UP Board Class 10 Mathematics 2024 Code : 822 (HX) Question Paper with Solutions

Time Allowed :3 Hours 15 Minutes	**Maximum Marks :**70	**Total Questions :**25

General Instructions

Read the following instructions very carefully and strictly follow them:

1. First 15 minutes are allotted for examinees to read this question paper.
2. All questions are compulsory.
3. This question paper has two Parts A and B.
4. Part - A contains 20 multiple choice type questions of 1 mark each that have to be answered on the OMR Answer Sheet only.
5. After giving answer on the OMR Answer Sheet do not cut it and do not use eraser, whitener, etc.
6. Part - B contains descriptive type questions of 50 marks.
7. There are 5 questions in Part - B.
8. In the beginning of each question, it has been clearly mentioned that how many parts of it are to be attempted.
9. Marks allotted to each question are mentioned against it.
10. Start from the first question and go up to the last question. Do not waste your time on the question you cannot solve.
11. If you need place for rough work, do it on the left page of your answer book and cross (x) the page. Do not write the solution on that page.
12. Draw neat and correct figure in solution of a question wherever it is necessary, otherwise in its absence the solution will be treated as incomplete and wrong.

Part A

Multiple Choise Question:

1. If LCM of 26, 156 is 156, then the value of HCF will be:
(A) 156
(B) 26
(C) 13
(D) 6
Correct Answer: (B) 26

Solution: We are given the LCM of 26 and 156, and we need to find their HCF. We use the relationship between LCM, HCF, and the product of two numbers:

$$\text{LCM} \times \text{HCF} = \text{Product of the two numbers.}$$

Let HCF $= h$. Then:

$$\text{LCM}(26, 156) \times h = 26 \times 156.$$

We are given that $\text{LCM}(26, 156) = 156$, so:

$$156 \times h = 26 \times 156.$$

Canceling out 156 from both sides:

$$h = 26.$$

Thus, the value of HCF is $\boxed{26}$.

> Quick Tip
>
> To find the HCF when the LCM is known, use the formula:
>
> $$\text{LCM} \times \text{HCF} = \text{Product of the numbers.}$$

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2. Which one of the following is a pair of co-prime numbers?
(A) (14, 35)
(B) (18, 25)
(C) (31, 93)
(D) (32, 62)
Correct Answer: (B) (18, 25)

Solution: Co-prime numbers are numbers whose HCF is 1. Let's check the HCF for each pair of numbers.
- For (14, 35), the factors of 14 are $1, 2, 7, 14$ and the factors of 35 are $1, 5, 7, 35$. The common factor is 7, so they are not co-prime.
- For (18, 25), the factors of 18 are $1, 2, 3, 6, 9, 18$ and the factors of 25 are $1, 5, 25$. The common factor is 1, so they are co-prime.
- For (31, 93), the factors of 31 are $1, 31$ and the factors of 93 are $1, 3, 31, 93$. The common factor is 31, so they are not co-prime.
- For (32, 62), the factors of 32 are $1, 2, 4, 8, 16, 32$ and the factors of 62 are $1, 2, 31, 62$. The common factor is 2, so they are not co-prime.
Thus, the correct pair of co-prime numbers is $\boxed{(18, 25)}$.

> Quick Tip
>
> To check if two numbers are co-prime, find their HCF. If the HCF is 1, the numbers are co-prime.

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3. The distance between the points $(5, 0)$ and $(-12, 0)$ will be:
(A) 5
(B) 7
(C) 13
(D) 17
Correct Answer: (D) 17

Solution: The distance between two points (x_1, y_1) and (x_2, y_2) is given by the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Given points: $(5, 0)$ and $(-12, 0)$,
Since both points lie on the x-axis ($y = 0$), the distance simplifies to:

$$d = |x_2 - x_1|$$

$$d = |-12 - 5|$$

$$d = |-17|$$

$$d = 17$$

Thus, the correct answer is $\boxed{17}$.

Quick Tip

For points on the x-axis or y-axis, the distance can be found by taking the absolute difference of their coordinates.

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4. The least number which when divided by 35, 56, and 91 leaves the same remainder 7 in each case will be:
(A) 3640
(B) 3645
(C) 3647
(D) 3740
Correct Answer: (C) 3647

Solution: We are asked to find the least number x that, when divided by 35, 56, and 91, leaves a remainder of 7 in each case. This means that:

$$x - 7 \text{ is divisible by 35, 56, and 91.}$$

Thus, we need to find the least common multiple (LCM) of 35, 56, and 91, and then add 7 to the result.
- The prime factorization of 35 is $35 = 5 \times 7$, - The prime factorization of 56 is $56 = 2^3 \times 7$, - The prime factorization of 91 is $91 = 7 \times 13$.

