VITEEE 2019 Question Paper with Solutions for Maths

Time Allowed :90 Minutes	Maximum Marks :80	Total Questions :80

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The question paper contains a total of 80 questions divided into four parts:
   - Part I: Physics (Questions 1 to 25)
   - Part II: Chemistry (Questions 26 to 50)
   - Part III: Mathematics (Questions 51 to 75)
   - Part IV: English & Logical Reasoning (Questions 76 to 80)
2. All questions are multiple-choice with four options, and only one of them is correct.
3. For each correct answer, the candidate will earn 1 mark.
4. There is no negative marking for incorrect answers.
5. The test duration is 12 hours.

1. If $G(x) = \begin{vmatrix} f(x)f(-x) & 0 & x^4 \\ 3 & f(x)-f(-x) & \cos x \\ x^4 & 2x & f(x)f(-x) \end{vmatrix}$**, then** $\int_{-2}^{2} x^4 G(x)dx$ **is equal to**

(A) -1
(B) 0
(C) 2
(D) 1

Correct Answer: (B) 0

Solution:
First, let's determine the parity (even or odd nature) of the function $G(x)$.

Let $A(x) = f(x)f(-x)$. Note that $A(-x) = f(-x)f(x) = A(x)$, so $A(x)$ is an even function.

Let $B(x) = f(x) - f(-x)$. Note that $B(-x) = f(-x) - f(x) = -(f(x) - f(-x)) = -B(x)$, so $B(x)$ is an odd function.

The determinant is $G(x) = \begin{vmatrix} A(x) & 0 & x^4 \\ 3 & B(x) & \cos x \\ x^4 & 2x & A(x) \end{vmatrix}$.

Expanding along the second row:

$$G(x) = -3\begin{vmatrix} 0 & x^4 \\ 2x & A(x) \end{vmatrix} + B(x)\begin{vmatrix} A(x) & x^4 \\ x^4 & A(x) \end{vmatrix} - \cos x\begin{vmatrix} A(x) & 0 \\ x^4 & 2x \end{vmatrix}.$$

Term 1: $-3(0 - 2x^5) = 6x^5$. This is an Odd function.

Term 2: $B(x)((A(x))^2 - x^8)$. Since B is Odd and A^2, x^8 are Even, Odd $\times$ Even = Odd function.

Term 3: $-\cos x(2xA(x) - 0) = -2x\cos xA(x)$. Since x is Odd and $\cos x, A(x)$ are Even, Odd $\times$ Even = Odd function.

Since $G(x)$ is the sum of three odd functions, $G(x)$ is an Odd function.

The integral is $I = \int_{-2}^{2} x^4 G(x)dx$.

The integrand is $x^4 \cdot G(x) = (\text{Even}) \cdot (\text{Odd}) = \text{Odd}$.

The definite integral of an odd function over a symmetric interval $[-a, a]$ is always 0.

Therefore, the value is 0.

Quick Tip

Always check for symmetry (even/odd) when integrating over limits like $[-a, a]$. If $f(-x) = -f(x)$, $\int_{-a}^{a} f(x)dx = 0$.

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2. If $1, \alpha_1, \alpha_2, \alpha_3$ are the fourth roots of unity, then the value of $(1+\alpha_1)(1+\alpha_2)(1+\alpha_3)$ is equal to
(A) -3
(B) -1
(C) 0
(D) 2
Correct Answer: (C) 0

Solution:
The fourth roots of unity are the roots of the equation $x^4 - 1 = 0$.

The roots are given as $1, \alpha_1, \alpha_2, \alpha_3$.

Since $x^4 - 1 = (x-1)(x+1)(x-i)(x+i)$, the roots are $1, -1, i, -i$.

So, the set $\{\alpha_1, \alpha_2, \alpha_3\}$ is equal to $\{-1, i, -i\}$.

We need to calculate $(1+\alpha_1)(1+\alpha_2)(1+\alpha_3)$.

One of these roots must be -1. Let $\alpha_1 = -1$.

Then the first factor is $(1+(-1)) = 0$.

Consequently, the entire product $(0)(1+i)(1-i)$ becomes 0.

Alternatively, using polynomial properties:
$\frac{x^4-1}{x-1} = x^3 + x^2 + x + 1 = (x-\alpha_1)(x-\alpha_2)(x-\alpha_3)$.

We want the value of $(1+\alpha_1)(1+\alpha_2)(1+\alpha_3)$. This is not directly $P(1)$ or $P(-1)$.

However, rewriting the expression: $(1+\alpha_1) = -((-1)-\alpha_1)$.
Product $= (-1)^3((-1)-\alpha_1)((-1)-\alpha_2)((-1)-\alpha_3) = -1 \cdot P(-1)$.

