



General Instructions

- (i) **Duration:** The total duration of the examination is 2.5 hours (150 minutes).
- (ii) **Total Marks:** The complete paper carries a maximum of 500 marks.
- (iii) **Structure:** The paper has 4 Sections:
 - **Part 1:** 35 Multiple Choice Questions (Physics).
 - **Part 2:** 35 Multiple Choice Questions (Chemistry).
 - **Part 3:** 40 Multiple Choice Questions (Mathematics/Biology).
 - **Part 4:** 10 Multiple Choice Questions (Aptitude).
 - **Part 4:** 5 Multiple Choice Questions (English)
- (iv) **Compulsory Questions:** All 125 questions are compulsory.
- (v) Each question has four options. Only **one** option is correct.
- (vi) **Correct Answer:** +4 marks.
- (vii) **Incorrect Answer:** -1 (Negative marking).
- (viii) **Unanswered/Marked for Review:** 0 marks.

1. Find the value of the integral $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$.

- (A) π
- (B) $\pi/2$
- (C) $\pi/4$
- (D) 0

Correct Answer: (C) $\pi/4$

Solution:

Step 1: Understanding the Question:

The objective is to evaluate a definite integral involving trigonometric functions in the denominator and numerator over the interval from 0 to $\pi/2$.

The structure of the integrand suggests the use of symmetry properties to simplify the calculation.

Step 2: Key Formula or Approach:

The primary tool used here is the Reflection Property (also known as King's Property) of definite integrals:

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

For the interval $[0, \pi/2]$, the substitution is $x \rightarrow \pi/2 - x$.

Step 3: Detailed Explanation:

Let the given integral be I :

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \dots(1)$$

Applying the property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$, we replace x with $\pi/2 - x$:

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\sin(\pi/2 - x)} + \sqrt{\cos(\pi/2 - x)}} dx$$

Using the trigonometric identities $\sin(\pi/2 - x) = \cos x$ and $\cos(\pi/2 - x) = \sin x$, we get:

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \dots(2)$$

Now, add equation (1) and equation (2):

$$2I = \int_0^{\pi/2} \left(\frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \right) dx$$

$$2I = \int_0^{\pi/2} 1 dx$$

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0$$

$$I = \frac{\pi}{4}$$

Step 4: Final Answer:

The value of the integral is $\pi/4$.

Quick Tip: Whenever you see an integral from 0 to $\pi/2$ with a symmetric structure like $\frac{\sin^n x}{\sin^n x + \cos^n x}$,

the answer is almost always half the upper limit, which is $\pi/4$.

This applies to any power n , including square roots or large integers.

2. If A is a 3×3 matrix and $|A| = 5$, find the value of $|adj(A)|$.

- (A) 5
- (B) 25
- (C) 125
- (D) 15

Correct Answer: (B) 25

Solution:

Step 1: Understanding the Question:

The problem asks for the determinant of the adjoint of a matrix A , given the determinant of the matrix itself and its order.

Step 2: Key Formula or Approach:

The relationship between the determinant of a matrix and the determinant of its adjoint is given by the formula:

$$|adj(A)| = |A|^{n-1}$$

where n is the order of the square matrix A .

Step 3: Detailed Explanation:

From the question, we have:

The determinant of matrix A , $|A| = 5$.

The order of the matrix, $n = 3$.

Applying the formula:

$$|adj(A)| = 5^{3-1}$$

$$|adj(A)| = 5^2$$

$$|adj(A)| = 25$$

Step 4: Final Answer:

The value of $|adj(A)|$ is 25.

Quick Tip: Memorize the property $|adj(A)| = |A|^{n-1}$ as it is a frequent favorite in competitive exams. If the question asks for $|adj(adj(A))|$, the formula becomes $|A|^{(n-1)^2}$.

3. Calculate the de Broglie wavelength of an electron accelerated through a potential difference of 100 V.

- (A) $1.227 \text{ \AA}$
- (B) $12.27 \text{ \AA}$
- (C) $0.1227 \text{ \AA}$
- (D) $122.7 \text{ \AA}$

Correct Answer: (A) $1.227 \text{ \AA}$

Solution:

Step 1: Understanding the Question:

We need to find the de Broglie wavelength of an electron when it is given kinetic energy by an accelerating electric potential V .

Step 2: Key Formula or Approach:

The de Broglie wavelength λ for an electron accelerated through a potential difference V (in Volts) can be calculated using the simplified formula:

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ \AA}$$

Alternatively, in nanometers, it is $\lambda = \frac{1.227}{\sqrt{V}} \text{ nm}$.

Step 3: Detailed Explanation:

Given the potential difference $V = 100\text{ V}$.

Using the formula:

$$\lambda = \frac{12.27}{\sqrt{100}} \text{ \AA}$$

$$\lambda = \frac{12.27}{10} \text{ \AA}$$

$$\lambda = 1.227 \text{ \AA}$$

Step 4: Final Answer:

The de Broglie wavelength is $1.227 \text{ \AA}$.

Quick Tip: For electrons, always use the shortcut $\lambda \approx \sqrt{150/V}$ or $12.27/\sqrt{V}$ to save time.

Ensure that the potential V is in Volts before plugging it into the square root.

4. Calculate the work done in moving a charge of $2\mu\text{C}$ between two points with a potential difference of 5 V .

- (A) 10 J
- (B) 10^{-5} J
- (C) 2.5 J
- (D) 0.4 J

Correct Answer: (B) 10^{-5} J

Solution:

Step 1: Understanding the Question:

The question asks for the energy or work required to transport a specific amount of electric charge across a known potential difference.

