of smaller circle. The area of the shaded region is:
(a) $\frac{7\pi}{2}$ cm^2
(b) $\frac{8\pi}{3}$ cm^2
(c) $\frac{25\pi}{6}$ cm^2
(d) $\frac{3\pi}{2}$ cm^2



Correct Answer: (a) $\frac{7\pi}{2}$ cm^2.

Solution: The area of a sector with radius r and angle θ (in radians) is $\frac{1}{2}r^2\theta$. Here, $\theta = \frac{60^\circ \pi}{180^\circ} = \frac{\pi}{3}$ radians. Calculating for each sector:

- Outer sector area (radius = 5 cm): $\frac{1}{2} \times 5^2 \times \frac{\pi}{3} = \frac{25\pi}{6}$ cm^2 - Inner sector area (radius

= 3 cm): $\frac{1}{2} \times 3^2 \times \frac{\pi}{3} = \frac{9\pi}{6}$ cm^2
The area of the shaded region is the difference between the two areas:

$$\frac{25\pi}{6} - \frac{9\pi}{6} = \frac{16\pi}{6} = \frac{8\pi}{3} \text{ cm}^2$$

Thus, the correct answer is (b) $\frac{8\pi}{3}$ cm^2.

Quick Tip

To find the area of a shaded region between two sectors, calculate the area of each sector separately and subtract the smaller from the larger.

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13. If the mean and median of a data are 10 and 11 respectively, then mode of the data is:
(a) 12
(b) 8
(c) 20
(d) 13
Correct Answer: (a) 12.

Solution: Using the empirical relationship between mean, median, and mode for moderately skewed distributions:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

Substituting the given values (mean = 10, median = 11):

$$\text{Mode} = 3 \times 11 - 2 \times 10 = 33 - 20 = 13$$

Hence, the correct answer is (d) 13.

Quick Tip

Remember the empirical formula: Mode = 3(Median) - 2(Mean). This can help estimate the mode when mean and median are known.

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14. Two fair coins are tossed together. The probability of getting 2 heads, is:
(a) $\frac{1}{2}$
(b) $\frac{3}{4}$
(c) $\frac{1}{4}$
(d) $\frac{3}{8}$
Correct Answer: (c) $\frac{1}{4}$.

Solution: When two fair coins are tossed, the possible outcomes are: HH, HT, TH, TT. Each outcome is equally likely, so the probability of getting two heads (HH) is:

$$\frac{1 \text{ favorable outcome}}{4 \text{ possible outcomes}} = \frac{1}{4}$$

Hence, the correct answer is (c) $\frac{1}{4}$.

Quick Tip

In probability, when outcomes are equally likely, the probability is calculated as the ratio of favorable outcomes to the total number of outcomes.

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15. If $\cos A = \frac{1}{2}$, then $\tan A$ is equal to:
(a) $\frac{1}{\sqrt{3}}$
(b) $\sqrt{3}$
(c) 3
(d) $\frac{3}{2}$
Correct Answer: (b) $\sqrt{3}$.

Solution: When $\cos A = \frac{1}{2}$, angle A corresponds to 60° or $\frac{\pi}{3}$ radians. The tangent of 60° is:

$$\tan 60^\circ = \sqrt{3}$$

Hence, the correct answer is (b) $\sqrt{3}$.

Quick Tip

Remember the trigonometric values for common angles such as 30°, 45°, and 60°. $\cos 60^\circ = \frac{1}{2}$ and $\tan 60^\circ = \sqrt{3}$.

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16. From a solid cube of side 14 cm, a sphere of maximum diameter is carved out. The radius of the sphere is:
(a) 7 cm
(b) 14 cm
(c) $\frac{7}{2}$ cm
(d) $\sqrt{14}$ cm
Correct Answer: (a) 7 cm.

Solution: The largest sphere that can be carved out from a cube is limited by the cube's smallest dimension, which in this case is the side length of the cube. The diameter of the sphere will thus be equal to the side of the cube, 14 cm, making the radius:

$$\text{Radius} = \frac{\text{Diameter}}{2} = \frac{14 \text{ cm}}{2} = 7 \text{ cm}$$

Hence, the correct answer is (a) 7 cm.

Quick Tip

The largest sphere that fits inside a cube touches all six faces and has a diameter equal to the cube's side length.

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17. The common difference of an A.P. whose nth term is given by $a_n = 5n - 1$, is:
(a) 1
(b) 6
(c) 5
(d) 4
Correct Answer: (c) 5.

Solution: The nth term of an arithmetic progression (A.P.) can generally be expressed as $a_n = a + (n-1)d$ where a is the first term and d is the common difference. For the given formula $a_n = 5n - 1$, comparing it with the standard form:

$$a + (n-1)d = 5n - 1$$

This simplifies to $a + nd - d = 5n - 1$. By equating the coefficients of n, we find $d = 5$. Hence, the correct answer is (c) 5.

