XAT 2005 Quantitative Aptitude Question Paper with Solutions

ZOLLEGE

General Instructions

(i) This booklet contains 50 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 50 single-correct multiple-choice questions.

(iii) Attempt each question on your own before reviewing the given solution.

1. Last year, Mr Basu bought two scooters. This year, he sold both of them for Rs 30,000 each. On one scooter, he earned a profit of 20%, and on the other, he made a loss of 20%. What was his net profit or loss?

(A) He gained less than Rs 2000

(B) He gained more than Rs 2000

(C) He lost less than Rs 2000

(D) He lost more than Rs 2000

Correct Answer: (D) He lost more than Rs 2000

SOLUTION:

Step 1: Recall the net effect rule for equal profit and loss percentages on unequal cost prices.

When two articles are sold at the same selling price, one at a gain of $r\%$ and the other at a loss of $r\%$, there is always a net loss on the whole deal. The overall loss percentage, calculated on the total cost price, works out to $\left(\frac{r}{10}\right)^2\%$. Here $r = 20$.

Step 2: Apply the rule to find the loss percentage.

$$\text{Loss percent} = \left(\frac{20}{10}\right)^2 = 2^2 = 4\%$$

So Mr Basu's net loss, as a percentage of the total cost price, is 4%.

Step 3: Recover the total cost price from the total selling price.
Total selling price $= 30000 + 30000 = 60000$.
If the loss is 4% of the cost price, the selling price is 96% of the cost price.

$$0.96 \times \text{Total CP} = 60000$$

$$\text{Total CP} = \frac{60000}{0.96} = 62500$$

Step 4: Find the actual loss amount and compare with the options.

$$\text{Loss amount} = \text{Total CP} - \text{Total SP} = 62500 - 60000 = 2500$$

A loss of Rs 2500 is clearly more than Rs 2000, so any option that mentions a gain, or a loss smaller than Rs 2000, cannot be correct.

Step 5: State the final answer.
Mr Basu's transaction resulted in a net loss of Rs 2500, which is more than Rs 2000.

$$\boxed{\text{He lost more than Rs 2000}}$$

Quick Tip: When the same rupee amount is earned by selling one item at a profit and another at an equal percentage loss, the two cost prices are not equal, so work out each cost price separately before comparing totals.

• • •

2. In an examination, the average marks obtained by students who passed was $x\%$, while the average of those who failed was $y\%$. The average marks of all the students taking the exam was $z\%$. Find, in terms of x, y and z, the percentage of students taking the exam who failed.

(A) $\dfrac{z-x}{y-x}$

(B) $\dfrac{x-z}{y-z}$

(C) $\dfrac{y-x}{z-y}$

(D) $\dfrac{y-z}{x-z}$

Correct Answer: (A) $\dfrac{z-x}{y-x}$

SOLUTION:

Step 1: Recognize this as a mixture (alligation) problem.

The overall average $z\%$ is a mixture of two groups: the passed students averaging $x\%$ and the failed students averaging $y\%$. The rule of alligation says the ratio in which the two groups are mixed is the inverse of how far each group's average lies from the overall average.

Step 2: Write the alligation ratio.

$$\frac{\text{number who passed}}{\text{number who failed}} = \frac{y-z}{z-x}$$

This follows the standard alligation rule: the ratio of the two quantities mixed equals the ratio of the distances of the other quantity's average from the mean, taken in reverse order.

Step 3: Convert the ratio into a fraction of the whole class.

If passed : failed $= (y - z) : (z - x)$, the fraction of the whole class that failed is the failed part divided by the sum of both parts.

$$\text{fraction failed} = \frac{z - x}{(y - z) + (z - x)}$$

Step 4: Simplify the denominator.

$$(y - z) + (z - x) = y - x$$

The z terms cancel out, leaving $y - x$.

$$\text{fraction failed} = \frac{z - x}{y - x}$$

Step 5: Match with the answer choices.

Since x, y and z are already percentages, this fraction is directly the percentage who failed, $\frac{z - x}{y - x}$. None of the other three options simplify to this expression, since each one pairs the wrong pair of quantities together.

Final Answer:

The percentage of students who failed is $\frac{z - x}{y - x}$.

$$\boxed{\frac{z-x}{y-x}}$$

Quick Tip: Write the overall average as a weighted average of the pass and fail group averages, then solve that equation for the fail fraction.

• • •

3. Three circles A, B and C have a common centre O. A is the inner circle, B is the middle circle, and C is the outer circle. P is a point on the outer circle C, and the radius OP cuts the inner circle at X and the middle circle at Y, such that $OX = XY = YP$. The ratio of the area of the region between the inner and middle circles to the area of the region between the middle and outer circles is:

(A) $\frac{1}{3}$

(B) $\frac{2}{5}$

(C) $\frac{3}{5}$

(D) $\frac{1}{5}$

Correct Answer: (C) $\frac{3}{5}$

SOLUTION:

Step 1: Express the three radii as parts.

Because $OX = XY = YP$, the point X lies at 1 part from the centre, Y lies at 2 parts, and P lies at 3 parts, taking one segment length as one "part". So the radii of the inner, middle and outer circles are in the ratio $1 : 2 : 3$.

Step 2: Recall that the area of a circle scales with the square of its

radius.
Since area $= \pi r^2$, radii in ratio $1:2:3$ give areas in ratio $1^2:2^2:3^2$ $= 1:4:9$.

Step 3: Write the three circle areas using this ratio.
Let one area unit be k. Then:
area of inner circle $= k$
area of middle circle $= 4k$
area of outer circle $= 9k$

Step 4: Find the two ring areas from these values.
Ring between inner and middle circles $= 4k - k = 3k$.
Ring between middle and outer circles $= 9k - 4k = 5k$.

Step 5: Take the ratio asked in the question.

$$\frac{\text{ring (inner-middle)}}{\text{ring (middle-outer)}} = \frac{3k}{5k} = \frac{3}{5}$$

Working purely with the squared-ratio numbers $1:4:9$, without writing out πr^2 for an actual radius, gives the same result as computing the real areas, since k cancels out either way.

Final Answer:
The ratio of the two ring areas is $3:5$.

$$\boxed{\frac{3}{5}}$$

Quick Tip: Since OX = XY = YP, the three circles have radii in the ratio 1:2:3; use area = pi times radius squared for each ring and take the ratio.

4. The sides of a rhombus ABCD measure 2 cm each, and the difference between two of its angles is 90°. Then the area of the rhombus is:

(A) $\sqrt{2}$ sq cm

(B) $2\sqrt{2}$ sq cm

(C) $3\sqrt{2}$ sq cm

(D) $4\sqrt{2}$ sq cm

Correct Answer: (B) $2\sqrt{2}$ sq cm

SOLUTION:

Step 1: Find the two distinct angles of the rhombus.

Adjacent angles of a rhombus add up to 180°, so if one angle is θ, the next one is $180° - \theta$. Their difference is given as 90°:

$$(180° - \theta) - \theta = 90°$$

$$\theta = 45°$$

So the rhombus has angles of 45° and 135°.

Step 2: Recall how the diagonals relate to the side and the angle.

The diagonals of a rhombus bisect each other at right angles and also bisect the vertex angles. At the vertex with angle 45°, the diagonal through it splits that angle into two parts of 22.5° each. In the right triangle formed by half of each diagonal and a side of length 2 cm:

$$\frac{p}{2} = 2\sin(22.5°), \qquad \frac{q}{2} = 2\cos(22.5°)$$

where p and q are the lengths of the two diagonals.

Step 3: Write the diagonals explicitly.

$$p = 4\sin(22.5^\circ), \qquad q = 4\cos(22.5^\circ)$$

Step 4: Use the diagonal formula for the area of a rhombus.

$$\text{Area} = \frac{1}{2}pq = \frac{1}{2} \times 4\sin(22.5^\circ) \times 4\cos(22.5^\circ) = 8\sin(22.5^\circ)\cos(22.5^\circ)$$

Step 5: Apply the double angle identity to simplify.
Using $2\sin\alpha\cos\alpha = \sin(2\alpha)$ with $\alpha = 22.5^\circ$:

$$8\sin(22.5^\circ)\cos(22.5^\circ) = 4 \times \big(2\sin(22.5^\circ)\cos(22.5^\circ)\big) = 4\sin(45^\circ)$$

$$= 4 \times \frac{\sqrt{2}}{2} = 2\sqrt{2}$$

Step 6: Confirm against the options.
This diagonal-based route lands on exactly the same value as the direct side-angle area formula, confirming the area is $2\sqrt{2}$ sq cm and ruling out the other three options.

Final Answer:

$$\boxed{2\sqrt{2}\text{ sq cm}}$$

Quick Tip: A rhombus has only two distinct angles that add up to 180 degrees; use their given difference to find each one, then apply Area = side squared times sine of the angle.

• • •

5. If S_n denotes the sum of the first n terms of an Arithmetic Progression, and $S_1 : S_4 = 1 : 10$, then the ratio of the first term to the fourth term is:

(A) 1 : 3

(B) 2 : 3

(C) 1 : 4

(D) 1 : 5

Correct Answer: (C) 1 : 4

SOLUTION:

Step 1: Pick convenient numbers that satisfy the given ratio, instead of working with letters throughout.
We are told $S_1 : S_4 = 1 : 10$. Since S_1 is always just the first term, assume the first term is $a = 1$ so that $S_1 = 1$, and work out what d must be so that $S_4 = 10$.

Step 2: Write S_4 using the sum formula.

$$S_4 = \frac{4}{2}(2a + 3d) = 2(2(1) + 3d) = 2(2 + 3d) = 4 + 6d$$

Step 3: Set this equal to 10 and solve for d.

$$4 + 6d = 10$$

$$6d = 6$$

$$d = 1$$

So with $a = 1$, the common difference is also 1, giving the AP $1, 2, 3, 4, 5, \ldots$

Step 4: Check this AP against the original ratio.
$S_1 = 1$. $S_4 = 1 + 2 + 3 + 4 = 10$. This gives $S_1 : S_4 = 1 : 10$, confirming this AP fits the given condition.

Step 5: Read off the first and fourth terms directly from this AP.
First term $= 1$. Fourth term $= 4$.

$$\frac{T_1}{T_4} = \frac{1}{4}$$

Step 6: Confirm the answer holds in general, not just for this one example.
The relation $a = d$ found this way holds for any starting value of a, since scaling a scales d in the same proportion. So the ratio $1 : 4$ is the answer for every AP satisfying the given condition, not only the one picked here.

Final Answer:

$$\boxed{1:4}$$

Quick Tip: Write S1 = a and S4 = 4a + 6d using the AP sum formula, then use the given ratio S1:S4 = 1:10 to relate a and d.

• • •

6. The curve $y = 4x^2$ and $y^2 = 2x$ meet at the origin O and at a point P, forming a loop. The straight line OP divides the loop into two parts. What is the ratio of the areas of the two parts of the loop?

(A) 3 : 1

(B) 3 : 2

(C) 2 : 1

(D) 1 : 1

Correct Answer: (D) 1 : 1

SOLUTION:

Step 1: Find the point of intersection P of the two curves.

Substituting $y = 4x^2$ into $y^2 = 2x$ gives $16x^4 = 2x$, so $x(8x^3 - 1) = 0$, leading to $x = 0$ or $x = \frac{1}{2}$. At $x = \frac{1}{2}$, $y = 1$, so $P = (\frac{1}{2}, 1)$ and the line OP is $y = 2x$.

Step 2: Rescale the x-axis so the intersection point becomes a simple reference point.

Let $X = 2x$ and $Y = y$, so that $P = (\frac{1}{2}, 1)$ maps to $(X, Y) = (1, 1)$.

Rescaling one axis by a constant factor changes the size of a region but does not change the ratio of areas between two parts of the same region, so any area ratio found in the new coordinates is exactly the same as in the original ones.

Step 3: Rewrite the two curves and the line in the new coordinates.
Curve $y = 4x^2$ becomes $Y = 4\left(\frac{X}{2}\right)^2 = X^2$.
Curve $y^2 = 2x$ becomes $Y^2 = 2 \cdot \frac{X}{2} = X$, i.e. $X = Y^2$.
Line OP ($y = 2x$) becomes $Y = 2 \cdot \frac{X}{2} = X$, the simple diagonal line $Y = X$.

Step 4: Notice the mirror symmetry of the rescaled picture.
The loop is now bounded by $Y = X^2$ and $X = Y^2$ between $(0, 0)$ and $(1, 1)$. Swapping X and Y in $Y = X^2$ gives exactly $X = Y^2$, so these two curves are mirror images of each other across the line $Y = X$. Since the dividing line OP is precisely $Y = X$, the axis of this mirror symmetry, it must split the loop into two halves that are exact reflections of each other.

Step 5: Conclude the two areas are equal.
A shape and its mirror image always cover the same amount of area, since reflecting a region does not change its area. So the two parts of the loop, on either side of the line OP, must be equal in area.

$$\text{Area}_1 : \text{Area}_2 = 1 : 1$$

This symmetry argument reaches the same conclusion as direct integration, without needing to compute either integral explicitly.

Final Answer:

$$\boxed{1 : 1}$$

Quick Tip: Find where the two curves meet, write both curves as x in terms of y, then integrate to compare the area on each side of the chord OP.

• • •

7. How many numbers between 1 and 1000 (both excluded) are both squares and cubes?

(A) None

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

SOLUTION:

A number is a perfect square when it can be written as some whole number raised to the power 2, and a perfect cube when it can be written as some whole number raised to the power 3. A number that is both at once must be expressible as a whole number raised to a power that is a multiple of both 2 and 3, and the smallest such power is 6. So the real question is how many perfect sixth powers lie strictly between 1 and 1000.

1. **None**: This would only be correct if no sixth power fell inside the range 2 to 999. Checking a few small sixth powers quickly shows this is false.
2. **1**: This undercounts. It would be correct only if just one sixth power lay strictly between 1 and 1000, but there are in fact two, as the next point shows.

3. **2**: Testing $k = 2, 3, 4, \ldots$ in k^6 gives $2^6 = 64$, $3^6 = 729$, and $4^6 = 4096$. Both 64 and 729 fall strictly between 1 and 1000, while 4096 is too large. So exactly 2 numbers qualify.
4. **3**: This overcounts. There is no third sixth power between 729 and 1000, since the next one after 729, namely 4^6, is already 4096.

Checking each candidate on its own: $64 = 8^2 = 4^3$ and $729 = 27^2 = 9^3$, so both numbers are genuinely perfect squares and perfect cubes at the same time. That gives exactly 2 numbers in the open interval from 1 to 1000, which is option (C) by direct calculation.

Let's summarize:

- A number that is both a square and a cube must be a perfect sixth power.
- The sixth powers near this range are 1, 64, 729 and 4096; only 64 and 729 lie strictly between 1 and 1000.

The direct count gives 2 qualifying numbers, 64 and 729. Note that the answer key printed with this paper marks option (B) as correct for this question, which does not match the count found by testing every sixth power in range; this looks like an error in the source key rather than in the method shown above, and should be checked before publishing.

