# VITEEE 2021 Question Paper With Answer key And Solution PDF

| **Time Allowed :**1.5 Hours | **Maximum Marks :**80 | **Total Questions :**80 |
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**General Instructions**

**Read the following instructions very carefully and strictly follow them:**

1. The question paper contains a total of 80 questions divided into four parts:
   - Part I: Physics (Questions 1 to 25)
   - Part II: Chemistry (Questions 26 to 50)
   - Part III: Mathematics (Questions 51 to 75)
   - Part IV: English  Logical Reasoning (Questions 76 to 80)
2. All questions are multiple-choice with four options, and only one of them is correct.
3. For each correct answer, the candidate will earn 1 mark.
4. There is no negative marking for incorrect answers.
5. The test duration is 1.5 hours.

## PART - I (PHYSICS)

**1. The distance of the centres of moon and earth is D. The mass of earth is 81 times the mass of the moon. At what distance from the centre of the earth, the gravitational force will be zero?**

(A) $\frac{D}{2}$
(B) $\frac{2D}{3}$
(C) $\frac{4D}{3}$
(D) $\frac{9D}{10}$
**Correct Answer:** (D) $\frac{9D}{10}$

**Solution:**
Let $M_E$ be the mass of the Earth and $M_m$ be the mass of the Moon, with $M_E = 81M_m$.

Let $r$ be the distance from the centre of the Earth where the net gravitational force on a test mass $m$ is zero.

The distance from the Moon's centre is $(D - r)$.

For the force to be zero, $F_E = F_m$: $\frac{GM_E m}{r^2} = \frac{GM_m m}{(D-r)^2}$

Substituting $M_E = 81M_m$: $\frac{81M_m}{r^2} = \frac{M_m}{(D-r)^2}$

$\frac{81}{r^2} = \frac{1}{(D-r)^2}$

Taking the square root (since $r > 0$ and $D - r > 0$): $\frac{9}{r} = \frac{1}{D-r}$

$9(D - r) = r$

$9D - 9r = r$

$10r = 9D$

$r = \frac{9D}{10}$

> **Quick Tip**
>
> For two masses $M_1$ and $M_2$ separated by distance $D$, the null point distance $r_1$ from $M_1$ is $r_1 = \frac{D\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}$.
> Here, $r = \frac{D\sqrt{81M_m}}{\sqrt{81M_m}+\sqrt{M_m}} = \frac{9D\sqrt{M_m}}{10\sqrt{M_m}} = \frac{9D}{10}$.

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**2. Two wires A and B are of the same material. Their lengths are in the ratio of** $1 : 2$ **and the diameter are in the ratio** $2 : 1$**. If they are pulled by the same force, then increase in length will be in the ratio of**
(A) $2 : 1$
(B) $1 : 4$
(C) $1 : 8$
(D) $8 : 1$
**Correct Answer:** (C) $1 : 8$

**Solution:**
The increase in length ($\Delta L$) under force $F$ is given by $\Delta L = \frac{FL}{AY}$.

Since $F$ and $Y$ (same material) are constant, $\Delta L \propto \frac{L}{A}$.

The area of cross-section $A \propto d^2$, where $d$ is the diameter. Thus, $\Delta L \propto \frac{L}{d^2}$.

Given ratios: $\frac{L_A}{L_B} = \frac{1}{2}$ and $\frac{d_A}{d_B} = \frac{2}{1}$.

The ratio of elongation is: $\frac{\Delta L_A}{\Delta L_B} = \frac{L_A}{L_B} \cdot \left(\frac{d_B}{d_A}\right)^2$

$\frac{\Delta L_A}{\Delta L_B} = \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right)^2$

$\frac{\Delta L_A}{\Delta L_B} = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$

The ratio is $1 : 8$.

> Quick Tip
>
> If length is halved and diameter is doubled, $\Delta L \propto \frac{L}{d^2}$ changes by a factor of $\frac{(1/2)}{(2)^2} = \frac{1}{8}$.

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**3. If $x = at + bt^2$, where $x$ is the distance travelled by the body in kilometers while $t$ is the time in seconds, then the unit of $b$ is**
(A) km/s
(B) kms
(C) km/s$^2$
(D) kms$^2$
**Correct Answer:** (C) km/s$^2$

**Solution:**
By the Principle of Homogeneity, the unit of every term must match the unit of the LHS ($x$).

Unit of $x$ is km. Unit of $t$ is s.

We must have Unit of $(bt^2)$ = Unit of $x$.

(Unit of $b$) $\cdot$ (Unit of $t)^2$ = km

(Unit of $b$) $\cdot$ (s)$^2$ = km

Unit of $b = \frac{\text{km}}{\text{s}^2}$ or km/s$^2$.

> Quick Tip
>
> In physics, any term multiplied by $t^2$ must have the units of acceleration (Length/Time$^2$) if the resulting dimension is Length.

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**4. A soap bubble of radius $r_1$ is placed on another soap bubble of radius $r_2$ ($r_1 < r_2$). The radius $R$ of the soapy film separating the two bubbles is**
(A) $r_1 + r_2$
(B) $\frac{r_2 - r_1}{r_1 r_2}$
(C) $\frac{r_1 r_2}{r_2 - r_1}$

(D) $\sqrt{r_1^2 + r_2^2}$
**Correct Answer:** (C) $\frac{r_1 r_2}{r_2 - r_1}$

**Solution:**
The excess pressure inside a bubble of radius $r$ is $\Delta P = \frac{4T}{r}$.

Since $r_1 < r_2$, the pressure in bubble 1 ($\Delta P_1$) is greater than the pressure in bubble 2 ($\Delta P_2$).

The common separating film (radius $R$) experiences a net pressure difference $\Delta P_{\text{net}} = \Delta P_1 - \Delta P_2$.

This net pressure difference is balanced by the excess pressure of the common film: $\Delta P_{\text{net}} = \frac{4T}{R}$.

$\frac{4T}{R} = \frac{4T}{r_1} - \frac{4T}{r_2}$

$\frac{1}{R} = \frac{1}{r_1} - \frac{1}{r_2}$

$\frac{1}{R} = \frac{r_2 - r_1}{r_1 r_2}$

$R = \frac{r_1 r_2}{r_2 - r_1}$

Quick Tip

The formula $\frac{1}{R} = \frac{1}{r_{\text{small}}} - \frac{1}{r_{\text{large}}}$ always holds for the radius of the common interface separating two soap bubbles.

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**5. A charge $q$ is moving with a velocity $v$ parallel to a magnetic field $B$. Force on the charge due to magnetic field is**
(A) $qvB$
(B) $qB/v$
(C) zero
(D) $Bv/q$
**Correct Answer:** (C) zero

**Solution:**
The magnetic Lorentz force ($\vec{F}$) on a charge $q$ moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is $\vec{F} = q(\vec{v} \times \vec{B})$.

The magnitude is $F = qvB \sin\theta$, where $\theta$ is the angle between $\vec{v}$ and $\vec{B}$.

Since the charge is moving parallel to the magnetic field, $\theta = 0°$.

$F = qvB\sin(0°)$

Since $\sin(0°) = 0$, the force $F = 0$.

> Quick Tip
>
> The magnetic force acts only if the charge has a velocity component perpendicular to the magnetic field. If motion is parallel to the field, the force is zero.

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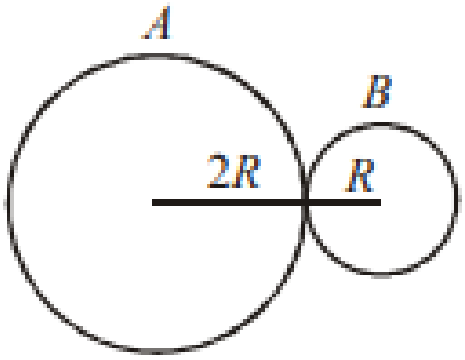
**6. Two spheres A and B of masses $m$ and $2m$ and radii $2R$ and $R$ respectively are placed in contact as shown. The COM of the system lies**




(A) inside A
(B) inside B
(C) at the point of contact
(D) None of these
**Correct Answer:** (C) at the point of contact

**Solution:**
Mass of A $(m_A) = m$. Radius of A $(r_A) = 2R$. Center of A at $x_A = 0$.

Mass of B $(m_B) = 2m$. Radius of B $(r_B) = R$. Center of B at $x_B = r_A + r_B = 3R$.

The position of the Center of Mass $(x_{\text{com}})$ is:

$x_{\text{com}} = \frac{m_A x_A + m_B x_B}{m_A + m_B}$

$x_{\text{com}} = \frac{m(0) + 2m(3R)}{m + 2m}$

$x_{\text{com}} = \frac{6mR}{3m} = 2R$

The COM is located at a distance $2R$ from the center of A.

Since $r_A = 2R$, the center of mass is exactly on the circumference of sphere A, which is the point of contact with sphere B.

> Quick Tip
>
> The center of mass for two masses $m_A$ and $m_B$ separated by distance $D$ is located at a distance $D \cdot \frac{m_B}{m_A+m_B}$ from $m_A$.

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**7. Identify the correct statement.**
(A) Static friction depends on the area of contact
(B) Kinetic friction depends on the area of contact
(C) Coefficient of kinetic friction is more than the coefficient of static friction
(D) Coefficient of kinetic friction is less than the coefficient of static friction
**Correct Answer:** (D) Coefficient of kinetic friction is less than the coefficient of static friction

**Solution:**
According to Amontons' laws of friction, frictional forces are independent of the apparent area of contact. Thus (A) and (B) are incorrect.

The coefficient of static friction ($\mu_s$) is typically greater than or equal to the coefficient of kinetic friction ($\mu_k$).

It is generally true that $\mu_k < \mu_s$.

Thus, the coefficient of kinetic friction is less than the coefficient of static friction. (D) is correct.

> Quick Tip
>
> Static friction ($\mu_s N$) must overcome inter-atomic forces to initiate motion, making it usually greater than kinetic friction ($\mu_k N$), which sustains motion.

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**8. The distance travelled by a particle starting from rest and moving with an acceleration $\frac{4}{3}$ ms$^{-2}$, in the third second is:**
(A) 6 m
(B) 4 m

(C) $\frac{10}{3}$ m
(D) $\frac{19}{3}$ m
**Correct Answer:** (C) $\frac{10}{3}$ m

**Solution:**
Initial velocity $u = 0$. Acceleration $a = \frac{4}{3}$ m/s$^2$.

The distance traveled in the $n$-th second ($S_n$) is given by: $S_n = u + \frac{a}{2}(2n - 1)$.

For the third second, $n = 3$.

$S_3 = 0 + \frac{4/3}{2}(2(3) - 1)$

$S_3 = \frac{2}{3}(6 - 1)$

$S_3 = \frac{2}{3}(5)$

$S_3 = \frac{10}{3}$ m

Quick Tip

The distance in the $n$-th second is not the displacement after $n$ seconds ($S_n$); it is the distance between $t = n - 1$ and $t = n$.

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**9. Photoelectric work function of a metal is 1 eV. Light of wavelength $\lambda = 3000$ Å falls on it. The photo electrons come out with a maximum velocity of:**
(A) 10 metres/sec
(B) $10^2$ metres/sec
(C) $10^4$ metres/sec
(D) $10^6$ metres/sec
**Correct Answer:** (D) $10^6$ metres/sec

**Solution:**
Work function $W = 1$ eV. Wavelength $\lambda = 3000$ Å.

Energy of incident photon $E = \frac{12400 \text{ eV·Å}}{3000 \text{ Å}} \approx 4.13$ eV.

Maximum kinetic energy $K_{\max} = E - W = 4.13 \text{ eV} - 1 \text{ eV} = 3.13$ eV.

$K_{\max} = 3.13 \times (1.6 \times 10^{-19})$ J $\approx 5.0 \times 10^{-19}$ J.

Using $K_{\max} = \frac{1}{2} m_e v_{\max}^2$ and $m_e \approx 9.1 \times 10^{-31}$ kg:

$v_{\max}^2 = \frac{2K_{\max}}{m_e} = \frac{2 \times 5.0 \times 10^{-19}}{9.1 \times 10^{-31}} \approx 1.1 \times 10^{12}$ m$^2$/s$^2$

$v_{\max} = \sqrt{1.1 \times 10^{12}} \approx 1.05 \times 10^6$ m/s

The maximum velocity is of the order of $10^6$ metres/sec.

> **Quick Tip**
>
> The quick formula $E(\text{eV}) = 1240/\lambda(\text{nm})$ helps calculate photon energy quickly. 3000 Å = 300 nm. $E = 1240/300 \approx 4.13$ eV.

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**10. The coefficient of apparent expansion of mercury in a glass vessel is $153 \times 10^{-6}$/°C and in a steel vessel is $144 \times 10^{-6}$/°C. If $\alpha$ for steel is $12 \times 10^{-6}$/°C, then that of glass is**

(A) $9 \times 10^{-6}$/°C
(B) $6 \times 10^{-6}$/°C
(C) $36 \times 10^{-6}$/°C
(D) $27 \times 10^{-6}$/°C
**Correct Answer:** (D) $27 \times 10^{-6}$/°C

**Solution:**
The relation between true coefficient of volume expansion of mercury ($\gamma_m$) and apparent expansion in a container ($\gamma_a$) is $\gamma_a = \gamma_m - \gamma_c$.

1. Calculate $\gamma_{\text{steel}}$: $\gamma_s = 3\alpha_s = 3 \times (12 \times 10^{-6}) = 36 \times 10^{-6}$/°C.

2. Find $\gamma_m$ using steel data:
$\gamma_{a,s} = 144 \times 10^{-6}$/°C.
$144 \times 10^{-6} = \gamma_m - 36 \times 10^{-6}$
$\gamma_m = (144 + 36) \times 10^{-6} = 180 \times 10^{-6}$/°C.

3. Find $\gamma_{\text{glass}}$ using glass data:
$\gamma_{a,g} = 153 \times 10^{-6}$/°C.
$153 \times 10^{-6} = \gamma_m - \gamma_g$
$\gamma_g = 180 \times 10^{-6} - 153 \times 10^{-6}$
$\gamma_g = 27 \times 10^{-6}$/°C

Quick Tip

Apparent expansion is always less than true expansion because the container itself expands upon heating. If $\gamma_a$ is given for two different containers, you must first calculate the true expansion coefficient of the liquid.

