WBBSE Madhyamik 2026 Mathematics Question Paper With Solutions

Time Allowed : 3 Hours 15 Minutes	**Maximum Marks :** 90	**Total questions :** 06

Important Instructions

1. Time allowed is **3 hours 15 minutes** and the maximum marks are **90**.
2. All questions are compulsory unless otherwise stated.
3. Question 1 contains Multiple Choice Questions (MCQs). Choose the correct answer from the given options.
4. Question 2,3,4 includes:
   - True/False questions
   - Fill in the blanks
   - Very short answer questions (one or two words)
5. Question 5,6,7,8 contains short answer type questions.
6. Question 9,10,11,12 contains long descriptive questions with internal choices.
7. Question 13,14,15 contains long answer questions. Attempt the required number as instructed.
8. Write answers only in the space provided or as instructed.

1. Choose the correct option in each case from the following questions :

(i). If a principal amount doubles itself in 10 years at simple interest, then the annual rate of interest is:

(A) 5%

(B) 10%

(C) 15%

(D) 20%

Correct Answer: (B) 10%

Solution:

Let the initial principal amount be P.

The problem states that the amount doubles itself, so the final amount (A) becomes $2P$.

The simple interest (SI) earned is calculated as: $SI = A - P$.

Substituting the values: $SI = 2P - P = P$.

The time period (T) is given as 10 years.

The formula for simple interest is: $SI = \frac{P \times R \times T}{100}$, where R is the annual rate of interest.

Substituting the known values into the formula: $P = \frac{P \times R \times 10}{100}$.

Divide both sides by P: $1 = \frac{R \times 10}{100}$.

Simplify the fraction: $1 = \frac{R}{10}$.

Rearrange to solve for R: $R = 1 \times 10 = 10$.

Therefore, the annual rate of simple interest is 10%.

> **Quick Tip**
>
> For any sum to double itself at simple interest, the rate of interest is given by the shortcut formula $R = \frac{100}{T}$. Here, $R = \frac{100}{10} = 10\%$.

(ii). For the quadratic equation $ax^2 + bx + c = 0$ where $a > 0$, the condition for the roots to be equal in magnitude but opposite in sign is:

(a) $b = 0, c = 0$

(b) $b = 0, c > 0$

(c) $b = 0, c < 0$

(d) $b > 0, c = 0$

Correct Answer: (c) $b = 0, c < 0$

Solution:

Let the two roots of the quadratic equation be α and $-\alpha$.

This satisfies the requirement that the roots are equal in magnitude but opposite in sign.

According to the relationship between roots and coefficients, the sum of the roots is given by $-b/a$.

Therefore, $\alpha + (-\alpha) = -b/a$.

$0 = -b/a \implies b = 0$.

Next, the product of the roots is given by c/a.

Therefore, $\alpha \cdot (-\alpha) = c/a$.

$-\alpha^2 = c/a$.

Since α is a real non-zero number, α^2 is positive, making $-\alpha^2$ a negative value.

This implies that $c/a < 0$.

Given the condition $a > 0$, for the fraction c/a to be negative, the numerator c must be less than zero ($c < 0$).

Thus, the required conditions for the roots to be equal in magnitude and opposite in sign are $b = 0$ and $c < 0$.

Quick Tip

When roots are equal in magnitude but opposite in sign, the sum of roots is zero, meaning the linear term coefficient b must be zero.

(iii). If the average of the numbers $6, 7, x, y, 16$ is 9, then which of the following represents the correct relationship between x and y?

(a) $x + y = 21$

(b) $x + y = 16$

(c) $x - y = 21$

(d) $x - y = 19$

Correct Answer: (b) $x + y = 16$

Solution:

The average (mean) of a set of observations is defined as the sum of all observations divided by the total number of observations.

The given set of numbers consists of five observations: $6, 7, x, y$, and 16.

The average of these numbers is provided as 9.

Using the formula: Average $= \frac{\text{Sum of observations}}{\text{Number of observations}}$.

Substituting the known values into the formula: $9 = \frac{6+7+x+y+16}{5}$.

Multiply both sides of the equation by 5 to isolate the sum: $9 \times 5 = 6 + 7 + x + y + 16$.

$45 = 29 + x + y$.

Rearrange the equation to solve for the sum of the unknown variables $(x+y)$: $x + y = 45 - 29$.

$x + y = 16$.

> Quick Tip
>
> To find the sum of any set of numbers quickly, multiply their arithmetic mean by the total count of the numbers in the set.

(iv). If an arc of a circle of length 121 cm subtends an angle of 77° at the center, then the radius of the circle will be:

(a) 110 cm

(b) 100 cm

(c) 90 cm

(d) 70 cm

Correct Answer: (c) 90 cm

Solution:
Let the radius of the circle be r cm.

The length of an arc (s) subtending an angle θ at the center is given by the formula $s = \frac{\theta}{360^\circ} \times 2\pi r$.

Given arc length $s = 121$ cm and central angle $\theta = 77°$.

Substitute these values into the formula: $121 = \frac{77°}{360°} \times 2 \times \frac{22}{7} \times r$.

Simplify the fraction $\frac{77}{7}$: $121 = \frac{11}{360} \times 2 \times 22 \times r$.

Rearrange to solve for r: $r = \frac{121 \times 360}{11 \times 2 \times 22}$.

Since $121 = 11 \times 11$, we have: $r = \frac{11 \times 360}{2 \times 22}$.

Since $22 = 11 \times 2$, we have: $r = \frac{360}{2 \times 2}$.

$r = \frac{360}{4} = 90$ cm.

Therefore, the radius of the circle is 90 cm.

> **Quick Tip**
>
> The length of an arc is directly proportional to its central angle. Use $s = r\theta$ if the angle is in radians, or $s = \frac{\theta}{360} \times 2\pi r$ if the angle is in degrees.

(v). If the length of a side of a cube is a units and the length of its diagonal is d units, then the relationship between a and d will be:

(a) $\sqrt{2}a = d$

(b) $\sqrt{3}a = d$

(c) $a = \sqrt{3}d$

(d) $a = \sqrt{2}d$

Correct Answer: (b) $\sqrt{3}a = d$

Solution:
Consider a cube with side length a.

The space diagonal of a cube connects two opposite vertices through the center of the cube.

Let the side of the cube be represented by the vector lengths in three dimensions as (a, a, a).

The length of the space diagonal d can be calculated using the Pythagorean theorem in 3D: $d = \sqrt{a^2 + a^2 + a^2}$.

$d = \sqrt{3a^2}$.

$d = \sqrt{3}a$.

Thus, the relationship is $\sqrt{3}a = d$.

Quick Tip

In a square of side a, the diagonal is $\sqrt{2}a$. In a cube of side a, the space diagonal is $\sqrt{3}a$.

(vi). In a circle with center O, $ABCD$ is a cyclic quadrilateral. BC is extended to E. If $\angle DCE = 96^\circ$, then what is the value of $\angle BOD$?

(a) 42°

(b) 84°

(c) 142°

(d) 168°

Correct Answer: (d) 168°

Solution:
In the cyclic quadrilateral $ABCD$, $\angle DCE$ is the exterior angle at vertex C.

A property of cyclic quadrilaterals is that the exterior angle at any vertex is equal to the interior opposite angle.

Therefore, $\angle BAD = \angle DCE = 96^\circ$.

Since A, B, C, and D are points on the circle, the vertices B, C, D form an arc BCD.

The sum of opposite angles in a cyclic quadrilateral is 180°, so $\angle BCD + \angle BAD = 180^\circ$.

$\angle BCD = 180^\circ - 96^\circ = 84^\circ$.

The arc BCD subtends $\angle BAD$ at the circumference, and the arc BAD subtends $\angle BCD$ at the circumference.

Consider the minor arc BCD which subtends $\angle BOD$ at the center O.

This arc subtends $\angle BAD$ on the alternate segment. However, the angle theorem states: Angle at center $= 2\times$ Angle at circumference subtended by the same arc.

Here, the angle $\angle BOD$ is subtended at the center by the arc that subtends $\angle BCD$ at the circumference (arc BAD).

So, $\angle BOD = 2 \times \angle BCD$.