Quick Tip: A number that is both a square and a cube must be a perfect sixth power, since 6 is the lowest common multiple of 2 and 3; check which sixth powers fall strictly between 1 and 1000.

8. An operation $ is defined as follows. For any two positive integers x and y,

$$x\$y = \sqrt{\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}}}$$

Which of the following is an integer?

(A) 4$9

(B) 4$16

(C) 4$4

(D) None of the above

Correct Answer: (D) None of the above

SOLUTION:

Step 1: Rewrite the rule in one variable.

Let $r = x/y$. Then $y/x = 1/r$, so the operation becomes a function of just r:

$$x\$y = \sqrt{\sqrt{r} + \frac{1}{\sqrt{r}}}$$

This is a cleaner way to test all three cases, since we only need the ratio x/y for each pair, not the two numbers separately.

Step 2: Find r for each option.

For 4$9, $r = 4/9$.

For 4$16, $r = 4/16 = 1/4$.

For 4$4, $r = 4/4 = 1$.

Step 3: Evaluate the function at $r = 4/9$.
$\sqrt{r} = 2/3$ and $1/\sqrt{r} = 3/2$.
Sum $= 2/3 + 3/2 = 13/6$, which is not a perfect square of a rational number, since 13 is prime and does not divide 6. So $\sqrt{13/6}$ is irrational, and 4$9 fails.

Step 4: Evaluate the function at $r = 1/4$.
$\sqrt{r} = 1/2$ and $1/\sqrt{r} = 2$.
Sum $= 1/2 + 2 = 5/2$. Again 5 and 2 share no square factor, so $\sqrt{5/2}$ is irrational. 4$16 fails too.

Step 5: Evaluate the function at $r = 1$.
$\sqrt{r} = 1$ and $1/\sqrt{r} = 1$.
Sum $= 1 + 1 = 2$, and $\sqrt{2}$ is a classic irrational number. 4$4 fails as well.

Step 6: Conclude.
All three specific cases give an irrational square root, never a whole number. So none of the given options works, and the answer has to be none of the above.

$$\boxed{\text{None of the above}}$$

Quick Tip: Plug the given pairs into the formula one at a time and simplify the nested square roots fully before deciding if the result is a whole number.

9. If $f(x) = \cos(x)$ then the 50th derivative of $f(x)$ is:

(A) $\sin x$

(B) $-\sin x$

(C) $\cos x$

(D) $-\cos x$

Correct Answer: (D) $-\cos x$

SOLUTION:

Step 1: Use the general derivative formula for cosine.

Instead of listing derivatives one at a time, there is a direct formula for the nth derivative of $\cos x$:

$$\frac{d^n}{dx^n}\cos x = \cos\left(x + \frac{n\pi}{2}\right)$$

This formula comes from the fact that each derivative shifts the cosine graph by a quarter turn, a phase of $\pi/2$, which matches the 4 step repeating pattern that cosine derivatives follow.

Step 2: Plug in $n = 50$.

$$f^{(50)}(x) = \cos\left(x + \frac{50\pi}{2}\right) = \cos(x + 25\pi)$$

Step 3: Simplify the angle 25π.

Cosine repeats every 2π, so we only care about 25π modulo 2π.

$$25\pi = 12(2\pi) + \pi$$

So 25π and π point to the same position on the circle, meaning cos(x

$+ 25\pi) = \cos(x + \pi)$.

Step 4: Use the identity for $\cos(x + \pi)$.

Adding π to an angle flips the cosine to its negative:

$$\cos(x + \pi) = -\cos x$$

Step 5: State the result.

So $f^{(50)}(x) = -\cos x$, which matches the value found by tracking the 4 step cycle directly.

$$\boxed{f^{(50)}(x) = -\cos x}$$

Quick Tip: Derivatives of cos x repeat every 4 steps, so divide 50 by 4 and match the remainder to that step in the cycle.

• • •

10. If a, b and c are three real numbers, then which of the following is NOT true?

(A) $|a + b| \leq |a| + |b|$

(B) $|a - b| \leq |a| + |b|$

(C) $|a - b| \leq |a| - |b|$

(D) $|a - c| \leq |a - b| + |b - c|$

Correct Answer: (C) $|a - b| \leq |a| - |b|$

SOLUTION:

This question tests how well we know the triangle inequality and its variations. The triangle inequality for real numbers p and q says $|p + q| \leq |p| + |q|$, and every option here is some version of that idea dressed up differently. Let's go through the four options one by one.

1. $|a+b| \leq |a| + |b|$: this is the triangle inequality stated directly, with no disguise. It holds for all real a and b, so this option is true.
2. $|a-b| \leq |a| + |b|$: replace b with -b in the triangle inequality. Since $|-b| = |b|$, we get $|a + (-b)| \leq |a| + |-b|$, which is exactly $|a-b| \leq |a| + |b|$. This holds for all real a and b.
3. $|a-b| \leq |a| - |b|$: check this with a=1, b=2. The left side is $|1-2| = 1$. The right side is $|1| - |2| = 1 - 2 = -1$. The claim $1 \leq -1$ is false, so this option breaks down. The correct fact is the reverse triangle inequality, $|a-b| \geq ||a| - |b||$, which points the opposite way, so option (C) as written is simply not a true statement.
4. $|a-c| \leq |a-b| + |b-c|$: think of a, b, c as three points on the number line. The distance from a to c can never be more than going from a to b and then b to c. Writing $a - c = (a-b) + (b-c)$ and applying the triangle inequality confirms this is always true.

So three of the four options, (A), (B) and (D), are genuine inequalities that hold for every choice of real numbers, while (C) fails, as the counterexample a=1, b=2 shows directly.

Let's summarize:

- The triangle inequality $|p+q| \leq |p| + |q|$ is the base rule behind options (A), (B) and (D).
- The correct relation between $|a-b|$ and $|a|, |b|$ is $|a-b| \geq ||a| - |b||$, the reverse of what option (C) states.

So the statement that is NOT true is option (C).

Quick Tip: Test option (C) with small numbers like a = 1 and b = 2 to see it break, then confirm the other three follow directly from the triangle inequality.

• • •

11. If $R = \{(1,1),(2,2),(1,2),(2,1),(3,3)\}$ and $S = \{(1,1),(2,2),(2,3),(3,2),(3,3)\}$ are two relations on the set $X = \{1,2,3\}$, the incorrect statement is:

(A) R and S are both equivalence relations

(B) $R \cap S$ is an equivalence relation

(C) $R^{-1} \cap S^{-1}$ is an equivalence relation

(D) $R \cup S$ is an equivalence relation

Correct Answer: (D) $R \cup S$ is an equivalence relation

SOLUTION:

Step 1: Think of equivalence relations as partitions.

Every equivalence relation on a set splits that set into disjoint groups, called equivalence classes, and every such grouping (partition) gives back an equivalence relation. This is a cleaner lens to check the four options.

Step 2: Identify the partitions for R and S.

R relates 1 and 2 to each other and leaves 3 alone, so R corresponds to the partition $\{1,2\},\{3\}$. This is a valid partition of $X = \{1,2,3\}$, so R is an equivalence relation.

S relates 2 and 3 to each other and leaves 1 alone, so S corresponds to the partition $\{1\},\{2,3\}$. This is also a valid partition, so S is an equivalence relation too.

Step 3: Intersections correspond to a common refinement.
$R \cap S$ keeps only the pairs both relations agree on, which is just $\{(1,1),(2,2),(3,3)\}$, everyone only related to themselves. That is the partition $\{1\},\{2\},\{3\}$, still a valid partition, so $R \cap S$ is an equivalence relation.
Since both R and S are symmetric, reversing all their pairs changes nothing, so $R^{-1} \cap S^{-1}$ gives the exact same set and is an equivalence relation for the same reason.

Step 4: Unions do not correspond to a partition in general.
$R \cup S$ tries to keep 1 and 2 together (from R) and 2 and 3 together (from S) at the same time. But grouping 1 with 2, and 2 with 3, forces 1 and 3 into the same group too if the result is to be a genuine partition, and the pair $(1,3)$ is missing from $R \cup S$.
So $R \cup S$ does not match any valid partition of X, which means it cannot be an equivalence relation. Concretely, it fails transitivity since $(1,2)$ and $(2,3)$ are present but $(1,3)$ is not.

Step 5: Match this back to the four statements.
Statement (A), about R and S, is correct. Statements (B) and (C), about the intersections, are correct. Statement (D), claiming $R \cup S$ is an equivalence relation, is the one that is wrong.

$$\boxed{\text{The incorrect statement is (D)}}$$

Quick Tip: Check reflexive, symmetric and transitive for each combination separately; the union of two equivalence relations is not always transitive.

• • •

12. If $x > 8$ and $y > -4$, then which one of the following is always true?

(A) $xy < 0$

(B) $x^2 < -y$

(C) $-x < 2y$

(D) $x > y$

Correct Answer: (C) $-x < 2y$

SOLUTION:

Step 1: Picture the conditions on a number line.

x is any number strictly to the right of 8, so x can be 8.0001, 100, or a million. y is any number strictly to the right of -4, so y can be -3.999, 0, or a million as well. Since neither variable has an upper bound, any option involving a fixed relationship between the sizes of x and y needs to be checked at extreme values, not just typical ones.

Step 2: Break options (A) and (D) using large values.

For (A), $xy < 0$ needs one of x, y to be negative. But x is always positive here (since $x > 8 > 0$), and y is free to be positive too (say y = 1), which makes xy positive. So (A) fails for that choice.

For (D), $x > y$ needs y to stay smaller than x always. But since y has no upper limit, picking y = 1,000,000 with x = 9 breaks $x > y$ right away.

Step 3: Break option (B) using the lower bounds.

The smallest x^2 can get close to is $8^2 = 64$ (approached but never reached, since x is strictly greater than 8), and the largest $-y$ can get close to is 4 (as y approaches -4 from above). Since x^2 is always above 64 and $-y$ is always below 4, the required inequality $x^2 < -y$ can never be satisfied.

Step 4: Confirm option (C) using the extreme boundary values.

Push x to its smallest allowed value, just above 8, and y to its smallest allowed value, just above -4. Even in this tightest case, $-x$ sits just below -8, and $2y$ sits just above -8, keeping $-x < 2y$ true.

Now push x and y to be very large instead: $-x$ becomes a huge negative number, while $2y$ becomes a huge positive number, so $-x < 2y$ is even more clearly true.

Since the inequality survives both the tightest boundary case and the most extreme case, it holds everywhere in between as well.

Step 5: Conclude.

Only $-x < 2y$, option (C), survives every test, so it is the one that is always true.

$$\boxed{-x < 2y \text{ is always true}}$$

Quick Tip: Turn the two given conditions into -x less than -8 and 2y more than -8, then chain them to see which option they force to be true.

• • •

13. For $n = 1, 2, 3, \ldots$, let $T_n = 1^3 + 2^3 + \cdots + n^3$. Which one of the following statements is correct?

(A) There is no value of n for which T_n is a positive power of 2.

(B) There is exactly one value of n for which T_n is a positive power of 2.

(C) There are exactly two values of n for which T_n is a positive power of 2.

(D) There are more than two values of n for which T_n is a positive power of 2.

Correct Answer: (A) There is no value of n for which T_n is a positive power of 2.

SOLUTION:

Step 1: Write T_n using the sum of cubes formula.

$T_n = 1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$, so every T_n is a perfect square.

Powers of 2 that are also perfect squares are $1, 4, 16, 64, 256, \ldots$, that is, $2^0, 2^2, 2^4, 2^6, \ldots$ (even exponents only).

Step 2: Compute the first several values of T_n directly.

$T_1 = 1$

$T_2 = 1 + 8 = 9$

$T_3 = 9 + 27 = 36$

$T_4 = 36 + 64 = 100$

$T_5 = 100 + 125 = 225$

$T_6 = 225 + 216 = 441$

None of $9, 36, 100, 225, 441$ is a power of 2, since they equal $3^2, 6^2, 10^2, 15^2, 21^2$, and 3, 6, 10, 15, 21 are not powers of 2. Only $T_1 = 1$ $= 2^0$ shows up as a power of 2, and its exponent is 0.

Step 3: Explain why this pattern never changes for larger n.

T_n being a power of 2 needs $t_n = n(n+1)/2$ to be a power of 2, since $T_n = t_n^2$ and squares of powers of 2 are powers of 2, and conversely. n and n+1 never share a common factor, so if their product $n(n+1)$ $= 2t_n$ is to have only 2 as a prime factor, both n and n+1 must separately be powers of 2.

Powers of 2 spread out fast: after 1 and 2, the gaps are 2, 4, 8, 16 and so on, so no two powers of 2 beyond $\{1, 2\}$ are ever consecutive integers. This confirms $n = 1$ is the only place t_n can be a power of 2, and it only gives 2^0.

Step 4: Apply the positive power condition.

The question specifically asks for a positive power of 2, meaning 2^k with $k \geq 1$, values like 2, 4, 8, 16 and so on. Since the one case found, $T_1 = 1 = 2^0$, has exponent 0, it does not count.

No other n gets close, as shown by both the direct computation for small n and the coprimality argument for all n.

Step 5: State the conclusion.

So there is no n at all for which T_n works out to a positive power of 2.

$$\boxed{\text{No such n exists, so statement (A) is correct}}$$

Quick Tip: Use T_n = (n(n+1)/2)^2 and note that n and n+1 are coprime, so both would have to be powers of 2 themselves for T_n to be a power of 2.

• • •

14. An equilateral triangle is formed by joining the midpoints of the sides of a given equilateral triangle. A third equilateral triangle is formed inside the second equilateral triangle in the same way, and so on. If this process continues indefinitely, then the sum of the areas of all such triangles, when the side of the first triangle is 16 cm, is:

(A) $256\sqrt{3}$ sq cm

(B) $\frac{256}{3}\sqrt{3}$ sq cm

(C) $\frac{64}{3}\sqrt{3}$ sq cm

(D) $64\sqrt{3}$ sq cm

Correct Answer: (B) $\frac{256}{3}\sqrt{3}$ sq cm

SOLUTION:

Step 1: List the side lengths of the shrinking triangles.

Starting from a side of 16, joining midpoints repeatedly gives sides 16, 8, 4, 2, 1, and so on forever, each one exactly half of the last, because the segment joining the midpoints of two sides of a triangle is always half the third side, a direct result of the midpoint theorem.

Step 2: Compute the area of each triangle one by one using $A = \frac{\sqrt{3}}{4}a^2$.

$$A_1 = \frac{\sqrt{3}}{4}(16^2) = \frac{\sqrt{3}}{4}(256) = 64\sqrt{3}$$
$$A_2 = \frac{\sqrt{3}}{4}(8^2) = \frac{\sqrt{3}}{4}(64) = 16\sqrt{3}$$
$$A_3 = \frac{\sqrt{3}}{4}(4^2) = \frac{\sqrt{3}}{4}(16) = 4\sqrt{3}$$
$$A_4 = \frac{\sqrt{3}}{4}(2^2) = \frac{\sqrt{3}}{4}(4) = \sqrt{3}$$

Step 3: Notice the pattern between consecutive areas.

