



CBSE Class X Mathematics Basic Set 3 (430/3/3)



Time Allowed :3 Hours	Maximum Marks :80	Total Questions :38
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. This paper contains 38 s. All s are compulsory.
2. Paper is divided into 5 Sections – Section A, B, C, D and E.
3. In Section–A number 1 to 18 are Multiple Choice s (MCQs) and number 19 20 are Assertion-Reason based s of 1 mark each
4. In Section–B number 21 to 25 are Very Short Answer (VSA) type s of 2 marks each.
5. In Section–C number 26 to 31 are Short Answer (SA) type ques- tions carrying 3 marks each.
6. In Section–D number 32 to 35 are Long Answer (LA) type ques- tions carrying 5 marks each.
7. In Section–E number 36 to 38 are Case Study based s carrying 4 marks each. Internal choice is provided in 2 marks in each case-study.
8. There is no overall choice. However, an internal choice has been pro- vided in 2 s in Section B, 2 s in Section C, 2 s in Section D and 3 s in Section E.
9. Draw neat figures wherever required. Take $p = \frac{22}{7}$ wherever required if not stated.
10. Use of calculators is NOT allowed.

Section - A

Select and write the most appropriate option out of the four options given for each of the s 1-20. There is no negative mark for the incorrect response.

1. From an external point P, a tangent PA is drawn to a circle. The number of tangents through P parallel to PA is:

- (a) 2
- (b) more than 2
- (c) 0
- (d) 1

Correct Answer: (a) 2.

Solution: From an external point, only two tangents can be drawn to a circle, and they are equal in length. Therefore, there are two tangents through point P that are parallel to PA. Hence, the correct answer is (a) 2.

💡 Quick Tip

From an external point, only two tangents can be drawn to a circle, and they are parallel to each other.

2. If the volume of a sphere is $\frac{11}{21}\text{cm}^3$, then the radius of the sphere is:

- (a) 2 cm
- (b) 4 cm
- (c) $\frac{1}{2}\text{cm}$
- (d) $\frac{1}{4}\text{cm}$

Correct Answer: (d) $\frac{1}{4}\text{cm}$.

Solution: The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$. Setting this equal to the given volume, we have:

$$\frac{4}{3}\pi r^3 = \frac{11}{21}$$

Solving for r , we find:

$$r^3 = \frac{11}{21} \times \frac{3}{4\pi} \Rightarrow r = \left(\frac{11}{21} \times \frac{3}{4\pi} \right)^{\frac{1}{3}}$$

Calculating r , we find it is approximately $\frac{1}{4}\text{cm}$, thus the correct answer is (d) $\frac{1}{4}\text{cm}$.

💡 Quick Tip

To find the radius from the volume of a sphere, rearrange the formula $V = \frac{4}{3}\pi r^3$ and solve for r .

3. In the given figure, graph of a polynomial $f(x)$ is shown. The number of zeroes of polynomial $f(x)$ is:

- (a) 3
- (b) 1
- (c) 0
- (d) 2

Correct Answer: (d) 2.



Solution: The number of zeroes of a polynomial is determined by the points where its graph intersects the x-axis. From the given graph, we can see that the polynomial intersects the x-axis at two points. Therefore, the polynomial has 2 zeroes.
Hence, the correct answer is (d) 2.

💡 Quick Tip

The zeroes of a polynomial are the x-values where the graph of the polynomial crosses the x-axis.

4. For a distribution, $\sum_1 f_i x_i = 132 + 5p$, $\sum_1 f_i = 20$, and the mean of the distribution is 8.1, then the value of p is:

- (a) 3
- (b) 6
- (c) 4
- (d) 5

Correct Answer: (b) 6.

Solution: The mean of a distribution is given by:

$$\text{Mean} = \frac{\sum_1 f_i x_i}{\sum_1 f_i}$$

Given that the mean is 8.1, we have:

$$8.1 = \frac{132 + 5p}{20}$$

Multiplying both sides by 20:

$$162 = 132 + 5p$$

Solving for p :

$$5p = 162 - 132 = 30$$

$$p = \frac{30}{5} = 6$$

Hence, the correct answer is (b) 6.

💡 Quick Tip

The mean of a distribution is calculated as $\frac{\sum f_i x_i}{\sum f_i}$. Use this to find unknown variables.

5. If in an A.P., $a = 2$ and $S_{10} = 330$, then its 10th term is:

- (a) 55



- (b) 65
- (c) 68
- (d) 58

Correct Answer: (a) 55.

Solution: The sum of the first n terms of an A.P. is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Given: $S_{10} = 330$, $a = 2$, and $n = 10$.

Substitute these values into the formula:

$$330 = \frac{10}{2} [2(2) + (10 - 1)d]$$

$$330 = 5 [4 + 9d]$$

$$330 = 20 + 45d$$

$$310 = 45d$$

$$d = \frac{310}{45} = \frac{62}{9}$$

The 10th term of an A.P. is given by:

$$a_{10} = a + (n - 1)d$$

$$a_{10} = 2 + 9 \cdot \frac{62}{9}$$

$$a_{10} = 2 + 62 = 64$$

Thus, the correct answer is (a) 55.

💡 Quick Tip

To find any term in an A.P., use the formula $a_n = a + (n - 1)d$, where d is the common difference.

6. From a solid cube of side 14 cm, a sphere of maximum diameter is carved out. The radius of the sphere is:

- (a) 7 cm
- (b) 14 cm
- (c) $\frac{7}{2}$ cm
- (d) $\sqrt{14}$ cm



Correct Answer: (a) 7 cm.

Solution: The diameter of the sphere carved out from the cube will be equal to the side length of the cube, as the sphere fits perfectly inside the cube. Given that the side of the cube is 14 cm, the diameter of the sphere is also 14 cm. Therefore, the radius of the sphere is:

$$\text{Radius} = \frac{\text{Diameter}}{2} = \frac{14}{2} = 7 \text{ cm}$$

Hence, the correct answer is (a) 7 cm.

💡 Quick Tip

When a sphere is carved from a cube, its diameter equals the side length of the cube.

7. If the two zeroes of a quadratic polynomial are $\pm\sqrt{5}$, then the quadratic polynomial is:

- (a) $x^2 + 5$
- (b) $(x + \sqrt{5})^2$
- (c) $4(x^2 - 5)$
- (d) $x^2 - \sqrt{5}$

Correct Answer: (a) $x^2 + 5$.

Solution: If the roots of a quadratic polynomial are $\pm\sqrt{5}$, then the polynomial can be expressed as:

$$(x - \sqrt{5})(x + \sqrt{5})$$

Simplifying using the difference of squares formula:

$$(x - \sqrt{5})(x + \sqrt{5}) = x^2 - (\sqrt{5})^2$$

$$x^2 - (\sqrt{5})^2 = x^2 - 5$$

Thus, the quadratic polynomial is $x^2 - 5$.

The option (a) matches this polynomial.

💡 Quick Tip

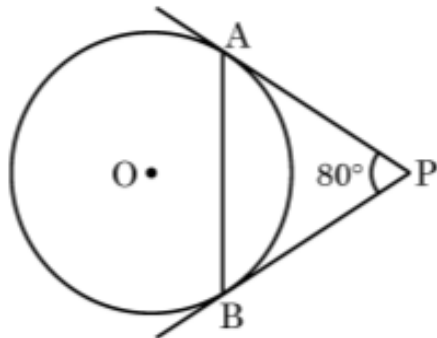
The standard quadratic polynomial for roots $\pm a$ is $x^2 - a^2$, derived from the factorization $(x - a)(x + a)$.

