XAT 2005 Quantitative Aptitude Question Paper

ZOLLEGE

General Instructions

(i) This question paper contains 50 questions. All questions are compulsory.

(ii) It comprises 50 single-correct multiple-choice questions.

(iii) Attempt every question; detailed solutions are provided in the companion solutions booklet.

1. Last year, Mr Basu bought two scooters. This year, he sold both of them for Rs 30,000 each. On one scooter, he earned a profit of 20%, and on the other, he made a loss of 20%. What was his net profit or loss?

(A) He gained less than Rs 2000

(B) He gained more than Rs 2000

(C) He lost less than Rs 2000

(D) He lost more than Rs 2000

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2. In an examination, the average marks obtained by students who passed was x%, while the average of those who failed was y%. The average marks of all the students taking the exam was z%. Find, in terms of x, y and z, the percentage of students taking the exam who failed.

(A) $\frac{z-x}{y-x}$

(B) $\frac{x-z}{y-z}$

(C) $\frac{y-x}{z-y}$

(D) $\frac{y-z}{x-z}$

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3. Three circles A, B and C have a common centre O. A is the inner circle, B is the middle circle, and C is the outer circle. P is a point on the outer circle C, and the radius OP cuts the inner circle at X and the middle circle at Y, such that $OX = XY = YP$. The ratio of the area of the region between the inner and middle circles to the area of the region between the middle and outer circles is:

(A) $\frac{1}{3}$

(B) $\frac{2}{5}$

(C) $\frac{3}{5}$

(D) $\frac{1}{5}$

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4. The sides of a rhombus ABCD measure 2 cm each, and the difference between two of its angles is $90°$. Then the area of the rhombus is:

(A) $\sqrt{2}$ sq cm

(B) $2\sqrt{2}$ sq cm

(C) $3\sqrt{2}$ sq cm

(D) $4\sqrt{2}$ sq cm

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5. If S_n denotes the sum of the first n terms of an Arithmetic Progression, and $S_1 : S_4 = 1 : 10$, then the ratio of the first term to the fourth term is:

(A) $1 : 3$

(B) $2 : 3$

(C) $1 : 4$

(D) $1 : 5$

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6. The curve $y = 4x^2$ and $y^2 = 2x$ meet at the origin O and at a point P, forming a loop. The straight line OP divides the loop into two parts. What is the ratio of the areas of the two parts of the loop?

(A) 3 : 1

(B) 3 : 2

(C) 2 : 1

(D) 1 : 1

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7. How many numbers between 1 and 1000 (both excluded) are both squares and cubes?

(A) None

(B) 1

(C) 2

(D) 3

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8. An operation \$ is defined as follows. For any two positive integers x and y,

$$x\$y = \sqrt{\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}}}$$

Which of the following is an integer?

(A) 4$9

(B) 4$16

(C) 4$4

(D) None of the above

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9. If $f(x) = \cos(x)$ then the 50th derivative of $f(x)$ is:

(A) $\sin x$

(B) $-\sin x$

(C) $\cos x$

(D) $-\cos x$

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10. If a, b and c are three real numbers, then which of the following is NOT true?

(A) $|a+b| \leq |a| + |b|$

(B) $|a-b| \leq |a| + |b|$

(C) $|a-b| \leq |a| - |b|$

(D) $|a-c| \leq |a-b| + |b-c|$

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11. If $R = \{(1,1),(2,2),(1,2),(2,1),(3,3)\}$ and $S = \{(1,1),(2,2),(2,3),(3,2),(3,3)\}$ are two relations on the set $X = \{1,2,3\}$, the incorrect statement is:

(A) R and S are both equivalence relations

(B) $R \cap S$ is an equivalence relation

(C) $R^{-1} \cap S^{-1}$ is an equivalence relation

(D) $R \cup S$ is an equivalence relation

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12. If $x > 8$ and $y > -4$, then which one of the following is always true?

(A) $xy < 0$

(B) $x^2 < -y$

(C) $-x < 2y$

(D) $x > y$

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13. For $n = 1, 2, 3, \ldots$, let $T_n = 1^3 + 2^3 + \cdots + n^3$. Which one of the following statements is correct?

