JEE Main 2024 April 9 Shift 2 Mathematics Question Paper

Time Allowed : 1 Hour	**Maximum Marks :** 100	**Total Questions :** 30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Mathematics

1. $\lim_{x\to 0} \frac{e^{-\frac{1}{2x}}(1+2x)^{\frac{1}{2x}}}{x}$ **is equal to:**
(1) e
(2) $-\frac{2}{e}$
(3) 0
(4) $e - e^2$

2. Consider the line L **passing through the points** $(1, 2, 3)$ **and** $(2, 3, 5)$**. The distance of the point** $\left(\frac{11}{3}, \frac{11}{3}, \frac{19}{3}\right)$ **from the line** L **along the line:**

$$\frac{3x-11}{2} = \frac{3y-11}{1} = \frac{3z-19}{2}$$

is equal to:
(1) 3
(2) 5
(3) 4
(4) 6

3. Let $\int_0^x \sqrt{1-(y'(t))^2}\,dt = \int_0^x y(t)\,dt$**,** $0 \le x \le 3, y \ge 0$**,**
$y(0) = 0$. Then at $x = 2$, $y'' + y + 1$ is equal to:

(1) 1
(2) 2
(3) $\sqrt{2}$
(4) 1/2

4. Let z be a complex number such that the real part of $\frac{z-2i}{z+2i}$ is zero. Then, the maximum value of $|z-(6+8i)|$ is equal to:
(1) 12
(2) ∞
(3) 10
(4) 8

5. The area (in square units) of the region enclosed by the ellipse $x^2+3y^2=18$ in the first quadrant below the line $y=x$ is:
(1) $\sqrt{3}\pi+\frac{3}{4}$
(2) $\sqrt{3}\pi$
(3) $\sqrt{3}\pi-\frac{3}{4}$
(4) $\sqrt{3}\pi+1$

6. Let the foci of a hyperbola H coincide with the foci of the ellipse $E:\frac{(x-1)^2}{100}+\frac{(y-1)^2}{75}=1$, and the eccentricity of the hyperbola H be the reciprocal of the eccentricity of the ellipse E. If the length of the transverse axis of H is α and the length of its conjugate axis is β, then $3\alpha^2+2\beta^2$ is equal to:
(1) 242
(2) 225
(3) 237
(4) 205

7. Two vertices of a triangle ABC are $A(3,-1)$ and $B(-2,3)$, and its orthocenter is $P(1,1)$. If the coordinates of the point C are (α,β) and the center of the circle circumscribing the triangle PAB is (h,k), then the value of $(\alpha+\beta)+2(h+k)$ equals:
(1) 51
(2) 81
(3) 5
(4) 15

8. If the variance of the frequency distribution is 160, then the value of $c \in N$ is:
(1) 5
(2) 8
(3) 7
(4) 6

9. Let the range of the function $f(x)=\frac{1}{2+\sin 3x+\cos 3x}$, where $x \in R$ and $x \in [a,b]$. If α and β are respectively the A.M. and G.M. of a and b, then $\frac{\alpha}{\beta}$ is equal to:
(1) $\sqrt{2}$
(2) 2

(3) $\sqrt{\pi}$
(4) π

10. Between the following two statements:
Statement-I: Let $\mathbf{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\mathbf{b} = 2\hat{i} + \hat{j} - \hat{k}$. Then the vector $\mathbf{r}$ satisfying $\mathbf{a} \times \mathbf{r} = \mathbf{a} \times \mathbf{b}$ and $\mathbf{a} \cdot \mathbf{r} = 0$ is of magnitude $\sqrt{10}$.
Statement-II: In a triangle ABC, $\cos 2A + \cos 2B + \cos 2C \geq -\frac{3}{2}$.
(1) Both Statement-I and Statement-II are incorrect
(2) Statement-I is incorrect but Statement-II is correct
(3) Both Statement-I and Statement-II are correct
(4) Statement-I is correct but Statement-II is incorrect

11. Evaluate the following limit:

$$\lim_{x \to \frac{\pi}{2}} \frac{\int_{x^3}^{\left(\frac{\pi}{2}\right)^3} \left(\sin(t^{1/3}) + \cos(t^{1/3})\right) dt}{\left(x - \frac{\pi}{2}\right)^2}$$

(1) $\frac{9\pi^2}{8}$
(2) $\frac{11\pi^2}{10}$
(3) $\frac{3\pi^2}{2}$
(4) $\frac{5\pi^2}{9}$

12. The sum of the coefficient of $x^{2/3}$ and $x^{-2/5}$ in the binomial expansion of $\left(x^{2/3} + \frac{1}{2}x^{-2/5}\right)^9$ is:
(1) $\frac{21}{4}$
(2) $\frac{69}{16}$
(3) $\frac{63}{16}$
(4) $\frac{19}{4}$