Step 2: Key Formula or Approach:

The relationship between work (W), charge (q), and potential difference (ΔV) is:

$$W = q \times \Delta V$$

Step 3: Detailed Explanation:

Given values:

Charge $q = 2\mu C = 2 \times 10^{-6} C$.

Potential difference $\Delta V = 5 V$.

Calculation:

$$W = (2 \times 10^{-6} C) \times (5 V)$$

$$W = 10 \times 10^{-6} J$$

$$W = 10^{-5} J$$

Step 4: Final Answer:

The work done is $10^{-5} J$.

Quick Tip: Pay close attention to unit prefixes like micro ($\mu = 10^{-6}$), nano ($n = 10^{-9}$), and milli ($m = 10^{-3}$).

Work done is also equal to the change in electrostatic potential energy.

5. Calculate the half-life of a first-order reaction if the rate constant is 0.693 min^{-1} .

- (A) 10 min
- (B) 1 min
- (C) 0.5 min
- (D) 6.93 min

Correct Answer: (B) 1 min

Solution:

Step 1: Understanding the Question:

We are required to determine the time taken for the concentration of a reactant to reduce to half its initial value for a first-order kinetics process.

Step 2: Key Formula or Approach:

For a first-order reaction, the half-life ($t_{1/2}$) is related to the rate constant (k) by the expression:

$$t_{1/2} = \frac{\ln(2)}{k} \approx \frac{0.693}{k}$$

Step 3: Detailed Explanation:

The given rate constant $k = 0.693 \text{ min}^{-1}$.

Plugging this into the half-life formula:

$$t_{1/2} = \frac{0.693}{0.693 \text{ min}^{-1}}$$

$$t_{1/2} = 1 \text{ min}$$

Step 4: Final Answer:

The half-life of the reaction is 1 minute.

Quick Tip: In first-order reactions, the half-life is constant and independent of the initial concentration. If the rate constant is exactly 0.693 or a multiple of it, the calculation becomes very simple.

6. What is the major product formed when Propene reacts with HBr in the presence of peroxide?

- (A) 2-Bromopropane
- (B) 1-Bromopropane
- (C) 1,2-Dibromopropane
- (D) Propane

Correct Answer: (B) 1-Bromopropane

Solution:

Step 1: Understanding the Question:

The question asks for the outcome of an addition reaction between an asymmetric alkene (propene) and hydrogen bromide under specific radical-forming conditions (peroxide).

Step 2: Key Formula or Approach:

Normally, addition follows Markovnikov's rule. However, in the presence of organic peroxides, HBr adds via a free-radical mechanism.

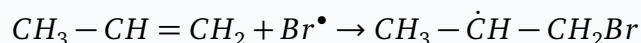
This is known as the Anti-Markovnikov addition or the Kharasch effect.

Step 3: Detailed Explanation:

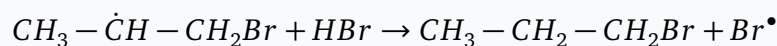
Propene structure: $\text{CH}_3 - \text{CH} = \text{CH}_2$.

In the peroxide-induced mechanism, the bromine radical attacks the terminal carbon to form a more stable secondary carbon radical.

1. Initiation: Peroxide decomposes to form radicals.
2. Propagation: $\text{Br}^\bullet$ adds to the terminal carbon of propene.



3. The resulting secondary radical then abstracts a hydrogen from another HBr molecule.



The final product is 1-bromopropane.

Step 4: Final Answer:

The major product is 1-bromopropane.

Quick Tip: The Peroxide Effect is only valid for HBr .

If the reagent was HCl or HI , even in the presence of peroxide, the product would still follow Markovnikov's rule (2-chloropropane or 2-iodopropane).

7. Complete the series: 3, 7, 15, 31, 63, ...

- (A) 95
- (B) 111
- (C) 127
- (D) 128

Correct Answer: (C) 127

Solution:

Step 1: Understanding the Question:

The goal is to identify the logical progression in the given sequence of numbers to find the next term.

Step 2: Key Formula or Approach:

Examine the difference between consecutive terms or look for a multiplier-addition pattern.

Pattern 1: $\text{Term}_n \times 2 + 1$.

Pattern 2: Difference increases as powers of 2 (4, 8, 16, 32...).

Step 3: Detailed Explanation:

Let's check the multiplier pattern:

$$3 \times 2 + 1 = 7$$

$$7 \times 2 + 1 = 15$$

$$15 \times 2 + 1 = 31$$

$$31 \times 2 + 1 = 63$$

The pattern is consistent. Following this for the next term:

$$63 \times 2 + 1 = 126 + 1 = 127$$

Alternatively, using differences:

$$7 - 3 = 4 = 2^2$$

$$15 - 7 = 8 = 2^3$$

$$31 - 15 = 16 = 2^4$$

$$63 - 31 = 32 = 2^5$$

The next difference should be $2^6 = 64$.

$$63 + 64 = 127$$

Step 4: Final Answer:

The next number in the series is 127.

Quick Tip: Notice that these numbers are all one less than a power of 2:

$$2^2 - 1, 2^3 - 1, 2^4 - 1, 2^5 - 1, 2^6 - 1.$$

The next term is $2^7 - 1 = 128 - 1 = 127$.