MA : MATHEMATICS

Duration: Three Hours *Maximum Marks*: 100

Read the following instructions carefully.

1. This question paper contains **24** pages including blank pages for rough work. Please check all pages and report discrepancy, if any.

2. Write your registration number, your name and name of the examination centre at the specified locations on the right half of the Optical Response Sheet (**ORS**).

3. Using HB pencil, darken the appropriate bubble under each digit of your registration number and the letters corresponding to your paper code.

4. All questions in this paper are of objective type.

5. Questions must be answered on the **ORS** by darkening the appropriate bubble (marked A, B, C, D) using HB pencil against the question number on the left hand side of the ORS. **For each question darken the bubble of the correct answer**. In case you wish to change an answer, erase the old answer completely. More than one answer bubbled against a question will be treated as an incorrect response.

6. There are a total of 65 questions carrying 100 marks.

7. Questions Q.1 – Q.25 will carry 1-mark each, and questions Q.26 – Q.55 will carry 2-marks each.

8. Questions Q.48 – Q.51 (2 pairs) are common data questions and question pairs (Q.52, Q.53) and (Q.54, Q.55) are linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is un-attempted, then the answer to the second question in the pair will not be evaluated.

9. Questions Q.56 – Q.65 belong to General Aptitude (GA). Questions Q.56 – Q.60 will carry 1-mark each, and questions Q.61 – Q.65 will carry 2-marks each. The GA questions will begin on a fresh page starting from page 13.

10. Un-attempted questions will carry zero marks.

11. Wrong answers will carry **NEGATIVE** marks. For Q.1 – Q.25 and Q.56 – Q.60, ⅓ mark will be deducted for each wrong answer. For Q.26 – Q.51 and Q.61 – Q.65, ⅔ mark will be deducted for each wrong answer. The question pairs (Q.52, Q.53), and (Q.54, Q.55) are questions with linked answers. There will be negative marks only for wrong answer to the first question of the linked answer question pair i.e. for Q.52 and Q.54, ⅔ mark will be deducted for each wrong answer. There is no negative marking for Q.53 and Q.55.

12. Calculator (without data connectivity) is allowed in the examination hall.

13. Charts, graph sheets or tables are NOT allowed in the examination hall.

14. Rough work can be done on the question paper itself. Additionally, blank pages are provided at the end of the question paper for rough work.

Notations and Symbols used

$X \setminus Y$	: $\{x \in X : x \notin Y\}$
$\mathbb{N}$	: The set of all natural numbers
$\mathbb{Z}$	: The set of all integers
$\mathbb{Z}_n$	: The set of all integers modulo n
$\mathbb{Q}$	: The set of all rational numbers
$\mathbb{R}$	: The set of all real numbers
$\mathbb{R}^n$	: The set of all n-tuples of real numbers
$\mathbb{C}$	: The set of all complex numbers
$C[0,1]$	: The set of all complex valued continuous functions on $[0,1]$
$P_n[a,b]$	: The set of all polynomials of degree at most n defined on $[a,b]$
$GL(2, \mathbb{R})$	: The group of all 2×2 real invertible matrices under multiplication
S_n	: The set of all permutations on n symbols
A_n	: Alternating group on n symbols
$P(E)$	: Probability of an event E
$P(E \mid F)$	: Conditional probability of E given F
$E(X)$	: Expectation of the random variable X
$E(X \mid Y = y)$	: Conditional expectation of X given $Y = y$

Q.1 – Q.25 carry one mark each.

Q.1 Let E and F be any two events with $P(E \cup F) = 0.8$, $P(E) = 0.4$ and $P(E \mid F) = 0.3$. Then $P(F)$ is

(A) $\frac{3}{7}$ (B) $\frac{4}{7}$ (C) $\frac{3}{5}$ (D) $\frac{2}{5}$

Q.2 Let X have a binomial distribution with parameters n and p, where n is an integer greater than 1 and $0 < p < 1$. If $P(X = 0) = P(X = 1)$, then the value of p is

(A) $\frac{1}{n-1}$ (B) $\frac{n}{n+1}$ (C) $\frac{1}{n+1}$ (D) $\frac{1}{1+n^{\frac{1}{n-1}}}$

Q.3 Let $u(x,y) = 2x(1-y)$ for all real x and y. Then a function $v(x,y)$, so that $f(z) = u(x,y) + i\,v(x,y)$ is analytic, is

(A) $x^2 - (y-1)^2$ (B) $(x-1)^2 - y^2$

(C) $(x-1)^2 + y^2$ (D) $x^2 + (y-1)^2$

Q.4 Let $f(z)$ be analytic on $D = \{z \in \mathbb{C} : |z-1| < 1\}$ such that $f(1) = 1$. If $f(z) = f(z^2)$ for all $z \in D$, then which one of the following statements is **NOT** correct?