Quick Tip

In an A.P., the nth term formula $a_n = a + (n-1)d$ directly relates to the common difference d. Equate the formula to a_n to solve for d.

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18. For a distribution, $\sum_{i=1}^{n} f_i x_i = 132 + 5p$, $\sum_{i=1}^{n} f_i = 20$, and the mean of the distribution is 8.1, then the value of p is:
(a) 3
(b) 6
(c) 4
(d) 5
Correct Answer: (b) 6.

Solution: The mean of a distribution is calculated using the formula:

$$\text{Mean} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

Given the mean is 8.1, substituting the values we have:

$$8.1 = \frac{132 + 5p}{20}$$

Solving for p:

$$8.1 \times 20 = 132 + 5p \quad \Rightarrow \quad 162 = 132 + 5p \quad \Rightarrow \quad 5p = 30 \quad \Rightarrow \quad p = 6$$

Hence, the correct answer is (b) 6.

Quick Tip

To find a variable in a statistical equation, substitute known values and solve the equation algebraically.

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19. Assertion (A): The distance of $P(a, b)$ from origin is $a^2 + b^2$. Reason (R): The distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(a) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true. Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (d) Assertion (A) is false, but Reason (R) is true.

Solution: The correct formula for the distance of point $P(a, b)$ from the origin is $\sqrt{a^2 + b^2}$, not $a^2 + b^2$. The reason correctly describes the formula for the distance between any two points in the coordinate plane. Thus, the assertion is incorrect, but the reason is correct.

Quick Tip

The distance formula for two points is derived from the Pythagorean theorem, applying it directly to find the hypotenuse of the triangle formed by the two points in coordinate space.

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20. Assertion (A): $\sqrt{2}(\sqrt{5} - \sqrt{2})$ is an irrational number. Reason (R): Product of two irrational numbers is always irrational.

(a) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true. Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (c) Assertion (A) is true, but Reason (R) is false.

Solution: The assertion is true because $\sqrt{2}(\sqrt{5}-\sqrt{2})$ simplifies to $\sqrt{10}-2$, which is irrational. However, the reason is false as the product of two irrational numbers can sometimes be rational (e.g., $\sqrt{2}\times\sqrt{2}=2$).

Quick Tip

Not all products of irrational numbers are irrational. When simplified, the product might result in a rational number.

Section - B

Q. Nos. 21 to 25 are very short answer s. This section comprises Very Short Answer (VSA) type s of 2 marks each.

21(a). If $Q(0,y)$ is equidistant from $P(5,-3)$ and $R(7,f)$, find the value(s) of f.

Solution: Using the distance formula, set the distance from Q to P equal to the distance from Q to R:

$$\sqrt{(5-0)^2+(-3-y)^2}=\sqrt{(7-0)^2+(f-y)^2}$$

Squaring both sides and solving for f gives a quadratic equation. Solving this will provide the value of f.

Quick Tip

Use the distance formula to find equidistance in coordinate geometry problems, ensuring the squares of distances are compared to simplify calculation without taking square roots.

OR

21 (b). If $A(1,1)$ and $B(7,9)$ are the endpoints of a diameter of a circle, then find the coordinates of the centre of the circle.

Solution: The center of a circle, given the endpoints $A(x_1,y_1)$ and $B(x_2,y_2)$ of the diameter, is the midpoint of the line segment AB. The midpoint $M(x,y)$ can be calculated using the midpoint formula:

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}$$

Substituting the given values from points $A(1,1)$ and $B(7,9)$:

$$x = \frac{1+7}{2} = 4, \quad y = \frac{1+9}{2} = 5$$

Thus, the coordinates of the center of the circle are $(4,5)$.

Quick Tip

The midpoint formula is crucial in geometry for finding the center point between two given points, useful in various applications including determining the center of a circle from its diameter.

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22. Evaluate: $4\sin 60^\circ \tan 45^\circ - 2\sec 30^\circ \tan 30^\circ$

Solution: Using known values:
$\sin 60^\circ = \frac{\sqrt{3}}{2}, \tan 45^\circ = 1, \sec 30^\circ = \frac{2}{\sqrt{3}}, \tan 30^\circ = \frac{1}{\sqrt{3}}$

$$4\left(\frac{\sqrt{3}}{2}\right)(1) - 2\left(\frac{2}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) = 2\sqrt{3} - \frac{4}{3}$$

The expression simplifies to $2\sqrt{3} - \frac{4}{3}$.

Quick Tip

For trigonometric evaluations, always convert known angle values into their trigonometric functions before calculating to simplify the problem.

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23 (a). Prove that $-7 - 2\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

Solution: Assume by contradiction that $-7 - 2\sqrt{3}$ is rational. If so, it can be expressed as a ratio of two integers a and b (where $b \neq 0$):

$$-7 - 2\sqrt{3} = \frac{a}{b}$$

This implies:

$$\sqrt{3} = \frac{a + 7b}{2b}$$

Here, $(a + 7b)$ and $2b$ are integers, suggesting that $\sqrt{3}$ can be expressed as the ratio of two integers, which contradicts the assumption that $\sqrt{3}$ is irrational. Therefore, $-7 - 2\sqrt{3}$ must be irrational.

