JEE Main 2024 April 6 Shift 2 Mathematics Question Paper

Time Allowed :1 Hour	**Maximum Marks :**100	**Total Questions :**30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. Let ABC be an equilateral triangle. A new triangle is formed by joining the midpoints of all sides of the triangle ABC, and the same process is repeated infinitely many times. If P is the sum of the perimeters and Q is the sum of areas of all the triangles formed in this process, then:
(1) $\mathrm{P}^2 = 36\sqrt{3}Q$
(2) $P^2 = 6\sqrt{3}Q$
(3) $P = 36\sqrt{3}Q^2$
(4) $P^2 = 72\sqrt{3}Q$

Question 2: Let $A = \{1, 2, 3, 4, 5\}$. Let R be a relation on A defined by xRy if and only if $4x \leq 5y$. Let n be the number of elements in R and m be the minimum number of elements from $A \times A$ that are required to be added to R to make it a symmetric relation. Then $m + n$ is equal to:

(1) 24
(2) 23
(3) 25
(4) 26

Question 3: If three letters can be posted to any one of 5 different addresses, then the probability that the three letters are posted to exactly two addresses is:

(1) $\frac{12}{25}$
(2) $\frac{18}{25}$
(3) $\frac{4}{25}$
(4) $\frac{6}{25}$

Question 4: Suppose the solution of the differential equation

$$\frac{dy}{dx} = \frac{(2+\alpha)x - \beta y + 2}{\beta x - 2\alpha y - (\beta y - 4\alpha)}$$

represents a circle passing through the origin. Then the radius of this circle is:

(1) $\sqrt{17}$
(2) $\frac{1}{2}$
(3) $\frac{\sqrt{17}}{2}$
(4) 2

Question 5: If the locus of the point, whose distances from the points $(2, 1)$ and $(1, 3)$ are in the ratio $5 : 4$, is

$$ax^2 + by^2 + cxy + dx + ey + 170 = 0,$$

then the value of $a^2 + 2b + 3c + 4d + e$ is equal to:

(1) 5
(2) 27
(3) 37
(4) 437

---

6. Evaluate the following limit:

$$\lim_{n\to\infty} \frac{(1^2-1)(n-1) + (2^2-2)(n-2) + \cdots + ((n-1)^2 - (n-1))}{(1^3 + 2^3 + \cdots + n^3) - (1^2 + 2^2 + \cdots + n^2)}$$

Options:
(1) $\frac{2}{3}$
(2) $\frac{1}{3}$
(3) $\frac{3}{4}$
(4) $\frac{1}{2}$

---

7. Given that $0 \le r \le n$ and the ratio of binomial coefficients is given by ${}^{n+1}C_{r+1} : {}^{n}C_r : {}^{n-1}C_{r-1} = 55 : 35 : 21$, find the value of $2n + 5r$:
Options:
(1) 60
(2) 62
(3) 50
(4) 55

---

8. A software company uses m computer systems to complete an assignment in 17 days. If, starting on the second day, 4 systems crash each day, causing the completion time to extend by 8 days, find the value of m:
Options:
(1) 125
(2) 150
(3) 180
(4) 160

---

9. If z_1 and z_2 are distinct complex numbers such that:

$$\left| \frac{z_1 - 2z_2}{\frac{1}{2} - 2z_1\overline{z_2}} \right| = 2,$$

then determine the relationship between z_1 and z_2.
Options:
(1) Either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$
(2) Either z_1 lies on a circle of radius $\frac{1}{2}$ or z_2 lies on a circle of radius 1
(3) z_1 lies on a circle of radius $\frac{1}{2}$ and z_2 lies on a circle of radius 1
(4) Both z_1 and z_2 lie on the same circle

---

10. If the function $f(x) = \left(\frac{1}{x}\right)^{2x}$; $x > 0$ attains the maximum value at $x = \frac{1}{e}$, then:

(1) $e^{\pi} < \pi^{e}$
(2) $e^{2\pi} < (2\pi)^{e}$
(3) $e^{\pi} > \pi^{e}$
(4) $(2e)^{\pi} > \pi^{2e}$

---

11. Let $\vec{a} = 6\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $|\vec{c}| \geq 6$, $\vec{a} \cdot \vec{c} = 6|\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$, and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is $60°$, then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is equal to:
(1) $\frac{9}{2}(6 - \sqrt{6})$
(2) $\frac{3}{2}\sqrt{3}$
(3) $\frac{3}{2}\sqrt{6}$
(4) $\frac{9}{2}(6 + \sqrt{6})$

---

12. If all the words with or without meaning made using all the letters of the word "NAGPUR" are arranged as in a dictionary, then the word at the 315^{th} position in this arrangement is:
(1) NRAGUP
(2) NRAGPU
(3) NRAPGU
(4) NRAPUG

---

13. Suppose for a differentiable function h, $h(0) = 0$, $h(1) = 1$, and $h'(0) = h'(1) = 2$. If $g(x) = h(e^x)e^{h(x)}$, then $g'(0)$ is equal to:
(1) 5

(2) 3
(3) 8
(4) 4

---

14. Let $P(\alpha, \beta, \gamma)$ be the image of the point $Q(3, -3, 1)$ in the line

$$\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$$

and let R be the point $(2, 5, -1)$. If the area of the triangle PQR is λ and $\lambda^2 = 14K\ ), then K$ is equal to:
(1) 36
(2) 72
(3) 18
(4) 81