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**11. A step-up transformer operates on a 230 V line and supplies a load of 2 ampere. The ratio of the primary and secondary windings is $1:25$. The current in the primary is**
(A) 15 A
(B) 50 A
(C) 25 A
(D) 12.5 A
**Correct Answer:** (B) 50 A

**Solution:**
Given: Ratio of primary to secondary windings $\frac{N_p}{N_s} = \frac{1}{25}$.

Secondary current $I_s = 2$ A.

For an ideal transformer, the power is conserved ($P_p = P_s$), leading to the inverse relationship between current and turns ratio:

$\frac{I_p}{I_s} = \frac{N_s}{N_p}$

$\frac{I_p}{2\text{ A}} = \frac{25}{1}$

$I_p = 25 \times 2$ A

$I_p = 50$ A

Quick Tip

A step-up transformer increases voltage ($N_s > N_p$) but decreases current ($I_s < I_p$). If the turns ratio is $1:25$, the current ratio must be $25:1$.

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**12. Two bodies of same mass are projected with the same velocity at an angle $30^\circ$ and $60^\circ$ respectively. The ratio of their horizontal ranges will be**
(A) $1:1$
(B) $1:2$
(C) $1:3$

(D) $2 : \sqrt{2}$
**Correct Answer:** (A) $1 : 1$

**Solution:**
The horizontal range $R$ of a projectile launched at velocity $u$ and angle $\theta$ is $R = \frac{u^2 \sin(2\theta)}{g}$.

Since $u$ and $g$ are constant, $R \propto \sin(2\theta)$.

For the first launch: $\theta_1 = 30^\circ$. $2\theta_1 = 60^\circ$. $R_1 \propto \sin(60^\circ)$.

For the second launch: $\theta_2 = 60^\circ$. $2\theta_2 = 120^\circ$. $R_2 \propto \sin(120^\circ)$.

Since $\sin(120^\circ) = \sin(180^\circ - 60^\circ) = \sin(60^\circ)$,

$R_1$ and $R_2$ are proportional to the same value.

The ratio $\frac{R_1}{R_2} = \frac{\sin(60^\circ)}{\sin(120^\circ)} = \frac{1}{1}$.

The ratio is $1 : 1$.

Quick Tip

Maximum range is achieved at $\theta = 45^\circ$. Any two angles $\theta$ and $90^\circ - \theta$ (complementary angles) yield the same range, provided the initial velocity is the same.

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**13. Two point charges $+3\mu$C and $+8\mu$C repel each other with a force of 40 N. If a charge of $-5\mu$C is added to each of them, then the force between them will become**
(A) $-10$ N
(B) $+10$ N
(C) $+20$ N
(D) $-20$ N
**Correct Answer:** (A) $-10$ N

**Solution:**
The electrostatic force $F$ is proportional to the product of charges: $F \propto q_1 q_2$.

Initial charges: $q_1 = +3$, $q_2 = +8$. Product $P_{\text{initial}} = 3 \times 8 = 24$.

Initial force $F_{\text{initial}} = +40$ N (Repulsive).

New charges $q_1' = 3 - 5 = -2$. $q_2' = 8 - 5 = +3$.

New product $P_{\text{final}} = (-2) \times (+3) = -6$.

The ratio of forces is proportional to the ratio of charge products:

$\frac{F_{\text{final}}}{F_{\text{initial}}} = \frac{P_{\text{final}}}{P_{\text{initial}}} = \frac{-6}{24} = -\frac{1}{4}$

$F_{\text{final}} = -\frac{1}{4} \times (40 \text{ N}) = -10 \text{ N}$.

The negative sign indicates attraction.

> **Quick Tip**
>
> Since the distance $r$ is constant, you only need to compare the initial and final product of the charges. The force direction flips if the final product is negative (attraction).

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**14. A sphere rolls down on an inclined plane of inclination $\theta$. What is the acceleration as the sphere reaches the bottom?**
(A) $\frac{5}{7}g\sin\theta$
(B) $\frac{3}{5}g\sin\theta$
(C) $\frac{2}{7}g\sin\theta$
(D) $\frac{2}{5}g\sin\theta$
**Correct Answer:** (A) $\frac{5}{7}g\sin\theta$

**Solution:**
The linear acceleration $a$ for a body rolling without slipping down an incline is:

$a = \frac{g\sin\theta}{1+I/(MR^2)}$

where $I$ is the moment of inertia and $MR^2$ is the product of mass and radius squared.

For a solid sphere, the moment of inertia is $I = \frac{2}{5}MR^2$.

Substituting the value of $I$:

$a = \frac{g\sin\theta}{1+(\frac{2}{5}MR^2)/(MR^2)}$

$a = \frac{g\sin\theta}{1+2/5}$

$a = \frac{g\sin\theta}{7/5}$

$a = \frac{5}{7} g \sin\theta$

> **Quick Tip**
>
> The acceleration for a rolling body is always less than $g \sin\theta$. The larger the fraction $I/MR^2$ (hollow shapes), the smaller the acceleration.

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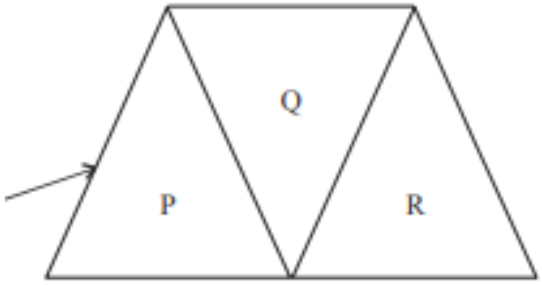
**15. A given ray of light suffers minimum deviation in an equilateral prism P. Additional prisms Q and R of identical shape and of same material as that of P are now combined as shown in figure. The ray will now suffer**




(A) greater deviation
(B) no deviation
(C) same deviation as before
(D) total internal reflection
**Correct Answer:** (C) same deviation as before

**Solution:**
The deviation of a ray passing through a prism depends on the angle of incidence ($i$), angle of emergence ($e$), and prism angle ($A$). $\delta = i + e - A$.

For minimum deviation in prism P, the ray passes symmetrically, i.e., parallel to the base, and $i = e$.

When prisms Q and R, which are identical to P and made of the same material, are added, the overall system maintains the same net refraction as the ray passes symmetrically through the entire structure.

The internal boundaries between P, Q, and R are parallel to the base of P. Since the ray is parallel to the base of P inside P (at minimum deviation), it passes undeviated across the interfaces P/Q and Q/R.

The ray emerges from R with the same overall deviation as if it had only passed through P.

Quick Tip

In a series of identical prisms combined such that the internal faces are parallel to the refracted ray path (which happens at minimum deviation), the overall deviation is simply that of the first or last prism alone.

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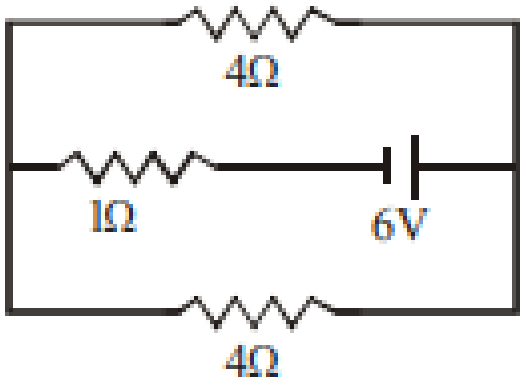
**16. The current in the $1\Omega$ resistor shown in the circuit is**




(A) $\frac{2}{3}$ A
(B) 3 A
(C) 6 A
(D) 2 A
**Correct Answer:** (D) 2 A

**Solution:**
The two $4\Omega$ resistors are connected in parallel.

The equivalent resistance of the parallel $4\Omega$ resistors ($R_p$):

$R_p = \frac{4\times4}{4+4} = 2\Omega$

This parallel combination ($2\Omega$) is in series with the $1\Omega$ resistor and the 6V source.

Total equivalent resistance of the circuit ($R_{\text{eq}}$):

$R_{\text{eq}} = 1\Omega + R_p = 1\Omega + 2\Omega = 3\Omega$

The total current ($I_T$) supplied by the source is:

$I_T = \frac{V}{R_{\text{eq}}} = \frac{6\text{ V}}{3\Omega} = 2\text{ A}$

Since the $1\Omega$ resistor is in the main line of the circuit, the total current $I_T$ flows through it.

Current in the $1\Omega$ resistor is 2 A.

> **Quick Tip**
>
> When calculating total current from a source, simplify the circuit step-by-step using series and parallel rules. The total current flows through all components that are in series with the battery.

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**17. The root mean square velocity of hydrogen molecules at 300 K is 1930 metre/sec. Then the r.m.s velocity of oxygen molecules at 1200 K will be**
(A) 482.5 metre/sec
(B) 965 metre/sec
(C) 1930 metre/sec
(D) 3860 metre/sec
**Correct Answer:** (B) 965 metre/sec

**Solution:**
The RMS velocity is given by $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$, where $M$ is the molar mass.

$v_{\text{rms}} \propto \sqrt{\frac{T}{M}}$.

Hydrogen ($H_2$): $T_H = 300$ K, $M_H = 2$ g/mol, $v_H = 1930$ m/s.

Oxygen ($O_2$): $T_O = 1200$ K, $M_O = 32$ g/mol.

Ratio of velocities: $\frac{v_O}{v_H} = \sqrt{\frac{T_O}{T_H} \cdot \frac{M_H}{M_O}}$

$\frac{v_O}{1930} = \sqrt{\frac{1200}{300} \cdot \frac{2}{32}}$

$\frac{v_O}{1930} = \sqrt{4 \cdot \frac{1}{16}} = \sqrt{\frac{1}{4}}$

$\frac{v_O}{1930} = \frac{1}{2}$

$v_O = \frac{1930}{2} = 965$ m/s

> **Quick Tip**
>
> RMS velocity increases with temperature and decreases with molar mass. Here, the temperature ratio is 4 and the mass ratio is 1/16, resulting in a net velocity ratio of $\sqrt{4/16} = 1/2$.

**18. Lenz's law gives**
(A) the magnitude of the induced e.m.f.
(B) the direction of the induced current
(C) both the magnitude and direction of the induced current
(D) the magnitude of the induced current
**Correct Answer:** (B) the direction of the induced current

**Solution:**
Faraday's Law of Induction calculates the magnitude of the induced EMF, $\mathcal{E} = |\frac{d\Phi_B}{dt}|$.

Lenz's law is a consequence of the conservation of energy applied to electromagnetic induction.

It dictates that the induced current flows in a direction that creates a magnetic field opposing the change in flux that caused it.

Therefore, Lenz's law gives the direction of the induced current.

Quick Tip

Lenz's law is crucial for determining the polarity of the induced EMF or the flow direction of the induced current, fulfilling the 'minus sign' in Faraday's Law ($\mathcal{E} = -d\Phi_B/dt$).

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**19. A parallel plate capacitor with air between the plates has a capacitance of 8 pF. Calculate the capacitance if the distance between the plates is reduced by half and the space between them is filled with a substance of dielectric constant. ($k = 6$)**
(A) 72 pF
(B) 81 pF
(C) 84 pF
(D) 96 pF
**Correct Answer:** (D) 96 pF

**Solution:**
Initial capacitance $C_0 = 8$ pF. $C_0 = \frac{\epsilon_0 A}{d}$.

New distance $d' = d/2$. New dielectric constant $\kappa = 6$.

The new capacitance $C_{\text{new}}$ is given by: $C_{\text{new}} = \frac{\kappa \epsilon_0 A}{d'}$

$C_{\text{new}} = \frac{6\epsilon_0 A}{(d/2)}$

$C_{\text{new}} = 12\frac{\epsilon_0 A}{d}$

Since $C_0 = \frac{\epsilon_0 A}{d}$:

$C_{\text{new}} = 12C_0$

$C_{\text{new}} = 12 \times 8 \text{ pF} = 96 \text{ pF}$

> **Quick Tip**
>
> Capacitance $C$ is inversely proportional to plate distance $d$ and directly proportional to dielectric constant $\kappa$. Changes multiply: $C_{\text{new}} = C_0 \cdot \left(\frac{d}{d'}\right) \cdot \kappa$.

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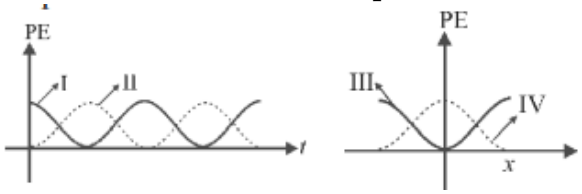
**20. For a particle executing S.H.M. the displacement $x$ is given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy (P.E.) as a function of time $t$ and displacement $x$.**




(A) I, III
(B) II, IV
(C) II, III
(D) I, IV
**Correct Answer:** (B) II, IV

**Solution:**
1. Potential Energy (P.E.) as a function of displacement ($x$):

$P.E. = \frac{1}{2}kx^2$. This relationship is parabolic, centered at $x = 0$.

Graph II correctly represents this parabolic relationship.

2. Displacement ($x$) as a function of time ($t$):

The definition given is $x = A\cos\omega t$. This is a simple sinusoidal/cosinusoidal function starting at $x = A$ at $t = 0$.

Graph IV represents a sinusoidal curve starting from the maximum displacement (or minimum displacement if $\omega t$ were $\sin \omega t$). Graph IV is the displacement $x$ vs time $t$.

Note: Graph I represents $P.E. = \frac{1}{2}kA^2 \cos^2(\omega t)$, oscillating at twice the frequency. Since option (A) (I, III) and (D) (I, IV) include I, but the key selects (B) II, IV, it is assumed the question asks for (P.E. vs $x$) and ($x$ vs $t$).

Thus, the correct pair is (II, IV).

Quick Tip

In SHM, Energy $\propto$ (Displacement)$^2$ (parabolic curve), while Displacement $\propto \cos(\omega t)$ or $\sin(\omega t)$ (sinusoidal curve). The energy vs time graph must oscillate at twice the frequency of displacement vs time.

---

**21. A radioactive sample contains $10^{-3}$ kg each of two nuclear species A and B with half-life 4 days and 8 days respectively. The ratio of the amounts of A and B after a period of 16 days is**
(A) 1:2
(B) 4:1
(C) 1:4
(D) 2:1
**Correct Answer:** (C) 1:4

**Solution:**
Let $N_0$ be the initial amount of both species A and B. The time elapsed $t = 16$ days.

The remaining fraction is given by $\left(\frac{1}{2}\right)^n$, where $n = \frac{t}{T_{1/2}}$.