Dividing each area by the one before it: $16\sqrt{3}/64\sqrt{3} = 1/4$, and $4\sqrt{3}/16\sqrt{3} = 1/4$, and $\sqrt{3}/4\sqrt{3} = 1/4$ again. So each area is exactly one quarter of the one before it, confirming a geometric series with first term $64\sqrt{3}$ and common ratio $1/4$.

Step 4: Add up the series using the infinite sum formula.

For a geometric series with $|r| < 1$, the sum to infinity is $\frac{\text{first term}}{1 - r}$:

$$\text{Sum} = \frac{64\sqrt{3}}{1 - 1/4} = \frac{64\sqrt{3}}{3/4}$$

Step 5: Simplify the fraction.

Dividing by 3/4 is the same as multiplying by 4/3:

$$\frac{64\sqrt{3}}{3/4} = 64\sqrt{3} \times \frac{4}{3} = \frac{256\sqrt{3}}{3}$$

Step 6: State the final total.

So the combined area of every triangle in this never ending sequence adds up to $\frac{256}{3}\sqrt{3}$ sq cm, matching option (B).

$$\boxed{\frac{256\sqrt{3}}{3} \text{ sq cm}}$$

Quick Tip: Each new triangle has half the side and one quarter the area of the one before it, so sum the resulting infinite geometric series with first term 64 root 3 and ratio 1/4.

• • •

15. The length of the sides of a triangle are $x + 1$, $9 - x$ and $5x - 3$. The number of values of x for which the triangle is isosceles is:

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

SOLUTION:

Step 1: Understanding the Concept.
A triangle is isosceles when two of its three sides are equal, but before that we always need the three given lengths to actually close into a triangle in the first place. That means every pair of sides must add up to more than the remaining side. We test each of the three possible equal pair equations one at a time and then filter out any case that cannot physically form a triangle.

Step 2: List all pairings and solve each.
Pairing 1: $x + 1 = 9 - x$ gives $2x = 8$, so $x = 4$.
Pairing 2: $x + 1 = 5x - 3$ gives $4x = 4$, so $x = 1$.
Pairing 3: $9 - x = 5x - 3$ gives $6x = 12$, so $x = 2$.
So algebraically there seem to be three candidate values: 1, 2 and 4.

Step 3: Test each candidate against the triangle rule.
For $x = 4$: sides are $5, 5, 17$. Smallest two add to 10, and $10 < 17$, so this figure is impossible. Two short sides of length 5 simply cannot stretch across a base of 17.
For $x = 1$: sides are $2, 8, 2$. Smallest two add to 4, and $4 < 8$, so this is impossible too.
For $x = 2$: sides are $3, 7, 7$. Check all pairs: $3 + 7 = 10 > 7$ and $7 + 7 = 14 > 3$. Every pair clears the third side, so this triangle is valid.

Step 4: Final Answer.
Only the pairing that gives $x = 2$ survives the triangle test, since the other two pairings produce side lengths that cannot form any triangle at all, let alone an isosceles one. So there is only one value of x that makes the triangle isosceles.

$$\boxed{1}$$

Quick Tip: First equate each pair of sides to find candidate x-values, then check the triangle inequality on each candidate since not every algebraic solution gives real side lengths.

• • •

16. The expression $\dfrac{x^2 - 2x + a^2 + b^2}{x^2 + 2x + a^2 + b^2}$ lies between:

(A) $\dfrac{\sqrt{a^2+b^2}+1}{\sqrt{a^2+b^2}-1}$ and $\dfrac{\sqrt{a^2+b^2}-1}{\sqrt{a^2+b^2}+1}$

(B) a and b

(C) $\dfrac{\sqrt{a^2+b^2}+1}{\sqrt{a^2+b^2}-1}$ and 1

(D) $\dfrac{\sqrt{a^2+b^2}-1}{\sqrt{a^2+b^2}+1}$

Correct Answer: (A) $\dfrac{\sqrt{a^2+b^2}+1}{\sqrt{a^2+b^2}-1}$ and $\dfrac{\sqrt{a^2+b^2}-1}{\sqrt{a^2+b^2}+1}$

SOLUTION:

Step 1: Understanding the Concept.

Instead of jumping straight to a discriminant, notice a neat symmetry in this expression first. Write $k = \sqrt{a^2+b^2}$ and $f(x) = \dfrac{x^2 - 2x + k^2}{x^2 + 2x + k^2}$. Replacing x by $-x$ swaps the numerator and denominator of f, so $f(-x) = \dfrac{1}{f(x)}$. This tells us the set of values f can take must be symmetric under taking the reciprocal, which already hints that the two boundary values of the range are reciprocals of each other.

Step 2: Key Formula or Approach.

To pin down the actual boundary values, treat $y = f(x)$ and rewrite the equation as a quadratic in x:

$$(y-1)x^2 + 2(y+1)x + (y-1)k^2 = 0$$

For a real x to exist, the discriminant of this quadratic (in x) must be non-negative.

Step 3: Detailed Explanation.
Discriminant condition:

$$4(y+1)^2 - 4k^2(y-1)^2 \geq 0$$

$$(y+1)^2 \geq k^2(y-1)^2$$

This reduces cleanly to the factored inequality

$$[y(1-k) + (1+k)]\,[y(1+k) + (1-k)] \geq 0$$

Solving $y(1-k) + (1+k) = 0$ gives $y = \dfrac{k+1}{k-1}$.
Solving $y(1+k) + (1-k) = 0$ gives $y = \dfrac{k-1}{k+1}$.
These two values are reciprocals of each other, exactly matching the symmetry noticed in Step 1. Since the leading coefficient of the quadratic in y is negative (because $k > 1$ makes $1 - k^2 < 0$), the expression is non-negative between its two roots, so y is confined between $\dfrac{k-1}{k+1}$ and $\dfrac{k+1}{k-1}$.

Step 4: Final Answer.
Substituting back $k = \sqrt{a^2 + b^2}$, the expression always lies between

$\frac{\sqrt{a^2+b^2}-1}{\sqrt{a^2+b^2}+1}$ and $\frac{\sqrt{a^2+b^2}+1}{\sqrt{a^2+b^2}-1}$, a reciprocal pair of bounds, confirming the symmetry argument from the start.

$$\boxed{\frac{\sqrt{a^2+b^2}-1}{\sqrt{a^2+b^2}+1} \le y \le \frac{\sqrt{a^2+b^2}+1}{\sqrt{a^2+b^2}-1}}$$

Quick Tip: Treat the expression as y, cross multiply to get a quadratic in x, and use the real-roots (discriminant greater than or equal to 0) condition to find the range of y.

• • •

17. What is the sum of the first 100 terms which are common to both the progressions $17, 21, 25, \ldots$ and $16, 21, 26, \ldots$?

(A) 100000

(B) 101100

(C) 111000

(D) 100110

Correct Answer: (B) 101100

SOLUTION:

Step 1: Understanding the Concept.

A number that lies in both progressions must satisfy both general term formulas at once. Instead of listing terms and spotting a pattern, we can set the two general terms equal and study what that forces.

Step 2: Key Formula or Approach.

First progression: term $= 17 + 4(p-1) = 4p + 13$ for $p = 1, 2, 3, \ldots$

Second progression: term $= 16 + 5(q - 1) = 5q + 11$ for $q = 1, 2, 3, \ldots$

A common term needs $4p + 13 = 5q + 11$, that is $4p + 2 = 5q$, so $q = \frac{4p+2}{5}$. Since q must be a whole number, $4p + 2$ must divide evenly by 5.

Step 3: Detailed Explanation.

Check $p = 1, 2, 3, \ldots$ for when $4p + 2$ is divisible by 5:

$p = 1 : 6$, not divisible.

$p = 2 : 10$, divisible by 5. This gives the term $4(2) + 13 = 21$.

The pattern of valid p repeats every 5 steps, since $4p + 2 \pmod 5$ cycles with period 5. The next valid value is $p = 7$, giving $4(7) + 13 = 41$.

So valid p values are $2, 7, 12, 17, \ldots$, an arithmetic sequence with common difference 5 in p, which through $4p + 13$ turns into a common difference of $4 \times 5 = 20$ in the actual term values. This matches the two step sizes 4 and 5 combining into $\text{lcm}(4, 5) = 20$, confirming the common terms form an AP with common difference 20 starting at 21.

Step 4: Final Answer.

The common-term sequence is $21, 41, 61, \ldots$ with first term 21 and common difference 20. Summing 100 terms:

$$S_{100} = \frac{100}{2}[2(21) + 99(20)] = 50(42 + 1980) = 50 \times 2022 = 101100$$

So the required sum is 101100, matching the pattern-based approach.

$$\boxed{101100}$$

Quick Tip: Common terms of two APs also form an AP, whose common difference is the LCM of the two original common differences.

• • •

18. Two people agree to meet on January 9, 2005 between 6:00 P.M. and 7:00 P.M., with the understanding that each will wait no longer than 20 minutes for the other. What is the probability that they will meet?

(A) $\frac{5}{9}$
(B) $\frac{7}{9}$
(C) $\frac{2}{9}$
(D) $\frac{4}{9}$

Correct Answer: (A) $\frac{5}{9}$

SOLUTION:

Step 1: Understanding the Concept.
This is a geometric probability problem: since arrival times are continuous and random over an hour, we cannot count outcomes one by one. Instead we compare areas on a square grid of possible arrival-time pairs.

Step 2: Key Formula or Approach.
Let x and y be the minutes past 6:00 P.M. at which the two people show up, each uniform on $[0, 60]$. Plot all possible pairs (x, y) as points inside a 60×60 square, giving total area 3600. They meet exactly when $|x - y| \leq 20$, which is the band around the diagonal $x = y$. Instead of shading that band directly, it is simpler to first shade the two corner triangles where they miss each other, since those have a clean right-

triangle shape.

Step 3: Detailed Explanation.

The miss each other region is $x - y > 20$ or $y - x > 20$.

For $x - y > 20$: the leftover corner beyond the line $x - y = 20$ is a right triangle with legs of length $60 - 20 = 40$ each.

Area of this triangle $= \frac{1}{2} \times 40 \times 40 = 800$.

The mirror-image triangle for $y - x > 20$ has the same area, 800, by the symmetry of the square.

Total miss area $= 800 + 800 = 1600$, so miss probability $= \frac{1600}{3600} = \frac{4}{9}$.

The meeting probability is everything else in the square:

$$P(\text{meet}) = 1 - \frac{4}{9} = \frac{5}{9}$$

Step 4: Final Answer.

Working from the complement confirms the same result as working with the meeting band directly: the two people meet with probability $\frac{5}{9}$.

$$\boxed{\frac{5}{9}}$$

Quick Tip: Model both arrival times as points in a 60 by 60 square and find the area where the gap between them is at most 20 minutes.

• • •

19. If the roots of the equation $\frac{x+a}{x+a+c} + \frac{x+b}{x+b+c} = 1$ are equal in magnitude but opposite in sign, then:

(A) $c \geq a$

(B) $a \geq c$

(C) $a + b = 0$

(D) $a = b$

Correct Answer: (C) $a + b = 0$

SOLUTION:

Step 1: Understanding the Concept.

Roots equal in magnitude but opposite in sign is a special condition on a quadratic equation: it always means the coefficient of x in the quadratic must vanish, because the sum of such roots is zero, and the sum of roots equals minus the coefficient of x once the quadratic is written with leading coefficient 1. So the real task is just to convert the given fractional equation into a plain quadratic in x and read off when that middle coefficient is zero.

Step 2: Key Formula or Approach.

Start directly from

$$\frac{x+a}{x+a+c} + \frac{x+b}{x+b+c} = 1$$

Bring the left side to a common denominator $(x+a+c)(x+b+c)$:

$$(x+a)(x+b+c) + (x+b)(x+a+c) = (x+a+c)(x+b+c)$$

Step 3: Detailed Explanation.

Expand the left side term by term:

$(x+a)(x+b+c) = x^2 + (a+b+c)x + ab + ac$

$(x+b)(x+a+c) = x^2 + (a+b+c)x + ab + bc$
Adding these: $2x^2 + 2(a+b+c)x + 2ab + ac + bc$.
Now expand the right side: $(x+a+c)(x+b+c) = x^2 + (a+b+2c)x + ab + ac + bc + c^2$.
Setting left equal to right and moving everything to one side:

$$2x^2 + 2(a+b+c)x + 2ab + ac + bc - x^2 - (a+b+2c)x - ab - ac - bc - c^2 = 0$$

$$x^2 + (a+b)x + ab - c^2 = 0$$

The extra c terms in the middle coefficient cancel to leave just $a+b$, and the constants simplify to $ab - c^2$. This matches the same reduced quadratic found through substitution, which is a good check that the algebra is consistent.

Step 4: Final Answer.
For the two roots of $x^2 + (a+b)x + (ab - c^2) = 0$ to be equal in size but opposite in sign, their sum must be zero. Since the sum of the roots is $-(a+b)$, we need

$$a + b = 0$$

This is the required condition.

$$\boxed{a + b = 0}$$

Quick Tip: Clear denominators to get a plain quadratic in x, then use the fact that roots equal in magnitude but opposite in sign must sum to zero.

20. Steel Express runs between Tatanagar and Howrah and has five stoppages in between. Find the number of different kinds of one-way second class tickets that Indian Railways will have to print to service all types of passengers who might travel by Steel Express.

(A) 49

(B) 42

(C) 21

(D) 7

Correct Answer: (B) 42

SOLUTION:

Step 1: Understanding the Concept.
A train ticket names one station as the start and a different station as the end. Two stations can be paired for travel in two distinct ways depending on which one is the origin, so this is really a two-part counting question: first count how many station-pairs exist, then account for direction.

Step 2: Key Formula or Approach.
Total stations $= 2$ terminal stations $+5$ stoppages $= 7$ stations.
The number of ways to pick any 2 different stations out of 7, without worrying about order, is given by the combination formula:

$$\binom{7}{2} = \frac{7 \times 6}{2 \times 1} = 21$$

Step 3: Detailed Explanation.

Each unordered pair of stations, say $\{P, Q\}$, actually needs two separate tickets printed: one for travel from P to Q, and a separate one for travel from Q to P, since these are different journeys with different origin and destination printed on the ticket. So the true number of distinct tickets is double the number of unordered pairs:

$$21 \times 2 = 42$$

Step 4: Final Answer.
Counting unordered station-pairs and then doubling for direction gives the same result as directly counting ordered pairs: 42 different kinds of tickets are needed.

$$\boxed{42}$$

Quick Tip: Count the total stations (2 end points plus 5 stops), then count ordered pairs of stations since a ticket from P to Q differs from one from Q to P.

• • •

21. The horizontal distance of a kite from the boy flying it is 30 m and 50 m of cord is out from the roll. If the wind moves the kite horizontally at the rate of 5 km per hour directly away from the boy, how fast is the cord being released?

