JEE Main 2024 April 5 Shift 1 Physics Question Paper with Solutions

Time Allowed : 1 Hour	Maximum Marks : 100	Total Questions : 30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Physics

Question 1: Light emerges out of a convex lens when a source of light is kept at its focus. The shape of the wavefront of the light is:

Options:
(1) Both spherical and cylindrical
(2) Cylindrical
(3) Spherical
(4) Plane

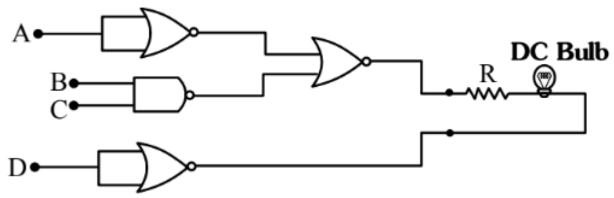
Correct Answer: (4): Plane

Solution:
When a source of light is placed at the focus of a convex lens, the lens causes the light rays to emerge as parallel rays. According to Huygens' principle, the shape of the wavefront depends on the direction of propagation of light rays: - For parallel light rays, the wavefront formed is planar (plane wavefront).
The convex lens converts the spherical wavefront from the source into a plane wavefront by refracting and aligning the light rays to travel parallel to each other. Hence, the shape of the wavefront is plane.

💡 Quick Tip

The wavefront produced by a convex lens is determined by the relative position of the light source. When the source is at the focus, the emerging wavefront is always plane.

Question 2: Following gate section is connected in a complete suitable circuit. For which of the following combinations, the bulb will glow (ON):



Options:
(1) $A = 0, B = 1, C = 1, D = 1$
(2) $A = 1, B = 0, C = 0, D = 0$
(3) $A = 0, B = 0, C = 0, D = 1$
(4) $A = 1, B = 1, C = 1, D = 0$

Correct Answer: (2): $A = 1, B = 0, C = 0, D = 0$

Solution:
To determine when the bulb will glow, we analyze the logic circuit step-by-step based on the given gates and their respective truth tables.
1. AND Gate Analysis: The output of an AND gate is 1 only if all its inputs are 1. - First AND gate: Inputs are A and B. Output = $A \cdot B$. - Second AND gate: Inputs are C and D. Output = $C \cdot D$.
2. OR Gate Analysis: The output of an OR gate is 1 if at least one of its inputs is 1. - First OR gate combines $A \cdot B$ and $C \cdot D$. Output = $(A \cdot B) + (C \cdot D)$.
3. NOT Gate Analysis: A NOT gate inverts its input: - B and C are inverted before being fed to the final logic gate.
4. Final AND Gate: The final AND gate combines the outputs of the previous gates: - Output = $(A \cdot B) + (C \cdot D)$.
5. Bulb Condition: The bulb will glow if the final output of the AND gate is 1. Substituting the given combinations: - For Option (1): $A = 0, B = 1, C = 1, D = 1 \rightarrow$ Output = 0 (bulb off). - For Option (2): $A = 1, B = 0, C = 0, D = 0 \rightarrow$ Output = 1 (bulb on). - For Option (3): $A = 0, B = 0, C = 0, D = 1 \rightarrow$ Output = 0 (bulb off). - For Option (4): $A = 1, B = 1, C = 1, D = 0 \rightarrow$ Output = 0 (bulb off).
Thus, the bulb glows for Option (2).

Quick Tip

When solving logic circuit problems, break down the circuit into its components (AND, OR, NOT gates) and analyze each step systematically. Use truth tables or direct substitution to simplify complex circuits.

Question 3: If G be the gravitational constant and u be the energy density, then which of the following quantities has the same dimension as $\sqrt{uG}$:

Options:
(1) Pressure gradient per unit mass
(2) Force per unit mass
(3) Gravitational potential
(4) Energy per unit mass

Correct Answer: (2): Force per unit mass

Solution:
Let us analyze the dimensional formula of $\sqrt{uG}$: - The gravitational constant G has the dimensional formula:

$$[G] = \left[M^{-1}L^3T^{-2}\right].$$

- The energy density u is energy per unit volume. Since energy has the dimensional formula $[ML^2T^{-2}]$, and volume is $[L^3]$, the dimensional formula of u is:

$$[u] = \left[ML^{-1}T^{-2}\right].$$

- Now, combining u and G:

$$uG = \left[ML^{-1}T^{-2}\right] \times \left[M^{-1}L^3T^{-2}\right] = \left[L^2T^{-4}\right].$$

- Taking the square root:

$$\sqrt{uG} = \left[LT^{-2}\right].$$

The dimensional formula of $\sqrt{uG}$ matches that of force per unit mass:

$$\text{Force per unit mass} = \frac{\text{Force}}{\text{Mass}} = \frac{[MLT^{-2}]}{[M]} = [LT^{-2}].$$

Thus, the quantity with the same dimension as $\sqrt{uG}$ is force per unit mass.

Quick Tip

When solving dimensional analysis problems, break each quantity into its base units and match the dimensions step by step. This approach avoids confusion and ensures accuracy.

Question 4: Given below are two statements:
Statement-I: When a capillary tube is dipped into a liquid, the liquid neither rises nor falls in the capillary. The contact angle may be $0°$.
Statement-II: The contact angle between a solid and a liquid is a property of the material of the solid and liquid as well.
In the light of the above statements, choose the correct answer from the options given below.

Options:
(1) Statement-I is false but Statement-II is true.
(2) Both Statement-I and Statement-II are true.
(3) Both Statement-I and Statement-II are false.
(4) Statement-I is true and Statement-II is false.

Correct Answer: (1): Statement-I is false but Statement-II is true.

Solution:
Analysis of Statement-I: The behavior of liquid in a capillary tube is governed by capillary action, which depends on surface tension, the density of the liquid, and the contact angle (θ). The liquid rises or falls based on the contact angle: - If $\theta < 90°$, the liquid rises. - If $\theta > 90°$, the liquid falls. - A $\theta = 0°$ implies maximum capillary rise.
The statement suggests that the liquid neither rises nor falls when the contact angle is $0°$. This is incorrect because $0°$ contact angle results in the liquid rising to its maximum height in the capillary. Thus, Statement-I is **false**.

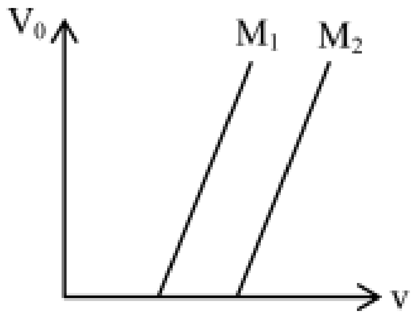
Analysis of Statement-II: The contact angle between a solid and a liquid is indeed a property of the materials involved. It depends on the intermolecular forces between the solid, liquid, and the surrounding medium (usually air). For example: - Hydrophilic surfaces have smaller contact angles. - Hydrophobic surfaces have larger contact angles.
This makes Statement-II **true**.

Conclusion: Statement-I is false, but Statement-II is true.
Hence, the correct answer is Option (1).

Quick Tip

Capillary action depends heavily on the contact angle. For maximum capillary rise, the contact angle should be $0°$, while a $90°$ or greater angle reduces the effect or causes the liquid to fall.