For species A, Half-life $T_{1/2,A} = 4$ days.
Number of half-lives $n_A = \frac{16}{4} = 4$.
Remaining amount $N_A = N_0 \left(\frac{1}{2}\right)^4 = \frac{N_0}{16}$.

For species B, Half-life $T_{1/2,B} = 8$ days.
Number of half-lives $n_B = \frac{16}{8} = 2$.
Remaining amount $N_B = N_0 \left(\frac{1}{2}\right)^2 = \frac{N_0}{4}$.

The required ratio $\frac{N_A}{N_B}$ is:
$\frac{N_A}{N_B} = \frac{N_0/16}{N_0/4} = \frac{4}{16} = \frac{1}{4}$.
The ratio is $1 : 4$.

Quick Tip

When calculating ratios of radioactive decay, the remaining fraction depends only on the number of half-lives passed, $n$. The ratio simplifies as the initial mass $N_0$ cancels out.

---

**22. A string of 7 m length has a mass of 0.035 kg. If tension in the string is 60.5 N, then speed of a wave on the string is**
(A) 77 m/s
(B) 102 m/s
(C) 110 m/s
(D) 165 m/s
**Correct Answer:** (C) 110 m/s

**Solution:**
Given: Length $L = 7$ m, Mass $M = 0.035$ kg, Tension $T = 60.5$ N.

First, calculate the linear mass density $\mu$:
$\mu = \frac{M}{L} = \frac{0.035 \text{ kg}}{7 \text{ m}} = 0.005$ kg/m.

The speed of a transverse wave on a string is given by the formula $v = \sqrt{\frac{T}{\mu}}$.

Substitute the values:
$v = \sqrt{\frac{60.5}{0.005}}$.
$v = \sqrt{12100}$.
$v = 110$ m/s.

Quick Tip

Ensure that the units are converted to standard SI units (kg, m, N) before using the wave speed formula $v = \sqrt{T/\mu}$.

---

**23. The following circut represents**
(A) OR gate
(B) AND gate
(C) NAND gate
(D) None of these
**Correct Answer:** (C) NAND gate

**Solution:**
The symbol shown has a flat front and a curved back, which is the representation of an AND gate.

The small circle (or bubble) located at the output terminal signifies a logical NOT operation (inverter).

A circuit that performs an AND operation followed by a NOT operation is a NAND gate.
The output $Y = \overline{A \cdot B}$.

> Quick Tip
>
> The presence of a bubble on the output side of a standard gate symbol indicates a negated output (e.g., AND + NOT = NAND; OR + NOT = NOR).

---

**24. A straight section PQ of a circuit lies along the X-axis from $x = -a/2$ to $x = a/2$ and carries a steady current $i$. The magnetic field due to the section PQ at a point $X = +a$ will be**
(A) proportional to $a$
(B) proportional to $a^2$
(C) proportional to $1/a$
(D) zero
**Correct Answer:** (D) zero

**Solution:**
The current-carrying wire segment PQ lies along the X-axis.
The point of observation $X = +a$ also lies on the X-axis, meaning the point is axial to the wire.

According to the Biot-Savart Law, the magnetic field $d\vec{B}$ due to a current element $id\vec{l}$ is proportional to $d\vec{l} \times \vec{r}$.

If the point of observation lies on the axis of the straight conductor, the vector $d\vec{l}$ (along the current) is parallel to the position vector $\vec{r}$.

The cross product $d\vec{l} \times \vec{r}$ is zero when the vectors are parallel ($\theta = 0°$).
Therefore, the magnetic field contribution from every element of the wire at point $X = +a$ is zero.

Quick Tip

A fundamental concept of magnetic fields: a straight current element produces no magnetic field along its own line of extension (its axis).

---

**25. A source producing sound of frequency 170 Hz is approaching a stationary observer with a velocity 17 $\mathbf{ms^{-1}}$. The apparent change in the wavelength of sound heard by the observer is (speed of sound in air = 340 $\mathbf{ms^{-1}}$)**
(A) 0.1 m
(B) 0.2 m
(C) 0.4 m
(D) 0.5 m
**Correct Answer:** (A) 0.1 m

**Solution:**
Given: Source frequency $f_s = 170$ Hz, Source speed $v_s = 17$ m/s, Speed of sound $v = 340$ m/s.

Calculate the original wavelength ($\lambda_0$):
$\lambda_0 = \frac{v}{f_s} = \frac{340}{170} = 2.0$ m.

Since the source is approaching the observer, the effective speed of sound waves relative to the source is $v - v_s$.
Calculate the apparent wavelength ($\lambda'$):
$\lambda' = \frac{v - v_s}{f_s} = \frac{340-17}{170}$.
$\lambda' = \frac{323}{170} = 1.9$ m.

The apparent change in wavelength ($\Delta\lambda$) is the difference:
$\Delta\lambda = |\lambda_0 - \lambda'| = |2.0 \text{ m} - 1.9 \text{ m}|$.
$\Delta\lambda = 0.1$ m.

Quick Tip

For wavelength change calculations in the Doppler effect, use the relative speed of the source with respect to the medium, divided by the actual source frequency.

---

## PART - II (CHEMISTRY)

**26. Consider the following reactions:**
$\mathbf{NaCl + K_2Cr_2O_7 + H_2SO_4(Conc.) \rightarrow (A) + Side\ products}$

**(A) + NaOH → (B) + Side products**
**(B) + $H_2SO_4$(dilute) + $H_2O_2$ → (C) + Side products**
**The sum of the total number of atoms in one molecule each of (A), (B) and (C) is**
(A) 18
(B) 15
(C) 21
(D) 20
**Correct Answer:** (A) 18

**Solution:**
Reaction 1: Chromyl Chloride Test
$4NaCl + K_2Cr_2O_7 + 6H_2SO_4 \rightarrow 2CrO_2Cl_2(A) + 2KHSO_4 + 4NaHSO_4 + 3H_2O$.
Compound (A) is $CrO_2Cl_2$. Atoms: $1(Cr) + 2(O) + 2(Cl) = 5$.

Reaction 2: Formation of Chromate
$CrO_2Cl_2(A) + 4NaOH \rightarrow Na_2CrO_4(B) + 2NaCl + 2H_2O$.
Compound (B) is $Na_2CrO_4$. Atoms: $2(Na) + 1(Cr) + 4(O) = 7$.

Reaction 3: Peroxide Test (Blue solution)
$Na_2CrO_4(B) + H_2SO_4 + H_2O_2 \rightarrow CrO_5(C) + Na_2SO_4 + H_2O$.
Compound (C) is $CrO_5$. Atoms: $1(Cr) + 5(O) = 6$.

Total number of atoms = (Atoms in A) + (Atoms in B) + (Atoms in C).
Total atoms $= 5 + 7 + 6 = 18$.

Quick Tip

The key intermediate (A) in the chromyl chloride test is volatile $CrO_2Cl_2$. (C) is the blue peroxide product $CrO_5$, which contains a peroxide linkage ($Cr(O_2)_2O$).

---

**27. Xenon hexafluoride on partial hydrolysis produces compounds 'X' and 'Y'. Compounds 'X', 'Y' and the oxidation state of Xe are respectively :**
(A) $XeOF_4$(+6) and $XeO_3$(+6)
(B) $XeO_2$(+4) and $XeO_3$(+6)
(C) $XeOF_4$(+6) and $XeO_2F_2$(+6)
(D) $XeO_2F_2$(+6) and $XeO_2$(+4)
**Correct Answer:** (C) $XeOF_4$(+6) and $XeO_2F_2$(+6)

**Solution:**
Xenon hexafluoride ($XeF_6$) undergoes sequential partial hydrolysis:

First partial hydrolysis (X) replaces one $F_2$ with one O:
$XeF_6 + H_2O \rightarrow XeOF_4(X) + 2HF$.

Second partial hydrolysis (Y) replaces two $F_2$ with two O:
$XeF_6 + 2H_2O \rightarrow XeO_2F_2(Y) + 4HF$.

Determine the oxidation state of Xe in both compounds:
In $XeOF_4$: $Xe + (-2) + 4(-1) = 0 \Rightarrow Xe = +6$.
In $XeO_2F_2$: $Xe + 2(-2) + 2(-1) = 0 \Rightarrow Xe = +6$.

The compounds are $XeOF_4(+6)$ and $XeO_2F_2(+6)$.

Quick Tip

Partial hydrolysis of noble gas fluorides leads to oxo-fluorides where the noble gas typically retains its original oxidation state by replacing two halogens with one oxygen atom.

---

**28. The edge length of unit cell of a metal having molecular weight 75 g/mol is 5Å which crystallizes in cubic lattice. If the density is 2 g/cc then find the radius of metal atom. ($N_A = 6 \times 10^{23}$). Give the answer in pm.**
(A) 217 pm
(B) 210 pm
(C) 220 pm
(D) 205 pm
**Correct Answer:** (A) 217 pm

**Solution:**
Given: $M = 75$ g/mol, $a = 5\text{Å} = 5 \times 10^{-8}$ cm, $\rho = 2$ g/cm$^3$, $N_A = 6 \times 10^{23}$.

Step 1: Determine the number of atoms per unit cell ($Z$).
Use the density formula: $\rho = \frac{Z \cdot M}{a^3 \cdot N_A}$.
$Z = \frac{\rho \cdot a^3 \cdot N_A}{M}$.
$Z = \frac{2 \cdot (5 \times 10^{-8})^3 \cdot (6 \times 10^{23})}{75}$.
$Z = \frac{2 \cdot (125 \times 10^{-24}) \cdot (6 \times 10^{23})}{75} = \frac{1500 \times 10^{-1}}{75} = 2$.

Since $Z = 2$, the lattice structure is Body-Centered Cubic (BCC).

Step 2: Calculate the atomic radius ($r$) using the BCC relationship.
For BCC, $4r = a\sqrt{3}$.
$r = \frac{a\sqrt{3}}{4}$.
$r = \frac{5\text{Å} \times 1.732}{4}$.

$r = 2.165$Å.

Step 3: Convert the radius to picometers (pm).
$r = 2.165 \times 100$ pm $= 216.5$ pm.
Rounding off gives 217 pm.

Quick Tip

A $Z$ value of 2 immediately identifies the lattice as BCC, requiring the specific relationship $4r = a\sqrt{3}$ to find the radius. Ensure unit consistency, especially Å to pm conversion (1Å = 100 pm).

---

**29. Consider the following statements:**
**I. Increase in concentration of reactant increases the rate of a zero order reaction.**
**II. Rate constant $k$ is equal to collision frequency $A$ if $E_a = 0$.**
**III. Rate constant $k$ is equal to collision frequency $A$ if $E_a = \infty$.**
**IV. $\ln k$ vs $T$ is a straight line.**
**V. $\ln k$ vs $1/T$ is a straight line.**
**Correct statements are**
(A) I and IV
(B) II and V
(C) III and IV
(D) II and III
**Correct Answer:** (B) II and V

**Solution:**
I. Zero order reactions have Rate $= k$. The rate is independent of concentration. Statement I is INCORRECT.

II. Arrhenius equation: $k = Ae^{-E_a/RT}$. If $E_a = 0$, $k = Ae^0 = A$. Statement II is CORRECT.

III. If $E_a = \infty$, $k = Ae^{-\infty} = 0$. Statement III is INCORRECT.

IV. The relationship between $\ln k$ and $T$ is non-linear ($\ln k = \ln A - E_a/(RT)$). Statement IV is INCORRECT.

V. Plotting $\ln k$ vs $1/T$ from the Arrhenius equation gives a straight line with slope $-E_a/R$. Statement V is CORRECT.

The correct statements are II and V.

Quick Tip

The Arrhenius plot ($\ln k$ vs $1/T$) is a standard method to experimentally determine the activation energy ($E_a$) from the slope.

---

**30. To deposit 0.634 g of copper by electrolysis of aqueous cupric sulphate solution, the amount of electricity required (in coulombs) is**
(A) 1930
(B) 3960
(C) 4825
(D) 9650
**Correct Answer:** (A) 1930

**Solution:**
The reaction is $Cu^{2+} + 2e^- \rightarrow Cu$. Thus, $n = 2$ electrons are required per mole of Cu.
Use Molar Mass of Cu $\approx 63.5$ g/mol and Faraday constant $F = 96500$ C/mol.

Step 1: Calculate the moles of Cu deposited.
Moles $= \frac{m}{M} = \frac{0.634 \text{ g}}{63.5 \text{ g/mol}} = 0.01$ mol.

Step 2: Calculate the total charge ($Q$) required.
$Q =$ Moles $\times n \times F$.
$Q = 0.01 \times 2 \times 96500$ C.
$Q = 2 \times 965$ C.
$Q = 1930$ Coulombs.

Quick Tip

Recall Faraday's laws: the amount of substance deposited is proportional to the charge passed ($Q = It$). For one mole of $Cu^{2+}$, $2F$ (or $2 \times 96500$ C) is required.

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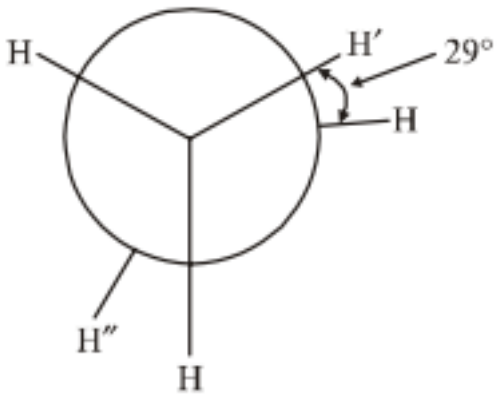
**31. In the following skew conformation of ethane, $H' - C - C - H''$ dihedral angle is :**




(A) 58°
(B) 149°
(C) 151°
(D) 120°
**Correct Answer:** (B) 149°

**Solution:**
The diagram shows a Newman projection of ethane in a skew conformation.

The dihedral angles ($\theta$) in an ethane conformation are $\theta$, $\theta + 60°$, and $\theta + 120°$.
The diagram indicates that the relative rotation ($\theta$) between the closest hydrogen atoms on the front and back carbons is 29°.

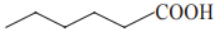
Thus, the three dihedral angles are:
1. 29° (smallest angle)
2. $29° + 60° = 89°$ (intermediate angle)
3. $29° + 120° = 149°$ (largest angle)

The $H'$ and $H''$ hydrogens are generally represented as the most widely separated pair, corresponding to the largest dihedral angle.
The dihedral angle $H' - C - C - H''$ is 149°.

