JEE MAIN 6 April Shift-1 Mathematics Question Paper

Mathematics

Q.1 If $f(x)=\begin{cases} x^3\sin\left(\frac{1}{x}\right), & x\neq 0, \\ 0, & x=0, \end{cases}$ then:

(1) $f''(0)=1$
(2) $f''\left(\frac{2}{\pi}\right)=\frac{24-\pi^2}{2\pi}$
(3) $f''\left(\frac{2}{\pi}\right)=\frac{12-\pi^2}{2\pi}$
(4) $f''(0)=0$

Q.2 If $A(3,1,-1)$, $B\left(\frac{5}{3},\frac{7}{3},\frac{1}{3}\right)$, $C(2,2,1)$, **and** $D\left(\frac{10}{3},\frac{2}{3},-\frac{1}{3}\right)$ **are the vertices of a quadrilateral** $ABCD$**, then its area is:**

(1) $\frac{4\sqrt{2}}{3}$
(2) $\frac{5\sqrt{2}}{3}$
(3) $2\sqrt{2}$
(4) $\frac{2\sqrt{2}}{3}$

Q.3 $\int_0^{\pi/4}\frac{\cos^2 x\sin^2 x}{\left(\cos^3 x+\sin^3 x\right)^2}\,dx$ **is equal to:**

(1) $1/12$
(2) $1/9$
(3) $1/6$
(4) $1/3$

Q.4 The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation by mistake was taken as 8 instead of 12. The correct standard deviation is:

(1) $\sqrt{3.86}$
(2) 1.8
(3) $\sqrt{3.96}$
(4) 1.94

Q.5 The function $f(x)=\frac{x^2+2x-15}{x^2-4x+9}$, $x\in R$, **is:**

(1) both one-one and onto.
(2) onto but not one-one.
(3) neither one-one nor onto.

(4) one-one but not onto.

Q.6 Let $A = \{n \in [100, 700] \cap N : n$ is neither a multiple of 3 nor a multiple of $4\}$. Then the number of elements in A is:

(1) 300
(2) 280
(3) 310
(4) 290

Q.7 Let C be the circle of minimum area touching the parabola $y = 6 - x^2$ and the lines $y = \sqrt{3}|x|$. Then, which one of the following points lies on the circle C?

(1) $(2, 4)$
(2) $(1, 2)$
(3) $(2, 2)$
(4) $(1, 1)$

Q.8 For $\alpha, \beta \in R$ and a natural number n, let

$$A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2} + \alpha \\ 2r & 2 & n^2 - \beta \\ 3r - 2 & 3 & n(3n-1)/2 \end{vmatrix}.$$

Then $2A_{10} - A_5$ is:

(1) $4\alpha + 2\beta$
(2) $2\alpha + 4\beta$
(3) $2n$
(4) 0

Q.9 The shortest distance between the lines:

$$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5} \quad \text{and} \quad \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3},$$

is:

(1) $6\sqrt{3}$
(2) $4\sqrt{3}$
(3) $5\sqrt{3}$
(4) $8\sqrt{3}$

Q.10 A company has two plants A and B to manufacture motorcycles. 60% **motorcycles are manufactured at plant A and the remaining are manufactured**

at plant B. 80% of the motorcycles manufactured at plant A are rated of the standard quality, while 90% of the motorcycles manufactured at plant B are rated of the standard quality. A motorcycle picked up randomly from the total production is found to be of the standard quality. If p is the probability that it was manufactured at plant B, then $126p$ is:

(1) 54
(2) 64
(3) 66
(4) 56

Q.11 Let α, β be the distinct roots of the equation:

$$x^2 - (t^2 - 5t + 6)x + 1 = 0,\ t \in R,$$

and $a_n = \alpha^n + \beta^n$. Then the minimum value of $\frac{a_{2023}+a_{2025}}{a_{2024}}$ is:

(1) $1/4$
(2) $-1/2$
(3) $-1/4$
(4) $1/2$

Q.12 Let the relations R_1 and R_2 on the set $X = \{1, 2, 3, \ldots, 20\}$ be given by: $R_1 = \{(x, y) : 2x - 3y = 2\}$ and $R_2 = \{(x, y) : -5x + 4y = 0\}$. If M and N be the minimum number of elements required to be added in R_1 and R_2, respectively, in order to make the relations symmetric, then $M + N$ equals:

(1) 8
(2) 16
(3) 12
(4) 10

Q.13 Let a variable line of slope $m > 0$ passing through the point $(4, -9)$ intersect the coordinate axes at the points A and B. The minimum value of the sum of the distances of A and B from the origin is:

(1) 25
(2) 30
(3) 15
(4) 10

Q.14 The interval in which the function $f(x) = x^x, x > 0$, is strictly increasing is:

(1) $\left(0, \frac{1}{e}\right]$
(2) $\left[\frac{1}{e^2}, 1\right)$

(3) $(0, \infty)$
(4) $\left[\frac{1}{e}, \infty\right)$

Q.15 A circle is inscribed in an equilateral triangle of side of length 12. If the area and perimeter of any square inscribed in this circle are m and n, respectively, then $m + n^2$ is equal to:
(1) 396
(2) 408
(3) 312
(4) 414