(A) There is no value of n for which T_n is a positive power of 2.

(B) There is exactly one value of n for which T_n is a positive power of 2.

(C) There are exactly two values of n for which T_n is a positive power of 2.

(D) There are more than two values of n for which T_n is a positive power of 2.

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14. An equilateral triangle is formed by joining the midpoints of the sides of a given equilateral triangle. A third equilateral triangle is formed inside the second equilateral triangle in the same way, and so on. If this process continues indefinitely, then the sum of the areas of all such triangles, when the side of the first triangle is 16 cm, is:

(A) $256\sqrt{3}$ sq cm

(B) $\frac{256}{3}\sqrt{3}$ sq cm

(C) $\frac{64}{3}\sqrt{3}$ sq cm

(D) $64\sqrt{3}$ sq cm

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15. The length of the sides of a triangle are $x+1$, $9-x$ and $5x-3$. The number of values of x for which the triangle is isosceles is:

(A) 0

(B) 1

(C) 2

(D) 3

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16. The expression $\frac{x^2-2x+a^2+b^2}{x^2+2x+a^2+b^2}$ lies between:

(A) $\frac{\sqrt{a^2+b^2}+1}{\sqrt{a^2+b^2}-1}$ and $\frac{\sqrt{a^2+b^2}-1}{\sqrt{a^2+b^2}+1}$

(B) a and b

(C) $\frac{\sqrt{a^2+b^2}+1}{\sqrt{a^2+b^2}-1}$ and 1

(D) $\frac{\sqrt{a^2+b^2}-1}{\sqrt{a^2+b^2}+1}$

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17. What is the sum of the first 100 terms which are common to both the progressions 17, 21, 25, ... and 16, 21, 26, ...?

(A) 100000

(B) 101100

(C) 111000

(D) 100110

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18. Two people agree to meet on January 9, 2005 between 6:00 P.M. and 7:00 P.M., with the understanding that each will wait no longer than 20 minutes for the other. What is the probability that they will meet?

(A) $\frac{5}{9}$

(B) $\frac{7}{9}$

(C) $\frac{2}{9}$

(D) $\frac{4}{9}$

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19. If the roots of the equation $\frac{x+a}{x+a+c} + \frac{x+b}{x+b+c} = 1$ are equal in magnitude but opposite in sign, then:

(A) $c \geq a$

(B) $a \geq c$

(C) $a + b = 0$

(D) $a = b$

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20. Steel Express runs between Tatanagar and Howrah and has five stoppages in between. Find the number of different kinds of one-way second class tickets that Indian Railways will have to print to service all types of passengers who might travel by Steel Express.

(A) 49

(B) 42

(C) 21

(D) 7

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21. The horizontal distance of a kite from the boy flying it is 30 m and 50 m of cord is out from the roll. If the wind moves the kite horizontally at the rate of 5 km per hour directly away from the boy, how fast is the cord being released?

(A) 3 km per hour

(B) 4 km per hour

(C) 5 km per hour

(D) 6 km per hour

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22. Suppose S and T are sets of vectors, where $S = \{(1,0,0),(0,0,-5),(0,3,4)\}$ and $T = \{(5,2,3),(5,-3,4)\}$, then:

(A) S and T both sets are linearly independent vectors

(B) S is a set of linearly independent vectors, but T is not

(C) T is a set of linearly independent vectors, but S is not

(D) Neither S nor T is a set of linearly independent vectors

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23. Suppose the function f satisfies the equation $f(x+y) = f(x)f(y)$ for all x and y. Here $f(x) = 1 + xg(x)$, where $\lim_{x\to 0} g(x) = T$, and T is a positive integer. If $f^n(x) = kf(x)$, where $f^n(x)$ denotes the nth derivative of f, then k is equal to:

(A) T

(B) T^n

(C) $\log T$

(D) $(\log T)^n$

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24. Set of real numbers 'x, y', satisfying the inequations $x - 3y \geq 0$, $x + y \geq -2$ and $3x - y \leq -2$ is:

(A) Empty

(B) Finite

(C) Infinite

(D) Cannot be determined

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25. $ABCD$ is a trapezium, such that AB, DC are parallel and BC is perpendicular to them. If $\angle DAB = 45°$, $BC = 2$ cm and $CD = 3$ cm, then AB =?

