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2010 – PH

Test Paper Code : PH

Time : 3 Hours **Max. Marks : 300**

INSTRUCTIONS

1. The question-cum-answer booklet has **32** pages and has **25** questions. Please ensure that the copy of the question-cum-answer booklet you have received contains all the questions.
2. Write your **Registration Number, Name and the name of the Test Centre** in the appropriate space provided on the right side.
3. Write the answers to the objective questions against each Question No. in the **Answer Table for Objective Questions**, provided on Page No. **9**. Do not write anything else on this page.
4. Each objective question has **4 choices** for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used :
   (a) For each correct answer, you will be awarded **6 (six)** marks.
   (b) For each wrong answer, you will be awarded **–2 (Negative two)** marks.
   (c) Multiple answers to a question will be treated as a wrong answer.
   (d) For each un-attempted question, you will be awarded **0 (Zero)** mark.
5. Answer the subjective question only in the space provided after each question.
6. Do not write more than one answer for the same question. In case you attempt a subjective question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing last only will be evaluated.
7. All answers must be written in blue/black/ blue-black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. No supplementary sheets will be provided to the candidates.
10. **Clip board, log tables, slide rule, calculator, cellular phone or electronic gadgets in any form are NOT allowed.**
11. The question-cum-answer booklet must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this booklet.
12. Refer to special instructions/useful data on the reverse.

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READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

REGISTRATION NUMBER						

Name :

Test Centre :

Do not write your Registration Number or Name anywhere else in this question-cum-answer booklet.

I have read all the instructions and shall abide by them.

.....

Signature of the Candidate

I have verified the information filled by the Candidate above.

.....

Signature of the Invigilator

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Special Instructions / Useful Data

Divergence of a vector field $\vec{A}$ in spherical polar coordinates

$$\nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\ A_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Boltzmann constant k_B	=	1.38×10^{-23} J/K
Electric permittivity of free space ε_0	=	8.85×10^{-12} F/m
Elementary charge e	=	1.60×10^{-19} C
Magnetic permeability of free space μ_0	=	1.26×10^{-6} H/m
Mass of electron m_e	=	9.11×10^{-31} kg
Molar gas constant R	=	8.31 J/mol-K
Planck constant h	=	6.63×10^{-34} J-s
Speed of light in vacuum	=	3×10^{8} m/s

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IMPORTANT NOTE FOR CANDIDATES

- **Attempt ALL the 25 questions.**
- **Questions 1-15 (objective questions) carry *six* marks each and questions 16-25 (subjective questions) carry *twenty one* marks each.**
- **Write the answers to the objective questions in the *Answer Table for Objective Questions* provided on page 9 only.**

Q.1 A matrix is given by $M = \frac{1}{\sqrt{2}}\begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}$. The eigenvalues of the M are

(A) real and positive
(B) purely imaginary with modulus 1
(C) complex with modulus 1
(D) real and negative

Q.2 The value of θ at which the first-order peak in X-ray ($\lambda = 1.53$ Å) diffraction corresponding to (111) plane of a simple cubic structure with the lattice constant, $a = 2.65$ Å, is approximately

(A) 15° (B) 30° (C) 45° (D) 60°

Q.3 The magnetic field associated with the electric field vector $\vec{E} = E_0 \sin(kz - \omega t)\hat{j}$ is given by

(A) $\vec{B} = -\frac{E_0}{c}\sin(kz - \omega t)\hat{i}$
(B) $\vec{B} = \frac{E_0}{c}\sin(kz - \omega t)\hat{i}$
(C) $\vec{B} = \frac{E_0}{c}\sin(kz - \omega t)\hat{j}$
(D) $\vec{B} = \frac{E_0}{c}\sin(kz - \omega t)\hat{k}$

Space for rough work

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Q.4 Consider the following truth table :