Question 5: Given below are two statements:



Statement-I: Figure shows the variation of stopping potential with frequency (ν) for the two photosensitive materials M_1 and M_2. The slope gives the value of $\frac{h}{e}$, where h is Planck's constant, e is the charge of the electron.
Statement-II: M_2 will emit photoelectrons of greater kinetic energy for the incident radiation having the same frequency.
In the light of the above statements, choose the most appropriate answer from the options given below:

Options:
(1) Statement-I is correct and Statement-II is incorrect.
(2) Statement-I is incorrect but Statement-II is correct.
(3) Both Statement-I and Statement-II are incorrect.
(4) Both Statement-I and Statement-II are correct.

Correct Answer: (1): Statement-I is correct and Statement-II is incorrect.

Solution:
1. Analysis of Statement-I: - The graph shows the variation of stopping potential V_0 with frequency ν. According to Einstein's photoelectric equation:

$$eV_0 = h\nu - \phi,$$

where h is Planck's constant, ϕ is the work function, and e is the charge of an electron. Rearranging gives:

$$V_0 = \frac{h}{e}\nu - \frac{\phi}{e}.$$

- The slope of the V_0 vs ν graph is $\frac{h}{e}$, which is correctly described in Statement-I.
2. Analysis of Statement-II: - The work function ϕ determines the threshold frequency ν_0 for photoemission, where $\nu_0 = \frac{\phi}{h}$. In the graph, M_2 has a higher threshold frequency ($\nu_{0,2}$) compared to M_1 ($\nu_{0,1}$), implying M_2 has a larger work function. - For the same incident frequency, the kinetic energy of the emitted photoelectrons is given by:

$$KE = h\nu - \phi.$$

Since ϕ is larger for M_2, the kinetic energy of photoelectrons emitted by M_2 will be smaller, not greater, for the same frequency. Thus, Statement-II is incorrect.
3. Conclusion: - Statement-I is correct as the slope gives $\frac{h}{e}$. - Statement-II is incorrect because M_2, having a higher work function, emits photoelectrons with lesser kinetic energy for the same frequency.

Quick Tip

The slope of the stopping potential vs frequency graph always represents $\frac{h}{e}$, and the intercepts relate to the work functions of the materials. Analyze carefully to determine which material has the higher work function.

Question 6: The angle between vector $\vec{Q}$ and the resultant of $(2\vec{Q}+2\vec{P})$ and $(2\vec{Q}-2\vec{P})$ is:

Options:
(1) $0°$
(2) $\tan^{-1}\left(\frac{2\vec{Q}-2\vec{P}}{2\vec{Q}+2\vec{P}}\right)$
(3) $\tan^{-1}\left(\frac{\vec{P}}{\vec{Q}}\right)$
(4) $\tan^{-1}\left(\frac{2\vec{Q}}{\vec{P}}\right)$

Correct Answer: (1): $0°$

Solution:
To determine the angle between $\vec{Q}$ and the resultant of $(2\vec{Q}+2\vec{P})$ and $(2\vec{Q}-2\vec{P})$, we proceed as follows:
1. Resultant Vector: Given:

$$\vec{A} = (2\vec{Q}+2\vec{P}), \quad \vec{B} = (2\vec{Q}-2\vec{P}).$$

The resultant of $\vec{A}$ and $\vec{B}$ is:

$$\vec{R} = \vec{A} + \vec{B}.$$

Simplifying:

$$\vec{R} = (2\vec{Q}+2\vec{P}) + (2\vec{Q}-2\vec{P}),$$
$$\vec{R} = 4\vec{Q}.$$

2. Direction of $\vec{R}$: Since $\vec{R} = 4\vec{Q}$, the resultant vector $\vec{R}$ is in the same direction as $\vec{Q}$. Hence, the angle between $\vec{Q}$ and $\vec{R}$ is:

$$\theta = 0°.$$

Thus, the correct answer is $0°$, corresponding to Option (1).

Quick Tip

When the resultant vector is a scalar multiple of one of the given vectors, the angle between them is $0°$ (if they are in the same direction) or $180°$ (if they are in opposite directions).

Question 7: In a hydrogen-like system, the ratio of Coulombian force and gravitational force between an electron and a proton is in the order of:

Options:
(1) 10^{39}
(2) 10^{19}
(3) 10^{29}
(4) 10^{36}

Correct Answer: (1): 10^{39}

Solution:
1. Analysis of Coulomb Force:
The Coulomb force between an electron and a proton is given by:

$$F_c = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r^2},$$

where e is the charge of the electron, r is the separation between the electron and the proton, and ϵ_0 is the permittivity of free space.
2. Analysis of Gravitational Force:
The gravitational force between an electron and a proton is given by:

$$F_g = G \cdot \frac{m_e \cdot m_p}{r^2},$$

where G is the gravitational constant, m_e is the mass of the electron, and m_p is the mass of the proton.
3. Ratio of Coulomb Force to Gravitational Force:
Taking the ratio of F_c to F_g:

$$\frac{F_c}{F_g} = \frac{\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r^2}}{G \cdot \frac{m_e \cdot m_p}{r^2}}.$$

Simplifying:

$$\frac{F_c}{F_g} = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{G \cdot m_e \cdot m_p}.$$

4. Substituting Known Values:
- $e = 1.6\times10^{-19}$ C, - $\frac{1}{4\pi\epsilon_0} = 9\times10^9$ N·m^2/C^2, - $G = 6.67\times10^{-11}$ N·m^2/kg^2, - $m_e = 9.1\times10^{-31}$ kg, - $m_p = 1.67 \times 10^{-27}$ kg.
Substituting these values:

$$\frac{F_c}{F_g} = \frac{9 \times 10^9 \cdot (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \cdot 9.1 \times 10^{-31} \cdot 1.67 \times 10^{-27}}.$$

Simplifying:

$$\frac{F_c}{F_g} \approx 2.3 \times 10^{39}.$$

The result is of the order of 10^{39}.

5. Conclusion:
The ratio of Coulombian force to gravitational force in a hydrogen-like system is of the order 10^{39}.

> Quick Tip
>
> In atomic systems, Coulombian forces are immensely stronger than gravitational forces. This ratio highlights why gravitational effects are negligible at atomic scales.

Question 8: In a co-axial straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is zero:

Options:
(1) inside the outer conductor
(2) in between the two conductors
(3) outside the cable
(4) inside the inner conductor

Correct Answer: (3): outside the cable

Solution:
To analyze the problem, we use Ampere's law and the superposition principle for magnetic fields.
1. Inside the Inner Conductor: The current in the central conductor produces a magnetic field proportional to the distance from its center. However, since the current is confined to the inner conductor, there is a magnetic field inside this region.
2. Between the Two Conductors: In the region between the central conductor and the outer conductor, the magnetic field arises from the current in the central conductor. The contribution from the outer conductor does not cancel it completely in this region. Thus, there is a net magnetic field in this region.
3. Inside the Outer Conductor: Within the material of the outer conductor, the current in the outer conductor contributes to the magnetic field, which combines with the field due to the central conductor. This results in a non-zero magnetic field inside the outer conductor.
4. Outside the Cable: Outside the cable, the currents in the central conductor and the outer conductor are equal in magnitude but opposite in direction. By Ampere's law, the net magnetic field in this region is zero, as the fields due to the two currents cancel each other out.
Thus, the magnetic field is zero only outside the cable, making Option (3) the correct answer.

> Quick Tip
>
> In co-axial cables with equal and opposite currents in the central and outer conductors, the magnetic field cancels out completely outside the cable. This property makes co-axial cables ideal for minimizing electromagnetic interference.