The LCM is obtained by taking the highest powers of all primes appearing in the factorizations:

$$\text{LCM}(35, 56, 91) = 2^3 \times 5 \times 7 \times 13 = 3640.$$

Thus, the least number is:

$$x = 3640 + 7 = 3647.$$

Therefore, the correct answer is $\boxed{3647}$.

Quick Tip

To find a number that leaves the same remainder when divided by multiple divisors, find the LCM of those divisors and add the remainder to it.

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5. The sum of the first 15 multiples of 8 will be:
(A) 960
(B) 980
(C) 984
(D) 990
Correct Answer: (A) 960

Solution: The first 15 multiples of 8 are:

$$8, 16, 24, \ldots, 8 \times 15 = 120.$$

The sum of the first n multiples of 8 is:

$$\text{Sum} = 8 \times (1 + 2 + 3 + \cdots + 15).$$

The sum of the first 15 natural numbers is given by the formula:

$$\text{Sum of first } n \text{ numbers} = \frac{n(n+1)}{2}.$$

For $n = 15$:

$$\text{Sum of first 15 numbers} = \frac{15 \times 16}{2} = 120.$$

Thus, the sum of the first 15 multiples of 8 is:

$$\text{Sum} = 8 \times 120 = 960.$$

Thus, the correct answer is $\boxed{960}$.

Quick Tip

To find the sum of the first n multiples of a number, first find the sum of the first n natural numbers and multiply by the number.

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6. If one root of the quadratic equation $x^2 + 2x - p = 0$ is -2, then the value of p will be:

(A) 0
(B) 1
(C) 2
(D) 3
Correct Answer: (A) 0

Solution: Since -2 is a root of the quadratic equation,

$$x^2 + 2x - p = 0$$

It must satisfy the equation. Substituting $x = -2$:

$$(-2)^2 + 2(-2) - p = 0$$

$$4 - 4 - p = 0$$

$$0 - p = 0$$

$$p = 0$$

Thus, the correct answer is $\boxed{0}$.

Quick Tip

To find an unknown coefficient in a quadratic equation when a root is given, substitute the root into the equation and solve for the unknown.

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7. The coordinates of the point on the x-axis and equidistant from the points $(5, -2)$ **and** $(-3, 2)$ **will be:**
(A) (2, 0)
(B) (2, 2)
(C) (2, 1)
(D) (1, 0)
Correct Answer:(D) (1, 0)

Solution:
Let the required point on the x-axis be $(x, 0)$. Since it is equidistant from $(5, -2)$ and $(-3, 2)$, we use the distance formula:

$$\sqrt{(x-5)^2 + (0+2)^2} = \sqrt{(x+3)^2 + (0-2)^2}$$

Squaring both sides:

$$(x-5)^2 + 4 = (x+3)^2 + 4$$

Canceling 4 on both sides:

$$(x-5)^2 = (x+3)^2$$

Expanding:

$$x^2 - 10x + 25 = x^2 + 6x + 9$$

Canceling x^2 on both sides:

$$-10x + 25 = 6x + 9$$

Solving for x:

$$25 - 9 = 6x + 10x$$

$$16 = 16x$$

$$x = 1$$

Thus, the required point is $(1, 0)$.

$$\boxed{(1, 0)}$$

Quick Tip

To find a point equidistant from two given points, use the distance formula and equate the distances.

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8. The nature of the roots of the equation $2x^2 - 5x + 4 = 0$ will be:
(A) Real and equal
(B) Imaginary (not real)
(C) Real and distinct
(D) None of these
Correct Answer: (C) Real and distinct

Solution: To determine the nature of the roots of a quadratic equation, we calculate the discriminant:

$$\Delta = b^2 - 4ac,$$

where $a = 2$, $b = -5$, and $c = 4$.
Substituting the values:

$$\Delta = (-5)^2 - 4(2)(4) = 25 - 32 = -7.$$

Since the discriminant is negative, the roots are real and distinct.
Thus, the nature of the roots is $\boxed{\text{Real and distinct}}$.

💡 Quick Tip

The discriminant $\Delta = b^2 - 4ac$ helps determine the nature of the roots. If $\Delta > 0$, the roots are real and distinct; if $\Delta = 0$, the roots are real and equal; if $\Delta < 0$, the roots are imaginary.

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9. Two triangles are said to be similar to each other:
(A) If their corresponding angles are equal
(B) If their corresponding sides are proportional
(C) Both (A) and (B)
(D) None of these
Correct Answer: (C) Both (A) and (B)

Solution: Two triangles are said to be similar if: 1. Their corresponding angles are equal. 2. Their corresponding sides are proportional.
Thus, the correct answer is $\boxed{\text{Both (A) and (B)}}$.