13. Let $B = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}$ and A be a 2×2 matrix such that $AB^{-1} = A^{-1}$. If $BCB^{-1} = A$ and $C^4 + \alpha C^2 + \beta$
(1) 16
(2) 2
(3) 8
(4) 10

14. If $\log_e y = 3 \sin^{-1} x$, then $(1-x)^2 y'' - xy'$ at $x = \frac{1}{2}$ is equal to:
(1) $9e^{\pi/6}$
(2) $3e^{\pi/6}$
(3) $3e^{\pi/2}$
(4) $9e^{\pi/2}$

15. The integral

$$\int_{1/4}^{3/4} \cos\left(2\cot^{-1}\left(\frac{1-x}{1+x}\right)\right) dx$$

is equal to:

(1) $-\frac{1}{2}$
(2) $\frac{1}{4}$
(3) $\frac{1}{2}$
(4) $-\frac{1}{4}$

16. Let $a, ar, ar^2, \ldots$ be an infinite geometric progression. If $\sum_{n=0}^{\infty} ar^n = 57$ and $\sum_{n=0}^{\infty} a3^r3^n = 9747$, the
(1) 27
(2) 46
(3) 38
(4) 31

17. If an unbiased die is rolled thrice, then the probability of getting a greater number on the i-th roll th
(1) $\frac{3}{54}$
(2) $\frac{2}{54}$
(3) $\frac{5}{54}$
(4) $\frac{1}{54}$

18. The value of the integral

$$\int_{-1}^{2} \left|\log_e \left(x + \sqrt{x^2+1}\right)\right| dx$$

is:
(1) $\sqrt{5} - \sqrt{2} + \log_e \left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$
(2) $\sqrt{2} - \sqrt{5} + \log_e \left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$
(3) $\sqrt{5} - \sqrt{2} + \log_e \left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$
(4) $\sqrt{2} - \sqrt{5} + \log_e \left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

19. Let $\alpha, \beta; \alpha > \beta$, be the roots of the equation $x^2 - \sqrt{2}x - \sqrt{3} = 0$. Let $P_n = \alpha^n - \beta^n$, where $n \in N$. T

$$(11\sqrt{3} - 10\sqrt{2})P_{10} + (11\sqrt{2} + 10)P_{11} - 11P_{12}$$

(1) $10\sqrt{2}P_9$
(2) $10\sqrt{3}P_9$
(3) $11\sqrt{2}P_9$
(4) $11\sqrt{3}P_9$

20. Let $\mathbf{a} = 2\hat{i} + \alpha\hat{j} + \hat{k}$, $\mathbf{b} = -\hat{i} + \hat{j} + \hat{k}$, and $\mathbf{c} = \beta\hat{j} - \hat{k}$, where α and β are integers and $\alpha\beta = -6$. Let
(1) 17
(2) 24
(3) 21
(4) 19

21. Consider the circle $C : x^2 + y^2 = 4$ and the parabola $P : y^2 = 8x$. If the set of all values of α, for whi

22. Let the set of all values of p, for which $f(x) = (p^2 - 6p + 8)\cos 4x + 2(2-p)x + 7$ does not have any

23. For a differentiable function $f : R \to R$, suppose

$$f'(x) = 3f(x) + \alpha, \quad \text{where} \quad \alpha \in R, \quad f(0) = 1 \quad \text{and} \quad \lim_{x \to \infty} f(x) = 7.$$

Then, $9f(-\log 3)$ is equal to:

24. The number of integers between 100 and 1000 having the sum of their digits equal to 14 is:

25. Let $A = \{(x, y) : 2x + 3y = 23,\ x, y \in N\}$ and $B = \{x : (x, y) \in A\}$. Then the number of one-to-one f

26. Let A, B, and C be three points on the parabola $y^2 = 6x$ and let the line segment AB meet the line

$$\left(\frac{AM \cdot BN}{CD}\right)^2 \text{ is equal to:}$$

27. The square of the distance of the image of the point $(6, 1, 5)$ in the line

$$\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-2}{4}, \text{ from the origin is:}$$

28. If

$$\left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \ldots + \frac{1}{\alpha+1012}\right) - \left(\frac{1}{2 \cdot 1} + \frac{1}{4 \cdot 3} + \frac{1}{6 \cdot 5} + \ldots + \frac{1}{2024 \cdot 2023}\right) = \frac{1}{2024},$$

then α is equal to:

29. Let the inverse trigonometric functions take principal values. The number of real solutions of the equ

$$2\sin^{-1} x + 3\cos^{-1} x = \frac{2\pi}{5} \text{ is:}$$

30. Consider the matrices

$$A = \begin{bmatrix} 2 & -5 \\ 3 & m \end{bmatrix}, \quad B = \begin{bmatrix} 20 \\ m \end{bmatrix}$$

and $X = \begin{bmatrix} x \\ y \end{bmatrix}$. Let the set of all m be such that the system of equations $AX = B$ has a negative solution
The interval (a, b) is given, and we are to compute:

$$\int_a^b |A|\, dm$$

is equal to —— .