(A) 3 km per hour

(B) 4 km per hour

(C) 5 km per hour

(D) 6 km per hour

Correct Answer: (A) 3 km per hour

SOLUTION:

Step 1: Understanding the Concept.

This is a related-rates problem: two quantities, the horizontal distance x and the cord length L, are both changing with time, but they stay linked by a fixed right triangle because the kite's height never changes as the wind blows it away only horizontally. Once we know that fixed height, we can connect the rate of change of x to the rate of change of L using the triangle's angle instead of differentiating the Pythagorean relation directly.

Step 2: Key Formula or Approach.

First find the fixed height using the 30-40-50 right triangle: since $50^2 - 30^2 = 2500 - 900 = 1600 = 40^2$, the height is $h = 40$ m.
Let θ be the angle the cord makes with the ground, so $\cos\theta = \frac{x}{L}$ (adjacent over hypotenuse) at any instant. At the given instant, $\cos\theta = \frac{30}{50} = \frac{3}{5}$.

Step 3: Detailed Explanation.

Because h is fixed, the rate at which the cord length grows is the horizontal rate scaled down by the cosine of the angle the cord makes with the ground:

$$\frac{dL}{dt} = \cos\theta \cdot \frac{dx}{dt}$$

This matches the usual $\frac{dL}{dt} = \frac{x}{L}\frac{dx}{dt}$ relation since $\cos\theta = x/L$.
Substituting the known values:

$$\frac{dL}{dt} = \frac{3}{5} \times 5 = 3$$

Step 4: Final Answer.

Using the angle of the cord with the ground instead of direct implicit differentiation gives the same result: the cord is let out at 3 km per hour.

$$\boxed{3 \text{ km per hour}}$$

Quick Tip: Find the fixed height using the 30-40-50 right triangle, then differentiate L squared = x squared + h squared with respect to time to relate dL/dt to dx/dt.

• • •

22. Suppose S and T are sets of vectors, where $S = \{(1, 0, 0), (0, 0, -5), (0, 3, 4)\}$ and $T = \{(5, 2, 3), (5, -3, 4)\}$, then:

(A) S and T both sets are linearly independent vectors

(B) S is a set of linearly independent vectors, but T is not

(C) T is a set of linearly independent vectors, but S is not

(D) Neither S nor T is a set of linearly independent vectors

Correct Answer: (A) S and T both sets are linearly independent vectors

SOLUTION:

Here is a different way to reach the same result, by solving the defining equations directly instead of computing a determinant.

1. **Set up the homogeneous system for S**: for $S = \{(1,0,0),(0,0,-5),(0,3,4)\}$, write $a(1,0,0) + b(0,0,-5) + c(0,3,4) = (0,0,0)$. Adding the components gives the vector $(a, 3c, -5b + 4c)$, so the equations are $a = 0$, $3c = 0$, and $-5b + 4c = 0$.
2. **Solve this system**: from $a = 0$ and $3c = 0$ we get $a = 0, c = 0$. Putting $c = 0$ into the third equation gives $-5b = 0$, so $b = 0$. The only solution is $a = b = c = 0$, the trivial one, which means S is linearly independent.
3. **Check T using ratios**: for $T = \{(5,2,3),(5,-3,4)\}$, two vectors are dependent only when every pair of matching coordinates is in the same ratio. Compare $5/5 = 1$, $2/(-3) \approx -0.67$, and $3/4 = 0.75$. Since these three ratios are not all equal, the vectors cannot be scalar multiples of each other.
4. **Conclude for T**: because no single scalar relates the two vectors, T is also linearly independent.

Both the system-of-equations check for S and the ratio check for T confirm that neither set contains a vector that depends on the others. So both S and T qualify as linearly independent sets.

Let's summarize:

- Solving $a(1,0,0) + b(0,0,-5) + c(0,3,4) = 0$ forces $a = b = c = 0$, so S is independent.
- The coordinate ratios of the two vectors in T are not all equal, so T is independent too.

So the correct choice is that both S and T are linearly independent, matching option (A).

Quick Tip: Check if the determinant formed by the three vectors of S is nonzero, and check whether either vector of T is a scalar multiple of the other.

• • •

23. Suppose the function f satisfies the equation $f(x+y) = f(x)f(y)$ for all x and y. Here $f(x) = 1 + xg(x)$, where $\lim_{x \to 0} g(x) = T$, and T is a positive integer. If $f^n(x) = kf(x)$, where $f^n(x)$ denotes the nth derivative of f, then k is equal to:

(A) T

(B) T^n

(C) $\log T$

(D) $(\log T)^n$

Correct Answer: (B) T^n

SOLUTION:

Here is a different way to reach the same result, using induction instead of solving the differential equation outright.

1. **Find f(0)**: putting $x = y = 0$ into $f(x+y) = f(x)f(y)$ gives $f(0) = f(0)^2$, so $f(0) = 0$ or $f(0) = 1$. Since $f(x) = 1 + xg(x)$ forces $f(0) = 1$, we keep $f(0) = 1$.
2. **Find f'(0)**: from $f(x) = 1 + xg(x)$, for small h we get $f(h) - 1 = hg(h)$, so $\frac{f(h)-1}{h} = g(h)$. Taking $h \to 0$ gives $f'(0) = \lim_{h \to 0} g(h) = T$. This is the instantaneous rate of change of f right at $x = 0$.
3. **Show f'(x) = Tf(x) everywhere**: differentiate $f(x+h) = f(x)f(h)$ with respect to h and then set $h = 0$. This gives $f'(x) = f(x)f'(0) = Tf(x)$ for every x, not only at $x = 0$.
4. **Induction base case**: for $n = 1$, $f'(x) = Tf(x)$ is already shown above, so $k = T^1 = T$ works for $n = 1$.

5. **Induction step**: assume $f^{(n)}(x) = T^n f(x)$ holds for some n. Differentiating both sides gives $f^{(n+1)}(x) = T^n f'(x) = T^n \cdot T f(x) = T^{n+1} f(x)$, so the formula also holds for $n + 1$.

By induction, $f^{(n)}(x) = T^n f(x)$ for every positive integer n. Comparing with the given relation $f^n(x) = k f(x)$, we read off $k = T^n$.

Let's summarize:

- The functional equation plus the given form of f pin down $f'(0) = T$.
- The same functional equation forces $f'(x) = T f(x)$ at every point, not just at 0.
- Repeating differentiation n times multiplies in a factor of T each time, giving $k = T^n$.

So $k = T^n$, matching option (B). A logarithm of T never enters this relation; that only appears if you try to solve for $f(x)$ itself, which is a different quantity from k.

Quick Tip: Differentiate the functional equation to show f'(x) = Tf(x), then repeat this n times to find the nth derivative in terms of f(x).

• • •

24. Set of real numbers 'x, y', satisfying the inequations $x - 3y \geq 0$, $x + y \geq -2$ and $3x - y \leq -2$ is:

(A) Empty

(B) Finite

(C) Infinite

(D) Cannot be determined

Correct Answer: (C) Infinite

SOLUTION:

Step 1: Turn each inequality into a bound on y.

Rewrite each condition so y is isolated:

From $x - 3y \geq 0$: $y \leq \frac{x}{3}$.

From $x + y \geq -2$: $y \geq -2 - x$.

From $3x - y \leq -2$: $y \geq 3x + 2$.

So for a given x, y must satisfy $\max(-2 - x,\ 3x + 2) \leq y \leq \frac{x}{3}$.

Step 2: Find which of the two lower bounds is larger, depending on x.

Compare $-2 - x$ and $3x + 2$: $-2 - x \geq 3x + 2 \iff -4 \geq 4x \iff x \leq -1$.

So for $x \leq -1$, the lower bound is $-2 - x$; for $x > -1$, the lower bound is $3x + 2$.

Step 3: Find the range of x for which the lower bound does not exceed the upper bound.

For $x \leq -1$: need $-2 - x \leq \frac{x}{3}$, which gives $-6 - 3x \leq x$, so $x \geq -\frac{3}{2}$.

Combined with $x \leq -1$, this gives $-\frac{3}{2} \leq x \leq -1$.

For $x > -1$: need $3x + 2 \leq \frac{x}{3}$, which gives $9x + 6 \leq x$, so $x \leq -\frac{3}{4}$.

Combined with $x > -1$, this gives $-1 < x \leq -\frac{3}{4}$.

Putting both pieces together, x ranges over the interval $\left[-\frac{3}{2}, -\frac{3}{4}\right]$, which is a proper interval containing infinitely many real values, not a single point.

Step 4: Check that each such x gives a real range of y, not just a single value.

Take $x = -1.2$, which lies inside the interval: lower bound $= -2 - (-1.2) = -0.8$, upper bound $= \frac{-1.2}{3} = -0.4$. Since $-0.8 < -0.4$, y can be any of the infinitely many real numbers between -0.8 and -0.4, and each choice gives a valid (x, y) pair.

Step 5: Conclude.

Since there is a whole interval of valid x values, and for each of these an entire interval of valid y values, the total set of pairs (x, y) satisfying all three inequalities is a two-dimensional region, not empty and not a finite list of isolated points.

$$\boxed{\text{The solution set is infinite}}$$

Quick Tip: Convert each inequality into a bound on y in terms of x, then check whether the resulting range of valid x and y values is empty, a single point, or a full region.

• • •

25. $ABCD$ is a trapezium, such that AB, DC are parallel and BC is perpendicular to them. If $\angle DAB = 45°$, $BC = 2$ cm and $CD = 3$ cm, then AB =?

(A) 5 cm

(B) 4 cm

(C) 3 cm

(D) 2 cm

Correct Answer: (A) 5 cm

SOLUTION:

Here is a more direct geometric way to solve this, without using coordinates.

Since DC is parallel to AB and BC is perpendicular to both, $ABCD$ is a right trapezium with right angles at B and C. Drop a perpendicular from D down to side AB, meeting it at point E. Since DC is parallel to AB and BC is already perpendicular to both, the figure $BCDE$ is a rectangle: $BE = CD = 3$ cm and $DE = BC = 2$ cm.

Now look at triangle ADE. Since E lies on AB between A and B (because AB is longer than CD), $AE = AB - BE = AB - 3$. The angle $\angle DAE$ is the same as $\angle DAB = 45°$, since E sits on segment AB.

Triangle ADE has a right angle at E, because DE is perpendicular to AB, being a side of rectangle $BCDE$. With a right angle at E and an acute angle of 45° at A, the third angle at D must also be 45°. This makes triangle ADE an isosceles right triangle, so its two legs are equal: $AE = DE$.

Since $DE = 2$ cm, the height of the trapezium, we get $AE = 2$ cm as well.

Let's summarize:

- $BE = CD = 3$ cm, being opposite sides of rectangle $BCDE$.
- $AE = DE = 2$ cm, because triangle ADE is a 45-45-90 right triangle.
- $AB = AE + EB = 2 + 3 = 5$ cm.

So $AB = 5$ cm, confirming option (A), and matching the coordinate-based calculation using a completely different, purely geometric route.

Quick Tip: Drop a perpendicular from D to AB and use the resulting 45-45-90 right triangle together with the rectangle formed by BC, CD, and part of AB.

• • •

26. If F is a differentiable function such that $F(3) = 6$ and $F(9) = 2$, then there must exist at least one number 'a' between 3 and 9, such that:

(A) $F'(a) = \frac{3}{2}$

(B) $F(a) = -\frac{3}{2}$

(C) $F'(a) = -\frac{3}{2}$

(D) $F'(a) = -\frac{2}{3}$

Correct Answer: (D) $F'(a) = -\frac{2}{3}$

SOLUTION:

Let's think about this using the geometric picture behind the Mean Value Theorem, instead of just quoting the formula.

Picture the graph of F between $x = 3$ and $x = 9$. We know the graph passes through the point $(3, 6)$ and the point $(9, 2)$. Draw the straight line joining these two points; this is called the chord. Its slope is the average rate at which F changes over this stretch.

1. **Slope of the chord**: rise over run gives $\frac{2-6}{9-3} = \frac{-4}{6} = -\frac{2}{3}$. This single number describes how the two endpoints compare, regardless of what F does in between.
2. **What LMVT adds**: because F is differentiable everywhere on $(3, 9)$, so its graph has no corners or breaks, somewhere along that stretch the curve must have a tangent line exactly parallel to this chord, meaning the same slope as the chord.
3. **Why this must happen**: if the curve's slope were always steeper than the chord's slope, F would change faster than the chord the whole way and overshoot the endpoint value; if it were always shallower, it would undershoot. So the curve's slope has to match the chord's slope at least once strictly inside the interval.

Since the tangent slope at that point a is $F'(a)$, and we found the chord's slope is $-\frac{2}{3}$, we get $F'(a) = -\frac{2}{3}$.

Let's summarize:

- The chord joining $(3,6)$ and $(9,2)$ has slope $-\frac{2}{3}$.
- LMVT guarantees a point where the tangent to F is parallel to this chord.
- Options naming a function value instead of a derivative, or giving a different number, do not match this slope.

So the correct statement is $F'(a) = -\frac{2}{3}$, which is option (D).

Quick Tip: Use the Lagrange Mean Value Theorem: the derivative at some point between 3 and 9 must equal the average rate of change (F(9)-F(3))/(9-3).

• • •

27. A conical tent of given capacity has to be constructed. The ratio of the height to the radius of the base for the minimum amount of canvas required for the tent is:

(A) $1:2$

(B) $2:1$

(C) $1:\sqrt{2}$

(D) $\sqrt{2}:1$

Correct Answer: (D) $\sqrt{2}:1$

SOLUTION:

Here is another way to solve this, by working with the ratio $k = h/r$ directly instead of minimising in terms of r first.

Let $k = \frac{h}{r}$, so $h = kr$. The volume is fixed: $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3 k$, which gives $r^3 = \frac{3V}{\pi k}$.

The curved surface area is $S = \pi r l = \pi r\sqrt{r^2 + h^2} = \pi r^2\sqrt{1 + k^2}$, using $h = kr$.

Now express r^2 in terms of k: from $r^3 = \frac{3V}{\pi k}$, we get $r = \left(\frac{3V}{\pi k}\right)^{1/3}$, so $r^2 = \left(\frac{3V}{\pi k}\right)^{2/3}$.

Substituting into S:

1. $S = \pi\left(\frac{3V}{\pi k}\right)^{2/3}\sqrt{1 + k^2}$.
2. Since V is fixed, minimising S over k is the same as minimising $f(k) = k^{-2/3}\sqrt{1 + k^2}$, because the other factors involving V and π do not depend on k.
3. Write $f(k) = k^{-2/3}(1 + k^2)^{1/2}$ and take the logarithm to make differentiation easier: $\ln f(k) = -\frac{2}{3}\ln k + \frac{1}{2}\ln(1 + k^2)$.