💡 Quick Tip

For two triangles to be similar, their corresponding angles must be equal, and their corresponding sides must be proportional.

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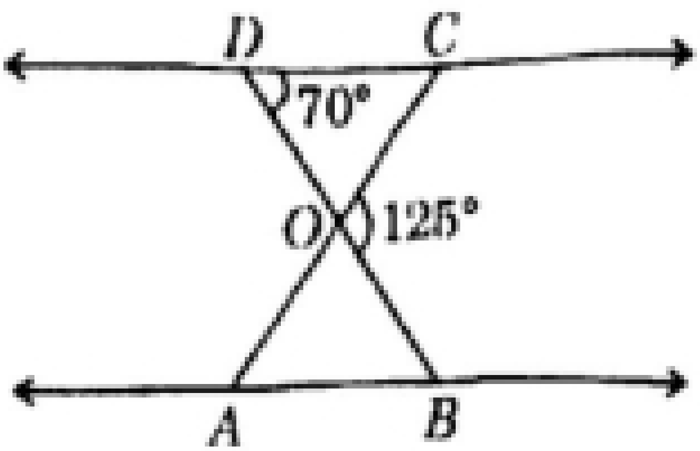
10. In the figure, $\triangle ODC \sim \triangle OBA$. If $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$, then the value of $\angle OAB$ will be:



(A) 55°
(B) 60°
(C) 65°
(D) 70°

Correct Answer: (A) 55°

Solution:
Given:

$$\angle BOC = 125^\circ, \quad \angle CDO = 70^\circ$$

Since $\triangle ODC \sim \triangle OBA$, their corresponding angles are equal.
From the straight line property:

$$\angle BOC + \angle BOA = 180^\circ$$

$$125^\circ + \angle BOA = 180^\circ$$

$$\angle BOA = 180^\circ - 125^\circ = 55^\circ$$

Since $\triangle ODC \sim \triangle OBA$, corresponding angles are equal:

$$\angle OAB = \angle BOA = 55^\circ$$

Thus, the required angle is:

$$\boxed{55^\circ}$$

Quick Tip

For two similar triangles, corresponding angles are equal, and corresponding sides are proportional.

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11. If $\sin\theta = \cos\theta$, $0^\circ \leq \theta \leq 90^\circ$, then the value of θ will be:
(A) 60°
(B) 45°
(C) 30°
(D) 0°
Correct Answer: (B) 45°

Solution: We are given that $\sin\theta = \cos\theta$, and we need to find the value of θ. We know that:

$$\sin\theta = \cos\theta \quad \Rightarrow \quad \tan\theta = 1.$$

The value of θ that satisfies $\tan\theta = 1$ within the range $0^\circ \leq \theta \leq 90^\circ$ is $\theta = 45^\circ$.
Thus, the value of θ is $\boxed{45^\circ}$.

Quick Tip

When $\sin\theta = \cos\theta$, the value of θ is 45°, since $\tan 45^\circ = 1$.

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12. The value of $\frac{\sin 15^\circ}{\cos 75^\circ}$ will be:
(A) 1
(B) 0
(C) 2
(D) -1
Correct Answer:(A) 1

Solution:
We know the trigonometric identity:

$$\cos(90^\circ - \theta) = \sin\theta$$

Using this identity,

$$\cos 75^\circ = \sin(90^\circ - 75^\circ) = \sin 15^\circ$$

Thus,

$$\frac{\sin 15^\circ}{\cos 75^\circ} = \frac{\sin 15^\circ}{\sin 15^\circ} = 1$$

Therefore, the correct answer is:

$$\boxed{1}$$

Quick Tip

Use the identity $\cos(90^\circ - \theta) = \sin\theta$ to simplify expressions involving complementary angles.

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13. If $\sin\theta - \cos\theta = 0$, then the value of $\sin^4\theta + \cos^4\theta$ will be:
(A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) $\frac{3}{4}$
(D) 1
Correct Answer: (B) $\frac{1}{2}$

Solution:
Given:

$$\sin\theta - \cos\theta = 0$$

This implies:

$$\sin\theta = \cos\theta$$

Dividing both sides by $\cos\theta$:

$$\tan\theta = 1$$

Thus,

$$\theta = 45^\circ$$

Now, we need to evaluate:

$$\sin^4 \theta + \cos^4 \theta$$

Using the identity:

$$\sin^4 \theta + \cos^4 \theta = (\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta$$

Since:

$$\sin^2 \theta + \cos^2 \theta = 1$$

And:

$$\sin^2 \theta = \cos^2 \theta = \frac{1}{2}$$

$$\sin^2 \theta \cos^2 \theta = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$$

Substituting these values:

$$\sin^4 \theta + \cos^4 \theta = 1 - 2 \times \frac{1}{4}$$

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

Thus, the required value is:

$$\boxed{\frac{1}{2}}$$

Quick Tip

Use the identity $\sin^4 \theta + \cos^4 \theta = 1 - 2 \sin^2 \theta \cos^2 \theta$ to simplify expressions.