Question 9: An electron rotates in a circle around a nucleus having positive charge Ze. The correct relation between the total energy (E) of the electron to its potential energy (U) is:

Options:
(1) $E = 2U$
(2) $2E = 3U$
(3) $E = U$
(4) $2E = U$

Correct Answer: (4): $2E = U$

Solution:
1. Expression for Potential Energy (U):
The potential energy of the electron in the Coulomb field of the nucleus is given by:

$$U = -\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r},$$

where Z is the atomic number, e is the charge of the electron, r is the radius of the electron's orbit, and ϵ_0 is the permittivity of free space.
2. Expression for Kinetic Energy (K):
The centripetal force required for the circular motion of the electron is provided by the electrostatic force:

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r^2} = \frac{mv^2}{r},$$

where m is the mass of the electron and v is its velocity. Simplifying:

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r}.$$

3. Relation between Total Energy (E) and Potential Energy (U):
The total energy of the electron is the sum of its kinetic energy and potential energy:

$$E = K + U.$$

Substituting $K = -\frac{U}{2}$ (as derived from the virial theorem):

$$E = -\frac{U}{2} + U = \frac{U}{2}.$$

4. Rewriting the Relation:
Multiplying through by 2, we get:

$$2E = U.$$

5. Conclusion:
The correct relation between the total energy and the potential energy of the electron is $2E = U$.

💡 Quick Tip

In systems governed by Coulomb forces, such as electrons in atoms, the total energy is always half the potential energy with opposite sign, as dictated by the virial theorem.

Question 10: If the collision frequency of hydrogen molecules in a closed chamber at $27°C$ is Z, then the collision frequency of the same system at $127°C$ is:

Options:
(1) $\frac{\sqrt{3}}{2}Z$
(2) $\frac{4}{3}Z$
(3) $\frac{2}{\sqrt{3}}Z$
(4) $\frac{3}{4}Z$

Correct Answer: (3): $\frac{2}{\sqrt{3}}Z$

Solution:
The collision frequency (Z) in a gas is proportional to the square root of the temperature, i.e.,

$$Z \propto \sqrt{T},$$

where T is the absolute temperature in Kelvin.
1. Initial Conditions at $27°C$: Convert $27°C$ to Kelvin:

$$T_1 = 27 + 273 = 300\,\text{K}.$$

2. New Conditions at $127°C$: Convert $127°C$ to Kelvin:

$$T_2 = 127 + 273 = 400\,\text{K}.$$

3. Ratio of Collision Frequencies: The ratio of collision frequencies at the two temperatures is:

$$\frac{Z_2}{Z_1} = \sqrt{\frac{T_2}{T_1}}.$$

Substituting the values of T_2 and T_1:

$$\frac{Z_2}{Z_1} = \sqrt{\frac{400}{300}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}.$$

4. Collision Frequency at $127°C$: The collision frequency at $127°C$ is:

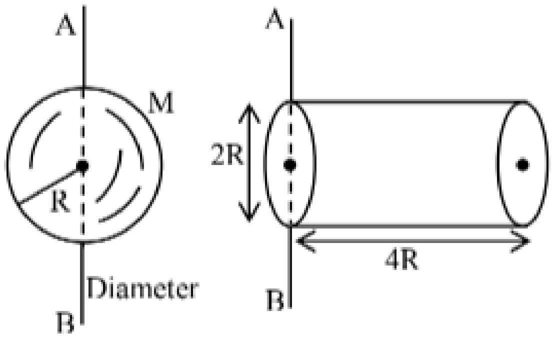
$$Z_2 = \frac{2}{\sqrt{3}}Z.$$

Thus, the correct answer is $\frac{2}{\sqrt{3}}Z$, corresponding to Option (3).

Quick Tip

The collision frequency of gas molecules is directly proportional to the square root of the absolute temperature. Always convert the temperatures to Kelvin when applying this proportionality.

Question 11: Ratio of the radius of gyration of a hollow sphere to that of a solid cylinder of equal mass, for the moment of inertia about their diameter axis AB as shown in the figure is $\sqrt{\frac{8}{x}}$. The value of x is:



Options:
(1) 34
(2) 17
(3) 67
(4) 51

Correct Answer: (3): 67

Solution:
1. Moment of Inertia of a Hollow Sphere:
The moment of inertia of a hollow sphere about its diameter is:

$$I_{\text{sphere}} = \frac{2}{3}MR^2 = Mk_1^2,$$

where k_1 is the radius of gyration of the hollow sphere.
2. Moment of Inertia of a Solid Cylinder:
The moment of inertia of a solid cylinder about its diameter is:

$$I_{\text{cylinder}} = \frac{1}{12}M(4R^2) + \frac{1}{4}MR^2 + M(2R)^2.$$

Simplifying:

$$I_{\text{cylinder}} = \frac{67}{12}MR^2 = Mk_2^2,$$

where k_2 is the radius of gyration of the solid cylinder.

3. Ratio of Radii of Gyration:
The ratio of radii of gyration is given by:

$$\frac{k_1}{k_2} = \sqrt{\frac{\frac{2}{3}}{\frac{67}{12}}}.$$

Simplify:

$$\frac{k_1}{k_2} = \sqrt{\frac{2}{3} \cdot \frac{12}{67}} = \sqrt{\frac{8}{67}}.$$

4. Expression for x:
From the problem, the ratio is given as:

$$\frac{k_1}{k_2} = \sqrt{\frac{8}{x}}.$$

Equating:

$$\sqrt{\frac{8}{x}} = \sqrt{\frac{8}{67}}.$$

Squaring both sides:

$$\frac{8}{x} = \frac{8}{67}.$$

Solve for x:

$$x = 67.$$

5. Conclusion:
The value of x is 67.

> **Quick Tip**
>
> When calculating radii of gyration, always use the relation $K = \sqrt{\frac{I}{M}}$. Simplify the moments of inertia carefully to maintain accuracy.

Question 12: Two conducting circular loops A and B are placed in the same plane with their centers coinciding as shown in the figure. The mutual inductance between them is:

Options:
(1) $\frac{\mu_0 \pi a^2}{2b}$
(2) $\frac{\mu_0 b^2}{2\pi a}$
(3) $\frac{\mu_0 \pi b^2}{2a}$
(4) $\frac{\mu_0}{2\pi} \frac{a^2}{b}$

Correct Answer: (1): $\frac{\mu_0 \pi a^2}{2b}$

Solution:
The mutual inductance between two circular loops is determined by the flux linked with one loop due to the magnetic field produced by the current in the other loop.
1. Magnetic Field Produced by Loop A: When a current I flows through loop A (radius a), the magnetic field at the center of the loop is given by:

$$B = \frac{\mu_0 I a^2}{2b^3},$$

where μ_0 is the permeability of free space, and b is the distance of loop B from the center of loop A (which here is also the radius of loop B).
2. Flux Through Loop B: The magnetic flux Φ through loop B due to the magnetic field B is given by:

$$\Phi = B \cdot A_B,$$

where $A_B = \pi b^2$ is the area of loop B. Substituting the value of B:

$$\Phi = \frac{\mu_0 I a^2}{2b^3} \cdot \pi b^2.$$

Simplify:

$$\Phi = \frac{\mu_0 \pi a^2 I}{2b}.$$

3. Mutual Inductance: By definition, the mutual inductance M is given by:

$$M = \frac{\Phi}{I}.$$

Substituting Φ:

$$M = \frac{\mu_0 \pi a^2}{2b}.$$

Thus, the mutual inductance between the two loops is $\frac{\mu_0 \pi a^2}{2b}$, which corresponds to Option (1).

Quick Tip

Mutual inductance between two circular loops depends on the radius of the loops, their separation, and the permeability of the medium. For coaxial loops, the smaller loop's area significantly influences the mutual inductance.