$\angle BOD = 2 \times 84° = 168°$.

> **Quick Tip**
>
> For a cyclic quadrilateral, the exterior angle equals the interior opposite angle. Also, remember the central angle theorem: the angle subtended by an arc at the center is twice the angle subtended by it at any point on the remaining part of the circle.

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2. Fill up the Blanks (Any Five)

(i). If in one year the ratio of principal and amount is $8 : 9$, then the annual rate of interest is ________.

Correct Answer: 12.5%

Solution:
Let the principal (P) be $8k$ and the amount (A) after one year be $9k$.

The interest (I) earned in one year is $I = A - P = 9k - 8k = k$.

The time period (t) is given as 1 year.

The formula for simple interest is $I = \frac{P\times R\times t}{100}$, where R is the annual rate of interest.

Substituting the values: $k = \frac{8k\times R\times 1}{100}$.

Canceling k from both sides: $1 = \frac{8R}{100}$.

$8R = 100 \implies R = \frac{100}{8} = 12.5$.

Therefore, the annual rate of interest is 12.5%.

> **Quick Tip**
>
> The rate of interest can be quickly found by taking the interest as a percentage of the principal for a one-year period: $R = \frac{\text{Interest}}{\text{Principal}} \times 100$.

(ii). The conjugate surd of $(\sqrt{3} - 5)$ is ________.

Correct Answer: $-\sqrt{3} - 5$

Solution:
A conjugate surd is obtained by changing the sign of the irrational part (the surd) of a binomial expression.

In the expression $(\sqrt{3} - 5)$, the irrational part is $\sqrt{3}$ and the rational part is -5.

Changing the sign of the irrational part $\sqrt{3}$ gives $-\sqrt{3}$.

Keeping the rational part -5 unchanged, the conjugate becomes $-\sqrt{3} - 5$.

When we multiply a surd by its conjugate, the result is always a rational number: $(\sqrt{3} - 5)(-\sqrt{3} - 5) = -(3) - 5\sqrt{3} + 5\sqrt{3} + 25 = 22$.

> **Quick Tip**
>
> For a mixed surd of the form $\sqrt{a} + b$, the conjugate is $-\sqrt{a} + b$, which rationalizes the expression upon multiplication.

(iii). The two tangents drawn at the ends of a diameter of a circle are ________ to each other.

Correct Answer: Parallel

Solution:
Consider a circle with center O and a diameter AB.

Let tangents be drawn at points A and B.

A tangent at any point on a circle is perpendicular to the radius through that point of contact.

Thus, the tangent at A is perpendicular to OA, and the tangent at B is perpendicular to OB.

Since AOB is a straight line (the diameter), both tangents are perpendicular to the same straight line segment.

In a plane, two lines perpendicular to the same line are parallel to each other.

Therefore, the two tangents are parallel.

> **Quick Tip**
>
> The distance between two parallel tangents of a circle is always equal to the length of the diameter.

(iv). If $x = a\sec\theta$ and $y = b\cot\theta$, then $\frac{x^2}{a^2} - \frac{b^2}{y^2} =$ ________.

Correct Answer: 1

Solution:
Given $x = a\sec\theta$, we can write $\frac{x}{a} = \sec\theta$.

Squaring both sides gives $\frac{x^2}{a^2} = \sec^2\theta$. — (Equation 1)

Given $y = b\cot\theta$, we can write $\frac{y}{b} = \cot\theta$.

Taking the reciprocal, $\frac{b}{y} = \frac{1}{\cot\theta} = \tan\theta$.

Squaring both sides gives $\frac{b^2}{y^2} = \tan^2\theta$. — (Equation 2)

Now, subtract Equation 2 from Equation 1: $\frac{x^2}{a^2} - \frac{b^2}{y^2} = \sec^2\theta - \tan^2\theta$.

Using the trigonometric identity $\sec^2\theta - \tan^2\theta = 1$.

Therefore, $\frac{x^2}{a^2} - \frac{b^2}{y^2} = 1$.

Quick Tip

Recall the fundamental Pythagorean identities: $\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, and $1 + \cot^2\theta = \csc^2\theta$.

(v). If the radius of a solid hemisphere is $3r$, then its total surface area is ________.

Correct Answer: $27\pi r^2$

Solution:
The total surface area (TSA) of a solid hemisphere is the sum of its curved surface area and its flat circular base area.

Formula: $TSA = 2\pi R^2 + \pi R^2 = 3\pi R^2$, where R is the radius.

The problem gives the radius $R = 3r$.

Substituting this into the formula: $TSA = 3\pi(3r)^2$.

$TSA = 3\pi(9r^2)$.

$TSA = 27\pi r^2$.

The total surface area is $27\pi r^2$.

Quick Tip

Always distinguish between Curved Surface Area ($2\pi R^2$) and Total Surface Area ($3\pi R^2$) for a solid hemisphere.

(vi). The frequencies of the numbers $1, 2, 3, 4, 5$ are $1, 2, 3, 4, f$ respectively. If their arithmetic mean is 4, then the value of f is ________.

Correct Answer: 10

Solution:
The arithmetic mean ($\bar{x}$) for a frequency distribution is calculated using the formula $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$.

Sum of values multiplied by frequencies ($\sum f_i x_i$) $= (1 \times 1) + (2 \times 2) + (3 \times 3) + (4 \times 4) + (5 \times f)$.

$\sum f_i x_i = 1 + 4 + 9 + 16 + 5f = 30 + 5f$.

Total frequency ($\sum f_i$) $= 1 + 2 + 3 + 4 + f = 10 + f$.

Given the mean is 4, we have: $4 = \frac{30+5f}{10+f}$.

Multiply both sides by $(10 + f)$: $4(10 + f) = 30 + 5f$.

$40 + 4f = 30 + 5f$.

Rearrange terms: $40 - 30 = 5f - 4f$.

$f = 10$.

The value of f is 10.

Quick Tip

In frequency distribution problems, ensure you calculate the weighted sum of values correctly by multiplying each value by its corresponding frequency.

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3. Write True or False:(Any Five)

(i). $\sin^2\theta = (\sin\theta)^2$, **where** $0° < \theta < 90°$.

Correct Answer: True

Solution:
In trigonometry, the notation $\sin^2\theta$ is the standard shorthand way to represent the square of the sine of an angle θ.

Mathematically, $\sin^2\theta$ means the entire value of $\sin\theta$ is multiplied by itself.

This can be written explicitly as $(\sin\theta) \times (\sin\theta)$ or $(\sin\theta)^2$.

This convention is followed for all trigonometric functions.

Therefore, the statement is true.

💡 Quick Tip

Note the difference: $\sin^2\theta = (\sin\theta)^2$, but this is NOT equal to $\sin(\theta^2)$, where only the angle is squared.

(ii). The side length of the largest cube inscribed in a sphere of radius 4 cm is $4\sqrt{2}$ cm.

Correct Answer: False

Solution:
When a cube is inscribed in a sphere, its vertices touch the inner surface of the sphere.

The space diagonal of the cube must be equal to the diameter of the sphere.

Diameter of the sphere $= 2 \times \text{radius} = 2 \times 4 = 8$ cm.

Let the side of the cube be a. The space diagonal of a cube is given by $a\sqrt{3}$.

Thus, $a\sqrt{3} = 8 \implies a = \frac{8}{\sqrt{3}} \approx 4.62$ cm.

The statement claims the side length is $4\sqrt{2} \approx 5.66$ cm.

Since $4.62 \neq 5.66$, the statement is false.

💡 Quick Tip

For a cube inscribed in a sphere of radius R, the side $a = \frac{2R}{\sqrt{3}}$.

(iii). An angle in a circular segment greater than a semicircle is an obtuse angle.

Correct Answer: False

Solution:
An angle in a circular segment is the angle subtended by an arc at any point on the corresponding segment.

An angle in a semicircle is exactly 90°.

If the segment is greater than a semicircle, it is called a major segment.

The arc subtending the angle in the major segment is a minor arc.

A property of circles is that the angle subtended by a minor arc in the major segment is always an acute angle ($< 90°$).

Conversely, an angle in a segment smaller than a semicircle (a minor segment) is obtuse.