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14. The value of $\sec A + \tan A(1 - \sin A)$ **will be:**
(A) $\sec A$
(B) $\sin A$
(C) $\csc A$
(D) $\cos A$
Correct Answer: (D) $\cos A$

Solution:
We start with the given expression:

$$(\sec A + \tan A)(1 - \sin A)$$

Expanding using trigonometric identities:

$$\sec A = \frac{1}{\cos A}, \quad \tan A = \frac{\sin A}{\cos A}$$

$$\sec A + \tan A = \frac{1 + \sin A}{\cos A}$$

Thus, multiplying by $(1 - \sin A)$:

$$(\sec A + \tan A)(1 - \sin A) = \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$$

Expanding:

$$= \frac{(1 + \sin A)(1 - \sin A)}{\cos A}$$

Using the identity:

$$(1 + \sin A)(1 - \sin A) = 1 - \sin^2 A = \cos^2 A$$

$$= \frac{\cos^2 A}{\cos A}$$

$$= \cos A$$

Thus, the required value is:

$$\boxed{\cos A}$$

Quick Tip

Use the identity $(1 + \sin A)(1 - \sin A) = \cos^2 A$ to simplify expressions involving sine and cosine.

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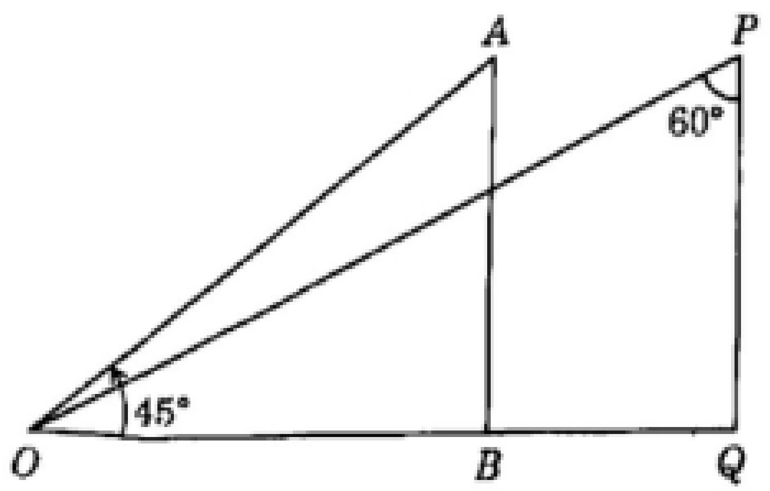
15. In the figure, the angles of depression of point O as seen from points A and P are:



(A) $30°, 45°$
(B) $45°, 30°$
(C) $45°, 60°$
(D) None of these
Correct Answer: (C) $45°, 60°$

Solution: From the diagram, we can see that point O is at the base of the figure, and points A and P are positioned higher. The angles of depression are given from these points to point O.

Step 1: Understand the angles of depression. The angle of depression from point A to point O is the angle formed between the line of sight from A to O and the horizontal line through A. The same applies for point P. Since these are angles of depression, they are measured downward from the horizontal line at points A and P.

Step 2: Use the concept of alternate interior angles. The angles of depression from points A and P will be congruent to the angles of elevation from point O to these points, because the lines OA and OP are parallel to the horizontal lines. This means that:

$$\text{Angle of depression from point } A = \text{Angle of elevation at point } O,$$

and

$$\text{Angle of depression from point } P = \text{Angle of elevation at point } O.$$

Step 3: Apply the given information. In the diagram, the angle of elevation at point O is $45°$ for point A and $60°$ for point P.

Thus, the angles of depression from points A and P are: - $45°$ from A, - $60°$ from P.
Thus, the correct answer is $\boxed{(C) 45°, 60°}$.

💡 Quick Tip

Angles of depression and elevation are often congruent because they are alternate interior angles formed by a horizontal line and a line of sight to a point.