(A) $f(z) = [f(z)]^2$ for all $z \in D$ (B) $f\left(\frac{z}{2}\right) = \frac{1}{2} f(z)$ for all $z \in D$

(C) $f(z^3) = [f(z)]^3$ for all $z \in D$ (D) $f'(1) = 0$

Q.5 The maximum number of linearly independent solutions of the differential equation $\frac{d^4 y}{dx^4} = 0$, with the condition $y(0) = 1$, is

(A) 4 (B) 3 (C) 2 (D) 1

Q.6 Which one of the following sets of functions is **NOT** orthogonal (with respect to the L^2-inner product) over the given interval?

(A) $\{\sin nx : n \in \mathbb{N}\}$, $-\pi < x < \pi$ (B) $\{\cos nx : n \in \mathbb{N}\}$, $-\pi < x < \pi$

(C) $\left\{x^{2n+\frac{1}{2}} : n \in \mathbb{N}\right\}$, $-1 < x < 1$ (D) $\{x^{2n+1} : n \in \mathbb{N}\}$, $-1 < x < 1$

Q.7 If $f : [1, 2] \to \mathbb{R}$ is a non-negative Riemann-integrable function such that $\int_1^2 \frac{f(x)}{\sqrt{x}}\,dx = k\int_1^2 f(x)\,dx \neq 0$, then k belongs to the interval

(A) $\left[0, \frac{1}{3}\right]$ (B) $\left(\frac{1}{3}, \frac{2}{3}\right]$ (C) $\left(\frac{2}{3}, 1\right]$ (D) $\left(1, \frac{4}{3}\right]$

Q.8 The set $X = \mathbb{R}$ with the metric $d(x, y) = \dfrac{|x-y|}{1+|x-y|}$ is

(A) bounded but not compact
(B) bounded but not complete
(C) complete but not bounded
(D) compact but not complete

Q.9 Let $f(x,y) = \begin{cases} \dfrac{xy}{(x^2+y^2)^{3/2}}[1-\cos(x^2+y^2)], & (x,y) \neq (0,0) \\ k, & (x,y) = (0,0). \end{cases}$

Then the value of k for which $f(x,y)$ is continuous at $(0,0)$ is

(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) $\frac{3}{2}$

Q.10 Let A and B be disjoint subsets of R and let m^* denote the Lebesgue outer measure on R.

Consider the statements:

$P: m^*(A \cup B) = m^*(A) + m^*(B)$

Q: Both A and B are Lebesgue measurable

R: One of A and B is Lebesgue measureable

Which one of the following is correct?

(A) If P is true, then Q is true
(B) If P is NOT true, then R is true
(C) If R is true, then P is NOT true
(D) If R is true, then P is true

Q.11 Let $f : \mathbb{R} \to [0, \infty)$ be a Lebesgue measurable function and E be a Lebesgue measurable subset of R such that $\int_E f\, dm = 0$, where m is the Lebesgue measure on R. Then

(A) $m(E) = 0$
(B) $\{x \in \mathbb{R} : f(x) = 0\} = E$
(C) $m(\{x \in E : f(x) \neq 0\}) = 0$
(D) $m(\{x \in E : f(x) = 0\}) = 0$

Q.12 If the nullity of the matrix $\begin{bmatrix} k & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 4 \end{bmatrix}$ is 1, then the value of k is

(A) -1
(B) 0
(C) 1
(D) 2

Q.13 If a 3×3 real skew-symmetric matrix has an eigenvalue $2i$, then one of the remaining eigenvalues is

(A) $\frac{1}{2i}$
(B) $-\frac{1}{2i}$
(C) 0
(D) 1

Q.14 For the linear programming problem

Minimize $z = x - y$, subject to $2x + 3y \le 6$, $0 \le x \le 3$, $0 \le y \le 3$,

the number of extreme points of its feasible region and the number of basic feasible solutions respectively, are

(A) 3 and 3 (B) 4 and 4 (C) 3 and 5 (D) 4 and 5

Q.15 Which one of the following statements is correct?

(A) If a Linear Programming Problem (LPP) is infeasible, then its dual is also infeasible
(B) If an LPP is infeasible, then its dual always has unbounded solution
(C) If an LPP has unbounded solution, then its dual also has unbounded solution
(D) If an LPP has unbounded solution, then its dual is infeasible

Q.16 Which one of the following groups is simple?

(A) S_3 (B) $GL(2, \mathbb{R})$ (C) $Z_2 \times Z_2$ (D) A_5

Q.17 Consider the algebraic extension $E = \mathbb{Q}\left(\sqrt{2}, \sqrt{3}, \sqrt{5}\right)$ of the field $\mathbb{Q}$ of rational numbers. Then $[E : \mathbb{Q}]$, the degree of E over $\mathbb{Q}$, is

(A) 3 (B) 4 (C) 7 (D) 8

Q.18 The general solution of the partial differential equation $\dfrac{\partial^2 z}{\partial x\, \partial y} = x + y$ is of the form

(A) $\frac{1}{2}xy(x+y) + F(x) + G(y)$ (B) $\frac{1}{2}xy(x-y) + F(x) + G(y)$

(C) $\frac{1}{2}xy(x-y) + F(x)G(y)$ (D) $\frac{1}{2}xy(x+y) + F(x)G(y)$

Q.19 The numerical value obtained by applying the two-point trapezoidal rule to the integral $\int_0^1 \frac{\ln(1+x)}{x}\, dx$ is

(A) $\frac{1}{2}(\ln 2 + 1)$ (B) $\frac{1}{2}$ (C) $\frac{1}{2}(\ln 2 - 1)$ (D) $\frac{1}{2}\ln 2$

Q.20 Let $l_k(x)$, $k = 0, 1, \ldots, n$ denote the Lagrange's fundamental polynomials of degree n for the nodes $x_0, x_1, \ldots, x_n$. Then the value of $\sum_{k=0}^{n} l_k(x)$ is

(A) 0 (B) 1 (C) $x^n + 1$ (D) $x^n - 1$

Q.21 Let X and Y be normed linear spaces and $\{T_n\}$ be a sequence of bounded linear operators from X to Y. Consider the statements:

$P: \{\|T_n x\| : n \in \mathbb{N}\}$ is bounded for each $x \in X$

$Q: \{\|T_n\| : n \in \mathbb{N}\}$ is bounded

Which one of the following is correct?

(A) If P implies Q, then both X and Y are Banach spaces
(B) If P implies Q, then only one of X and Y is a Banach space
(C) If X is a Banach space, then P implies Q
(D) If Y is a Banach space, then P implies Q

Q.22 Let $X = C[0,1]$ with the norm $\|x\|_1 = \int_0^1 |x(t)|\,dt$, $x \in C[0,1]$ and $\Omega = \{f \in X' : \|f\| = 1\}$, where X' denotes the dual space of X. Let $C(\Omega)$ be the linear space of continuous functions on Ω with the norm $\|u\| = \sup_{s \in \Omega} |u(s)|$, $u \in C(\Omega)$. Then

(A) X is linearly isometric with $C(\Omega)$
(B) X is linearly isometric with a proper subspace of $C(\Omega)$
(C) there does not exist a linear isometry from X into $C(\Omega)$
(D) every linear isometry from X to $C(\Omega)$ is onto

Q.23 Let $X = \mathbb{R}$ equipped with the topology generated by open intervals of the form (a,b) and sets of the form $(a,b) \cap \mathbb{Q}$. Then which one of the following statements is correct?