Differentiate both sides with respect to k, using logarithmic differentiation, $\frac{f'(k)}{f(k)} = \frac{d}{dk}[\ln f(k)]$:

$$\frac{f'(k)}{f(k)} = -\frac{2}{3k} + \frac{k}{1 + k^2}$$

Set this to zero, since $f(k) \neq 0$, the bracket must vanish:

$$-\frac{2}{3k} + \frac{k}{1 + k^2} = 0 \implies \frac{k}{1 + k^2} = \frac{2}{3k} \implies 3k^2 = 2(1 + k^2) \implies 3k^2 - 2k^2 = 2 \implies k^2 = 2$$

So $k = \sqrt{2}$, taking the positive root since $k = h/r$ must be positive.

Let's summarize:

- Working directly with the ratio $k = h/r$ turns this into a one-variable minimisation.

- Logarithmic differentiation avoids messy product and chain rule steps.
- Both this method and the direct r-based method agree: $k = h/r = \sqrt{2}$.

So the ratio of height to radius is $\sqrt{2} : 1$, confirming option (D).

Quick Tip: Fix the volume, write the curved surface area as a function of one variable, either r or the ratio h/r, and minimise it using calculus.

• • •

28.

If n is a positive integer, let $S(n)$ denote the sum of the positive divisors of n, including n itself, and let $G(n)$ be the greatest divisor of n. If $H(n) = \frac{G(n)}{S(n)}$, then which of the following is the largest?

(A) $H(2009)$

(B) $H(2010)$

(C) $H(2011)$

(D) $H(2012)$

Correct Answer: (C) $H(2011)$

SOLUTION:

Step 1: Rewrite H(n) using a reciprocal trick.

The greatest divisor of n is n itself, so $G(n) = n$ and $H(n) = n/S(n)$.

Divide the numerator and denominator by n:

$$H(n) = \frac{1}{S(n)/n} = \frac{1}{\sum_{d|n} \frac{1}{d}}$$

since if d runs over every divisor of n, so does n/d, which means $S(n)/n$ is just the sum of $1/d$ over every divisor d of n. So H(n) is largest exactly when this reciprocal sum is smallest.

Step 2: Figure out which kind of number makes that reciprocal sum small.
Every number has 1 and itself as divisors, contributing $1 + 1/n$ to the sum. Any extra divisor d with $1 < d < n$ adds a further positive term $1/d$ on top of that. A number with no divisors besides 1 and itself, that is, a prime number, has the smallest possible reciprocal sum. Composite numbers always add extra positive terms and push the sum up.

Step 3: Check which of 2009, 2010, 2011, 2012 is prime.
2010 is even, so it is divisible by 2 right away, hence composite.
2009 factors as $7 \times 287 = 7 \times 7 \times 41$, so it is composite.
2012 is even too, in fact $2012 = 4 \times 503$, so it is composite.
2011 is odd, its digit sum is 4 so it is not divisible by 3, it does not end in 0 or 5, and testing every prime up to $\sqrt{2011} \approx 44.8$ (7, 11, 13, up to 43) leaves no exact division, so 2011 is prime.

Step 4: Conclude directly from the reciprocal-sum rule.
Since 2011 is prime, its reciprocal sum is just $1 + \frac{1}{2011} \approx 1.0005$, the smallest of the four choices by a wide margin. 2009, 2010 and 2012 are all composite and carry extra divisors like 2, 3, 7 or 41, each adding a noticeably larger extra term (for example $1/2 = 0.5$ for 2010, or $1/7 \approx 0.143$ for 2009), so their reciprocal sums sit well above 1.0005.

Final Answer:

The smallest reciprocal-divisor sum belongs to 2011, so H(2011) is the largest of the four.

$$\boxed{\text{Option (C): } H(2011)}$$

Quick Tip: The greatest divisor of any n is n itself, so H(n) = n/S(n) is largest when n has the fewest extra divisors, which happens when n is prime.

• • •

29.

If the ratio of the roots of the equation $x^2 - 2ax + b = 0$ is equal to that of the roots of the equation $x^2 - 2cx + d = 0$, then:

(A) $a^2b = c^2d$

(B) $a^2c = b^2d$

(C) $a^2d = c^2b$

(D) $d^2b = c^2a$

Correct Answer: (C) $a^2d = c^2b$

SOLUTION:

Step 1: Write the roots using the quadratic formula.
For $x^2 - 2ax + b = 0$, the roots are $a + \sqrt{a^2 - b}$ and $a - \sqrt{a^2 - b}$. Call the ratio of the roots λ, taking the larger root over the smaller one:

$$\lambda = \frac{a + \sqrt{a^2 - b}}{a - \sqrt{a^2 - b}}$$

Step 2: Apply componendo and dividendo.

This is a standard trick for ratios written as (sum)/(difference): if $\dfrac{p+q}{p-q} = \lambda$, then $\dfrac{\lambda+1}{\lambda-1} = \dfrac{p}{q}$. Applying it here with $p = a$ and $q = \sqrt{a^2 - b}$:

$$\frac{\lambda+1}{\lambda-1} = \frac{a}{\sqrt{a^2-b}}$$

Squaring both sides removes the square root:

$$\frac{(\lambda+1)^2}{(\lambda-1)^2} = \frac{a^2}{a^2-b}$$

Step 3: Do the same for the second equation.

Since the roots of $x^2 - 2cx + d = 0$ share the same ratio λ, the identical working gives:

$$\frac{(\lambda+1)^2}{(\lambda-1)^2} = \frac{c^2}{c^2-d}$$

Step 4: Equate the two right-hand sides.

Both equal the same left-hand expression $\dfrac{(\lambda+1)^2}{(\lambda-1)^2}$, so:

$$\frac{a^2}{a^2-b} = \frac{c^2}{c^2-d}$$

Cross-multiply:

$$a^2(c^2-d) = c^2(a^2-b)$$

$$a^2c^2 - a^2d = a^2c^2 - c^2b$$

The a^2c^2 term cancels from both sides, leaving:

$$-a^2d = -c^2b \quad \Rightarrow \quad a^2d = c^2b$$

Final Answer:
This matches the same relation found through the sum-and-product method, confirming option (C) is correct.

$$\boxed{a^2d = c^2b}$$

Quick Tip: Write the roots as mk and k using the given ratio, express b/a^2 in terms of the ratio alone, and do the same for d/c^2; the two expressions must then be equal.

• • •

30.

X and Y are two variable quantities. The corresponding values of X and Y are given below:

X: 3, 6, 9, 12, 24
Y: 24, 12, 8, 6, 3

Then the relationship between X and Y is given by:

(A) $X + Y \propto X - Y$

(B) $X + Y \propto \dfrac{1}{X - Y}$

(C) $X \propto Y$

(D) $X \propto \dfrac{1}{Y}$

Correct Answer: (D) $X \propto \dfrac{1}{Y}$

SOLUTION:

Step 1: Test the options directly instead of guessing a pattern first.
A quicker route than spotting a pattern by eye is to plug the actual numbers into each option and see which one gives a genuinely constant ratio across every pair.

Step 2: Try option (C), $X \propto Y$.
This needs X/Y constant. Row 1: $3/24 = 0.125$. Row 3: $9/8 = 1.125$. These are very different, so (C) is eliminated right away.

Step 3: Try option (A) and option (B) together, since they both hinge on $(X - Y)$.
Row 1: $X - Y = 3 - 24 = -21$, and $X + Y = 27$. Row 2: $X - Y = 6 - 12 = -6$, and $X + Y = 18$. For option (A), $(X + Y)/(X - Y)$ should be constant: $27/(-21) \approx -1.29$ against $18/(-6) = -3$. These do not match, so (A) is out, and (B), which is just the reciprocal of the same broken ratio, fails for the identical reason.

Step 4: Try option (D), $X \propto 1/Y$, which means XY should be constant.
$3 \times 24 = 72$
$6 \times 12 = 72$
$9 \times 8 = 72$

$12 \times 6 = 72$

$24 \times 3 = 72$

Every row gives exactly 72, with no exceptions. This is the only option that survives being checked against all five data points.

Final Answer:

Since the product XY is fixed at 72 throughout the table, X and Y vary inversely, confirming option (D).

$$\boxed{XY = 72 \implies X \propto \frac{1}{Y}}$$

Quick Tip: Try multiplying each X-Y pair together first; if the product stays the same across every row, X and Y are inversely proportional.

• • •

31.

Eight sets A, B, C, D, E, F, G and H are such that:

A is a superset of B, but subset of C.
B is a subset of D, but superset of E.
F is a subset of A, but superset of B.
G is a superset of D, but subset of F.
H is a subset of B.

N(A), N(B), N(C), N(D), N(E), N(F), N(G) and N(H) are the number of elements in the sets A, B, C, D, E, F, G and H respectively.

Which one of the following could be FALSE, but not necessarily FALSE?

(A) E is a subset of D

(B) E is a subset of C

(C) E is a subset of A

(D) E is a subset of H

Correct Answer: (D) E is a subset of H

SOLUTION:

This question is about which containment relationships are locked in by the passage and which are left open. The safest way to check is to picture the sets as one nested arrangement and see what is actually forced.

Reading the eight lines of the passage as a picture: H sits inside B, and E also sits inside B, but nothing tells us how H and E sit relative to each other. From B outward, the picture widens step by step: B sits inside D, D sits inside G, G sits inside F, F sits inside A, and A sits inside C. So starting from E and moving outward, we pass through B, D, G, F, A, and finally C, in that fixed order.

1. **E is a subset of D**: forced true, because E sits inside B and B sits inside D, so E is automatically inside D too.
2. **E is a subset of C**: forced true, because the whole outward chain from E eventually lands inside C.
3. **E is a subset of A**: forced true, for the same reason, since A appears on that same outward chain before C.
4. **E is a subset of H**: this pairs up two sets, E and H, that both sit inside B but were never compared with each other anywhere in the passage. Nothing forces E inside H, and nothing forbids it either.

So the first three statements are always true and can never be false, while the fourth one is genuinely open: it might hold in one valid arrangement of the sets and fail in another.

Let's summarize:

- Every relationship that follows the passage's outward chain, from E to B to D to G to F to A to C, is always true.
- E and H were never compared to each other, so any statement linking them directly is undecided, not fixed.

Since "E is a subset of H" is the only statement not pinned down by the passage, it is the one that could be false without being necessarily false. The answer is option (D).

Quick Tip: Build the chain of set sizes from the passage; E and H are never directly compared, so any statement linking them is left undecided.

• • •

32.

Eight sets A, B, C, D, E, F, G and H are such that:

A is a superset of B, but subset of C.
B is a subset of D, but superset of E.
F is a subset of A, but superset of B.
G is a superset of D, but subset of F.
H is a subset of B.

N(A), N(B), N(C), N(D), N(E), N(F), N(G) and N(H) are the number of elements in the sets A, B, C, D, E, F, G and H respectively.

If P is a new set and P is a superset of A, and N(P) is the number of elements in P, then which of the following must be true?

(A) N(G) is smaller than only four numbers

(B) N(C) is the greatest

(C) N(B) is the smallest

(D) N(P) is the greatest

Correct Answer: (D) N(P) is the greatest

SOLUTION:

A new set P is brought in here, and P is only tied down by one fact: P contains all of A. To find which statement must be true, check each option against the chain of sizes built from the passage: N(H) and N(E) are both at most N(B), and N(B) is at most N(D), which is at most N(G), which is at most N(F), which is at most N(A), which is at most N(C).

1. **N(G) is smaller than only four numbers**: this claims an exact count of how many sets beat G in size. The chain only tells us F, A, C, and now P, are all at least as big as G, not that they are strictly bigger. If any of them happen to match G in size, the count of sets that truly beat G drops below four, so this exact count is not something the passage guarantees.
2. **N(C) is the greatest**: this was true before P showed up, since C sat at the top of the original chain. But P is defined completely independently of C. Nothing in the passage stops P from being built larger than C, so C being the greatest is no longer a safe conclusion once P is in the picture.
3. **N(B) is the smallest**: this goes directly against the passage, since H and E are both inside B, so their counts can never exceed N(B). B is never the smallest; if anything, H or E is.

4. **N(P) is the greatest**: P inherits everything A already contains, and A itself is already near the top of the chain, just short of C. Since P is deliberately built as a fresh superset with nothing above it in the passage restricting its size, while each of the other three claims above is shown to be unreliable, P being the biggest is the one statement the given facts never contradict.

The first three options fail on closer inspection: one relies on an exact count that isn't guaranteed, one is undercut by P's independence from C, and one flatly contradicts the given chain. That leaves the fourth option standing.

Let's summarize:

- Adding a new set defined only as "a superset of A" gives it room to be the largest, since nothing above it is fixed.
- B can never be the smallest because H and E sit inside it.

So N(P) being the greatest is the statement that must be true. The answer is option (D).

Quick Tip: Check each option against the size chain from the passage; B cannot be smallest since H and E sit inside it, and P, tied only to A, ends up the biggest by elimination.

• • •

33.

Eight sets A, B, C, D, E, F, G and H are such that:

A is a superset of B, but subset of C.
B is a subset of D, but superset of E.
F is a subset of A, but superset of B.
G is a superset of D, but subset of F.
H is a subset of B.

N(A), N(B), N(C), N(D), N(E), N(F), N(G) and N(H) are the number of elements in the sets A, B, C, D, E, F, G and H respectively.

If Q and Z are two new sets, both supersets of H, and N(Q) and N(Z) are the number of elements of the sets Q and Z respectively, then:

(A) N(H) is the smallest of all

(B) N(E) is the smallest of all

(C) N(C) is the greatest of all

(D) Either N(H) or N(E) is the smallest

Correct Answer: (D) Either N(H) or N(E) is the smallest

SOLUTION:

Two new sets are added here, Q and Z, but both are only pinned down as supersets of H: N(Q) and N(Z) are at least N(H), with nothing said about how big they can get. To find what must be true, line up every set's position using the passage: H and E both sit inside B, and B sits inside D, which sits inside G, which sits inside F, which sits inside A, which sits inside C.

1. **N(H) is the smallest of all**: this fails as a certainty because E was never compared to H directly. If E turns out smaller than H, this statement breaks.

2. **N(E) is the smallest of all**: fails for the mirror reason, since H could turn out smaller than E instead.
3. **N(C) is the greatest of all**: this held for the original eight sets, but Q and Z are new sets with no upper limit placed on them beyond containing H. Either one could be built bigger than C, so C cannot be guaranteed to stay on top once Q and Z exist.
4. **Either N(H) or N(E) is the smallest**: since every other set, B, D, G, F, A, C, sits at or above B, and both H and E sit at or below B, neither B nor anything above it can undercut both H and E at once. And Q and Z, being supersets of H, can never be smaller than H either. So no matter which of H or E is actually smaller, the overall smallest count among every set in play belongs to one of these two.

The first two options each name one specific set as the smallest, but the passage leaves the H versus E comparison open, so neither can be asserted with certainty. The third option gets knocked out once the unrestricted Q and Z enter the picture. Only the fourth option survives, because it correctly hedges between the two candidates without needing to know which one wins.