(A) 5 cm

(B) 4 cm

(C) 3 cm

(D) 2 cm

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26. If F is a differentiable function such that $F(3) = 6$ and $F(9) = 2$, then there must exist at least one number 'a' between 3 and 9, such that:

(A) $F'(a) = \frac{3}{2}$

(B) $F(a) = -\frac{3}{2}$

(C) $F'(a) = -\frac{3}{2}$

(D) $F'(a) = -\frac{2}{3}$

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27. A conical tent of given capacity has to be constructed. The ratio of the height to the radius of the base for the minimum amount of canvas required for the tent is:

(A) $1 : 2$

(B) $2 : 1$

(C) $1 : \sqrt{2}$

(D) $\sqrt{2} : 1$

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28.

If n is a positive integer, let $S(n)$ denote the sum of the positive divisors of n, including n itself, and let $G(n)$ be the greatest divisor of n. If $H(n) = \frac{G(n)}{S(n)}$, then which of the following is the largest?

(A) $H(2009)$

(B) $H(2010)$

(C) $H(2011)$

(D) $H(2012)$

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29.

If the ratio of the roots of the equation $x^2 - 2ax + b = 0$ is equal to that of the roots of the equation $x^2 - 2cx + d = 0$, then:

(A) $a^2b = c^2d$

(B) $a^2c = b^2d$

(C) $a^2d = c^2b$

(D) $d^2b = c^2a$

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30.

X and Y are two variable quantities. The corresponding values of X and Y are given below:

X: 3, 6, 9, 12, 24
Y: 24, 12, 8, 6, 3

Then the relationship between X and Y is given by:

(A) $X + Y \propto X - Y$

(B) $X + Y \propto \frac{1}{X - Y}$

(C) $X \propto Y$

(D) $X \propto \frac{1}{Y}$

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31.

Eight sets A, B, C, D, E, F, G and H are such that:

A is a superset of B, but subset of C.
B is a subset of D, but superset of E.
F is a subset of A, but superset of B.
G is a superset of D, but subset of F.
H is a subset of B.

N(A), N(B), N(C), N(D), N(E), N(F), N(G) and N(H) are the number of elements in the sets A, B, C, D, E, F, G and H respectively.

Which one of the following could be FALSE, but not necessarily FALSE?

(A) E is a subset of D

(B) E is a subset of C

(C) E is a subset of A

(D) E is a subset of H

• • •

32.

Eight sets A, B, C, D, E, F, G and H are such that:

A is a superset of B, but subset of C.
B is a subset of D, but superset of E.
F is a subset of A, but superset of B.
G is a superset of D, but subset of F.
H is a subset of B.

N(A), N(B), N(C), N(D), N(E), N(F), N(G) and N(H) are the number of elements in the sets A, B, C, D, E, F, G and H respectively.

If P is a new set and P is a superset of A, and N(P) is the number of elements in P, then which of the following must be true?

(A) N(G) is smaller than only four numbers

(B) N(C) is the greatest

(C) N(B) is the smallest

(D) N(P) is the greatest

• • •

33.

Eight sets A, B, C, D, E, F, G and H are such that:

A is a superset of B, but subset of C.
B is a subset of D, but superset of E.
F is a subset of A, but superset of B.
G is a superset of D, but subset of F.
H is a subset of B.

N(A), N(B), N(C), N(D), N(E), N(F), N(G) and N(H) are the number of elements in the sets A, B, C, D, E, F, G and H respectively.

If Q and Z are two new sets, both supersets of H, and N(Q) and N(Z) are the number of elements of the sets Q and Z respectively, then:

(A) N(H) is the smallest of all

(B) N(E) is the smallest of all

(C) N(C) is the greatest of all

(D) Either N(H) or N(E) is the smallest

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34.

Eight sets A, B, C, D, E, F, G and H are such that:

A is a superset of B, but subset of C.
B is a subset of D, but superset of E.
F is a subset of A, but superset of B.
G is a superset of D, but subset of F.
H is a subset of B.

N(A), N(B), N(C), N(D), N(E), N(F), N(G) and N(H) are the number of elements in the sets A, B, C, D, E, F, G and H respectively.

