JEE (Main) 2020

COMPUTER BASED TEST (CBT)
Questions & Solutions

Date: 05 September, 2020 (SHIFT-1) | TIME: (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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SECTION – 1 : (Maximum Marks : 80)

Straight Objective Type  (सीधे वस्तुनिष्ठ प्रश्न)

This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Part: Mathematics

1. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If \( x \) denotes the percentage of them, who like both coffee and tea, then \( x \) cannot be:

(1) 36  (2) 63  (3) 38  (4) 54

**Ans. (1)**

**Sol.**

\[ n(C) = 73, \quad n(T) = 65, \quad n(C \cap T) = x \]

\[ n(C \cup T) \leq 100 \]

\[ \Rightarrow \quad n(C) + n(T) - n(C \cap T) \leq 100 \]

\[ \Rightarrow \quad x \geq 38 \]

\[ n(C \cap T) \leq \min(n(C), n(T)) \quad \Rightarrow \quad x \leq 65 \]

\[ 38 \leq x \leq 65 \]

2. If the function \( f(x) = \begin{cases} k_1(x-\pi)^2 - 1, & x \leq \pi \\ k_2 \cos x, & x > \pi \end{cases} \) is twice differentiable, then the ordered pair \((k_1, k_2)\) is equal to:

(1) \(\left(\frac{1}{2}, -1\right)\)  (2) \(\left(\frac{1}{2}, 1\right)\)  (3) \(1, 0\)  (4) \(1, 1\)

**Ans. (2)**

**Sol.**

\( f(x) \) is differentiable then will also continuous

\[ f(\pi) = -1, \quad f(\pi^+) = -k_2 \]

\( k_2 = 1 \)

Now

\[ f'(x) = \begin{cases} 2k_1(x-\pi), & x \leq \pi \\ -k_2 \sin x, & x > \pi \end{cases} \]

then

\[ f'(\pi^-) = f'(\pi^+) = 0 \]

\[ f''(x) = \begin{cases} 2k_1, & x \leq \pi \\ -k_2 \cos x, & x > \pi \end{cases} \]

then

\[ 2k_1 = k_2 \]

\[ k_1 \cdot \frac{1}{2} \]

3. If \(3^2\sin 2\alpha - 1\), \(14\), and \(3^4 - 2\sin 2\alpha\) are the first three terms of an A.P. for some \(\alpha\), then the sixth term of this A.P. is:

(1) 81  (2) 65  (3) 66  (4) 78

**Ans. (3)**

**Sol.**

\( a, b, c \) are in AP then

\[ 2b = a + c \]

\[ 28 = 3^2\sin 2\alpha - 1 + 3^4 - 2\sin 2\alpha \]

Put \( 3^2\sin 2\alpha = x \)

\[ 28 = \frac{x}{3} \cdot \frac{81}{x} \Rightarrow x^2 - 84x + 243 = 0 \]

\((x - 3)(x - 81) = 0\)
$3^{2 \sin 2 \theta} = 3$ or $3^4$
$2\sin 2\theta = 1$ or $4$
$\sin 2\theta = \frac{1}{2}$

Terms are $1, 14, 27, \ldots$ then $T_6 = 1 + 5 = 6$ (13)

4. If $S$ is the sum of the first 10 terms of the series $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \ldots$, then $\tan(S)$ is equal to:

$\begin{align*}
\text{(1)} & \quad \frac{\pi}{4} \\
\text{(2)} & \quad \frac{5}{11} \\
\text{(3)} & \quad \frac{5}{6} \\
\text{(4)} & \quad \frac{10}{11}
\end{align*}$

Ans. (3)
Sol. $S = \tan^{-1}(1) + \tan^{-1}(1) + \tan^{-1}(1) + \ldots$ up to 10 term
$S = \tan^{-1}\left(1\right) + \tan^{-1}\left(2\right) + \tan^{-1}\left(3\right) + \ldots + \tan^{-1}\left(10\right)$
$S = (\tan^{-1}1 - \tan^{-1}1) + (\tan^{-1}2 - \tan^{-1}1) + (\tan^{-1}3 - \tan^{-1}2) + \ldots + (\tan^{-1}10 - \tan^{-1}9)$
$S = \tan^{-1}(1) - 10 $ $\tan(1) = \frac{\pi}{4}$
$\tan(S) = \frac{5}{6}$