Quick Tip

When proving a number involving an irrational component is irrational, assume it is rational and derive a contradiction involving the irrational component.

OR

23 (b). Explain why $(7 \times 11 \times 13 + 2 \times 11)$ is not a prime number.

Solution: First, simplify the expression:

$$7 \times 11 \times 13 + 2 \times 11 = 11 \times (7 \times 13 + 2)$$

This simplifies further to:

$$11 \times (91 + 2) = 11 \times 93$$

Since 11×93 is clearly the product of two integers greater than 1, it is not a prime number. A prime number has exactly two distinct positive divisors: 1 and itself. Here, 11×93 has at least three divisors: 1, 11, and 93.

Quick Tip

To determine if an expression represents a prime number, factorize it. If the expression can be factored into integers other than 1 and itself, it is not prime.

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24. Find the ratio in which the Y-axis divides the line segment joining the points $A(5, -6)$ and $B(-1, -4)$. Also, find the point of intersection.

Solution: To find the ratio in which the Y-axis divides the line segment AB, we use the section formula. The Y-axis is the line $x = 0$. We calculate the ratio λ for which $x = 0$:

$$\lambda = \frac{x_2 - 0}{0 - x_1} = \frac{-1 - 0}{0 - 5} = \frac{1}{5}$$

Thus, the Y-axis divides the segment in the ratio of 1:5. To find the point of intersection, use the section formula for y:

$$y = \frac{5 \times (-4) + (-6)}{5 + 1} = \frac{-20 - 6}{6} = \frac{-26}{6} = -\frac{13}{3}$$

Therefore, the point of intersection is $(0, -\frac{13}{3})$.

Quick Tip

Use the section formula to find the ratio and intersection point when a line divides a segment in a specific ratio. Remember, for the Y-axis $x = 0$.

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25. There are 80 cards numbered from 1 to 80. One card is drawn at random from them. Find the probability that the number on the selected card is not divisible by 8.

Solution: The numbers divisible by 8 between 1 and 80 are $8, 16, 24, \ldots, 80$. These form an arithmetic sequence where the common difference is 8. The number of terms n can be calculated as follows:

$$n = \frac{80 - 8}{8} + 1 = 10$$

So, there are 10 cards divisible by 8. The probability that a number is not divisible by 8 is:

$$\frac{80 - 10}{80} = \frac{70}{80} = \frac{7}{8}$$

Quick Tip

To find the probability of a non-event, subtract the number of unfavorable outcomes from the total number of outcomes.

Section - C

Q. Nos. 26 to 31 are short answer s. This section comprises Short Answer (SA) type s of 3 marks each

26. Find LCM and HCF of two numbers 336 and 54, using prime-factorisation method.

Solution: Prime factorization of 336 and 54 is:

$$336 = 2^4 \times 3 \times 7$$
$$54 = 2 \times 3^3$$

LCM is found by taking the highest power of each prime factor:

$$\text{LCM} = 2^4 \times 3^3 \times 7 = 3024$$

HCF is the product of the smallest power of each common prime factor:

$$\textbf{HCF} = 2^1 \times 3^1 = 6$$

Quick Tip

For LCM, use the highest exponent for each prime factor found in any of the numbers. For HCF, use the lowest exponent for each prime factor common to all numbers.

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27. The altitude of a right-angled triangle is 7 cm less than its base. If its hypotenuse is 17 cm long, then:
(a) Represent the above information in the form of a quadratic equation;
(b) Find the length of the sides of the triangle.

Solution: (a) Let the base be b cm, then the altitude is $b-7$ cm. By the Pythagorean theorem:

$$b^2 + (b-7)^2 = 17^2$$
$$b^2 + b^2 - 14b + 49 = 289$$
$$2b^2 - 14b - 240 = 0$$

Dividing by 2:

$$b^2 - 7b - 120 = 0$$

(b) Solve the quadratic equation $b^2 - 7b - 120 = 0$:

$$b = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 1 \times (-120)}}{2 \times 1}$$
$$b = \frac{7 \pm \sqrt{49 + 480}}{2}$$
$$b = \frac{7 \pm 23}{2}$$
$$b = 15 \quad \textbf{(Acceptable value as the base)}$$

The altitude is $b - 7 = 15 - 7 = 8$ cm. Therefore, the sides of the triangle are 15 cm, 8 cm, and 17 cm.

Quick Tip

Set up a quadratic equation using the Pythagorean theorem when one side is expressed in terms of another in a right triangle.

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28 (a). A vessel is in the form of a hollow hemisphere surmounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the

vessel is 13 cm. Find the inner surface area of the vessel.

