JEE Main 2024 April 6 Shift 1 PHYSICS Question Paper

Time Allowed :1 Hour	Maximum Marks :100	Total Questions :30

PHYSICS

Q1. To find the spring constant (k) of a spring experimentally, a student commits a 2% positive error in the measurement of time and a 1% negative error in the measurement of mass. The percentage error in determining the value of k is:

(1) 3%   (2) 1%   (3) 4%   (4) 5%

Answer: (4)

Solution

Step 1: Formula for spring constant k

The spring constant k is related to the time period T and mass m as:

$$k = \frac{4\pi^2 m}{T^2}.$$

Step 2: Propagation of errors

Let the percentage errors in m and T be $\Delta m\%$ and $\Delta T\%$, respectively. Then, the percentage error in k is given by:

$$\Delta k\% = \Delta m\% + 2 \cdot \Delta T\%.$$

Step 3: Substitute the given values

The given errors are:

$$\Delta T = +2\%, \quad \Delta m = -1\%.$$

Substitute into the formula:

$$\Delta k\% = (-1\%) + 2 \cdot (2\%).$$

Simplify:

$$\Delta k\% = -1 + 4 = 3\%.$$

Thus, the percentage error in determining k is 5%.
Final Answer: 5%.

> Quick Tip
>
> When calculating percentage errors in derived quantities: 1. Identify the formula and its dependencies. 2. Add errors linearly for direct terms and multiply errors by the exponent for squared terms. 3. Account for positive and negative errors correctly.

Q2. A bullet of mass 50 g is fired with a speed 100 m/s on a plywood and emerges with 40 m/s. The percentage loss of kinetic energy is:

(1) 32% (2) 44% (3) 16% (4) 84%

Answer: (4)

Solution

Step 1: Kinetic energy formula

The kinetic energy of the bullet is given by:

$$KE = \frac{1}{2}mv^2,$$

where m is the mass of the bullet and v is its velocity.

Step 2: Initial and final kinetic energy

The mass of the bullet is $m = 50\,\text{g} = 0.05\,\text{kg}$.
Initial kinetic energy:

$$KE_{\text{initial}} = \frac{1}{2}mv_{\text{initial}}^2 = \frac{1}{2}(0.05)(100)^2.$$

Simplify:

$$KE_{\text{initial}} = 0.025 \cdot 10000 = 250\,\text{J}.$$

Final kinetic energy:

$$KE_{\text{final}} = \frac{1}{2}mv_{\text{final}}^2 = \frac{1}{2}(0.05)(40)^2.$$

Simplify:

$$KE_{\text{final}} = 0.025 \cdot 1600 = 40\,\text{J}.$$

Step 3: Loss of kinetic energy

The loss of kinetic energy is:

$$\Delta KE = KE_{\text{initial}} - KE_{\text{final}} = 250 - 40 = 210\,\text{J}.$$

Step 4: Percentage loss of kinetic energy

The percentage loss of kinetic energy is:

$$\text{Percentage loss} = \frac{\Delta KE}{KE_{\text{initial}}} \cdot 100.$$

Substitute the values:

$$\text{Percentage loss} = \frac{210}{250} \cdot 100 = 84\%.$$

Final Answer: 84%.

Quick Tip

To calculate the percentage loss in kinetic energy: 1. Use the formula $\Delta KE = KE_{\text{initial}} - KE_{\text{final}}$. 2. Divide the loss by the initial kinetic energy and multiply by 100. 3. Always convert mass to kilograms before using it in kinetic energy calculations.

Q3. The ratio of the shortest wavelength of Balmer series to the shortest wavelength of Lyman series for hydrogen atom is:

(1) 4 : 1 (2) 1 : 2 (3) 1 : 4 (4) 2 : 1

Answer: (1)

Solution

Step 1: Formula for wavelength in the hydrogen spectrum

The wavelength of spectral lines in the hydrogen spectrum is given by:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right),$$

where: - R_H is the Rydberg constant, - n_1 and n_2 are the principal quantum numbers ($n_2 > n_1$), - λ is the wavelength of the emitted photon.

Step 2: Shortest wavelength in Balmer series

For the Balmer series, $n_1 = 2$, and the shortest wavelength corresponds to the transition from $n_2 = \infty$ to $n_1 = 2$. Substitute these values into the formula:

$$\frac{1}{\lambda_{\text{Balmer}}} = R_H \left(\frac{1}{2^2} - \frac{1}{\infty^2} \right).$$

Simplify:

$$\frac{1}{\lambda_{\text{Balmer}}} = R_H \left(\frac{1}{4} - 0 \right) = \frac{R_H}{4}.$$

Step 3: Shortest wavelength in Lyman series

For the Lyman series, $n_1 = 1$, and the shortest wavelength corresponds to the transition from $n_2 = \infty$ to $n_1 = 1$. Substitute these values:

$$\frac{1}{\lambda_{\text{Lyman}}} = R_H \left(\frac{1}{1^2} - \frac{1}{\infty^2} \right).$$

Simplify:

$$\frac{1}{\lambda_{\text{Lyman}}} = R_H (1 - 0) = R_H.$$

Step 4: Ratio of the shortest wavelengths

The ratio of the shortest wavelength of the Balmer series to the shortest wavelength of the Lyman series is:

$$\frac{\lambda_{\text{Balmer}}}{\lambda_{\text{Lyman}}} = \frac{\frac{1}{\frac{R_H}{4}}}{\frac{1}{R_H}} = \frac{\frac{4}{R_H}}{\frac{1}{R_H}} = 4 : 1.$$

Final Answer: 4 : 1.

Quick Tip

For shortest wavelengths in spectral series: 1. Use the Rydberg formula with $n_2 = \infty$. 2. The wavelength is inversely proportional to the energy difference between levels. 3. Ratios of wavelengths can be directly derived from the n_1^2 terms.

Q4. To project a body of mass m from Earth's surface to infinity, the required kinetic energy is (assume, the radius of Earth is R_E, g = acceleration due to gravity on the surface of Earth):

$$(1)\ 2mgR_E \quad (2)\ mgR_E \quad (3)\ \frac{1}{2}mgR_E \quad (4)\ 4mgR_E$$

Answer: (2)

Solution

Step 1: Energy required to escape Earth's gravitational pull

The total energy required to project a body from the Earth's surface to infinity is the work needed to overcome the Earth's gravitational potential energy. The gravitational potential energy at a distance r from the Earth's center is:

$$U = -\frac{GMm}{r},$$

where: - G is the gravitational constant, - M is the mass of the Earth, - r is the distance from the Earth's center, - m is the mass of the object.