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16. In a triangle ABC, $\angle C = 90°$ and $\tan A = \frac{1}{\sqrt{3}}$. The value of $\sin A \cos B + \cos A \sin B$ will be:
(A) 0
(B) $\frac{1}{\sqrt{2}}$
(C) 1
(D) $\sqrt{2}$
Correct Answer: (C) 1

Solution:
We are given that:

$$\tan A = \frac{1}{\sqrt{3}}$$

From the definition of tangent:

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

Let the opposite side of A be 1 and the adjacent side be $\sqrt{3}$. Using the Pythagorean theorem:

$$\text{hypotenuse} = \sqrt{(1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

Now, we calculate the trigonometric values:

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2}, \quad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$$

Since $\angle B = 90^\circ - A$, we have:

$$\sin B = \cos A = \frac{\sqrt{3}}{2}, \quad \cos B = \sin A = \frac{1}{2}$$

Now, calculating the given expression:

$$\sin A \cos B + \cos A \sin B$$

$$= \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

Thus, the required value is:

$$\boxed{1}$$

Quick Tip

The identity $\sin A \cos B + \cos A \sin B = \sin(A+B)$ simplifies to $\sin 90^\circ = 1$.

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17. The length of the minute hand of a clock is r cm. The area of the sector swept by the minute hand in one minute will be:

(A) $\frac{\pi r^2}{60^\circ}$

(B) $\frac{\pi r^2}{180^\circ}$

(C) $\frac{\pi r^2}{360^\circ}$

(D) $\frac{\pi r^2}{90^\circ}$

Correct Answer: (A) $\frac{\pi r^2}{60^\circ}$

Solution:

The area of a sector of a circle is given by:

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

where θ is the angle swept by the sector.
Since the minute hand of a clock moves 360° in 60 minutes, the angle swept in one minute is:

$$\theta = \frac{360^\circ}{60} = 6^\circ$$

Thus, the area swept in one minute is:

$$A = \frac{6^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{60} \times \pi r^2$$

$$= \frac{\pi r^2}{60}$$

Thus, the required area is:

$$\boxed{\frac{\pi r^2}{60}}$$

Quick Tip

To find the area of a sector, use the formula $\frac{\theta}{360^\circ} \times \pi r^2$, where θ is the angle in degrees.

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18. The whole surface area of a solid hemisphere of diameter $\frac{1}{2}$ cm will be:
(A) $\frac{1}{8}\pi$ cm^2
(B) $3 \times \frac{1}{16}\pi$ cm^2
(C) $\frac{3}{16}\pi$ cm^2
(D) $\frac{3}{2}\pi$ cm^2
Correct Answer: (C) $\frac{3}{16}\pi$ cm^2

Solution: The total surface area of a solid hemisphere is the sum of the curved surface area and the area of the circular base.
The surface area of the curved part of the hemisphere is given by:

$$\text{Curved surface area} = 2\pi r^2.$$

The area of the base is:

$$\text{Base area} = \pi r^2.$$

Thus, the total surface area is:

$$\text{Total surface area} = 2\pi r^2 + \pi r^2 = 3\pi r^2.$$

The diameter is given as $\frac{1}{2}$ cm, so the radius $r = \frac{1}{4}$ cm. Substituting into the formula:

$$\text{Total surface area} = 3\pi \times \left(\frac{1}{4}\right)^2 = 3\pi \times \frac{1}{16} = \frac{3}{16}\pi \,\text{cm}^2.$$

Thus, the correct answer is $\boxed{\frac{3}{16}\pi \, \text{cm}^2}$.

Quick Tip

For a solid hemisphere, the total surface area is the sum of the curved surface area and the area of the circular base, given by $3\pi r^2$.

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19. The mean and median of a frequency distribution are 26.1 and 25.8 respectively. The value of mode for the distribution will be:
(A) 24.2
(B) 25.1
(C) 25.2
(D) 26.4
Correct Answer: (C) 25.2

Solution:
Using the empirical relation between mean, median, and mode:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

Substituting the given values:

$$\text{Mode} = 3 \times 25.8 - 2 \times 26.1$$

$$= 77.4 - 52.2$$

$$= 25.2$$

Thus, the required mode is:

$$\boxed{25.2}$$

Quick Tip

The empirical formula Mode $= 3 \times$ Median $- 2 \times$ Mean is useful when the mode is not directly available from the data.

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20. The measure of central tendency is:
(A) Frequency
(B) Mean
(C) Median
(D) Both (B) and (C)
Correct Answer: (D) Both (B) and (C)

Solution: The *measure of central tendency* is a statistical term that refers to the central or typical value for a set of data. Common measures of central tendency include:

1. Mean – the arithmetic average of the data.
2. Median – the middle value of the data when it is ordered.
3. Mode – the value that appears most frequently in the data.

Given the options, both the mean and the median are measures of central tendency. Therefore, the correct answer is:

$$\boxed{\text{(D) Both (B) and (C)}}.$$

Quick Tip

In statistics, the mean and median are the most commonly used measures of central tendency. The mean is used when data is symmetrically distributed, while the median is preferred when data has outliers or is skewed.

Part B

Descriptive Questions:

21. Do all the parts:

(a) Given that HCF$(99, 153) = 9$, find the value of LCM$(99, 153)$.