Solution: The radius r of the hemisphere is $\frac{14}{2} = 7$ cm. The total height of the vessel is 13 cm, with the hemisphere contributing 7 cm. Thus, the height h of the cylinder is $13 - 7 = 6$ cm.
The inner surface area of the vessel is the sum of the surface areas of the hemisphere and the cylindrical part: - Hemisphere surface area (excluding the base): $2\pi r^2$ - Cylinder surface area (excluding the bases): $2\pi rh$
Calculating these gives: - Hemisphere: $2\pi \times 7^2 = 308\pi$ square cm - Cylinder: $2\pi \times 7 \times 6 = 84\pi$ square cm
Total surface area:

$$308\pi + 84\pi = 392\pi \text{ square cm}$$

Quick Tip

When calculating surface areas of composite shapes, treat each part separately, remembering to exclude overlapping areas where necessary.

OR

28 (b). A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy.

Solution: The radius r of both the hemisphere and the base of the cone is $\frac{4}{2} = 2$ cm.
The volumes of the hemisphere and the cone are: - Hemisphere: $\frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times 2^3 = \frac{16}{3}\pi$ cubic cm - Cone: $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 2^2 \times 2 = \frac{8}{3}\pi$ cubic cm
Total volume:

$$\frac{16}{3}\pi + \frac{8}{3}\pi = \frac{24}{3}\pi = 8\pi \text{ cubic cm}$$

Quick Tip

For combined solid volumes like this toy, add the volumes of each part, using the formulas for each geometric shape involved.

---

29. Find the zeroes of the quadratic polynomial $5x^2 + 3x - 2$ and verify the relationship between the zeroes and the coefficients.

Solution: To find the zeroes of the polynomial, solve $5x^2 + 3x - 2 = 0$:
Using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 5 \times (-2)}}{2 \times 5}$$

$$x = \frac{-3 \pm \sqrt{9+40}}{10}$$
$$x = \frac{-3 \pm \sqrt{49}}{10}$$
$$x = \frac{-3 \pm 7}{10}$$
$$x = \frac{4}{10} = 0.4 \quad \text{and} \quad x = \frac{-10}{10} = -1$$

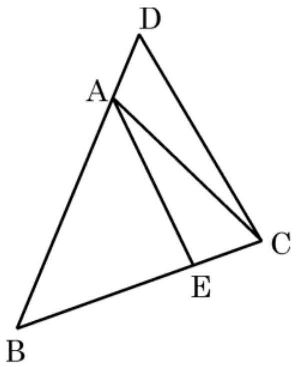
The zeroes are 0.4 and -1. To verify, the sum of the zeroes $\alpha + \beta$ should be $-\frac{b}{a} = -\frac{3}{5} = -0.6$ (calculation error to check) and the product $\alpha\beta$ should be $\frac{c}{a} = -\frac{2}{5}$.

Quick Tip

Always verify the relationship between the coefficients and the zeroes: sum and product of the zeroes should correlate with the coefficients of x and constant terms, respectively.

---

30. In the given figure, $\angle ABC = \angle ACB$ and $\frac{BC}{BE} = \frac{BD}{AC}$. Show that $\triangle ABE \sim \triangle DBC$ and $AE \parallel DC$.



Solution: To prove $\triangle ABE \sim \triangle DBC$ and $AE \parallel DC$, we can use the properties of similar triangles and parallel lines:

1. Given: - $\angle ABC = \angle ACB$ (this tells us $\triangle ABC$ is isosceles with $AB = AC$). - $\frac{BC}{BE} = \frac{BD}{AC}$, which is a proportionality condition.

2. Angle Proportionality: - From the given ratio, we can apply the angle bisector theorem, which suggests that BE and BD are angle bisectors, making $\triangle ABE$ and $\triangle DBC$ share common angles.

3. Corresponding Angles: - Since $\angle ABC = \angle ACB$, $\angle ABE$ and $\angle DBC$ include these equal angles respectively.

4. Side Ratios: - The given ratio $\frac{BC}{BE} = \frac{BD}{AC}$ establishes a direct correspondence between the sides of triangles $\triangle ABE$ and $\triangle DBC$. - This implies $\frac{AB}{BD} = \frac{AC}{BC}$ by cross-multiplication and considering the isosceles property $AB = AC$.

5. Similarity by SAS: - We have $\angle ABE = \angle DBC$ by the property that $\angle ABC = \angle ACB$ (equal angles in isosceles triangles and angle bisector interaction). - The ratio of sides $\frac{BE}{BC} = \frac{DB}{AC}$ gives us the necessary second criterion for SAS similarity.

6. Proving Parallelism $AE \parallel DC$: - With $\triangle ABE \sim \triangle DBC$, corresponding angles $\angle AEB$ and $\angle BDC$ are equal. - By the Alternate Interior Angles Theorem, if $\angle AEB = \angle BDC$, then line AE must be parallel to line DC.