At the Earth's surface, $r = R_E$ (Earth's radius):

$$U_{\text{surface}} = -\frac{GMm}{R_E}.$$

At infinity, the gravitational potential energy is zero:

$$U_\infty = 0.$$

Step 2: Kinetic energy required

To move the object from Earth's surface to infinity, the object must overcome the entire gravitational potential energy at Earth's surface:

$$\Delta U = U_\infty - U_{\text{surface}} = 0 - \left(-\frac{GMm}{R_E}\right) = \frac{GMm}{R_E}.$$

Thus, the required kinetic energy to project the object is:

$$KE = \frac{GMm}{R_E}.$$

Step 3: Relate GM to g

The acceleration due to gravity g on Earth's surface is given by:

$$g = \frac{GM}{R_E^2}.$$

Rearranging:

$$GM = gR_E^2.$$

Substitute $GM = gR_E^2$ into the expression for KE:

$$KE = \frac{gR_E^2 \cdot m}{R_E} = mgR_E.$$

Final Answer: $\mathbf{mgR_E}$.

Quick Tip

For escape velocity or energy problems: 1. Use the formula for gravitational potential energy $U = -\frac{GMm}{r}$. 2. Remember that $GM = gR_E^2$ simplifies calculations for Earth's surface conditions. 3. Escape energy is the magnitude of gravitational potential energy at the surface.

Q5. Electromagnetic waves travel in a medium with a speed of 1.5×10^8 ms^{-1}. The relative permeability of the medium is 2.0. The relative permittivity will be:

(1) 5 (2) 1 (3) 4 (4) 2

Answer: (4)

Solution

Step 1: Formula for the speed of electromagnetic waves in a medium

The speed of electromagnetic waves in a medium is related to the relative permeability μ_r and relative permittivity ε_r of the medium by the formula:

$$v = \frac{1}{\sqrt{\mu_r \varepsilon_r}} \cdot c,$$

where: - v is the speed of electromagnetic waves in the medium, - c is the speed of light in vacuum, $c = 3.0 \times 10^8\,\mathrm{ms}^{-1}$, - μ_r is the relative permeability, - ε_r is the relative permittivity.

Step 2: Substitute the given values

The speed of electromagnetic waves in the medium is $v = 1.5 \times 10^8\,\mathrm{ms}^{-1}$, and the relative permeability is $\mu_r = 2.0$. Substitute these values into the formula:

$$1.5 \times 10^8 = \frac{1}{\sqrt{2.0 \cdot \varepsilon_r}} \cdot 3.0 \times 10^8.$$

Step 3: Solve for ε_r

Rearrange the equation:

$$\sqrt{2.0 \cdot \varepsilon_r} = \frac{3.0 \times 10^8}{1.5 \times 10^8}.$$

Simplify:

$$\sqrt{2.0 \cdot \varepsilon_r} = 2.$$

Square both sides:

$$2.0 \cdot \varepsilon_r = 4.$$

Solve for ε_r:

$$\varepsilon_r = \frac{4}{2.0} = 2.$$

Final Answer: 2.

Quick Tip

For electromagnetic waves in a medium: 1. Use the formula $v = \frac{1}{\sqrt{\mu_r \varepsilon_r}} \cdot c$. 2. Rearrange to solve for the unknown relative permittivity or permeability. 3. Ensure the correct values for the speed of light and wave speed are substituted.

Q6. Which of the following phenomena does not explain by wave nature of light:
(A) Reflection (B) Diffraction (C) Photoelectric effect
(D) Interference (E) Polarization

Choose the **most appropriate** answer from the options given below:

(1) E only (2) C only (3) B, D only (4) A, C only

Answer: (2)

Solution

Step 1: Wave nature of light

The wave nature of light is responsible for the following phenomena:

- **Reflection:** Light waves bounce off a surface, following the laws of reflection.
- **Diffraction:** Light bends around obstacles and spreads out after passing through small openings, explained by its wave nature.
- **Interference:** When two or more light waves overlap, they form constructive and destructive interference patterns, explained by wave superposition.
- **Polarization:** Light waves oscillate in specific planes, which is a property of transverse waves and is explained by the wave nature of light.

Step 2: Phenomenon not explained by wave nature

The **photoelectric effect** involves the emission of electrons from a metal surface when light of sufficient energy shines on it. This phenomenon is explained by the **particle nature of light** (photons) and not by its wave nature.

Step 3: Correct answer

The phenomenon that does not depend on the wave nature of light is:

$$\textbf{C. Photoelectric effect}.$$

Thus, the most appropriate answer is (2).

Quick Tip
Wave nature explains phenomena like reflection, diffraction, interference, and polarization. However, the photoelectric effect is explained by the particle nature of light.

Q7. While measuring the diameter of a wire using a screw gauge, the following readings were noted:

- Main scale reading $= 1\,\text{mm}$,
- Circular scale reading $= 42$ divisions,
- Pitch of screw gauge $= 1\,\text{mm}$,
- Number of divisions on circular scale $= 100$.

The diameter of the wire is $\frac{x}{50}$ mm. The value of x is:

(1) 142 (2) 71 (3) 42 (4) 21

Answer: (2)

Solution

Step 1: Formula for diameter of the wire

The diameter of the wire measured using a screw gauge is given by:

$$\text{Diameter} = \text{Main scale reading} + (\text{Circular scale reading}) \times (\text{Least count}).$$

Step 2: Least count of the screw gauge

The least count (LC) of the screw gauge is:

$$\text{Least count} = \frac{\text{Pitch of screw gauge}}{\text{Number of divisions on circular scale}}.$$

Substitute the given values:

$$\text{Least count} = \frac{1\,\text{mm}}{100} = 0.01\,\text{mm}.$$

Step 3: Circular scale contribution

The contribution from the circular scale reading is:

$$\text{Circular scale contribution} = \text{Circular scale reading} \times \text{Least count}.$$

Substitute the values:

$$\text{Circular scale contribution} = 42 \times 0.01 = 0.42\,\text{mm}.$$

Step 4: Total diameter of the wire

The total diameter is:

$$\text{Diameter} = \text{Main scale reading} + \text{Circular scale contribution}.$$

Substitute the values:

$$\text{Diameter} = 1 + 0.42 = 1.42\,\text{mm}.$$

Step 5: Convert diameter to the given format

The diameter is expressed as:

$$\text{Diameter} = \frac{x}{50}\,\text{mm}.$$

Equate:

$$\frac{x}{50} = 1.42.$$

Solve for x:

$$x = 1.42 \times 50 = 71.$$

Final Answer: 71.

Quick Tip

For screw gauge problems: 1. Use the formula: Diameter = Main scale reading + (Circular scale reading) × (Least count). 2. Calculate the least count by dividing the pitch by the number of divisions on the circular scale. 3. Ensure proper conversion to the required units or formats.