(A) X is regular
(B) X is normal
(C) $X \setminus \mathbb{Q}$ is dense in X
(D) $\mathbb{Q}$ is dense in X

Q.24 Let H, T and V denote the Hamiltonian, the kinetic energy and the potential energy respectively of a mechanical system at time t. If H contains t explicitly, then $\frac{\partial H}{\partial t}$ is equal to

(A) $\frac{\partial T}{\partial t} + \frac{\partial V}{\partial t}$
(B) $\frac{\partial T}{\partial t} - \frac{\partial V}{\partial t}$
(C) $\frac{\partial V}{\partial t} - \frac{\partial T}{\partial t}$
(D) $-\frac{\partial V}{\partial t} - \frac{\partial T}{\partial t}$

Q.25 The Euler's equation for the variational problem: Minimize $I[y(x)] = \int_0^1 (2x - xy - y')y'\,dx$, is

(A) $2y'' - y = 2$
(B) $2y'' + y = 2$
(C) $y'' + 2y = 0$
(D) $2y'' - y = 0$

Q.26 – Q.55 carry two marks each.

Q.26 Let X have a binomial distribution with parameters n and p, $n=3$. For testing the hypothesis $H_0: p=\frac{2}{3}$ against $H_1: p=\frac{1}{3}$, let a test be: "Reject H_0 if $X \geq 2$ and accept H_0 if $X \leq 1$". Then the probabilities of Type I and Type II errors respectively are

(A) $\frac{20}{27}$ and $\frac{20}{27}$ (B) $\frac{7}{27}$ and $\frac{20}{27}$ (C) $\frac{20}{27}$ and $\frac{7}{27}$ (D) $\frac{7}{27}$ and $\frac{7}{27}$

Q.27 Let $I=\int_C \frac{f(z)}{(z-1)(z-2)}\,dz$, where $f(z)=\sin\frac{\pi z}{2}+\cos\frac{\pi z}{2}$ and C is the curve $|z|=3$ oriented anti-clockwise. Then the value of I is

(A) $4\pi i$ (B) 0 (C) $-2\pi i$ (D) $-4\pi i$

Q.28 Let $\sum_{n=-\infty}^{\infty} b_n z^n$ be the Laurent series expansion of the function $\frac{1}{z\sinh z}$, $0<|z|<\pi$. Then which one of the following is correct?

(A) $b_{-2}=1,\ b_0=-\frac{1}{6},\ b_2=\frac{7}{360}$.
(B) $b_{-3}=1,\ b_{-1}=-\frac{1}{6},\ b_1=\frac{7}{360}$.
(C) $b_{-2}=0,\ b_0=-\frac{1}{6},\ b_2=\frac{7}{360}$.
(D) $b_0=1,\ b_2=-\frac{1}{6},\ b_4=\frac{7}{360}$.

Q.29 Under the transformation $w=\sqrt{\frac{1-iz}{z-i}}$, the region $D=\{z\in \mathbb{C}:|z|<1\}$ is transformed to

(A) $\{z\in \mathbb{C}: 0<\arg z<\pi\}$
(B) $\{z\in \mathbb{C}: -\pi<\arg z<0$
(C) $\{z\in \mathbb{C}: 0<\arg z<\frac{\pi}{2}$ or $\pi<\arg z<\frac{3\pi}{2}\}$
(D) $\{z\in \mathbb{C}: \frac{\pi}{2}<\arg z<\pi$ or $\frac{3\pi}{2}<\arg z<2\pi\}$

Q.30 Let $y(x)$ be the solution of the initial value problem

$$y'''-y''+4y'-4y=0,\quad y(0)=y'(0)=2,\ y''(0)=0.$$

Then the value of $y(\frac{\pi}{2})$ is

(A) $\frac{1}{5}\left(4e^{\frac{\pi}{2}}-6\right)$ (B) $\frac{1}{5}\left(6e^{\frac{\pi}{2}}-4\right)$ (C) $\frac{1}{5}\left(8e^{\frac{\pi}{2}}-2\right)$ (D) $\frac{1}{5}\left(8e^{\frac{\pi}{2}}+2\right)$

Q.31 Let $y(x)$ be the solution of the initial value problem

$$x^2y'' + xy' + y = x, \quad y(1) = y'(1) = 1.$$

Then the value of $y(e^{\frac{\pi}{2}})$ is

(A) $\frac{1}{2}(1-e^{\frac{\pi}{2}})$ (B) $\frac{1}{2}(1+e^{\frac{\pi}{2}})$ (C) $\frac{1}{2}+\frac{\pi}{4}$ (D) $\frac{1}{2}-\frac{\pi}{4}$