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

The logic expression for F is

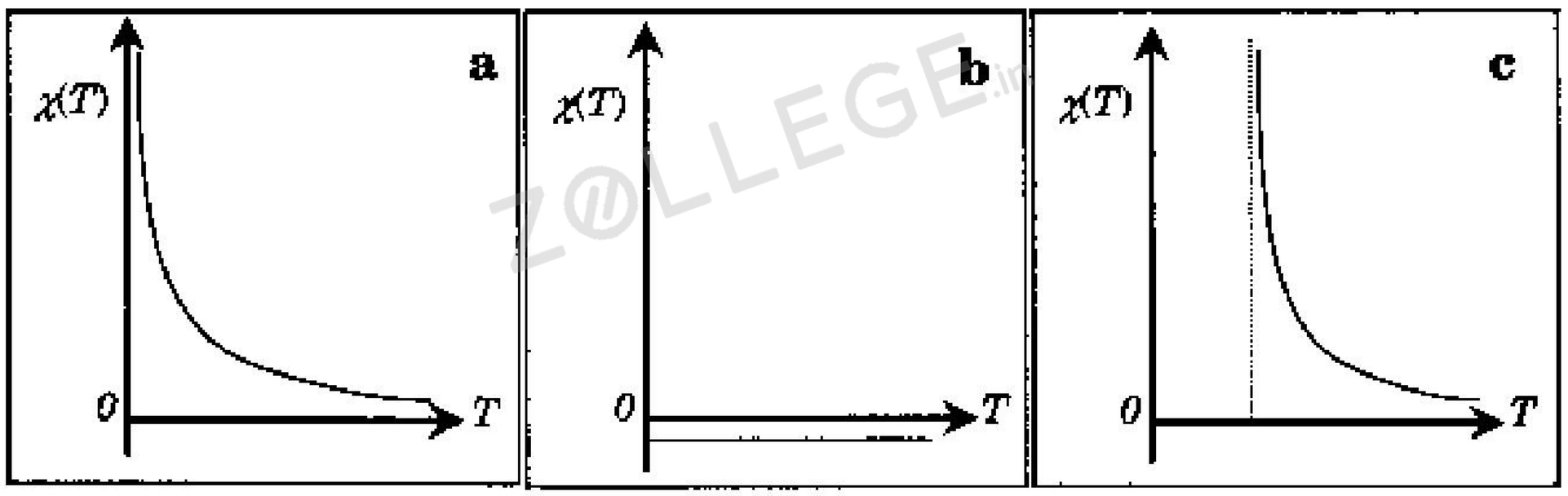
(A) $AB + BC + CA$

(B) $\overline{A}B + A\overline{C} + \overline{B}C$

(C) $\overline{\overline{C}\,\overline{A}\,\overline{B}} + A\overline{B}$

(D) $\overline{C}(A + \overline{B}) + A\overline{B}$

Q.5 The following are the plots of the temperature dependence of the magnetic susceptibility for three different samples.



The plots **a**, **b** and **c** correspond to

(A) ferromagnet, paramagnet and diamagnet, respectively

(B) paramagnet, diamagnet and ferromagnet, respectively

(C) ferromagnet, diamagnet and paramagnet, respectively

(D) diamagnet, paramagnet and ferromagnet, respectively

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Q.6 The equation of a surface of revolution is

$$z = \pm\sqrt{\frac{3}{2}x^2 + \frac{3}{2}y^2}$$

The unit normal to the surface at the point $A\left(\sqrt{\frac{2}{3}}, 0, 1\right)$ is

(A) $\sqrt{\frac{3}{5}}\hat{i} + \frac{2}{\sqrt{10}}\hat{k}$

(B) $\sqrt{\frac{3}{5}}\hat{i} - \frac{2}{\sqrt{10}}\hat{k}$

(C) $\sqrt{\frac{3}{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{k}$

(D) $\sqrt{\frac{3}{10}}\hat{i} + \frac{2}{\sqrt{10}}\hat{j}$

Q.7 A gas of molecules each having mass m is in thermal equilibrium at a temperature T. Let v_x, v_y, v_z be the Cartesian components of velocity, $\vec{v}$, of a molecule. The mean value of $(v_x - \alpha v_y + \beta v_z)^2$ is

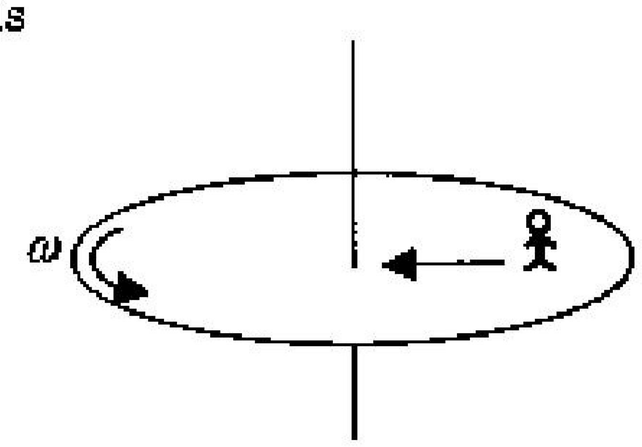
(A) $(1 + \alpha^2 + \beta^2)\frac{k_B T}{m}$

(B) $(1 - \alpha^2 + \beta^2)\frac{k_B T}{m}$

(C) $(\beta^2 - \alpha^2)\frac{k_B T}{m}$

(D) $(\alpha^2 + \beta^2)\frac{k_B T}{m}$

Q.8 A circular platform is rotating with a uniform angular speed ω counterclockwise about an axis passing through its centre and perpendicular to its plane as shown in the figure. A person of mass m walks radially inward with a uniform speed v on the platform. The magnitude and the direction of the Coriolis force (with respect to the direction along which the person walks) is