$P(-1) = (-1)^3 + (-1)^2 + (-1) + 1 = -1 + 1 - 1 + 1 = 0$.

Thus, the value is 0.

Quick Tip

Recognize that if roots are n-th roots of unity, one of the roots is -1 (if n is even). Factors involving $(1 + \text{root})$ will become zero.

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3. A conic has focus $(1, 0)$ and corresponding directrix $x + y = 5$. If the eccentricity of the conic is 2, then its equation is
(A) $x^2 + 4xy + y^2 + 18x - 20y + 49 = 0$
(B) $x^2 - 4xy + y^2 - 18x - 20y + 49 = 0$
(C) $x^2 + 4xy + y^2 - 18x + 20y + 49 = 0$
(D) $x^2 + 4xy + y^2 - 18x - 20y + 49 = 0$
Correct Answer: (D) $x^2 + 4xy + y^2 - 18x - 20y + 49 = 0$

Solution:
Let $P(x, y)$ be any point on the conic.

The definition of a conic is $\frac{SP}{PM} = e$, where S is the focus, M is the perpendicular distance to the directrix, and e is the eccentricity.

Here, $S = (1, 0)$, Directrix line $L : x + y - 5 = 0$, and $e = 2$.

$SP^2 = e^2 \cdot PM^2$.

$(x-1)^2 + (y-0)^2 = 2^2 \cdot \left(\frac{x+y-5}{\sqrt{1^2+1^2}}\right)^2$.

$(x-1)^2 + y^2 = 4 \cdot \frac{(x+y-5)^2}{2}$.

$x^2 - 2x + 1 + y^2 = 2(x^2 + y^2 + 25 + 2xy - 10x - 10y)$.

$x^2 + y^2 - 2x + 1 = 2x^2 + 2y^2 + 50 + 4xy - 20x - 20y$.

Rearranging all terms to the right side:
$(2x^2 - x^2) + 4xy + (2y^2 - y^2) + (-20x + 2x) + (-20y) + (50 - 1) = 0$.

$x^2 + 4xy + y^2 - 18x - 20y + 49 = 0$.

Quick Tip

For any conic section problem given Focus, Directrix, and Eccentricity, directly apply the definition $SP^2 = e^2PM^2$.

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4. Let $\vec{u}, \vec{v}, \vec{w}$ be three vectors such that $|\vec{u}| = 1, |\vec{v}| = 2, |\vec{w}| = 3$ and $\vec{v}$ and $\vec{w}$ are mutually perpendicular. If projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ then $|\vec{u} - \vec{v} + \vec{w}|$ equals to
(A) $\sqrt{7}$
(B) 14
(C) 2
(D) $\sqrt{14}$
Correct Answer: (D) $\sqrt{14}$

Solution:
Given: $|\vec{u}| = 1, |\vec{v}| = 2, |\vec{w}| = 3$.

$\vec{v} \perp \vec{w} \implies \vec{v} \cdot \vec{w} = 0$.

Projection of $\vec{v}$ along $\vec{u}$ = Projection of $\vec{w}$ along $\vec{u}$.
$\frac{\vec{v}\cdot\vec{u}}{|\vec{u}|} = \frac{\vec{w}\cdot\vec{u}}{|\vec{u}|} \implies \vec{v} \cdot \vec{u} = \vec{w} \cdot \vec{u}$.

Let $k = \vec{v} \cdot \vec{u} = \vec{w} \cdot \vec{u}$.

We need to find $|\vec{u} - \vec{v} + \vec{w}|$. Let's calculate its square.
$|\vec{u} - \vec{v} + \vec{w}|^2 = (\vec{u} - \vec{v} + \vec{w}) \cdot (\vec{u} - \vec{v} + \vec{w})$.

$= |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 - 2(\vec{u} \cdot \vec{v}) + 2(\vec{u} \cdot \vec{w}) - 2(\vec{v} \cdot \vec{w})$.

Substitute the known values:
$= 1^2 + 2^2 + 3^2 - 2k + 2k - 2(0)$.

$= 1 + 4 + 9 = 14$.

Therefore, $|\vec{u} - \vec{v} + \vec{w}| = \sqrt{14}$.

> 💡 Quick Tip
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> Expand the square of the magnitude $|\vec{A}+\vec{B}+\vec{C}|^2 = A^2+B^2+C^2+2(\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{C}+\vec{C}\cdot\vec{A})$.

