JEE Main 2024 April 5 Shift 2 Mathematics Question Paper

Time Allowed :1 Hour	**Maximum Marks :**100	**Total Questions :**30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Question 1: Let $f : [-1, 2] \to R$ be given by

$$f(x) = 2x^2 + x + \lfloor x^2 \rfloor - \lfloor x \rfloor,$$

where $\lfloor t \rfloor$ denotes the greatest integer less than or equal to t. The number of points where f is not continuous is:

Options:
(1) 6
(2) 3
(3) 4
(4) 5

Question 2: The differential equation of the family of circles passing the origin and having center at the line $y = x$ is:

Options:
(1) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 + 2xy)dy$
(2) $(x^2 + y^2 + 2xy)dx = (x^2 + y^2 - 2xy)dy$
(3) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 - 2xy)dy$
(4) $(x^2 + y^2 - 2xy)dx = (x^2 + y^2 + 2xy)dy$

Question 3: Let $S_1 = \{z \in C : |z| \leq 5\}$**,** $S_2 = \left\{z \in C : \text{Im}\left(\frac{z+(1-\sqrt{3}i)}{1-\sqrt{3}i}\right) \geq 0\right\}$, and $S_3 = \{z \in C : \text{Re}(z) \geq 0\}$. Then:

Options:
(1) $\frac{125\pi}{6}$
(2) $\frac{125\pi}{24}$
(3) $\frac{125\pi}{4}$
(4) $\frac{125\pi}{12}$

Question 4: The area enclosed between the curves $y = x|x|$ **and** $y = x - |x|$ **is:**

Options:
(1) $\frac{8}{3}$
(2) $\frac{2}{3}$
(3) 1
(4) $\frac{4}{3}$

Question 5: 60 words can be made using all the letters of the word BHBJO, with or without meaning. If these words are written as in a dictionary, then the 50th word is:

Options:
(1) OBBHJ
(2) HBBJO
(3) OBBJH
(4) JBBOH

Question 6: Let $\vec{a} = 2\hat{i} + 5\hat{j} - \hat{k}$**,** $\vec{b} = 2\hat{i} - 2\hat{j} + 2\hat{k}$**, and** $\vec{c}$ **be three vectors such that**

$$(\vec{c} + \hat{i}) \times (\vec{a} + \vec{b} + \hat{i}) = \vec{a} \times (\vec{c} + \hat{i}),$$

$$\vec{a} \cdot \vec{c} = -29.$$

Then $\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k})$ **is equal to:**

Options:
(1) 10
(2) 5
(3) 15
(4) 12

Question 7: Consider three vectors $\vec{a}, \vec{b}, \vec{c}$**. Let** $|\vec{a}| = 2, |\vec{b}| = 3$ **and** $\vec{a} = \vec{b} \times \vec{c}$**. If** $\alpha \in \left[0, \frac{\pi}{3}\right]$

is the angle between the vectors $\vec{b}$ and $\vec{c}$, then the minimum value of $27|\vec{c}-\vec{a}|^2$ is equal to:

Options:
(1) 110
(2) 105
(3) 124
(4) 121

Question 8: Let $A(-1,1)$ and $B(2,3)$ be two points and P be a variable point above the line AB such that the area of $\triangle PAB$ is 10. If the locus of P is $ax+by=15$, then $5a+2b$ is:

Options:
(1) $-\frac{12}{5}$
(2) $-\frac{6}{5}$
(3) 4
(4) 6

Question 9: Let (α, β, γ) be the point $(8,5,7)$ in the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{5}$. Then $\alpha+\beta+\gamma$ is equal to:

Options:
(1) 16
(2) 18
(3) 14
(4) 20

Question 10: If the constant term in the expansion of

$$\left(\frac{\sqrt[3]{5}}{x}+\frac{2x}{\sqrt[3]{5}}\right)^{12}, \ x \neq 0,$$

is $\alpha x^2 \times \sqrt[3]{5}$, then 25α is equal to:

Options:
(1) 639
(2) 724
(3) 693
(4) 742

Question 11: Let $f, g : R \to R$ be defined as: $f(x) = |x-1|$ and

$$g(x) = \begin{cases} e^x, & x \geq 0 \\ x+1, & x \leq 0 \end{cases}. \text{ Then the function } f(g(x)) \text{ is}$$

Options:

(1) neither one-one nor onto.
(2) one-one but not onto.
(3) both one-one and onto.
(4) onto but not one-one.

Question 12: Let the circle $C_1 : x^2+y^2-2(x+y)+1 = 0$ and C_2 be a circle having centre at $(-1, 0)$ and radius 2. If the line of the common chord of C_1 and C_2 intersects the y-axis at the point P, then the square of the distance of P from the centre of C_1 is:

Options:
(1) 2
(2) 1
(3) 6
(4) 4

Question 13: Let the set $S = \{2, 4, 8, 16, \ldots, 512\}$ be partitioned into 3 sets A, B, C with an equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \emptyset$. The maximum number of such possible partitions of S is equal to:

Options:
(1) 1680
(2) 1520
(3) 1710
(4) 1640

Question 14: The values of m, n for which the system of equations

$$x + y + z = 4,$$

$$2x + 5y + 5z = 17,$$

$$x + 2y + mz = n$$

has infinitely many solutions, satisfy the equation:

Options:
(1) $m^2 + n^2 - m - n = 46$
(2) $m^2 + n^2 + m + n = 64$
(3) $m^2 + n^2 + mn = 68$

(4) $m^2 + n^2 - mn = 39$

Question 15: The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are from the set $\{1, 2, 3, 4, 5, 6\}$. If the probability of this equation having one real root bigger than the other is p, then $216p$ equals:

Options:
(1) 57
(2) 38
(3) 19
(4) 76

Question 16: Let $ABCD$ and $AEFG$ be squares of side 4 and 2 units, respectively. The point E is on the line segment AB and the point F is on the diagonal AC. Then the radius r of the circle passing through the point F and touching the line segments BC and CD satisfies:

Options:
(1) $r = 1$
(2) $r^2 - 8r + 8 = 0$
(3) $2r^2 - 4r + 1 = 0$
(4) $2r^2 - 8r + 7 = 0$

Question 17: Let $\beta(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}\, dx$, $m, n > 0$. If

$$\int_0^1 (1 - x^{10})^{20}\, dx = a \times \beta(b, c),$$

then $100(a + b + x)$ equals:

Options:
(1) 1021
(2) 1120
(3) 2012
(4) 2120

Question 18: Let $\alpha\beta \neq 0$ and $\mathrm{A} = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$. If

$\mathrm{B} = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$ is the matrix of cofactors of the elements of A, then $\det(AB)$ is equal to:

Options:

(1) 343
(2) 125
(3) 64
(4) 216

Question 19: If

$$y(\theta) = \frac{2\cos\theta + \cos 2\theta - 1}{4\cos^3\theta + 8\cos^2\theta + 5\cos\theta + 2},$$

then at $\theta = \frac{\pi}{2}$, $y'' + y' + y$ is equal to:

Options:
(1) $\frac{3}{2}$
(2) 1
(3) $\frac{1}{2}$
(4) 2

Question 20: For $x \geq 0$, the least value of K for which $4^{1+x} + 4^{1-x}$, $\frac{K}{2}$, $16^x + 16^{-x}$ are three consecutive terms of an arithmetic progression (A.P.) is equal to:

Options:
(1) 10
(2) 4
(3) 8
(4) 16

Question 21: Let the mean and the standard deviation of the probability distribution given by:

X	α	1	0	-3
$P(X)$	$\frac{1}{3}$	K	$\frac{1}{6}$	$\frac{1}{4}$

be μ and σ, respectively. If $\sigma - \mu = 2$, then $\sigma + \mu$ is equal to:

Question 22: Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} + \frac{2x}{(1+x^2)}y = xe^{\frac{1}{1+x^2}}, \quad y(0) = 0.$$

Then the area enclosed by the curve

$$f(x) = y(x)e^{\frac{1}{1+x^2}}$$

and the line $y - x = 4$ is equal to:

Question 23: The number of solutions of

$$\sin^2 x + (2 + 2x - x^2)\sin x - 3(x-1)^2 = 0, \quad \text{where } -\pi \le x \le \pi,$$

is:

Options:
(1) 1
(2) 2
(3) 3
(4) 4

Question 24: Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines

$$\frac{x+2}{-3} = \frac{y-2}{4} = \frac{z-5}{2} \quad \text{and} \quad \frac{x+2}{-1} = \frac{y+6}{2} = \frac{z-1}{0}.$$

Then $(\alpha - \beta)^2$ is equal to:

Question 25: If

$$1 + \frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}} + \frac{5-2\sqrt{6}}{18} + \frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}} + \frac{49-20\sqrt{6}}{180} + \ldots$$

up to $\infty = 2\left(\sqrt{\frac{b}{a}+1}\right)\log_e\left(\frac{a}{b}\right)$, where a and b are integers with $\gcd(a,b) = 1$, then $11a + 18b$ is equal to:

Question 26: Let $a > 0$ be a root of the equation

$$2x^2 + x - 2 = 0.$$

If

$$\lim_{x \to 1/a} 16\left(\frac{1-\cos(2+x-2x^2)}{1-ax^2}\right) = \alpha + \beta\sqrt{17},$$

where $\alpha, \beta \in Z$, then $\alpha + \beta$ is equal to:

Question 27: If

$$f(t) = \int_0^\pi \frac{2x\,dx}{1-\cos^2 t \sin^2 x}, \quad 0 < t < \pi,$$

then the value of

$$\int_0^{\frac{\pi}{2}} \frac{\pi^2\,dt}{f(t)}$$

is equal to:

Question 28: Let the maximum and minimum values of

$$\left(\sqrt{8x - x^2 - 12 - 4}\right)^2 + (x-7)^2, \quad x \in R$$

be M and m respectively. Then $M^2 - m^2$ is equal to:

Question 29: Let a line perpendicular to the line $2x - y = 10$ touch the parabola

$$y^2 = 4(x - 9)$$

at the point P. The distance of the point P from the centre of the circle

$$x^2 + y^2 - 14x - 8y + 56 = 0$$

is equal to:

Question 30: The number of real solutions of the equation

$$x|x + 5| + 2|x + 7| - 2 = 0$$

is:
