Section A: Q.1 – Q.10 Carry ONE mark each.

Q.1 A competitive firm can sell any output at price $P = 1$. Production depends on capital alone, and the production function $y = f(K)$ is twice continuously differentiable, with

$$f(0) = 0,\ \ f' > 0,\ \ f'' < 0,\ \ \lim_{K\to 0} f'(K) = \infty\ ,\ \ \lim_{K\to\infty} f'(K) = 0.$$

The firm has positive capital stock $\overline{K}$ to start with, and can buy and sell capital at price r per unit of capital. If the firm is maximizing profit then which of the following statements is NOT CORRECT?

(A) If $\overline{K}$ is large enough, profit maximizing $y = 0$ and the profit is $r\overline{K}$

(B) If $f'(\overline{K}) > r$, the firm will buy additional capital

(C) If $f'(\overline{K}) < r$, the firm will sell some of its capital

(D) If $f'(\overline{K}) = r$, the firm will neither buy nor sell any capital

Q.2 Let $f, g : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x+2, & x \leq 1 \\ 2x+1, & x > 1 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 2x, & x \leq 2 \\ x+2, & x > 2. \end{cases}$$

Then

(A) f is convex and g is concave

(B) f is concave and g is convex

(C) both f and g are concave

(D) both f and g are convex

Q.3 Let S be a feasible set of a linear programming problem (P). If the dual problem of (P) is unbounded then

(A) (P) is unbounded

(B) S is empty

(C) S is unbounded

(D) (P) has multiple optimal solutions

Q.4 Which of the following is NOT CORRECT?

(A) A quasiconcave function is necessarily a concave function

(B) A concave function is necessarily a quasiconcave function

(C) A quasiconcave function can also be a quasiconvex function

(D) A quasiconcave function can also be a convex function

Q.5 Among the following statements which one is CORRECT?

S1: $x^2 + y^2 = 6$ is a level curve of

$$f(x, y) = \sqrt{x^2 + y^2} - x^2 - y^2 + 2$$

S2: $x^2 - y^2 = -3$ is a level curve of

$$g(x, y) = e^{-x^2} e^{y^2} + x^4 - 2 - 2x^2\, y^2 + y^4$$

(A) both S1 and S2

(B) only S1

(C) only S2

(D) neither S1 nor S2

Q.6 Which of the following is NOT a component of Gross Domestic Product?

(A) Investment

(B) Rental Income

(C) Transfer Payments

(D) Wages and Salaries

Q.7 Which of the following are the direct instruments exercised by the Reserve Bank of India to control the money supply?

(i) Cash Reserve Ratio
(ii) Open Market Operations
(iii) Foreign Exchange Rate
(iv) Statutory Liquidity Ratio

(A) (i, ii, iii)

(B) (i, ii, iv)

(C) (ii, iii, iv)

(D) (i, iii, iv)

Q.8 Which of the following committees for the first time recommended for India

(i) use of implicit prices derived from quantity and value data collected in household consumer expenditure surveys for computing and updating the poverty lines

(ii) Mixed Reference Period (MRP) in estimating poverty lines

(A) Y K Alagh Committee

(B) D T Lakdawala Committee

(C) S D Tendulkar Committee

(D) C Rangarajan Committee

Q.9 Which of the following Five Year Plans focused on rapid industrialization-heavy and basic industries, and advocated for a socialistic pattern of society as the goal of economic policy?

(A) 1^{st} Five Year Plan (1951-56)

(B) 2^{nd} Five Year Plan (1956-61)

(C) 3^{rd} Five Year Plan (1961-66)

(D) 4^{th} Five Year Plan (1969-74)

Q.10 Let M and N be events defined on the sample space S. If $P(M) = \frac{1}{3}$ and $P(N^c) = \frac{1}{4}$ then which one of the following is necessarily CORRECT?

(A) M and N are disjoint

(B) M and N are not disjoint

(C) M and N are independent

(D) M and N are not independent

Section A: Q.11 – Q.30 Carry TWO marks each.