Q8. σ is the uniform surface charge density of a thin spherical shell of radius R. The electric field at any point on the surface of the spherical shell is:

$$(1)\ \frac{\sigma}{\varepsilon_0 R} \quad (2)\ \frac{\sigma}{2\varepsilon_0} \quad (3)\ \frac{\sigma}{\varepsilon_0} \quad (4)\ \frac{\sigma}{4\varepsilon_0}$$

Answer: (3)

Solution

Step 1: Formula for electric field of a charged spherical shell

The electric field on the surface of a thin spherical shell of radius R and uniform surface charge density σ is given by:

$$E = \frac{\sigma}{\varepsilon_0}.$$

Step 2: Derivation using Gauss's law

Using Gauss's law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}.$$

For a spherical shell, the charge enclosed is:

$$q_{\text{enclosed}} = \sigma \cdot A,$$

where $A = 4\pi R^2$ is the surface area of the sphere.
Substitute q_{enclosed}:

$$\oint \vec{E} \cdot d\vec{A} = \frac{\sigma \cdot 4\pi R^2}{\varepsilon_0}.$$

Since $\vec{E}$ is constant over the spherical surface, we can simplify:

$$E \cdot 4\pi R^2 = \frac{\sigma \cdot 4\pi R^2}{\varepsilon_0}.$$

Simplify further:

$$E = \frac{\sigma}{\varepsilon_0}.$$

Step 3: Verify units and value

The electric field is proportional to the surface charge density σ and inversely proportional to ε_0. This is consistent with the SI units.
Final Answer: $\frac{\sigma}{\varepsilon_0}$.

> Quick Tip
>
> For spherical shells: 1. Use Gauss's law to derive the electric field on the surface. 2. The field on the surface of a uniformly charged spherical shell depends only on the surface charge density σ and the permittivity ε_0. 3. Inside a spherical shell, the electric field is zero.

Q9. The value of the unknown resistance (x) for which the potential difference between B and D will be zero in the arrangement shown, is:

(1) $3\,\Omega$ (2) $9\,\Omega$ (3) $6\,\Omega$ (4) $42\,\Omega$

Answer: (3)

Solution

Step 1: Condition for zero potential difference between B and D

For the potential difference between B and D to be zero, the bridge must be balanced. This is a Wheatstone bridge condition, which is satisfied when:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4},$$

where R_1, R_2, R_3, and R_4 are the resistances in the four branches of the bridge.

Step 2: Identify the resistances in the Wheatstone bridge

From the diagram:

- $R_1 = 24\,\Omega$,
- $R_2 = x\,\Omega$ (unknown resistance),
- $R_3 = 12\,\Omega$,
- $R_4 = 1\,\Omega$.

Step 3: Apply the Wheatstone bridge condition

Using the condition for a balanced bridge:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}.$$

Substitute the known values:

$$\frac{24}{x} = \frac{12}{1}.$$

Step 4: Solve for x

Cross-multiply:

$$24 \cdot 1 = 12 \cdot x.$$

Simplify:

$$x = \frac{24}{12} = 6\,\Omega.$$

Final Answer: $6\,\Omega$.

💡 Quick Tip

For Wheatstone bridge problems: 1. Use the condition $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ when the bridge is balanced. 2. Carefully identify the resistances in the branches. 3. Solve algebraically for the unknown resistance.

Q10. The specific heat at constant pressure of a real gas obeying $P^2V = RT$ equation is:

$$(1)\ C_V + R \quad (2)\ \frac{R}{3} + C_V \quad (3)\ R \quad (4)\ C_V + \frac{R}{2V}$$

Answer: (4)

Solution

Step 1: Relationship between specific heats

The specific heat at constant pressure C_P is related to the specific heat at constant volume C_V by the formula:

$$C_P = C_V + T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P.$$

Step 2: Given equation of state

The gas follows the equation of state:

$$PV^2 = RT.$$

Differentiate with respect to T while keeping V constant to find $\left(\frac{\partial P}{\partial T}\right)_V$:

$$\frac{\partial}{\partial T}(PV^2) = \frac{\partial}{\partial T}(RT).$$

Since V is constant, V^2 comes out as a factor:

$$V^2 \frac{\partial P}{\partial T} = R.$$

Solve for $\left(\frac{\partial P}{\partial T}\right)_V$:

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V^2}.$$

Step 3: Find $\left(\frac{\partial V}{\partial T}\right)_P$

From the given equation:

$$PV^2 = RT \implies V^2 = \frac{RT}{P}.$$

Taking partial derivative with respect to T at constant P:

$$2V\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P}.$$

Solve for $\left(\frac{\partial V}{\partial T}\right)_P$:

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{2VP}.$$

Step 4: Substitute into the expression for C_P

Substitute $\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V^2}$ and $\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{2VP}$ into the formula for C_P:

$$C_P = C_V + T \cdot \frac{R}{V^2} \cdot \frac{R}{2VP}.$$

Simplify:

$$C_P = C_V + \frac{TR^2}{2V^3P}.$$

From the ideal gas law $PV^2 = RT$, substitute $P = \frac{RT}{V^2}$:

$$C_P = C_V + \frac{TR^2}{2V^3 \cdot \frac{RT}{V^2}}.$$

Simplify further:

$$C_P = C_V + \frac{R}{2V}.$$

Final Answer: $\mathbf{C_V + \frac{R}{2V}}$.

💡 Quick Tip

For real gases: 1. Use the modified equation of state to calculate partial derivatives. 2. Substitute the derivatives into the expression for C_P. 3. Simplify carefully using relationships from the gas equation.

Q11. Match List I with List II:

List I	List II
A. Torque	I. $[M^1L^2T^{-2}]$
B. Magnetic field	II. $[L^2A^{-1}]$
C. Magnetic moment	III. $[M^1T^{-2}A^{-1}]$
D. Permeability of free space	IV. $[M^1L^1T^{-2}A^{-2}]$

Choose the correct answer from the options given below:
(1) A-I, B-III, C-II, D-IV (2) A-IV, B-III, C-II, D-I
(3) A-III, B-I, C-II, D-IV (4) A-IV, B-II, C-III, D-I
Answer: (2)

Solution

Step 1: Dimensional analysis for each quantity

1. **Torque:** Torque is the product of force and distance. The dimensional formula for force is $[M^1L^1T^{-2}]$, and multiplying by distance $[L]$:

$$\text{Dimensional formula for Torque: } [M^1L^2T^{-2}].$$

2. **Magnetic field (B):** The dimensional formula for magnetic field is derived from the Lorentz force equation $F = qvB$, where:

$$B = \frac{F}{qv}.$$

Substitute dimensions:

$$[B] = \frac{[M^1L^1T^{-2}]}{[A^1T^1][L^1T^{-1}]} = [M^1T^{-2}A^{-1}].$$

3. **Magnetic moment (M_m):** The magnetic moment is the product of current (I) and area (A):

$$[M_m] = [A^1][L^2] = [L^2A^1].$$

4. **Permeability of free space (μ_0):** From the equation $F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$, the dimensional formula for μ_0 is:

$$[\mu_0] = [M^1L^1T^{-2}A^{-2}].$$

Step 2: Match List I with List II

$$\begin{aligned} \text{A. Torque} &: \text{I. } [M^1L^2T^{-2}] \\ \text{B. Magnetic field} &: \text{III. } [M^1T^{-2}A^{-1}] \\ \text{C. Magnetic moment} &: \text{II. } [L^2A^1] \\ \text{D. Permeability of free space} &: \text{IV. } [M^1L^1T^{-2}A^{-2}] \end{aligned}$$

Step 3: Correct option

The correct matching is:

$$\text{A-I, B-III, C-II, D-IV.}$$

Final Answer: (2).