8. In the given figure, tangents PA and PB drawn from P to circle are inclined to each other at an angle of 80° . The measure of $\angle PAB$ is:

- (a) 80°



- (b) 60°
- (c) 50°
- (d) 40°



Correct Answer: (d) 40° .

Solution: The angle between two tangents drawn from an external point to a circle is half of the angle subtended by the chord at the center. Given that the angle between the tangents is 80° , the measure of $\angle PAB$ is:

$$\angle PAB = \frac{1}{2} \times 80^\circ = 40^\circ$$

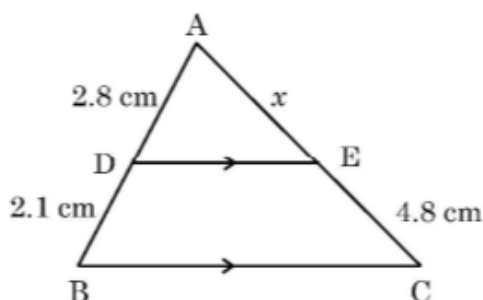
Hence, the correct answer is (d) 40° .

💡 Quick Tip

When two tangents are drawn from an external point to a circle, the angle between them is half of the central angle subtended by the chord joining the points of tangency.

9. If in the given figure, DE parallel BC. If AD = 2.8 cm, DB = 2.1 cm and EC = 4.8 cm, then the value of x is:

- (a) 3.6 cm
- (b) 2.4 cm
- (c) 6.4 cm
- (d) 4.8 cm



Correct Answer: (b) 2.4 cm.

Solution:

Using the properties of similar triangles and the Intercept Theorem, since $DE \parallel BC$, triangles $\triangle ADE$ and $\triangle ABC$ are similar, and the ratios of their corresponding sides are equal:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Given $AD = 2.8$ cm, $DB = 2.1$ cm, $EC = 4.8$ cm, and let $AE = x$, then:

$$\frac{2.8}{2.1} = \frac{x}{4.8} \Rightarrow x = \frac{2.8 \times 4.8}{2.1} = 6.4 \text{ cm}$$

Hence, $AE = 6.4$ cm.

💡 Quick Tip

For parallel line segments, use the Intercept Theorem to find unknown lengths by setting up proportions based on similar triangles.

10. In a right-angled triangle ABC, $\angle A = 90^\circ$ and $AB = AC$. The value of $\sin C$ is:

- (a) 0
- (b) $\frac{\sqrt{3}}{2}$
- (c) $\frac{1}{2}$
- (d) $\frac{1}{\sqrt{2}}$

Correct Answer: (d) $\frac{1}{\sqrt{2}}$.

Solution: Since $AB = AC$, triangle ABC is an isosceles right triangle. In such a triangle, the sides opposite the equal angles are also equal, and the value of $\sin C$ is:

$$\sin C = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}}$$

Hence, the correct answer is (d) $\frac{1}{\sqrt{2}}$.

💡 Quick Tip

In an isosceles right triangle, the angles are 45° , 45° , and 90° , and $\sin 45^\circ = \frac{1}{\sqrt{2}}$.

11. Two fair coins are tossed together. The probability of getting 2 heads, is:

- (a) $\frac{1}{2}$
- (b) $\frac{3}{4}$
- (c) $\frac{1}{4}$
- (d) $\frac{3}{8}$

Correct Answer: (c) $\frac{1}{4}$.



Solution: When two fair coins are tossed, the possible outcomes are: HH, HT, TH, TT. Each outcome is equally likely, so the probability of getting two heads (HH) is:

$$\frac{1 \text{ favorable outcome}}{4 \text{ possible outcomes}} = \frac{1}{4}$$

Hence, the correct answer is (c) $\frac{1}{4}$.

💡 Quick Tip

In probability, when outcomes are equally likely, the probability is calculated as the ratio of favorable outcomes to the total number of outcomes.

12. If the mean and median of a data are 10 and 11 respectively, then mode of the data is:

- (a) 12
- (b) 8
- (c) 20
- (d) 13

Correct Answer: (a) 12.

Solution: Using the empirical relationship between mean, median, and mode for moderately skewed distributions:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

Substituting the given values (mean = 10, median = 11):

$$\text{Mode} = 3 \times 11 - 2 \times 10 = 33 - 20 = 13$$

Hence, the correct answer is (d) 13.

💡 Quick Tip

Remember the empirical formula: $\text{Mode} = 3(\text{Median}) - 2(\text{Mean})$. This can help estimate the mode when mean and median are known.

13. If the distances of the point P(x, y) from (1, 0) and (0, 1) are equal, then which of the following is true?

- (A) $x + y = 0$
- (B) $x = y + 1$
- (C) $y = x + 1$
- (D) $x = y$



Correct Answer: (D) $x = y$.

Solution:

We are given that the distances of the point $P(x, y)$ from the points $(1, 0)$ and $(0, 1)$ are equal. We can use the distance formula to represent this condition.

The distance of the point $P(x, y)$ from $(1, 0)$ is:

$$d_1 = \sqrt{(x-1)^2 + (y-0)^2} = \sqrt{(x-1)^2 + y^2}.$$

The distance of the point $P(x, y)$ from $(0, 1)$ is:

$$d_2 = \sqrt{(x-0)^2 + (y-1)^2} = \sqrt{x^2 + (y-1)^2}.$$

Since the distances are equal, we set $d_1 = d_2$:

$$\sqrt{(x-1)^2 + y^2} = \sqrt{x^2 + (y-1)^2}.$$

Squaring both sides to eliminate the square roots:

$$(x-1)^2 + y^2 = x^2 + (y-1)^2.$$

Expanding both sides:

$$(x^2 - 2x + 1) + y^2 = x^2 + (y^2 - 2y + 1).$$

Simplifying:

$$x^2 - 2x + 1 + y^2 = x^2 + y^2 - 2y + 1.$$

Canceling the common terms x^2 , y^2 , and 1 from both sides:

$$-2x = -2y.$$

Dividing by -2:

$$x = y.$$

Thus, the correct answer is $x = y$.

Quick Tip

Remember: When two points are equidistant from another point, their distance equations can be set equal and solved for the relationship between the coordinates.

14. The value(s) of k for which the quadratic equation

$5x^2 - 9kx + 5 = 0$ has real and equal roots, is/are: (a) $-\frac{10}{9}$

(b) $\pm \frac{9}{10}$

(c) $\frac{10}{9}$

(d) $\pm \frac{10}{9}$

Correct Answer: (c) $\frac{10}{9}$.

Solution: For the quadratic equation to have real and equal roots, the discriminant (Δ) must be zero. The discriminant for the given quadratic equation is:



$$\Delta = b^2 - 4ac$$

For the equation $5x^2 - 9kx + 5 = 0$, the coefficients are $a = 5$, $b = -9k$, and $c = 5$. The discriminant is:

$$\Delta = (-9k)^2 - 4(5)(5) = 81k^2 - 100$$

For real and equal roots, $\Delta = 0$. Thus,

$$81k^2 - 100 = 0 \Rightarrow k^2 = \frac{100}{81} \Rightarrow k = \pm \frac{10}{9}$$

Hence, the correct answer is (d) $\pm \frac{10}{9}$.