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5. A plane at a unit distance from the origin intersects the coordinate axes at P, Q and R. If the locus of the centroid of ΔPQR satisfies the equation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$, then the value of k is
(A) 3
(B) 4
(C) 9
(D) 16
Correct Answer: (C) 9

Solution:
Let the equation of the plane be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

The intercepts are $P(a, 0, 0)$, $Q(0, b, 0)$, and $R(0, 0, c)$.

The distance of this plane from the origin $(0, 0, 0)$ is given as 1.
$\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} = 1 \implies \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = 1$.
Let the centroid of ΔPQR be (X, Y, Z).
$X = \frac{a+0+0}{3} \implies a = 3X$.
$Y = \frac{0+b+0}{3} \implies b = 3Y$.
$Z = \frac{0+0+c}{3} \implies c = 3Z$.

Substitute these into the distance equation:
$\frac{1}{(3X)^2} + \frac{1}{(3Y)^2} + \frac{1}{(3Z)^2} = 1$.

$\frac{1}{9X^2} + \frac{1}{9Y^2} + \frac{1}{9Z^2} = 1$.

Multiply by 9: $\frac{1}{X^2} + \frac{1}{Y^2} + \frac{1}{Z^2} = 9$.

Comparing with the given equation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$, we get $k = 9$.

> 💡 Quick Tip
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> The centroid of a triangle with vertices (x_1, y_1, z_1) etc., is $(\frac{\Sigma x}{3}, \frac{\Sigma y}{3}, \frac{\Sigma z}{3})$.

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6. If g be an inverse function of f and $f'(x) = \frac{1}{1+x^5}$, then $g'(x)$ will be :

(A) $1 + x^5$
(B) $1 + (g(x))^5$
(C) $\left(\frac{1}{1+g(x)}\right)^5$
(D) $(g(x))^5$

Correct Answer: (B) $1 + (g(x))^5$

Solution:
Since g is the inverse function of f, we have $f(g(x)) = x$.

Differentiating both sides with respect to x using the chain rule:
$f'(g(x)) \cdot g'(x) = 1$.

Therefore, $g'(x) = \frac{1}{f'(g(x))}$.

We are given $f'(x) = \frac{1}{1+x^5}$.

To find $f'(g(x))$, substitute $g(x)$ for x in the expression for f':
$f'(g(x)) = \frac{1}{1+(g(x))^5}$.

Now substitute this back into the equation for $g'(x)$:
$g'(x) = \frac{1}{\frac{1}{1+(g(x))^5}} = 1 + (g(x))^5$.

> 💡 Quick Tip
>
> If $g = f^{-1}$, then $g'(x) = \frac{1}{f'(g(x))}$. This is a standard property of derivatives of inverse functions.

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7. The area enclosed between the curves $y = |x^3|$ and $x = y^3$ is

(A) $\frac{1}{2}$
(B) $\frac{1}{4}$
(C) $\frac{1}{8}$
(D) $\frac{1}{16}$

Correct Answer: (A) $\frac{1}{2}$

Solution:
The curves are $C_1 : y = |x^3|$ and $C_2 : x = y^3 \implies y = x^{1/3}$.

Let's find the points of intersection.
For $x \geq 0$, $y = x^3$. So $x^3 = x^{1/3} \implies x^9 = x \implies x(x^8 - 1) = 0$.
The real roots are $x = 0$ and $x = 1$. The points are $(0, 0)$ and $(1, 1)$.

In the interval $(0, 1)$, take $x = 1/8$. $y_1 = (1/8)^3 = 1/512$. $y_2 = (1/8)^{1/3} = 1/2$.
So $y = x^{1/3}$ lies above $y = x^3$.

For $x < 0$, $y = -x^3$ (since $|x^3| = -x^3$) and $y = x^{1/3}$ (which is negative).
Intersection: $-x^3 = x^{1/3} \implies -x^9 = x \implies x(1 + x^8) = 0$.
The only real root is $x = 0$. The curves do not enclose a finite area in the 2nd/3rd quadrants as they only meet at the origin and diverge.

Thus, the enclosed area is only in the first quadrant.
Area $= \int_0^1 (x^{1/3} - x^3)dx$.

$= \left[\frac{x^{4/3}}{4/3} - \frac{x^4}{4}\right]_0^1$.

$= \left[\frac{3}{4}x^{4/3} - \frac{1}{4}x^4\right]_0^1$.

$= (\frac{3}{4} - \frac{1}{4}) - 0 = \frac{2}{4} = \frac{1}{2}$.

> Quick Tip
>
> Always visualize or sketch the curves to check which lies above the other and to verify the bounds of integration.