(A) $2m\omega v$ towards his left

(B) $2m\omega v$ towards his front

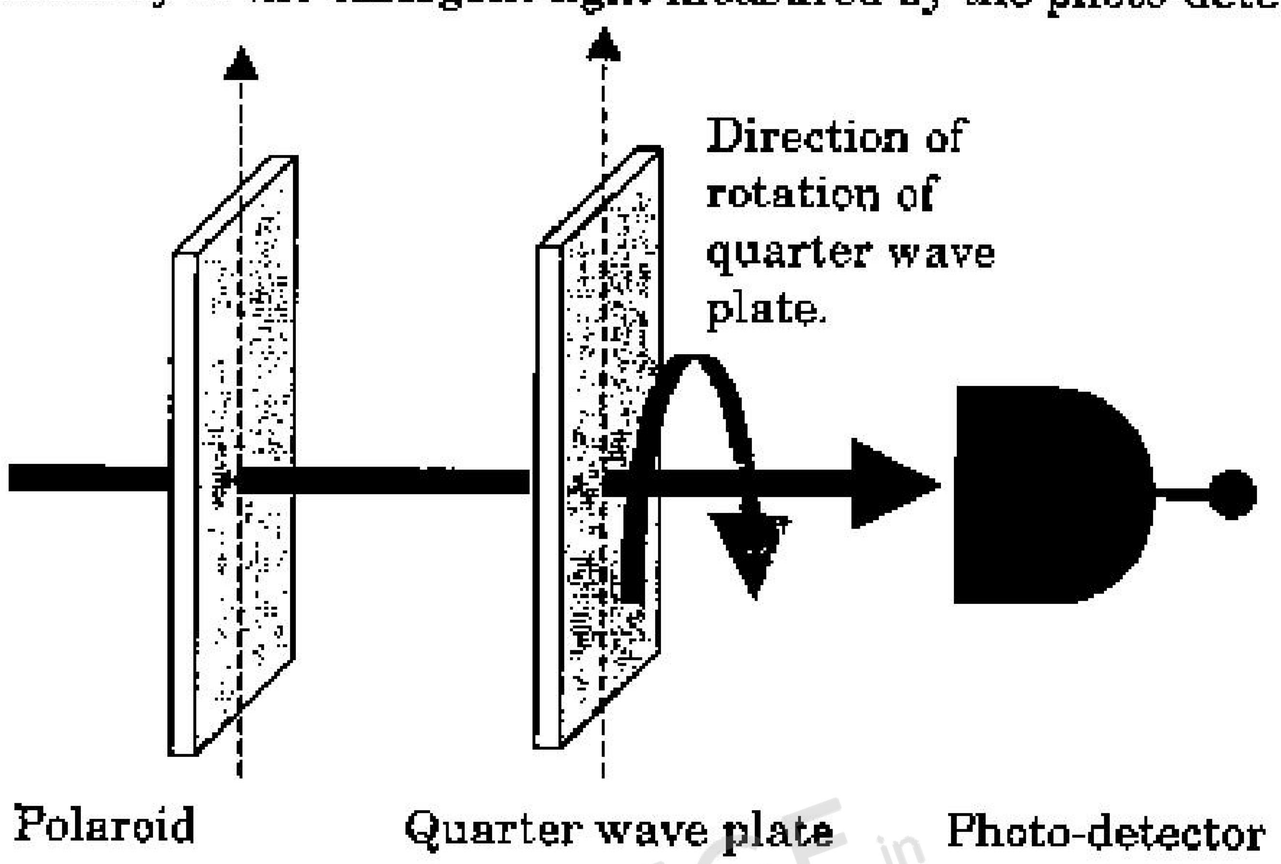
(C) $2m\omega v$ towards his right

(D) $2m\omega v$ towards his back

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Q.9 A quarter-wave plate is placed in between a polarizer and a photo-detector. When the optic axis of the quarter-wave plate is kept initially parallel to the pass axis of the polarizer and perpendicular to the direction of light propagation. The intensity of light passing through the quarter-wave plate is measured to be I_0 (see figure). If the quarter wave plate is now rotated by 45° about an axis parallel to the light propagation, what would be the intensity of the emergent light measured by the photo-detector?



(A) $\frac{I_0}{\sqrt{2}}$ (B) $\frac{I_0}{2}$ (C) $\frac{I_0}{2\sqrt{2}}$ (D) I_0

Q.10 A particle of mass m, moving with a velocity $\vec{v} = v_0(\hat{i}+\hat{j})$, collides elastically with another particle of mass $2m$ which is at rest initially. Here, v_0 is a constant. Which of the following statements is correct?