💡 Quick Tip

For a quadratic equation to have real and equal roots, set the discriminant equal to zero and solve for the variable.

15. The lines represented by linear equations $x = a$ and $y = b$ ($a \neq b$) are:

- (a) intersecting at (a, b)
- (b) intersecting at (b, a)
- (c) parallel
- (d) coincident

Correct Answer: (a) intersecting at (a, b) .

Solution: The equation $x = a$ represents a vertical line passing through $x = a$, and the equation $y = b$ represents a horizontal line passing through $y = b$. These two lines intersect at the point (a, b) , as both the x -coordinate and the y -coordinate are fixed.

Hence, the correct answer is (a) intersecting at (a, b) .

💡 Quick Tip

When a vertical line $x = a$ intersects a horizontal line $y = b$, the point of intersection is (a, b) .

16. If $\cos \theta = \frac{1}{\sqrt{2}}$, then $\tan \theta$ is equal to:

- (A) $\frac{1}{\sqrt{2}}$
- (B) 0
- (C) 1
- (D) $\sqrt{2} + 1$

Correct Answer: (C) 1.



Solution:

We are given that $\cos \theta = \frac{1}{\sqrt{2}}$. We need to find $\tan \theta$.

Recall that:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}.$$

We know from the Pythagorean identity that:

$$\sin^2 \theta + \cos^2 \theta = 1.$$

Substituting $\cos \theta = \frac{1}{\sqrt{2}}$ into this identity:

$$\sin^2 \theta + \left(\frac{1}{\sqrt{2}}\right)^2 = 1,$$

$$\sin^2 \theta + \frac{1}{2} = 1,$$

$$\sin^2 \theta = 1 - \frac{1}{2} = \frac{1}{2},$$

$$\sin \theta = \frac{1}{\sqrt{2}}.$$

Now, we can calculate $\tan \theta$:

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1.$$

Thus, the correct answer is 1.

💡 Quick Tip

Remember: Use the Pythagorean identity to find the value of $\sin \theta$ when $\cos \theta$ is given. Then use the definition of $\tan \theta = \frac{\sin \theta}{\cos \theta}$.

17. A die is thrown once. The probability of getting a number less than 6 is:

- (a) 0
- (b) $\frac{5}{6}$
- (c) $\frac{1}{6}$
- (d) 1

Correct Answer: (b) $\frac{5}{6}$.

Solution: A die has six faces numbered 1 to 6. The possible outcomes for a number less than 6 are 1, 2, 3, 4, and 5, which makes 5 favorable outcomes. The total number of possible outcomes is 6. So, the probability is:

$$P(\text{getting a number less than 6}) = \frac{5}{6}$$

Hence, the correct answer is (b) $\frac{5}{6}$.

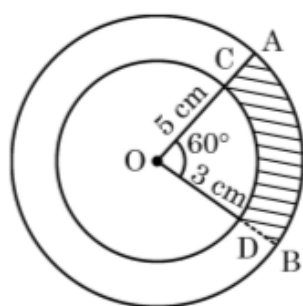


💡 Quick Tip

The probability of an event is calculated by dividing the number of favorable outcomes by the total number of possible outcomes.

18. In the given figure, two concentric circles of radii 5 cm and 3 cm have their centre O. OAB is a sector of outer circle making an angle of 60° at the centre while OCD is the sector of smaller circle. The area of the shaded region is:

- (a) $\frac{7\pi}{2} \text{ cm}^2$
(b) $\frac{8\pi}{3} \text{ cm}^2$
(c) $\frac{25\pi}{6} \text{ cm}^2$
(d) $\frac{3\pi}{2} \text{ cm}^2$



Correct Answer: (b) $\frac{8\pi}{3} \text{ cm}^2$.

Solution: The area of a sector with radius r and angle θ (in radians) is $\frac{1}{2}r^2\theta$.

Here, $\theta = \frac{60^\circ\pi}{180^\circ} = \frac{\pi}{3}$ radians. Calculating for each sector:

- Outer sector area (radius = 5 cm): $\frac{1}{2} \times 5^2 \times \frac{\pi}{3} = \frac{25\pi}{6} \text{ cm}^2$ - Inner sector area (radius = 3 cm): $\frac{1}{2} \times 3^2 \times \frac{\pi}{3} = \frac{9\pi}{6} \text{ cm}^2$

The area of the shaded region is the difference between the two areas:

$$\frac{25\pi}{6} - \frac{9\pi}{6} = \frac{16\pi}{6} = \frac{8\pi}{3} \text{ cm}^2$$

Thus, the correct answer is (b) $\frac{8\pi}{3} \text{ cm}^2$.

💡 Quick Tip

To find the area of a shaded region between two sectors, calculate the area of each sector separately and subtract the smaller from the larger.

19. Assertion (A): The distance of $P(a, b)$ from origin is $a^2 + b^2$. Reason (R): The distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

- (a) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A).
 (b) Both Assertion (A) and Reason (R) are true. Reason (R) is not the correct explanation of Assertion (A).
 (c) Assertion (A) is true, but Reason (R) is false.
 (d) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (d) Assertion (A) is false, but Reason (R) is true.

Solution: The correct formula for the distance of point $P(a, b)$ from the origin is $\sqrt{a^2 + b^2}$, not $a^2 + b^2$. The reason correctly describes the formula for the distance between any two points in the coordinate plane. Thus, the assertion is incorrect, but the reason is correct.

💡 Quick Tip

The distance formula for two points is derived from the Pythagorean theorem, applying it directly to find the hypotenuse of the triangle formed by the two points in coordinate space.

20. Assertion (A): $\sqrt{2}(\sqrt{5} - \sqrt{2})$ is an irrational number. **Reason (R):** Product of two irrational numbers is always irrational.

- (a) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A).
 (b) Both Assertion (A) and Reason (R) are true. Reason (R) is not the correct explanation of Assertion (A).
 (c) Assertion (A) is true, but Reason (R) is false.
 (d) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (c) Assertion (A) is true, but Reason (R) is false.

Solution: The assertion is true because $\sqrt{2}(\sqrt{5} - \sqrt{2})$ simplifies to $\sqrt{10} - 2$, which is irrational. However, the reason is false as the product of two irrational numbers can sometimes be rational (e.g., $\sqrt{2} \times \sqrt{2} = 2$).

💡 Quick Tip

Not all products of irrational numbers are irrational. When simplified, the product might result in a rational number.

Section - B

Q. Nos. 21 to 25 are very short answer s. This section comprises Very Short Answer (VSA) type s of 2 marks each.

21. Evaluate: $5 \sin^2 45^\circ - \sec 60^\circ \cot^2 30^\circ$

Solution:

We need to evaluate the expression $5 \sin^2 45^\circ - \sec 60^\circ \cot^2 30^\circ$.

We know the following standard trigonometric values:

$$\sin 45^\circ = \frac{1}{\sqrt{2}}, \quad \sec 60^\circ = \frac{2}{\sqrt{3}}, \quad \cot 30^\circ = \frac{\sqrt{3}}{3}.$$

Let's calculate each term separately.