Therefore, the statement is false.

> **Quick Tip**
>
> Angle in major segment is acute; Angle in minor segment is obtuse; Angle in semicircle is right.

(iv). The arithmetic mean of $x-3, x-1, 7, x, 2x-1, 3x-5$ is 7.5, then their median will be 3.

Correct Answer: False

Solution:
Sum of the 6 observations $= (x-3)+(x-1)+7+x+(2x-1)+(3x-5) = 8x-3$.

Mean $= \frac{\text{Sum}}{\text{Count}} = \frac{8x-3}{6} = 7.5$.

$8x-3 = 45 \implies 8x = 48 \implies x = 6$.

Substitute $x = 6$ into the observations: $3, 5, 7, 6, 11, 13$.

Sort the observations in ascending order: $3, 5, 6, 7, 11, 13$.

For $n = 6$ observations, the median is the average of the 3^{rd} and 4^{th} terms.

Median $= \frac{6+7}{2} = 6.5$.

The statement claims the median is 3, which is incorrect. Therefore, it is false.

> **Quick Tip**
>
> Always arrange data in ascending or descending order before calculating the median.

(v). If $x \propto \frac{1}{y}$, then $(xy)^{10}$ is a constant.

Correct Answer: True

Solution:
The notation $x \propto \frac{1}{y}$ represents inverse variation between x and y.

This means $x = \frac{k}{y}$, where k is a non-zero variation constant.

Multiplying both sides by y, we get $xy = k$.

Since k is a constant, any power of k will also be a constant.

Therefore, $(xy)^{10} = k^{10}$, which is a constant.

The statement is true.

> **Quick Tip**
>
> In inverse variation, the product of the two variables remains constant ($xy =$ constant).

(vi). In a business, the ratio of capitals of Raju and Asif is $5 : 4$. If Raju gets a profit of 80 rupees, Asif gets 100 rupees.

Correct Answer: False

Solution:
In a partnership business, if time periods are the same, profit is distributed in the ratio of capitals.

$\frac{\text{Profit of Raju}}{\text{Profit of Asif}} = \frac{\text{Capital of Raju}}{\text{Capital of Asif}} = \frac{5}{4}$.

Given Raju's profit $= 80$ rupees.

Let Asif's profit be x rupees.

$\frac{80}{x} = \frac{5}{4} \implies 5x = 320 \implies x = \frac{320}{5} = 64$.

Asif should get 64 rupees, not 100 rupees.

Therefore, the statement is false.

Quick Tip

Profit $\propto$ Investment $\times$ Time. If the time is equal, profit is directly proportional to the investment.

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4. Answer Any ten Question :

(i). A and B started a business by investing Rs. 15,000 and Rs. 45,000 respectively. After 6 months B got a profit of Rs. 3,030, what is the profit of A?

Correct Answer: Rs. 1,010

Solution:
In a business partnership, profit is distributed in the ratio of the products of capitals and their respective time periods.

Given investment of $A =$ Rs. $15,000$ and investment of $B =$ Rs. $45,000$.

The ratio of investments $A : B = 15,000 : 45,000 = 1 : 3$.

Since the time period is not specified differently for A, it is assumed to be the same as B.

Thus, the profit ratio is also $1 : 3$.

Let the profit of A be x and given profit of $B =$ Rs. $3,030$.

Therefore, $\frac{x}{3{,}030} = \frac{1}{3}$.

$x = \frac{3{,}030}{3} = 1,010$.

The profit of A is Rs. $1,010$.

Quick Tip

When time periods are equal, the profit share is directly proportional to the amount invested.

(ii). In a triangle ABC, a straight line parallel to BC intersects AB and AC respectively at P and Q. If $AP = 4$ cm, $QC = 9$ cm and if $PB = AQ$ then find the value of PB.

Correct Answer: 6 cm

Solution:
According to the Basic Proportionality Theorem (Thales' Theorem), if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

In $\triangle ABC$, since $PQ \parallel BC$, we have $\frac{AP}{PB} = \frac{AQ}{QC}$.

Let the length of PB be x. Given that $PB = AQ$, then $AQ = x$.

Substituting the given values: $\frac{4}{x} = \frac{x}{9}$.

Cross-multiplying, we get $x^2 = 4 \times 9 = 36$.

Taking the square root on both sides, $x = \sqrt{36} = 6$.

The value of PB is 6 cm.

Quick Tip

In parallel line geometry within triangles, the ratio of corresponding segments created on the sides is always equal.

(iii). Two Chords AB and CD are equidistant from the centre of a circle O. If $\angle AOB = 60°$ and $CD = 6$ cm, find the radius of the circle.

Correct Answer: 6 cm

Solution:
Chords that are equidistant from the center of a circle are equal in length.

Since AB and CD are equidistant from center O, $AB = CD = 6$ cm.

In $\triangle AOB$, $OA = OB$ as they are both radii (r) of the circle.

This means $\triangle AOB$ is an isosceles triangle, and the base angles $\angle OAB$ and $\angle OBA$ are equal.

$\angle OAB = \angle OBA = \frac{180^\circ - 60^\circ}{2} = 60^\circ$.

Since all angles of $\triangle AOB$ are 60°, it is an equilateral triangle.

Therefore, $OA = OB = AB = 6$ cm.

The radius of the circle is 6 cm.

Quick Tip
An isosceles triangle with one angle equal to 60° is always an equilateral triangle.

(iv). If $\tan\theta + \cot\theta = 2$ then find the value of $\tan^7\theta + \cot^7\theta$.

Correct Answer: 2

Solution:
The given expression is $\tan\theta + \cot\theta = 2$.

We can write $\cot\theta$ as $\frac{1}{\tan\theta}$. So, $\tan\theta + \frac{1}{\tan\theta} = 2$.

Multiplying throughout by $\tan\theta$, we get $\tan^2\theta - 2\tan\theta + 1 = 0$.

This is in the form $(x-1)^2 = 0$ where $x = \tan\theta$. So, $(\tan\theta - 1)^2 = 0$.

Therefore, $\tan\theta = 1$, which implies $\theta = 45^\circ$.

Then, $\cot\theta = \frac{1}{1} = 1$.

The value of $\tan^7\theta + \cot^7\theta = (1)^7 + (1)^7 = 1 + 1 = 2$.

Quick Tip
For any positive real number x, if $x + 1/x = 2$, then x must be equal to 1.

(v). If x and y are positive real numbers then $\sec\theta = \frac{x}{y}$ is correct or not? Give answer with reason.

Correct Answer: Correct if $x \geq y$

Solution:
In a right-angled triangle, the secant of an angle θ is defined as the ratio of the hypotenuse to the adjacent side.

$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}}$.

In any right-angled triangle, the hypotenuse is the longest side, meaning Hypotenuse $\geq$ Base.

Consequently, the value of $\sec\theta$ for any real angle θ is always such that $|\sec\theta| \geq 1$.

Given $\sec\theta = \frac{x}{y}$ where $x, y > 0$, we must have $\frac{x}{y} \geq 1$.

This simplifies to $x \geq y$.

The statement is correct only under the condition that $x \geq y$.

Quick Tip

The range of the secant function is $(-\infty, -1] \cup [1, \infty)$. It can never take values between -1 and 1.

(vi). For two right circular cylinders, if ratio of their heights be $1:2$ and ratio of the circumference of the base be $3:4$ then find the ratio of their volume.

Correct Answer: $9:32$

Solution:
Let the heights of the two cylinders be h_1 and h_2, and their radii be r_1 and r_2.

The ratio of heights is given as $\frac{h_1}{h_2} = \frac{1}{2}$.

The circumference of the base is $2\pi r$. Thus, the ratio of circumferences is $\frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2} = \frac{3}{4}$.

The volume of a cylinder is $V = \pi r^2 h$.

The ratio of their volumes is $\frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$.

Substituting the ratios: $\frac{V_1}{V_2} = \left(\frac{3}{4}\right)^2 \times \frac{1}{2}$.

$\frac{V_1}{V_2} = \frac{9}{16} \times \frac{1}{2} = \frac{9}{32}$.

The ratio of their volumes is $9 : 32$.