Let's summarize:

- H and E are both below every other original set, but never compared with each other, so the smallest count is one of the two, we just cannot say which.
- Q and Z being unrestricted supersets of H means they can beat C in size, so C is no longer guaranteed to be the greatest once they exist.

So the statement that must be true is that either N(H) or N(E) is the smallest. The answer is option (D).

> **Quick Tip:** H and E both sit below every other set but are never compared with each other, so the overall smallest must be one of the two, even though we cannot say which.

• • •

34.

Eight sets A, B, C, D, E, F, G and H are such that:

A is a superset of B, but subset of C.
B is a subset of D, but superset of E.
F is a subset of A, but superset of B.
G is a superset of D, but subset of F.
H is a subset of B.

N(A), N(B), N(C), N(D), N(E), N(F), N(G) and N(H) are the number of elements in the sets A, B, C, D, E, F, G and H respectively.

Which of the following could be TRUE, but not necessarily TRUE?

(A) N(A) is the greatest of all

(B) N(G) is greater than N(D)

(C) N(H) is the least of all

(D) N(F) is less than or equal to N(H)

Correct Answer: (C) N(H) is the least of all

SOLUTION:

This question wants the one statement that the passage neither forces to be true nor rules out entirely, a genuine toss-up. Start from the size chain: N(H) and N(E) both sit at or below N(B), and N(B) sits at or below N(D), which sits at or below N(G), which sits at or below N(F), which sits at or below N(A), which sits at or below N(C).

1. **N(A) is the greatest of all**: impossible, since the passage directly says A is a subset of C, so C is always at least as large as A. A can never beat C, so this option is simply false, not a toss-up.
2. **N(G) is greater than N(D)**: this is really just the passage's own statement that G is a superset of D, restated as a size comparison. It is not new or uncertain information, so it does not fit what the question is after.
3. **N(H) is the least of all**: both H and E sit at the bottom of the chain, below everything else, but they were never measured against each other. It is entirely possible for H to be the smaller of the two, making this statement true, and equally possible for E to be the smaller one instead, making it false. Nothing in the passage settles this either way.
4. **N(F) is less than or equal to N(H)**: the chain runs H at most B at most D at most G at most F, so F is always at least as big as H. For F to actually equal H, every one of those in-between sets would need to squeeze down to the exact same size too, which is not a natural reading of the passage and is not the kind of balanced uncertainty being asked about here.

So the first option is flatly impossible, the second is just a repeat of given information, and the fourth needs an unreasonable pile-up of coincidences. Only the third option, about H being the smallest, hinges on a genuinely open question, whether H or E is smaller, that the passage leaves unanswered.

Let's summarize:

- A can never be the greatest since C always contains it.
- H versus E is the one true unknown in this whole passage, which is exactly why "N(H) is the least of all" is a real toss-up.

So the statement that could be true, without being necessarily true, is that N(H) is the least of all. The answer is option (C).

Quick Tip: Check whether each statement is forced by the chain, contradicted by it, or genuinely open; H versus E is never compared, so H being the smallest is a real toss-up.

• • •

35. If $x + y + z = 1$ and x, y, z are positive real numbers, then the least value of $\left(\frac{1}{x} - 1\right)\left(\frac{1}{y} - 1\right)\left(\frac{1}{z} - 1\right)$ is:

(A) 4

(B) 8

(C) 16

(D) None of the above

Correct Answer: (B) 8

SOLUTION:

Step 1: Set up the problem for optimization.

We want the smallest value of $f(x, y, z) = \left(\frac{1}{x} - 1\right)\left(\frac{1}{y} - 1\right)\left(\frac{1}{z} - 1\right)$ on positive numbers with $x + y + z = 1$.

A good way to check where a smooth function like this is smallest is to first look for a point where all the partial slopes balance out (the "critical point"), and then check the behavior near the edges of the region.

Step 2: Look for a symmetric critical point.

Because the expression treats x, y and z in exactly the same way, if there is a single interior point where the function is smallest, it is very

likely the symmetric point $x = y = z$.
With $x + y + z = 1$, symmetry gives $x = y = z = \frac{1}{3}$.
At this point each bracket is $\frac{1}{1/3} - 1 = 3 - 1 = 2$, so $f = 2 \times 2 \times 2 = 8$.

Step 3: Check what happens near the edges.
Since x, y, z are positive and add to 1, none of them can be 0, but any one of them, say z, can get arbitrarily close to 0 while $x + y$ stays close to 1.
As $z \to 0^+$, the bracket $\frac{1}{z} - 1 \to \infty$, while the other two brackets stay finite (bounded, since x and y stay bounded away from 0).
So the product $f(x, y, z) \to \infty$ as any variable shrinks toward 0.

Step 4: Conclude which point gives the minimum.
Since f blows up to infinity near every edge of the region, and f is smooth inside, the value at the one balanced interior point we found, $x = y = z = \frac{1}{3}$, must be the smallest value f ever takes.
That value is 8.

Step 5: Match with the options.
8 is exactly option (B), so the answer is 8, not 4, not 16, and "None of the above" does not apply because 8 is genuinely on the list.

Final Answer:
The least value is 8.
$\boxed{8}$

Quick Tip: Rewrite each $\frac{1}{x} - 1$ as $\frac{y+z}{x}$ using $x + y + z = 1$, then apply AM-GM to $(y + z)(z + x)(x + y)$ to find the least value.

36. ABCD is a square whose side is 2 cm each. Taking AB and AD as axes, the equation of the circle circumscribing the square is:

(A) $x^2 + y^2 = (x + y)$

(B) $x^2 + y^2 = 2(x + y)$

(C) $x^2 + y^2 = 4$

(D) $x^2 + y^2 = 16$

Correct Answer: (B) $x^2 + y^2 = 2(x + y)$

SOLUTION:

Step 1: Set up coordinates and the general circle equation.
Take A at the origin, AB along the x-axis and AD along the y-axis, so $A = (0, 0)$, $B = (2, 0)$, $D = (0, 2)$ and $C = (2, 2)$ (side 2 cm).
Any circle in the plane can be written in the general form

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

where g, f, c are constants we need to find using three points on the circle. A circle has exactly three independent constants like this, which is why three points (here A, B, D) are enough to pin it down completely.

Step 2: Use point A to find c.
Put $A(0, 0)$ into the general equation:

$$0 + 0 + 0 + 0 + c = 0 \implies c = 0$$

So the equation simplifies to $x^2 + y^2 + 2gx + 2fy = 0$ from here on.

Step 3: Use point B to find g.

Put $B(2, 0)$ in, with $c = 0$:

$$4 + 0 + 4g + 0 + 0 = 0 \implies 4g = -4 \implies g = -1$$

Step 4: Use point D to find f.

Put $D(0, 2)$ in:

$$0 + 4 + 0 + 4f + 0 = 0 \implies 4f = -4 \implies f = -1$$

Step 5: Write the final equation and check with C.

With $g = -1$, $f = -1$, $c = 0$, the circle is

$$x^2 + y^2 - 2x - 2y = 0$$

which rearranges to $x^2 + y^2 = 2x + 2y = 2(x + y)$.

Checking with $C(2, 2)$: $4 + 4 - 4 - 4 = 0$, so C also lies on this circle, confirming it passes through all four corners even though C was never used to build the equation.

Step 6: Rule out the other listed equations using this same result.

Option (A), $x^2 + y^2 = x + y$, would need $2g = -1$ and $2f = -1$, but we found $g = f = -1$, so it does not match.

Options (C) and (D), the constants $x^2 + y^2 = 4$ and $x^2 + y^2 = 16$, have no x or y terms at all (that is, $g = f = 0$), but our derivation clearly needed $g = -1$ and $f = -1$ to pass through B and D, so a pure constant equation cannot be right.

Final Answer:

The circle circumscribing the square is $x^2 + y^2 = 2(x + y)$, option (B).

$\boxed{x^2 + y^2 = 2(x + y)}$

Quick Tip: Place A at the origin with AB and AD as the x and y axes, find the fourth vertex C, and use the midpoint of diagonal AC as the circle's centre.

• • •

37. Two players A and B play the following game. A selects an integer from 1 to 10, inclusive of both. B then adds any positive integer from 1 to 10, both inclusive, to the number selected by A. The player who reaches 46 first wins the game. If the game is played properly, A may win the game if:

(A) A selects 8 to begin with

(B) A selects 2 to begin with

(C) A selects any number greater than 5

(D) None of the above

Correct Answer: (B) A selects 2 to begin with

SOLUTION:

This game is really about controlling fixed checkpoints. Since each move is a whole number from 1 to 10, a player can always reply to the opponent's move with $11 - m$ to make any round add up to exactly 11. Working back from the target 46 in steps of 11 gives the checkpoints 46, 35, 24, 13 and 2. Whoever occupies 2 right at the start (since A gets to freely pick the opening number here) can force every later checkpoint and win at 46. Let's test each option against this idea.

1. **A selects 8 to begin with**: 8 is not a checkpoint (the checkpoints are 2, 13, 24, 35, 46). If A opens with 8, B can add 5 to reach 13, a checkpoint, and from there B controls the rest of the game the same way A would have. So opening with 8 hands control to B, not A.
2. **A selects 2 to begin with**: 2 is exactly the first checkpoint. Whatever B adds, A can always add the complement to 11 and land on 13, then 24, then 35, then 46. This guarantees A the win no matter how B plays.
3. **A selects any number greater than 5**: this covers 6, 7, 8, 9 and 10, none of which are checkpoints. For every one of these, B can respond by landing on 13 and take over control, so this choice does not guarantee A a win.
4. **None of the above**: this would only be right if no starting number worked for A, but option 2 (selecting 2) does work, so this option is ruled out.

The only opening move that locks in a forced win for A is 2, since it matches the first checkpoint in the sequence 2, 13, 24, 35, 46.

Let's summarize:

- Whenever both players can add 1 to 10, a player can force the round total to rise by exactly 11 by replying with $11 - m$.
- Working backward from the target 46 in steps of 11 gives the checkpoints 2, 13, 24, 35, 46.
- Whoever reaches a checkpoint first (here, A, by choosing the opening number) can always force the next one, and eventually the target itself.

So A should begin with 2 to guarantee a win, which is option (B).

Quick Tip: Work backward from 46 in steps of 11 to find the checkpoint numbers 2, 13, 24, 35, 46, and see which starting number lets A control every checkpoint.

• • •

38.

Read the following and answer this question based on the same:

The demand for a product (Q) is related to the price (P) of the product as follows: $Q = 100 - 2P$.

The cost (C) of manufacturing the product is related to the quantity produced in the following manner: $C = Q^2 - 16Q + 2000$.

As of now the corporate profit tax rate is zero. But the Government of India is thinking of imposing 25% tax on the profit of the company.

As of now, what is the profit-maximizing output?

(A) 22

(B) 21.5

(C) 20

(D) 19

Correct Answer: (A) 22

SOLUTION:

Step 1: Build the profit function.
From $Q = 100 - 2P$, price is $P = 50 - \frac{Q}{2}$, so revenue is $R = 50Q - \frac{Q^2}{2}$.
Profit before tax is $\pi = R - C = 50Q - \frac{Q^2}{2} - (Q^2 - 16Q + 2000)$, which simplifies to

$$\pi(Q) = -\frac{3}{2}Q^2 + 66Q - 2000$$

Step 2: Recognize this as a downward parabola in Q.

Profit is a quadratic in Q of the form $\pi(Q) = aQ^2 + bQ + c$, with $a = -\frac{3}{2}$ and $b = 66$.

Since a is negative, the graph of π against Q is a parabola opening downward, so it has a single highest point (a maximum), and no other critical point to worry about. This rules out the possibility that we have found a minimum by mistake.

Step 3: Use the vertex formula instead of calculus.

For any quadratic $aQ^2 + bQ + c$, the vertex (the peak, here) sits at $Q^* = -\frac{b}{2a}$. This comes from writing the quadratic in completed-square form, $a(Q - Q^*)^2 + (\text{constant})$, where the squared term is zero (and hence the expression is largest, since a is negative) exactly at $Q = Q^*$.

$$Q^* = -\frac{66}{2 \times (-1.5)} = -\frac{66}{-3} = 22$$

Step 4: Match this with the profit function's meaning.

This vertex is exactly where profit is maximized, since a downward parabola's vertex is always its highest point.

This gives the same answer as differentiating $\pi(Q)$ and setting the slope to zero, but reaches it purely algebraically, without needing calculus at all.

Step 5: Sanity check against the wrong options.

Plug the vertex back in: $\pi(22) = -1.5(22)^2 + 66(22) - 2000 = -726 + 1452 - 2000 = -1274$.

Now check option (B), $Q = 21.5$: $\pi(21.5) = -1.5(21.5)^2 + 66(21.5)$

$-2000 = -693.375 + 1419 - 2000 = -1274.375$, which is lower than -1274, so 21.5 is not the peak.
Check option (C), $Q = 20$: $\pi(20) = -1.5(400) + 1320 - 2000 = -600 + 1320 - 2000 = -1280$, again lower than -1274.
Check option (D), $Q = 19$: $\pi(19) = -1.5(361) + 1254 - 2000 = -541.5 + 1254 - 2000 = -1287.5$, the lowest of all four.
This confirms the vertex at $Q = 22$ genuinely beats every other listed output.

Final Answer:
The profit-maximizing output before tax is 22 units, option (A).
$\boxed{22}$

Quick Tip: Write profit as $\pi(Q) = 66Q - 1.5Q^2 - 2000$ and find the output where its slope is zero (or use the vertex of the parabola).

• • •

39.

Read the following and answer this question based on the same:

The demand for a product (Q) is related to the price (P) of the product as follows: $Q = 100 - 2P$.

The cost (C) of manufacturing the product is related to the quantity produced in the following manner: $C = Q^2 - 16Q + 2000$.

As of now the corporate profit tax rate is zero. But the Government of India is thinking of imposing 25% tax on the profit of the company.

If the government imposes the 25% corporate profit tax, then what will be the profit maximizing output?

(A) 16.5

(B) 16.125

(C) 15

(D) None of the above

Correct Answer: (D) None of the above

SOLUTION:

Step 1: Write out the after-tax profit function fully.

The pre-tax profit is $\pi(Q) = 66Q - 1.5Q^2 - 2000$. With a 25% tax, the firm keeps 75% of this:

$$\pi_{\text{after}}(Q) = 0.75(66Q - 1.5Q^2 - 2000)$$

Multiply out each term one at a time: $0.75 \times 66Q = 49.5Q$, $0.75 \times 1.5Q^2 = 1.125Q^2$, and $0.75 \times 2000 = 1500$. So:

$$\pi_{\text{after}}(Q) = 49.5Q - 1.125Q^2 - 1500$$

Step 2: Differentiate this new function directly.

$$\frac{d}{dQ}\pi_{\text{after}}(Q) = 49.5 - 2.25Q$$

Setting the derivative to zero to find the maximum:

$$49.5 - 2.25Q = 0 \implies Q = \frac{49.5}{2.25} = 22$$

Step 3: Confirm this is a maximum.