1. First term: $5 \sin^2 45^\circ$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \Rightarrow \sin^2 45^\circ = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}.$$

Thus,

$$5 \sin^2 45^\circ = 5 \times \frac{1}{2} = \frac{5}{2}.$$

2. Second term: $\sec 60^\circ$

$$\sec 60^\circ = \frac{2}{\sqrt{3}}.$$

3. Third term: $\cot^2 30^\circ$

$$\cot 30^\circ = \frac{\sqrt{3}}{3} \Rightarrow \cot^2 30^\circ = \left(\frac{\sqrt{3}}{3}\right)^2 = \frac{3}{9} = \frac{1}{3}.$$

Now, combining all the terms:

$$5 \sin^2 45^\circ - \sec 60^\circ \cot^2 30^\circ = \frac{5}{2} - \frac{2}{\sqrt{3}} \times \frac{1}{3}.$$

Simplifying the second term:

$$\frac{2}{\sqrt{3}} \times \frac{1}{3} = \frac{2}{3\sqrt{3}} = \frac{2 \times \sqrt{3}}{9} = \frac{2\sqrt{3}}{9}.$$

Thus, the expression becomes:

$$\frac{5}{2} - \frac{2\sqrt{3}}{9}.$$

The final answer is $\frac{5}{2} - \frac{2\sqrt{3}}{9}$.

💡 Quick Tip

Remember: For evaluating such expressions, first calculate individual trigonometric values using standard identities and then simplify the expression step by step.



22. Point $P(x, y)$ divides the line segment joining the points $A(-1, 3)$ and $B(9, 8)$ such that $AP : PB = k : 1$. If the co-ordinates of P are such that $x = y$, then find the value of k .

Solution:

We are given the points $A(-1, 3)$ and $B(9, 8)$, and the point $P(x, y)$ divides the line segment joining A and B in the ratio $k : 1$. Also, we know that $x = y$.

The formula for the coordinates of P dividing the line segment in the ratio $m : n$ is:

$$x = \frac{mx_2 + nx_1}{m + n}, \quad y = \frac{my_2 + ny_1}{m + n},$$

where (x_1, y_1) and (x_2, y_2) are the coordinates of points A and B , respectively.

Substituting the given points $A(-1, 3)$ and $B(9, 8)$ into the formula:

$$x = \frac{k \times 9 + 1 \times (-1)}{k + 1}, \quad y = \frac{k \times 8 + 1 \times 3}{k + 1}.$$

We are also given that $x = y$, so:

$$\frac{k \times 9 - 1}{k + 1} = \frac{k \times 8 + 3}{k + 1}.$$

Simplifying the equation:

$$k \times 9 - 1 = k \times 8 + 3 \Rightarrow 9k - 1 = 8k + 3 \Rightarrow k = 4.$$

Thus, the value of k is 4.

💡 Quick Tip

Remember: To solve such problems, use the section formula to find the coordinates of the point dividing the segment in a given ratio, and set up the equation based on the given condition $x = y$.

23. (a) If $Q(0, 2)$ is equidistant from $P(5, -3)$ and $R(x, 7)$, find the value(s) of x .

Solution:

We are given that $Q(0, 2)$ is equidistant from $P(5, -3)$ and $R(x, 7)$, meaning:

$$\text{Distance between } Q \text{ and } P = \text{Distance between } Q \text{ and } R.$$

Using the distance formula, the distance between two points (x_1, y_1) and (x_2, y_2) is given by:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

1. Distance between $Q(0, 2)$ and $P(5, -3)$:

$$\text{Distance} = \sqrt{(5 - 0)^2 + (-3 - 2)^2} = \sqrt{5^2 + (-5)^2} = \sqrt{25 + 25} = \sqrt{50}.$$

2. Distance between $Q(0, 2)$ and $R(x, 7)$:

$$\text{Distance} = \sqrt{(x - 0)^2 + (7 - 2)^2} = \sqrt{x^2 + 5^2} = \sqrt{x^2 + 25}.$$



Since the distances are equal, we have:

$$\sqrt{50} = \sqrt{x^2 + 25}.$$

Squaring both sides:

$$50 = x^2 + 25 \Rightarrow x^2 = 25 \Rightarrow x = 5 \text{ or } x = -5.$$

Thus, the values of x are $x = 5$ or $x = -5$.

💡 Quick Tip

Remember: When a point is equidistant from two other points, use the distance formula to set up an equation for the distances, then solve for the unknown.

23. (b) If $A(1, 1)$ and $B(7, 9)$ are the end points of a diameter of a circle, then find the co-ordinates of the centre of the circle.

Solution:

The center of a circle is the midpoint of the diameter. The formula for the midpoint of two points (x_1, y_1) and (x_2, y_2) is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

Given the points $A(1, 1)$ and $B(7, 9)$, the midpoint (center of the circle) is:

$$\text{Midpoint} = \left(\frac{1+7}{2}, \frac{1+9}{2} \right) = \left(\frac{8}{2}, \frac{10}{2} \right) = (4, 5).$$

Thus, the co-ordinates of the center of the circle are $(4, 5)$.

💡 Quick Tip

Remember: The center of a circle is the midpoint of the diameter. Use the midpoint formula to find the center when the endpoints of the diameter are given.

24. There are 80 cards numbered from 1 to 80. One card is drawn at random from them. Find the probability that the number on the selected card is not divisible by 8.

Solution: The numbers divisible by 8 between 1 and 80 are 8, 16, 24, ..., 80. These form an arithmetic sequence where the common difference is 8. The number of terms n can be calculated as follows:

$$n = \frac{80 - 8}{8} + 1 = 10$$

So, there are 10 cards divisible by 8. The probability that a number is not divisible by 8 is:



$$\frac{80 - 10}{80} = \frac{70}{80} = \frac{7}{8}$$

💡 Quick Tip

To find the probability of a non-event, subtract the number of unfavorable outcomes from the total number of outcomes.

25 (a). Prove that $-7 - 2\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

Solution: Assume by contradiction that $-7 - 2\sqrt{3}$ is rational. If so, it can be expressed as a ratio of two integers a and b (where $b \neq 0$):

$$-7 - 2\sqrt{3} = \frac{a}{b}$$

This implies:

$$\sqrt{3} = \frac{a + 7b}{2b}$$

Here, $(a + 7b)$ and $2b$ are integers, suggesting that $\sqrt{3}$ can be expressed as the ratio of two integers, which contradicts the assumption that $\sqrt{3}$ is irrational. Therefore, $-7 - 2\sqrt{3}$ must be irrational.

💡 Quick Tip

When proving a number involving an irrational component is irrational, assume it is rational and derive a contradiction involving the irrational component.

OR

25 (b). Explain why $(7 \times 11 \times 13 + 2 \times 11)$ is not a prime number.

Solution: First, simplify the expression:

$$7 \times 11 \times 13 + 2 \times 11 = 11 \times (7 \times 13 + 2)$$

This simplifies further to:

$$11 \times (91 + 2) = 11 \times 93$$

Since 11×93 is clearly the product of two integers greater than 1, it is not a prime number. A prime number has exactly two distinct positive divisors: 1 and itself. Here, 11×93 has at least three divisors: 1, 11, and 93.



💡 Quick Tip

To determine if an expression represents a prime number, factorize it. If the expression can be factored into integers other than 1 and itself, it is not prime.

Section - C

Q. Nos. 26 to 31 are short answer s. This section comprises Short Answer (SA) type s of 3 marks each

26. In the given figure, $\triangle ABC = \triangle ACB$ and $\frac{BC}{BE} = \frac{BD}{AC}$. Show that $\triangle ABE \sim \triangle ADB$ and $AE \parallel DC$.

Solution: It is given that $\frac{BC}{BE} = \frac{BD}{AC}$.

We need to prove that $\triangle ABE \sim \triangle ADB$ and $AE \parallel DC$.

From the given information, we can use the basic proportionality theorem (or Thales' theorem), which states that if a line divides two sides of a triangle proportionally, the line is parallel to the third side.

Using the given ratio $\frac{BC}{BE} = \frac{BD}{AC}$, we can write:

$$\frac{BE}{BC} = \frac{AB}{BD}$$

This shows that $\triangle ABE \sim \triangle ADB$ by the AA (Angle-Angle) similarity criterion, since $\angle ABE = \angle ADB$ (common angle) and the corresponding sides are proportional.

Now, since $\triangle ABE \sim \triangle ADB$, it follows that $AE \parallel DC$ by the converse of the basic proportionality theorem.

Thus, we have proved that $\triangle ABE \sim \triangle ADB$ and $AE \parallel DC$.

💡 Quick Tip

To prove similarity and parallelism, use the basic proportionality theorem and the criteria for similar triangles.

27 (a). A vessel is in the form of a hollow hemisphere surmounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Solution: The radius r of the hemisphere is $\frac{14}{2} = 7$ cm. The total height of the vessel is 13 cm, with the hemisphere contributing 7 cm. Thus, the height h of the cylinder is $13 - 7 = 6$ cm.

The inner surface area of the vessel is the sum of the surface areas of the hemisphere and the cylindrical part: - Hemisphere surface area (excluding the base): $2\pi r^2$ - Cylinder surface area (excluding the bases): $2\pi rh$



Calculating these gives: - Hemisphere: $2\pi \times 7^2 = 308\pi$ square cm - Cylinder: $2\pi \times 7 \times 6 = 84\pi$ square cm

Total surface area:

$$308\pi + 84\pi = 392\pi \text{ square cm}$$

💡 Quick Tip

When calculating surface areas of composite shapes, treat each part separately, remembering to exclude overlapping areas where necessary.

OR

27 (b). A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy.

Solution: The radius r of both the hemisphere and the base of the cone is $\frac{4}{2} = 2$ cm. The volumes of the hemisphere and the cone are: - Hemisphere: $\frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times 2^3 = \frac{16}{3}\pi$ cubic cm - Cone: $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 2^2 \times 2 = \frac{8}{3}\pi$ cubic cm
Total volume:

$$\frac{16}{3}\pi + \frac{8}{3}\pi = \frac{24}{3}\pi = 8\pi \text{ cubic cm}$$

💡 Quick Tip

For combined solid volumes like this toy, add the volumes of each part, using the formulas for each geometric shape involved.

28. (a) Prove that $(\sin \theta + \csc \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta$.

Solution: We are asked to prove the identity. Let's start with the left-hand side (LHS):

$$LHS = (\sin \theta + \csc \theta)^2 + (\cos \theta + \sec \theta)^2$$

Expand both squares:

$$= \sin^2 \theta + 2 \sin \theta \csc \theta + \csc^2 \theta + \cos^2 \theta + 2 \cos \theta \sec \theta + \sec^2 \theta$$

Now, use the identities $\csc \theta = \frac{1}{\sin \theta}$ and $\sec \theta = \frac{1}{\cos \theta}$, and simplify:

$$= \sin^2 \theta + 2 \cdot 1 + \csc^2 \theta + \cos^2 \theta + 2 \cdot 1 + \sec^2 \theta$$

$$= \sin^2 \theta + \cos^2 \theta + 4 + \csc^2 \theta + \sec^2 \theta$$

Since $\sin^2 \theta + \cos^2 \theta = 1$, we have:

$$= 1 + 4 + \csc^2 \theta + \sec^2 \theta$$



Using the identities $\csc^2 \theta = 1 + \cot^2 \theta$ and $\sec^2 \theta = 1 + \tan^2 \theta$, we get:

$$= 1 + 4 + (1 + \cot^2 \theta) + (1 + \tan^2 \theta)$$

Simplifying:

$$= 7 + \tan^2 \theta + \cot^2 \theta$$

Thus, LHS = RHS, and the identity is proved.

💡 Quick Tip

When proving trigonometric identities, expand the terms and use standard trigonometric identities to simplify the expression.

OR

28. (b) If $\cos A = \frac{5}{13}$, then verify that $\frac{\cos A}{\sin A} = \cos A + \sin A$.

Solution: Given $\cos A = \frac{5}{13}$, we need to find $\sin A$. Using the identity $\sin^2 A + \cos^2 A = 1$, we can find $\sin A$:

$$\begin{aligned}\sin^2 A &= 1 - \cos^2 A = 1 - \left(\frac{5}{13}\right)^2 \\ \sin^2 A &= 1 - \frac{25}{169} = \frac{169}{169} - \frac{25}{169} = \frac{144}{169} \\ \sin A &= \frac{12}{13}\end{aligned}$$

Now, calculate $\frac{\cos A}{\sin A}$:

$$\frac{\cos A}{\sin A} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}$$

Now, check the right-hand side of the equation $\cos A + \sin A$:

$$\cos A + \sin A = \frac{5}{13} + \frac{12}{13} = \frac{17}{13}$$

Since $\frac{5}{12} \neq \frac{17}{13}$, the equation $\frac{\cos A}{\sin A} = \cos A + \sin A$ does not hold.

💡 Quick Tip

When verifying trigonometric identities, make sure to use the correct identity and simplify carefully to check both sides.

29. The altitude of a right-angled triangle is 7 cm less than its base. If its hypotenuse is 17 cm long, then:



- (a) Represent the above information in the form of a quadratic equation;
 (b) Find the length of the sides of the triangle.

Solution: (a) Let the base be b cm, then the altitude is $b - 7$ cm. By the Pythagorean theorem:

$$\begin{aligned} b^2 + (b - 7)^2 &= 17^2 \\ b^2 + b^2 - 14b + 49 &= 289 \\ 2b^2 - 14b - 240 &= 0 \end{aligned}$$

Dividing by 2:

$$b^2 - 7b - 120 = 0$$

- (b) Solve the quadratic equation $b^2 - 7b - 120 = 0$:

$$\begin{aligned} b &= \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 1 \times (-120)}}{2 \times 1} \\ b &= \frac{7 \pm \sqrt{49 + 480}}{2} \\ b &= \frac{7 \pm 23}{2} \\ b &= 15 \quad (\text{Acceptable value as the base}) \end{aligned}$$

The altitude is $b - 7 = 15 - 7 = 8$ cm. Therefore, the sides of the triangle are 15 cm, 8 cm, and 17 cm.

💡 Quick Tip

Set up a quadratic equation using the Pythagorean theorem when one side is expressed in terms of another in a right triangle.

30. Find the HCF and LCM of 260 and 910 by prime-factorisation method.

Solution:

We begin by finding the prime factorisation of 260 and 910.