Which of the following could be TRUE, but not necessarily TRUE?

(A) N(A) is the greatest of all

(B) N(G) is greater than N(D)

(C) N(H) is the least of all

(D) N(F) is less than or equal to N(H)

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35. If $x + y + z = 1$ and x, y, z are positive real numbers, then the least value of $\left(\frac{1}{x} - 1\right)\left(\frac{1}{y} - 1\right)\left(\frac{1}{z} - 1\right)$ is:

(A) 4

(B) 8

(C) 16

(D) None of the above

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36. ABCD is a square whose side is 2 cm each. Taking AB and AD as axes, the equation of the circle circumscribing the square is:

(A) $x^2 + y^2 = (x + y)$

(B) $x^2 + y^2 = 2(x + y)$

(C) $x^2 + y^2 = 4$

(D) $x^2 + y^2 = 16$

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37. Two players A and B play the following game. A selects an integer from 1 to 10, inclusive of both. B then adds any positive integer from 1 to 10, both inclusive, to the number selected by A. The player who reaches 46 first wins the game. If the game is played properly, A may win the game if:

(A) A selects 8 to begin with

(B) A selects 2 to begin with

(C) A selects any number greater than 5

(D) None of the above

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38.

Read the following and answer this question based on the same:

The demand for a product (Q) is related to the price (P) of the product as follows: $Q = 100 - 2P$.

The cost (C) of manufacturing the product is related to the quantity produced in the following manner: $C = Q^2 - 16Q + 2000$.

As of now the corporate profit tax rate is zero. But the Government of India is thinking of imposing 25% tax on the profit of the company.

As of now, what is the profit-maximizing output?

(A) 22

(B) 21.5

(C) 20

(D) 19

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39.

Read the following and answer this question based on the same:

The demand for a product (Q) is related to the price (P) of the product as follows: $Q = 100 - 2P$.

The cost (C) of manufacturing the product is related to the quantity produced in the following manner: $C = Q^2 - 16Q + 2000$.

As of now the corporate profit tax rate is zero. But the Government of India is thinking of imposing 25% tax on the profit of the company.

If the government imposes the 25% corporate profit tax, then what will be the profit maximizing output?

(A) 16.5

(B) 16.125

(C) 15

(D) None of the above

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40. If $X = \frac{a}{1+r} + \frac{a}{(1+r)^2} + \cdots + \frac{a}{(1+r)^n}$, then what is the value of $a + a(1+r) + a(1+r)^2 + \cdots + a(1+r)^{n-1}$?

(A) $X\left[(1+r)+(1+r)^2+\cdots+(1+r)^n\right]$

(B) $X(1+r)^n$

(C) $X \cdot \dfrac{(1+r)^n-1}{r}$

(D) $X(1+r)^{n-1}$

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41. The first negative term in the expansion $\sqrt{(1+2x)^7}$ is the:

(A) 4th term

(B) 5th term

(C) 6th term

(D) 7th term

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42. The sum of the numbers from 1 to 100, which are not divisible by 3 and 5.

(A) 2946

(B) 2732

(C) 2632

(D) 2317

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43. Five numbers A, B, C, D and E are to be arranged in an array in such a manner that they have a common prime factor between two consecutive numbers. These integers are such that: A has a prime factor P. B has two prime factors Q and R. C has two prime factors Q and S. D has two prime factors P and S. E has two prime factors P and R.

Which of the following is an acceptable order, from left to right, in which the numbers can be arranged?

(A) D, E, B, C, A

(B) B, A, E, D, C

(C) B, C, D, E, A

(D) B, C, E, D, A

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44. Five numbers A, B, C, D and E are to be arranged in an array in such a manner that they have a common prime factor between two consecutive numbers. These integers are such that: A has a prime factor P. B has two prime factors Q and R. C has two prime factors Q and S. D has two prime factors P and S. E has two prime factors P and R.