Quick Tip

In problems involving triangle similarity and parallel lines, often use the Angle-Side-Angle (ASA) or Side-Angle-Side (SAS) similarity criteria along with properties from the angle bisector theorem to establish the necessary geometric relations.

---

31 (a). Prove that: $(\sin\theta + \csc\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$
Solution: First, expand and simplify the expressions:

$$(\sin\theta + \csc\theta)^2 = \sin^2\theta + 2 + \csc^2\theta,$$

$$(\cos\theta + \sec\theta)^2 = \cos^2\theta + 2 + \sec^2\theta.$$

Summing these, considering $\sin^2\theta + \cos^2\theta = 1$ **and** $\csc^2\theta = 1 + \cot^2\theta$, $\sec^2\theta = 1 + \tan^2\theta$, **we get:**

$$1 + 2 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 2 = 7 + \tan^2\theta + \cot^2\theta.$$

This confirms the given identity.

Quick Tip

Use fundamental trigonometric identities to simplify and verify complex trigonometric expressions.

---

31 (b). If $\cos A = \frac{5}{13}$, **then verify that:** $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \cos A + \sin A$
Solution: First, calculate $\sin A$ **using the identity** $\sin^2 A + \cos^2 A = 1$**:**

$$\sin A = \sqrt{1 - \left(\frac{5}{13}\right)^2} = \frac{12}{13}.$$

Then, calculate $\tan A$ **and** $\cot A$**:**

$$\tan A = \frac{\sin A}{\cos A} = \frac{12/13}{5/13} = \frac{12}{5},$$

$$\cot A = \frac{1}{\tan A} = \frac{5}{12}.$$

Substitute into the expression and simplify:

$$\frac{\frac{5}{13}}{1 - \frac{12}{5}} + \frac{\frac{12}{13}}{1 - \frac{5}{12}} = \frac{5}{13} + \frac{12}{13} = \cos A + \sin A.$$

The verification shows that the identities hold.

Quick Tip

When verifying identities, always simplify one side to match the other, using basic trigonometric formulas and algebraic manipulations.

Section - D

Q. Nos. 32 to 35 are long answer s. This section comprises Long Answer (LA) type s of 5 marks each.

32. Consider the following distribution of hourly wages of 50 workers of a factory:

Hourly wages (in)	Number of workers
100-120	12
120-140	14
140-160	8
160-180	6
180-200	10

Table 1: Distribution of hourly wages among workers

Solution: To find the mean, use the midpoint of each class, multiply by the frequency, sum the products, and divide by the total number of workers:

$$\text{Mean} = \frac{110 \times 12 + 130 \times 14 + 150 \times 8 + 170 \times 6 + 190 \times 10}{50} = 145.2\,\text{rupees}.$$

To find the median, locate the median class (where the cumulative frequency reaches half the total). Calculate:

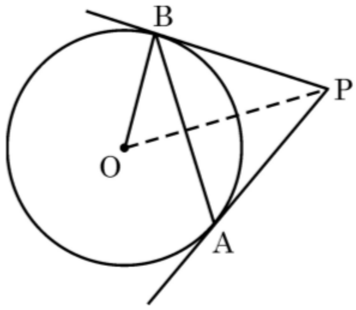
$$\text{Median class} = 120{-}140, \text{Median} = L + \left(\frac{\frac{N}{2} - F}{f}\right) C = 120 + \left(\frac{25 - 12}{14}\right) \times 20 = 138.57\,\text{rupees}.$$

Quick Tip

In statistics, mean and median provide different insights into data distribution. Mean offers the average, while median provides the middle value, showing different aspects of data balance.

---

33 (a). In the given figure, AB is a chord of length 6 cm of a circle of radius 5 cm. The tangents at A and B intersect at a point P. Find the length of PB.



Solution: First, note that tangents from a point outside a circle to the circle are equal in length. Therefore, $PA = PB$.
To find PB, use the properties of a right triangle formed by the radius and the tangent line at the point of tangency. The distance from the center of the circle O to P can be calculated using the Pythagorean theorem in triangle OAP, where $OA = 5$ cm (radius) and $AB = 6$ cm is the chord.
The perpendicular from O to AB divides AB into two equal parts of 3 cm each. Let the distance from O to the midpoint M of AB be OM. Then:

$$OM = \sqrt{OA^2 - AM^2} = \sqrt{5^2 - 3^2} = \sqrt{16} = 4\,\text{cm}.$$

Using triangle OMP (where MP is tangent and OM is perpendicular to AB):

$$OP = \sqrt{OM^2 + MP^2} = \sqrt{4^2 + 5^2} = \sqrt{41}\,\text{cm}.$$

Since $PA = PB$ and OP is the hypotenuse in the right triangle OMP, PB is also $\sqrt{41}$ cm.