> **Quick Tip**
>
> Volume of similar-based shapes scales linearly with height and quadratically with the radius.

(vii). Arithmetic mean of $x_1, x_2, \ldots, x_n$ is $\bar{x}$. Prove that $\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$.

Solution:
The arithmetic mean is defined as $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, which implies $\sum_{i=1}^{n} x_i = n\bar{x}$.

Expanding the expression on the left-hand side: $\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n}(x_i^2 - 2x_i\bar{x} + \bar{x}^2)$.

By the properties of summation, this can be split as: $\sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} 2x_i\bar{x} + \sum_{i=1}^{n} \bar{x}^2$.

Since $2\bar{x}$ is constant with respect to index i: $\sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} x_i + n\bar{x}^2$.

Substitute $\sum_{i=1}^{n} x_i$ with $n\bar{x}$: $\sum_{i=1}^{n} x_i^2 - 2\bar{x}(n\bar{x}) + n\bar{x}^2$.

$\sum_{i=1}^{n} x_i^2 - 2n\bar{x}^2 + n\bar{x}^2$.

$\sum_{i=1}^{n} x_i^2 - n\bar{x}^2$.

The identity is proved.

> **Quick Tip**
>
> This formula is the standard computational method for calculating variance in statistics.

(viii). If the rate of interest increases from 5.5% to 6% then the yearly interest increased by Rs. 49.50. Find the capital.

Correct Answer: Rs. 9,900

Solution:
Let the capital (Principal) be P.

The increase in the rate of interest is $6\% - 5.5\% = 0.5\%$.

Given that the yearly interest increase is Rs. 49.50.

The increase in interest is calculated as: Increase $= P \times \frac{\text{Increase in Rate}}{100} \times$ Time.

$49.50 = P \times \frac{0.5}{100} \times 1$.

$P = \frac{49.50 \times 100}{0.5} = \frac{4{,}950}{0.5}$.

$P = 9,900$.

The capital is Rs. 9,900.

Quick Tip

Interest is directly proportional to the rate; a 0.5% increase in rate implies a 0.5% increase in the absolute interest for the same principal.

(ix). If the sum of the roots of the equation $x^2 - 4x = K(x-1) - 5$ is 7 then find the value of K.

Correct Answer: 3

Solution:
First, rewrite the given equation in the standard quadratic form $ax^2 + bx + c = 0$.

$x^2 - 4x = Kx - K - 5$.

$x^2 - 4x - Kx + K + 5 = 0$.

$x^2 - (4 + K)x + (K + 5) = 0$.

For a quadratic equation $ax^2 + bx + c = 0$, the sum of roots is given by $-\frac{b}{a}$.

Here, sum of roots $= \frac{-(-(4+K))}{1} = 4 + K$.

Given that the sum of roots is 7, we have $4 + K = 7$.

$K = 3$.

The value of K is 3.

> Quick Tip
>
> Vieta's formulas state that for $ax^2 + bx + c = 0$, sum of roots $= -b/a$ and product of roots $= c/a$.

(x). If $(a+b) : \sqrt{ab} = 2 : 1$ then find $a : b$.

Correct Answer: $1 : 1$

Solution:
The given ratio is $\frac{a+b}{\sqrt{ab}} = \frac{2}{1}$.

Multiplying both sides by $\sqrt{ab}$, we get $a + b = 2\sqrt{ab}$.

Rearranging the terms: $a - 2\sqrt{ab} + b = 0$.

The left side can be written as $(\sqrt{a})^2 - 2\sqrt{a}\sqrt{b} + (\sqrt{b})^2 = 0$.

This is the expansion of a square: $(\sqrt{a} - \sqrt{b})^2 = 0$.

Taking the square root on both sides, $\sqrt{a} - \sqrt{b} = 0$, which means $\sqrt{a} = \sqrt{b}$.

Squaring again, we find $a = b$.

Therefore, the ratio $a : b = 1 : 1$.

> Quick Tip
>
> If the arithmetic mean of two numbers equals their geometric mean, then the numbers must be equal.

(xi). If the radius of a sphere be increased by 50% find the percentage increased of the volume.

Correct Answer: 237.5%

Solution:
Let the initial radius of the sphere be r. Initial volume $V_1 = \frac{4}{3}\pi r^3$.

The radius is increased by 50%. New radius $R = r + 0.5r = 1.5r$.

New volume $V_2 = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(1.5r)^3$.

$V_2 = \frac{4}{3}\pi \times 3.375 \times r^3 = 3.375V_1$.

Increase in volume $= V_2 - V_1 = 3.375V_1 - V_1 = 2.375V_1$.

Percentage increase $= \frac{\text{Increase}}{\text{Initial Volume}} \times 100\% = \frac{2.375V_1}{V_1} \times 100\%$.

Percentage increase $= 237.5\%$.

Quick Tip

For a change in a linear dimension by factor k, volume changes by factor k^3. Here $k = 1.5$, so $k^3 = 3.375$.

(xii). ABCD be a cyclic quadrilateral. If $AD = AB, \angle DAC = 60^\circ$ and $\angle BDC = 50^\circ$ then find the $\angle ACD$.

Correct Answer: 35°

Solution:
In a cyclic quadrilateral, angles subtended by the same arc at the circumference are equal.

$\angle DAC$ and $\angle DBC$ both subtend arc DC, so $\angle DBC = \angle DAC = 60^\circ$.

Similarly, $\angle BDC$ and $\angle BAC$ both subtend arc BC, so $\angle BAC = \angle BDC = 50^\circ$.

Total angle at A, $\angle DAB = \angle DAC + \angle BAC = 60^\circ + 50^\circ = 110^\circ$.

In a cyclic quadrilateral, opposite angles are supplementary. Thus, $\angle BCD = 180^\circ - \angle DAB = 180^\circ - 110^\circ = 70^\circ$.

Given $AD = AB$ in circle $ABCD$, the arcs subtended by these chords are equal (arc AD = arc AB).

Angles subtended by equal arcs at any point on the circumference are equal.

Arc AD subtends $\angle ACD$ and arc AB subtends $\angle ACB$ at vertex C.

Therefore, $\angle ACD = \angle ACB$.

Since $\angle BCD = \angle ACD + \angle ACB = 70^\circ$ and both parts are equal, $2 \times \angle ACD = 70^\circ$.

$\angle ACD = 35^\circ$.

Quick Tip
Use properties of arcs and angles in circles to relate non-adjacent angles in cyclic quadrilaterals.

5.Answer any one question

(i). If the rate of compound interest be 4% in the 1^{st} year and 5% in the 2^{nd} year, then find the interest of Rs. 25,000 for two years.

Solution:

The initial principal amount (P) is given as Rs. 25,000.

Let the rate of interest for the first year be $r_1 = 4\%$ and for the second year be $r_2 = 5\%$.

The total amount (A) after two years with different interest rates is calculated using the formula:

$A = P\left(1 + \frac{r_1}{100}\right)\left(1 + \frac{r_2}{100}\right)$.

Substituting the given values into the formula:

$A = 25,000 \times \left(1 + \frac{4}{100}\right) \times \left(1 + \frac{5}{100}\right)$.

$A = 25,000 \times \frac{104}{100} \times \frac{105}{100}$.

$A = 25,000 \times 1.04 \times 1.05 = 27,300$.

The compound interest (CI) earned is the difference between the final amount and the principal:

$CI = A - P$.

$CI = 27,300 - 25,000 = 2,300.$

Therefore, the compound interest for two years is Rs. 2,300.

Quick Tip

When rates vary annually, the final amount is the product of the principal and the growth factors for each individual year.

(ii). Three friends invest Rs. 4,800, Rs. 6,600 and Rs. 9,600 respectively in a business. 1^{st} person received $\frac{1}{8}^{th}$ of the profit as salary for looking after the business and the remaining profit was distributed among them in the ratio of their capitals. If after one year 1^{st} person received Rs. 780, find the amount received by the other two.

Solution:

First, determine the ratio of the capitals of the three friends: 4800 : 6600 : 9600.

Dividing each term by 600, the ratio simplifies to 8 : 11 : 16.