Question 13: Match List-I with List-II:

List-I	List-II
(A) Kinetic energy of planet	(I) $-\frac{GMm}{a}$
(B) Gravitational potential energy of Sun-planet system	(II) $\frac{GMm}{2a}$
(C) Total mechanical energy of planet	(III) $\frac{GMm}{r}$
(D) Escape energy at the surface of planet for unit mass object	(IV) $\frac{GMm}{2a}$

(Where a = radius of planet orbit, r = radius of planet, M = mass of Sun, m = mass of planet)
Choose the correct answer from the options given below:

Options:
(1) (A) – II, (B) – I, (C) – IV, (D) – III
(2) (A) – III, (B) – IV, (C) – I, (D) – II
(3) (A) – I, (B) – IV, (C) – II, (D) – III
(4) (A) – I, (B) – II, (C) – III, (D) – IV

Correct Answer: (1): (A) – II, (B) – I, (C) – IV, (D) – III

Solution:
1. Kinetic Energy of the Planet (A):
The kinetic energy of a planet moving in a circular orbit is:

$$K = \frac{1}{2}mv^2,$$

where $v^2 = \frac{GM}{a}$ (from the centripetal force acting on the planet). Substituting:

$$K = \frac{1}{2}m \cdot \frac{GM}{a} = \frac{GMm}{2a}.$$

Hence, (A) matches with (II).
2. Gravitational Potential Energy of Sun-Planet System (B):
The gravitational potential energy of the Sun-planet system is:

$$U = -\frac{GMm}{a}.$$

Hence, (B) matches with (I).
3. Total Mechanical Energy of the Planet (C):
The total mechanical energy of the planet is the sum of its kinetic energy and potential energy:

$$E = K + U = \frac{GMm}{2a} - \frac{GMm}{a}.$$

Simplifying:

$$E = -\frac{GMm}{2a}.$$

Hence, (C) matches with (IV).
4. Escape Energy at the Surface of Planet for Unit Mass Object (D):
The escape energy is:

$$E_{\text{escape}} = \frac{GM}{r}.$$

Hence, (D) matches with (III).
5. Conclusion:
The correct matching is:

$$\text{(A) – II, (B) – I, (C) – IV, (D) – III.}$$

Quick Tip

When solving matching problems, carefully recall and apply the standard formulas for kinetic energy, potential energy, and total energy in gravitational systems.

Question 14: A wooden block of mass 5 kg rests on a soft horizontal floor. When an iron cylinder of mass 25 kg is placed on top of the block, the floor yields, and the block and the cylinder together go down with an acceleration of $0.1\,\mathrm{ms}^{-2}$. The action force of the system on the floor is equal to:

Options:
(1) 297 N
(2) 294 N
(3) 291 N
(4) 196 N

Correct Answer: (3): 291 N

Solution:
To find the action force of the system on the floor, we calculate the effective force exerted due to the combined weight of the block and cylinder, adjusted for the downward acceleration.
1. Given Data: - Mass of wooden block $(m_1) = 5$ kg - Mass of iron cylinder $(m_2) = 25$ kg - Combined mass $(m) = m_1 + m_2 = 5 + 25 = 30\,\mathrm{kg}$ - Acceleration due to gravity $(g) = 9.8\,\mathrm{ms}^{-2}$ - Downward acceleration $(a) = 0.1\,\mathrm{ms}^{-2}$
2. Net Force on the Floor (Action Force): The total force exerted by the system on the floor is the apparent weight, given by:

$$F = m(g - a),$$

where $g - a$ accounts for the reduced effective gravitational acceleration due to the downward motion.
Substituting the values:

$$F = 30(9.8 - 0.1) = 30 \times 9.7 = 291\,\mathrm{N}.$$

3. Conclusion: The action force exerted by the system on the floor is $291\,\mathrm{N}$, which corresponds to Option (3).

Quick Tip

When an object is accelerating downwards, the effective weight reduces by a factor proportional to the acceleration. Use $F = m(g - a)$ to calculate the apparent force in such cases.

Question 15: A simple pendulum doing small oscillations at a place R height above Earth's surface has time period of $T_1 = 4\,\mathrm{s}$. T_2 would be its time period if it is brought to a point which is at a height $2R$ from Earth's surface. Choose the correct relation [$R =$ radius of Earth]:

Options:
(1) $T_1 = T_2$

(2) $2T_1 = 3T_2$
(3) $3T_1 = 2T_2$
(4) $2T_1 = T_2$

Correct Answer: (3): $3T_1 = 2T_2$

Solution:
1. Time Period of a Simple Pendulum: The time period of a pendulum depends on the acceleration due to gravity g at the location:

$$T \propto \frac{1}{\sqrt{g}}.$$

2. Variation of g with Height: The acceleration due to gravity g at a height h above the Earth's surface is:

$$g_h = g_0 \left(\frac{R}{R+h}\right)^2,$$

where g_0 is the acceleration due to gravity at the Earth's surface, R is the radius of the Earth, and h is the height above the surface.
3. Relating T_1 and T_2: For height $h = R$ (corresponding to T_1):

$$g_R = g_0 \left(\frac{R}{R+R}\right)^2 = g_0 \left(\frac{1}{2}\right)^2 = \frac{g_0}{4}.$$

For height $h = 2R$ (corresponding to T_2):

$$g_{2R} = g_0 \left(\frac{R}{R+2R}\right)^2 = g_0 \left(\frac{1}{3}\right)^2 = \frac{g_0}{9}.$$

The time period is inversely proportional to $\sqrt{g}$:

$$T_1 \propto \frac{1}{\sqrt{g_R}} \quad \text{and} \quad T_2 \propto \frac{1}{\sqrt{g_{2R}}}.$$

Taking the ratio of T_1 and T_2:

$$\frac{T_1}{T_2} = \sqrt{\frac{g_{2R}}{g_R}} = \sqrt{\frac{\frac{g_0}{9}}{\frac{g_0}{4}}} = \sqrt{\frac{4}{9}} = \frac{2}{3}.$$

Therefore:

$$T_1 = \frac{2}{3}T_2 \implies 3T_1 = 2T_2.$$

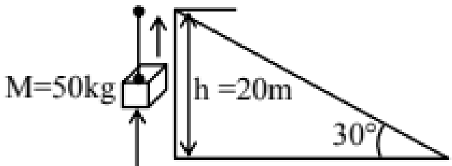
4. Conclusion: The correct relation is:

$$3T_1 = 2T_2.$$

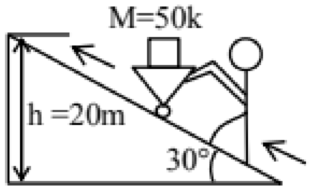
Quick Tip

The time period of a pendulum increases with altitude because the acceleration due to gravity decreases with height. Use the relationship $g_h = g_0 \left(\frac{R}{R+h}\right)^2$ to compute g at different heights.