Q.32 Let $T : \mathrm{R}^3 \to \mathrm{R}^3$ be a linear transformation defined by $T(x, y, z) = (x+y, y+z, z-x)$. Then, an orthonormal basis for the range of T is

(A) $\left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\right\}$ (B) $\left\{\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)\right\}$

(C) $\left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)\right\}$ (D) $\left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)\right\}$

Q.33 Let $T : P_3[0,1] \to P_2[0,1]$ be defined by $(Tp)(x) = p''(x) + p'(x)$. Then the matrix representation of T with respect to the bases $\{1, x, x^2, x^3\}$ and $\{1, x, x^2\}$ of $P_3[0,1]$ and $P_2[0,1]$ respectively is

(A) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 2 & 0 \\ 0 & 6 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 2 & 1 & 0 \\ 6 & 2 & 0 & 0 \\ 3 & 0 & 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 2 \\ 3 & 6 & 0 \end{bmatrix}$

Q.34 Consider the basis $\{u_1, u_2, u_3\}$ of R^3, where $u_1 = (1,0,0)$, $u_2 = (1,1,0)$, $u_3 = (1,1,1)$. Let $\{f_1, f_2, f_3\}$ be the dual basis of $\{u_1, u_2, u_3\}$ and f be a linear functional defined by $f(a,b,c) = a+b+c$, $(a,b,c) \in \mathrm{R}^3$. If $f = \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3$, then $(\alpha_1, \alpha_2, \alpha_3)$ is

(A) (1,2,3) (B) (1,3,2) (C) (2,3,1) (D) (3,2,1)

Q.35 The following table gives the cost matrix of a transportation problem

4	5	6
3	2	2
1	1	2

The basic feasible solution given by $x_{11} = 3, x_{13} = 1, x_{23} = 6, x_{31} = 2, x_{32} = 5$ is

(A) degenerate and optimal (B) optimal but not degenerate

(C) degenerate but not optimal (D) neither degenerate nor optimal

Q.36 If z^* is the optimal value of the linear programming problem

$$\text{Maximize } z = 5x_1 + 9x_2 + 4x_3$$
$$\text{subject to } x_1 + x_2 + x_3 = 5$$
$$4x_1 + 3x_2 + 2x_3 = 12$$
$$x_1, x_2, x_3 \geq 0,$$

then

(A) $0 \leq z^* < 10$ (B) $10 \leq z^* < 20$ (C) $20 \leq z^* < 30$ (D) $30 \leq z^* < 40$

Q.37 Let G_1 be an abelian group of order 6 and $G_2 = S_3$. For $j = 1, 2$, let P_j be the statement: "G_j has a unique subgroup of order 2". Then

(A) both P_1 and P_2 hold (B) neither P_1 nor P_2 holds
(C) P_1 holds but not P_2 (D) P_2 holds but not P_1

Q.38 Let G be the group of all symmetries of the square. Then the number of conjugate classes in G is

(A) 4 (B) 5 (C) 6 (D) 7

Q.39 Consider the polynomial ring $\mathbb{Q}[x]$. The ideal of $\mathbb{Q}[x]$ generated by $x^2 - 3$ is

(A) maximal but not prime (B) prime but not maximal
(C) both maximal and prime (D) neither maximal nor prime

Q.40 Consider the wave equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$, $0 < x < \pi$, $t > 0$, with $u(0,t) = u(\pi,t) = 0$, $u(x,0) = \sin x$ and $\frac{\partial u}{\partial t} = 0$ at $t = 0$. Then $u\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

(A) 2 (B) 1 (C) 0 (D) -1

Q.41 Let $I = \int_C \frac{e^y}{x}\,dx + (e^y \ln x + x)\,dy$, where C is the positively oriented boundary of the region enclosed by $y = 1 + x^2$, $y = 2$, $x = \frac{1}{2}$. Then the value of I is

(A) $\frac{1}{8}$ (B) $\frac{5}{24}$ (C) $\frac{7}{24}$ (D) $\frac{3}{8}$

Q.42 Let $\{f_n\}$ be a sequence of real valued differentiable functions on $[a,b]$ such that $f_n(x) \to f(x)$ as $n \to \infty$ for every $x \in [a,b]$ and for some Riemann-integrable function $f : [a,b] \to \mathbb{R}$. Consider the statements