The sum of the ratio terms is $8 + 11 + 16 = 35$.

Let the total profit for the year be x rupees.

The first person receives a salary of $\frac{1}{8}$ of the total profit, which is $\frac{x}{8}$.

The remaining profit to be distributed among them is $x - \frac{x}{8} = \frac{7x}{8}$.

The first person's share from this remaining profit is $\frac{8}{35} \times \frac{7x}{8} = \frac{x}{5}$.

Total amount received by the first person = salary + share = $\frac{x}{8} + \frac{x}{5} = \frac{13x}{40}$.

According to the problem, $\frac{13x}{40} = 780$.

Solving for x: $x = \frac{780 \times 40}{13} = 60 \times 40 = 2400$.

The remaining profit for distribution is $\frac{7 \times 2400}{8} = 2100$.

Amount received by the second person $= \frac{11}{35} \times 2100 = 11 \times 60 = 660$.

Amount received by the third person $= \frac{16}{35} \times 2100 = 16 \times 60 = 960$.

Therefore, the other two persons received Rs. 660 and Rs. 960 respectively.

Quick Tip
Salary is a flat deduction from the gross profit. The "net profit" after this deduction is what gets shared according to investment ratios.

6.Answer any one question

(i). Solve the quadratic equation: $b(c-a)x^2 + c(a-b)x + a(b-c) = 0$**.**

Solution:

Let the given quadratic equation be of the form $Ax^2 + Bx + C = 0$.

Here, $A = b(c-a)$, $B = c(a-b)$, and $C = a(b-c)$.

Check the sum of the coefficients: $A + B + C = b(c-a) + c(a-b) + a(b-c)$.

Expanding the terms: $bc - ab + ac - bc + ab - ac = 0$.

In a quadratic equation, if the sum of all coefficients is zero, then $x = 1$ is always one of the roots.

Let the two roots of the equation be α and β. We have $\alpha = 1$.

The product of the roots is given by the formula $\alpha\beta = \frac{C}{A}$.

Substituting the values of α, C, and A: $1 \times \beta = \frac{a(b-c)}{b(c-a)}$.

$\beta = \frac{a(b-c)}{b(c-a)}$.

Therefore, the solutions for x are 1 and $\frac{a(b-c)}{b(c-a)}$.

Quick Tip
Identifying that the sum of coefficients is zero is the fastest way to solve complex algebraic quadratics without using the quadratic formula.

(ii). In a two-digit number, the digit in the tens place is less by 3 than the digit in the unit place. The product of the digits is less than the number by 15. Find the

number.

Solution:

Let the digit in the unit's place be y and the digit in the ten's place be x.

According to the first condition, $x = y - 3$.

The two-digit number can be represented as $10x + y$.

According to the second condition, the product of the digits is 15 less than the number: $xy = (10x + y) - 15$.

Substituting $x = y - 3$ into the equation: $(y - 3)y = 10(y - 3) + y - 15$.

Expanding the terms: $y^2 - 3y = 10y - 30 + y - 15$.

$y^2 - 3y = 11y - 45$.

Rearranging into standard quadratic form: $y^2 - 14y + 45 = 0$.

Factoring the quadratic: $(y - 9)(y - 5) = 0$.

Case 1: If $y = 9$, then $x = 9 - 3 = 6$. The number is $10(6) + 9 = 69$.

Case 2: If $y = 5$, then $x = 5 - 3 = 2$. The number is $10(2) + 5 = 25$.

Therefore, the possible numbers are 25 and 69.

Quick Tip
A two-digit number with digits x and y is always mathematically expressed as $10x + y$.

7.Answer any one question

(i). If $(x^3 + y^3) \propto (x^3 - y^3)$, then prove that $(x^2 + y^2) \propto xy$.

Solution:

Given that $(x^3 + y^3) \propto (x^3 - y^3)$.

This can be written as $\frac{x^3+y^3}{x^3-y^3} = k$, where k is a non-zero variation constant.

Using the property of Componendo and Dividendo: $\frac{(x^3+y^3)+(x^3-y^3)}{(x^3+y^3)-(x^3-y^3)} = \frac{k+1}{k-1}$.

$\frac{2x^3}{2y^3} = \frac{k+1}{k-1}$.

$\frac{x^3}{y^3} = m$ (where m is another constant).

Taking the cube root of both sides, $\frac{x}{y} = \sqrt[3]{m} = c$ (constant).

So, $x = cy$. Now consider the expression $\frac{x^2+y^2}{xy}$.

Substituting $x = cy$ into the expression: $\frac{(cy)^2+y^2}{(cy)y} = \frac{y^2(c^2+1)}{cy^2} = \frac{c^2+1}{c}$.

Since c is a constant, $\frac{c^2+1}{c}$ is also a constant, say K.

Therefore, $\frac{x^2+y^2}{xy} = K$, which implies $(x^2 + y^2) \propto xy$.

> Quick Tip
>
> When one homogeneous expression of variables is proportional to another of the same degree, the ratio of the individual variables is always constant.

(ii). If $x(2-\sqrt{3}) = y(2+\sqrt{3}) = 1$, then find the value of $3x^2 - 5xy + 3y^2$.

Solution:

From $x(2-\sqrt{3}) = 1$, we have $x = \frac{1}{2-\sqrt{3}}$.

Rationalizing the denominator: $x = \frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{2+\sqrt{3}}{4-3} = 2 + \sqrt{3}$.

Similarly, from $y(2+\sqrt{3}) = 1$, we have $y = \frac{1}{2+\sqrt{3}}$.

Rationalizing the denominator: $y = \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$.

Calculate the sum of x and y: $x + y = (2+\sqrt{3}) + (2-\sqrt{3}) = 4$.

Calculate the product of x and y: $xy = (2+\sqrt{3})(2-\sqrt{3}) = 4 - 3 = 1$.

We need to find $3x^2 - 5xy + 3y^2 = 3(x^2 + y^2) - 5xy$.

Using the identity $x^2 + y^2 = (x + y)^2 - 2xy$:

Expression $= 3[(x+y)^2 - 2xy] - 5xy = 3(x+y)^2 - 6xy - 5xy = 3(x+y)^2 - 11xy$.

Substituting the values: $3(4)^2 - 11(1) = 3(16) - 11 = 48 - 11 = 37$.

The value of the given expression is 37.

> 💡 Quick Tip
>
> Conjugate surds always yield clean integers for their sum and product. Use this to simplify algebraic expressions involving such surds.

8.Answer any one question

(i). If $\frac{a+b-c}{a+b} = \frac{b+c-a}{b+c} = \frac{c+a-b}{c+a}$ and $a+b+c \neq 0$, then prove that $a = b = c$.

Solution:

Let the given equal ratios be equal to k: $\frac{a+b-c}{a+b} = \frac{b+c-a}{b+c} = \frac{c+a-b}{c+a} = k$.

This can be written as: $1 - \frac{c}{a+b} = 1 - \frac{a}{b+c} = 1 - \frac{b}{c+a} = k$.

Subtracting 1 from each term: $\frac{-c}{a+b} = \frac{-a}{b+c} = \frac{-b}{c+a}$.

Multiplying by -1: $\frac{c}{a+b} = \frac{a}{b+c} = \frac{b}{c+a}$.

By the law of equal ratios (Addendo): Each ratio $= \frac{c+a+b}{(a+b)+(b+c)+(c+a)}$.

Each ratio $= \frac{a+b+c}{2(a+b+c)} = \frac{1}{2}$ (since $a+b+c \neq 0$).

So, $\frac{c}{a+b} = \frac{1}{2} \implies 2c = a+b$. (1)

$\frac{a}{b+c} = \frac{1}{2} \implies 2a = b+c$. (2)

$\frac{b}{c+a} = \frac{1}{2} \implies 2b = c+a$. (3)

Subtracting (2) from (1): $2(c-a) = a-c \implies 3c = 3a \implies a = c$.

Similarly, using (2) and (3), we find $a = b$.

Therefore, $a = b = c$ is proved.

💡 Quick Tip

The "sum of numerators / sum of denominators" property is the most direct way to prove equality in symmetric ratio problems.

(ii). If $x = \frac{8ab}{a+b}$, then find the value of $\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b}$.