Question 16: **A body of mass 50 kg is lifted to a height of 20 m from the ground in two different ways as shown in the figures. The ratio of work done against gravity in both respective cases will be:**



Case-1: Pulled straight up



Case-2: Along the ramp

Options:
(1) 1 : 1
(2) 2 : 1
(3) $\sqrt{3}$: 2
(4) 1 : 2

Correct Answer: (1): 1 : 1

Solution:
The work done against gravity depends only on the change in gravitational potential energy and is independent of the path taken. For both cases, we calculate the work done:
1. Case 1: Vertical Lifting The body is lifted directly upward to a height $h = 20\,\text{m}$. The work done is:

$$W_1 = mgh,$$

where: - $m = 50\,\text{kg}$ (mass of the body), - $g = 9.8\,\text{ms}^{-2}$ (acceleration due to gravity), - $h = 20\,\text{m}$ (height). Substituting the values:

$$W_1 = 50 \times 9.8 \times 20 = 9800\,\text{J}.$$

2. Case 2: Lifting Along the Ramp The body is moved along the inclined plane to the same height $h = 20\,\text{m}$. The work done against gravity is:

$$W_2 = mgh,$$

where h is the vertical height gained. Substituting the values:

$$W_2 = 50 \times 9.8 \times 20 = 9800\,\text{J}.$$

3. Ratio of Work Done: The work done in both cases is equal since both involve lifting the

body to the same height against gravity:

$$\frac{W_1}{W_2} = \frac{9800}{9800} = 1 : 1.$$

Conclusion: The ratio of work done in the two cases is 1 : 1, corresponding to Option (1).

Quick Tip

Work done against gravity depends only on the vertical height gained, not the path taken. Whether lifted vertically or along an inclined plane, the work done is the same as long as the vertical displacement remains the same.

Question 17: Time periods of oscillation of the same simple pendulum measured using four different measuring clocks were recorded as 4.62 s, 4.632 s, 4.6 s, and 4.64 s. The arithmetic mean of these readings in correct significant figures is:

Options:
(1) 4.623 s
(2) 4.62 s
(3) 4.6 s
(4) 5 s

Correct Answer: (3): 4.6 s

Solution:
1. Arithmetic Mean Calculation:
The arithmetic mean is calculated as:

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}.$$

Substituting the given values:

$$\text{Sum} = 4.62 + 4.632 + 4.6 + 4.64 = 18.4.$$

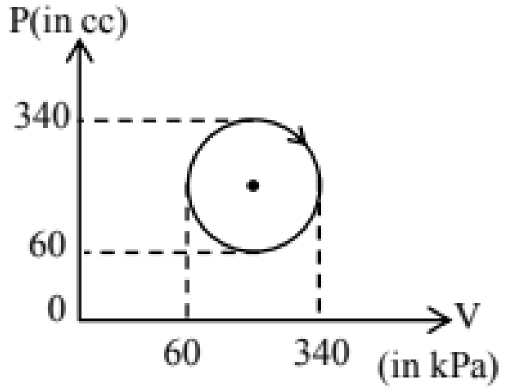
The number of observations is 4. Thus:

$$\text{Mean} = \frac{18.4}{4} = 4.6\,\text{s}.$$

Quick Tip

Always round the final result to the same precision as the least precise measurement in the data set. Significant figures are crucial for ensuring accurate reporting of results.

Question 18: The heat absorbed by a system in going through the given cyclic process is:



Options:
(1) 61.6 J
(2) 431.2 J
(3) 616 J
(4) 19.6 J

Correct Answer: (1): 61.6 J

Solution:
1. Heat Absorbed in a Cyclic Process: For a cyclic process, the change in internal energy (ΔU) is zero. According to the first law of thermodynamics:
$\Delta U = q + w$
where:
q is the heat absorbed by the system. w is the work done on the system.
Since $\Delta U = 0$ for a cyclic process: $q = -w$ For a cyclic process, the magnitude of the total work done is equal to the area of the cycle and the total heat absorbed by the system will be equal to the amount of work done by the system.

2. Work Done (Area of the Circle):
The given cyclic process is represented by a circle on a P-V diagram. The work done is equal to the area enclosed by the circle:

$$w = -\pi r^2$$

(Work done is negative as the cycle is clockwise, meaning work is done *by* the system, not on it). Here r represents radius of the circle which represents the cyclic process, in the given figure the radius can be calculated as follows:

$$r = \frac{340 - 60}{2} \text{ cc} = 140 \text{ cc}$$

Also the y-axis represents pressure, the units are cc which is cm^3 i.e. $1\text{cc} = 1 \text{ cm}^3 = 10^{-6} m^3$ and the x-axis represent volume in kPa. Thus, work done is given by,

$$w = -\pi(140 \times 10^{-6} m^3)^2 (180 \times 10^3 \text{ Pa}) \approx -0.0616 \text{ J}$$

$$\text{Area} = \pi r^2 = \pi(\frac{340-60}{2})^2 = \pi(140)^2 = 19600\pi \approx 61575 \text{ kPa} \cdot \text{cc (approx.)}$$

3. Converting Units for Work Done: We have the area in kPacc. To convert this to Joules:
1 kPacc $= 10^3\text{Pa} \times 10^{-6}\text{m}^3 = 10^{-3}\text{J}$
Work done (w) $\approx -61575 \times 10^{-3}$ J ≈ -61.6 J

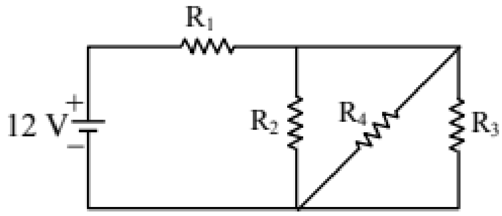
4. Heat Absorbed (q):
q = -w $\approx$ 61.6 J.

5. Conclusion: The heat absorbed by the system is approximately 61.6 J, so option (1) is the correct answer.

Quick Tip

Always ensure unit conversions when calculating work done in $P-V$ graphs. Work is the enclosed area, so ensure both pressure and volume are in SI units for consistency.

Question 19: In the given figure $R_1 = 10\,\Omega$, $R_2 = 8\,\Omega$, $R_3 = 4\,\Omega$, and $R_4 = 8\,\Omega$. The battery is ideal with an emf of $12\,\text{V}$. The equivalent resistance of the circuit and the current supplied by the battery are, respectively:



Options:
(1) $12\,\Omega$ and $1.14\,\text{A}$
(2) $10.5\,\Omega$ and $1.14\,\text{A}$
(3) $10.5\,\Omega$ and $1\,\text{A}$
(4) $12\,\Omega$ and $1\,\text{A}$

Correct Answer: (4): $12\,\Omega$ **and** $1\,\text{A}$

Solution:
1. Identify the Resistor Configuration:
- $R_1 = 10\,\Omega$ is in series with the parallel combination of R_2, R_3, and R_4.
2. Equivalent Resistance of Parallel Resistors (R_2, R_3, R_4):
The total resistance of the parallel combination is given by:

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}.$$

Substituting the given values:

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{8} + \frac{1}{4} + \frac{1}{8}.$$

Simplify:

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{8} + \frac{2}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}.$$

Therefore:

$$R_{\text{parallel}} = 2\,\Omega.$$

3. Total Equivalent Resistance of the Circuit:
The total resistance is the sum of R_1 and R_{parallel}:

$$R_{\text{total}} = R_1 + R_{\text{parallel}} = 10\,\Omega + 2\,\Omega = 12\,\Omega.$$

4. Current Supplied by the Battery:
The current supplied by the battery is given by Ohm's law:

$$I = \frac{\text{Voltage}}{\text{Total Resistance}} = \frac{12\,\text{V}}{12\,\Omega} = 1\,\text{A}.$$

5. Conclusion:
The equivalent resistance of the circuit is $12\,\Omega$, and the current supplied by the battery is $1\,\text{A}$.

> Quick Tip
>
> When dealing with circuits, always simplify the parallel and series combinations step by step. Double-check your calculations for equivalent resistance and apply Ohm's law to find the current.