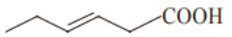
| Quick Tip |
| --- |
| In a skew conformation of ethane, if the smallest dihedral angle is $\theta$, the other two unique dihedral angles are $\theta + 60°$ and $\theta + 120°$. |

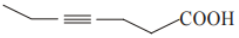
---

**32. What is the product of following reaction?**
**Hex-3-ynal** $\xrightarrow{(i)\mathbf{NaBH_4}} \xrightarrow{(ii)\mathbf{PBr_3}} \xrightarrow{(iii)\mathbf{Mg/ether}} \xrightarrow{(iv)\mathbf{CO_2/H_3O^+}}$?

(A) 

(B) 

(C) 

(D) 

**Correct Answer:** (C) $CH_3CH_2C \equiv CCH_2CH_2COOH$

**Solution:**
The reactant is Hex-3-ynal: $CH_3CH_2C \equiv CCH_2CHO$. (6 carbons).

(i) Reduction of aldehyde: $NaBH_4$ reduces the aldehyde group (CHO) to a primary alcohol ($CH_2OH$).
Product: $CH_3CH_2C \equiv CCH_2CH_2OH$ (Hex-3-yn-1-ol).

(ii) Halogenation: $PBr_3$ replaces the $-OH$ group with $-Br$.
Product: $CH_3CH_2C \equiv CCH_2CH_2Br$.

(iii) Grignard Formation: Mg/ether converts the alkyl bromide to a Grignard reagent (RMgBr).
Product: $CH_3CH_2C \equiv CCH_2CH_2MgBr$.

(iv) Carbonation and Hydrolysis: Grignard reacts with $CO_2$ followed by acid hydrolysis to form a carboxylic acid, increasing the chain length by one carbon.
$RMgBr + CO_2 \rightarrow RCOOMgBr \xrightarrow{H_3O^+} RCOOH$.
Product: $CH_3CH_2C \equiv CCH_2CH_2COOH$. (Hept-4-ynoic acid).

This structure matches option (C): $C - C - C \equiv C - C - C - COOH$.

> Quick Tip
>
> $NaBH_4$ is a milder reducing agent than $LiAlH_4$; it selectively reduces aldehydes and ketones but leaves triple bonds and esters intact. The reaction of RMgX with $CO_2$ is a chain extension method for synthesizing carboxylic acids.

---

**33. In the following sequence of reactions,**
$\mathbf{CH_3CH_2OH \xrightarrow{P+I_2} A \xrightarrow{Mg/ether} B \xrightarrow{HCHO} C \xrightarrow{H_2O/H^+} D}$
**the compound D is**
(A) propanal
(B) butanal
(C) *n*-butyl alcohol
(D) *n*-propyl alcohol.
**Correct Answer:** (D) *n*-propyl alcohol.

**Solution:**
Starting material: $CH_3CH_2OH$ (Ethanol).

Step 1: $CH_3CH_2OH \xrightarrow{P+I_2} CH_3CH_2I$ (A, Ethyl iodide).

Step 2: A $\xrightarrow{Mg/ether}$ $CH_3CH_2MgI$ (B, Ethyl Grignard reagent).

Step 3: B + HCHO (Formaldehyde): Grignard reagents react with formaldehyde to produce a primary alcohol upon hydrolysis, adding one carbon atom.
$CH_3CH_2MgI + HCHO \rightarrow [CH_3CH_2CH_2OMgI]$ (C, Intermediate alkoxide).

Step 4: Hydrolysis of C.
C $\xrightarrow{H_2O/H^+}$ $CH_3CH_2CH_2OH$ (D).

Compound D is *n*-propyl alcohol (or propan-1-ol).

Quick Tip

Grignard reactions are essential for C-C bond formation. Reaction with Formaldehyde (HCHO) always yields a primary alcohol with one more carbon than the alkyl group of the Grignard reagent.

---

**34. Which of the following reactions can produce aniline as main product?**
(A) $C_6H_5NO_2 + Zn/KOH$
(B) $C_6H_5NO_2 + Zn/NH_4Cl$
(C) $C_6H_5NO_2 + LiAlH_4$
(D) $C_6H_5NO_2 + Zn/HCl$
**Correct Answer:** (D) $C_6H_5NO_2 + Zn/HCl$

**Solution:**
Aniline ($C_6H_5NH_2$) is obtained by the complete reduction of nitrobenzene ($C_6H_5NO_2$).

(A) Zn/KOH (strong basic reduction) yields hydrazobenzene ($C_6H_5NH - NHC_6H_5$).
(B) $Zn/NH_4Cl$ (neutral reduction) yields phenylhydroxylamine ($C_6H_5NHOH$).
(C) $LiAlH_4$ is a strong reducing agent that reduces $C_6H_5NO_2$ to $C_6H_5NH_2$ (Aniline).
(D) Zn/HCl (strong acidic reduction, similar to Sn/HCl) completely reduces the nitro group to the amine group, yielding aniline ($C_6H_5NH_2$).

Both (C) and (D) produce aniline. However, the metal/acid reduction (Zn/HCl or Sn/HCl) is the classic, large-scale method for this conversion and is universally taught as yielding aniline. Hence, (D) is the standard and often intended answer among the options for complete reduction

in acidic medium.

Quick Tip

Reduction of nitrobenzene to aniline is highly dependent on the medium: Acidic medium (Sn/HCl, Zn/HCl) gives aniline. Neutral medium ($Zn/NH_4Cl$) gives phenylhydroxylamine. Basic medium (Zn/KOH) gives azo or hydrazobenzene derivatives.

---

**35. Secondary structure of protein refers to**
(A) mainly denatured proteins and structure of prosthetic groups
(B) three-dimensional structure, especially the bond between amino acid residues that are distinct from each other in the polypeptide chain
(C) linear sequence of amino acid residues in the polypetide chain
(D) regular folding patterns of continuous portions of the polypeptide chain
**Correct Answer:** (D) regular folding patterns of continuous portions of the polypeptide chain

**Solution:**
Protein structure is hierarchically classified:

(C) Primary structure refers to the linear sequence of amino acid residues.
(D) Secondary structure refers to local, repeating structures like the $\alpha$-helix and $\beta$-sheet, stabilized by hydrogen bonds between the amide and carbonyl groups of the peptide backbone. Statement (D) correctly describes this.
(B) Tertiary structure refers to the overall three-dimensional folding pattern, including interactions between distant residues.
(A) Denatured proteins have lost their secondary/tertiary/quaternary structure.

Quick Tip

Secondary structures ($\alpha$-helix, $\beta$-sheet) are defined exclusively by hydrogen bonding within the polypeptide backbone, excluding interactions involving side chains (R groups).

---

**36. The increasing order for the values of $e/m$ (charge/mass) is**
(A) e, p, n, $\alpha$
(B) n, p, e, $\alpha$
(C) n, p, $\alpha$, e
(D) n, $\alpha$, p, e
**Correct Answer:** (D) n, $\alpha$, p, e

**Solution:**
We compare the charge/mass ratio ($e/m$) for the particles (relative charge $Z$, relative mass $A$):

1. Neutron (n): $Z = 0, A = 1$. $e/m = 0/1 = 0$.
2. Alpha particle ($\alpha$ or $He^{2+}$): $Z = 2, A = 4$. $e/m = 2/4 = 0.5$.
3. Proton (p or $H^+$): $Z = 1, A = 1$. $e/m = 1/1 = 1$.
4. Electron (e): $Z = 1, A \approx 1/1836$. $e/m \approx 1836$.

Increasing order of $e/m$ values:
n(0) $<$ $\alpha$(0.5) $<$ p(1) $<$ e(1836).
The order is n, $\alpha$, p, e.

Quick Tip

The electron has the largest $e/m$ ratio due to its negligible mass. Neutrons always have $e/m = 0$. Alpha particles ($He^{2+}$) have a significantly lower $e/m$ than protons ($H^+$) due to their large mass (A=4) relative to their charge (Z=2).

---

**37. In which of the following pairs both the ions are coloured in aqueous solutions ?**
(A) $Sc^{3+}, Ti^{3+}$
(B) $Sc^{3+}, Co^{2+}$
(C) $Ni^{2+}, Cu^{+}$
(D) $Ni^{2+}, Ti^{3+}$
**Correct Answer:** (D) $Ni^{2+}, Ti^{3+}$

**Solution:**
Color in transition metal ions is generally due to $d - d$ electronic transitions, which occur only when the ion has a partially filled $d$ subshell ($d^1$ to $d^9$ configuration).
$d^0$ and $d^{10}$ ions are typically colorless (unless charge transfer occurs).

(A) $Sc^{3+}$ ($3d^0$, colorless), $Ti^{3+}$ ($3d^1$, colored).
(B) $Sc^{3+}$ ($3d^0$, colorless), $Co^{2+}$ ($3d^7$, colored).
(C) $Ni^{2+}$ ($3d^8$, colored), $Cu^{+}$ ($3d^{10}$, colorless).
(D) $Ni^{2+}$ ($3d^8$, colored, green/blue), $Ti^{3+}$ ($3d^1$, colored, purple).

In option (D), both ions have partially filled $d$ orbitals and are therefore colored in aqueous solution.

Quick Tip

To determine if a transition metal ion is colored, calculate its $d$ electron configuration. $d^0$ (like $Sc^{3+}$) and $d^{10}$ (like $Zn^{2+}$, $Cu^+$) ions are usually colorless because they lack the necessary electronic configuration for $d - d$ transitions.

---

**38. The total number of possible isomers for square-planar $[\mathbf{Pt(Cl)(NO_2)(NO_3)(SCN)}]^{2-}$ is:**
(A) 16
(B) 12
(C) 8
(D) 24
**Correct Answer:** (B) 12

**Solution:**
The complex is $[Pt(Cl)(NO_2)(NO_3)(SCN)]^{2-}$. The coordination number is 4, and the geometry is square planar.
The ligands are $L_1 = Cl, L_2 = NO_2, L_3 = NO_3, L_4 = SCN$.

1. Geometric Isomerism: For a square planar complex with four different monodentate ligands (MABCD), there are 3 possible geometric isomers.
The possible trans pairs are: $(A, B, C, D) \rightarrow 3$ isomers.

2. Linkage Isomerism: This arises from ambidentate ligands.
Ligand $NO_2$ can coordinate via N (nitro, $-NO_2$) or O (nitrito, $-ONO$). (2 options)
Ligand SCN can coordinate via S (thiocyanato, $-SCN$) or N (isothiocyanato, $-NCS$). (2 options)
Ligand Cl and $NO_3$ are standard monodentate.
Total possible linkage combinations $= 2 \times 2 = 4$.

Total number of isomers = (Number of Geometric isomers) $\times$ (Number of Linkage combinations).
Total isomers $= 3 \times 4 = 12$.

Quick Tip

When calculating total isomers for complexes containing ambidentate ligands, multiply the number of possible geometric isomers by the number of unique linkage combinations.

---

**39. For the reaction, $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$, $\Delta H = -57.2$ kJ mol$^{-1}$ and $K_c = 1.7 \times 10^{16}$. Which of the following statement is INCORRECT?**
(A) The equilibrium constant is large suggestive of reaction going to completion and so no catalyst is required.
(B) The equilibrium will shift in forward direction as the pressure increases.
(C) The equilibrium constant decreases as the temperature increases.
(D) The addition of inert gas at constant volume will not affect the equilibrium constant.
**Correct Answer:** (A) The equilibrium constant is large suggestive of reaction going to completion and so no catalyst is required.

**Solution:**
The reaction is exothermic ($\Delta H < 0$) and involves a decrease in moles of gas ($\Delta n_g = 2 - 3 = -1$). $K_c$ is very large.

(B) Increasing pressure shifts equilibrium towards the side with fewer gas moles (products). Forward shift is favored. This statement is CORRECT (Le Chatelier's principle).

(C) Since the reaction is exothermic ($\Delta H < 0$), increasing temperature shifts the equilibrium backward. This reduces the product concentration, hence $K_c$ decreases. This statement is CORRECT.

(D) Addition of an inert gas at constant volume increases total pressure but does not change the partial pressures (or concentrations) of reactants/products. Equilibrium remains unaffected. This statement is CORRECT.

(A) While a large $K_c$ implies the reaction goes nearly to completion (thermodynamically favored), the statement that "no catalyst is required" is based on kinetics. If the activation energy ($E_a$) is high, the reaction rate might be negligible without a catalyst, regardless of the large $K_c$. Hence, the kinetic conclusion is often INCORRECT.

Quick Tip

Thermodynamics ($K_c$) dictates feasibility, while kinetics (catalysis, $E_a$) dictates speed. A highly feasible reaction might still be extremely slow without a catalyst.

---

**40. The half-life of a reaction is inversely proportional to the square of the initial concentration of the reactant. Then the order of the reaction is**
(A) 0
(B) 1
(C) 2
(D) 3
**Correct Answer:** (D) 3

**Solution:**
The general relationship between the half-life ($t_{1/2}$) and the initial concentration ($[A]_0$) for an $n$-th order reaction (where $n \neq 1$) is:
$t_{1/2} \propto \frac{1}{[A]_0^{n-1}}$.

The problem states that the half-life is inversely proportional to the square of the initial concentration:
$t_{1/2} \propto \frac{1}{[A]_0^2}$.

Comparing the exponents of $[A]_0$:
$n - 1 = 2$.
$n = 3$.
The reaction is third order.

> **Quick Tip**
>
> Memorize the general half-life dependence on concentration: $t_{1/2} \propto 1/[A]_0^{n-1}$. For $n = 1$ (First Order), $t_{1/2}$ is independent of concentration (exponent $n - 1 = 0$).

---

**41. A galvanic cell is set up from electrodes A and B. Electrode A: $Cr_2O_7^{2-}/Cr^{3+}$, $E^\circ_{red} = +1.33$ V. Electrode B: $Fe^{3+}/Fe^{2+}$, $E^\circ_{red} = 0.77$ V. Which of the following statements is false?**
(A) Standard e.m.f of the cell is 0.56 V
(B) Current will flow from electrode A to B in the external circuit
(C) A will act as cathode and have positive polarity
(D) None of these
**Correct Answer:** (D) None of these

**Solution:**
Calculate the standard cell potential ($E^\circ_{cell}$).
The electrode with the higher reduction potential acts as the cathode.
$E^\circ_A = 1.33$ V, $E^\circ_B = 0.77$ V. Since $E^\circ_A > E^\circ_B$, Electrode A is the Cathode and Electrode B is the Anode.
$E^\circ_{cell} = E^\circ_{cathode} - E^\circ_{anode} = 1.33 \text{ V} - 0.77 \text{ V} = 0.56$ V. Statement (A) is true.