Quick Tip

To solve dimensional analysis problems: 1. Recall or derive the physical formula for the quantity. 2. Break down each term into its fundamental dimensions (M, L, T, A). 3. Use the correct relationships between quantities, such as force, magnetic field, and current.

Q12. Given below are two statements:
Statement I: In an LCR series circuit, current is maximum at resonance.
Statement II: Current in a purely resistive circuit can never be less than that in a series LCR circuit when connected to the same voltage source.
In the light of the above statements, choose the **correct** answer from the options given below:

(1) Statement I is true but Statement II is false.

(2) Statement I is false but Statement II is true.

(3) Both Statement I and Statement II are true.

(4) Both Statement I and Statement II are false.

Answer: (3)

Solution

Step 1: Analyze Statement I

In an LCR series circuit, the impedance (Z) is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2},$$

where: - R is the resistance, - $X_L = \omega L$ is the inductive reactance, - $X_C = \frac{1}{\omega C}$ is the capacitive reactance.
At resonance:

$$X_L = X_C \implies Z = R.$$

Since Z is minimum at resonance, the current is maximum (as $I = \frac{V}{Z}$).
Thus, **Statement I is true**.

Step 2: Analyze Statement II

In a purely resistive circuit, the impedance $Z = R$, and the current is:

$$I = \frac{V}{R}.$$

For a series LCR circuit, the impedance is always $Z \geq R$ (as the square term $(X_L - X_C)^2$ adds to R^2 in Z). This means the current in a purely resistive circuit is greater than or equal to the current in an LCR circuit connected to the same voltage source.
Thus, **Statement II is true**.

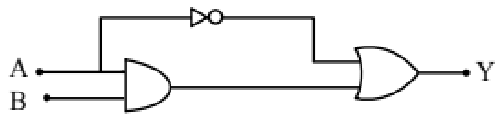
Step 3: Conclusion

Both Statement I and Statement II are correct.
Final Answer: (3).

Quick Tip

For LCR circuits: 1. At resonance, impedance is minimum ($Z = R$), and current is maximum. 2. In a purely resistive circuit, impedance is R, and current is always greater than or equal to that in a series LCR circuit for the same voltage.

Q13. The correct truth table for the following logic circuit is:
Options:



A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(1)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(2)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(3)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(4)

Answer: (2)

Solution

Step 1: Analyze the circuit

1. The input A passes through a NOT gate, producing $\bar{A}$. 2. The output of the NOT gate ($\bar{A}$) and B are inputs to the AND gate, producing $\bar{A} \cdot B$. 3. The OR gate takes two inputs: A and $\bar{A} \cdot B$.
The final output is:

$$Y = A + (\bar{A} \cdot B).$$

Step 2: Simplify the expression

Using Boolean algebra:

$$Y = A + (\bar{A} \cdot B) = A + B \cdot \bar{A}.$$

Simplify further:

$$Y = A + B \quad \text{(distributive property)}.$$

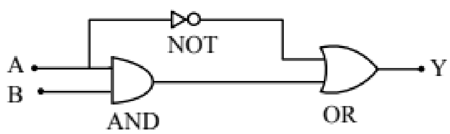
Step 3: Create the truth table

Substitute the values of A and B into the simplified expression $Y = A + B$:

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

This matches the truth table given in **Option (2)**.
Final Answer: (2)

Sol.



💡 Quick Tip

For logic circuits: 1. Simplify the Boolean expression using gate operations step-by-step. 2. Create the truth table by substituting all input combinations. 3. Double-check the circuit diagram to match inputs and outputs correctly.

Q14. A sample contains a mixture of helium and oxygen gas. The ratio of root mean square speed of helium and oxygen in the sample is:

$$(1)\ \frac{1}{32} \quad (2)\ \frac{2\sqrt{2}}{1} \quad (3)\ \frac{1}{4} \quad (4)\ \frac{1}{2\sqrt{2}}$$

Answer: (2)

Solution

Step 1: Root mean square speed formula

The root mean square (RMS) speed of a gas is given by:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}},$$

where:

- R is the universal gas constant,
- T is the absolute temperature,
- M is the molar mass of the gas.

Step 2: Ratio of RMS speeds

Let the RMS speed of helium be v_{He} and the RMS speed of oxygen be v_{O_2}. The ratio is:

$$\frac{v_{\text{He}}}{v_{\text{O}_2}} = \sqrt{\frac{M_{\text{O}_2}}{M_{\text{He}}}}.$$

Step 3: Molar masses of helium and oxygen

The molar mass of helium is:

$$M_{\text{He}} = 4\,\text{g/mol}.$$

The molar mass of oxygen (O_2) is:

$$M_{\text{O}_2} = 32\,\text{g/mol}.$$

Step 4: Substitute values into the ratio

Substitute $M_{\text{He}} = 4$ and $M_{\text{O}_2} = 32$:

$$\frac{v_{\text{He}}}{v_{\text{O}_2}} = \sqrt{\frac{32}{4}}.$$

Simplify:

$$\frac{v_{\text{He}}}{v_{\text{O}_2}} = \sqrt{8} = 2\sqrt{2}.$$

Final Answer: $\mathbf{\frac{2\sqrt{2}}{1}}$.

💡 Quick Tip

The RMS speed ratio of two gases depends on the inverse square root of their molar masses:

$$\frac{v_{\text{rms}_1}}{v_{\text{rms}_2}} = \sqrt{\frac{M_2}{M_1}}.$$

Lighter gases have higher RMS speeds at the same temperature.