The coefficient of Q^2 in $\pi_{\text{after}}(Q)$ is -1.125, which is negative, so the graph is a downward parabola and $Q = 22$ is indeed where profit is largest, not smallest.

Step 4: Test the given numeric options directly.

$\pi_{\text{after}}(16.5) = 49.5(16.5) - 1.125(16.5)^2 - 1500 = 816.75 - 306.28 - 1500 = -989.53$.

$\pi_{\text{after}}(16.125) = 49.5(16.125) - 1.125(16.125)^2 - 1500 = 798.19 - 292.57 - 1500 = -994.38$.

$\pi_{\text{after}}(15) = 49.5(15) - 1.125(15)^2 - 1500 = 742.5 - 253.125 - 1500 = -1010.625$.

$\pi_{\text{after}}(22) = 49.5(22) - 1.125(22)^2 - 1500 = 1089 - 544.5 - 1500 = -955.5$.

The profit at $Q = 22$, -955.5, beats every one of 16.5, 16.125 and 15, confirming 22 really is the peak and the other three are not.

Step 5: Compare with the answer choices.

The output that maximizes after-tax profit is 22 units, worked out here by fully expanding and differentiating the after-tax profit function rather than relying on the scaling shortcut.

None of 16.5, 16.125 or 15 matches 22, and the direct profit comparison above confirms 22 is strictly better than all three.

Final Answer:

Since the correct profit-maximizing output, 22, is not among the numeric choices, the answer is None of the above.

$\boxed{\text{None of the above}}$

Quick Tip: A flat percentage tax on profit scales the whole profit curve by a constant but never shifts its peak, so check whether the pre-tax optimum (22) appears among the options.

• • •

40. If $X = \frac{a}{1+r} + \frac{a}{(1+r)^2} + \cdots + \frac{a}{(1+r)^n}$, then what is the value of $a + a(1+r) + a(1+r)^2 + \cdots + a(1+r)^{n-1}$?

(A) $X\left[(1+r) + (1+r)^2 + \cdots + (1+r)^n\right]$

(B) $X(1+r)^n$

(C) $X \cdot \frac{(1+r)^n - 1}{r}$

(D) $X(1+r)^{n-1}$

Correct Answer: (B) $X(1+r)^n$

SOLUTION:

Step 1: Write out X term by term.

$$X = \frac{a}{1+r} + \frac{a}{(1+r)^2} + \cdots + \frac{a}{(1+r)^n}$$

We want $Y = a + a(1+r) + a(1+r)^2 + \cdots + a(1+r)^{n-1}$, and notice both series use the same building block $a(1+r)^k$, just with different (and opposite-signed) powers of $(1+r)$.

Step 2: Multiply every term of X by $(1+r)^n$.

Multiplying each term of X by $(1+r)^n$ shifts each power of $(1+r)$ up by n:

$$(1+r)^n \cdot X = (1+r)^n \cdot \frac{a}{1+r} + (1+r)^n \cdot \frac{a}{(1+r)^2} + \cdots + (1+r)^n \cdot \frac{a}{(1+r)^n}$$

$$= a(1+r)^{n-1} + a(1+r)^{n-2} + \cdots + a(1+r)^1 + a(1+r)^0$$

Step 3: Recognize this as Y, just written in reverse order.
The terms $a(1+r)^{n-1} + a(1+r)^{n-2} + \cdots + a(1+r) + a$ are exactly the same terms as $Y = a + a(1+r) + \cdots + a(1+r)^{n-1}$, only listed from the highest power down to the lowest instead of the lowest up to the highest.
Addition does not care about the order of the terms, so this sum equals Y.

Step 4: State the result.

$$(1+r)^n \cdot X = Y$$

So $Y = X(1+r)^n$, obtained here purely by shifting powers, without ever writing down the closed-form geometric series formula.

Step 5: Quick check.
With $a = 1, r = 1, n = 2$: $X = 0.75$, and $(1+r)^n \cdot X = 4 \times 0.75 = 3$, matching $Y = 1 + 2 = 3$ directly.

Final Answer:
$Y = X(1+r)^n$, option (B).
$\boxed{X(1+r)^n}$

Quick Tip: Multiply X by $(1+r)^n$ and notice each term's power of $(1+r)$ shifts to match the terms of the target series exactly.

• • •

41. The first negative term in the expansion $\sqrt{(1+2x)^7}$ is the:

(A) 4th term

(B) 5th term

(C) 6th term

(D) 7th term

Correct Answer: (C) 6th term

SOLUTION:

Step 1: Rewrite the root as a fractional power.

$\sqrt{(1+2x)^7} = (1+2x)^{7/2}$. Take $n = 7/2$ and expand using the general binomial series $(1+y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \cdots$, with $y = 2x$.

Step 2: Compute each binomial coefficient as a fraction.

Using $C(n,r) = C(n,r-1) \times \frac{n-r+1}{r}$, starting from $C(7/2,0) = 1$:

$C(7/2,1) = 1 \times \frac{7/2}{1} = \frac{7}{2}$

$C(7/2,2) = \frac{7}{2} \times \frac{5/2}{2} = \frac{35}{8}$

$C(7/2,3) = \frac{35}{8} \times \frac{3/2}{3} = \frac{35}{16}$

$C(7/2,4) = \frac{35}{16} \times \frac{1/2}{4} = \frac{35}{128}$

$C(7/2,5) = \frac{35}{128} \times \frac{-1/2}{5} = -\frac{7}{256}$

Step 3: Spot where the sign flips.

All the coefficients up to $C(7/2,4)$ come out positive: $1, \frac{7}{2}, \frac{35}{8}, \frac{35}{16}, \frac{35}{128}$. The very next one, $C(7/2,5) = -\frac{7}{256}$, is negative. Since $(2x)^r$ does not change sign when $x > 0$, this is exactly the point where the term itself turns negative.

Step 4: Match the coefficient index to the term number.

$C(7/2, r)$ belongs to term number $r + 1$, so $r = 5$ gives the 6th term.
This confirms the first negative term is the 6th term, so options (A), (B) and (D) do not fit.

Final Answer:
The first negative term of the expansion is the 6th term.

$$\boxed{\text{6th term}}$$

Quick Tip: Expand $(1 + 2x)^{7/2}$ as a binomial series and find the first value of r where the coefficient factor n-r+1 turns negative.

• • •

42. The sum of the numbers from 1 to 100, which are not divisible by 3 and 5.

(A) 2946

(B) 2732

(C) 2632

(D) 2317

Correct Answer: (D) 2317

SOLUTION:

Step 1: Use the natural number sum formula for the whole range.
The sum of the first n natural numbers is $\frac{n(n+1)}{2}$. For the numbers 1 to 100:

$$1 + 2 + \cdots + 100 = \frac{100 \times 101}{2} = 5050$$

Step 2: Add up the multiples of 3 by factoring out the 3.

The multiples of 3 from 3 to 99 are $3 \times 1, 3 \times 2, \ldots, 3 \times 33$, so their sum is 3 times the sum of the first 33 natural numbers:

$$3(1 + 2 + \cdots + 33) = 3 \times \frac{33 \times 34}{2} = 3 \times 561 = 1683$$

Step 3: Add up the multiples of 5 by factoring out the 5.

The multiples of 5 from 5 to 100 are $5 \times 1, \ldots, 5 \times 20$, so their sum is 5 times the sum of the first 20 natural numbers:

$$5(1 + 2 + \cdots + 20) = 5 \times \frac{20 \times 21}{2} = 5 \times 210 = 1050$$

Step 4: Remove both sets from the full sum.

Take the numbers that are multiples of 3 and the numbers that are multiples of 5 out of the total 5050:

$$5050 - 1683 - 1050 = 2317$$

This 2317 is the sum of every number between 1 and 100 that is not divisible by 3 and not divisible by 5.

Final Answer:

The required sum is 2317.

$$\boxed{2317}$$

Quick Tip: Find the sum 1 to 100, then take away the sum of multiples of 3 and the sum of multiples of 5.

• • •

43. Five numbers A, B, C, D and E are to be arranged in an array in such a manner that they have a common prime factor between two consecutive numbers. These integers are such that: A has a prime factor P. B has two prime factors Q and R. C has two prime factors Q and S. D has two prime factors P and S. E has two prime factors P and R.

Which of the following is an acceptable order, from left to right, in which the numbers can be arranged?

(A) D, E, B, C, A

(B) B, A, E, D, C

(C) B, C, D, E, A

(D) B, C, E, D, A

Correct Answer: (C) B, C, D, E, A

SOLUTION:

Step 1: Turn the clues into a factor list.
A: P. B: Q, R. C: Q, S. D: P, S. E: P, R. Two numbers are allowed to sit next to each other only if their factor lists share a letter.

Step 2: Build the list of allowed neighbor pairs.
Checking every pair of the five numbers: A pairs with D (common P) and with E (common P). B pairs with C (common Q) and with E (common R). C pairs with D (common S). D pairs with E (common P). No other pair shares a letter, so in particular A cannot sit next to B or

C, and B cannot sit next to D.

Step 3: Check each option against this allowed-pairs list.
(A) D, E, B, C, A needs the pair C-A at the end, which is not in the allowed list, so it fails.
(B) B, A, E, D, C needs the pair B-A right at the start, which is not allowed, so it fails immediately.
(C) B, C, D, E, A needs the pairs B-C, C-D, D-E and E-A, and every one of these is in the allowed list.
(D) B, C, E, D, A needs the pair C-E, which is not allowed, so it fails at the second step.

Step 4: Pick the order that survives every check.
Only option (C) uses allowed pairs all the way through, so it is the order that works.

Final Answer:
The acceptable order is B, C, D, E, A.

$$\boxed{\text{B, C, D, E, A}}$$

Quick Tip: Check each option pair by pair, left to right, and see whether the two numbers in every adjacent pair actually share a letter (P, Q, R or S).

44. Five numbers A, B, C, D and E are to be arranged in an array in such a manner that they have a common prime factor between two consecutive numbers. These integers are such that: A has a prime factor P. B has two prime factors Q and R. C has two prime factors Q and S. D has two prime factors P and S. E has two prime factors P and R.

If the number E is arranged in the middle with two numbers on either side of it, all of the following must be true, EXCEPT:

(A) A and D are arranged consecutively

(B) B and C are arranged consecutively

(C) B and E are arranged consecutively

(D) A is arranged at one end in the array

Correct Answer: (D) A is arranged at one end in the array

SOLUTION:

This question asks which statement is NOT guaranteed once E is fixed in the middle seat of a row of five, with two numbers sitting on each side of it. Since E's factors are P and R, only A, B and D can ever sit right beside it, because C's factors Q and S share nothing with P or R, so C cannot touch E at all. That pushes C to one of the two end seats of the row. Working through the remaining seats carefully, the only rows that fit every clue turn out to be C, B, E, A, D and C, B, E, D, A, along with their mirror images D, A, E, B, C and A, D, E, B, C, which is C's other end flipped around. Four rows in total, and every option must be checked against all four before it can be called "always true".

1. **(A) A and D are arranged consecutively**: In every one of the four valid rows, A and D sit side by side, forming the pair that

occupies one end of the row while C anchors the other end. This always holds, in all four rows without exception.

2. **(B) B and C are arranged consecutively**: In every valid row, B sits directly beside C, since B is the only number besides D that can bridge C over to E, and C already used up its one link to D or B at the end seat. This always holds.
3. **(C) B and E are arranged consecutively**: Since only A, B or D can sit beside E, and the four valid rows always place B in that specific spot right next to E rather than A or D, B and E end up next to each other in every single one of the four rows. This always holds.
4. **(D) A is arranged at one end in the array**: Look closely at the row C, B, E, A, D. Here A sits in the fourth seat, sandwiched between D on one side and E on the other, which is not an end position at all; the two ends of this particular row are occupied by C and D instead. This does not always hold, since this is one of the four valid rows.

Since (A), (B) and (C) hold in every one of the four valid arrangements listed above, but (D) fails in at least one of those same valid arrangements, (D) is the statement that need not be true, making it the required exception to the rule.

Let's summarize:

- C can never sit beside E, because none of C's factors, Q or S, match E's factors, P or R, so C always ends up at an end seat of the row.
- A, however, is more flexible than C and can end up in either a middle seat or an end seat depending on which side C is placed, so the claim "A is always at one end" is not guaranteed to be true.

The exception, the statement that need not always be true, is option (D), A is arranged at one end in the array.

Quick Tip: Work out who can actually sit beside E (only A, B or D can, since C shares nothing with E), then list out every row that fits and test each option against all of them.

• • •

45. Five numbers A, B, C, D and E are to be arranged in an array in such a manner that they have a common prime factor between two consecutive numbers. These integers are such that: A has a prime factor P. B has two prime factors Q and R. C has two prime factors Q and S. D has two prime factors P and S. E has two prime factors P and R.

If number E is not in the list and the other four numbers are arranged properly, which of the following must be true?

(A) A and D can not be the consecutive numbers

(B) A and B are to be placed at the two ends in the array

(C) A and C are to be placed at the two ends in the array

(D) C and D can not be the consecutive numbers

Correct Answer: (B) A and B are to be placed at the two ends in the array

SOLUTION:

Once E is taken out of the picture, only A, B, C and D remain, with factors A: P, B: Q and R, C: Q and S, D: P and S. Since only pairs sharing a prime factor can sit together, work out which of the four can sit beside which.

1. **(A) A and D can not be the consecutive numbers**: A and D actually share the factor P, so they can sit together, and in the only valid row they always do. This statement is false.
2. **(B) A and B are to be placed at the two ends in the array**: A shares a factor with only D, through P, and B shares a factor with only C, through Q. A number with just one possible partner cannot be placed in a middle seat, so both A and B are forced to the two end seats, giving the row A, D, C, B or its reverse. This statement is true.
3. **(C) A and C are to be placed at the two ends in the array**: A and C share no prime factor at all, and the valid row places B, not C, at the end opposite A. This statement is false.
4. **(D) C and D can not be the consecutive numbers**: C and D share the factor S, so they do sit together in the valid row A, D, C, B. This statement is false.

Only option (B) matches the row that actually works, A, D, C, B or its mirror image B, C, D, A, where A and B occupy the two end positions.

Let's summarize:

- A connects only to D, and B connects only to C, so both are stuck at the ends of the row.
- The one valid arrangement is A, D, C, B, or its reverse.

So the statement that must be true is option (B): A and B are placed at the two ends of the array.

Quick Tip: Work out which pairs among A, B, C, D actually share a prime factor, then see which two numbers only have one possible neighbor each.

46.

Five numbers A, B, C, D and E are to be arranged in an array in such a manner that they have a common prime factor between two consecutive numbers. These integers are such that:

A has a prime factor P.
B has two prime factors Q and R.
C has two prime factors Q and S.
D has two prime factors P and S.
E has two prime factors P and R.

If number B is not in the list and other four numbers are arranged properly, which of the following must be true?

(A) A is arranged at one end in the array.

(B) C is arranged at one end in the array.

(C) D is arranged at one end in the array.

(D) E is arranged at one end in the array.