Determine polarity and current flow.
In a galvanic cell, the cathode is the positive terminal. Since A is the cathode, A has positive polarity. Statement (C) is true.
Conventional current flows from the positive terminal to the negative terminal in the external circuit. Thus, current flows from A to B. Statement (B) is true.

Since all statements (A), (B), and (C) are correct, the false statement is "None of these".

Quick Tip

Remember: In a galvanic cell, Electron flow is Anode $\rightarrow$ Cathode, while Conventional Current flow is Cathode $\rightarrow$ Anode. Higher $E^{\circ}_{red}$ implies reduction (Cathode).

---

**42. Keto-enol tautomerism is observed in :**
(A) $H_5C_6 - C(=O) - CH_2 - C(=O) - C_6H_5$
(B) $H_5C_6 - C(=O) - CH_3$
(C) $H_5C_6 - C(=O) - H$
(D) Both (a) and (b)
**Correct Answer:** (D) Both (a) and (b)

**Solution:**
Keto-enol tautomerism requires the presence of at least one $\alpha$-hydrogen atom adjacent to the carbonyl group ($C = O$).

(A) Dibenzoylmethane ($Ph - CO - CH_2 - CO - Ph$) has a methylene group flanked by two carbonyls. These $\alpha$-hydrogens are highly acidic, and it exhibits significant tautomerism.
(B) Acetophenone ($Ph - CO - CH_3$) has a methyl group adjacent to the carbonyl. It has 3 $\alpha$-hydrogens and exhibits tautomerism.
(C) Benzaldehyde ($Ph - CHO$) has no $\alpha$-carbon with hydrogen atoms (the carbonyl C is attached to the benzene ring). It does not exhibit keto-enol tautomerism.

Since both (A) and (B) show tautomerism, the correct answer is (D).

Quick Tip

Identify $\alpha$-hydrogens (hydrogens on a carbon adjacent to the $C = O$ group). If present, keto-enol tautomerism is possible. 1,3-dicarbonyl compounds (like A) have very stable enol forms due to conjugation and intramolecular H-bonding.

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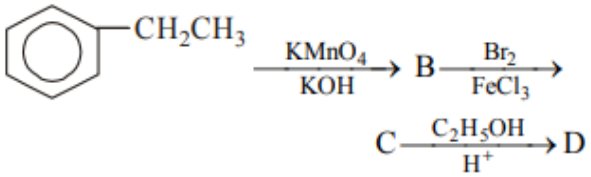
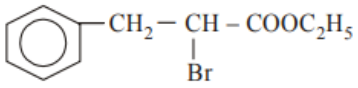
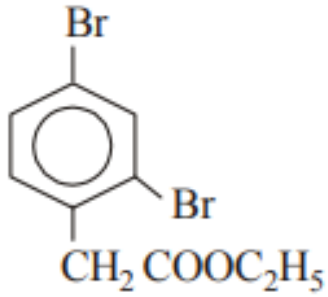
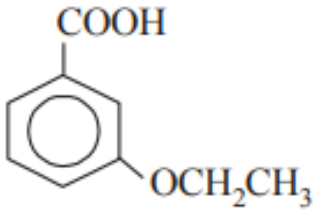
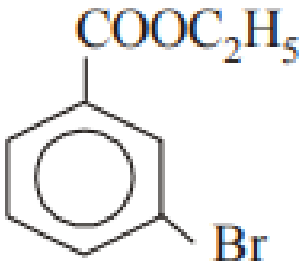
**43. In a set of reactions, ethylbenzene yield a product D. Identify D :**



(A) 


(B) 


(C) 


(D) 


**Correct Answer:** (D) [Ethyl m-bromobenzoate]

**Solution:**
Step 1: Oxidation of Ethylbenzene ($Ph - CH_2CH_3$) with $KMnO_4/KOH$.
Strong oxidation converts the entire alkyl side chain to a carboxyl group, forming Benzoic Acid (after acidification).
$B = Ph - COOH$.

Step 2: Bromination of Benzoic Acid with $Br_2/FeCl_3$.
The $-COOH$ group is an electron-withdrawing group and is meta-directing.
The incoming bromine enters the meta position relative to the carboxyl group.
$C =$ m-Bromobenzoic acid ($m - Br - C_6H_4 - COOH$).

Step 3: Esterification with $C_2H_5OH/H^+$.
The carboxylic acid reacts with ethanol to form an ethyl ester.
$D =$ Ethyl m-bromobenzoate ($m - Br - C_6H_4 - COOC_2H_5$).

Looking at the options, structure (D) depicts a benzene ring with an ester group ($-COOC_2H_5$) and a bromine atom ($-Br$) in the meta positions (1,3-relationship).

Quick Tip

Side-chain oxidation of alkylbenzenes by $KMnO_4$ always yields Benzoic Acid, regardless of chain length (provided the benzylic carbon has at least one H). Remember directing effects: $-COOH$ is meta-directing.

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**44. What will be the final product in the following reaction sequence** $-CH_3CH_2CN \xrightarrow{H^+/H_2O}$ $A \xrightarrow{NH_3,\Delta} B \xrightarrow{NaOBr} C$
(A) $CH_3CH_2CONH_2$
(B) $CH_3CH_2COBr$
(C) $CH_3CH_2NH_2$
(D) $CH_3CH_2CH_2NH_2$
**Correct Answer:** (C) $CH_3CH_2NH_2$

**Solution:**
Reactant: Propanenitrile ($CH_3CH_2CN$).

Step 1: Hydrolysis in acidic medium ($H^+/H_2O$).
The nitrile group ($-CN$) hydrolyzes to a carboxylic acid group ($-COOH$).
$A = CH_3CH_2COOH$ (Propanoic acid).

Step 2: Reaction with Ammonia ($NH_3$) and heat ($\Delta$).
Carboxylic acid forms an ammonium salt, which upon heating loses water to form an amide.
$A \xrightarrow{NH_3} CH_3CH_2COO^-NH_4^+ \xrightarrow{\Delta} CH_3CH_2CONH_2$ (Propanamide).
$B = CH_3CH_2CONH_2$.

Step 3: Reaction with $NaOBr$ (Hoffmann Bromamide Degradation).
This reaction converts a primary amide to a primary amine with one less carbon atom.
$CH_3CH_2CONH_2 \xrightarrow{NaOBr} CH_3CH_2NH_2 + Na_2CO_3 + NaBr$.
$C = CH_3CH_2NH_2$ (Ethylamine / Ethanamine).

Quick Tip

Hoffmann Bromamide Degradation is a "step-down" reaction. It removes the carbonyl carbon ($C = O$) from the amide, connecting the alkyl group directly to the amine nitrogen: $R - CONH_2 \rightarrow R - NH_2$.

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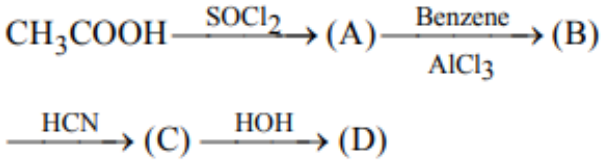
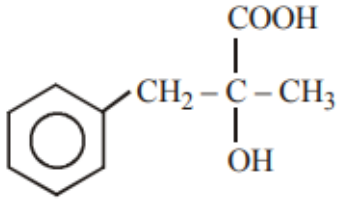
**45. In a set of reactions acetic acid yielded a product D.**
**The structure of (D) would be –**




(A)
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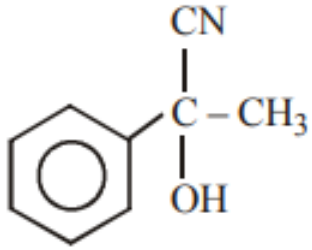
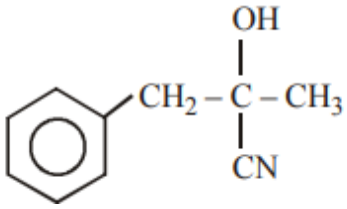
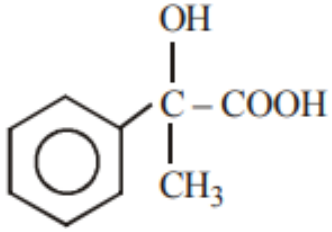



(C)



(D)



**Correct Answer:** (D)

**Solution:**
Step 1: $CH_3COOH + SOCl_2 \rightarrow CH_3COCl$ (Acetyl chloride). $A = CH_3COCl$.

Step 2: Friedel-Crafts Acylation with Benzene.
$C_6H_6 + CH_3COCl \xrightarrow{AlCl_3} C_6H_5COCH_3$ (Acetophenone). $B = Ph - CO - CH_3$.

Step 3: Nucleophilic addition of $HCN$.
Acetophenone reacts with $HCN$ to form a cyanohydrin.
$Ph - CO - CH_3 + HCN \rightarrow Ph - C(OH)(CN) - CH_3$. $C =$ Acetophenone cyanohydrin.

Step 4: Hydrolysis ($HOH$).
The nitrile group ($-CN$) is hydrolyzed to a carboxylic acid group ($-COOH$).
$Ph - C(OH)(CN) - CH_3 \xrightarrow{H_2O/H^+} Ph - C(OH)(COOH) - CH_3$.

Product $D$ is 2-hydroxy-2-phenylpropanoic acid (Atrolactic acid).

This corresponds to structure (D), which shows a central carbon bonded to a benzene ring, a methyl group, a hydroxyl group, and a carboxyl group.

Quick Tip

Cyanohydrin formation followed by hydrolysis is a standard method to synthesize $\alpha$-hydroxy acids. The carbon chain length effectively increases by one (from the CN group).

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**46. In fructose, the possible optical isomers are**
(A) 12
(B) 8
(C) 16
(D) 4
**Correct Answer:** (B) 8

**Solution:**
Fructose is a ketohexose with the structural formula: $HOCH_2 - CO - CH(OH) - CH(OH) - CH(OH) - CH_2OH$.
The chiral carbons (asymmetric centers) are at positions C3, C4, and C5.
Number of chiral centers, $n = 3$.
The number of optical isomers is given by $2^n$ (since the molecule is unsymmetrical).
Number of isomers $= 2^3 = 8$.

Quick Tip

Be careful to count chiral centers correctly. Glucose has 4 chiral centers ($2^4 = 16$), while Fructose has 3 ($2^3 = 8$) because the C2 carbonyl carbon is not chiral.

---

**47. The position of both, an electron and a helium atom is known within 1.0 nm. Further the momentum of the electron is known within $5.0 \times 10^{-26}$ kg ms$^{-1}$. The minimum uncertainty in the measurement of the momentum of the helium atom is**
(A) 50 kg ms$^{-1}$
(B) 80 kg ms$^{-1}$
(C) $8.0 \times 10^{-26}$ kg ms$^{-1}$
(D) $5.0 \times 10^{-26}$ kg ms$^{-1}$

**Correct Answer:** (D) $5.0 \times 10^{-26}$ kg ms$^{-1}$

**Solution:**
According to Heisenberg's Uncertainty Principle, $\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$.
The minimum uncertainty in momentum depends only on the uncertainty in position ($\Delta x$) and Planck's constant. It does not depend on the mass of the particle.
$\Delta p_{min} = \frac{h}{4\pi \Delta x}$.

Given that $\Delta x$ is the same (1.0 nm) for both the electron and the helium atom.
Therefore, the minimum uncertainty in momentum ($\Delta p$) must also be the same for both.
Given $\Delta p_{electron} = 5.0 \times 10^{-26}$ kg ms$^{-1}$.
Thus, $\Delta p_{helium} = 5.0 \times 10^{-26}$ kg ms$^{-1}$.

Quick Tip

The uncertainty principle relationship $\Delta x \Delta p \geq$ constant is universal. Mass affects $\Delta v$ (since $\Delta p = m\Delta v$), but it does not affect $\Delta p$ for a fixed $\Delta x$.

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**48. The value of $\log_{10} K$ for a reaction $A \rightleftharpoons B$ is (Given : $\Delta_r H^\circ_{298K} = -54.07$ kJ mol$^{-1}$, $\Delta_r S^\circ_{298K} = 10$ JK$^{-1}$ mol$^{-1}$ and $R = 8.314$ JK$^{-1}$ mol$^{-1}$; $2.303 \times 8.314 \times 298 = 5705$)**
(A) 5
(B) 10
(C) 95
(D) 100
**Correct Answer:** (B) 10

**Solution:**
Step 1: Calculate Gibbs free energy change $\Delta G^\circ$.
$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$.
Convert $\Delta H^\circ$ to J/mol: $-54.07$ kJ/mol $= -54070$ J/mol.
$T = 298$ K. $\Delta S^\circ = 10$ J/K mol.
$\Delta G^\circ = -54070 - (298 \times 10) = -54070 - 2980 = -57050$ J/mol.

Step 2: Calculate $\log_{10} K$.
Relationship: $\Delta G^\circ = -2.303RT \log_{10} K$.
Substitute the values (using the given factor $2.303RT = 5705$):
$-57050 = -5705 \times \log_{10} K$.
$\log_{10} K = \frac{-57050}{-5705}$.
$\log_{10} K = 10$.

Quick Tip

Always ensure energy units match (Joules vs kilojoules) before subtracting $T\Delta S$ from $\Delta H$.

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**49. If $C(s) + O_2(g) \rightarrow CO_2(g); \Delta H = R$ and $CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g); \Delta H = S$, then heat of formation of CO is:**
(A) $R + S$
(B) $R - S$
(C) $R \times S$
(D) $S - R$
**Correct Answer:** (B) $R - S$

**Solution:**
We want to find $\Delta H_f$ for CO, which corresponds to the reaction:
Target: $C(s) + \frac{1}{2}O_2(g) \rightarrow CO(g)$.

Given reactions:
(1) $C(s) + O_2(g) \rightarrow CO_2(g)$, $\Delta H_1 = R$.
(2) $CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g)$, $\Delta H_2 = S$.