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8. Let f(x) be a differential function such that $f'(x) = f(x) + \int_0^2 f(x)dx$ and $f(0) = \frac{(4-e^2)}{3}$. Then $f(x)$ is:
(A) $e^x - \frac{(e^2-1)}{3}$
(B) $e^x - \frac{(e^2-1)}{4}$
(C) $e^x - \frac{(e^2+1)}{3}$
(D) $e^x - \frac{(4-e^2)}{3}$
Correct Answer: (A) $e^x - \frac{(e^2-1)}{3}$

Solution:
Let $K = \int_0^2 f(x)dx$. Note that K is a constant.
The differential equation becomes $f'(x) = f(x) + K$.

$\frac{dy}{dx} - y = K$. This is a linear differential equation.
Integrating Factor (I.F.) $= e^{\int -1 dx} = e^{-x}$.
$y \cdot e^{-x} = \int K e^{-x} dx = -K e^{-x} + C$.
$y = f(x) = Ce^x - K$.

Given $f(0) = \frac{4-e^2}{3}$.
Substituting $x = 0$: $C - K = \frac{4-e^2}{3}$ ...(1)

Now use the definition of K: $K = \int_0^2 (Ce^x - K)dx$.
$K = [Ce^x - Kx]_0^2 = (Ce^2 - 2K) - (C - 0)$.
$K = Ce^2 - 2K - C$.
$3K = C(e^2 - 1) \implies K = C\frac{e^2-1}{3}$.

Substitute K into (1):
$C - C\frac{e^2-1}{3} = \frac{4-e^2}{3}$.
$C\left(1 - \frac{e^2-1}{3}\right) = \frac{4-e^2}{3}$.
$C\left(\frac{3-e^2+1}{3}\right) = \frac{4-e^2}{3}$.
$C\left(\frac{4-e^2}{3}\right) = \frac{4-e^2}{3}$.
Since $e^2 \approx 7.4$, $4 - e^2 \neq 0$, so $C = 1$.

Then $K = \frac{e^2-1}{3}$.
The function is $f(x) = 1 \cdot e^x - \frac{e^2-1}{3}$.

Quick Tip

When an integral with fixed limits appears in an equation for $f(x)$, treat it as a constant (K) and solve for it at the end.

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9. A coin is tossed n times. The maximum value of n such that the probability of getting no head is greater than 1/16 is
(A) 4
(B) 3
(C) 5
(D) 2
Correct Answer: (B) 3

Solution:
Let X be the random variable representing the number of heads.
The probability of getting a head in a single toss is $p = 1/2$. Probability of tails is $q = 1/2$.

The probability of getting no head in n tosses is $P(X=0) = \binom{n}{0} p^0 q^n = (1/2)^n$.

We are given the condition: $P(X=0) > \frac{1}{16}$.
$\left(\frac{1}{2}\right)^n > \frac{1}{16}$.
$\left(\frac{1}{2}\right)^n > \left(\frac{1}{2}\right)^4$.

Since the base (1/2) is less than 1, the inequality sign reverses for the exponents:
$n < 4$.

Since n must be a positive integer (number of tosses), the possible values are 1, 2, 3.
The maximum value of n satisfying the condition is 3.

Quick Tip

Remember that for $0 < b < 1$, $b^x > b^y \implies x < y$.

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10. Suppose 5- digit numbers are formed by the digits 1,2,3,4 and 5 without repetition. If they are arranged in an ascending order, then 100^{th} number is
(A) 51243
(B) 51423
(C) 51234
(D) 51342
Correct Answer: (D) 51342

Solution:
Total numbers formed using digits 1, 2, 3, 4, 5 without repetition $= 5! = 120$.
We need to find the 100^{th} number in ascending order.

Number of numbers starting with 1: $4! = 24$. (Ranks 1-24)
Number of numbers starting with 2: $4! = 24$. (Ranks 25-48)
Number of numbers starting with 3: $4! = 24$. (Ranks 49-72)
Number of numbers starting with 4: $4! = 24$. (Ranks 73-96)
Total so far $= 24 \times 4 = 96$.

The 100^{th} number must start with 5.
Next, consider the second digit (ascending order of remaining digits 1, 2, 3, 4).
Numbers starting with 51... : Remainder 3 digits (2,3,4). $3! = 6$ numbers.
Ranks 97 to $96 + 6 = 102$.
So the 100^{th} number lies in the block 51...

Let's list these numbers starting from rank 97:
Digits available: 2, 3, 4. Ascending order: 2, 3, 4.

97: 51234
98: 51243
99: 51324
100: 51342

The 100^{th} number is 51342.

> **Quick Tip**
>
> Break the problem down by fixing the first digit and counting the permutations ($n!$) until you get close to the target rank.