(A) The direction along which the centre of mass moves before collision is $-\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(B) The speed of the particle of mass m before collision in the center of mass frame is $\sqrt{2}\, v_0$

(C) After collision the speed of the particle with mass $2m$ in the centre of mass frame is $\frac{\sqrt{2}}{3} v_0$

(D) The speed of the particle of mass $2m$ before collision in the center of mass frame is $\sqrt{2}\, v_0$

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Q.11 A trapped air bubble of volume V_0 is released from a depth h measured from the water surface in a large water tank. The volume of the bubble grows to $2V_0$ as it reaches just below the surface. The temperature of the water and the pressure above the surface of water (10^5 N/m^2) remain constant throughout the process. If the density of water is 1000 kg/m^3 and the acceleration due to gravity is 10 m/s^2, then the depth h is

(A) 1 m (B) 10 m (C) 50 m (D) 100 m

Q.12 A particle of mass m is confined in a two-dimensional infinite square well potential of side a. The eigen-energy of the particle in a given state is $E = \frac{25\pi^2\hbar^2}{ma^2}$. The state is

(A) 4-fold degenerate (B) 3-fold degenerate
(C) 2-fold degenerate (D) Non-degenerate

Q.13 For a wave in a medium the angular frequency ω and the wave vector $\vec{k}$ are related by, $\omega^2 = (\omega_0^2 + c^2 k^2)$, where ω_0 and c are constants. The product of group and phase velocities, i.e. $v_g v_p$ is

(A) 0.25 c^2 (B) 0.4 c^2 (C) 0.5 c^2 (D) c^2

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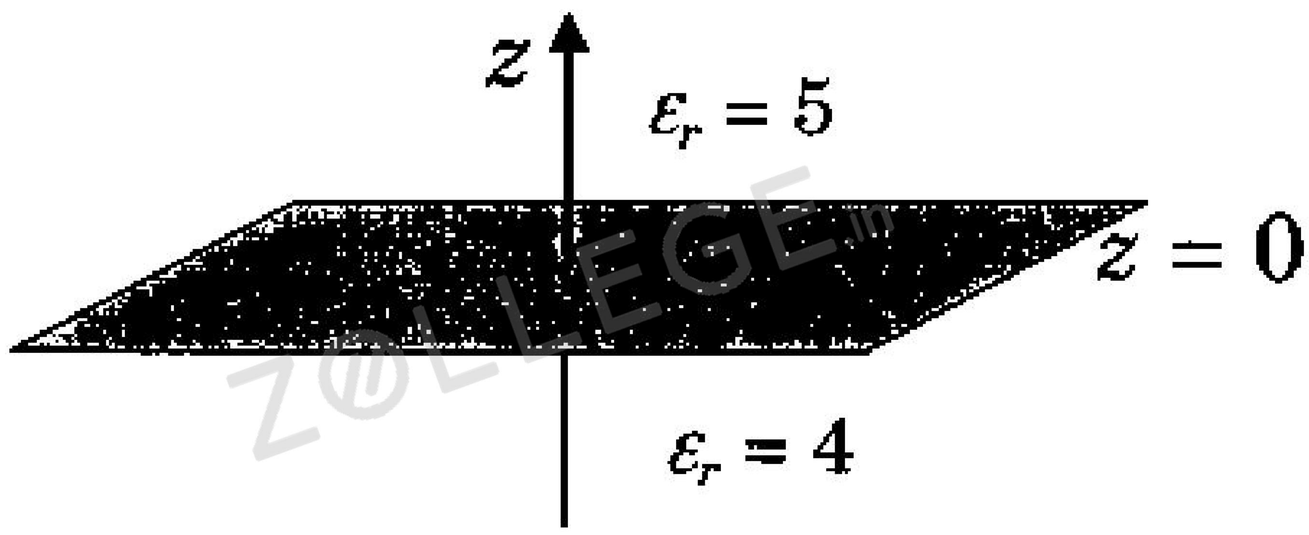
Q.14 Three identical non-interacting particles, each of spin ½ and mass m, are moving in a one-dimensional infinite potential well given by,

$$V(x)=\begin{cases}0 & \text{for } 0<x<a\\ \infty & \text{for } x\le 0 \text{ and } x\ge a\end{cases}$$

The energy of the lowest energy state of the system is

(A) $\dfrac{\pi^2\hbar^2}{ma^2}$ (B) $\dfrac{2\pi^2\hbar^2}{ma^2}$ (C) $\dfrac{3\pi^2\hbar^2}{ma^2}$ (D) $\dfrac{5\pi^2\hbar^2}{2ma^2}$

Q.15 Assume that $z=0$ plane is the interface between two linear and homogeneous dielectrics (see figure). The relative permittivities are $\epsilon_r=5$ for $z>0$ and $\epsilon_r=4$ for $z<0$. The electric field in the region $z>0$ is $\vec{E}_1=\left(3\hat{i}-5\hat{j}+4\hat{k}\right)$ kV/m. If there are no free charges on the interface, the electric field in the region $z<0$ is given by



(A) $\vec{E}_2=\left(\dfrac{3}{4}\hat{i}-\dfrac{5}{4}\hat{j}+\hat{k}\right)$ kV/m (B) $\vec{E}_2=\left(3\hat{i}-5\hat{j}+\hat{k}\right)$ kV/m

(C) $\vec{E}_2=\left(3\hat{i}-5\hat{j}-5\hat{k}\right)$ kV/m (D) $\vec{E}_2=\left(3\hat{i}-5\hat{j}+5\hat{k}\right)$ kV/m

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Answer Table for Objective Questions

Write the Code of your chosen answer only in the 'Answer' column against each Question No. Do not write anything else on this page.