Q.11 Consider a 2-agent, 2-good exchange economy where agent i has utility function $u_i(x_i, y_i) = \max\{x_i, y_i\}, i = 1,2$. The initial endowments of goods X and Y that the agents have are $(\overline{x_1}, \overline{y_1}, \overline{x_2}, \overline{y_2}) = (25, 5, 5, 5)$. Then select the CORRECT choice below where the price vector (p_x, p_y) specified is part of a competitive equilibrium.

(A) $(p_x, p_y) = (2,1)$

(B) $(p_x, p_y) = (2,2)$

(C) $(p_x, p_y) = (1,2)$

(D) $(p_x, p_y) = (4,2)$

Q.12 For a firm operating in a perfectly competitive market which of the following statements is CORRECT?

(A) Profit function is convex and homogeneous of degree 1 in prices

(B) Profit function is concave and homogeneous of degree 1 in prices

(C) Profit function is convex but not homogeneous in prices

(D) Profit function is neither concave nor convex in prices

Q.13 A firm is operating in a perfectly competitive environment. A change in the market condition leads to an increase in the firm's profit by an amount K. Which of the following describes the change in the Producer's Surplus due to the above change in the market condition?

(A) The Producer's Surplus increases by K

(B) The Producer's Surplus increases by less than K but greater than 0

(C) The Producer's Surplus changes but it is not possible to know the direction of the change

(D) The Producer's Surplus doesn't change

Q.14 Two people, 1 and 2, are engaged in a joint project. Person $i \in \{1,2\}$ puts in effort x_i $(0 \leq x_i \leq 1)$, and incurs cost $C_i(x_i) = x_i$. The monetary outcome of the project is $4x_1x_2$ which is split equally between them. Considering the situation as a strategic game, the set of all Nash Equilibria in pure strategies is

(A) $\{(0,0),(1,1)\}$

(B) $\left\{(0,0),\left(\frac{1}{4},\frac{1}{4}\right),\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{3}{4},\frac{3}{4}\right),(1,1)\right\}$

(C) $\left\{(0,0),\left(\frac{1}{2},\frac{1}{2}\right),(1,1)\right\}$

(D) a null set

Q.15 Two firms, *X* and *Y*, are operating in a perfectly competitive market. The price elasticity of supply of *X* and *Y* are respectively 0.5 and 1.5. Then

(A) if the market price increases by 1 %, *X* supplies 0.5 % less quantity

(B) *Y* experiences a slower increase in marginal cost in comparison to *X*

(C) if market price increases by 0.5 %, *X* supplies 1 % more quantity

(D) *Y* experiences a rapid increase in marginal cost in comparison to *X*

Q.16 Let $y = y(x)$ be a solution curve of the differential equation

$$x\frac{dy}{dx} = y\ \ln\left(\frac{y}{x}\right), \qquad y > x > 0.$$

If $y(1) = e^2$ and $y(2) = \alpha$, then the value of $\frac{dy}{dx}$ at $(2, \alpha)$ is equal to

(A) α

(B) $\frac{\alpha}{2}$

(C) 2α

(D) $\frac{3\alpha}{2}$

Q.17 Let $2z = -3 + \sqrt{3}\,i$, $i = \sqrt{-1}$. Then $2z^8$ is equal to

(A) $-81\,(1 + \sqrt{3}\,i)$

(B) $81\,(-1 + \sqrt{3}\,i)$

(C) $81\,(\sqrt{3} + i)$

(D) $9\,(-\sqrt{3} + i)$

Q.18 Let $a_n = \left(1 + \frac{1}{n}\right)^{\frac{n}{2}}$ be the n^{th} term of the sequence $\langle a_n \rangle, n = 1, 2, 3, \ldots.$

Then which one of the following is NOT CORRECT?

(A) $\langle a_n \rangle$ is bounded

(B) $\langle a_n \rangle$ is increasing

(C) $\sum_{n=1}^{\infty} \ln(a_n)$ is a convergent series

(D) $\lim_{n\to\infty} \left(\frac{1}{n} \sum_{k=1}^{n} a_k \right) = \sqrt{e}$

Q.19 Consider a linear programming problem (P)

$$\begin{aligned} &\min\ z = 4x_1 + 6x_2 + 6x_3 \\ &\text{subject to} \\ &\qquad x_1 + 3x_2 \geq 3 \\ &\qquad x_1 + 2x_3 \geq 5 \\ &\qquad x_1,\ x_2,\ x_3 \geq 0 \end{aligned}$$