Solution:
We can use the relationship between the *HCF* and *LCM* of two numbers, which is given by the formula:

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b.$$

Here, we are given: - HCF$(99, 153) = 9$, - $a = 99$, - $b = 153$.
We need to find LCM$(99, 153)$. Using the formula, we have:

$$9 \times \text{LCM}(99, 153) = 99 \times 153.$$

Now, calculate 99×153:

$$99 \times 153 = 15147.$$

Thus:

$$9 \times \text{LCM}(99, 153) = 15147,$$
$$\text{LCM}(99, 153) = \frac{15147}{9} = 1683.$$

Thus, the value of LCM$(99, 153)$ is $\boxed{1683}$.

Quick Tip

Use the formula HCF$(a, b) \times$ LCM$(a, b) = a \times b$ to relate the HCF and LCM of two numbers.

(b) If $2\cos^2 45° - 1 = \cos\theta$, then find the value of θ.

Solution:
We are given the equation:

$$2\cos^2 45° - 1 = \cos\theta.$$

We know that $\cos 45° = \frac{1}{\sqrt{2}}$, so:

$$\cos^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}.$$

Substituting this into the equation:

$$2 \times \frac{1}{2} - 1 = \cos\theta,$$

$$1 - 1 = \cos\theta,$$

$$\cos\theta = 0.$$

The value of θ for which $\cos\theta = 0$ is:

$$\theta = 90° \text{ or } 270°.$$

Since θ is between $0° \leq \theta \leq 90°$, the value of θ is $\boxed{90°}$.

Quick Tip

When solving trigonometric equations, remember to use known values for standard angles and identities.

(c) The distance between the points $(4, y)$ and $(12, 3)$ is 10 units. Find the value of y.

Solution: We are given the distance between the points $(4, y)$ and $(12, 3)$ is 10 units. The distance formula is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Substitute the given points $(x_1, y_1) = (4, y)$ and $(x_2, y_2) = (12, 3)$ into the formula:

$$10 = \sqrt{(12 - 4)^2 + (3 - y)^2}.$$

Simplifying:

$$10 = \sqrt{8^2 + (3 - y)^2},$$

$$10 = \sqrt{64 + (3 - y)^2}.$$

Squaring both sides:

$$100 = 64 + (3 - y)^2.$$

Subtracting 64 from both sides:

$$36 = (3 - y)^2.$$

Taking the square root of both sides:

$$6 = |3 - y|.$$

Thus, $3 - y = 6$ or $3 - y = -6$.
Solving these: 1. $3 - y = 6 \quad \Rightarrow \quad y = -3$, 2. $3 - y = -6 \quad \Rightarrow \quad y = 9$.
Thus, the two possible values for y are $\boxed{-3}$ and $\boxed{9}$.

> Quick Tip
>
> When using the distance formula, make sure to square both sides and solve for the unknown.

(d) Find the relation between x and y such that the point (x, y) is equidistant from the points $(3, 6)$ and $(-3, 4)$.

Solution:
The distance between two points (x_1, y_1) and (x_2, y_2) is given by the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Since the point (x, y) is equidistant from $(3, 6)$ and $(-3, 4)$, we equate the distances:

$$\sqrt{(x - 3)^2 + (y - 6)^2} = \sqrt{(x + 3)^2 + (y - 4)^2}$$

Squaring both sides:

$$(x - 3)^2 + (y - 6)^2 = (x + 3)^2 + (y - 4)^2$$

Expanding:

$$x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$$

Cancel x^2 and y^2 on both sides:

$$-6x + 9 - 12y + 36 = 6x + 9 - 8y + 16$$

$$-6x - 12y + 45 = 6x - 8y + 25$$

Rearranging:

$$-6x - 12y - 6x + 8y = 25 - 45$$

$$-12x - 4y = -20$$

Dividing by -4:

$$3x + y - 5 = 0$$

Thus, the required relation is:

$$\boxed{3x + y - 5 = 0}$$

> **Quick Tip**
>
> To find the locus of points equidistant from two given points, use the distance formula and equate the two distances.

(e) The radius of the base of a right circular cone is 3.5 cm and height is 12 cm. Find the volume of the cone.

Solution:
The volume V of a cone is given by the formula:

$$V = \frac{1}{3}\pi r^2 h,$$

where r is the radius and h is the height.
We are given: - $r = 3.5$ cm, - $h = 12$ cm.
Substitute the values into the formula:

$$V = \frac{1}{3}\pi(3.5)^2 \times 12 = \frac{1}{3}\pi \times 12.25 \times 12.$$

Simplifying:

$$V = \frac{1}{3}\pi \times 147 = 49\pi \, \text{cm}^3.$$

Thus, the volume of the cone is $\boxed{49\pi \, \text{cm}^3}$, or approximately $153.94 \, \text{cm}^3$.