Question No.	Answer	Do not write in this column
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FOR EVALUATION ONLY

No. of correct answers		Marks	(+)
No. of incorrect answers		Marks	(–)
Total marks in question nos. 1-15			( )

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Q.16 A particle of mass 2/3 kg is subjected to a potential energy function $V(x) = \left(3x^2 - 2x^3\right)$ J, where $x \geq 0$ and expressed in meters.

(a) Sketch $V(x)$ with respect to x in the range 0 to 2 m. Mark the positions of all the maxima and minima. What is the maximum value of the potential energy? (12)

(b) Supposing the particle is released at $x = 4/3$ m, find its velocity when it reaches $x = 3/2$ m. (9)

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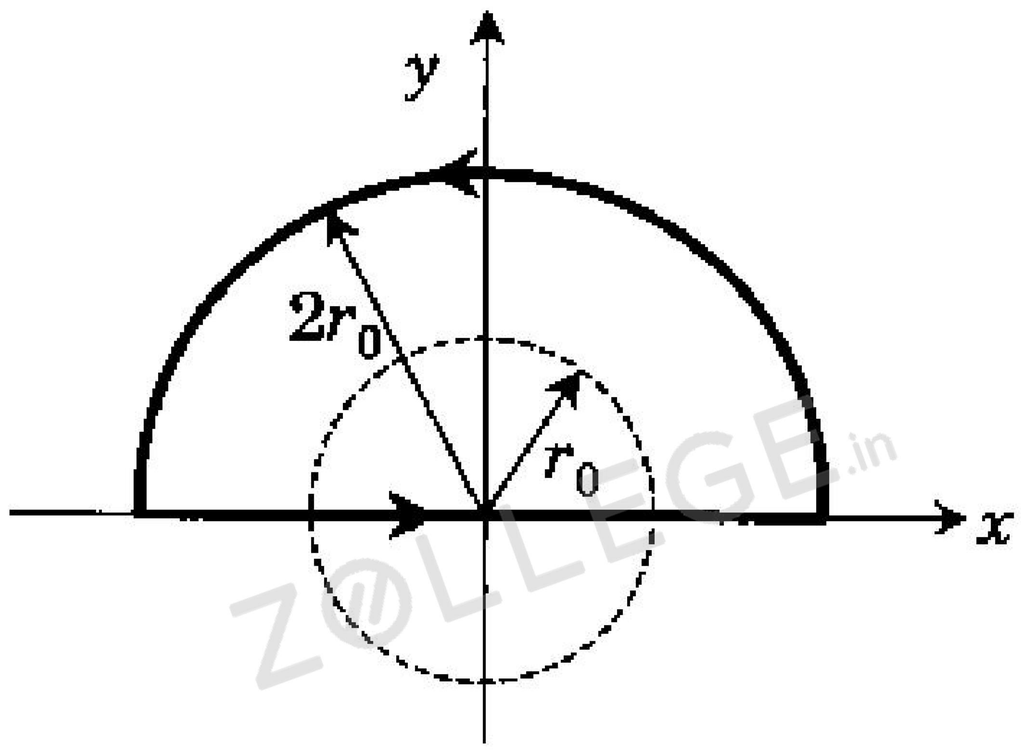
Q.17 A vector field is given by,

$$\vec{F}(\vec{r}) = \begin{cases} \alpha\left(x\hat{j} - y\hat{i}\right) & \text{for } (x^2+y^2) \le r_0^2 \text{ (region – I)} \\ \alpha r_0^2 \dfrac{\left(x\hat{j} - y\hat{i}\right)}{(x^2+y^2)} & \text{for } (x^2+y^2) > r_0^2 \text{ (region – II)} \end{cases}$$

Here α and r_0 are two constants.

(a) Find the curl of this field in both the regions. (9)

(b) Find the line integral $\oint \vec{F} \cdot d\vec{l}$ along the closed semicircular path of radius $2r_0$ as shown in the figure below. (12)



