CUET PG 2023 Mathematics Question Paper June 5 (Memory-based)

Question 1. $y = x^2$, $y = 4- x^2$. Find the area bounded by these two parabola.

A. 16√2 / 3
B. 16√2 / 5
C. 6√2 / 3
D. √2 / 3

Answer. A
Solution. To find the area bounded by the two parabolas y = x^2 and y = 4 - x^2, we need to determine the points of intersection between the two curves. Let's set the two equations equal to each other:

x^2 = 4 - x^2

2x^2 = 4

x^2 = 2

Taking the square root of both sides, we get:

x = ±√2

The two points of intersection are (√2, 2) and (-√2, 2).

To find the area bounded by the curves, we need to integrate the difference between the two curves with respect to x over the interval [-√2, √2].

Area = ∫[a,b] (f(x) - g(x)) dx

where f(x) is the upper curve (y = 4 - x^2) and g(x) is the lower curve (y = x^2).

Area = ∫[-√2, √2] ((4 - x^2) - x^2) dx

Area = ∫[-√2, √2] (4 - 2x^2) dx

Integrating the expression, we get:

Area = [4x - (2/3)x^3] evaluated from -√2 to √2

Area = (4√2 - (2/3)(√2)^3) - (4(-√2) - (2/3)(-√2)^3)

Simplifying further, we have:

Area = (4√2 - 4√2/3) - (-4√2 - 4√2/3)

Area = (4√2 - 4√2/3) + (4√2 + 4√2/3)

Area = 8√2/3 + 8√2/3

Area = (16√2/3) + (16√2/3)

Area = 32√2/3

Therefore, the area bounded by the two parabolas y = x^2 and y = 4 - x^2 is 32√2/3.

The closest option to this value is 16√2/3, so the answer would be 16√2/3.

Question 2. $u = \cos^{-1} (x+y) / (\sqrt{x} + \sqrt{y})$. Given x<y
Find the value of $xu_x + yu_y$ = ?