Solution:

Given $x = \frac{8ab}{a+b}$.

We can write this as $\frac{x}{4a} = \frac{2b}{a+b}$.

Applying Componendo and Dividendo: $\frac{x+4a}{x-4a} = \frac{2b+(a+b)}{2b-(a+b)} = \frac{3b+a}{b-a}$. (1)

Again, from the given expression, $\frac{x}{4b} = \frac{2a}{a+b}$.

Applying Componendo and Dividendo: $\frac{x+4b}{x-4b} = \frac{2a+(a+b)}{2a-(a+b)} = \frac{3a+b}{a-b} = \frac{-(3a+b)}{b-a}$. (2)

Adding equations (1) and (2):

$\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b} = \frac{3b+a}{b-a} - \frac{3a+b}{b-a}$.

Expression $= \frac{3b+a-3a-b}{b-a} = \frac{2b-2a}{b-a} = \frac{2(b-a)}{b-a} = 2$.

The value of the expression is 2.

💡 Quick Tip

Expressions of the form $\frac{x+ma}{x-ma} + \frac{x+mb}{x-mb}$ where x is a harmonic mean related to a and b frequently result in the integer value 2.

9.Answer any one question

(i). Prove that the angle formed at the centre of a circle by an arc is double of the angle formed by the same arc at any point of the circle.

Solution:

Let us consider a circle with centre O.

Let arc AB subtend $\angle AOB$ at the centre and $\angle ACB$ at any point C on the remaining part of the circle.

Join CO and extend it to a point D outside the circle.

In $\triangle AOC$, $OA = OC$ (radii of the same circle).

Therefore, $\angle OAC = \angle OCA$ (angles opposite to equal sides are equal).

In $\triangle AOC$, exterior angle $\angle AOD = \angle OAC + \angle OCA = 2\angle OCA$ — (1).

Similarly, in $\triangle BOC$, $OB = OC$ (radii of the same circle).

Therefore, $\angle OBC = \angle OCB$.

In $\triangle BOC$, exterior angle $\angle BOD = \angle OBC + \angle OCB = 2\angle OCB$ — (2).

Adding equations (1) and (2), we get: $\angle AOD + \angle BOD = 2\angle OCA + 2\angle OCB$.

$\angle AOB = 2(\angle OCA + \angle OCB) = 2\angle ACB$.

Thus, the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Quick Tip

This central angle theorem is the most fundamental property of circle geometry and serves as the basis for proving that angles in the same segment are equal.

(ii). If two circles touch each other, prove that the point of contact lies on the line joining the centre of two circles.

Solution:

Let two circles with centres A and B touch each other at a point P.

The circles can touch each other either internally or externally.

Since they touch at point P, there exists a common tangent line L passing through P.

The radius AP of the first circle drawn to the point of contact P is perpendicular to the tangent L ($\angle APL = 90^\circ$).

Similarly, the radius BP of the second circle drawn to the point of contact P is also perpendicular to the tangent L ($\angle BPL = 90^\circ$).

At a point P on a line L, there can be only one line perpendicular to L.

Therefore, the segments AP and BP must lie on the same straight line.

This means the points A, P, and B are collinear.

Thus, the point of contact P lies on the line segment joining the centres A and B.

> Quick Tip
>
> When two circles touch externally, the distance between centres is $r_1 + r_2$; when they touch internally, it is $|r_1 - r_2|$.

10.Answer any one question

(i). In the isosceles triangle ABC, $\angle B$ is a right angle. Bisector of the $\angle BAC$ intersects BC at D. Prove that $CD^2 = 2BD^2$.

Solution:

In $\triangle ABC$, $\angle B = 90^\circ$ and since it is isosceles, $AB = BC$.

Let $AB = BC = x$. Then by Pythagoras theorem, $AC^2 = AB^2 + BC^2 = x^2 + x^2 = 2x^2$.

Therefore, $AC = \sqrt{2}x = \sqrt{2}AB$.

AD is the bisector of $\angle BAC$ meeting BC at D.

According to the Interior Angle Bisector Theorem, the bisector of an angle of a triangle divides the opposite side into segments that are proportional to the adjacent sides.

$\frac{BD}{CD} = \frac{AB}{AC}$.

Substituting the value of AC: $\frac{BD}{CD} = \frac{AB}{\sqrt{2}AB} = \frac{1}{\sqrt{2}}$.

Squaring both sides of the ratio: $\left(\frac{BD}{CD}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2$.

$\frac{BD^2}{CD^2} = \frac{1}{2} \implies CD^2 = 2BD^2$.

Hence, the result is proved.

> Quick Tip
>
> The Angle Bisector Theorem relates the lengths of the segments of the opposite side to the other two sides of the triangle.

(ii). O is a point inside a rectangle $ABCD$. Prove that $OA^2 + OC^2 = OD^2 + OB^2$.

Solution:

Let $ABCD$ be a rectangle. Draw a line PQ passing through O such that PQ is parallel to AB and CD.

Let P lie on AD and Q lie on BC.

Since $PQ \parallel AB \parallel CD$ and $AD \perp AB$, we have $PQ \perp AD$ and $PQ \perp BC$.

This makes $\triangle OPA, \triangle OPD, \triangle OQB$ and $\triangle OQC$ right-angled triangles.

In $\triangle OPA$: $OA^2 = OP^2 + AP^2$ — (1).

In $\triangle OQC$: $OC^2 = OQ^2 + CQ^2$ — (2).

Adding (1) and (2): $OA^2 + OC^2 = OP^2 + AP^2 + OQ^2 + CQ^2$.

Since $APQB$ is a rectangle, $AP = BQ$ and since $PQCD$ is a rectangle, $CQ = DP$.

So, $OA^2 + OC^2 = OP^2 + BQ^2 + OQ^2 + DP^2$.

Rearranging: $OA^2 + OC^2 = (OQ^2 + BQ^2) + (OP^2 + DP^2)$.

In $\triangle OQB$: $OB^2 = OQ^2 + BQ^2$.

In $\triangle OPD$: $OD^2 = OP^2 + DP^2$.

Substituting these: $OA^2 + OC^2 = OB^2 + OD^2$.

Hence proved.

Quick Tip

This property is known as the British Flag Theorem, and it holds for any point O relative to the rectangle (inside, outside, or on the boundary).

11.Answer any one question

(i). In $\triangle ABC$ the base $BC = 6$ cm, $\angle ABC = 60°$ and $AB = 8$ cm. Draw the circumcircle of the triangle.

Solution:

Step 1: Draw a line segment AB of length 8 cm.

Step 2: At point B, draw an angle $\angle ABX = 60°$ using a protractor or compass.

Step 3: From the ray BX, cut off a segment $BC = 6$ cm. Join AC to complete $\triangle ABC$.

Step 4: Construct the perpendicular bisectors of any two sides of the triangle, for example, AB and BC.

Step 5: Let these two perpendicular bisectors intersect at a point O. This point O is the circumcenter of the triangle.

Step 6: Taking O as the centre and OA (or OB or OC) as the radius, draw a circle.

The circle will pass through all three vertices A, B and C. This is the required circumcircle.

Quick Tip

The circumcenter is the unique point equidistant from all vertices of a triangle, formed by the intersection of the perpendicular bisectors of its sides.

(ii). Construct a square of equal area of an equilateral triangle of side 6 cm.

Solution:

Step 1: Construct an equilateral triangle of side 6 cm. Let its area be $A_T = \frac{\sqrt{3}}{4} \times 6^2 = 9\sqrt{3}$ cm^2.

Step 2: Construct a rectangle of the same area as the triangle. The area of a triangle is $\frac{1}{2} \times \text{base} \times \text{height}$.

Draw a rectangle with one side equal to half the base (3 cm) and the other side equal to the altitude of the triangle ($3\sqrt{3}$ cm ≈ 5.2 cm).

Step 3: Let the sides of the rectangle be a and b. To construct a square of equal area, we need to find the side x such that $x^2 = ab$, where x is the mean proportional.

Step 4: Draw a line segment of length $(a + b)$. On this segment as diameter, construct a semi-circle.