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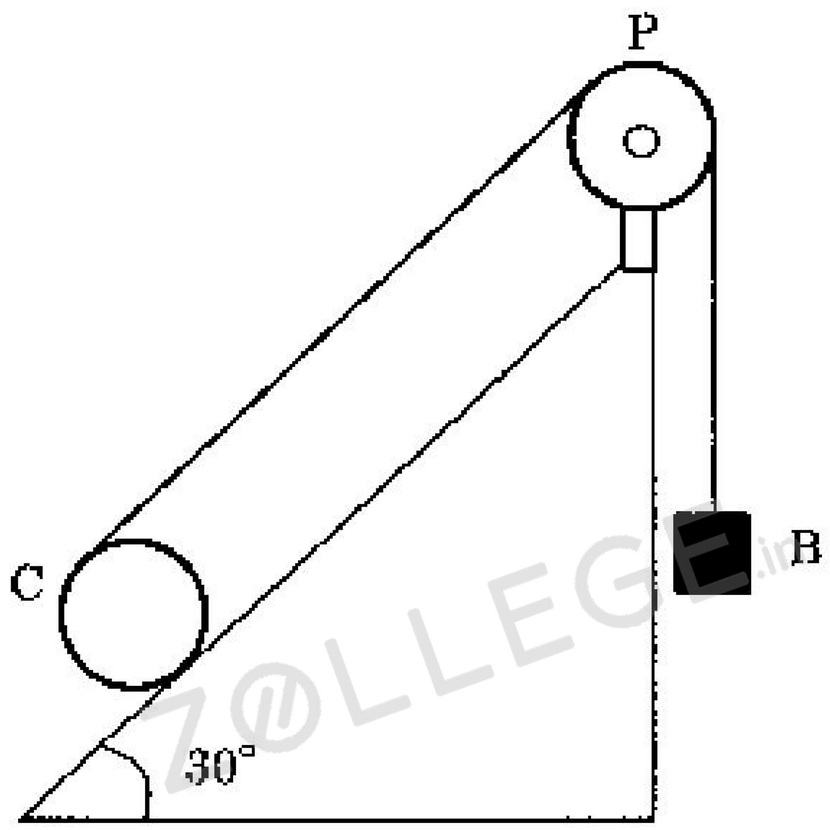
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Q.18 A solid cylinder C of mass 10 kg rolls without slipping in an inclined plane which offers friction. The angle of the inclined plane with the horizontal is 30° as shown in the figure below. A massless inextensible string is wrapped around the cylinder and it passes over a frictionless pulley, P. The other end of the string holds a block B of mass 2 kg as shown in the figure. Take $g = 10 \text{ ms}^{-2}$.

(a) Draw the free body diagram of the cylinder and the block. If the block B moves down by 0.01 m, how much distance does C move along the incline? (9)

(b) If the block B moves with an acceleration of $0.05\,g$ upward, find the magnitude of the frictional force. (12)



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Q.19 (a) A photon of initial momentum p_0 collides with an electron of rest mass m_0 moving with relativistic momentum P and energy E. The change in wavelength of the photon after scattering by an angle θ is given by, $\Delta\lambda = 2c\lambda_0 \frac{p_0 + P}{E - cP}\sin^2\frac{\theta}{2}$, where c is the speed of light and λ_0 is the wavelength of the incident photon before scattering. What will be the value of $\Delta\lambda$ when the electron is moving in a direction opposite to that of the incident photon with momentum P and energy E? Show that the value of $\Delta\lambda$ becomes independent of the wavelength of the incident photon when the electron is at rest before collision. (12)

(b) In a Compton experiment, the ultraviolet light of wavelength 2000 Å is scattered from an electron at rest. What should be the minimum resolving power of an optical instrument to measure the Compton shift, if the observation is made at 90° with respect to the direction of the incident light? (9)

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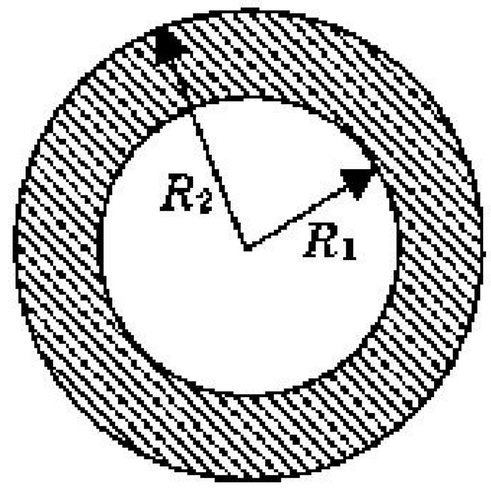
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Q.20 A conducting spherical shell of radius R_1 carries a total charge Q. A spherical layer of a linear, homogeneous and isotropic dielectric of dielectric constant K and outer radius R_2 ($> R_1$) covers the shell as shown in the figure.



(a) Find the electric field and the polarization vector $\vec{P}$ inside the dielectric. From this $\vec{P}$, calculate the surface bound charge density, σ_b, on the outer surface of the dielectric layer and the volume bound charge density, ρ_b, inside the dielectric. (12)

(b) Calculate the electrostatic energy stored in the region $R_1 \leq r \leq R_2$. (9)