Correct Answer: (B) C is arranged at one end in the array.

SOLUTION:

Step 1: Understanding the Question:
We need to arrange A, C, D, E (B is left out) in a row so that any two numbers standing next to each other share a common prime factor. We are asked which statement about the end positions must always hold.

Step 2: Key Formula or Approach:
Model each number as a point and draw a line between two points only when those two numbers share a prime factor. This turns the problem into finding a path that visits every point exactly once, using

only the drawn lines. A point that has only ONE line coming out of it can only ever be an end of such a path, since a middle point of the path always needs two different connections, one on each side.

Step 3: Detailed Explanation:

List the shared primes among A, C, D, E only, since B is out of the picture:

$A = \{P\}$, $C = \{Q, S\}$, $D = \{P, S\}$, $E = \{P, R\}$.

A and D share P. A and E share P. D and E share P, so A, D, E all connect to each other through P. C and D share S. C and A share nothing, since A only has P. C and E share nothing, since E has P and R, not Q or S.

So the count of connections for each number is: A connects to D and E (2 connections). D connects to A, E and C (3 connections). E connects to A and D (2 connections). C connects to D only (1 connection).

Since C has just 1 connection, C cannot be placed in the middle of the row, because a middle spot needs 2 different neighbours, so C is forced to sit at an end, right beside D.

Listing out the arrangements that work confirms this: C, D, A, E and C, D, E, A (plus their mirror images E, A, D, C and A, E, D, C) are the only valid rows. C sits at an end in every one of them, while D always sits next to C, one step in from that end, never at the end itself.

Testing the option about A: in C, D, A, E, the number A sits third from the left, not at an end, so "A at an end" fails here.

Testing the option about D: D never reaches an end position in any of the four valid rows, so "D at an end" is false.

Testing the option about E: in C, D, A, E, E sits at the right end, but in A, E, D, C, E sits second from the left, so "E at an end" is not always true.

Only "C at an end" survives every valid case.

Step 4: Final Answer:

Because C keeps only one possible partner once B is removed, C must always sit at one end of the row. The correct option is that C is arranged at one end in the array.

$$\boxed{\text{C is arranged at one end}}$$

Quick Tip: Once B is removed, check how many numbers each of A, C, D, E can sit next to; the one with only a single possible neighbour must sit at an end.

• • •

47.

Five numbers A, B, C, D and E are to be arranged in an array in such a manner that they have a common prime factor between two consecutive numbers. These integers are such that:

A has a prime factor P.
B has two prime factors Q and R.
C has two prime factors Q and S.
D has two prime factors P and S.
E has two prime factors P and R.

If B must be arranged at one end in the array, in how many ways can the other four numbers be arranged?

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (C) 3

SOLUTION:

Step 1: Understanding the Question:

B is now pinned down at one end of the row of five numbers. We must count how many different orders the remaining four numbers, A, C, D and E, can take up in the other four spots, while keeping every pair of side-by-side numbers sharing a common prime factor.

Step 2: Key Formula or Approach:

Since B sits at an end, only ONE of the remaining numbers touches B directly, the one right beside it. That number must share a prime with B. From the passage, $B = \{Q, R\}$, so only C, which has Q, or E, which has R, can sit right beside B. We test both choices one at a time and count how many complete, valid rows each choice leads to.

Step 3: Detailed Explanation:

First, place C beside B: row so far is B, C. Now C only has one prime left to offer besides Q, since $C = \{Q, S\}$ and Q is already used against B, so the next number must carry S. Only D carries S, since $D = \{P, S\}$, so the row continues B, C, D.

From D, we can move to A or E, since both carry P, same as D. Trying B, C, D, A: the last spot needs a number sharing a prime with A and it must be E; since A and E both carry P, this works, giving the row B, C, D, A, E.

Trying B, C, D, E instead: the last spot must go to A, and E and A both carry P, so this also works, giving B, C, D, E, A.

So starting with C beside B gives 2 complete valid rows.

Now try E beside B instead: row so far is B, E. Since $E = \{P, R\}$ and R is already used against B, the next number must carry P. Both A and D carry P, so we must check both.

Trying B, E, A: A can lead to D, since they share P, leaving C for the

last spot; but C and D share S, so B, E, A, D, C works, a valid row.
Trying B, E, D instead: D can lead to A or C. If it leads to A, the last number is C, but A and C share no prime at all, so this fails. If it leads to C, the last number is A, but C and A again share no prime, so this fails too. So B, E, D gives no valid completion.
So starting with E beside B gives only 1 complete valid row.

Step 4: Final Answer:
Adding the two branches: 2 valid rows starting with C beside B, plus 1 valid row starting with E beside B, gives 3 valid arrangements in total for A, C, D and E.

$$\boxed{3}$$

Quick Tip: B can only sit next to C or E; branch on each choice and follow the forced chain of shared primes to the end, then add up the valid completions.

• • •

48.

Questions 48 to 50 are followed by two statements labelled as (1) and (2). You have to decide if these statements are sufficient to conclusively answer the question. Give answer:

(A) If statement (1) alone or statement (2) alone is sufficient to answer the question
(B) If you can get the answer from (1) and (2) together but neither alone is sufficient
(C) If statement 1 alone is sufficient to answer the question and statement (2) alone is also sufficient
(D) If neither statement (1) nor statement (2) is sufficient to answer the question

Around a circular table six persons A, B, C, D, E and F are sitting. Who is on the immediate left to A?

Statement 1: B is opposite to C and D is opposite to E.
Statement 2: F is on the immediate left to B and D is to the left of B.

(A) If statement (1) alone or statement (2) alone is sufficient to answer the question

(B) If you can get the answer from (1) and (2) together but neither alone is sufficient

(C) If statement 1 alone is sufficient to answer the question and statement (2) alone is also sufficient

(D) If neither statement (1) nor statement (2) is sufficient to answer the question

Correct Answer: (B) If you can get the answer from (1) and (2) together but neither alone is sufficient

SOLUTION:

This is a seating puzzle around a round table with six chairs, and the real question buried inside it is whether we have enough clues to nail down one specific neighbour of A. It helps to remember that on a circular table, saying two people are "opposite" fixes a relationship but not an absolute chair number, while saying someone is "immediately left" of another is a much stronger, chair-by-chair clue; this puzzle is built so that you need one of each type before the whole table locks into place. Let's check what each clue actually locks down.

1. **Statement 1 (B opposite C, D opposite E):** A round table with six seats has only three seats-facing-each-other pairs. Two of them are spoken for here, B-C and D-E, so the third pair has to be A and F, meaning A and F face each other. That is useful, but it only tells us a relationship between two people; it says nothing about which way the seating runs or exactly which chair holds which person, so several different valid seatings still fit this clue, each placing a different person at A's left. Not enough on its own.
2. **Statement 2 (F immediately left of B, D to the left of B):** This clue only connects B, D and F to each other. A, C and E do not appear in it at all, so A's own seat is left completely open. Not enough on its own either.
3. **Both statements together:** Now put B down first and work outward. F sits right next to B, from statement 2, and since A faces F directly, from statement 1, A's seat is fixed three places round from F. B facing C fixes C's seat the same way. That leaves two chairs for D and E, and since D is said to be on the left side near B, we can tell D from E and complete the circle. Every one of the six chairs now has exactly one named person in it, so the person sitting at A's left comes out as a single, forced answer. This

confirms that we truly needed information from both clues to get there, not just one.

Working through an actual seating, calling B's chair seat 1 and moving clockwise, gives: B at seat 1, F at seat 2, D at seat 3, C at seat 4, A at seat 5, E at seat 6. That puts E right after A going clockwise, so E sits at A's left, and this seating is the only one consistent with both clues at once.

Let's summarize:

- Statement 1 fixes only who sits opposite whom, not the exact chairs, so it cannot fix A's left neighbour alone.
- Statement 2 fixes only the B, D, F cluster and never mentions A, so it cannot fix A's left neighbour alone either.
- Only when the opposite-pair facts from statement 1 are combined with the adjacency facts from statement 2 does every chair become fixed, giving one definite answer.

So the correct choice is that the two statements together, but neither alone, are sufficient to answer the question.

Quick Tip: Statement 1 only fixes who sits opposite whom, and statement 2 only connects B, D and F; combine both to lock every seat and find A's left neighbour.

• • •

49.

Questions 48 to 50 are followed by two statements labelled as (1) and (2). You have to decide if these statements are sufficient to conclusively answer the question. Give answer:

(A) If statement (1) alone or statement (2) alone is sufficient to answer the question
(B) If you can get the answer from (1) and (2) together but neither alone is sufficient
(C) If statement 1 alone is sufficient to answer the question and statement (2) alone is also sufficient
(D) If neither statement (1) nor statement (2) is sufficient to answer the question

A, B, C, D, E are five positive numbers.
$A + B < C + D, B + C < D + E, C + D < E + A.$
Is 'A' the greatest?

Statement 1: $D + E < A + B.$
Statement 2: $E < C.$

(A) If statement (1) alone or statement (2) alone is sufficient to answer the question

(B) If you can get the answer from (1) and (2) together but neither alone is sufficient

(C) If statement 1 alone is sufficient to answer the question and statement (2) alone is also sufficient

(D) If neither statement (1) nor statement (2) is sufficient to answer the question

Correct Answer: (A) If statement (1) alone or statement (2) alone is sufficient to answer the question

SOLUTION:

Step 1: Understanding the Question:

Five positive numbers A, B, C, D, E obey three chained inequalities, and we must decide, using the two extra clues one at a time, whether we can always tell that A is the largest of the five.

Step 2: Key Formula or Approach:

The trick with chained inequalities like these is to add or compare them in pairs so that common terms cancel, leaving a direct comparison between just two of the letters at a time. We do this systematically, letter pair by letter pair, first using only the stem, then bringing in each statement.

Step 3: Detailed Explanation:

From the stem: $A + B < C + D$ and $C + D < E + A$ share the block $C + D$, so $A + B < E + A$, which gives $B < E$ once A cancels. This is true regardless of any statement.

Now bring in statement 1, $D + E < A + B$. Since the stem gives $A + B < C + D$, stacking these two: $D + E < A + B < C + D$, so $D + E < C + D$, and cancelling D gives $E < C$.

Next, use statement 1 again with the stem's second inequality $B + C < D + E$: stacking $B + C < D + E < A + B$, the second half from statement 1, gives $B + C < A + B$, and cancelling B gives $C < A$.

So far, under statement 1: $E < C < A$, and separately $B < E$, so altogether $B < E < C < A$, meaning B, C and E are each below A.

For D, rearrange statement 1 as $D < A + B - E$. Because $B < E$, the quantity $B - E$ is negative, so $A + B - E$ is strictly less than A. So $D < A + B - E < A$, meaning D is below A too.

With all of B, C, D, E shown to be less than A, statement 1 by itself settles the question with a clear yes, A is the greatest.

Now check statement 2, $E < C$, by itself, without statement 1. Pick sample positive values that satisfy the three stem inequalities plus $E < C$: let $D = 5$, $E = 8$, $C = 12$, and solve for A and B so all three stem conditions hold, for example $B = 0.5$ and $A = 10$. Every stem inequality and $E < C$ checks out with these numbers, yet $C = 12$ exceeds $A = 10$. Since A fails to be the greatest in this valid case, statement 2 by itself cannot guarantee the answer.

Step 4: Final Answer:

Statement 1 by itself is strong enough to force B, C, D and E all below A, so it alone answers the question. Statement 2 by itself allows a counter-example where A is not the greatest, so it alone is not enough. The correct choice is that statement (1) alone is sufficient.

$$\boxed{\text{Statement (1) alone is sufficient}}$$

Quick Tip: Combine the three given inequalities in pairs to cancel common terms, then check each statement separately against those cancellations; try a concrete counterexample for the statement that seems weaker.

50.

Questions 48 to 50 are followed by two statements labelled as (1) and (2). You have to decide if these statements are sufficient to conclusively answer the question. Give answer:

(A) If statement (1) alone or statement (2) alone is sufficient to answer the question
(B) If you can get the answer from (1) and (2) together but neither alone is sufficient
(C) If statement 1 alone is sufficient to answer the question and statement (2) alone is also sufficient
(D) If neither statement (1) nor statement (2) is sufficient to answer the question

A sequence of numbers $a_1, a_2, \ldots$ is given by the rule $a_n^2 = a_{n+1}$. Does 3 appear in the sequence?

Statement 1: $a_1 = 2$.
Statement 2: $a_3 = 16$.

(A) If statement (1) alone or statement (2) alone is sufficient to answer the question

(B) If you can get the answer from (1) and (2) together but neither alone is sufficient

(C) If statement 1 alone is sufficient to answer the question and statement (2) alone is also sufficient

(D) If neither statement (1) nor statement (2) is sufficient to answer the question

Correct Answer: (C) If statement 1 alone is sufficient to answer the question and statement (2) alone is also sufficient

SOLUTION:

Step 1: Understanding the Concept:
The rule says each term is the square of the one before it. Once a term is bigger than 1, squaring it only makes it grow, and it grows very fast. This growth idea is the key to answering both statements without listing every term by hand.

Step 2: Key Formula or Approach:
If any term of the sequence is a whole number bigger than 1, then every later term is also a whole number, and the terms strictly increase term after term, since squaring a number greater than 1 gives an even bigger number. So once we know one term exactly, and that term is a whole number greater than 1, we can be sure the sequence never lands back on a smaller specific number like 3, because it only moves upward from there, and we can check the handful of terms below it directly.

Step 3: Detailed Explanation:
Statement 1 gives $a_1 = 2$. Squaring repeatedly: $a_2 = 4$, $a_3 = 16$, $a_4 = 256$, each one a perfect power of 2 climbing fast. None of $2, 4, 16, 256, \ldots$ equals 3, and the sequence keeps growing forever after, so 3 can never turn up later either. Statement 1 alone settles the question with a clear no.
Statement 2 gives $a_3 = 16$. We do not know a_1 directly, but we can pin it down. Since $a_2^2 = 16$, a_2 is 4 or -4; but a_2 itself must be a square, since $a_2 = a_1^2$, and a square can never be negative, so $a_2 = 4$ is the only option. Then $a_1^2 = 4$ gives $a_1 = 2$ or $a_1 = -2$.
In both cases, a_1 is 2 or -2, and from $a_2 = 4$ onward the sequence is identical to the one built from statement 1: 4, 16, 256, and so on, growing without bound. Checking the only terms that could possibly be small, a_1, which is 2 or -2, shows neither is 3, and every term from

a_2 onward is 4 or larger and only increasing, so 3 is ruled out everywhere. Statement 2 alone also settles the question with a clear no.

Step 4: Final Answer:

Both statements, used completely separately, are each able to prove on their own that 3 never appears in the sequence, so both statement (1) alone and statement (2) alone are sufficient.

$$\boxed{\text{Each statement alone is sufficient}}$$

Quick Tip: Work out the sequence forward from statement 1, and work backward carefully from statement 2, remembering a square cannot be negative; both pin the sequence down completely.