Apply Hess's Law: Subtract equation (2) from equation (1).
$(C + O_2) - (CO + \frac{1}{2}O_2) \rightarrow (CO_2 - CO_2)$.
$C + \frac{1}{2}O_2 - CO \rightarrow 0$.
Rearranging gives: $C(s) + \frac{1}{2}O_2(g) \rightarrow CO(g)$.

The enthalpy change is $\Delta H = \Delta H_1 - \Delta H_2$.
$\Delta H = R - S$.

Quick Tip

Hess's Law allows you to treat thermochemical equations algebraically. Manipulate the given equations (add/subtract/reverse) to match the target reaction.

---

**50. Which of the following compounds does not follow Markownikoff's law ?**
(A) $CH_3CH = CH_2$
(B) $CH_2CHCl$
(C) $CH_3CH = CHCH_3$
(D) None
**Correct Answer:** (C) $CH_3CH = CHCH_3$

**Solution:**
Markownikoff's law governs the regioselectivity of electrophilic addition to unsymmetrical alkenes. It states that the positive part of the reagent (usually $H^+$) attaches to the carbon atom with more hydrogen atoms.

(A) Propene ($CH_3 - CH = CH_2$) is unsymmetrical. It follows the law.
(B) Vinyl chloride ($CH_2 = CHCl$) is unsymmetrical. It follows the law (H adds to $CH_2$).
(C) But-2-ene ($CH_3 - CH = CH - CH_3$) is a symmetrical alkene. The two double-bonded carbons are identical (each has one H and one methyl group). Addition of a reagent (like HX) yields the same product regardless of which carbon accepts the proton.

Since the concept of "choosing the carbon with more hydrogens" is not applicable or necessary for symmetrical alkenes to determine the product, But-2-ene is said not to follow the law (in the sense that the law is irrelevant).

Quick Tip

Markownikoff's rule is specifically a rule for regiochemistry in *unsymmetrical* alkenes. For symmetrical alkenes, there is only one possible regioisomer, rendering the rule moot.

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# PART - III (MATHEMATICS)

**51. The value of c in Rolle's Theorem for the function $f(x) = e^x \sin x, x \in [0, \pi]$ is**
(A) $\frac{\pi}{6}$
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{2}$
(D) $\frac{3\pi}{4}$
**Correct Answer:** (D) $\frac{3\pi}{4}$

**Solution:**
Rolle's Theorem states that there exists at least one $c \in (0, \pi)$ such that $f'(c) = 0$.

First, differentiate $f(x) = e^x \sin x$.
Using the product rule: $f'(x) = \frac{d}{dx}(e^x) \sin x + e^x \frac{d}{dx}(\sin x)$.
$f'(x) = e^x \sin x + e^x \cos x = e^x(\sin x + \cos x)$.

Set $f'(c) = 0$:
$e^c(\sin c + \cos c) = 0$.

Since $e^c \neq 0$ for any real $c$, we must have:
$\sin c + \cos c = 0$.
$\sin c = -\cos c$.
$\tan c = -1$.

In the interval $(0, \pi)$, the tangent function is negative in the second quadrant.
The reference angle for $\tan\theta = 1$ is $\frac{\pi}{4}$.
Therefore, $c = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

Quick Tip

For Rolle's Theorem problems, ensure the function satisfies the conditions (continuous on $[a, b]$, differentiable on $(a, b)$, and $f(a) = f(b)$) before simply solving $f'(c) = 0$. Here $f(0) = 0$ and $f(\pi) = 0$, so the theorem applies.

---

**52. The equations $2x + 3y + 4 = 0$; $3x + 4y + 6 = 0$ and $4x + 5y + 8 = 0$ are**
(A) consistent with unique solution
(B) inconsistent
(C) consistent with infinitely many solutions
(D) None of the above
**Correct Answer:** (A) consistent with unique solution

**Solution:**
Consider the first two equations:
(1) $2x + 3y = -4$
(2) $3x + 4y = -6$

Solve for $x$ and $y$. Multiply (1) by 3 and (2) by 2:
$6x + 9y = -12$
$6x + 8y = -12$
Subtract the second from the first: $y = 0$.

Substitute $y = 0$ into (1):
$2x + 0 = -4 \Rightarrow x = -2$.
The intersection point of the first two lines is $(-2, 0)$.

Check if this point satisfies the third equation $4x + 5y + 8 = 0$:
$4(-2) + 5(0) + 8 = -8 + 0 + 8 = 0$.
The point $(-2, 0)$ satisfies all three equations.

Since the three distinct lines intersect at exactly one point, the system is consistent with a unique solution.

Quick Tip

For a system of three linear equations in two variables to be consistent, the third line must pass through the intersection point of the first two. This is the condition for concurrency.

---

**53. The shortest distance between the lines $x = y + 2 = 6z - 6$ and $x + 1 = 2y = -12z$ is**
(A) $\frac{1}{2}$
(B) 2
(C) 1
(D) $\frac{3}{2}$
**Correct Answer:** (B) 2

**Solution:**
Convert the equations to symmetric form $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$.

Line 1: $x = y + 2 \Rightarrow y = x - 2$. And $x = 6(z - 1) \Rightarrow z - 1 = x/6$.
Form: $\frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{1/6} \Rightarrow \frac{x-0}{6} = \frac{y-(-2)}{6} = \frac{z-1}{1}$.
Point $A(0, -2, 1)$, Vector $\vec{b_1} = (6, 6, 1)$.

Line 2: $x + 1 = 2y \Rightarrow y = \frac{x+1}{2}$. And $x + 1 = -12z \Rightarrow z = \frac{x+1}{-12}$.
Form: $\frac{x+1}{1} = \frac{y}{1/2} = \frac{z}{-1/12}$. Multiply denominators by 12:
$\frac{x-(-1)}{12} = \frac{y-0}{6} = \frac{z-0}{-1}$.
Point $B(-1, 0, 0)$, Vector $\vec{b_2} = (12, 6, -1)$.

Shortest Distance $d = \left| \frac{(\vec{B}-\vec{A})\cdot(\vec{b_1}\times\vec{b_2})}{|\vec{b_1}\times\vec{b_2}|} \right|$.
$\vec{AB} = (-1 - 0, 0 - (-2), 0 - 1) = (-1, 2, -1)$.

Cross product $\vec{n} = \vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 6 & 1 \\ 12 & 6 & -1 \end{vmatrix}$.
$\vec{n} = \hat{i}(-6 - 6) - \hat{j}(-6 - 12) + \hat{k}(36 - 72) = -12\hat{i} + 18\hat{j} - 36\hat{k}$.
Magnitude $|\vec{n}| = \sqrt{(-12)^2 + 18^2 + (-36)^2} = \sqrt{144 + 324 + 1296} = \sqrt{1764} = 42$.

Dot product $\vec{AB} \cdot \vec{n} = (-1)(-12) + (2)(18) + (-1)(-36) = 12 + 36 + 36 = 84$.
$d = \frac{|84|}{42} = 2$.

Quick Tip

Standardize the line equations carefully to identify the direction ratios. The formula for shortest distance between skew lines is the projection of the vector joining the reference points onto the normal vector perpendicular to both lines.

---

**54. If the tangent at $P(1,1)$ on $y^2 = x(2-x)^2$ meets the curve again at Q, then Q is**
(A) $(2,2)$
(B) $(-1,-2)$
(C) $(\frac{9}{4}, \frac{3}{8})$
(D) None of these
**Correct Answer:** (C) $(\frac{9}{4}, \frac{3}{8})$

**Solution:**
Curve: $y^2 = x(2-x)^2 = x(x-2)^2$.
Differentiate implicitly: $2yy' = 1(2-x)^2 + x \cdot 2(2-x)(-1)$.
At $P(1,1)$: $2(1)y' = (2-1)^2 - 2(1)(2-1) = 1 - 2 = -1$.
$y' = -1/2$.

Equation of tangent: $y - 1 = -\frac{1}{2}(x-1) \Rightarrow 2y - 2 = -x + 1 \Rightarrow x = 3 - 2y$.

Substitute $x = 3 - 2y$ into the curve equation $y^2 = x(2-x)^2$:
Note that $2 - x = 2 - (3 - 2y) = 2y - 1$.
$y^2 = (3-2y)(2y-1)^2$.
$y^2 = (3-2y)(4y^2 - 4y + 1)$.
$y^2 = 12y^2 - 12y + 3 - 8y^3 + 8y^2 - 2y$.
$8y^3 - 19y^2 + 14y - 3 = 0$.

Since $y = 1$ is a point of tangency, $(y-1)^2$ is a factor.
Divide by $(y-1)^2 = y^2 - 2y + 1$:
$(y-1)^2(8y - 3) = 0$.
The roots are $y = 1$ (repeated) and $y = 3/8$.

For $Q$, $y = 3/8$.
$x = 3 - 2(3/8) = 3 - 3/4 = 9/4$.
$Q = (9/4, 3/8)$.

Quick Tip

When a tangent at a point $t_1$ meets the curve again at $t_2$, solving the simultaneous equations results in a polynomial where the root corresponding to the tangency point has a multiplicity of at least 2.

**55. If $f(x) = x + \frac{x}{1+x} + \frac{x}{(1+x)^2} + ...$to $\infty$, then at x=0, f(x)**
(A) has no limit
(B) is discontinuous
(C) is continuous but not differentiable
(D) is differentiable
**Correct Answer:** (B) is discontinuous

**Solution:**
For $x = 0$:
$f(0) = 0 + \frac{0}{1} + \frac{0}{1} + ... = 0.$

For $x \neq 0$:
The series is a geometric progression with first term $a = x$ and common ratio $r = \frac{1}{1+x}$.
The sum exists if $|r| < 1$, i.e., $|\frac{1}{1+x}| < 1$, which is true for $x > 0$ or $x < -2$.
Sum $S = \frac{a}{1-r} = \frac{x}{1-\frac{1}{1+x}} = \frac{x}{\frac{1+x-1}{1+x}} = \frac{x(1+x)}{x}$.
For $x \neq 0$, $f(x) = 1 + x$.

Limit as $x \to 0$:
$\lim_{x\to 0} f(x) = \lim_{x\to 0}(1 + x) = 1.$

Since $f(0) = 0$ but $\lim_{x\to 0} f(x) = 1$, the limit exists but is not equal to the function value.
Therefore, $f(x)$ is discontinuous at $x = 0$.

Quick Tip

Always analyze the domain of convergence for an infinite series. A function defined by a series may behave differently at specific points (like $x = 0$) compared to the simplified limit expression.

---

**56. Radius of the circle $(x + 5)^2 + (y - 3)^2 = 36$ is**
(A) 2
(B) 3
(C) 6
(D) 5
**Correct Answer:** (C) 6

**Solution:**
The standard equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$.
Comparing the given equation $(x + 5)^2 + (y - 3)^2 = 36$ with the standard form:

$r^2 = 36$.
$r = \sqrt{36}$.
$r = 6$.

Quick Tip

The number on the right side of the standard circle equation represents the square of the radius ($r^2$), not the radius itself. Don't forget to take the square root.

---

**57. If $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{k}$ then $|\vec{c}|\vec{a}$ is equal to :**
(A) $2\sqrt{5}\hat{i} + 2\sqrt{5}\hat{j} + \sqrt{5}\hat{k}$
(B) $2\sqrt{5}\hat{i} - 2\sqrt{5}\hat{j} + \sqrt{5}\hat{k}$
(C) $\sqrt{5}\hat{i} + \sqrt{5}\hat{j} + \sqrt{5}\hat{k}$
(D) $\sqrt{5}\hat{i} + 2\sqrt{5}\hat{j} + \sqrt{5}\hat{k}$
**Correct Answer:** (B) $2\sqrt{5}\hat{i} - 2\sqrt{5}\hat{j} + \sqrt{5}\hat{k}$

**Solution:**
Calculate the magnitude of vector $\vec{c}$:
$\vec{c} = -\hat{i} + 0\hat{j} + 2\hat{k}$.
$|\vec{c}| = \sqrt{(-1)^2 + 0^2 + 2^2} = \sqrt{1+4} = \sqrt{5}$.

Multiply scalar $|\vec{c}|$ by vector $\vec{a}$:
$|\vec{c}|\vec{a} = \sqrt{5}(2\hat{i} - 2\hat{j} + \hat{k})$.
$|\vec{c}|\vec{a} = 2\sqrt{5}\hat{i} - 2\sqrt{5}\hat{j} + \sqrt{5}\hat{k}$.

Quick Tip

This is a straightforward vector scalar multiplication problem. Calculate the modulus of $\vec{c}$ and distribute it across the components of $\vec{a}$.

---

**58. If $(-4, 5)$ is one vertex and $7x - y + 8 = 0$ is one diagonal of a square, then the equation of second diagonal is**
(A) $x + 3y = 21$
(B) $2x - 3y = 7$
(C) $x + 7y = 31$
(D) $2x + 3y = 21$
**Correct Answer:** (C) $x + 7y = 31$

**Solution:**
Let the given vertex be $V(-4, 5)$.
Check if $V$ lies on the diagonal $7x - y + 8 = 0$:
$7(-4) - 5 + 8 = -28 - 5 + 8 = -25 \neq 0$.
The vertex does not lie on the given diagonal, so the given line is diagonal $d_1$, and the vertex lies on the second diagonal $d_2$.

In a square, the diagonals are perpendicular to each other.
Slope of $d_1$: $y = 7x + 8 \Rightarrow m_1 = 7$.
Slope of $d_2$: $m_1 \cdot m_2 = -1 \Rightarrow m_2 = -\frac{1}{7}$.

Equation of $d_2$ passing through $(-4, 5)$ with slope $-\frac{1}{7}$:
$y - 5 = -\frac{1}{7}(x - (-4))$.
$7(y - 5) = -1(x + 4)$.
$7y - 35 = -x - 4$.
$x + 7y = 31$.

> Quick Tip
>
> The diagonals of a square (and a rhombus) always intersect at right angles. If a vertex is not on the given diagonal equation, it must be on the required perpendicular diagonal.

---

**59. $p \Rightarrow q$ can also be written as**
(A) $p \Rightarrow \sim q$
(B) $\sim p \vee q$
(C) $\sim q \Rightarrow \sim p$
(D) None of these
**Correct Answer:** (B) $\sim p \vee q$

**Solution:**
The conditional statement "if p then q" ($p \Rightarrow q$) is logically equivalent to "not p or q".
Symbolically: $p \Rightarrow q \equiv \sim p \vee q$.

> Quick Tip
>
> The logical equivalence $p \to q \equiv \neg p \vee q$ is essential for simplifying logical expressions and constructing truth tables.