> **Quick Tip**
>
> The volume of a cone is calculated using the formula $V = \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height.

(f) Find the mean from the following table:

Class Interval	Frequency
$0 - 10$	3
$10 - 20$	10
$20 - 30$	11
$30 - 40$	9
$40 - 50$	7

Solution:

The formula for the mean of a grouped frequency distribution is given by:

$$\text{Mean} = \frac{\sum(f \cdot x)}{\sum f}$$

where:
- f is the frequency of each class,
- x is the midpoint of each class interval,
- $\sum f$ is the sum of the frequencies.

Step 1: Calculate the midpoints of each class interval.
For each class interval, the midpoint x is calculated as:

$$x = \frac{\text{lower limit} + \text{upper limit}}{2}.$$

Thus, the midpoints are: - For $0 - 10$, the midpoint $x = \frac{0+10}{2} = 5$,
- For $10 - 20$, the midpoint $x = \frac{10+20}{2} = 15$,
- For $20 - 30$, the midpoint $x = \frac{20+30}{2} = 25$,
- For $30 - 40$, the midpoint $x = \frac{30+40}{2} = 35$,
- For $40 - 50$, the midpoint $x = \frac{40+50}{2} = 45$.

Step 2: Multiply each frequency by the corresponding midpoint.
Now, we compute $f \cdot x$ for each class interval:
- For $0 - 10$, $f \cdot x = 3 \times 5 = 15$,
- For $10 - 20$, $f \cdot x = 10 \times 15 = 150$,
- For $20 - 30$, $f \cdot x = 11 \times 25 = 275$,
- For $30 - 40$, $f \cdot x = 9 \times 35 = 315$,
- For $40 - 50$, $f \cdot x = 7 \times 45 = 315$.

Step 3: Calculate the sum of $f \cdot x$ and the sum of the frequencies.
Sum of $f \cdot x$:

$$\sum(f \cdot x) = 15 + 150 + 275 + 315 + 315 = 1070.$$

Sum of the frequencies:

$$\sum f = 3 + 10 + 11 + 9 + 7 = 40.$$

Step 4: Compute the mean.
Now, use the formula to compute the mean:

$$\text{Mean} = \frac{1070}{40} = 26.75.$$

Thus, the mean is $\boxed{26.75}$.

Quick Tip

To find the mean for a grouped data set, always calculate the midpoints of the class intervals, multiply each midpoint by the corresponding frequency, sum those products, and then divide by the total frequency.

22. Do any five parts:
(a) Find the zeros of the quadratic polynomial $x^2 + 12x + 7$ and verify the relationship between the zeros and the coefficients.

Solution:
Step 1. We are given the quadratic polynomial:

$$p(x) = x^2 + 12x + 7.$$

To find the zeros of this polynomial, we solve the equation:

$$x^2 + 12x + 7 = 0.$$

This is a quadratic equation in standard form $ax^2 + bx + c = 0$, where: - $a = 1$, - $b = 12$, - $c = 7$.
Step 2. We will use the quadratic formula to find the zeros. The quadratic formula is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Substitute the values of a, b, and c into the formula:

$$x = \frac{-12 \pm \sqrt{12^2 - 4(1)(7)}}{2(1)} = \frac{-12 \pm \sqrt{144 - 28}}{2} = \frac{-12 \pm \sqrt{116}}{2}.$$

Step 3. We can simplify $\sqrt{116}$ as:

$$\sqrt{116} = \sqrt{4 \times 29} = 2\sqrt{29}.$$

Thus, the zeros are:

$$x = \frac{-12 \pm 2\sqrt{29}}{2}.$$

Simplifying further:

$$x = -6 \pm \sqrt{29}.$$

Therefore, the zeros of the polynomial are:

$$x_1 = -6 + \sqrt{29}, \quad x_2 = -6 - \sqrt{29}.$$

Verification of the Relationship Between the Zeros and Coefficients:
For a quadratic equation $ax^2 + bx + c = 0$, the relationship between the zeros α and β and the coefficients is given by Vieta's formulas: - The sum of the zeros $\alpha + \beta = -\frac{b}{a}$, - The product of the zeros $\alpha \times \beta = \frac{c}{a}$.
For the equation $x^2 + 12x + 7 = 0$, we have: - The sum of the zeros:

$$\alpha + \beta = (-6 + \sqrt{29}) + (-6 - \sqrt{29}) = -12,$$

which matches $-\frac{b}{a} = -\frac{12}{1} = -12$. - The product of the zeros:

$$\alpha \times \beta = (-6 + \sqrt{29})(-6 - \sqrt{29}) = (-6)^2 - (\sqrt{29})^2 = 36 - 29 = 7,$$

which matches $\frac{c}{a} = \frac{7}{1} = 7$.