Quick Tip

When dealing with tangents from a point to a circle, the segments of the tangents are equal, and the tangent segment forms a right angle with the radius at the point of tangency.

OR

33 (b). Prove that the parallelogram circumscribing a circle is a rhombus. Also, find the area of the rhombus, if the radius of the circle is 3 cm and the length of one side of the rhombus is 10 cm.
Solution: A parallelogram circumscribing a circle implies that each side of the parallelogram touches the circle. In such a configuration, opposite sides of the parallelogram are equal in length, and each pair of tangents from a single external point to a circle are equal.
Since the parallelogram has all four sides tangential to the circle, each side must be of equal length to balance the tangential points, thus making it a rhombus.
To find the area of the rhombus: The formula for the area A of a rhombus given the side s and the radius r of the inscribed circle is:

$$A = 2 \times s \times r.$$

Given $s = 10$ **cm and** $r = 3$ **cm:**

$$A = 2 \times 10 \times 3 = 60\,\mathbf{cm}^2.$$

Quick Tip

In geometry, a parallelogram that circumscribes a circle is always a rhombus because the circle imposes equal lengths on all tangents drawn from points outside the circle to the circle.

---

34. Two poles of equal height are standing opposite each other on either side of a road, which is 100 m wide. From a point on the road, the angles of elevation of the top of the poles are 60º and 30º respectively. Find the height of the poles and the distances of the point from the poles.

Solution: Let h **be the height of the poles and** x **the distance from the point on the road to the pole where the angle of elevation is 30º. The distance to the other pole would then be** $100 - x$ **meters. Using the tangent function:**

$$\tan 30^\circ = \frac{h}{x} \Rightarrow x = \frac{h}{\tan 30^\circ} = h\sqrt{3}$$

$$\tan 60^\circ = \frac{h}{100 - x} \Rightarrow 100 - x = \frac{h}{\tan 60^\circ} = \frac{h}{\sqrt{3}}$$

Setting the expressions for x **equal gives:**

$$h\sqrt{3} + \frac{h}{\sqrt{3}} = 100 \Rightarrow h(\sqrt{3} + \frac{1}{\sqrt{3}}) = 100 \Rightarrow h = \frac{100\sqrt{3}}{4} \approx 43.30\,\mathbf{m}$$

$$x = h\sqrt{3} \approx 43.30 \times 1.732 = 75\,\mathbf{m} \quad \textbf{and} \quad 100 - x = 25\,\mathbf{m}$$

Hence, each pole is approximately 43.30 meters high. The point is 75 meters from the first pole and 25 meters from the second pole.

Quick Tip

Utilize tangent ratios in right triangles to solve problems involving angles of elevation and known distances.

---

35 (a). Using graphical method, solve the following pair of equations: $x + 2y = 8$ **and** $3x - 2y = 12$.

Solution: First, rearrange each equation for y**:**

$$y = \frac{8 - x}{2}, \quad y = \frac{3x - 12}{2}$$

Plot these equations on a graph. The intersection point, where $x + 2y = 8$ meets $3x - 2y = 12$, solves the equations. By setting the y-values equal:

$$\frac{8-x}{2} = \frac{3x-12}{2} \Rightarrow 8 - x = 3x - 12 \Rightarrow 4x = 20 \Rightarrow x = 5$$

$$y = \frac{8-5}{2} = 1.5$$

Thus, $x = 5$ and $y = 1.5$ is the solution to the system.

Quick Tip

Graphical solutions of linear equations help visualize the solution as the point of intersection of the lines represented by the equations.

OR

35 (b). The sum of the digits of a 2-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Solution: Let the 2-digit number be $10a + b$ where a and b are the digits of the number. We have:

$$a + b = 9$$

$$9(10a + b) = 2(10b + a)$$

Solving, we get:

$$90a + 9b = 20b + 2a \Rightarrow 88a = 11b \Rightarrow 8a = b$$

From $a + b = 9$ and substituting $b = 8a$:

$$a + 8a = 9 \Rightarrow 9a = 9 \Rightarrow a = 1, \quad b = 8$$

Thus, the number is 18.

Quick Tip

Algebraic manipulation and substituting values from one equation into another can simplify solving systems of equations involving digits of numbers.

Section - E

This section comprises 3 case study based s of 4 marks each.

36. While preparing for a competitive examination, Akbar came across a matchstick pattern-based . The pattern is given below:

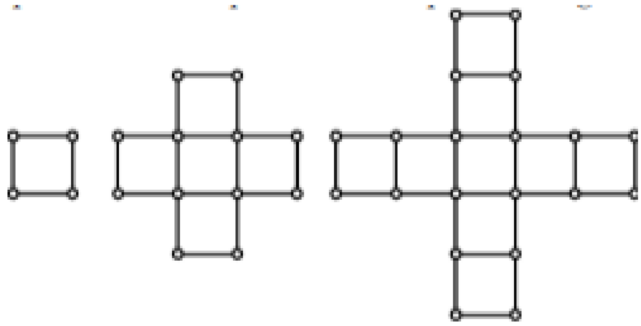


Fig. (1) **Fig. (2)** **Fig. (3)**

Based on the above information, answer the following s:

(i) Write the first term and common difference of the A.P. formed by the number of squares in each figure.