5. If $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + \ldots + 2 \cdot 3^9 + 3^{10} = 3^{10} = S - 2^{11}$ then $S$ is equal to:

$\begin{align*}
\text{(1)} & \quad 3^{11} \\
\text{(2)} & \quad 2 \cdot 3^{11} \\
\text{(3)} & \quad \frac{3^{11}}{2} + 2^{10} \\
\text{(4)} & \quad 3^{11} - 2^{12}
\end{align*}$

Ans. (1)
Sol. $S' = 2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + \ldots + 2 \cdot 3^9 + 3^{10}$
G.P. $\rightarrow a = 2^{10}, r = \frac{3}{2}, n = 11$
$S' = 2^{10} \cdot \left(\frac{3^{11}}{2^{11}} - 1\right)$
$= 3^{11} - 2^{11}$

6. If the common tangent to the parabolas, $y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2 + y^2 = c^2$, then $c$ is equal to:

$\begin{align*}
\text{(1)} & \quad \frac{1}{\sqrt{2}} \\
\text{(2)} & \quad \frac{1}{2\sqrt{2}} \\
\text{(3)} & \quad \frac{1}{2} \\
\text{(4)} & \quad \frac{1}{4}
\end{align*}$

Ans. (1)
Sol. $y^2 = 4x \& x^2 = 4y$
Any tangent of $y^2 = 4x$ is $y = mx + \frac{1}{m}$
It also tangent for $x^2 = 4y$
7. If $y = y(x)$ is the solution of the differential equation $\frac{5 + e^x}{2 + y} \frac{dy}{dx} + e^x = 0$ satisfying $y(0) = 1$, then a value of $y(\log_{13} e)$ is:

(1) $-1$
(2) $0$
(3) $2$
(4) $1$

Ans. (1)

Sol. Given $\frac{dy}{dx} = \frac{-e^x dx}{2 + y}$

$\ln(2 + y) = -\ln(5 + e^x) + \ln C$

$y = \frac{5 + e^x}{e^x - 2}$

$y(0) = 1$ \quad \therefore \quad C = 18$

$y = \left(\log_{13} e\right) = -1$

8. If $\int (e^{2x} + 2e^x - e^{-x} - 1)e^{(e^x + e^{-x})} dx = g(x)e^{(e^x + e^{-x})} + c$, where $c$ is a constant of integration, then $g(0)$ is equal to:

(1) $1$
(2) $e$
(3) $e^2$
(4) $2$

Ans. (4)

Sol. $I = \int (e^{2x} + 2e^x - e^{-x} - 1)e^{(e^x + e^{-x})} dx$

$I = \int (e^{2x} + e^x - 1)e^{(e^x + e^{-x})} dx + \int (e^x - e^{-x})e^{(e^x + e^{-x})} dx$

$I = \int (e^{x+1} - e^{-x})e^{e^x + e^{-x}} dx + e^{e^x + e^{-x}}$

$\left(\frac{x^2 - x - 2}{x + \alpha - 4}\right)$ is equal to:

(1) $\frac{3}{2}$
(2) $\frac{3}{\sqrt{2}}$
(3) $\frac{1}{\sqrt{2}}$
(4) $\frac{1}{2}$

Ans. (2)
10. If the co-ordinates of two points A and B are \((\sqrt{7}, 0)\) and \((-\sqrt{7}, 0)\) respectively and P is any point on the conic, \(9x^2 + 16y^2 = 144\), then \(PA + PB\) is equal to:

Ans. (1)

Sol. For ellipse \(\frac{x^2}{16} + \frac{y^2}{9} = 1\), \(a = 4, b = 3\), \(e = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}\)

A and B are foci
then \(PA + PB = 2a = 2(4) = 8\)