If $x^* = (x_1^*, x_2^*, x_3^*)$ is an optimal solution and z^* is an optimal value of (P) and $w^* = (w_1^*, w_2^*)$ is an optimal solution of the dual of (P) then

(A) $x_2^* + x_3^* = w_1^* + w_2^*$

(B) $z^* = 4(x_1^* + w_2^*)$

(C) $z^* = 6(w_1^* + x_3^*)$

(D) $x_1^* + x_3^* = w_1^* + w_2^*$

Q.20 For α , $\beta \in \mathbb{R}$, consider the system of linear equations

$$x + y + z = 1$$
$$3x + y + 2z = 2$$
$$5x + \alpha y + \beta z = 3$$

Then

(A) for every (α, β), $\alpha = \beta$, the system is consistent

(B) there exists (α, β), satisfying $\alpha - 2\beta + 5 = 0$, for which the system has a unique solution

(C) there exists a unique pair (α, β) for which the system has infinitely many solutions

(D) for every (α, β), $\alpha \neq \beta$, satisfying $\alpha - 2\beta + 5 = 0$, the system has infinitely many solutions

Q.21 For a positively sloped LM curve, which of the following statements is CORRECT?

(A) A decrease in the price level will shift the LM curve to the left

(B) A lower nominal money supply will shift the LM curve to the right

(C) An increase in the price level will shift the LM curve to the right

(D) A higher nominal money supply will shift the LM curve to the right

Q.22 Consider an Economy that produces only Apples and Bananas. The following Table contains per unit price (in INR) and quantity (in kg) of these goods. Assuming 2010 as the Base Year and using GDP deflator to calculate the annual inflation rate, which of the following options is CORRECT?

Year	Price of Apple	Quantity of Apple	Price of Banana	Quantity of Banana
2010	1	100	2	50
2011	1	200	2	100
2012	2	200	4	100

(A) GDP deflator for the year 2011 is 100 and the inflation rate for the year 2011 is 0 %

(B) GDP deflator for the year 2012 is 50 and the inflation rate for the year 2012 is 100 %

(C) GDP deflator for the year 2011 is 50 and the inflation rate for the year 2011 is 0 %

(D) GDP deflator for the year 2012 is 100 and the inflation rate for the year 2012 is 100 %

Q.23 Which of the following statements is NOT CORRECT in the context of an Open Economy IS-LM Model under Floating Exchange Rate (with fixed price) and Perfect Capital Mobility?

(A) An expansionary fiscal policy would appreciate the domestic currency value

(B) An expansionary monetary policy would depreciate the domestic currency value

(C) Exchange rate has significant impact on determining the equilibrium level of income and employment

(D) Monetary policy is fully effective in determining income and employment whereas fiscal policy is ineffective

Q.24 Among the following statements which one is CORRECT?

S1: Structural unemployment arises in between two jobs, the first job which an individual has quit in order to find the second job

S2: Frictional unemployment arises due to the mismatch of vacancies and skills of the individual

(A) only S1

(B) only S2

(C) both S1 and S2

(D) neither S1 nor S2

Q.25 Matching List-I and List-II, choose the CORRECT option.

List-I	List-II
(a) Fiscal Deficit	(i) Difference between Government revenue expenditure and Government revenue receipts
(b) Revenue Deficit	(ii) Difference between Government total expenditure and Government total non-debt receipts minus interest payments
(c) Primary Deficit	(iii) Difference between Government total expenditure and Government total non-debt receipts

(A) (a, iii), (b, ii), (c, i)

(B) (a, iii), (b, i), (c, ii)

(C) (a, i), (b, iii), (c, ii)

(D) (a, ii), (b, i), (c, iii)

Q.26 A production function at time t is given by

$$Y_t = A_t\, K_t^{\alpha}\, L_t^{1-\alpha}, \quad \alpha \in (0, 1), \quad \alpha \neq 0.5,$$

where Y is output, K is capital, L is labour and A is the level of Total Factor Productivity. Define per capita output as $y_t \equiv \frac{Y_t}{L_t}$ and capital-output ratio as $k_t \equiv \frac{K_t}{Y_t}$. For any variable x_t, denote $\frac{dx_t}{dt}$ by $\dot{x}$. The per capita output growth rate is