Question 20: An alternating voltage of amplitude 40 V and frequency 4 kHz is applied directly across a capacitor of $12\,\mu F$. The maximum displacement current between the plates of the capacitor is nearly:

Options:
(1) 13 A
(2) 8 A
(3) 10 A
(4) 12 A

Correct Answer: (4): 12 A

Solution:

Current in the diode: $\mathrm{i}_d = i_c = \frac{V}{x_c} = \omega CV$

Substituting values: $i_d = 40 \times 2\pi \times 4 \times 10^3 \times 12 \times 10^{-6}$

Simplifying: $i_d \approx 12A$

Therefore, the correct option is (4).

💡 Quick Tip

The displacement current in a capacitor depends on the angular frequency (ω), capacitance (C), and the maximum voltage ($V_{\max}$). Ensure all units are consistent when applying the formula $I_{\max} = V_{\max} \cdot \omega \cdot C$.

Question 21: In Young's double slit experiment, carried out with light of wavelength 5000Å, the distance between the slits is 0.3 mm and the screen is at 200 cm from the slits. The central maximum is at x = 0 cm. The value of x for the third maxima is . . . mm.

Correct Answer: (10)

Solution:

1. Young's Double Slit Experiment: In Young's double slit experiment, the position of the bright fringes (maxima) is given by:

$$x = \frac{n\lambda D}{d}$$

where:
x is the distance from the central maximum to the nth maximum.
n is the order of the maximum (n = 0 for the central maximum, n = 1 for the first maximum, etc.).
λ is the wavelength of the light.
D is the distance between the slits and the screen.
d is the distance between the slits.

2. Convert Units: * Wavelength (λ) = 5000 Å = 5000×10^{-10} m = 5×10^{-7} m * Distance between slits (d) = 0.3 mm = 0.3×10^{-3} m * Distance to screen (D) = 200 cm = 2 m

3. Calculate x for the Third Maxima (n=3):

$$x = \frac{3 \times (5 \times 10^{-7}\ \text{m}) \times 2\ \text{m}}{0.3 \times 10^{-3}\ \text{m}}$$

$$x = 10 \times 10^{-3}\ \text{m} = 10\ \text{mm}$$

4. Conclusion: The value of x for the third maxima is 10 mm.

💡 Quick Tip

Ensure consistent units when using the formula for fringe position in Young's double slit experiment.

Question 22: A 2A current carrying straight metal wire of resistance 1 Ω, resistivity 2×10^{-6} Ωm, area of cross-section 10 mm^2 and mass 500 g is suspended horizontally in mid-air by applying a uniform magnetic field $\vec{B}$. The magnitude of B is . . . $\times 10^{-1}$ T (given, g = 10 m/s^2).

Correct Answer: (5)

Solution:

1. Magnetic Force on a Current-Carrying Wire: The magnetic force (F) on a straight current-carrying wire in a uniform magnetic field is given by:

$$F = BIL \sin\theta$$

where:
B is the magnitude of the magnetic field.
I is the current in the wire.
L is the length of the wire.
θ is the angle between the magnetic field and the direction of the current.

2. Equilibrium Condition: The wire is suspended horizontally in mid-air, meaning the magnetic force balances the gravitational force (weight) of the wire:

$$F_{\text{magnetic}} = F_{\text{gravitational}}$$
$$BIL \sin\theta = mg$$

Since the wire is horizontal and we want the minimum magnitude of B which means the magnetic field must be perpendicular to current. Thus $\theta = 90^\circ$ hence,

$$BIL = mg$$

3. Resistivity and Resistance: The resistance (R) of the wire is related to its resistivity (ρ), length (L), and area of cross-section (A):

$$R = \frac{\rho L}{A}$$

$$L = \frac{RA}{\rho}$$

4. Substitute and Solve for B: Substituting the expression for L into the equilibrium equation:

$$BI\left(\frac{RA}{\rho}\right) = mg$$

$$B = \frac{mg\rho}{IRA}$$

5. Convert Units and Calculate B: * m = 500 g = 0.5 kg * A = 10 mm^2 = 10×10^{-6} m^2 * $\rho = 2 \times 10^{-6}$ Ωm * I = 2 A * R=1 Ω * g = 10 m/s^2

$$B = \frac{(0.5 \text{ kg})(10 \text{ m/s}^2)(2 \times 10^{-6} \Omega\text{m})}{(2 \text{ A})(1\Omega)(10 \times 10^{-6} \text{ m}^2)}$$

$$B = 0.5 \text{ T} = 5 \times 10^{-1} \text{ T}$$

6. Conclusion: The magnitude of the magnetic field B is 5×10^{-1} T.

Quick Tip

In problems involving forces and equilibrium, ensure all forces are considered and that units are consistent.

Question 23: The electric field between the two parallel plates of a capacitor of 1.5 μF capacitance drops to one-third of its initial value in 6.6 μs when the plates are connected by a thin wire. The resistance of this wire is . . . Ω. (Given, log 3 = 1.1)

Correct Answer: (4)

Solution:
1. Discharging a Capacitor: When the plates of a charged capacitor are connected by a wire, the capacitor discharges through the wire. The electric field between the plates is proportional to the charge on the plates. The decay of charge (and therefore electric field) follows an exponential decay:

$$E(t) = E_0 e^{-t/RC}$$

where:
$E(t)$ is the electric field at time t.
E_0 is the initial electric field.
t is the time.
R is the resistance of the wire.
C is the capacitance of the capacitor.

2. Given Information: We are given that the electric field drops to one-third of its initial value, so $E(t) = \frac{1}{3}E_0$. The time taken for this to happen is $t = 6.6\,\mu s = 6.6 \times 10^{-6}$ s. The capacitance is $C = 1.5\,\mu F = 1.5 \times 10^{-6}$ F.

3. Set up the Equation: Substituting the given values into the equation for exponential decay:

$$\frac{1}{3}E_0 = E_0 e^{-t/RC}$$

$$\frac{1}{3} = e^{-(6.6\times10^{-6})/(R\times1.5\times10^{-6})}$$

4. Solve for R: Take the natural logarithm of both sides:

$$\ln\left(\frac{1}{3}\right) = \frac{-6.6\times10^{-6}}{R\times1.5\times10^{-6}}$$

$$-\ln 3 = \frac{-6.6}{1.5R}$$

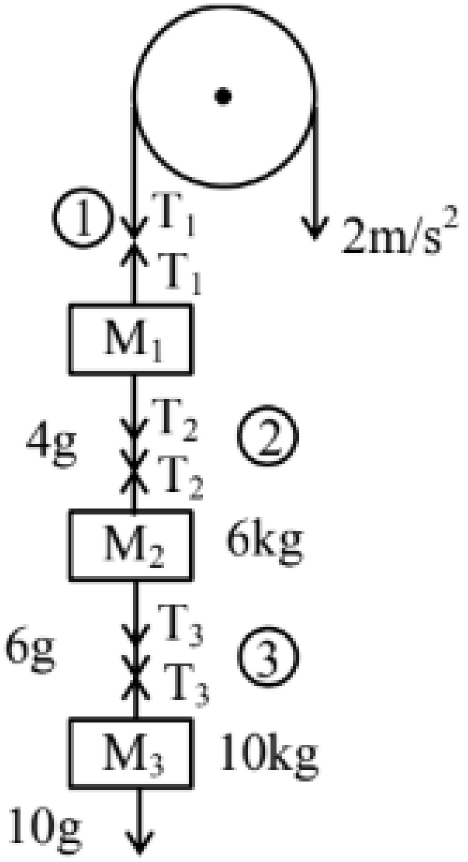
$$-1.0986 \approx \frac{-4.4}{R}$$

$$R \approx \frac{4.4}{1.0986} \approx 4\Omega$$

5. Conclusion: The resistance of the wire is approximately 4 Ω.

> Quick Tip
>
> Discharging a capacitor through a resistor follows an exponential decay. The time constant ($\tau = RC$) is the time it takes for the charge (or electric field) to decrease to approximately 37% of its initial value.