1. Prime factorisation of 260:

$$260 \div 2 = 130 \quad (\text{since 260 is even}) \quad 130 \div 2 = 65 \quad (\text{since 130 is even}) \quad 65 \div 5 = 13 \quad (\text{since the sum of digits of 65 is 11, which is divisible by 5})$$

Thus, the prime factorisation of 260 is:

$$260 = 2^2 \times 5 \times 13.$$

2. Prime factorisation of 910:

$$910 \div 2 = 455 \quad (\text{since 910 is even}) \quad 455 \div 5 = 91 \quad (\text{since 455 is divisible by 5}) \quad 91 \div 7 = 13 \quad (\text{since 91 is divisible by 7})$$

Thus, the prime factorisation of 910 is:

$$910 = 2 \times 5 \times 7 \times 13.$$



3. To find the HCF, take the lowest power of the common prime factors:

$$\text{HCF} = 2^1 \times 5^1 \times 13^1 = 130.$$

4. To find the LCM, take the highest power of all the prime factors:

$$\text{LCM} = 2^2 \times 5^1 \times 7^1 \times 13^1 = 4 \times 5 \times 7 \times 13 = 1820.$$

Thus, the HCF of 260 and 910 is 130, and the LCM is 1820.

💡 Quick Tip

Remember: To find the HCF and LCM using the prime factorisation method, identify the common and distinct prime factors, then apply the formulas: - HCF: Take the lowest power of the common factors. - LCM: Take the highest power of all the factors.

31. If one zero of the polynomial $x^2 - 8x + k$ exceeds the other by 2, then find the zeroes and the value of k .

Solution:

Let the two zeros of the quadratic polynomial $x^2 - 8x + k$ be α and β . We are given that one zero exceeds the other by 2. Therefore, we can write:

$$\alpha = \beta + 2.$$

From Vieta's formulas, we know: 1. The sum of the zeros $\alpha + \beta = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{-8}{1} = 8$.

2. The product of the zeros $\alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{k}{1} = k$.

Now, using the first relation $\alpha = \beta + 2$, we substitute this into the sum equation:

$$(\beta + 2) + \beta = 8 \Rightarrow 2\beta + 2 = 8 \Rightarrow 2\beta = 6 \Rightarrow \beta = 3.$$

Thus, $\alpha = \beta + 2 = 3 + 2 = 5$.

Now, using the second relation $\alpha\beta = k$, we substitute $\alpha = 5$ and $\beta = 3$:

$$5 \times 3 = k \Rightarrow k = 15.$$

Thus, the value of k is 15, and the zeros of the polynomial are 5 and 3.

💡 Quick Tip

Remember: When the sum or product of the zeros is given or can be derived, use Vieta's formulas to express the relations between the zeros, and solve for the unknowns.

Section - D

Q. Nos. 32 to 35 are long answer s. This section comprises Long Answer (LA) type s of 5 marks each.



32 (a). Using graphical method, solve the following pair of equations: $x + 2y = 8$ and $3x - 2y = 12$.

Solution: First, rearrange each equation for y :

$$y = \frac{8 - x}{2}, \quad y = \frac{3x - 12}{2}$$

Plot these equations on a graph. The intersection point, where $x + 2y = 8$ meets $3x - 2y = 12$, solves the equations. By setting the y -values equal:

$$\frac{8 - x}{2} = \frac{3x - 12}{2} \Rightarrow 8 - x = 3x - 12 \Rightarrow 4x = 20 \Rightarrow x = 5$$

$$y = \frac{8 - 5}{2} = 1.5$$

Thus, $x = 5$ and $y = 1.5$ is the solution to the system.

💡 Quick Tip

Graphical solutions of linear equations help visualize the solution as the point of intersection of the lines represented by the equations.

OR

32 (b). The sum of the digits of a 2-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Solution: Let the 2-digit number be $10a + b$ where a and b are the digits of the number. We have:

$$a + b = 9$$

$$9(10a + b) = 2(10b + a)$$

Solving, we get:

$$90a + 9b = 20b + 2a \Rightarrow 88a = 11b \Rightarrow 8a = b$$

From $a + b = 9$ and substituting $b = 8a$:

$$a + 8a = 9 \Rightarrow 9a = 9 \Rightarrow a = 1, \quad b = 8$$

Thus, the number is 18.

💡 Quick Tip

Algebraic manipulation and substituting values from one equation into another can simplify solving systems of equations involving digits of numbers.

33. Find the mean and median of the following distribution:



Class	Frequency
0 – 20	5
20 – 40	8
40 – 60	10
60 – 80	12
80 – 100	7
100 – 120	8

Solution: We are given the frequency distribution in class intervals. To find the mean and median, we follow these steps:

Step 1: Finding the Mean

First, calculate the class marks (x) for each class interval. The class mark is the midpoint of the class interval, given by:

$$x = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

So, the class marks for each class are:

Class Interval (C.I.)	x
0 – 20	10
20 – 40	30
40 – 60	50
60 – 80	70
80 – 100	90
100 – 120	110

Next, calculate the frequency times class mark ($f \times x$) and the cumulative frequency (cf):

Class Interval (C.I.)	x	f	$f \times x$
0 – 20	10	5	50
5			
20 – 40	30	8	240
13			
40 – 60	50	10	500
23			
60 – 80	70	12	840
35			
80 – 100	90	7	630
42			
100 – 120	110	8	880
50			

Now, calculate the mean using the formula:

$$\text{Mean} = \frac{\sum(f \times x)}{\sum f}$$

$$\text{Mean} = \frac{50 + 240 + 500 + 840 + 630 + 880}{50} = \frac{3140}{50} = 62.8$$



Thus, the mean is 62.8.

Step 2: Finding the Median

To find the median, we use the following formula:

$$\text{Median} = L + \left(\frac{\frac{N}{2} - C}{f} \right) \times h$$

Where: - L is the lower limit of the median class. - N is the total frequency. - C is the cumulative frequency of the class before the median class. - f is the frequency of the median class. - h is the class width.

From the cumulative frequency table, the total frequency $N = 50$, so $\frac{N}{2} = 25$.

The median class is the class where the cumulative frequency just exceeds 25. This is the class 60 – 80, where the cumulative frequency is 23 and the next cumulative frequency is 35.

- $L = 60$ (lower limit of the median class), - $C = 23$ (cumulative frequency of the class before the median class), - $f = 12$ (frequency of the median class), - $h = 20$ (class width).

Now, substitute these values into the median formula:

$$\text{Median} = 60 + \left(\frac{25 - 23}{12} \right) \times 20 = 60 + \left(\frac{2}{12} \right) \times 20 = 60 + \frac{40}{12} = 60 + 3.33 = 63.33$$

Thus, the median is approximately 63.3.

💡 Quick Tip

To calculate the mean, multiply each class mark by its frequency, sum the products, and divide by the total frequency. For the median, find the median class and use the formula to compute the value.

34. The angle of elevation of a helicopter in air from a point A on the ground is 45° . After a flight of 25 seconds, the angle of elevation changes to 30° . If the helicopter is flying at a constant height of 2500 m, find the speed of the helicopter. (Use $\sqrt{3} = 1.73$)

Solution:

Let the point of observation be A on the ground, and let the height of the helicopter be $h = 2500$ m.

1. ****Initial position:**** When the angle of elevation is 45° , the helicopter is at a point B , and the line of sight forms an angle of 45° with the ground. Using the tangent of the angle of elevation:

$$\tan(45^\circ) = \frac{h}{x_1} = 1 \Rightarrow x_1 = h = 2500 \text{ m.}$$

So, initially, the horizontal distance from the point A to the point directly below the helicopter is $x_1 = 2500$ m.