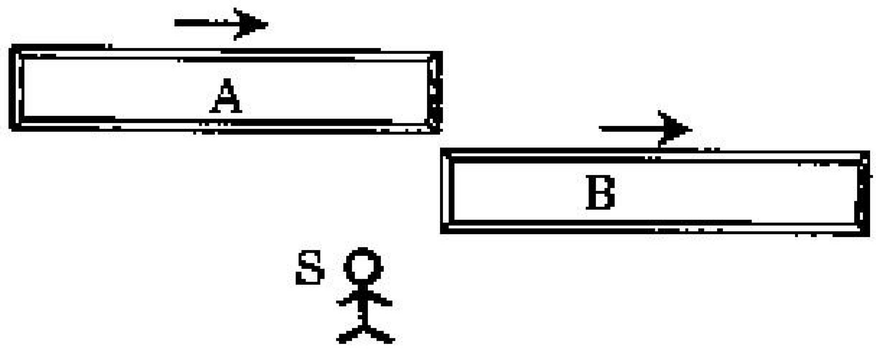
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Q.21 Two spaceships A and B of proper length 50 m each are moving with relativistic speeds 0.8 c and 0.6 c, respectively, in the same direction with respect to a stationary observer S. Here c is the speed of light. At $t = 0$, the spaceship A is just behind the spaceship B as shown below.



(a) Find the time taken by the spaceship A to completely overtake the spaceship B (that is the back of A is in line with the front of B) as seen by the observer S. (9)

(b) Find the time taken by the spaceship A to completely overtake the spaceship B as seen by an observer in the spaceship B. (12)

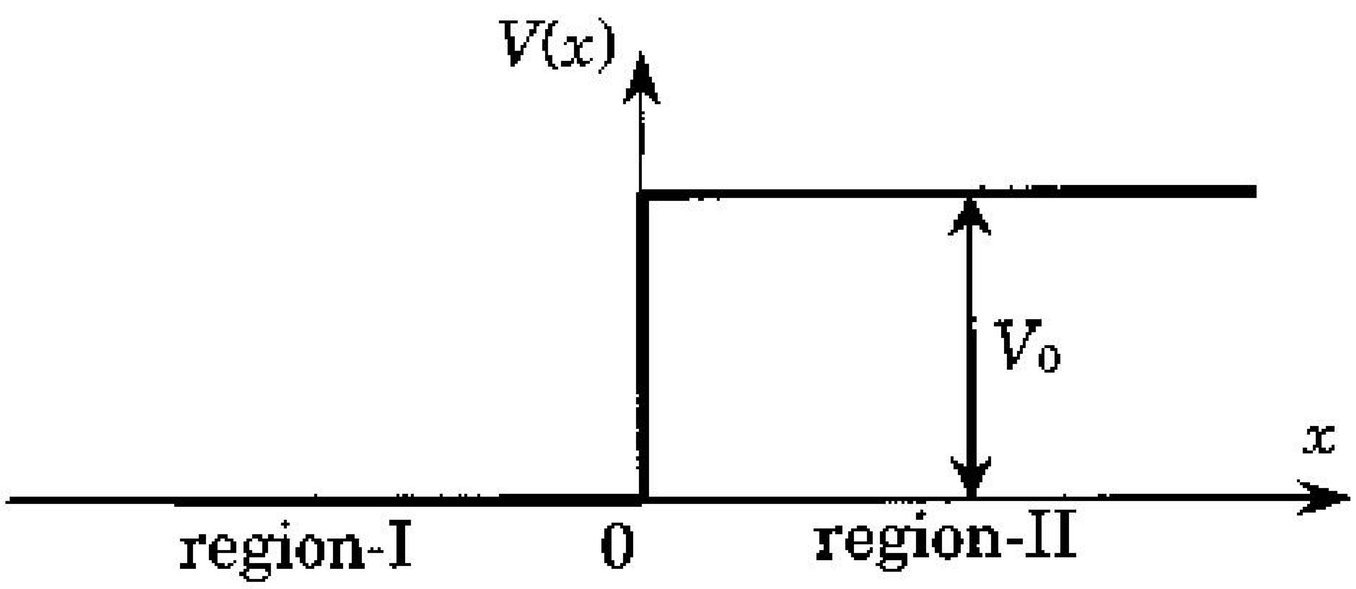
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Q.22 A free particle of mass m with energy $V_0/2$ is incident from left on a step potential of height V_0 as shown in the figure below.



Writing down the time independent Schrödinger equation in both the regions, obtain the corresponding general solutions. Apply the boundary conditions to find the wave functions in both the regions. Determine the reflection coefficient R. What is the transmission coefficient T? (21)