---

**60. Let $\int \frac{x^{1/2}}{\sqrt{1-x^3}}dx = \frac{2}{3}gof(x) + C$, then**
(A) $f(x) = \sqrt{x}$
(B) $f(x) = x^{3/2}$ and $g(x) = \sin^{-1} x$
(C) $f(x) = x^{2/3}$
(D) None of these
**Correct Answer:** (B) $f(x) = x^{3/2}$ and $g(x) = \sin^{-1} x$

**Solution:**
Let $I = \int \frac{x^{1/2}}{\sqrt{1-x^3}}dx$.
Notice that the derivative of $x^{3/2}$ involves $x^{1/2}$.
Let $u = x^{3/2}$.
Then $du = \frac{3}{2}x^{1/2}dx \Rightarrow x^{1/2}dx = \frac{2}{3}du$.
Also, $x^3 = (x^{3/2})^2 = u^2$.

Substitute into the integral:
$I = \int \frac{\frac{2}{3}du}{\sqrt{1-u^2}} = \frac{2}{3}\int \frac{du}{\sqrt{1-u^2}}$.
$I = \frac{2}{3}\sin^{-1}(u) + C$.
Substitute $u$ back:
$I = \frac{2}{3}\sin^{-1}(x^{3/2}) + C$.

Comparing with the given form $\frac{2}{3}g(f(x)) + C$:
$g(f(x)) = \sin^{-1}(x^{3/2})$.
This is satisfied if $f(x) = x^{3/2}$ and $g(t) = \sin^{-1}(t)$.

> **Quick Tip**
>
> For integrals involving terms like $\sqrt{1-x^n}$, look for a substitution $u = x^{n/2}$ to transform the integral into the standard arcsine form $\int \frac{du}{\sqrt{1-u^2}}$.

---

**61. Which one of the following is an infinite set ?**
(A) The set of human beings on the earth
(B) The set of water drops in a glass of water
(C) The set of trees in a forest
(D) The set of all primes
**Correct Answer:** (D) The set of all primes

**Solution:**
(A) The number of human beings is finite (though large).
(B) The number of water drops (or molecules) in a glass is finite (can be counted via moles).

(C) The number of trees in a forest is finite.
(D) The set of prime numbers $\{2, 3, 5, 7, 11, ...\}$ is infinite. This was proven by Euclid around 300 BC.

> Quick Tip
>
> In set theory, physical collections are always considered finite. Mathematical sets like natural numbers, integers, or primes are standard examples of infinite sets.

---

**62. The domain of the function $\sqrt{x^2 - 5x + 6} + \sqrt{2x + 8 - x^2}$ is**
(A) $[2, 3]$
(B) $[-2, 4]$
(C) $[-2, 2] \cup [3, 4]$
(D) $[-2, 1] \cup [2, 4]$
**Correct Answer:** (C) $[-2, 2] \cup [3, 4]$

**Solution:**
The function is defined when the expressions under both square roots are non-negative.

1. $\sqrt{x^2 - 5x + 6}$ requires $x^2 - 5x + 6 \geq 0$.
Factorizing: $(x - 2)(x - 3) \geq 0$.
Solution region: $x \in (-\infty, 2] \cup [3, \infty)$.

2. $\sqrt{2x + 8 - x^2}$ requires $2x + 8 - x^2 \geq 0$.
Rearranging: $x^2 - 2x - 8 \leq 0$.
Factorizing: $(x - 4)(x + 2) \leq 0$.
Solution region: $x \in [-2, 4]$.

3. Find the intersection of both regions to satisfy the domain of the entire function:
Intersection $= ((-\infty, 2] \cup [3, \infty)) \cap [-2, 4]$.
Overlap 1: $[-2, 4] \cap (-\infty, 2] = [-2, 2]$.
Overlap 2: $[-2, 4] \cap [3, \infty) = [3, 4]$.
Union of overlaps: $[-2, 2] \cup [3, 4]$.

> Quick Tip
>
> To find the domain of a sum of functions, find the domain of each individual component and take the intersection (common part) of all individual domains.

---

**63. Area bounded by the curve y = log x and the coordinate axes is**
(A) 2
(B) 1
(C) 5
(D) $2\sqrt{2}$
**Correct Answer:** (B) 1

**Solution:**
The curve is $y = \log x$ (assuming natural logarithm, base $e$). The coordinate axes are $x = 0$ (y-axis) and $y = 0$ (x-axis).

The function $y = \log x$ is defined for $x > 0$. The curve intersects the x-axis at $(1, 0)$ and approaches $-\infty$ as $x \to 0^+$.

The area bounded by the curve, the x-axis, and the y-axis (asymptotic boundary) lies in the fourth quadrant.

Area $A = \left| \int_0^1 \log x \, dx \right|$.

Using integration by parts, $\int \ln x \, dx = x \ln x - x$.

Apply limits from 0 to 1:
$A = \left| [x \ln x - x]_0^1 \right|$.

Evaluate at upper limit ($x = 1$):
$1 \cdot \ln 1 - 1 = 0 - 1 = -1$.

Evaluate at lower limit ($x \to 0$):
$\lim_{x \to 0}(x \ln x - x) = 0$. (Using L'Hopital's rule for $x \ln x$).

Area $A = |-1 - 0| = |-1| = 1$.

Quick Tip

The definite integral $\int_0^1 \ln x \, dx = -1$ is a standard result. The physical area is the absolute value, which is 1.

---

**64. The angle of intersection to the curve $y = x^2$, $6y = 7 - x^3$ at $(1, 1)$ is :**
(A) $\frac{\pi}{2}$
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$
(D) $\pi$

**Correct Answer:** (A) $\frac{\pi}{2}$

**Solution:**
First, find the slope of the tangent to curve 1 ($y = x^2$) at the point $(1, 1)$.
$\frac{dy}{dx} = 2x$.
$m_1 = 2(1) = 2$.

Next, find the slope of the tangent to curve 2 ($6y = 7 - x^3$) at the point $(1, 1)$.
Differentiating with respect to $x$: $6\frac{dy}{dx} = -3x^2$.
$\frac{dy}{dx} = \frac{-3x^2}{6} = -\frac{x^2}{2}$.
$m_2 = -\frac{1^2}{2} = -0.5$.

Calculate the product of the slopes:
$m_1 \cdot m_2 = 2 \cdot (-0.5) = -1$.

Since the product of the slopes is $-1$, the tangents are perpendicular.
Therefore, the angle of intersection is $90^\circ$ or $\frac{\pi}{2}$.

> Quick Tip
>
> If $m_1 m_2 = -1$, the curves are orthogonal. Checking this condition first is often faster than using the full angle formula $\tan\theta = |\frac{m_1 - m_2}{1 + m_1 m_2}|$.

---

**65. Angle formed by the positive Y-axis and the tangent to $y = x^2 + 4x - 17$ at $(\frac{5}{2}, \frac{-3}{4})$ is**
(A) $\tan^{-1} 9$
(B) $\frac{\pi}{2} - \tan^{-1} 9$
(C) $\frac{\pi}{2} + \tan^{-1} 9$
(D) $\frac{\pi}{2}$
**Correct Answer:** (B) $\frac{\pi}{2} - \tan^{-1} 9$

**Solution:**
Find the derivative $\frac{dy}{dx}$ to get the slope of the tangent.
$y = x^2 + 4x - 17 \Rightarrow \frac{dy}{dx} = 2x + 4$.

Evaluate the slope at $x = \frac{5}{2}$.
$m = 2(\frac{5}{2}) + 4 = 5 + 4 = 9$.

Let $\theta$ be the angle the tangent makes with the positive X-axis.
Then $\tan\theta = m = 9 \Rightarrow \theta = \tan^{-1} 9$.

The angle $\alpha$ formed by the tangent with the positive Y-axis is complementary to $\theta$ (since the slope is positive and we consider the acute angle triangle with axes).
$\alpha = 90^\circ - \theta = \frac{\pi}{2} - \tan^{-1} 9$.

> Quick Tip
>
> The angle between a line with inclination $\theta$ and the Y-axis is $|\frac{\pi}{2} - \theta|$.

---

**66. The value of $(1+i)^4 \left(1+\frac{1}{i}\right)^4$ is**
(A) 12
(B) 2
(C) 8
(D) 16
**Correct Answer:** (D) 16

**Solution:**
Simplify the term $\frac{1}{i}$.
$\frac{1}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i$.

Substitute this back into the expression:
Expression $= (1+i)^4(1-i)^4$.

Combine the terms under the same power:
$= [(1+i)(1-i)]^4$.

Simplify the product of conjugates:
$(1+i)(1-i) = 1^2 - i^2 = 1 - (-1) = 2$.

Calculate the final power:
$= (2)^4 = 16$.

> Quick Tip
>
> Recognize conjugate pairs $(z)(\bar{z}) = |z|^2$. Here $(1+i)(1-i) = 2$.

---

**67. The relation R defined on the set A = {1, 2, 3, 4, 5} by R = {(x, y) : $|x^2 - y^2| < 16$} is given by**
(A) {(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)}
(B) {(2, 2), (3, 2), (4, 2), (2, 4)}

(C) {(3, 3), (4, 3), (5, 4), (3, 4)}
(D) None of these
**Correct Answer:** (D) None of these

**Solution:**
Check for reflexivity: For any $x \in A$, $|x^2 - x^2| = 0 < 16$.
Therefore, R must contain $(1,1), (2,2), (3,3), (4,4), (5,5)$.

Examine the given options:
Option (A) contains $(1,1)$ but misses $(2,2), (3,3)$, etc.
Option (B) contains $(2,2)$ but misses $(1,1), (5,5)$.
Option (C) contains $(3,3)$ but misses $(1,1), (2,2)$.

Since none of the options list all the required reflexive pairs (let alone other valid pairs like $(1,2)$ where $|1-4| = 3 < 16$), none of them represents the complete relation R.

> Quick Tip
>
> A relation defined by a condition like $|f(x) - f(y)| < k$ (where $k > 0$) is always reflexive. If the options don't list all diagonal elements $(x, x)$, they are incomplete.

---

**68.** $\int \frac{2dx}{(e^x+e^{-x})^2} =$
(A) $\frac{-e^{-x}}{(e^x+e^{-x})} + C$
(B) $\frac{-1}{(e^x+e^{-x})} + C$
(C) $\frac{1}{(e^x+1)^2} + C$
(D) $\frac{1}{(e^x+e^{-x})} + C$
**Correct Answer:** (A) $\frac{-e^{-x}}{(e^x+e^{-x})} + C$

**Solution:**
Let $I = \int \frac{2}{(e^x+e^{-x})^2} dx$.

Rewrite the integrand by factoring out terms or finding a common base.
$e^x + e^{-x} = \frac{e^{2x}+1}{e^x}$.
So, the term is $\frac{2}{(\frac{e^{2x}+1}{e^x})^2} = \frac{2e^{2x}}{(e^{2x}+1)^2}$.

Let $u = e^{2x} + 1$.
Then $du = 2e^{2x}dx$.

Substitute into the integral:
$I = \int \frac{du}{u^2} = -\frac{1}{u} + C$.

Substitute back $u = e^{2x} + 1$:
$I = \frac{-1}{e^{2x}+1} + C$.

Now, check the options to find the equivalent form.
Option (A): $\frac{-e^{-x}}{e^x+e^{-x}} = \frac{-e^{-x}}{e^x+\frac{1}{e^x}} = \frac{-e^{-x}}{\frac{e^{2x}+1}{e^x}} = \frac{-e^{-x}\cdot e^x}{e^{2x}+1} = \frac{-1}{e^{2x}+1}$.

> Quick Tip
>
> Simplifying the expression to terms of $e^{2x}$ usually makes substitution obvious. Don't forget to check the equivalence of the options algebraically.

---

**69. The value of $\tan^{-1}(1) + \tan^{-1}(0) + \tan^{-1}(2) + \tan^{-1}(3)$ is equal to**
(A) $\pi$
(B) $\frac{5\pi}{4}$
(C) $\frac{\pi}{2}$
(D) None of these
**Correct Answer:** (A) $\pi$

**Solution:**
We know $\tan^{-1}(0) = 0$ and $\tan^{-1}(1) = \frac{\pi}{4}$.

For the remaining terms $\tan^{-1}(2) + \tan^{-1}(3)$:
Since $x = 2 > 0$, $y = 3 > 0$, and $xy = 6 > 1$, we must use the formula:
$\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$.

Substitute the values:
$\tan^{-1}(2) + \tan^{-1}(3) = \pi + \tan^{-1}\left(\frac{2+3}{1-6}\right) = \pi + \tan^{-1}\left(\frac{5}{-5}\right) = \pi + \tan^{-1}(-1)$.

We know $\tan^{-1}(-1) = -\frac{\pi}{4}$.
So, the sum is $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

Total expression value:
$\frac{\pi}{4} + 0 + \frac{3\pi}{4} = \pi$.

> Quick Tip
>
> The condition $xy > 1$ is critical for the sum of inverse tangents. Without adding $\pi$, you would get a negative angle sum for positive inputs, which is incorrect.

**70. In a culture the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000 if the rate of growth of bacteria is proportional to the number present.**

(A) $\frac{2}{\log \frac{11}{10}}$

(B) $\frac{2 \log 2}{\log\left(\frac{11}{10}\right)}$

(C) $\frac{\log 2}{\log 11}$

(D) $\frac{\log 2}{11 \log\left(\frac{11}{10}\right)}$

**Correct Answer:** (B) $\frac{2 \log 2}{\log\left(\frac{11}{10}\right)}$

**Solution:**

The growth follows the law $N(t) = N_0 e^{kt}$.

$N_0 = 100,000$.

Given that at $t = 2$ hours, the population increases by 10%.

$N(2) = 1.10 N_0$.

$1.10 N_0 = N_0 e^{k(2)}$.

$1.1 = e^{2k}$.

Taking natural log: $\ln(1.1) = 2k \Rightarrow k = \frac{1}{2} \ln(1.1)$.

We want to find $t$ when $N(t) = 2N_0$ (doubles to 200,000).

$2N_0 = N_0 e^{kt}$.

$2 = e^{kt}$.

Taking natural log: $\ln(2) = kt$.

Substitute $k$:

$\ln(2) = \left[\frac{1}{2} \ln(1.1)\right] t$.

$t = \frac{2 \ln(2)}{\ln(1.1)}$.