Question 24: Three blocks M_1, M_2, M_3 having masses 4 kg, 6 kg, and 10 kg respectively are hanging from a smooth pulley using ropes 1, 2, and 3 as shown in the figure. The tension in the rope 1, T_1, when they are moving upward with acceleration of 2 m/s^2 is . . . N (if g = 10 m/s^2).



Correct Answer: (240)

Solution:

Let's analyze the forces acting on each block. For the system as a whole (masses M_1, M_2, and M_3 together) moving upwards with an acceleration $a = 2m/s^2$

Total mass = 20 kg

Total weight = 200 N

Since the entire system is accelerating upwards, the net force F required to produce this acceleration is given by:

F = Ma = 20 × 2 = 40 N

The weight and the additional force required for acceleration:

$= 200 + 40$

$= 240N$

> Quick Tip
>
> When dealing with multiple connected objects, analyze the forces acting on each object individually and consider the relationships between their accelerations. Be mindful of pseudo forces when working in non-inertial frames of reference.

Question 25: The density and breaking stress of a wire are 6×10^4 kg/m^3 and 1.2×10^8 N/m^2 respectively. The wire is suspended from a rigid support on a planet where acceleration due to gravity is $\frac{1}{3}$ of the value on the surface of Earth. The maximum length of the wire with breaking is . . . m (take, g = 10 m/s^2).

Correct Answer: (600)

Solution:

1. Breaking Stress: Breaking stress is the maximum stress a material can withstand before it breaks. It is defined as the force per unit area:

$$\text{Breaking stress} = \frac{\text{Force}}{\text{Area}}$$

2. Force Due to Gravity: The force acting on the wire due to gravity is its weight, which is given by:

$$F = mg$$

where:
m is the mass of the wire.
g is the acceleration due to gravity on the planet.

3. Mass and Density: The mass (m) of the wire can be expressed in terms of its density (ρ), volume (V), and length (L):

$$m = \rho V = \rho A L$$

where A is the cross-sectional area of the wire.

4. Acceleration Due to Gravity on the Planet: The acceleration due to gravity on the planet is $\frac{1}{3}$g, where g is the acceleration due to gravity on Earth (10 m/s^2).

$$g_{\text{planet}} = \frac{1}{3}(10 \text{ m/s}^2) = \frac{10}{3} \text{ m/s}^2$$

5. Set up the Equation and Solve for L: At the breaking point, the breaking stress equals the weight of the wire divided by the cross-sectional area:

$$\text{Breaking stress} = \frac{m g_{\text{planet}}}{A} = \frac{(\rho A L) g_{\text{planet}}}{A}$$

$$\text{Breaking stress} = \rho L g_{\text{planet}}$$

$$L = \frac{\text{Breaking stress}}{\rho g_{\text{planet}}}$$

$$L = \frac{1.2 \times 10^8 \text{ N/m}^2}{(6 \times 10^4 \text{ kg/m}^3)(\frac{10}{3} \text{ m/s}^2)}$$

$$L = \frac{1.2 \times 10^8 \times 3}{6 \times 10^5}$$
$$L = 600 \text{ m}$$

6. Conclusion: The maximum length of the wire before breaking is 600 m.

Quick Tip

Breaking stress is an intrinsic property of a material, independent of its shape or size. However, the maximum length a wire can support before breaking depends on its density and the gravitational field.

Question 26: A body moves on a frictionless plane starting from rest. If S_n is distance moved between t = n − 1 and t = n and S_{n-1} is distance moved between t = n − 2 and t = n − 1, then the ratio $\frac{S_{n-1}}{S_n}$ is $\left(1 - \frac{2}{x}\right)$ for n = 10. The value of x is . . .

Correct Answer: (19)

Solution:

1. Distance and Uniform Acceleration: Since the plane is frictionless, we can assume constant acceleration. The distance traveled by a body starting from rest under uniform acceleration (a) is given by:

$$S = ut + \frac{1}{2}at^2$$

Since the body starts from rest, u = 0, so:

$$S = \frac{1}{2}at^2$$

2. Distance in Given Time Intervals: * S_n: Distance moved between t = n - 1 and t = n.

$$S_n = \frac{1}{2}a(n^2 - (n-1)^2) = \frac{1}{2}a(2n - 1)$$

* S_{n-1}: Distance moved between t = n - 2 and t = n - 1.

$$S_{n-1} = \frac{1}{2}a((n-1)^2 - (n-2)^2) = \frac{1}{2}a(2n - 3)$$

3. Calculate the Ratio $\frac{S_{n-1}}{S_n}$:

$$\frac{S_{n-1}}{S_n} = \frac{\frac{1}{2}a(2n-3)}{\frac{1}{2}a(2n-1)} = \frac{2n-3}{2n-1}$$

4. Given Information: We are given that n = 10 and the ratio is equal to $1 - \frac{2}{x}$.

5. Solve for x: Substitute n = 10 into the ratio:

$$\frac{2(10)-3}{2(10)-1} = \frac{17}{19} = 1 - \frac{2}{x}$$
$$\frac{2}{x} = 1 - \frac{17}{19} = \frac{2}{19}$$
$$x = 19$$

6. Conclusion: The value of x is 19.

> 💡 Quick Tip
>
> Carefully define the distances based on the given time intervals. Remember the equations of motion for uniform acceleration.

Question 27: If three helium nuclei combine to form a carbon nucleus then the energy released in this reaction is . . . $\times 10^{-2}$ MeV. (Given 1 u = 931 MeV/c^2, atomic mass of helium = 4.002603 u)

Correct Answer: (727)

Solution:

1. Nuclear Reaction: The nuclear reaction is:
$3\,{}^4\text{He} \to {}^{12}\text{C}$

2. Mass Defect: The mass defect (Δm) is the difference between the total mass of the reactants and the total mass of the products.
Atomic Mass of ^{12}C = 12 u
Δm = (3 $\times$ mass of ^{4}He) - (mass of ^{12}C) Δm = (3 $\times$ 4.002603 u) - 12 u Δm = 12.007809 u - 12 u Δm = 0.007809 u

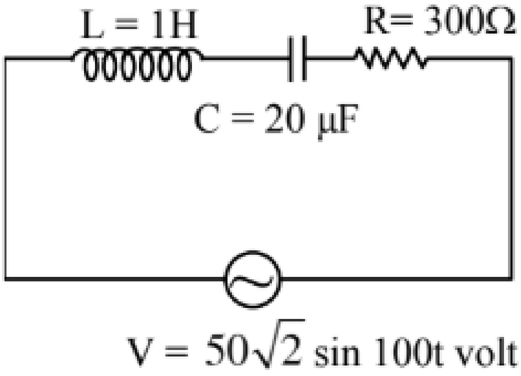
3. Energy Released (Einstein's Mass-Energy Equivalence):

The energy released (E) is related to the mass defect by Einstein's famous equation:
$E = \Delta m c^2$
where c is the speed of light. Using the conversion given that 1u = 931 MeV/c^2: $E = 0.007809 \text{ u} \times \frac{931 \text{ MeV}}{1\text{u c}^2} \times c^2$ $E = 0.007809 \times 931$ MeV $E \approx 7.27$ MeV $= 727 \times 10^{-2}$ MeV

4. Conclusion: The energy released in this reaction is approximately 727×10^{-2} MeV.

> 💡 Quick Tip
>
> In nuclear reactions, the mass defect is converted into energy. Einstein's equation, E = mc^2, relates the mass defect to the energy released.