$P_1 : \{f_n\}$ converges uniformly

$P_2 : \{f_n'\}$ converges uniformly

$P_3 : \int_a^b f_n(x)\,dx \to \int_a^b f(x)\,dx$

$P_4 : f$ is differentiable

Then which one of the following need **NOT** be true

(A) P_2 implies P_1 (B) P_2 implies P_3 (C) P_2 implies P_4 (D) P_3 implies P_1

Q.43 Let $f_n(x) = \frac{x^n}{1+x}$ and $g_n(x) = \frac{x^n}{1+nx}$ for $x \in [0,1]$ and $n \in \mathbb{N}$. Then on the interval $[0,1]$,

(A) both $\{f_n\}$ and $\{g_n\}$ converge uniformly

(B) neither $\{f_n\}$ nor $\{g_n\}$ converges uniformly

(C) $\{f_n\}$ converges uniformly but $\{g_n\}$ does not converge uniformly

(D) $\{g_n\}$ converges uniformly but $\{f_n\}$ does not converge uniformly

Q.44 Consider the power series $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ and $\sum_{n=1}^{\infty} \frac{x^n}{n}$. Then

(A) both converge on $(-1,1]$

(B) both converge on $[-1,1)$

(C) exactly one of them converges on $(-1,1]$

(D) none of them converges on $[-1,1)$

Q.45 Let $X = \mathbb{N}$ be equipped with the topology generated by the basis consisting of sets $A_n = \{n, n+1, n+2, \ldots\}$, $n \in \mathbb{N}$. Then X is

(A) Compact and connected

(B) Hausdorff and connected

(C) Hausdorff and compact

(D) Neither compact nor connected

Q.46 Four weightless rods form a rhombus $PQRS$ with smooth hinges at the joints. Another weightless rod joins the midpoints E and F of PQ and PS respectively. The system is suspended from P and a weight $2W$ is attached to R. If the angle between the rods PQ and PS is 2θ, then the thrust in the rod EF is

(A) $W \tan\theta$

(B) $2W \tan\theta$

(C) $2W \cot\theta$

(D) $4W \tan\theta$

Q.47 For a continuous function $f(t)$, $0 \le t \le 1$, the integral equation $y(t) = f(t) + 3\int_0^1 ts\, y(s)\, ds$ has

(A) a unique solution if $\int_0^1 sf(s)\, ds \ne 0$

(B) no solution if $\int_0^1 sf(s)\, ds = 0$

(C) infinitely many solutions if $\int_0^1 sf(s)\, ds = 0$

(D) infinitely many solutions if $\int_0^1 sf(s)\, ds \ne 0$

Common Data Questions

Common Data for Questions 48 and 49:

Let X and Y be continuous random variables with the joint probability density function

$$f(x,y)=\begin{cases} a\,e^{-2y}, & 0<x<y<\infty, \\ 0, & \text{otherwise.} \end{cases}$$

Q.48 The value of a is

(A) 4 (B) 2 (C) 1 (D) 0.5

Q.49 The value of $E(X \mid Y=2)$ is

(A) 4 (B) 3 (C) 2 (D) 1

Common Data for Questions 50 and 51:

Let $X = \mathbb{N} \times \mathbb{Q}$ with the subspace topology of the usual topology on $\mathbb{R}^2$ and $P = \left\{ \left(n, \frac{1}{n}\right) : n \in \mathbb{N} \right\}$.

Q.50 In the space X,

(A) P is closed but not open
(B) P is open but not closed
(C) P is both open and closed
(D) P is neither open nor closed

Q.51 The boundary of P in X is

(A) an empty set (B) a singleton set (C) P (D) X

Linked Answer Questions

Statement for Linked Answer Questions 52 and 53:

For a differentiable function $f(x)$, the integral $\int_0^h f(x)\,dx$ is approximated by the formula $h\,[a_0 f(0) + a_1 f(h)] + h^2[b_0 f'(0) + b_1 f'(h)]$, which is exact for all polynomials of degree at most 3.