Q15. A light string passing over a smooth light pulley connects two blocks of masses m and m (where $m_2 > m_1$). If the acceleration of the system is $\frac{g}{\sqrt{2}}$, then the ratio of the masses $\frac{m_1}{m_2}$ is:

$$(1)\ \frac{\sqrt{2}-1}{\sqrt{2}+1} \quad (2)\ \frac{1+\sqrt{5}}{\sqrt{5}-1} \quad (3)\ \frac{1+\sqrt{5}}{\sqrt{2}-1} \quad (4)\ \frac{\sqrt{3}+1}{\sqrt{2}-1}$$

Answer: (1)

Solution

Step 1: Free-body diagram and equations of motion

For the heavier mass m_2:

$$m_2 g - T = m_2 a,$$

where T is the tension in the string and a is the acceleration of the system.
For the lighter mass m_1:

$$T - m_1 g = m_1 a.$$

Step 2: Solve for tension T

From the first equation:

$$T = m_2 g - m_2 a.$$

From the second equation:

$$T = m_1 g + m_1 a.$$

Equating the two expressions for T:

$$m_2 g - m_2 a = m_1 g + m_1 a.$$

Step 3: Simplify the equation

Rearrange terms:

$$m_2 g - m_1 g = m_1 a + m_2 a.$$

Factor out g and a:

$$g(m_2 - m_1) = a(m_1 + m_2).$$

Substitute $a = \frac{g}{\sqrt{2}}$:

$$g(m_2 - m_1) = \frac{g}{\sqrt{2}}(m_1 + m_2).$$

Simplify by canceling g:

$$m_2 - m_1 = \frac{1}{\sqrt{2}}(m_1 + m_2).$$

Step 4: Solve for $\frac{m_1}{m_2}$

Expand and rearrange terms:

$$\sqrt{2}m_2 - \sqrt{2}m_1 = m_1 + m_2.$$

Combine terms for m_1 and m_2:

$$(\sqrt{2} - 1)m_2 = (\sqrt{2} + 1)m_1.$$

Solve for $\frac{m_1}{m_2}$:

$$\frac{m_1}{m_2} = \frac{\sqrt{2} - 1}{\sqrt{2} + 1}.$$

Final Answer: $\boldsymbol{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$.

> Quick Tip
>
> For pulley problems: 1. Write separate equations of motion for each mass. 2. Solve for the tension T and use the condition that the string's acceleration is the same for both masses. 3. Substitute given accelerations and simplify systematically.

Q16. Four particles A, B, C, D of mass $\frac{m}{2}$, m, $2m$, $4m$ have the same momentum, respectively. The particle with maximum kinetic energy is:

(1) D (2) C (3) A (4) B

Answer: (3)

Solution

Step 1: Kinetic energy in terms of momentum

The kinetic energy K is expressed as:

$$K = \frac{p^2}{2m},$$

where: - p is the momentum, - m is the mass of the particle.

Step 2: Relationship between mass and kinetic energy

For particles with the same momentum:

$$K \propto \frac{1}{m}.$$

This means that for the same momentum, a particle with smaller mass will have larger kinetic energy.

Step 3: Compare the masses of the particles

The masses of the particles are:

$$\text{A: } \frac{m}{2}, \quad \text{B: } m, \quad \text{C: } 2m, \quad \text{D: } 4m.$$

The particle with the smallest mass is A ($\frac{m}{2}$).
Final Answer: (3) A.

> Quick Tip
>
> For particles with the same momentum, kinetic energy is inversely proportional to mass.
> Lighter particles have more kinetic energy.

Q17. A train starting from rest first accelerates uniformly up to a speed of 80 km/h for time t, then it moves with a constant speed for time $3t$. The average speed of the train for this duration of journey will be (in km/h):

(1) 80 (2) 70 (3) 30 (4) 40

Answer: (2)

Solution

Step 1: Formula for average speed

The average speed is given by:

$$v_{\text{avg}} = \frac{\text{Total distance}}{\text{Total time}}.$$

Step 2: Calculate total distance

The motion has two phases:

- **Phase 1 (Acceleration):** The train accelerates from rest to 80 km/h in time t. The average speed during this phase is:

$$v_{\text{avg},1} = \frac{0+80}{2} = 40\,\text{km/h}.$$

  The distance covered in this phase is:

$$d_1 = v_{\text{avg},1} \cdot t = 40t.$$

- **Phase 2 (Constant speed):** The train moves at a constant speed of 80 km/h for time $3t$. The distance covered in this phase is:

$$d_2 = 80 \cdot 3t = 240t.$$

The total distance is:

$$d_{\text{total}} = d_1 + d_2 = 40t + 240t = 280t.$$

Step 3: Calculate total time

The total time is:

$$t_{\text{total}} = t + 3t = 4t.$$

Step 4: Calculate average speed

The average speed is:

$$v_{\text{avg}} = \frac{d_{\text{total}}}{t_{\text{total}}} = \frac{280t}{4t} = 70\,\text{km/h}.$$

Final Answer: (2) 70 km/h.

Quick Tip

For problems involving multiple phases of motion: 1. Compute distances for each phase separately. 2. Use the total distance and total time to calculate the average speed.

Q18. An element $\Delta l = \Delta x i$ is placed at the origin and carries a large current $I = 10$ A. The magnetic field on the y-axis at a distance of 0.5 m from the element Δx of 1 cm length is:

(1) 4×10^{-8} T (2) 8×10^{-8} T (3) 12×10^{-8} T (4) 10×10^{-8} T

Answer: (1)

Solution

Step 1: Formula for the magnetic field due to a current element

The magnetic field due to a current element is given by the Biot-Savart law:

$$dB = \frac{\mu_0}{4\pi} \frac{I\,|\Delta\vec{l} \times \vec{r}|}{r^3},$$

where:

- $\mu_0 = 4\pi \times 10^{-7}\,\text{T m/A}$ is the permeability of free space,
- $I = 10\,\text{A}$ is the current,
- $\Delta\vec{l} = \Delta x\,\hat{i}$ is the current element vector,
- $\vec{r}$ is the position vector from the element to the point of interest,
- $r = 0.5\,\text{m}$ is the distance between the element and the point.

Step 2: Determine $|\Delta\vec{l} \times \vec{r}|$

The current element is along the x-axis: $\Delta\vec{l} = \Delta x\,\hat{i}$, and the position vector to the point on the y-axis is $\vec{r} = 0.5\,\hat{j}$. The cross product is:

$$\Delta\vec{l} \times \vec{r} = \Delta x\,\hat{i} \times 0.5\,\hat{j}.$$

Using the cross product rule:

$$\hat{i} \times \hat{j} = \hat{k}.$$

Thus:

$$\Delta\vec{l} \times \vec{r} = 0.5\Delta x\,\hat{k}.$$

The magnitude is:

$$|\Delta\vec{l} \times \vec{r}| = 0.5\Delta x.$$

Step 3: Substitute into Biot-Savart law

The magnetic field is:

$$dB = \frac{\mu_0}{4\pi} \frac{I\,|\Delta\vec{l} \times \vec{r}|}{r^3}.$$

Substitute $|\Delta\vec{l} \times \vec{r}| = 0.5\Delta x$ and $r = 0.5\,\text{m}$:

$$dB = \frac{\mu_0}{4\pi} \frac{I \cdot 0.5\Delta x}{(0.5)^3}.$$

Step 4: Simplify the expression

Substitute $\mu_0 = 4\pi \times 10^{-7}\,\text{T m/A}$:

$$dB = \frac{4\pi \times 10^{-7}}{4\pi} \cdot \frac{10 \cdot 0.5 \cdot \Delta x}{0.125}.$$

Simplify:

$$dB = 10^{-7} \cdot \frac{10 \cdot 0.5 \cdot \Delta x}{0.125}.$$

$$dB = 4 \times 10^{-6} \cdot \Delta x.$$

Step 5: Substitute $\Delta x = 1\,\mathbf{cm} = 0.01\,\mathbf{m}$

Substitute $\Delta x = 0.01$:

$$dB = 4 \times 10^{-6} \cdot 0.01 = 4 \times 10^{-8}\,\text{T}.$$

Final Answer: $\mathbf{4 \times 10^{-8}}\,\text{T}$.