11. The product of the roots of the equation \(9x^2 - 18|x| + 5 = 0\) is:

Ans. (1)

Sol. \(x^2 = |x|^2 = t\) let

\(9t^2 - 18t + 5 = 0\)

\(3t - 1) (3t - 5) = 0\)

\(|x| = \frac{1}{3} \times \frac{5}{3} = \frac{5}{3}\)

product of roots = \(\frac{1}{3} \times \frac{5}{3} \times \frac{-5}{3} = \frac{25}{81}\)

12. If the volume of a parallelepiped, whose conterminous edges are given by the vectors \(\vec{a} = \hat{i} + \hat{j} + \hat{k}\), \(\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}\) and \(\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}\) \((n \geq 0)\), is 158 cu-units, then:

Ans. (3)

Sol. Volume of parallelepiped \(v = |\vec{a} \cdot \vec{b} \cdot \vec{c}|\)

\[\begin{vmatrix}
1 & 1 & n \\
2 & 4 & -n \\
1 & n & 3
\end{vmatrix} = \pm 158\]

\(1(12 + n^2) - 1(6 + n) + n(2n - 4) = \pm 158\)
If the point P on the curve, \(4x^2 + 5y^2 = 20\) is farthest from the point Q(0, -4), then \(PQ^2\) is equal to:

1. (1) 29
2. (2) 48
3. (3) 21
4. (4) 36

**Ans. (4)**

**Sol.**

Equation \(\frac{x^2}{5} + \frac{y^2}{4} = 1\) then \(P\left(\sqrt{5}\cos \theta, 2\sin \theta\right)\)

\[
(PQ)^2 = 5\cos^2 \theta + 4(\sin \theta + 2)^2 = \cos^2 \theta + 16\sin \theta + 20 = -\sin^2 \theta + 16\sin \theta + 21
\]

\[
= 85 - (\sin \theta - 8)^2
\]

\[= (PO)_{\text{max}}^2 = 85 - 49 = 36, \quad \therefore (\sin \theta - 8)^2 \in [49, 81]
\]

If (a, b, c) is the image of the point (1, 2, -3) in the line,

\[
\frac{x + 1}{2} = \frac{y - 3}{2} = \frac{z}{-1} = r
\]

\[
\text{line: } x + 1 = y - 3 = z = r
\]

**Ans. (3)**

**Sol.**

\[
R(-1 + 2r, 3 - 2r, -r)
\]

Dr's of PR are \((2 - 2r, -1 + 2r, -3 + r)\)

Then \(2(2 - 2r) + 2(1 - 2r) + 1(3 - r) = 0\)

\(9 - 9r = 0 \quad \Rightarrow \quad r = 1\)

R(1, 1, -1)

\[
a + 1 = 2, \quad b + 2 = 2, \quad c - 3 = -2
\]

\[
a = 1, \quad b = 0, \quad c = 1
\]

\[\therefore a + b + c = 2
\]

If the four complex numbers \(z, z, z - 2\text{Re}(z)\) and \(z - 2\text{Re}(z)\) represent the vertices of a square of side 4 units in the Argand plane, then \(|z|\) is equal to:

1. (1) 4
2. (2) 2
3. (3) \(4\sqrt{2}\)
4. (4) \(2\sqrt{2}\)

**Ans. (4)**
16. If the minimum and the maximum values of the function $f: \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \to \mathbb{R}$, defined by

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1 & 1 \\ -\cos^2\theta & -1 & 1 \\ 12 & 10 & -2 \end{vmatrix} = 4(\cos^2\theta - \sin^2\theta) = 4(\cos 2\theta), \quad \theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$$

are $m$ and $M$ respectively, then the ordered pair $(m, M)$ is equal to:

(1) $(-4, 4)$  
(2) $(0, 2\sqrt{2})$  
(3) $(-4, 0)$  
(4) $(0, 4)$

Ans. (3)