(A) $\dfrac{\dot{y}}{y} = \dfrac{1}{(1-\alpha)}\dfrac{\dot{A}}{A} + \dfrac{\alpha}{(1-\alpha)}\dfrac{\dot{k}}{k}$

(B) $\dfrac{\dot{y}}{y} = \dfrac{\alpha}{(1-\alpha)}\dfrac{\dot{A}}{A} + \dfrac{1}{(1-\alpha)}\dfrac{\dot{k}}{k}$

(C) $\dfrac{\dot{y}}{y} = (1-\alpha)\dfrac{\dot{A}}{A} + \alpha\dfrac{\dot{k}}{k}$

(D) $\dfrac{\dot{y}}{y} = \alpha\dfrac{\dot{A}}{A} + (1-\alpha)\dfrac{\dot{k}}{k}$

Q.27 Matching List-I and List-II, choose the CORRECT option.

List-I (Regulatory and Supervisory Financial Institutions)	List-II (Established as statutory bodies via Parliamentary Acts in year)
(a) Reserve Bank of India	(i) 2016
(b) Security and Exchange Board of India	(ii) 1934
(c) Insurance Regulatory Development Authority of India	(iii) 1992
(d) Insolvency and Bankruptcy Board of India	(iv) 1999

(A) (a, ii), (b, iv), (c, iii), (d, i)

(B) (a, iii), (b, ii), (c, iv), (d, i)

(C) (a, ii), (b, iii), (c, i), (d, iv)

(D) (a, ii), (b, iii), (c, iv), (d, i)

Q.28 Let $X \sim Normal(0, 1)$ and $Y = |X|$. If the probability density function of Y is $f_Y(y)$ then for $y > 0$, $\sqrt{\frac{\pi}{2}}\, f_Y(y)$ is

(A) $e^{-\frac{y^2}{2}}$

(B) $e^{\frac{y^2}{2}}$

(C) e^{-y^2}

(D) $e^{-\frac{y}{2}}$

Q.29 Let the probability density function of the continuous random variable X be

$$f_X(x,\lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise}, \end{cases}$$

where $\lambda > 0$ is a parameter. If the observed sample values of X are

$$x_1 = 1.75,\ \ x_2 = 2.25,\ \ x_3 = 2.50,\ \ x_4 = 2.75,\ \ x_5 = 3.25,$$

then the Maximum Likelihood Estimator of λ is

(A) $\frac{5}{2}$

(B) $\frac{1}{5}$

(C) $\frac{5}{12}$

(D) $\frac{2}{5}$

Q.30 From a set comprising of 10 students, four girls $G_i, i = 1, \ldots, 4,$ and six boys $B_j, \ j = 1, \ldots, 6,$ a team of five students is to be formed. The probability that a randomly selected team comprises of 2 girls and 3 boys, with at least one of them to be B_1 or B_2, is equal to

(A) $\frac{3}{7}$

(B) $\frac{6}{7}$

(C) $\frac{8}{21}$

(D) $\frac{5}{21}$

Section B: Q.31 – Q.40 Carry TWO marks each.

Q.31 Suppose that the utility function $u: \mathbb{R}_+^n \rightarrow \mathbb{R}_+$ represents a complete, transitive and continuous preference relation over all bundles of n goods. Then select the choices below in which the function also represents the same preference relation.

(A) $f(x_1, x_2, \dots, x_n) = u(x_1, x_2, \dots, x_n) + (u(x_1, x_2, \dots, x_n))^3$

(B) $g(x_1, x_2, \dots, x_n) = u(x_1, x_2, \dots, x_n) + \sum_{i=1}^{n} x_i$

(C) $h(x_1, x_2, \dots, x_n) = (u(x_1, x_2, \dots, x_n))^{\frac{1}{n}}$

(D) $m(x_1, x_2, \dots, x_n) = u(x_1, x_2, \dots, x_n) + (x_1^2 + x_2^2 + \dots + x_n^2)^{0.5}$