Q.16 The number of triangles whose vertices are at the vertices of a regular octagon but none of whose sides is a side of the octagon is:
(1) 24
(2) 56
(3) 16
(4) 48

Q.17 Let $y = y(x)$ be the solution of the differential equation:

$$(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}, \quad y(1) = 0.$$

Then $y(0)$ is:
(1) $\frac{1}{4}(e^{\pi/2} - 1)$
(2) $\frac{1}{2}\left(1 - e^{-\pi/2}\right)$
(3) $\frac{1}{4}\left(1 - e^{\pi/2}\right)$
(4) $\frac{1}{2}\left(e^{\pi/2} - 1\right)$

Q.18 Let $y = y(x)$ be the solution of the differential equation:

$$(2x \ln x)\frac{dy}{dx} + 2y = \frac{3}{x} \ln x, \quad x > 0,$$

and $y(e^{-1}) = 0$. Then, $y(e)$ is equal to:
(1) $-\frac{3}{2e}$
(2) $-\frac{2}{3e}$
(3) $-\frac{3}{e}$
(4) $-\frac{2}{e}$

Q.19 Let the area of the region enclosed by the curves $y = 3x$, $2y = 27 - 3x$, and $y = 3x - x\sqrt{x}$ be A. Then $10A$ is equal to:
(1) 184
(2) 154
(3) 172
(4) 162

Q.20 Let $f : (-\infty, \infty) \to R \setminus \{0\}$ be a differentiable function such that:

$$f'(1) = \lim_{a\to\infty} a^2 f\left(\frac{1}{a}\right).$$

Then:

$$\lim_{a\to\infty} \frac{a(a+1)}{2} \tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2\ln a$$

is equal to:
(1) $\frac{3}{2} + \frac{\pi}{4}$
(2) $\frac{3}{8} + \frac{\pi}{4}$
(3) $\frac{5}{2} + \frac{\pi}{8}$
(4) $\frac{3}{4} + \frac{\pi}{8}$

Q.21 Let $\alpha\beta\gamma = 45$; $\alpha, \beta, \gamma \in R$. If:

$$x(\alpha, 1, 2) + y(1, \beta, 2) + z(2, 3, \gamma) = (0, 0, 0),$$

for some $x, y, z \in R$, $xyz \neq 0$, then $6\alpha + 4\beta + \gamma$ is equal to:
(1)
(2)
(3)
(4)

Q.22 Let a conic C pass through the point $(4, -2)$ and $P(x, y)$, $x \geq 3$, be any point on C. Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and $(3, -5)$. If the focal distance of the point $(7, 1)$ on C is d, then $12d$ equals:

Q.23 Let

$$I_k = \int_0^1 (1-x)^k dx,\ k \in N.$$

Then the value of

$$\sum_{k=1}^{10} \frac{1}{7(I_k - 1)}$$

is equal to ______.

Q.24 Let x_1, x_2, x_3, x_4 be the solution of the equation

$$4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$$

and

$$\left(4 + x_1^2\right)\left(4 + x_2^2\right)\left(4 + x_3^2\right)\left(4 + x_4^2\right) = \frac{125}{16}m.$$

Then the value of m is ______.

Q.25 Let L_1, L_2 be the lines passing through the point $P(0,1)$ and touching the parabola

$$9x^2 + 12x + 18y - 14 = 0.$$

Let Q and R be the points on the lines L_1 and L_2 such that the $\triangle PQR$ is an isosceles triangle with base QR. If the slopes of the lines QR are m_1 and m_2, then

$$16\left(m_1^2 + m_2^2\right)$$

is equal to:

Q26. If the second, third and fourth terms in the expansion of $(x+y)^n$ are 135, 30 and $\frac{10}{3}$, respectively, then $6(n^3 + x^2 + y)$ is equal to ____

Q27.Let the first term of a series be $T_1 = 6$ and its r^{th} term $T_r = 3T_{r-1} + 6^r$, $r = 2, 3, \ldots, n$. If the sum of the first n terms of this series is $\frac{1}{5}\left(n^2 - 12n + 39\right)\left(4.6^n - 5.3^n + 1\right)$, then n is equal to __.

Q28.For $n \in N$, if $\cot^{-1} 3 + \cot^{-1} 4 + \cot^{-1} 5 + \cot^{-1} n = \frac{\pi}{4}$, then n is equal to __.

Q.29 Let $P(10, -2, -1)$ and Q be the foot of the perpendicular drawn from the point $R(1, 7, 6)$ on the line passing through the points $(2, -5, 11)$ and $(-6, 7, -5)$. Then the length of the line segment PQ is equal to __.

Q.30. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$, and a vector $\vec{c}$ be such that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times \vec{c} = \hat{i} + 8\hat{j} + 13\hat{k}$. If $\vec{a} \cdot \vec{c} = 13$, then $(24 - \vec{b} \cdot \vec{c})$ is equal to ______.