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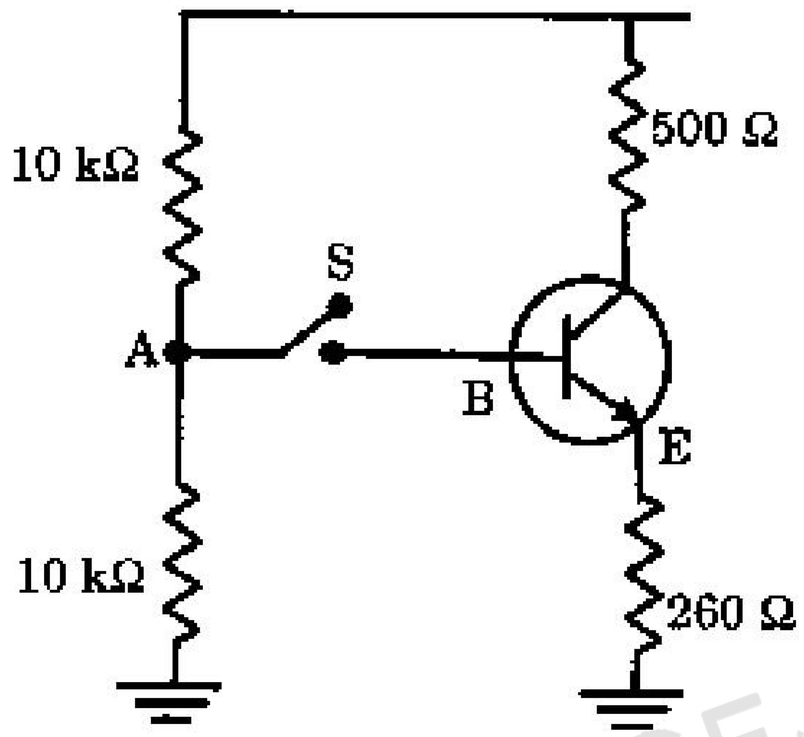
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Q.23 For the transistor shown in the figure, the dc current gain $\beta_{dc} = 50$ and $V_{BE} = 0.7$ V. The switch S is initially open.

(a) Calculate the voltage at point A. If the switch S is now closed, what would be the voltage at point A? (12)

(b) Draw the dc load line and find the Q-point of the circuit with the switch S remaining closed. (9)



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Q.24 Consider a capacitor placed in free space, consisting of two concentric circular parallel plates of radii r. The separation z between the plates oscillates with a constant frequency ω, i.e., $z(t) = z_0 + z_1 \cos \omega t$. Here z_0 and $z_1 (< z_0)$ are constants. The separation $z(t)$ $(<< r)$ is varied in such a way that the voltage V_0 across the capacitor remains constant.

(a) Calculate the displacement current density and the displacement current between the plates through a concentric circle of radius $r/2$. (12)

(b) Calculate the magnetic field vector ($\vec{H}$) between the plates at a distance $r/2$ from the axis of the capacitor. (9)

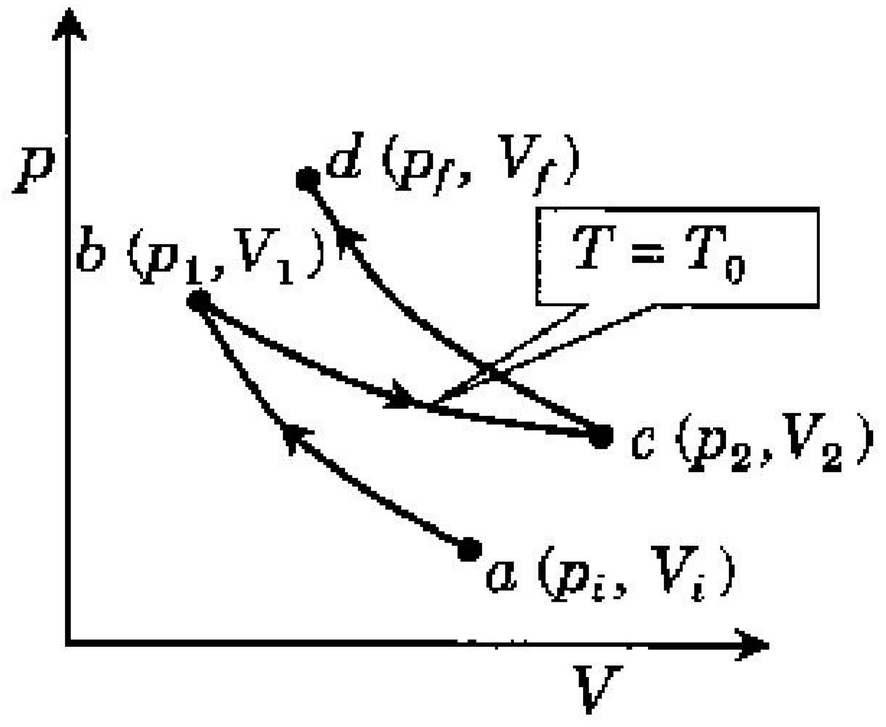
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Q.25 One mole of an ideal monatomic gas in an initial state a with pressure, p_i, and volume, V_i, is to be taken to a final state d with $p_f = B^2 p_i$ and $V_f = V_i / B$ through the path $a \to b \to c \to d$ as shown in the figure below for a particular value of $B\,(>1)$. Here $a \to b$ and $c \to d$ are adiabatic paths while $b \to c$ is an isotherm with temperature T_0. States b and c correspond to (p_1, V_1) and (p_2, V_2), respectively.



Find the ratio $\frac{V_2}{V_1}$ and the total work done by the gas in terms of p_i, V_i, T_0 and B. (21)

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