💡 Quick Tip

To solve magnetic field problems using Biot-Savart law: 1. Identify the current element $\Delta\vec{l}$ and position vector $\vec{r}$. 2. Use the cross product rule to calculate $|\Delta\vec{l} \times \vec{r}|$. 3. Substitute all values into the Biot-Savart formula and simplify carefully.

Q19. A small ball of mass m and density ρ is dropped in a viscous liquid of density ρ . After some time, the ball falls with constant velocity. The viscous force on the ball is:

$$(1)\ mg\left(\frac{\rho_0}{\rho} - 1\right) \quad (2)\ mg\left(1 + \frac{\rho}{\rho_0}\right) \quad (3)\ mg(1 - \rho\rho_0) \quad (4)\ mg\left(1 - \frac{\rho_0}{\rho}\right)$$

Answer: (4)

Solution

Step 1: Forces acting on the ball

When the ball reaches constant velocity, the net force acting on it is zero. The forces acting on the ball are:

- Weight of the ball: $F_g = mg$,
- Buoyant force due to the liquid: $F_b = V\rho_0 g$, where V is the volume of the ball,
- Viscous force: F_v (to be determined).

At terminal velocity:

$$F_g = F_b + F_v.$$

Step 2: Express the forces in terms of mass and density

The mass of the ball is related to its density and volume as:

$$m = \rho V.$$

Substitute $V = \frac{m}{\rho}$ into the buoyant force:

$$F_b = \left(\frac{m}{\rho}\right)\rho_0 g.$$

Step 3: Solve for the viscous force

From the equilibrium condition:

$$F_v = F_g - F_b.$$

Substitute $F_g = mg$ and $F_b = \left(\frac{m}{\rho}\right)\rho_0 g$:

$$F_v = mg - \left(\frac{m}{\rho}\right)\rho_0 g.$$

Factorize:

$$F_v = mg\left(1 - \frac{\rho_0}{\rho}\right).$$

Final Answer: $\mathbf{mg\left(1 - \frac{\rho_0}{\rho}\right)}$.

> 💡 Quick Tip
>
> For terminal velocity problems: 1. Use the equilibrium condition: Weight = Buoyant force + Viscous force. 2. Express buoyant force using the displaced fluid's density. 3. Solve algebraically for the unknown force or quantity.

Q20. In a photoelectric experiment, energy of 2.48 eV irradiates a photo-sensitive material. The stopping potential was measured to be 0.5 V. The work function of the photo-sensitive material is:

(1) 0.5 eV (2) 1.68 eV (3) 2.48 eV (4) 1.98 eV

Answer: (4)

Solution

Step 1: Photoelectric equation

The photoelectric equation relates the energy of the incident photon to the work function and the maximum kinetic energy of the ejected electrons:

$$E_{\text{photon}} = \phi + K_{\text{max}},$$

where:

- E_{photon} is the energy of the incident photon,
- ϕ is the work function of the material,
- K_{max} is the maximum kinetic energy of the ejected electrons.

Step 2: Relationship between K_{max} and stopping potential

The maximum kinetic energy of the electrons is related to the stopping potential V_s as:

$$K_{\text{max}} = eV_s,$$

where:

- e is the charge of the electron (in this case, taken as unity since energy is in eV),
- $V_s = 0.5\,\text{V}$ is the stopping potential.

Thus:

$$K_{\text{max}} = 0.5\,\text{eV}.$$

Step 3: Solve for the work function

Using the photoelectric equation:

$$\phi = E_{\text{photon}} - K_{\text{max}}.$$

Substitute the given values:

$$\phi = 2.48\,\text{eV} - 0.5\,\text{eV}.$$

Simplify:

$$\phi = 1.98\,\text{eV}.$$

Final Answer: (4) 1.98 **eV.**

💡 Quick Tip

For photoelectric problems: 1. Use the photoelectric equation $E_{\text{photon}} = \phi + K_{\max}$. 2. Relate the stopping potential to the maximum kinetic energy using $K_{\max} = eV_s$. 3. Subtract $K_{\max}$ from the photon energy to find the work function.

SECTION-B

Q21. If the radius of the earth is reduced to three-fourth of its present value without changing its mass, then the value of the duration of the day of the earth will be $_{hours 30 minutes.}$

Answer: (13)

Solution

Step 1: Conservation of angular momentum

The angular momentum of the earth is conserved:

$$I_1\omega_1 = I_2\omega_2,$$

where:

- $I = \frac{2}{5}MR^2$ is the moment of inertia of the earth,
- $\omega = \frac{2\pi}{T}$ is the angular velocity,
- T is the time period of rotation.

Substitute $I = \frac{2}{5}MR^2$ and $\omega = \frac{2\pi}{T}$:

$$\frac{2}{5}MR_1^2 \cdot \frac{2\pi}{T_1} = \frac{2}{5}MR_2^2 \cdot \frac{2\pi}{T_2}.$$

Step 2: Relation between T_1 and T_2

Simplify the equation:

$$\frac{R_1^2}{T_1} = \frac{R_2^2}{T_2}.$$

Since $R_2 = \frac{3}{4}R_1$:

$$\frac{R_1^2}{T_1} = \frac{\left(\frac{3}{4}R_1\right)^2}{T_2}.$$

Simplify further:

$$\frac{1}{T_1} = \frac{\frac{9}{16}}{T_2}.$$

Rearrange:

$$\frac{1}{T_2} = \frac{9}{16} \cdot \frac{1}{T_1}.$$

Step 3: Calculate T_2

Substitute $T_1 = 24\,\text{hours}$:

$$T_2 = \frac{16}{9} \cdot T_1 = \frac{16}{9} \cdot 24 = \frac{384}{27} = 13.5\,\text{hours}.$$

Convert 0.5 hours to minutes:

$$13.5\,\text{hours} = 13\,\text{hours}\,30\,\text{minutes}.$$

Final Answer: (13 hours 30 minutes).

Quick Tip

For problems involving angular momentum conservation: 1. Use $I_1\omega_1 = I_2\omega_2$. 2. Relate the moment of inertia to radius as $I \propto R^2$. 3. Solve for the new time period or angular velocity.