Q.32 Consider a 2-agent, 2-good economy with an aggregate endowment of 30 units of good X and 10 units of good Y. Agent i has utility function

$$u_i(x_i, y_i) = \max\{x_i, y_i\}, \quad i = 1, 2.$$

Select the choices below in which the specified allocation of the goods to the agents is Pareto optimal for this economy

(A) $(x_1, y_1, x_2, y_2) = (5, 5, 25, 5)$

(B) $(x_1, y_1, x_2, y_2) = (10, 10, 20, 0)$

(C) $(x_1, y_1, x_2, y_2) = (30, 0, 0, 10)$

(D) $(x_1, y_1, x_2, y_2) = (0, 10, 30, 0)$

Q.33 In a 3-player game, player 1 can choose either Up or Down as strategies. Player 2 can chose either Left or Right as strategies. Player 3 can choose either Table 1 or Table 2 as strategies.

		Player 2					Player 2	
		Left	Right				Left	Right
Player 1	Up	3, 2, 5	4, 1, 3		Player 1	Up	2, 3, 4	4, 5, 7
	Down	2, 6, 1	5, 4, 6			Down	6, 4, 0	3, 3, 3
	Table 1					Table 2		
Player 3								

Which of the following strategy profile(s) is/are Nash Equilibrium?

(A) (Up, Left, Table 1)

(B) (Down, Right, Table 1)

(C) (Down, Left, Table 2)

(D) (Up, Right, Table 2)

Q.34 Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ be the function defined by

$$f(x,y) = \begin{cases} \dfrac{x^2 - y^3}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Then

(A) f is not continuous at $(0,0)$

(B) $f_x(0,0) = 0$

(C) $f_y(0,0) = -1$

(D) $f_x(0,0)$ does not exists

Q.35 For $\alpha, \beta \in \mathbb{R}$, $\alpha \neq \beta$, if -2 and 5 are the eigenvalues of the matrix

$$M = \begin{bmatrix} 1-\alpha & 1+\beta \\ \beta & \alpha+\beta \end{bmatrix}$$

and $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is an eigenvector of M associated to -2, then

(A) $2x_1 + x_2 = 0$

(B) $\beta - \alpha = 5$

(C) $\alpha^2 - \beta^2 = 5$

(D) $x_1 + 3x_2 = 0$

Q.36 Which of the following statements is/are CORRECT in the context of the Absolute Income Hypothesis?

(A) The marginal propensity to consume (MPC) is a constant

(B) As income increases, the average propensity to consume (APC) tends to approach the marginal propensity to consume (MPC)

(C) Average propensity to consume (APC) increases as income increases

(D) Current saving/dis-saving has no bearing on future consumption

Q.37 GDP_F = Gross Domestic Product at Factor Cost; GDP_M = Gross Domestic Product at Market Price; NNP_F = Net National Product at Factor Cost; C = Consumption; I = Investment; G = Government Expenditure; X = Export; M = Import; T = Tax; S = Saving; D = Depreciation; NIA = Net Income from Abroad

Which of the following expressions is/are CORRECT?

(A) $GDP_F = C + I + G + X - M$

(B) $GDP_M = C + I + G + X - M$

(C) $NNP_F = C + I + G + X - M - T + S - D + NIA$

(D) $NNP_F = C + I + G + X - M - T + S - D$

Q.38 Which of the following major developments have been undertaken after the initiation of structural reforms in 1991 of the Indian Economy?

(A) A general deregulation of interest rates and a greater role for market forces in the determination of both interest and exchange rates

(B) The phase out of ad hoc Treasury Bill, which puts a check on the automatic monetization of the fiscal deficit

(C) An exchange rate anchor under a Proportional Reserve System

(D) A commitment to the Fiscal Responsibility and Budget Management (FRBM) which sought to put ceiling on the overall fiscal deficit

Q.39 Which of the following functions qualify to be a cumulative density function of a random variable X ?