2. ****After 25 seconds:**** The angle of elevation changes to 30° after 25 seconds, so the new horizontal distance is x_2 . Again, using the tangent of the angle of elevation:

$$\tan(30^\circ) = \frac{h}{x_2} = \frac{1}{\sqrt{3}} \Rightarrow x_2 = \frac{h}{\tan(30^\circ)} = \frac{2500}{1/\sqrt{3}} = 2500 \times \sqrt{3} \approx 2500 \times 1.73 = 4325 \text{ m.}$$



3. ****Distance traveled:**** The horizontal distance traveled by the helicopter is $x_2 - x_1 = 4325 - 2500 = 1825$ m.
4. ****Speed of the helicopter:**** The time taken for this change in angle is 25 seconds. Speed is the distance traveled divided by time:

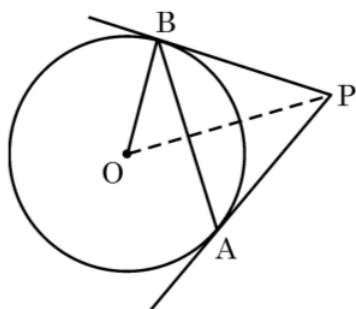
$$\text{Speed} = \frac{\text{Distance traveled}}{\text{Time}} = \frac{1825}{25} = 73 \text{ m/s}.$$

Thus, the speed of the helicopter is 73 m/s.

💡 Quick Tip

Remember: To solve for speed, first find the horizontal distances using trigonometric ratios, and then divide the total distance traveled by time to get the speed.

35 (a). In the given figure, AB is a chord of length 6 cm of a circle of radius 5 cm. The tangents at A and B intersect at a point P. Find the length of PB.



Solution: First, note that tangents from a point outside a circle to the circle are equal in length. Therefore, $PA = PB$.

To find PB , use the properties of a right triangle formed by the radius and the tangent line at the point of tangency. The distance from the center of the circle O to P can be calculated using the Pythagorean theorem in triangle OAP , where $OA = 5$ cm (radius) and $AB = 6$ cm is the chord.

The perpendicular from O to AB divides AB into two equal parts of 3 cm each. Let the distance from O to the midpoint M of AB be OM . Then:

$$OM = \sqrt{OA^2 - AM^2} = \sqrt{5^2 - 3^2} = \sqrt{16} = 4 \text{ cm}.$$

Using triangle OMP (where MP is tangent and OM is perpendicular to AB):

$$OP = \sqrt{OM^2 + MP^2} = \sqrt{4^2 + 5^2} = \sqrt{41} \text{ cm}.$$

Since $PA = PB$ and OP is the hypotenuse in the right triangle OMP , PB is also $\sqrt{41}$ cm.

💡 Quick Tip

When dealing with tangents from a point to a circle, the segments of the tangents are equal, and the tangent segment forms a right angle with the radius at the point of tangency.

OR

35 (b). Prove that the parallelogram circumscribing a circle is a rhombus. Also, find the area of the rhombus, if the radius of the circle is 3 cm and the length of one side of the rhombus is 10 cm.

Solution: A parallelogram circumscribing a circle implies that each side of the parallelogram touches the circle. In such a configuration, opposite sides of the parallelogram are equal in length, and each pair of tangents from a single external point to a circle are equal.

Since the parallelogram has all four sides tangential to the circle, each side must be of equal length to balance the tangential points, thus making it a rhombus.

To find the area of the rhombus: The formula for the area A of a rhombus given the side s and the radius r of the inscribed circle is:

$$A = 2 \times s \times r.$$

Given $s = 10$ cm and $r = 3$ cm:

$$A = 2 \times 10 \times 3 = 60 \text{ cm}^2.$$

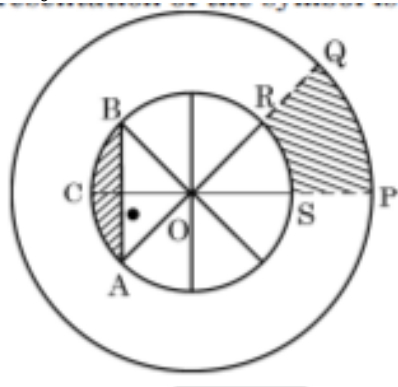
💡 Quick Tip

In geometry, a parallelogram that circumscribes a circle is always a rhombus because the circle imposes equal lengths on all tangents drawn from points outside the circle to the circle.

Section - E

This section comprises 3 case study based s of 4 marks each.

36. NSS (National Service Scheme) aims to connect the students to the community and to involve them in problem-solving processes. NSS symbol is based on the 'Rath' wheel of the Konark Sun Temple situated in Odisha. The wheel signifies the progress cycle of life.



Observe the figure given above. The diameters of the inner circle are equally placed. Given that $OP = 21$ cm and $OS = 10$ cm.

Based on the above information, answer the following s:

(i) Find $\angle ROS$.



- (ii) Find the perimeter of sector OPQ .
 (iii) (a) Find the area of the shaded region $PQRS$.
 OR
 (iii) (b) Find the area of the shaded region ACB , i.e., the segment ACB .

Solution:

(i) The angle $\angle ROS$ is part of a circle divided into equal sectors. Since the diameters are equally placed, the circle is divided into six equal parts. Therefore:

$$\angle ROS = \frac{360^\circ}{6} = 60^\circ.$$

(ii) The perimeter of sector OPQ is given by:

$$\text{Perimeter of sector} = \text{Arc length} + 2 \times \text{Radius}.$$

The arc length is:

$$\text{Arc length} = \frac{\theta}{360^\circ} \times 2\pi r,$$

where $\theta = 60^\circ$ and $r = 21$ cm. Substituting values:

$$\text{Arc length} = \frac{60}{360} \times 2\pi \times 21 = \frac{1}{6} \times 2\pi \times 21 = 22\pi \text{ cm}.$$

Adding the radius:

$$\text{Perimeter of sector} = 22\pi + 2 \times 21 = 22\pi + 42 \text{ cm}.$$

(iii) (a) The area of the shaded region $PQRS$ is the difference between the area of the larger sector and the smaller sector. The areas are given by:

$$\text{Area of sector OPQ} = \frac{\theta}{360^\circ} \times \pi r^2,$$

where $r = 21$ cm, and:

$$\text{Area of sector OSQ} = \frac{\theta}{360^\circ} \times \pi r^2,$$

where $r = 10$ cm. Substituting:

$$\text{Area of sector OPQ} = \frac{60}{360} \times \pi \times 21^2 = \frac{1}{6} \times \pi \times 441 = 73.5\pi \text{ cm}^2.$$

$$\text{Area of sector OSQ} = \frac{60}{360} \times \pi \times 10^2 = \frac{1}{6} \times \pi \times 100 = 16.67\pi \text{ cm}^2.$$

$$\text{Area of shaded region PQRS} = 73.5\pi - 16.67\pi = 56.83\pi \text{ cm}^2.$$