Q22. Three infinitely long charged thin sheets are placed as shown in the figure. The magnitude of the electric field at the point P is $\frac{x\sigma}{\epsilon_0}$. The value of x is $_{(all quantities are measured in SI units).}$

Answer: (2)

Solution

Step 1: Electric field due to an infinite sheet of charge

The electric field due to an infinite charged sheet with surface charge density σ is:

$$E = \frac{\sigma}{2\epsilon_0}.$$

Step 2: Net electric field at point P

The configuration of the sheets and the fields they produce:

- The sheet at $X = -a$ has charge density $-\sigma$. The field at P due to this sheet is:

$$E_1 = \frac{-\sigma}{2\epsilon_0}.$$

- The sheet at $X = a$ has charge density $-\sigma$. The field at P due to this sheet is:

$$E_2 = \frac{-\sigma}{2\epsilon_0}.$$

- The sheet at $X = 3a$ has charge density $+\sigma$. The field at P due to this sheet is:

$$E_3 = \frac{\sigma}{2\epsilon_0}.$$

Step 3: Calculate the net field

Add the contributions at P:

$$E_{\text{net}} = E_1 + E_2 + E_3.$$

Substitute the values:

$$E_{\text{net}} = \frac{-\sigma}{2\epsilon_0} + \frac{-\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}.$$

Simplify:

$$E_{\text{net}} = \frac{-\sigma}{2\epsilon_0}.$$

Step 4: Compare with the given expression

The given expression for the field is:

$$E_{\text{net}} = \frac{x\sigma}{\epsilon_0}.$$

Equating the two expressions:

$$\frac{-\sigma}{2\epsilon_0} = \frac{x\sigma}{\epsilon_0}.$$

Solve for x:

$$x = -\frac{1}{2}.$$

Final Answer: (2).

💡 Quick Tip

For problems involving electric fields due to infinite sheets: 1. Use $E = \frac{\sigma}{2\epsilon_0}$ for the field due to a single sheet. 2. Account for the direction of the field using the sign of the charge. 3. Add fields vectorially to find the net field.

Q23. A big drop is formed by coalescing 1000 small droplets of water. The ratio of surface energy of 1000 droplets to that of the energy of the big drop is $\frac{10}{x}$. The value of x is

Answer: (1)

Solution

Step 1: Volume conservation

The volume of 1000 small droplets equals the volume of the big drop:

$$1000 \cdot \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3,$$

where r is the radius of a small droplet and R is the radius of the big drop. Simplify:

$$R^3 = 1000r^3.$$

Taking the cube root:

$$R = 10r.$$

Step 2: Surface energy comparison

The surface energy of 1000 small droplets is:

$$\text{S.E.}_{\text{small}} = 1000 \cdot 4\pi r^2 \cdot T,$$

where T is the surface tension of water.
The surface energy of the big drop is:

$$\text{S.E.}_{\text{big}} = 4\pi R^2 \cdot T = 4\pi(10r)^2 \cdot T.$$

Simplify:

$$\text{S.E.}_{\text{big}} = 4\pi \cdot 100r^2 \cdot T = 400\pi r^2 \cdot T.$$

Step 3: Calculate the ratio

The ratio of surface energies is:

$$\frac{\text{S.E.}_{\text{small}}}{\text{S.E.}_{\text{big}}} = \frac{1000 \cdot 4\pi r^2 \cdot T}{400\pi r^2 \cdot T}.$$

Simplify:

$$\frac{\text{S.E.}_{\text{small}}}{\text{S.E.}_{\text{big}}} = \frac{1000}{400} = \frac{10}{x}.$$

Equating:

$$\frac{10}{x} = \frac{1000}{400}.$$

Solve for x:

$$x = 4.$$

Final Answer: 1.

> 💡 Quick Tip
>
> For problems involving coalescence of droplets: 1. Use volume conservation: $N \cdot V_{\text{small}} = V_{\text{big}}$. 2. Relate the surface areas and calculate the ratio of surface energies.

Q24. When a DC voltage of 100 V is applied to an inductor, a DC current of 5 A flows through it. When an AC voltage of 200 V peak value is connected to the inductor, its inductive reactance is found to be $20\sqrt{3}\,\Omega$. The power dissipated in the circuit is $_{w}$.

Answer: (250)

Solution

Step 1: DC circuit resistance

For a DC circuit, the inductor behaves as a pure resistor. Using Ohm's law:

$$R = \frac{V}{I},$$

where:

- $V = 100\,\text{V}$,
- $I = 5\,\text{A}$.

Substitute the values:

$$R = \frac{100}{5} = 20\,\Omega.$$

Step 2: AC circuit power dissipation

In an AC circuit, the power dissipated is given by:

$$P = I_{\text{rms}}^2 R.$$

The RMS value of the current is:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z},$$

where:

- $V_{\text{rms}} = \frac{V_{\text{peak}}}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 100\sqrt{2}\,\text{V}$,
- Z is the total impedance. Here, $Z = \sqrt{R^2 + X_L^2}$,
- $R = 20\,\Omega$, $X_L = 20\sqrt{3}\,\Omega$.

Step 3: Calculate impedance

Substitute R and X_L:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(20)^2 + (20\sqrt{3})^2}.$$

Simplify:

$$Z = \sqrt{400 + 1200} = \sqrt{1600} = 40\,\Omega.$$

Step 4: Calculate RMS current

The RMS current is:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{100\sqrt{2}}{40} = 2.5\sqrt{2}\,\text{A}.$$

Step 5: Calculate power dissipated

Substitute I_{rms} and R:

$$P = I_{\text{rms}}^2 R = (2.5\sqrt{2})^2 \cdot 20.$$

Simplify:

$$P = 2.5^2 \cdot 2 \cdot 20 = 6.25 \cdot 2 \cdot 20 = 250\,\text{W}.$$

Final Answer: 250 W.

💡 Quick Tip
For AC circuits with inductors: 1. Calculate the impedance Z using $Z = \sqrt{R^2 + X_L^2}$. 2. Use RMS values of voltage and current to calculate power dissipation as $P = I_{\text{rms}}^2 R$.

Q25. The refractive index of a prism is μ =$\sqrt{3}$ and the ratio of the angle of minimum deviation to the angle of the prism is one. The value of the angle of the prism is

Answer: (60°)

Solution

Step 1: Prism formula

The relationship between the refractive index (μ), angle of the prism (A), and the angle of minimum deviation (D_m) is given by:

$$\mu = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}.$$

Step 2: Ratio condition

It is given that the ratio of the angle of minimum deviation to the angle of the prism is one:

$$\frac{D_m}{A} = 1.$$

Thus:

$$D_m = A.$$

Step 3: Substitute $D_m = A$ into the prism formula

Substitute $D_m = A$ into the formula:

$$\mu = \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin A}{\sin\left(\frac{A}{2}\right)}.$$

Step 4: Given refractive index

The refractive index is given as $\mu = \sqrt{3}$. Substitute this value:

$$\sqrt{3} = \frac{\sin A}{\sin\left(\frac{A}{2}\right)}.$$

Step 5: Simplify using trigonometric identities

Using the identity $\sin A = 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)$:

$$\sqrt{3} = \frac{2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)}.$$

Cancel $\sin\left(\frac{A}{2}\right)$ (non-zero):

$$\sqrt{3} = 2\cos\left(\frac{A}{2}\right).$$

Step 6: Solve for $\cos\left(\frac{A}{2}\right)$

$$\cos\left(\frac{A}{2}\right) = \frac{\sqrt{3}}{2}.$$

Step 7: Find $\frac{A}{2}$ and A

The angle whose cosine is $\frac{\sqrt{3}}{2}$ is $30°$:

$$\frac{A}{2} = 30°.$$

Multiply by 2 to find A:

$$A = 60°.$$

Final Answer: 60°.