(A) $F(x) = \begin{cases} 1- e^{-x}, & x \in (0, \infty) \\ 0, & \text{otherwise} \end{cases}$

(B) $F(x) = (1+ e^{-x})^{-1}, \quad x \in (-\infty, \infty)$

(C) $F(x) = \begin{cases} 1 - x^{-1}\ln(x), & x \in (e, \infty) \\ 0, & \text{otherwise} \end{cases}$

(D) $F(x) = \begin{cases} 1 - (\ln(x))^{-1}, & x \in (e, \infty) \\ 0, & \text{otherwise} \end{cases}$

Q.40 Let the joint probability density function of the random variables X and Y be

$$f(x,y) = \begin{cases} 1, & 0 < x < 1, \quad x < y < x+1 \\ 0, & \text{otherwise.} \end{cases}$$

Let the marginal density of X and Y be $f_X(x)$ and $f_Y(y)$, respectively. Which of the following is/are CORRECT?

(A) $f_X(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ and $f_Y(y) = \begin{cases} 2-y, & 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$

(B) $f_X(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ and $f_Y(y) = \begin{cases} y, & 0 < y < 1 \\ 2-y, & 1 \le y < 2 \\ 0, & \text{otherwise} \end{cases}$

(C) $E(X) = \frac{1}{2}, \quad Var(X) = \frac{1}{12}$

(D) $E(Y) = 1, \quad Var(Y) = \frac{1}{6}$

Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41 Let $X \sim Uniform(8, 20)$ and $Z \sim Uniform(0, 6)$ be independent random variables. Let $Y = X + Z$ and $W = X - Z$. Then $Cov(Y, W)$ is ____________ (*in integer*).

Q.42 Let $Y \sim Normal(3, 1)$, $W \sim Normal(1, 2)$ and $X \sim Bernoulli\ (p = 0.9)$ where $X = 1$ is success and $X = 0$ is failure. Let $S = XY + (1 - X)W$. Then $E(S) =$ ____________ (*round off to 1 decimal place*).

Q.43 If X denotes the sum of the numbers appearing on a throw of two fair six-faced dice then the probability

$P(7 < X < 10) =$ ____________ (*round off to 2 decimal places*).

Q.44 Using the following table,

Year	Population of the Economy	GDP of the Economy (in crore)
2010	20,000	25,000
2020	25,000	40,000

the average growth rate (compounded annually) of per capita GDP in an economy during the period 2010-2020 is ____________ (*in percent, round off to 2 decimal places*).

Q.45 Consider a Keynesian Cross Model with following features,

Consumption Function: $C = C_0 + b(Y-T)$

Tax Function: $T = T_0 + tY$

Income Identity: $Y = C + I_0 + G_0$

Where, C = Consumption; Y = Real Income; T = Tax; I = Investment; G = Government Expenditure; b = Parameter; t = Tax Rate (The subscript 0 (zero) indicates that the concerned variable is autonomous)

If $b = 0.7$ and $t = 0.2$, value of the Keynesian multiplier is ________________ (*round off to 2 decimal places*).

Q.46 Let $[t]$ denote the greatest integer $\leq t$. The number of points of discontinuity of the function $f(x) = [\,x^2 - 3x + 2\,]$ for $x \in [0, 4]$ is _______ (*in integer*).

Q.47 Let E be the area of the region bounded by the curves $y = x^2$ and $y = 8\sqrt{x}$, $x \geq 0$. Then $30E$ is equal to ____________ (*round off to 1 decimal place*).

Q.48 A firm has production function $y = K^{0.5}L^{0.5}$ and faces wage rate $w = 4$ and rental rate of capital $r = 4$. The firm's marginal cost is equal to ___________ (*in integer*).

Q.49 Let $\hat{y} = 5.5 + 3.2\,x$ be an estimated regression equation using a large sample. The 95% confidence interval of the coefficient of x is $[0.26,\ 6.14]$ and $R^2 = 0.26$. The standard error of the estimated coefficient is ____________ (*round off to 1 decimal place*).

Q.50 Let π be the proportion of a population vaccinated against a disease. An estimate $\hat{\pi} = 0.64$ is found using a sample of 100 individuals from the population. The z test statistic for the null hypothesis $H_0 : \pi = 0.58$ is ____________ (*round off to 2 decimal places*).

Section C: Q.51 – Q.60 Carry TWO marks each.