Sol. $C_2 \to C_2 - C_1$

$$f(\theta)_{\text{max}} = M = 0$$
$$f(\theta)_{\text{min}} = m = -4$$

17. The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to:

(1) $(\sim x \land y) \lor (\sim x \land \sim y)$  
(2) $(x \land y) \lor (\sim x \land \sim y)$  
(3) $(x \land \sim y) \land (\sim x \land y)$  
(4) $(x \land y) \land (\sim x \land \sim y)$

Ans. (2)

Sol. Negation of $x \leftrightarrow \sim y$

$$= \sim(x \leftrightarrow \sim y)$$
$$= x \leftrightarrow \sim(\sim y)$$
$$= x \leftrightarrow y$$
$$= (x \land y) \lor (\sim x \land \sim y)$$
18. Let $\lambda \in \mathbb{R}$. The system of linear equations

\[
\begin{align*}
2x_1 - 4x_2 + \lambda x_3 &= 1 \\
x_1 - 6x_2 + x_3 &= 2 \\
\lambda x_1 - 10x_2 + 4x_3 &= 3
\end{align*}
\]

is inconsistent for:

(1) every value of $\lambda$  
(2) exactly two values of $\lambda$  
(3) exactly one positive value of $\lambda$  
(4) exactly one negative value of $\lambda$

**Ans.** (4)

**Sol.**
\[D = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = (\lambda - 3)(3\lambda + 2)\]

\[D = 0 \implies \lambda = 3, -\frac{2}{3}\]

\[D_1 = \begin{vmatrix} 2 & -4 & 1 \\ 1 & -6 & 1 \\ 3 & -10 & 4 \end{vmatrix} = 2(3 - \lambda)\]

\[\lambda = -\frac{2}{3}, D_1 \neq 0\]

19. The mean and variance of 7 observations are 8 and 16, respectively. If five observation are 2, 4, 10, 12, 14 then the absolute difference of the remaining two observations is:

(1) 1  
(2) 2  
(3) 3  
(4) 4

**Ans.** (2)

**Sol.**
\[\bar{x} = \frac{2 + 4 + 10 + 12 + 14 + x + y}{7} = 8 \implies 42 + x + y = 56 \implies x + y = 14\]

\[\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2\]

\[16 = \frac{4 + 16 + 100 + 144 + x^2 + y^2}{7} - (8)^2\]

\[\implies 16 + 64 = \frac{460 + x^2 + y^2}{7} \implies 560 = 460 + x^2 + y^2 \implies x^2 + y^2 = 100 \quad \text{...(2)} \implies xy = 48\]

\[(x - y)^2 = (x + y)^2 - 4xy = 4\]

\[|x - y| = 2\]
20. The value of \( \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} \, dx \) is:

(1) \( \frac{\pi}{4} \)  
(2) \( \frac{\pi}{2} \)  
(3) \( \frac{3\pi}{2} \)  
(4) \( \pi \)

Ans. (2)

Sol. 
\[
I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} \, dx
\]
\[
= \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}}{1+e^{\sin x}} \, dx
\]
Replace \( x \rightarrow (a+b+x) \)
\[
b \int (f(x)) \, dx = \int (a+b+x) \, dx
\]
\[
= \int_0^1 (a+b+x) \, dx
\]
\[
2I = \int_{-\pi/2}^{\pi/2} \frac{x}{2} \, dx
\]
\[
\Rightarrow I = \frac{\pi}{2}
\]

SECTION – 2 : (Maximum Marks : 20)

- This section contains FIVE (05) questions. The answer to each question is NUMERICAL VALUE with two digit integer and decimal upto one digit.
- If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.
  - Full Marks : +4 If ONLY the correct option is chosen.
  - Zero Marks : 0 In all other cases

खंड 2 (अविकल्म अंक: 20)
- इस खंड में पांच (05) प्रश्न हैं। प्रत्येक प्रश्न का उत्तर संख्यात्मक मान (NUMERICAL VALUE) हैं, जो द्वि-अंकीय पूर्णाक तथा दशमलव एकल-अंक में हैं।
- यदि संख्यात्मक मान में दो से अधिक दशमलव स्थान हैं, तो संख्यात्मक मान को दशमलव के दो स्थानों तक ट्रूपेट/रांपड ऑफ (truncate/round-off) करें।
- अंकन योजना:
  - पूर्ण अंक : +4 यदि शीर्ष सही विकल्प ही चुना गया है।
  - शून्य अंक : 0 अन्य सभी परिस्थितियों में।

21. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is __________.

