VITEEE 2018 Question Paper with Solutions for Maths

Time Allowed :90 Minutes	Maximum Marks :80	Total Questions :80

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The question paper contains a total of 80 questions divided into four parts:
   - Part I: Physics (Questions 1 to 25)
   - Part II: Chemistry (Questions 26 to 50)
   - Part III: Mathematics (Questions 51 to 75)
   - Part IV: English & Logical Reasoning (Questions 76 to 80)
2. All questions are multiple-choice with four options, and only one of them is correct.
3. For each correct answer, the candidate will earn 1 mark.
4. There is no negative marking for incorrect answers.
5. The test duration is 12 hours.

1. If A is a non-singular matrix and $(A-2I)(A-4I)=[0]$, then $\frac{1}{6}A+\frac{4}{3}A^{-1}$ is
(A) $[0]$
(B) I
(C) $2I$
(D) $6I$
Correct Answer: (B) I

Solution:
We are given the matrix equation: $(A-2I)(A-4I)=0$.

Expand the equation: $A^2-4A-2IA+8I^2=0$.

Simplify: $A^2-6A+8I=0$.

Since A is non-singular, multiply the entire equation by A^{-1}:

$A^2A^{-1}-6AA^{-1}+8IA^{-1}=0$.

$A-6I+8A^{-1}=0$.

Rearrange the equation: $A+8A^{-1}=6I$.

Divide both sides by 6: $\frac{1}{6}A + \frac{8}{6}A^{-1} = \frac{6I}{6}$.

Simplify the coefficients: $\frac{1}{6}A + \frac{4}{3}A^{-1} = I$.

> 💡 Quick Tip
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> To find an expression involving a matrix A and its inverse A^{-1} from a polynomial equation in A, multiply the polynomial by A^{-1}. Remember that $AA^{-1} = I$ and $IA^{-1} = A^{-1}$.

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2. The amplitude of the complex number $Z = \frac{-1+i\sqrt{3}}{2}$ is
(A) $\frac{\pi}{6}$
(B) $\frac{\pi}{3}$
(C) $\frac{2\pi}{3}$
(D) $\frac{4\pi}{3}$
Correct Answer: (C) $\frac{2\pi}{3}$

Solution:
The complex number is $Z = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$.

Let $x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$.

Since $x < 0$ and $y > 0$, Z lies in the second quadrant.

Calculate the reference angle $\alpha = \arctan\left|\frac{y}{x}\right|$.

$\alpha = \arctan\left|\frac{\sqrt{3}/2}{-1/2}\right| = \arctan(\sqrt{3})$.

$\alpha = \frac{\pi}{3}$.

The amplitude θ in the second quadrant is $\theta = \pi - \alpha$.

$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

> 💡 Quick Tip
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> When a complex number $Z = x + iy$ lies in the second quadrant $(x < 0, y > 0)$, its amplitude θ is calculated as π minus the reference angle $\alpha = \arctan(|y/x|)$.

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3. The eccentricity of ellipse $4x^2 + 9y^2 - 16x = 20$ is
(A) $\frac{\sqrt{5}}{3}$
(B) $\frac{2}{3}$
(C) $\frac{1}{3}$
(D) $\frac{4}{3}$
Correct Answer: (A) $\frac{\sqrt{5}}{3}$

Solution:
Rewrite the equation by completing the square for x: $4x^2 - 16x + 9y^2 = 20$.

$4(x^2 - 4x) + 9y^2 = 20$.

$4(x^2 - 4x + 4) - 16 + 9y^2 = 20$.

$4(x-2)^2 + 9y^2 = 36$.

Divide by 36 to get the standard form: $\frac{(x-2)^2}{9} + \frac{y^2}{4} = 1$.

We have $a^2 = 9$ and $b^2 = 4$. Since $a^2 > b^2$, the major axis is horizontal.

Eccentricity e is defined by $b^2 = a^2(1 - e^2)$.

$4 = 9(1 - e^2)$.

$1 - e^2 = \frac{4}{9}$.

$e^2 = 1 - \frac{4}{9} = \frac{5}{9}$.

$e = \frac{\sqrt{5}}{3}$.

> Quick Tip
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> The key to identifying properties of non-standard conic section equations is completing the square to isolate the variables and determine a^2 and b^2. The eccentricity formula $e = \sqrt{1 - b^2/a^2}$ applies when $a > b$.

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4. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$ then $\sin\frac{\theta}{2}$ is equal to
(A) 1
(B) $\frac{1}{2}|\vec{a} - \vec{b}|$
(C) 0
(D) $\frac{1}{2}|\vec{a} + \vec{b}|$

Correct Answer: (B) $\frac{1}{2}|\vec{a}-\vec{b}|$

Solution:
Given $|\vec{a}| = 1$ and $|\vec{b}| = 1$.

We start with the magnitude squared of the difference vector:

$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a}\cdot\vec{b})$.

$|\vec{a}-\vec{b}|^2 = 1^2 + 1^2 - 2|\vec{a}||\vec{b}|\cos\theta$.

$|\vec{a}-\vec{b}|^2 = 2 - 2\cos\theta = 2(1-\cos\theta)$.

Using the half-angle identity $1-\cos\theta = 2\sin^2\frac{\theta}{2}$:

$|\vec{a}-\vec{b}|^2 = 2\left(2\sin^2\frac{\theta}{2}\right) = 4\sin^2\frac{\theta}{2}$.

Taking the square root of both sides: $|\vec{a}-\vec{b}| = 2\sin\frac{\theta}{2}$.

Solving for $\sin\frac{\theta}{2}$: $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a}-\vec{b}|$.

Quick Tip

Remember the cosine rule for triangles applied to vectors, which leads directly to the half-angle relation: the length of the vector difference $|\vec{a}-\vec{b}|$ is proportional to $\sin(\theta/2)$, where θ is the angle between $\vec{a}$ and $\vec{b}$.

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5. The image of the point $(1,2,4)$ in the plane $2x-y+z+2=0$ is
(A) $(-3,4,2)$
(B) $(3,-4,2)$
(C) $(-3,-4,2)$
(D) $(-3,4,-2)$
Correct Answer: (A) $(-3,4,2)$

Solution:
Let $P(1,2,4)$ and the plane coefficients be $(A,B,C) = (2,-1,1)$, $D = 2$.

The formula for the image $P'(x',y',z')$ is:
$\frac{x'-x_1}{A} = \frac{y'-y_1}{B} = \frac{z'-z_1}{C} = k$, where $k = -2\frac{Ax_1+By_1+Cz_1+D}{A^2+B^2+C^2}$.

Calculate the numerator of the fraction:
$Ax_1 + By_1 + Cz_1 + D = 2(1) - 1(2) + 1(4) + 2 = 2 - 2 + 4 + 2 = 6.$

Calculate the denominator of the fraction:
$A^2 + B^2 + C^2 = 2^2 + (-1)^2 + 1^2 = 4 + 1 + 1 = 6.$

Substitute to find k: $k = -2\frac{6}{6} = -2.$

Solve for x': $\frac{x'-1}{2} = -2 \implies x' = 1 + 2(-2) = -3.$

Solve for y': $\frac{y'-2}{-1} = -2 \implies y' = 2 + (-1)(-2) = 4.$

Solve for z': $\frac{z'-4}{1} = -2 \implies z' = 4 + 1(-2) = 2.$

The image point is $(-3, 4, 2)$.

> Quick Tip
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> Be sure to substitute all components correctly, paying careful attention to signs (especially $B = -1$ and the overall factor of -2 in the image formula). This ensures accurate calculation of the ratio k.

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6. $\lim_{x\to 0}[1 + x\sin(\pi - x)]^{\frac{1}{x}}$ **is equal to**
(A) 0
(B) e
(C) 1
(D) π
Correct Answer: (C) 1

Solution:
The expression is $L = \lim_{x\to 0}[1 + x\sin(\pi - x)]^{\frac{1}{x}}$.

Use the identity $\sin(\pi - x) = \sin x$.

$L = \lim_{x\to 0}[1 + x\sin x]^{\frac{1}{x}}$.

This is of the indeterminate form 1^∞. We use the limit rule $L = e^{\lim_{x\to a} g(x)[f(x)-1]}$.

Here $f(x) = 1 + x\sin x$ and $g(x) = \frac{1}{x}$.

The exponent E is: $E = \lim_{x\to 0} \frac{1}{x}[(1 + x\sin x) - 1]$.

$E = \lim_{x \to 0} \frac{x \sin x}{x}$.

$E = \lim_{x \to 0} \sin x$.

$E = \sin(0) = 0$.

Therefore, $L = e^E = e^0 = 1$.

Quick Tip

Always simplify trigonometric functions before applying limit identities. The appearance of $\sin(\pi - x)$ should immediately signal simplification to $\sin x$. For 1^∞ limits, the conversion to exponential form e^E is standard.

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7. $\int_0^\pi \log(\sin^2 x)dx$ is equal to
(A) $2\pi \log_e(\frac{1}{2})$
(B) $2\pi \log_e(2)$
(C) $\pi \log_e(\frac{1}{2})$
(D) $\pi \log_e(2)$
Correct Answer: (A) $2\pi \log_e(\frac{1}{2})$

Solution:
Let $I = \int_0^\pi \log(\sin^2 x)dx$.