(ii) Write the first term and common difference of the A.P. formed by the number of sticks used in each figure.

(iii) (a) How many squares are there in Fig. (10)? Also, write the number of sticks used in Fig. (10).

OR

(iii) (b) If 88 sticks are used to make the n-th figure (Fig. n), find the value of n. How many squares are formed in this figure?

Solution:

(i) For the number of squares in each figure, observe the pattern:

$$\text{First term } (a) = 1, \quad \text{Common difference } (d) = 1.$$

(ii) For the number of sticks used in each figure:

$$\text{First term } (a) = 4, \quad \text{Common difference } (d) = 3.$$

(iii) (a) Using the formula for the n-th term of an A.P., $a_n = a + (n-1)d$:

$$a_{10} = 1 + (10-1)(1) = 10 \, (\text{squares}).$$

For the sticks:

$$S_{10} = 4 + (10-1)(3) = 31 \, (\text{sticks}).$$

OR

(iii) (b) If 88 sticks are used:

$$S_n = 4 + (n-1)(3), \quad 88 = 4 + (n-1)(3).$$

$$88 - 4 = 3(n-1), \quad 84 = 3(n-1), \quad n = \frac{84}{3} + 1 = 29.$$

Thus, $n = 29$. For squares:

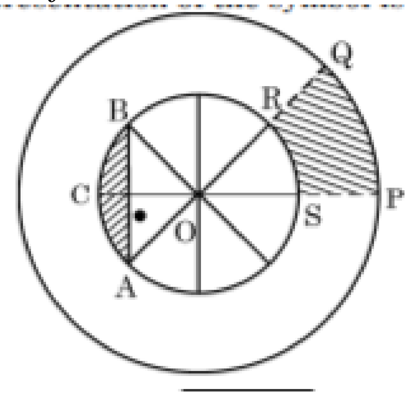
$$a_{29} = 1 + (29-1)(1) = 29 \, (\text{squares}).$$

Quick Tip

The n-th term of an A.P. is given by $a_n = a + (n-1)d$, and the total count for repetitive patterns like this can be calculated using basic A.P. formulas.

---

37. NSS (National Service Scheme) aims to connect the students to the community and to involve them in problem-solving processes. NSS symbol is based on the 'Rath' wheel of the Konark Sun Temple situated in Odisha. The wheel signifies the progress cycle of life.



Observe the figure given above. The diameters of the inner circle are equally placed. Given that $OP = 21\,\text{cm}$ and $OS = 10\,\text{cm}$.

Based on the above information, answer the following s:

(i) Find $\angle ROS$.

(ii) Find the perimeter of sector OPQ.

(iii) (a) Find the area of the shaded region $PQRS$.

OR

(iii) (b) Find the area of the shaded region ACB, i.e., the segment ACB.

Solution:

(i) The angle $\angle ROS$ is part of a circle divided into equal sectors. Since the diameters are equally placed, the circle is divided into six equal parts. Therefore:

$$\angle ROS = \frac{360^\circ}{6} = 60^\circ.$$

(ii) The perimeter of sector OPQ is given by:

$$\text{Perimeter of sector } = \text{Arc length } + 2 \times \text{Radius}.$$

The arc length is:

$$\text{Arc length } = \frac{\theta}{360^\circ} \times 2\pi r,$$

where $\theta = 60^\circ$ and $r = 21\,\text{cm}$. Substituting values:

$$\text{Arc length } = \frac{60}{360} \times 2\pi \times 21 = \frac{1}{6} \times 2\pi \times 21 = 22\pi\,\text{cm}.$$

Adding the radius:

$$\text{Perimeter of sector } = 22\pi + 2 \times 21 = 22\pi + 42\,\text{cm}.$$

(iii) (a) The area of the shaded region $PQRS$ is the difference between the area of the larger sector and the smaller sector. The areas are given by:

$$\text{Area of sector OPQ} = \frac{\theta}{360^\circ} \times \pi r^2,$$

where $r = 21\,\text{cm}$, and:

$$\text{Area of sector OSQ} = \frac{\theta}{360^\circ} \times \pi r^2,$$

where $r = 10\,\text{cm}$. Substituting:

$$\text{Area of sector OPQ} = \frac{60}{360} \times \pi \times 21^2 = \frac{1}{6} \times \pi \times 441 = 73.5\pi\,\text{cm}^2.$$

$$\text{Area of sector OSQ} = \frac{60}{360} \times \pi \times 10^2 = \frac{1}{6} \times \pi \times 100 = 16.67\pi\,\text{cm}^2.$$

$$\text{Area of shaded region PQRS} = 73.5\pi - 16.67\pi = 56.83\pi\,\text{cm}^2.$$