Q.51 An industry has 3 firms (1, 2 and 3) in Cournot competition. They have no fixed costs, and their constant marginal costs are respectively

$$c_1 = \frac{9}{30}, \qquad c_2 = \frac{10}{30}, \qquad c_3 = \frac{11}{30}.$$

They face an industry inverse demand function $P = 1 - Q$, where P is the market price and Q is the industry output (sum of outputs of the 3 firms). Suppose that Q^c is the industry output under Cournot-Nash equilibrium. Then $(Q^c)^{-1}$ is equal to ____________ (*in integer*).

Q.52 A consumer has utility function

$$u(x_1, x_2) = \max\ \{0.5\, x_1,\ 0.5\, x_2\} + \min\{x_1\,, x_2\}.$$

She has some positive income y, and faces positive prices p_1, p_2 for goods 1 and 2 respectively. Suppose $p_2 = 1$. There exists a lowest price $\overline{p_1}$ such that if $p_1 > \overline{p_1}$ then the unique utility maximizing choice is to buy ONLY good 2. Then $\overline{p_1}$ is ____________ (*in integer*).

Q.53 An economy has three firms: *X, Y* and *Z*. Every unit of output that *X* produces creates a benefit of INR 700 for *Y* and a cost of INR 300 for *Z*. Firm *X*'s cost curve is

$$C(Q_X) = 2Q_X^2 + 10$$

where C represents cost and Q_X is the output. The market price for the output of *X* is INR 1600 per unit. The difference between the socially optimal output and private profit maximizing output of firm *X* (in INR) is ____________ (*in integer*).

Q.54 Let $\int \sin^9 x \cos(11x)\, dx = \cos(10x)\ f(x) + c$, where c is a constant. If $f''\left(\frac{\pi}{4}\right) - k\, f'\left(\frac{\pi}{4}\right) = 0$, then k is equal to ______ (*in integer*).

Q.55 Let $M = \begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}$ and I_3 be the identity matrix of order 3. If the rank of the matrix $10\, I_3 - M$ is 2 then k is equal to ____________ (*in integer*).

Q.56 In a two period model, a consumer is maximizing the present discounted utility

$$W_t = \ln(c_t) + \frac{1}{1+\theta}\ln(c_{t+1})$$

with respect to c_t and c_{t+1} and subject to the following budget constraint

$$c_t + \frac{c_{t+1}}{1+r} \leq y_t + \frac{y_{t+1}}{1+r}$$

where c_i and y_i are the consumption and income in period i $(i = t, t+1)$ respectively, $\theta \in [0, \infty)$ is the time discount rate and $r \in [0, \infty)$ is the rate of interest. Suppose, consumer is in the interior equilibrium and $\theta = 0.05$ and $r = 0.08$. In equilibrium, the ratio $\frac{c_{t+1}}{c_t}$ is equal to ____________ (*round off to 2 decimal places*).

Q.57 The portfolio of an investment firm comprises of two risky assets, S and T, whose returns are denoted by random variables R_s and R_T respectively. The mean, the variance and the covariance of the returns are

$$E(R_s) = 0.08,\ Var(R_s) = 0.07,$$

$$E(R_T) = 0.05,\ Var(R_T) = 0.05,\ Cov(R_s, R_T) = 0.04.$$

Let w be the proportion of assets allotted to S so that the return from the portfolio is $R = wR_s + (1-w)R_T$. The value of w which minimizes $Var(R)$ is ____________ (*round off to 2 decimal places*).

Q.58 A number x is randomly chosen from the set of the first 100 natural numbers. The probability that x satisfies the condition $x + \frac{300}{x} > 65$ is ____________ (*round off to 2 decimal places*).

Q.59 For $k \in \mathbb{R}$, let $f(x) = x^4 + 2x^3 + kx^2 - k,\ x \in \mathbb{R}$. If $x = \frac{3}{2}$ is a point of local minima of f and m is the global minimum value of f then $f(0) - m$ is equal to ____________ (*in integer*).

Q.60 If (x^*, y^*) is the optimal solution of the problem

$$\begin{array}{ll} \text{maximize} & f(x,y) = 100 - e^{-x} - e^{-y} \\ \text{subject to} & ex + y = \frac{e}{e-1}, \quad x \geq 0, \quad y \geq 0. \end{array}$$

Then $\sqrt{\frac{y^*}{x^*}}$ is equal to __________ (*round off to 2 decimal places*).