If the number E is arranged in the middle with two numbers on either side of it, all of the following must be true, EXCEPT:

(A) A and D are arranged consecutively

(B) B and C are arranged consecutively

(C) B and E are arranged consecutively

(D) A is arranged at one end in the array

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45. Five numbers A, B, C, D and E are to be arranged in an array in such a manner that they have a common prime factor between two consecutive numbers. These integers are such that: A has a prime factor P. B has two prime factors Q and R. C has two prime factors Q and S. D has two prime factors P and S. E has two prime factors P and R.

If number E is not in the list and the other four numbers are arranged properly, which of the following must be true?

(A) A and D can not be the consecutive numbers

(B) A and B are to be placed at the two ends in the array

(C) A and C are to be placed at the two ends in the array

(D) C and D can not be the consecutive numbers

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46.

Five numbers A, B, C, D and E are to be arranged in an array in such a manner that they have a common prime factor between two consecutive numbers. These integers are such that:

A has a prime factor P.
B has two prime factors Q and R.
C has two prime factors Q and S.
D has two prime factors P and S.
E has two prime factors P and R.

If number B is not in the list and other four numbers are arranged properly, which of the following must be true?

(A) A is arranged at one end in the array.

(B) C is arranged at one end in the array.

(C) D is arranged at one end in the array.

(D) E is arranged at one end in the array.

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47.

Five numbers A, B, C, D and E are to be arranged in an array in such a manner that they have a common prime factor between two consecutive numbers. These integers are such that:

A has a prime factor P.
B has two prime factors Q and R.
C has two prime factors Q and S.
D has two prime factors P and S.
E has two prime factors P and R.

If B must be arranged at one end in the array, in how many ways can the other four numbers be arranged?

(A) 1

(B) 2

(C) 3

(D) 4

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48.

Questions 48 to 50 are followed by two statements labelled as (1) and (2). You have to decide if these statements are sufficient to conclusively answer the question. Give answer:

(A) If statement (1) alone or statement (2) alone is sufficient to answer the question
(B) If you can get the answer from (1) and (2) together but neither alone is sufficient
(C) If statement 1 alone is sufficient to answer the question and statement (2) alone is also sufficient
(D) If neither statement (1) nor statement (2) is sufficient to answer the question

Around a circular table six persons A, B, C, D, E and F are sitting. Who is on the immediate left to A?

Statement 1: B is opposite to C and D is opposite to E.
Statement 2: F is on the immediate left to B and D is to the left of B.

(A) If statement (1) alone or statement (2) alone is sufficient to answer the question

(B) If you can get the answer from (1) and (2) together but neither alone is sufficient

(C) If statement 1 alone is sufficient to answer the question and statement (2) alone is also sufficient

(D) If neither statement (1) nor statement (2) is sufficient to answer the question

49.

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(A) If statement (1) alone or statement (2) alone is sufficient to answer the question
(B) If you can get the answer from (1) and (2) together but neither alone is sufficient
(C) If statement 1 alone is sufficient to answer the question and statement (2) alone is also sufficient
(D) If neither statement (1) nor statement (2) is sufficient to answer the question

A, B, C, D, E are five positive numbers.
$A + B < C + D, B + C < D + E, C + D < E + A.$
Is 'A' the greatest?

Statement 1: $D + E < A + B.$
Statement 2: $E < C.$

(A) If statement (1) alone or statement (2) alone is sufficient to answer the question

(B) If you can get the answer from (1) and (2) together but neither alone is sufficient

(C) If statement 1 alone is sufficient to answer the question and statement (2) alone is also sufficient

(D) If neither statement (1) nor statement (2) is sufficient to answer the question

50.

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(B) If you can get the answer from (1) and (2) together but neither alone is sufficient
(C) If statement 1 alone is sufficient to answer the question and statement (2) alone is also sufficient
(D) If neither statement (1) nor statement (2) is sufficient to answer the question

A sequence of numbers $a_1, a_2, \ldots$ is given by the rule $a_n^2 = a_{n+1}$. Does 3 appear in the sequence?

Statement 1: $a_1 = 2$.
Statement 2: $a_3 = 16$.

(A) If statement (1) alone or statement (2) alone is sufficient to answer the question

(B) If you can get the answer from (1) and (2) together but neither alone is sufficient

(C) If statement 1 alone is sufficient to answer the question and statement (2) alone is also sufficient

(D) If neither statement (1) nor statement (2) is sufficient to answer the question