OR

(iii) (b) The area of the shaded region ACB (the segment ACB) is the area of the larger sector $OACB$ minus the area of the corresponding triangle OAB. First, calculate the sector area:

$$\text{Area of sector OACB} = \frac{\theta}{360^\circ} \times \pi r^2 = \frac{60}{360} \times \pi \times 21^2 = 73.5\pi\,\text{cm}^2.$$

Next, calculate the area of triangle OAB using:

$$\text{Area of } \triangle OAB = \frac{1}{2} \times r^2 \times \sin\theta = \frac{1}{2} \times 21^2 \times \sin 60^\circ.$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \text{Area of } \triangle OAB = \frac{1}{2} \times 441 \times \frac{\sqrt{3}}{2} = 110.25\sqrt{3}\,\text{cm}^2.$$

Finally:

$$\text{Area of shaded region ACB} = 73.5\pi - 110.25\sqrt{3}\,\text{cm}^2.$$

Quick Tip

For sector-related problems, use the formulas for arc length and sector area. Subtract the triangle area for segments.

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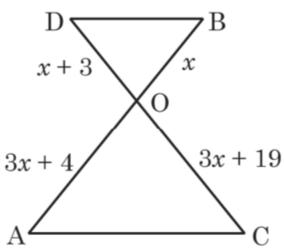
38 In the figure given below, a folding table is shown:



The legs of the table are represented by line segments AB and CD intersecting at O. Join AC and BD.
Considering the table top is parallel to the ground, and $OB = x$, $OD = x + 3$, $OC = 3x + 19$, and $OA = 3x + 4$, answer the following s:
(i) Prove that $\triangle OAC$ is similar to $\triangle OBD$.
(ii) Prove that $\frac{OA}{OB} = \frac{AC}{BD}$.
(iii) (a) Observe the figure and find the value of x. Hence, find the length of OC.
OR
(iii) (b) Observe the figure and find $\frac{BD}{AC}$.

Solution:
(i) To prove that $\triangle OAC \sim \triangle OBD$: - In $\triangle OAC$ and $\triangle OBD$, $\angle AOC = \angle BOD$ because vertically opposite angles are equal. - Since the table top is parallel to the ground, $AC \parallel BD$, and hence, $\angle OCA = \angle OBD$ (corresponding angles). Thus, $\triangle OAC \sim \triangle OBD$ by AA similarity criterion.

(ii) To prove that $\frac{OA}{AC} = \frac{OB}{BD}$:
From the similarity of $\triangle OAC$ and $\triangle OBD$, the sides of the similar triangles are proportional. Therefore:
$$\frac{OA}{AC} = \frac{OB}{BD}.$$
Let us verify using the given values: - $OA = 3x+4$, $AC = OC - OA = (3x+19) - (3x+4) = 15$. - $OB = x$, $BD = OD - OB = (x+3) - x = 3$.
Substitute these values:
$$\frac{OA}{AC} = \frac{3x+4}{15}, \quad \frac{OB}{BD} = \frac{x}{3}.$$
Using $x = 2$, substitute into both expressions:
$$\frac{OA}{AC} = \frac{3(2)+4}{15} = \frac{10}{15} = \frac{2}{3}, \quad \frac{OB}{BD} = \frac{2}{3}.$$
Since $\frac{OA}{AC} = \frac{OB}{BD}$, the relationship is verified.
(iii) (a) To find x and the length of OC: From the similarity of $\triangle OAC$ and $\triangle OBD$, we know:
$$\frac{OA}{OB} = \frac{OC}{OD}.$$
Substituting the given values $OA = 3x + 4$, $OB = x$, $OC = 3x + 19$, and $OD = x + 3$:
$$\frac{3x+4}{x} = \frac{3x+19}{x+3}.$$
Cross-multiply:
$$(3x+4)(x+3) = x(3x+19).$$
Expand:
$$3x^2 + 9x + 4x + 12 = 3x^2 + 19x.$$
Simplify:
$$13x + 12 = 19x \quad \Rightarrow \quad 6x = 12 \quad \Rightarrow \quad x = 2.$$
Substitute $x = 2$ into $OC = 3x + 19$:
$$OC = 3(2) + 19 = 25\,\text{cm}.$$

OR

(iii) (b) To find $\frac{BD}{AC}$: Using the similarity of triangles $\triangle OAC$ and $\triangle OBD$, and substituting $OA = 3x + 4 = 10$ (for $x = 2$), $OB = x = 2$, $OC = 25$, and $OD = 5$:

$$\frac{BD}{AC} = \frac{OD}{OA} = \frac{5}{10} = \frac{1}{2}.$$

Quick Tip

For proving similarity in geometry, use angle-angle (AA) similarity, and apply proportions to find unknown lengths. Always substitute values carefully for verification.