Q.52 The values of a_1 and b_1 respectively are

(A) $\frac{1}{2}$ and $-\frac{1}{12}$ (B) $-\frac{1}{12}$ and $\frac{1}{2}$ (C) $\frac{1}{2}$ and $\frac{1}{12}$ (D) $\frac{1}{12}$ and $\frac{1}{2}$

Q.53 The values of a_0 and b_0 respectively are

(A) $\frac{1}{2}$ and $\frac{1}{2}$ (B) $\frac{1}{12}$ and $-\frac{1}{12}$ (C) $\frac{1}{2}$ and $\frac{1}{12}$ (D) $\frac{1}{2}$ and $-\frac{1}{12}$

Statement for Linked Answer Questions 54 and 55:

Let $X = C[0,1]$ with the inner product $\langle x, y\rangle = \int_0^1 x(t)\overline{y(t)}\, dt, \quad x, y \in C[0,1]$,

$X_0 = \left\{ x \in X : \int_0^1 t^2 x(t)\, dt = 0 \right\}$, and $X_0^{\perp}$ be the orthogonal complement of X_0.

Q.54 Which one of the following statements is correct?

(A) Both X_0 and $X_0^{\perp}$ are complete

(B) Neither X_0 nor $X_0^{\perp}$ is complete

(C) X_0 is complete but $X_0^{\perp}$ is not complete

(D) $X_0^{\perp}$ is complete but X_0 is not complete

Q.55 Let $y(t) = t^3, t \in [0,1]$ and $x_0 \in X_0^{\perp}$ be the best approximation of y. Then $x_0(t),\ t \in [0,1]$, is

(A) $\frac{4}{5}t^2$

(B) $\frac{5}{6}t^2$

(C) $\frac{6}{7}t^2$

(D) $\frac{7}{8}t^2$

General Aptitude (GA) Questions

Q.56 – Q.60 carry one mark each.

Q.56 *Which of the following options is the closest in meaning to the word below:*
Circuitous

(A) cyclic
(B) indirect
(C) confusing
(D) crooked

Q.57 *The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.*
Unemployed : Worker

(A) fallow : land
(B) unaware : sleeper
(C) wit : jester
(D) renovated : house

Q.58 *Choose the most appropriate word from the options given below to complete the following sentence:*
If we manage to _____________ our natural resources, we would leave a better planet for our children.

(A) uphold
(B) restrain
(C) cherish
(D) conserve

Q.59 *Choose the most appropriate word from the options given below to complete the following sentence:*
His rather casual remarks on politics _____________ his lack of seriousness about the subject.

(A) masked
(B) belied
(C) betrayed
(D) suppressed

Q.60 25 persons are in a room. 15 of them play hockey. 17 of them play football and 10 of them play both hockey and football. Then the number of persons playing neither hockey nor football is:

(A) 2 (B) 17 (C) 13 (D) 3

Q.61 – Q.65 carry two marks each.

Q.61 **Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in military establishments who think that chemical agents are useful tools for their cause.**

Which of the following statements best sums up the meaning of the above passage:

(A) Modern warfare has resulted in civil strife.
(B) Chemical agents are useful in modern warfare.
(C) Use of chemical agents in warfare would be undesirable.
(D) People in military establishments like to use chemical agents in war.

Q.62 If 137 + 276 = 435 how much is 731 + 672?

(A) 534 (B) 1403 (C) 1623 (D) 1513

Q.63 5 skilled workers can build a wall in 20 days; 8 semi-skilled workers can build a wall in 25 days; 10 unskilled workers can build a wall in 30 days. If a team has 2 skilled, 6 semi-skilled and 5 unskilled workers, how long will it take to build the wall?

(A) 20 days (B) 18 days (C) 16 days (D) 15 days

Q.64 Given digits 2, 2, 3, 3, 3, 4, 4, 4, 4 how many distinct 4 digit numbers greater than 3000 can be formed?

(A) 50 (B) 51 (C) 52 (D) 54

Q.65 Hari (H), Gita (G), Irfan (I) and Saira (S) are siblings (i.e. brothers and sisters). All were born on 1st January. The age difference between any two successive siblings (that is born one after another) is less than 3 years. Given the following facts:

i. Hari's age + Gita's age > Irfan's age + Saira's age.
ii. The age difference between Gita and Saira is 1 year. However, Gita is not the oldest and Saira is not the youngest.
iii. There are no twins.

In what order were they born (oldest first)?

(A) HSIG (B) SGHI (C) IGSH (D) IHSG

END OF THE QUESTION PAPER

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