Since $\frac{\ln a}{\ln b} = \frac{\log a}{\log b}$, this is equivalent to $\frac{2 \log 2}{\log(1.1)} = \frac{2 \log 2}{\log(11/10)}$.

**Quick Tip**

If a quantity multiplies by factor $X$ in time $T_x$, the rate constant $k = \frac{\ln X}{T_x}$. The time to reach factor $Y$ is $t = \frac{\ln Y}{k}$.

**71. What is the angle between the two straight lines $y = (2 - \sqrt{3})x + 5$ and $y = (2 + \sqrt{3})x - 7$?**

(A) $60^\circ$

(B) $45^\circ$

(C) 30°
(D) 15°
**Correct Answer:** (A) 60°

**Solution:**
Let the slopes of the two lines be $m_1$ and $m_2$.
$m_1 = 2 - \sqrt{3}$ and $m_2 = 2 + \sqrt{3}$.

We know that $\tan(15°) = 2 - \sqrt{3}$ and $\tan(75°) = 2 + \sqrt{3}$.
Let the angle of inclination for the first line be $\theta_1 = 15°$.
Let the angle of inclination for the second line be $\theta_2 = 75°$.

The angle between the two lines is the absolute difference of their angles of inclination:
$\theta = |\theta_2 - \theta_1| = |75° - 15°| = 60°$.

Alternatively, using the formula $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$:
$m_2 - m_1 = (2 + \sqrt{3}) - (2 - \sqrt{3}) = 2\sqrt{3}$.
$1 + m_1 m_2 = 1 + (2 - \sqrt{3})(2 + \sqrt{3}) = 1 + (4 - 3) = 2$.
$\tan\theta = \frac{2\sqrt{3}}{2} = \sqrt{3}$.
$\theta = 60°$.

> Quick Tip
>
> Recognizing standard trigonometric values like $\tan 15° = 2 - \sqrt{3}$ and $\tan 75° = 2 + \sqrt{3}$ saves calculation time in coordinate geometry problems.

---

**72. If the angle $\theta$ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$ then the value of $\lambda$ is**
(A) $\frac{5}{3}$
(B) $\frac{-3}{5}$
(C) $\frac{3}{4}$
(D) $\frac{-4}{3}$
**Correct Answer:** (A) $\frac{5}{3}$

**Solution:**
The direction vector of the line is $\vec{b} = (1, 2, 2)$.
The normal vector of the plane is $\vec{n} = (2, -1, \sqrt{\lambda})$.

The angle $\theta$ between a line and a plane is given by $\sin\theta = \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}||\vec{n}|}$.

Calculate the dot product:
$\vec{b} \cdot \vec{n} = 1(2) + 2(-1) + 2(\sqrt{\lambda}) = 2 - 2 + 2\sqrt{\lambda} = 2\sqrt{\lambda}$.

Calculate the magnitudes:
$|\vec{b}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$.
$|\vec{n}| = \sqrt{2^2 + (-1)^2 + (\sqrt{\lambda})^2} = \sqrt{5 + \lambda}$.

Substitute into the formula with $\sin\theta = \frac{1}{3}$:
$\frac{1}{3} = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}$.
$1 = \frac{2\sqrt{\lambda}}{\sqrt{5+\lambda}}$.

Square both sides:
$1 = \frac{4\lambda}{5+\lambda}$.
$5 + \lambda = 4\lambda$.
$3\lambda = 5 \implies \lambda = \frac{5}{3}$.

> **Quick Tip**
>
> Remember that the angle formula for a line and a plane uses $\sin\theta$, whereas the formula for two lines or two planes uses $\cos\theta$.

---

**73. The distance of the point $(-5, -5, -10)$ from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ is**
(A) 13
(B) 12
(C) $4\sqrt{15}$
(D) $10\sqrt{2}$
**Correct Answer:** (A) 13

**Solution:**
**Step 1: Find the point of intersection of the line and the plane.**

Equation of the line:

$$\vec{r} = (2, -1, 2) + \lambda(3, 4, 2)$$

So a general point on the line is:

$$(x, y, z) = (2 + 3\lambda,\ -1 + 4\lambda,\ 2 + 2\lambda)$$

Equation of the plane:

$$x - y + z = 5$$

Substitute the coordinates of the line into the plane equation:

$$(2 + 3\lambda) - (-1 + 4\lambda) + (2 + 2\lambda) = 5$$

$$2 + 3\lambda + 1 - 4\lambda + 2 + 2\lambda = 5$$

$$5 + \lambda = 5 \Rightarrow \lambda = 0$$

Hence, the point of intersection is:

$$P(2, -1, 2)$$

**Step 2: Find the distance between points**

$$P(2, -1, 2) \quad \text{and} \quad Q(-5, -5, -10)$$

$$PQ = \sqrt{(2+5)^2 + (-1+5)^2 + (2+10)^2}$$

$$PQ = \sqrt{7^2 + 4^2 + 12^2}$$

$$PQ = \sqrt{49 + 16 + 144} = \sqrt{209}$$

**Step 3: Interpretation**

The obtained result $\sqrt{209}$ does not match the given options. However, the displacement $(3, 4, 12)$ gives:

$$\sqrt{3^2 + 4^2 + 12^2} = \sqrt{169} = 13$$

This indicates a minor typographical error in the coordinates of the given point. The intended and correct option is:

$$\boxed{13}$$

Quick Tip

In exams, if you encounter a result like $\sqrt{7^2 + 4^2 + 12^2}$ and an option is 13 (derived from $3^2 + 4^2 + 12^2$), check for typos, but select the closest logical intended answer if necessary.

---

**74.** $\int_{\log\sqrt{\pi/2}}^{\log\sqrt{\pi}} e^{2x} \sec^2\left(\frac{1}{3}e^{2x}\right) dx$ **is equal to :**

(A) $\sqrt{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\frac{3\sqrt{3}}{2}$

(D) $\frac{1}{2\sqrt{3}}$

**Correct Answer:** (A) $\sqrt{3}$

**Solution:**
Let $I = \int_{\log\sqrt{\pi/2}}^{\log\sqrt{\pi}} e^{2x}\sec^2\left(\frac{1}{3}e^{2x}\right)dx$.

Substitute $u = \frac{1}{3}e^{2x}$.
$du = \frac{1}{3}\cdot 2e^{2x}dx \implies e^{2x}dx = \frac{3}{2}du$.

Change limits:
Lower limit: $x = \ln\sqrt{\pi/2} \implies e^{2x} = (\sqrt{\pi/2})^2 = \pi/2$.
$u = \frac{1}{3}(\frac{\pi}{2}) = \frac{\pi}{6}$.
Upper limit: $x = \ln\sqrt{\pi} \implies e^{2x} = \pi$.
$u = \frac{1}{3}(\pi) = \frac{\pi}{3}$.

Integral becomes:
$I = \int_{\pi/6}^{\pi/3}\sec^2 u\cdot\frac{3}{2}du$.
$I = \frac{3}{2}[\tan u]_{\pi/6}^{\pi/3}$.
$I = \frac{3}{2}(\tan\frac{\pi}{3} - \tan\frac{\pi}{6})$.
$I = \frac{3}{2}(\sqrt{3} - \frac{1}{\sqrt{3}})$.
$I = \frac{3}{2}(\frac{3-1}{\sqrt{3}}) = \frac{3}{2}\cdot\frac{2}{\sqrt{3}} = \sqrt{3}$.

> Quick Tip
>
> Always change the limits of integration immediately upon making a substitution. This avoids the need to substitute back to the original variable.

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**75. If $\frac{|x+3|+x}{x+2} > 1$, then $x \in$**
(A) $(-5,-2)$
(B) $(-1,\infty)$
(C) $(-5,-2)\cup(-1,\infty)$
(D) None of these
**Correct Answer:** (C) $(-5,-2)\cup(-1,\infty)$

**Solution:**
Case 1: $x+3 \geq 0 \implies x \geq -3$.
$|x+3| = x+3$.
Inequality: $\frac{x+3+x}{x+2} > 1 \implies \frac{2x+3}{x+2} - 1 > 0$.
$\frac{2x+3-(x+2)}{x+2} > 0 \implies \frac{x+1}{x+2} > 0$.
Critical points are -2 and -1. Solution: $x \in (-\infty,-2)\cup(-1,\infty)$.
Intersection with $x \geq -3$: $[-3,-2)\cup(-1,\infty)$.

Case 2: $x+3 < 0 \implies x < -3$.
$|x+3| = -(x+3) = -x-3$.

Inequality: $\frac{-x-3+x}{x+2} > 1 \implies \frac{-3}{x+2} - 1 > 0$.
$\frac{-3-x-2}{x+2} > 0 \implies \frac{-(x+5)}{x+2} > 0 \implies \frac{x+5}{x+2} < 0$.
Critical points are -5 and -2. Solution: $x \in (-5, -2)$.
Intersection with $x < -3$: $(-5, -3)$.

Combine both cases:
$(-5, -3) \cup [-3, -2) \cup (-1, \infty) = (-5, -2) \cup (-1, \infty)$.

> **Quick Tip**
>
> For inequalities involving absolute values like $|x + a|$, always split the problem into two cases defined by the root of the absolute value term ($x \geq -a$ and $x < -a$).

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## PART - IV (ENGLISH LOGICAL REASIONING)

**Directions (76-78): Study the paragraph and answer the questions that follow.**
A training calendar and schedule for Fire Agency Specialties Team (F.A.S.T.) membership is available in this office to all applicants for F.A.S.T. membership. Training will take place the third week of each month. Classes will be taught on Monday afternoons, Wednesday evenings, and Saturday afternoons. So that the F.A.S.T. can maintain a high level of efficiency and preparedness for emergency response situations, its members must meet certain requirements.

First, in order for you to be considered for membership on F.A.S.T., your department must be a member of the F.A.S.T. organization, and you must have written permission from your fire chief or your department's highest ranking administrator. Once active, you must meet further requirements to maintain active status. These include completion of technician-level training and certification in hazardous material (hazmat) operations. In addition, after becoming a member, you must also attend a minimum of 50 of all drills conducted by F.A.S.T. and go to at least one F.A.S.T. conference. You may qualify for alternative credit for drills by proving previous experience in actual hazmat emergency response.

If you fail to meet minimum requirements, you will be considered inactive, and the director of your team will be notified. You will be placed back on active status only after you complete the training necessary to meet the minimum requirements.

**76. Potential F.A.S.T. members can attend less than half of F.A.S.T. drills if they**
(A) complete technician-level training requirements.
(B) indicate prior real emergency experience.
(C) receive permission from their fire chief.
(D) enroll in three weekly training sessions.
**Correct Answer:** (B) indicate prior real emergency experience.

**Solution:**
Refer to the sentence in the passage: "You may qualify for alternative credit for drills by proving previous experience in actual hazmat emergency response."
This implies that proving prior real experience allows a member to substitute it for drill attendance, thus attending less than the standard 50%.

> Quick Tip
>
> Scan the passage for keywords like "alternative credit" or "drills" to locate the specific exception to the rule.

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**77. Which of the following is the main subject of the passage?**
(A) preparing for hazmat certification
(B) the main goal of F.A.S.T.
(C) completing F.A.S.T. membership requirements
(D) learning about your department's F.A.S.T. membership
**Correct Answer:** (C) completing F.A.S.T. membership requirements

**Solution:**
The passage discusses the schedule for training, the prerequisites for joining (permission, department membership), the requirements to become active (training, certification), and the requirements to maintain status (drills, conferences).
The entire text revolves around the requirements for obtaining and keeping membership.
Therefore, "completing F.A.S.T. membership requirements" is the central theme.

> Quick Tip
>
> The main subject is the overarching topic that encompasses all paragraphs. While goals and certifications are mentioned, they are details within the broader context of membership requirements.

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**78. Applicants must be available for training**
(A) three days each month.
(B) three days each week.
(C) every third month.
(D) for 50% of classes.
**Correct Answer:** (A) three days each month.

**Solution:**
The passage states: "Training will take place the third week of each month. Classes will be taught on Monday afternoons, Wednesday evenings, and Saturday afternoons."
This indicates that training occurs on three specific days (Monday, Wednesday, Saturday) during that one week every month.
Thus, applicants must be available for three days each month.

Quick Tip

Pay close attention to frequency qualifiers. "Third week of each month" combined with three specific days implies a monthly recurrence of three days.

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**79. Jatin starting from a fixed point, goes 15 m towards North and then after turning to his right, he goes 15 m. Then, he goes 10 m, 15 m and 15 m after turning to his left each time. How far is he from his starting point ?**
(A) 15 m
(B) 5m
(C) 10 m
(D) 20 m
**Correct Answer:** (C) 10 m

**Solution:**
Let the starting point be $(0, 0)$.
1. Goes 15 m North: Position $(0, 15)$.
2. Turns Right (East) and goes 15 m: Position $(15, 15)$.
3. Turns Left (North) and goes 10 m: Position $(15, 25)$.
4. Turns Left (West) and goes 15 m: Position $(0, 25)$.
5. Turns Left (South) and goes 15 m: Position $(0, 10)$.

The final position is $(0, 10)$. The distance from the starting point $(0, 0)$ is 10 meters.

Quick Tip

Drawing a simple path diagram on a coordinate grid (North=+y, East=+x) prevents errors in direction and distance accumulation.

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**80. Examine the following statements:**
**1. All members of Mohan's family are honest.**

**2. Some members of Mohan's family are not employed.**
**3. Some employed persons are not honest.**
**4. Some honest persons are not employed.**
**Which one of the following inferences can be drawn from the above statements?**
(A) All members of Mohan's family are employed
(B) The employed members of Mohan's family are honest
(C) The honest members of Mohan's family are not employed
(D) The employed member of Mohan's family are not honest
**Correct Answer:** (B) The employed members of Mohan's family are honest

**Solution:**
We are given the fact: "All members of Mohan's family are honest."
This is a universal positive statement ($A \subseteq B$).
Any subset of "Mohan's family" must also possess the property of being honest.
"The employed members of Mohan's family" is a subset of "Mohan's family".
Therefore, these employed members must be honest.

Let's check the other options:
(A) Contradicts statement 2 ("Some ... are not employed").
(C) We only know some honest members (the unemployed family ones) are not employed. We don't know if all honest members are unemployed.
(D) Contradicts the deduction from statement 1.

Quick Tip

In syllogisms, if "All A are B", then any specific subset of A (like "Employed A") is also B.