Step 5: At the point where segments a and b meet, erect a perpendicular line to meet the semi-circle at a point P.

Step 6: The length of this perpendicular segment is $x = \sqrt{ab}$, which is the side of the required square.

Step 7: Using this length x, construct a square. This square has the same area as the equilateral triangle.

Quick Tip

The geometric construction of the mean proportional ($\sqrt{ab}$) is the standard way to transform any rectangle (and thus any polygon) into a square of equal area.

12.Answer any two question

(i). If the ratio of three angles of a triangle is $2 : 3 : 4$ then determine the circular value of the greatest angle.

Solution:

Let the three angles of the triangle be $2x, 3x$, and $4x$.

According to the angle sum property of a triangle, the sum of all interior angles is 180°.

$2x + 3x + 4x = 180^\circ$.

$9x = 180^\circ \implies x = 20^\circ$.

The greatest angle is $4x = 4 \times 20^\circ = 80^\circ$.

To find the circular value (value in radians), we use the conversion factor π radians $= 180^\circ$.

Value in radians $= 80 \times \frac{\pi}{180}$.

Simplifying the fraction by dividing by 20: $\frac{4\pi}{9}$.

The circular value of the greatest angle is $\frac{4\pi}{9}$ radians.

> Quick Tip
>
> To convert degrees to radians, multiply by $\frac{\pi}{180}$. To convert radians to degrees, multiply by $\frac{180}{\pi}$.

(ii). If $\tan\theta = \frac{4}{3}$, find the value of $\sin\theta + \cos\theta$.

Solution:

Given $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{4}{3}$.

Let Perpendicular $(P) = 4k$ and Base $(B) = 3k$.

Using Pythagoras theorem, Hypotenuse $(H) = \sqrt{P^2 + B^2}$.

$H = \sqrt{(4k)^2 + (3k)^2} = \sqrt{16k^2 + 9k^2} = \sqrt{25k^2} = 5k$.

Now, $\sin\theta = \frac{P}{H} = \frac{4k}{5k} = \frac{4}{5}$.

And, $\cos\theta = \frac{B}{H} = \frac{3k}{5k} = \frac{3}{5}$.

The required value is $\sin\theta + \cos\theta = \frac{4}{5} + \frac{3}{5}$.

$\sin\theta + \cos\theta = \frac{7}{5}$.

> Quick Tip
>
> The $(3, 4, 5)$ Pythagorean triplet is the most common ratio in trigonometry problems. If $\tan\theta = 4/3$, then $\sin\theta = 4/5$ and $\cos\theta = 3/5$ instantly.

(iii). If A and B are two complementary angles then prove that $(\sin A + \cos A)^2 = 1 + 2\sin A \sin B$.

Solution:

Two angles A and B are complementary if their sum is 90°.

$A + B = 90^\circ \implies B = 90^\circ - A.$

Starting from the Left Hand Side (LHS):

$\text{LHS} = (\sin A + \cos A)^2.$

Expanding using the identity $(a+b)^2 = a^2 + b^2 + 2ab$:

$\text{LHS} = \sin^2 A + \cos^2 A + 2\sin A \cos A.$

Using the fundamental identity $\sin^2\theta + \cos^2\theta = 1$:

$\text{LHS} = 1 + 2\sin A \cos A.$

Now, consider the relation between complementary angles: $\sin(90^\circ - \theta) = \cos\theta$.

Since $B = 90^\circ - A$, we have $\sin B = \sin(90^\circ - A) = \cos A$.

Substitute $\cos A = \sin B$ into the LHS expression:

$\text{LHS} = 1 + 2\sin A \sin B.$

$\text{LHS} = \text{RHS}.$

Hence, the result is proved.

Quick Tip
For complementary angles, the sine of one is equal to the cosine of the other ($\sin A = \cos B$ and $\cos A = \sin B$).

13.Answer any one question :

(i). From the roof of the building the angle of depression of the top and foot

of a lamp post are 30° and θ° respectively. If the ratio of the height of the building and the height of the lamp post be $3:2$, then find the value of θ.

Solution:
Let the height of the building be H and the height of the lamp post be h.

According to the given ratio, $H : h = 3 : 2$. Let $H = 3k$ and $h = 2k$.

Let the distance between the building and the lamp post be x.

The top of the building is at a height $H - h = 3k - 2k = k$ above the top of the lamp post.

Given the angle of depression of the top of the lamp post is 30°: $\tan 30^\circ = \frac{k}{x}$.

Since $\tan 30^\circ = \frac{1}{\sqrt{3}}$, we have $\frac{1}{\sqrt{3}} = \frac{k}{x} \implies x = k\sqrt{3}$.

Given the angle of depression of the foot of the lamp post is θ: $\tan\theta = \frac{H}{x}$.

Substituting the values: $\tan\theta = \frac{3k}{k\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$.

We know that $\tan 60^\circ = \sqrt{3}$. Therefore, $\theta = 60^\circ$.

Quick Tip

Angles of depression are numerically equal to angles of elevation. Drawing a horizontal line from the observation point helps visualize the right-angled triangles easily.

(ii). From the foot of a Tilla the angle of elevation of its top is 45°. By moving 100 m towards the Tilla along a slope of 30°, the angle of elevation of the top becomes 60°. Find the height of the Tilla.

Solution:
Let the height of the Tilla be h. From the initial position, the angle of elevation is 45°.

In the first triangle: $\tan 45^\circ = \frac{h}{\text{base}} \implies 1 = \frac{h}{\text{base}} \implies \text{base} = h$.

Now, the person moves 100 m along a 30° slope.

Horizontal distance covered $= 100 \times \cos 30^\circ = 100 \times \frac{\sqrt{3}}{2} = 50\sqrt{3}$ m.

Vertical distance covered $= 100 \times \sin 30^\circ = 100 \times \frac{1}{2} = 50$ m.

The new position relative to the top: Height $= h - 50$ and Horizontal distance $= h - 50\sqrt{3}$.

For the new angle of elevation: $\tan 60° = \frac{h-50}{h-50\sqrt{3}}$.

$\sqrt{3} = \frac{h-50}{h-50\sqrt{3}} \implies h\sqrt{3} - 150 = h - 50$.

$h(\sqrt{3} - 1) = 100 \implies h = \frac{100}{\sqrt{3}-1}$.

Rationalizing: $h = \frac{100(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{100(\sqrt{3}+1)}{3-1} = 50(\sqrt{3} + 1)$ m.

> **Quick Tip**
>
> When movement occurs along a slope, decompose the slant distance into horizontal and vertical components to find the change in the observer's relative position.

14.Answer any two question

(i). The ratio of the length, breadth and height of a solid rectangular parallelepiped is $4 : 3 : 2$ and area of the whole surface is 468 sq. cm. Find the volume of the parallelepiped.

Solution:
Let the length (l), breadth (b), and height (h) be $4k$, $3k$, and $2k$ respectively.

The formula for the total surface area (TSA) is $2(lb + bh + lh)$.

$2[(4k)(3k) + (3k)(2k) + (4k)(2k)] = 468$.

$2[12k^2 + 6k^2 + 8k^2] = 468$.

$2[26k^2] = 468 \implies 52k^2 = 468$.

$k^2 = \frac{468}{52} = 9 \implies k = 3$.

Now, find the dimensions: $l = 4(3) = 12$ cm, $b = 3(3) = 9$ cm, and $h = 2(3) = 6$ cm.

Volume $= l \times b \times h = 12 \times 9 \times 6 = 648$ cm^3.

Quick Tip

Always assign a common variable (like k) to ratio terms to convert them into actual dimensions for substitution in geometric formulas.

(ii). The internal and external radius of a hollow cylinder of height 20 cm are 4 cm and 5 cm respectively. By melting this cylinder a solid cone of height equal to one third height of the cylinder is formed. Find the diameter of the base of the cone.

Solution:
Volume of the material in the hollow cylinder $= \pi h(R^2 - r^2)$.

Substituting values: $V = \pi \times 20 \times (5^2 - 4^2) = 20\pi \times (25 - 16) = 180\pi$ cm^3.

The height of the cone (h_c) is one-third of the cylinder's height: $h_c = \frac{1}{3} \times 20 = \frac{20}{3}$ cm.