Ans. (240.00)

Sol. SYLLABUS
S-2, L-2, A, B, Y, U

Required = \( \frac{2! \cdot 5!}{2!} \) = 210 \times \frac{24}{2} = 240
22. The natural number \( m \), for which the coefficient of \( x \) in the binomial expansion of \( \left( x^m + \frac{1}{x^2} \right)^{22} \) is 1540, is

Ans. \( 13 \) (00)

Sol. 
\[
T_{r+1} = \binom{22}{r} x^{22-r} \cdot \left( \frac{1}{x^2} \right)^r = \binom{22}{r} \cdot x^{22-3r}
\]

\( 22m - mr - 2r = 1 \)
\[
r = \frac{22m - 1}{m + 2}
\]
\[
r = \frac{22m + 44 - 45}{m + 2}
\]
so possible value of \( m = 1, 3, 7, 13, 43 \)
but \( \binom{20}{r} = 1540 \) only possible condition is \( m = 13 \)

23. If the line, \( 2x - y + 3 = 0 \) is at a distance \( \frac{1}{\sqrt{5}} \) and \( \frac{2}{\sqrt{5}} \) from the lines \( 4x - 2y + \alpha = 0 \) and \( 6x - 3y + \beta = 0 \), respectively, then the sum of all possible values of \( \alpha \) and \( \beta \) is

Ans. \( 30 \) (00)

Sol. 
\[
2x - y + 3 = 0 \quad \text{.....(i)}
\]
\[
4x - 2y + \alpha = 0 \quad \Rightarrow \quad 2x - y + \frac{\alpha}{2} = 0 \quad \text{.....(ii)}
\]
\[
6x - 3y + \beta = 0 \quad \Rightarrow \quad 2x - y + \frac{\beta}{3} = 0 \quad \text{.....(iii)}
\]
\[
d_1 = \frac{|\alpha - 3|}{\sqrt{2^2 + 1^2}} = \frac{1}{\sqrt{5}} \quad \Rightarrow \quad |\alpha - 6| = 2 \quad \Rightarrow \quad \alpha - 6 = 2, -2 \quad \Rightarrow \quad \alpha = 8, 4
\]
\[
d_2 = \frac{|\beta - 3|}{\sqrt{2^2 + 1^2}} = \frac{2}{\sqrt{5}} \quad \Rightarrow \quad |\beta - 9| = 6 \quad \Rightarrow \quad \beta - 9 = 6, -6 \quad \Rightarrow \quad \beta = 15, 3
\]
Sum of all values of \( \alpha \) and \( \beta \) = 30.

24. Let \( f(x) = x \cdot \left[ \frac{x}{2} \right] \), for \(-10 < x < 10\), where \([t]\) denotes the greatest integer function. Then the number of points of discontinuity of \( f \) is equal to

Ans. \( 08 \) (00)

Sol. 
\[
-5 < \frac{x}{2} < 5
\]
\[
\Rightarrow \quad \left[ \frac{x}{2} \right] = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4
\]
Hence, function is discontinuous at \(-4, -3, -2, -1, 1, 2, 3, 4\)
Number of values is 8.
25. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is __________

Ans. (11.00)

Sol. \[ P(\text{at least 2 show 3 or 5}) = ^4C_2 \cdot \left( \frac{2}{6} \right)^2 \cdot \left( \frac{4}{6} \right)^2 + ^4C_3 \cdot \left( \frac{2}{6} \right)^3 \cdot \left( \frac{4}{6} \right) + ^4C_4 \cdot \left( \frac{2}{6} \right)^4 \]

\[ = \frac{384 + 128 + 16}{6^4} = \frac{11}{27} \]

\( n = 27 \)

\[ \therefore \text{ expectation of number of times} = np \]

\[ = 27 \cdot \frac{11}{27} = 11 \]
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