> 💡 Quick Tip
>
> For prism problems involving refractive index: 1. Use the prism formula $\mu = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$. 2. Substitute known values and simplify using trigonometric identities. 3. Pay attention to the given ratios or relationships between angles.

Q26. A wire of resistance R and radius r is stretched till its radius becomes r/2. If the new resistance of the stretched wire is xR, then the value of x is

Answer: (16)

Solution

Step 1: Formula for resistance

The resistance of a wire is given by:

$$R = \rho\frac{l}{A},$$

where:

- ρ is the resistivity,
- l is the length of the wire,

- A is the cross-sectional area of the wire.

For a wire of radius r, the cross-sectional area is:

$$A = \pi r^2.$$

Thus, resistance is inversely proportional to the square of the radius:

$$R \propto \frac{l}{r^2}.$$

Step 2: Volume conservation during stretching

When the wire is stretched, its volume remains constant:

$$V_i = V_f.$$

The initial volume is:

$$V_i = \pi r^2 l.$$

The final volume is:

$$V_f = \pi \left(\frac{r}{2}\right)^2 l_f.$$

Equate the two volumes:

$$\pi r^2 l = \pi \left(\frac{r}{2}\right)^2 l_f.$$

Simplify:

$$l_f = 4l.$$

Step 3: New resistance of the wire

The new resistance is proportional to the new length and inversely proportional to the square of the new radius:

$$R_{\text{new}} = \rho \frac{l_f}{\left(\frac{r}{2}\right)^2}.$$

Substitute $l_f = 4l$:

$$R_{\text{new}} = \rho \frac{4l}{\frac{r^2}{4}} = \rho \frac{4l \cdot 4}{r^2}.$$

Simplify:

$$R_{\text{new}} = 16 \cdot \rho \frac{l}{r^2}.$$

Since $R = \rho \frac{l}{r^2}$, the new resistance is:

$$R_{\text{new}} = 16R.$$

Final Answer: 16.

Quick Tip
When a wire is stretched: 1. Use volume conservation to relate initial and final lengths. 2. Remember that resistance is proportional to the length and inversely proportional to the square of the radius.

Q27. The radius of a certain orbit of the hydrogen atom is 8.48 ˚ A. If the energy of the electron in this orbit is E/x, then $x =$ ______.
(Given $a_0 = 0.529\,\text{Å}$, $E =$ energy of the electron in ground state).
Answer: (16)

Solution

Step 1: Formula for the radius of the n-th orbit

The radius of the n-th orbit in the hydrogen atom is given by:

$$r_n = n^2 a_0,$$

where:

- r_n is the radius of the n-th orbit,
- n is the principal quantum number,
- $a_0 = 0.529\,\text{Å}$ is the Bohr radius.

Step 2: Calculate the principal quantum number n

The given radius is:

$$r_n = 8.48\,\text{Å}.$$

Substitute r_n into the formula:

$$8.48 = n^2 \cdot 0.529.$$

Solve for n^2:

$$n^2 = \frac{8.48}{0.529} = 16.$$

Take the square root:

$$n = 4.$$

Step 3: Energy of the electron in the n-th orbit

The energy of the electron in the n-th orbit is given by:

$$E_n = \frac{E}{n^2},$$

where E is the energy of the electron in the ground state.
Substitute $n^2 = 16$:

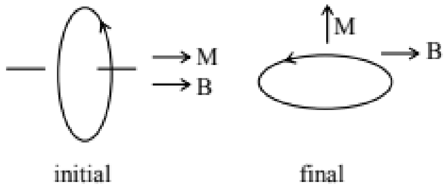
$$E_n = \frac{E}{16}.$$

Thus, $x = 16$.
Final Answer: 16.

> Quick Tip
>
> For hydrogen atom problems: 1. Use $r_n = n^2 a_0$ to calculate the principal quantum number. 2. Use $E_n = \frac{E}{n^2}$ to calculate the energy in higher orbits.

Q28. A circular coil having 200 turns, 2.5$\times 10^{-4}\,\text{m}^2$ area and carrying $100\,\mu\text{A}$ current is placed in a uniform magnetic field of $1\,\text{T}$. Initially, the magnetic dipole moment ($\vec{M}$) was directed along $\vec{B}$. The amount of work required to rotate the coil through $90°$ from its initial orientation such that $\vec{M}$ becomes perpendicular to $\vec{B}$ is μJ.
Answer: (5)

Sol.



Step 1: Magnetic dipole moment of the coil

The magnetic dipole moment of a coil is given by:

$$M = NIA,$$

where:

- $N = 200$ is the number of turns,
- $I = 100\,\mu\text{A} = 100 \times 10^{-6}\,\text{A}$ is the current,
- $A = 2.5 \times 10^{-4}\,\text{m}^2$ is the area of the coil.

Substitute the values:

$$M = 200 \cdot (100 \times 10^{-6}) \cdot (2.5 \times 10^{-4}).$$

Simplify:

$$M = 200 \cdot 10^{-4} \cdot 2.5 \times 10^{-4} = 5 \times 10^{-3}\,\text{A} \cdot \text{m}^2.$$

Step 2: Work done to rotate the dipole

The work required to rotate a magnetic dipole in a magnetic field is given by:

$$W = MB(\cos\theta_1 - \cos\theta_2),$$

where:

- $M = 5 \times 10^{-3}\,\text{A} \cdot \text{m}^2$ is the magnetic dipole moment,
- $B = 1\,\text{T}$ is the magnetic field,
- $\theta_1 = 0^\circ$ (initial orientation),
- $\theta_2 = 90^\circ$ (final orientation).

Substitute the angles:

$$W = (5 \times 10^{-3}) \cdot (1) \cdot (\cos 0^\circ - \cos 90^\circ).$$

Simplify:

$$W = (5 \times 10^{-3}) \cdot (1) \cdot (1 - 0) = 5 \times 10^{-3}\,\text{J}.$$

Convert to microjoules:

$$W = 5\,\mu\text{J}.$$

Step 3: Condition for $\vec{A} \cdot (\vec{B} \times \vec{C}) = 0$

Set the expression to zero:

$$-15x + 64 = 0.$$

Solve for x:

$$15x = 64.$$

Finally:

$$x = 4.$$

Final Answer: 4

Quick Tip

For problems involving cross and dot products: 1. Expand cross products using determinants. 2. Simplify the resultant vector before calculating the dot product. 3. Equate to zero and solve for unknowns when necessary.