Let the radius of the cone be R_c. Volume of the cone $= \frac{1}{3}\pi R_c^2 h_c$.

Equating the volumes: $\frac{1}{3}\pi R_c^2 \times \frac{20}{3} = 180\pi$.

$\frac{20}{9}R_c^2 = 180 \implies R_c^2 = \frac{180\times 9}{20} = 81$.

$R_c = \sqrt{81} = 9$ cm.

Diameter of the cone's base $= 2 \times R_c = 2 \times 9 = 18$ cm.

Quick Tip

In melting and recasting problems, the volume of the material remains constant. Equate the formulas for the initial and final shapes to find the unknown dimensions.

14(iii). A hemispherical bowl of internal radius 9 cm is full of water. How many cylindrical bottles of diameter 3 cm and height 4 cm are required to fill this water?

Solution:
The volume of water in the hemispherical bowl $= \frac{2}{3}\pi R^3 = \frac{2}{3}\pi \times 9^3 = 486\pi$ cm^3.

For the cylindrical bottles, the diameter is 3 cm, so the radius (r) is 1.5 cm. Height (h) is 4 cm.

The volume of one cylindrical bottle $= \pi r^2 h = \pi \times (1.5)^2 \times 4 = 9\pi$ cm^3.

Number of bottles required $= \frac{\text{Total Volume}}{\text{Volume of one bottle}}$.

Number of bottles $= \frac{486\pi}{9\pi} = 54$.

> 💡 Quick Tip
>
> To find the number of containers needed, simply divide the total volume of the liquid by the volume of a single container.

15.Answer any two question

(i). Find the arithmetic mean of the following distribution:

Class	5-14	15-24	25-34	35-44	45-54	55-64
Frequency	3	6	18	20	10	3

Solution:

First, calculate the mid-values (x) of each class:
9.5, 19.5, 29.5, 39.5, 49.5, 59.5.

Now, calculate $f \times x$ for each class:
$(3 \times 9.5) = 28.5$.
$(6 \times 19.5) = 117.0$.
$(18 \times 29.5) = 531.0$.
$(20 \times 39.5) = 790.0$.
$(10 \times 49.5) = 495.0$.
$(3 \times 59.5) = 178.5$.

Sum of frequencies $(\sum f) = 3 + 6 + 18 + 20 + 10 + 3 = 60$.

Sum of $f \times x$ $(\sum fx) = 28.5 + 117 + 531 + 790 + 495 + 178.5 = 2140$.

Arithmetic Mean $= \frac{\sum fx}{\sum f} = \frac{2140}{60} \approx 35.67$.

> 💡 Quick Tip
>
> When using the direct method for mean, the mid-value of a class interval is found by taking the average of the upper and lower limits.

(ii). Making cumulative frequency (greater than type) distribution table of given data, draw Ogive on graph paper:

Class	100-120	120-140	140-160	160-180	180-200
Frequency	8	14	10	12	4

Solution:
To construct the greater than type cumulative frequency table, we sum the frequencies from the bottom up or subtract from the total.

The total number of observations (N) is $8 + 14 + 10 + 12 + 4 = 48$.

We use the lower class limits to define the "greater than or equal to" categories.

For the class 100-120, the lower limit is 100. There are 48 observations ≥ 100.

For the class 120-140, the lower limit is 120. There are $48 - 8 = 40$ observations ≥ 120.

For the class 140-160, the lower limit is 140. There are $40 - 14 = 26$ observations ≥ 140.

For the class 160-180, the lower limit is 160. There are $26 - 10 = 16$ observations ≥ 160.

For the class 180-200, the lower limit is 180. There are $16 - 12 = 4$ observations ≥ 180.

Finally, for the upper limit of the last class, there are $4 - 4 = 0$ observations ≥ 200.

The points (x, y) to be plotted on the graph for the Ogive are:
$(100, 48), (120, 40), (140, 26), (160, 16), (180, 4)$, and $(200, 0)$.

Plotting these points on a graph paper and joining them with a smooth freehand curve gives the required greater than type Ogive.

> **Quick Tip**
>
> In a "greater than type" Ogive, the curve is always downward sloping because as the lower limit increases, the number of observations exceeding that limit naturally decreases.

(iii). Find the mode of the following frequency distribution:

Marks obtained	less than 10	less than 20	less than 30	less than 40	less than 50	less than 60
Number of Students	8	15	29	42	60	70

Solution:

The given data is in a cumulative "less than" form. First, we must convert it into a standard class-frequency distribution.

Class 0-10: Frequency $= 8$.

Class 10-20: Frequency $= 15 - 8 = 7$.

Class 20-30: Frequency $= 29 - 15 = 14$.

Class 30-40: Frequency $= 42 - 29 = 13$.

Class 40-50: Frequency $= 60 - 42 = 18$.

Class 50-60: Frequency $= 70 - 60 = 10$.

The highest frequency is 18, which corresponds to the modal class 40-50.

Lower limit of modal class $(l) = 40$.

Frequency of modal class $(f_1) = 18$.

Frequency of class preceding modal class $(f_0) = 13$.

Frequency of class following modal class $(f_2) = 10$.

Width of class interval $(h) = 10$.

Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$.

Mode $= 40 + \left(\frac{18-13}{2(18)-13-10}\right) \times 10$.

Mode $= 40 + \left(\frac{5}{36-23}\right) \times 10 = 40 + \frac{50}{13}$.

Mode $\approx 40 + 3.846 = 43.85$.

> 💡 Quick Tip
>
> When data is given in a cumulative frequency form (like "less than"), always convert it back to discrete class frequencies before applying the standard mode or median formulas.

[**Alternative Question for Sightless Candidates**]

11. Answer any one question :

(i). Describe the process of drawing a circumcircle of a triangle.

Solution:

Step 1: First, draw the given triangle ABC accurately based on the provided dimensions of sides or measures of angles.

Step 2: Construct the perpendicular bisectors of at least two sides of the triangle, for example, sides AB and BC.

Step 3: To draw a perpendicular bisector, use a compass to draw arcs of the same radius (greater than half the side's length) from both endpoints of the side.

Step 4: Join the two points where these arcs intersect. This line is the perpendicular bisector of that side.

Step 5: Mark the point where the two perpendicular bisectors intersect. This unique point is known as the circumcenter, denoted as O.

Step 6: Place the needle of the compass on the circumcenter O and adjust the pencil to touch any one of the vertices (A, B, or C).

Step 7: This distance (OA) is the circumradius. With O as the center, draw a circle that will pass through all three vertices of the triangle.

This circle is the required circumcircle of triangle ABC.

> 💡 Quick Tip
>
> For an acute-angled triangle, the circumcenter lies inside the triangle; for a right-angled triangle, it lies on the midpoint of the hypotenuse; for an obtuse-angled triangle, it lies outside.

(ii). Describe the process of drawing a square of equal area of an equilateral triangle.

Solution:

Step 1: Construct the given equilateral triangle ABC with the specified side length.

Step 2: Draw an altitude from vertex A to the base BC, meeting it at point D. Point D is the midpoint of BC.

Step 3: Construct a rectangle with one side equal to the altitude AD and the adjacent side equal to half of the base BC (i.e., BD).

Step 4: The area of this rectangle is (Altitude) $\times$ ($\frac{1}{2}$ $\times$ Base), which is equal to the area of the triangle. Let the sides of this rectangle be a and b.

Step 5: Extend the side of length a by a distance equal to length b along a straight line, forming a single segment of length $(a + b)$.

Step 6: Bisect this combined segment $(a + b)$ to find its midpoint and draw a semi-circle with this segment as the diameter.

Step 7: At the junction point where segments a and b meet, draw a perpendicular line upwards to intersect the circumference of the semi-circle.

Step 8: The length of this perpendicular segment represents the mean proportional $\sqrt{ab}$, which is the side length of the required square.

Step 9: Construct a square using this perpendicular segment as its side. This square has an area exactly equal to the original equilateral triangle.

Quick Tip

This geometric method utilizes the theorem of mean proportion in a circle ($h^2 = p \cdot q$) to convert any rectangular area into a square area.