Question 15: If $P(6, 1)$ is the orthocenter of a triangle with vertices $A(5, -2)$, $B(8, 3)$, and $C(h, k)$, then the point C lies on the circle:

Options:
(1) $x^2 + y^2 - 65 = 0$
(2) $x^2 + y^2 - 74 = 0$
(3) $x^2 + y^2 - 61 = 0$
(4) $x^2 + y^2 - 52 = 0$

---

16. Let $f(x) = \frac{1}{7-\sin 5x}$ be a function defined on R. Then the range of the function $f(x)$ is equal to:
(1) $\left[\frac{1}{8}, \frac{1}{5}\right]$
(2) $\left[\frac{1}{7}, \frac{1}{6}\right]$
(3) $\left[\frac{1}{7}, \frac{1}{5}\right]$
(4) $\left[\frac{1}{8}, \frac{1}{6}\right]$

---

17. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = ((\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i}) \times \hat{i}$. Then the square of the projection of $\vec{a}$ on $\vec{b}$ is:
(1) $\frac{1}{5}$
(2) 2
(3) $\frac{1}{3}$
(4) $\frac{2}{3}$

---

18. If the area of the region

$$\left\{(x, y) : \frac{a}{x^2} \leq y \leq \frac{1}{x},\ 1 \leq x \leq 2,\ 0 < a < 1\right\}$$

is

$$(\log_2 2) - \frac{1}{7}$$

then the value of $7a-3$ is equal to:
(1) 2
(2) 0
(3) -1
(4) 1

---

19. If

$$\textbf{If} \int \frac{1}{a^2\sin^2 x + b^2\cos^2 x}\,dx = \frac{1}{12}\tan^{-1}(3\tan x) + \textbf{ constant,}$$

then the maximum value of $a\sin x + b\cos x$ is:

(1) $\sqrt{40}$
(2) $\sqrt{39}$
(3) $\sqrt{42}$
(4) $\sqrt{41}$

---

20. If A is a square matrix of order 3 such that

$$\det(A) = 3 \quad \text{and} \quad \det\left(\text{adj}\left(-4\,\text{adj}\left(-3\,\text{adj}\left(3\,\text{adj}\,(2A)^{-1}\right)\right)\right)\right) = 2m3^n,$$

then $m + |2n|$ is equal to:
(1) 3
(2) 2
(3) 4
(4) 6

---

21. Let $\lfloor t \rfloor$ denote the greatest integer less than or equal to t. Let $f : [0, \infty) \to R$ be a function defined by

$$f(x) = \left\lfloor \frac{x}{2} + 3 \right\rfloor - \left\lfloor \sqrt{x} \right\rfloor.$$

Let S be the set of all points in the interval $[0, 8]$ at which f is not continuous. Then $\sum_{a\in S} a$ is equal to _____.

---

22. The length of the latus rectum and directrices of a hyperbola with eccentricity e are 9 and

$$x = \pm\frac{4}{\sqrt{3}},$$

respectively. Let the line

$$y - \sqrt{3}x + \sqrt{3} = 0$$

touch this hyperbola at (x_0, y_0). If m is the product of the focal distances of the point (x_0, y_0), then $4e^2 + m$ is equal to _____.

---

23. If $S(x) = (1+x)+2(1+x)^2+3(1+x)^3+\ldots+60(1+x)^{60}$, $x \neq 0$, and $(60)^2 S(60) = a(b)^b + b$, where $a, b \in N$, then $(a+b)$ is equal to ______.

---

24. Let $\lfloor t \rfloor$ denote the largest integer less than or equal to t. If

$$\int_0^3 \left(\lfloor x^2 \rfloor + \left\lfloor \frac{x^2}{2} \right\rfloor \right) dx = a + b\sqrt{2} - \sqrt{3} - \sqrt{5} + c\sqrt{6} - \sqrt{7},$$

where $a, b, c \in Z$, then $a + b + c$ is equal to ______.

---

25. From a lot of 12 items containing 3 defectives, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Let items in the sample be drawn one by one without replacement. If the variance of X is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then $n - m$ is equal to _____.

---

26. In a triangle ABC, $BC = 7$, $AC = 8$, $AB = \alpha \in N$, and $\cos A = \frac{2}{3}$. If $49\cos(3C) + 42 = \frac{m}{n}$, where $\gcd(m, n) = 1$, then $m + n$ is equal to _____.

---

27. If the shortest distance between the lines

$$\frac{x - \lambda}{3} = \frac{y - 2}{-1} = \frac{z - 1}{1}$$

and

$$\frac{x + 2}{-3} = \frac{y + 5}{2} = \frac{z - 4}{4}$$

is

$$\frac{44}{\sqrt{30}},$$

then the largest possible value of $|\lambda|$ is equal to _____.

---

28. Let α, β be the roots of $x^2 + \sqrt{2}x - 8 = 0$. If $U_n = \alpha^n + \beta^n$, then

$$\frac{U_{10} + \sqrt{12}U_9}{2U_8}$$

is equal to _____.

---

29. If the system of equations

$$2x + 7y + \lambda z = 3, \quad 3x + 2y + 5z = 4, \quad x + \mu y + 32z = -1$$

has infinitely many solutions, then $(\lambda - \mu)$ is equal to _____.

---

30. If the solution $y(x)$ of the given differential equation

$$(e^y + 1)\cos x \, dx + e^y \sin x \, dy = 0$$

passes through the point $\left(\frac{\pi}{2}, 0\right)$, then the value of $e^{y\left(\frac{\pi}{6}\right)}$ is equal to _____.