Question 28: An ac source is connected in a given series LCR circuit. The rms potential difference across the capacitor of 20 μF is . . . V.



Correct Answer: (50)

Solution:

1. RMS Voltage of the Source: The given ac source voltage is $V = 50\sqrt{2}\sin 100t$ volts. The peak voltage (V_0) is $50\sqrt{2}$ volts. The RMS voltage (V_{rms}) is given by:

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

$$V_{\text{rms}} = \frac{50\sqrt{2}}{\sqrt{2}} = 50 \text{ V}$$

2. Angular Frequency (ω): The angular frequency (ω) of the ac source is given by the coefficient of t in the sine function:

$$\omega = 100 \text{ rad/s}$$

3. Capacitive Reactance (X_C): The capacitive reactance (X_C) is given by:

$$X_C = \frac{1}{\omega C} = \frac{1}{(100 \text{ rad/s})(20 \times 10^{-6} \text{ F})} = 500\,\Omega$$

4. Impedance (Z): The impedance (Z) of the series LCR circuit is:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where $X_L = \omega L$ is the inductive reactance.

$$X_L = (100 \text{ rad/s})(1 \text{ H}) = 100\,\Omega$$

$$Z = \sqrt{(300)^2 + (100 - 500)^2} = \sqrt{90000 + 160000} = \sqrt{250000} = 500\,\Omega$$

5. RMS Current (I_{rms}):

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{50\text{ V}}{500\,\Omega} = 0.1\text{ A}$$

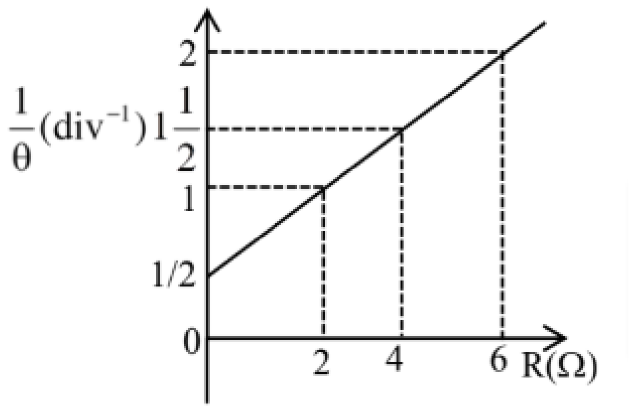
6. RMS Voltage across Capacitor (V_C):

$$V_C = I_{\text{rms}} X_C = (0.1\text{ A})(500\,\Omega) = 50\text{ V}$$

7. Conclusion: The RMS potential difference across the capacitor is 50 V.

> Quick Tip
>
> In ac circuits, impedance plays the role of resistance in dc circuits. The RMS values of current and voltage are used to calculate power and other quantities.

Question 29: In the experiment to determine the galvanometer resistance by the half-deflection method, the plot of $\frac{1}{\theta}$ vs the resistance (R) of the resistance box is shown in the figure. The figure of merit of the galvanometer is . . . $\times 10^{-1}$ A/division. [The source has emf 2V]



Correct Answer: (5)

Solution:

1. Half-Deflection Method and Figure of Merit:
In the half-deflection method, we use the following relationship:

$$G = R - S$$

where G is the galvanometer resistance, R is the resistance for half deflection and S is the shunt resistance. Also, the figure of merit is given by
$k = \frac{E}{(R+G)\theta}$
When $R = 0$, let deflection be θ_0, $\frac{1}{\theta_0} = \frac{G}{Ek} = 0.5$

When $R = R$, and $\theta = \theta_0/2$, $\frac{1}{\theta_0/2} = \frac{R+G}{Ek} = 2\frac{G}{Ek} + \frac{R}{Ek} = 1$

Thus, $1 = 1 + \frac{R}{Ek}$ $R = 2$.

Thus $k = \frac{E}{2G}$, $Ek = 1$, thus $G = 1$.

Since $G = R - S$, where R is resistance for half deflection, $1 = 2 - S$, $S = 1$.

Also from the graph, the slope is given by

$$\frac{2 - \frac{1}{2}}{6 - 0} = \frac{\frac{3}{2}}{6} = \frac{1}{4} = \frac{1}{Ek}$$

Since $E = 2$, thus $k = 0.5$. Thus, figure of merit is 5×10^{-1}.

2. Finding the slope and Figure of merit:

The graph provided is of $1/\theta$ vs R and represents the linear equation: $1/\theta = (1/Ek)R + G/(Ek)$ where E is emf, k is figure of merit and G is galvanometer resistance.

From the graph, the slope is $(2 - 1/2)/(6 - 0) = 1/4 = 1/(Ek)$.

Since emf is given as 2V, thus, k = 0.5. Thus the figure of merit is 5×10^{-1}.

3. Conclusion: The figure of merit of the galvanometer is 5×10^{-1} A/division.

> Quick Tip
>
> The half-deflection method and the figure of merit are important concepts in determining the characteristics of a galvanometer.

Question 30: Three capacitors of capacitances 25 μF, 30 μF, and 45 μF are connected in parallel to a supply of 100 V. Energy stored in the above combination is E. When these capacitors are connected in series to the same supply, the stored energy is $\frac{9}{x}$E. The value of x is . . .

Correct Answer: (86)

Solution:

1. Energy Stored in a Capacitor: The energy (E) stored in a capacitor is given by:

$$E = \frac{1}{2}CV^2$$

where:
C is the capacitance.
V is the voltage across the capacitor.

2. Parallel Combination: When capacitors are connected in parallel, the equivalent capacitance (C_p) is the sum of the individual capacitances:

$$C_p = C_1 + C_2 + C_3$$
$$C_p = 25\,\mu\text{F} + 30\,\mu\text{F} + 45\,\mu\text{F} = 100\,\mu\text{F}$$

The energy stored in the parallel combination (E_p) is:

$$E_p = \frac{1}{2}C_pV^2 = \frac{1}{2}(100 \times 10^{-6}\text{ F})(100\text{ V})^2 = 0.5\text{ J} = E$$

3. Series Combination: When capacitors are connected in series, the reciprocal of the equivalent capacitance (C_s) is the sum of the reciprocals of the individual capacitances:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$
$$\frac{1}{C_s} = \frac{1}{25} + \frac{1}{30} + \frac{1}{45} = \frac{18+15+10}{450} = \frac{43}{450}\,\mu\text{F}^{-1}$$
$$C_s = \frac{450}{43}\,\mu\text{F}$$

The energy stored in the series combination (E_s) is:

$$E_s = \frac{1}{2}C_sV^2 = \frac{1}{2}\left(\frac{450}{43} \times 10^{-6}\text{ F}\right)(100\text{ V})^2 = \frac{225}{43} \times 10^{-2}\text{ J}$$

4. Relationship between $\mathbf{E_p}$ and $\mathbf{E_s}$: We are given that $E_s = \frac{9}{x}E$, and we know that $E_p = E$.

$$\frac{9}{x}E = E_s$$
$$\frac{9}{x}(0.5) = \frac{225}{43} \times 10^{-2}$$
$$\frac{9}{x} = \frac{450}{43} \times 10^{-2} = \frac{4.5}{43}$$
$$x = \frac{9 \times 43}{4.5} = 2 \times 43 = 86$$

5. Conclusion: The value of x is 86.

Quick Tip

Remember the formulas for equivalent capacitance and energy stored for both parallel and series combinations of capacitors.