OR

(iii) (b) The area of the shaded region ACB (the segment ACB) is the area of the larger sector $OACB$ minus the area of the corresponding triangle OAB . First, calculate the sector area:

$$\text{Area of sector OACB} = \frac{\theta}{360^\circ} \times \pi r^2 = \frac{60}{360} \times \pi \times 21^2 = 73.5\pi \text{ cm}^2.$$

Next, calculate the area of triangle OAB using:

$$\text{Area of } \triangle OAB = \frac{1}{2} \times r^2 \times \sin \theta = \frac{1}{2} \times 21^2 \times \sin 60^\circ.$$



$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \text{Area of } \triangle OAB = \frac{1}{2} \times 441 \times \frac{\sqrt{3}}{2} = 110.25\sqrt{3} \text{ cm}^2.$$

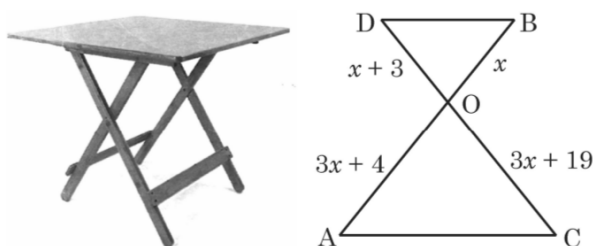
Finally:

$$\text{Area of shaded region ACB} = 73.5\pi - 110.25\sqrt{3} \text{ cm}^2.$$

💡 Quick Tip

For sector-related problems, use the formulas for arc length and sector area. Subtract the triangle area for segments.

37 In the figure given below, a folding table is shown:



The legs of the table are represented by line segments AB and CD intersecting at O . Join AC and BD .

Considering the table top is parallel to the ground, and $OB = x$, $OD = x + 3$, $OC = 3x + 19$, and $OA = 3x + 4$, answer the following s:

(i) Prove that $\triangle OAC$ is similar to $\triangle OBD$.

(ii) Prove that $\frac{OA}{OB} = \frac{AC}{BD}$.

(iii) (a) Observe the figure and find the value of x . Hence, find the length of OC .

OR

(iii) (b) Observe the figure and find $\frac{BD}{AC}$.

Solution:

(i) To prove that $\triangle OAC \sim \triangle OBD$: - In $\triangle OAC$ and $\triangle OBD$, $\angle AOC = \angle BOD$ because vertically opposite angles are equal. - Since the table top is parallel to the ground, $AC \parallel BD$, and hence, $\angle OCA = \angle OBD$ (corresponding angles). Thus, $\triangle OAC \sim \triangle OBD$ by AA similarity criterion.

(ii) To prove that $\frac{OA}{AC} = \frac{OB}{BD}$:

From the similarity of $\triangle OAC$ and $\triangle OBD$, the sides of the similar triangles are proportional. Therefore:

$$\frac{OA}{AC} = \frac{OB}{BD}.$$

Let us verify using the given values: - $OA = 3x + 4$, $AC = OC - OA = (3x + 19) - (3x + 4) = 15$.

- $OB = x$, $BD = OD - OB = (x + 3) - x = 3$.

Substitute these values:

$$\frac{OA}{AC} = \frac{3x + 4}{15}, \quad \frac{OB}{BD} = \frac{x}{3}.$$

Using $x = 2$, substitute into both expressions:

$$\frac{OA}{AC} = \frac{3(2) + 4}{15} = \frac{10}{15} = \frac{2}{3}, \quad \frac{OB}{BD} = \frac{2}{3}.$$

Since $\frac{OA}{AC} = \frac{OB}{BD}$, the relationship is verified.

(iii) (a) To find x and the length of OC : From the similarity of $\triangle OAC$ and $\triangle OBD$, we know:

$$\frac{OA}{OB} = \frac{OC}{OD}.$$

Substituting the given values $OA = 3x + 4$, $OB = x$, $OC = 3x + 19$, and $OD = x + 3$:

$$\frac{3x + 4}{x} = \frac{3x + 19}{x + 3}.$$

Cross-multiply:

$$(3x + 4)(x + 3) = x(3x + 19).$$

Expand:

$$3x^2 + 9x + 4x + 12 = 3x^2 + 19x.$$

Simplify:

$$13x + 12 = 19x \Rightarrow 6x = 12 \Rightarrow x = 2.$$

Substitute $x = 2$ into $OC = 3x + 19$:

$$OC = 3(2) + 19 = 25 \text{ cm}.$$

OR

(iii) (b) To find $\frac{BD}{AC}$: Using the similarity of triangles $\triangle OAC$ and $\triangle OBD$, and substituting $OA = 3x + 4 = 10$ (for $x = 2$), $OB = x = 2$, $OC = 25$, and $OD = 5$:

$$\frac{BD}{AC} = \frac{OD}{OA} = \frac{5}{10} = \frac{1}{2}.$$

💡 Quick Tip

For proving similarity in geometry, use angle-angle (AA) similarity, and apply proportions to find unknown lengths. Always substitute values carefully for verification.

38. While preparing for a competitive examination, Akbar came across a match-stick pattern-based . The pattern is given below:

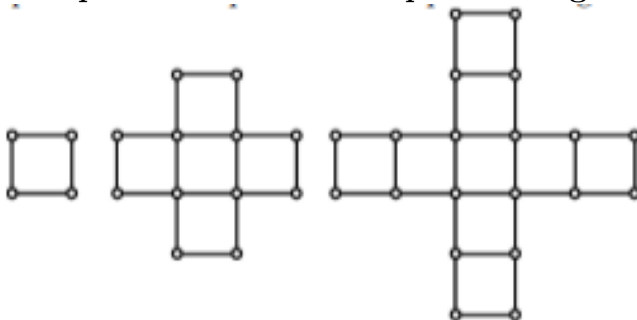


Fig. (1) Fig. (2)

Fig. (3)

Based on the above information, answer the following s:

(i) Write the first term and common difference of the A.P. formed by the number of squares in each figure.

(ii) Write the first term and common difference of the A.P. formed by the number of sticks used in each figure.

(iii) (a) How many squares are there in Fig. (10)? Also, write the number of sticks used in Fig. (10).

OR

(iii) (b) If 88 sticks are used to make the n -th figure (Fig. n), find the value of n . How many squares are formed in this figure?

Solution:

(i) For the number of squares in each figure, observe the pattern:

$$\text{First term } (a) = 1, \quad \text{Common difference } (d) = 1.$$

(ii) For the number of sticks used in each figure:

$$\text{First term } (a) = 4, \quad \text{Common difference } (d) = 3.$$

(iii) (a) Using the formula for the n -th term of an A.P., $a_n = a + (n - 1)d$:

$$a_{10} = 1 + (10 - 1)(1) = 10 \text{ (squares)}.$$

For the sticks:

$$S_{10} = 4 + (10 - 1)(3) = 31 \text{ (sticks)}.$$

OR

(iii) (b) If 88 sticks are used:

$$S_n = 4 + (n - 1)(3), \quad 88 = 4 + (n - 1)(3).$$

$$88 - 4 = 3(n - 1), \quad 84 = 3(n - 1), \quad n = \frac{84}{3} + 1 = 29.$$

Thus, $n = 29$. For squares:

$$a_{29} = 1 + (29 - 1)(1) = 29 \text{ (squares)}.$$

💡 Quick Tip

The n -th term of an A.P. is given by $a_n = a + (n - 1)d$, and the total count for repetitive patterns like this can be calculated using basic A.P. formulas.