Use the logarithm property: $I = 2\int_0^\pi \log(\sin x)dx$.

Use the property $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$ when $f(2a - x) = f(x)$.

Here $2a = \pi$, so $a = \pi/2$. Since $\sin(\pi - x) = \sin x$, the property holds.

$I = 2\left[2\int_0^{\pi/2} \log(\sin x)dx\right] = 4\int_0^{\pi/2} \log(\sin x)dx$.

Recall the standard integral result: $\int_0^{\pi/2} \log(\sin x)dx = -\frac{\pi}{2}\log 2$.

$I = 4\left(-\frac{\pi}{2}\log 2\right) = -2\pi \log 2$.

Rewrite the result using logarithm properties to match the options:

$I = 2\pi(-\log 2) = 2\pi(\log 2^{-1}) = 2\pi \log\left(\frac{1}{2}\right)$.

Quick Tip

Problems involving $\int \log(\sin x)$ or $\int \log(\cos x)$ often require repeated use of standard integral properties, specifically $\int_0^{\pi/2} \log(\sin x)dx = -\frac{\pi}{2}\log 2$ and the $f(2a-x)$ property.

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8. The general solution of the differential equation $2x + \frac{dy}{dx} - y = 3$ **is**
(A) $y = 2x - 1$
(B) $x^2 + y^2 = 2x - 1$
(C) $y = C_1 e^x + 2x - 1$
(D) $y^2 = C_1 e^x + 2x - 1$
Correct Answer: (C) $y = C_1 e^x + 2x - 1$

Solution:
Rearrange the equation into the linear first-order form $\frac{dy}{dx} + P(x)y = Q(x)$:

$\frac{dy}{dx} - y = 3 - 2x.$

Here $P(x) = -1$ and $Q(x) = 3 - 2x$.

Calculate the Integrating Factor (IF): $IF = e^{\int P(x)dx} = e^{\int -1dx} = e^{-x}$.

The general solution form is $y \cdot IF = \int Q(x) \cdot IF dx + C_1$.

$ye^{-x} = \int (3-2x)e^{-x}dx + C_1.$

Using integration by parts, we find $\int (3-2x)e^{-x}dx = (2x-1)e^{-x}$.

Substitute back: $ye^{-x} = (2x-1)e^{-x} + C_1$.

Multiply by e^x: $y = (2x-1) + C_1 e^x$.

$y = C_1 e^x + 2x - 1.$

Quick Tip

A linear first-order differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is solved by multiplying by the Integrating Factor $e^{\int P(x)dx}$ and then integrating the right side. The integration $\int (3-2x)e^{-x}dx$ can be efficiently solved using tabular integration or careful application of parts.

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9. A die is thrown 100 times. Getting an even number is considered as a success, the variance of number of successes is
(A) 50
(B) 25
(C) 10
(D) 100
Correct Answer: (B) 25

Solution:
This follows a binomial distribution $B(n, p)$.

Number of trials $n = 100$.

Success (p) is getting an even number $\{2, 4, 6\}$ on a standard die.

Probability of success $p = \frac{3}{6} = \frac{1}{2}$.

Probability of failure $q = 1 - p = \frac{1}{2}$.

The variance $Var(X)$ for a binomial distribution is given by $Var(X) = npq$.

$Var(X) = 100 \times \frac{1}{2} \times \frac{1}{2}$.

$Var(X) = 100 \times \frac{1}{4} = 25$.

Quick Tip

In statistical problems involving repeated independent trials with two outcomes (success/failure), identify the binomial distribution parameters n and p. Variance is a quick calculation once n, p, q are known.

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10. In the set of integers under the operation $a \times b = a + b - ab$ the identity element is
(A) 0
(B) 1
(C) a
(D) b
Correct Answer: (A) 0

Solution:
Let e be the identity element in the set of integers.

By the definition of the identity element, $a \times e = a$ must hold for all a.

Substitute the operation: $a + e - ae = a$.

Subtract a from both sides: $e - ae = 0$.

Factor out e: $e(1 - a) = 0$.

For this equation to be true for all integers a, the identity element e must be 0.

(If $e = 0$, $0(1 - a) = 0$, which is true for all a).

The identity element is 0.

Quick Tip

The identity element must work for *all* elements in the set. If $e(1 - a) = 0$, and $(1 - a)$ is not universally zero, e must be zero. Always verify that $e * a = a$ also holds (here $0 + a - 0a = a$).