Thus, the relationship between the zeros and the coefficients is verified.

> **Quick Tip**
>
> Vieta's formulas provide a way to find the sum and product of the roots of a quadratic equation without explicitly solving for the roots.

(b) The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Solution:
Let the larger number be x and the smaller number be y.
We are given the two conditions:

$$x^2 - y^2 = 180$$

$$y^2 = 8x$$

Step 1: Substituting $y^2 = 8x$ into the first equation

$$x^2 - 8x = 180$$

Rearrange the equation:

$$x^2 - 8x - 180 = 0$$

Step 2: Solving the Quadratic Equation
Using the quadratic formula:

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-180)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 + 720}}{2}$$

$$x = \frac{8 \pm \sqrt{784}}{2}$$

$$x = \frac{8 \pm 28}{2}$$

Solving for x:

$$x = \frac{8+28}{2} = \frac{36}{2} = 18$$

$$x = \frac{8-28}{2} = \frac{-20}{2} = -10 \quad \text{(not valid for positive numbers)}$$

Thus, $x = 18$.
Step 3: Finding y

$$y^2 = 8(18) = 144$$

$$y = \sqrt{144} = 12$$

Thus, the two numbers are:

$$\boxed{18, 12}$$

Quick Tip

To solve problems involving differences of squares, use algebraic identities and substitution.

(c) Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact to the centre.

Solution:
Step 1: Understanding the Problem

Let O be the center of the circle, and let P be an external point from which two tangents PA and PB are drawn to the circle, touching it at points A and B, respectively.

We need to prove that:

$$\angle APB + \angle AOB = 180^\circ$$

where $\angle APB$ is the angle between the two tangents, and $\angle AOB$ is the angle subtended at the center by the line segments joining the points of contact.

Step 2: Recognizing Key Properties

1. The tangents from an external point to a circle are equal in length:

$$PA = PB$$

2. The radius is perpendicular to the tangent at the point of contact:

$$OA \perp PA, \quad OB \perp PB$$

3. $\triangle OAP$ and $\triangle OBP$ are congruent by the RHS (Right-Angle Hypotenuse Side) rule.

4. The quadrilateral $OAPB$ is a cyclic quadrilateral because all its vertices lie on the same circle.

Step 3: Using the Exterior Angle Theorem

Since $OAPB$ is a cyclic quadrilateral, the exterior angle at P (i.e., $\angle APB$) is equal to the supplementary angle of the interior opposite angle at O (i.e., $\angle AOB$):

$$\angle APB + \angle AOB = 180^\circ$$

Step 4: Conclusion

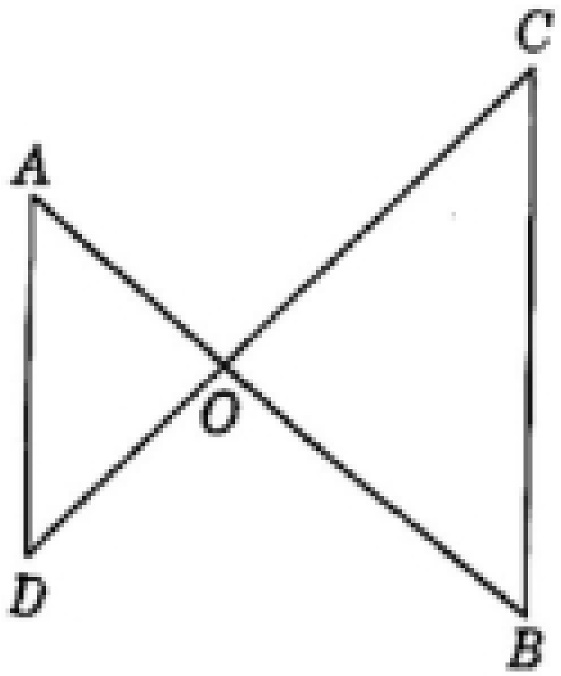
Thus, we have proved that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact to the center.

$$\boxed{\angle APB + \angle AOB = 180^\circ}$$

Quick Tip

The sum of opposite angles in a cyclic quadrilateral is always 180°, which is key to proving this theorem.

(d) In the figure, $OA = OB = OC = OD$**. Show that** $\angle A = \angle C$ **and** $\angle B = \angle D$**.**



Solution:
We are given that $OA = OB = OC = OD$, meaning that all four segments from the point O to points A, B, C, and D are equal. This suggests that triangles OAB, OBC, and ODC are isosceles triangles, as they have two equal sides.
Step 1: *Prove that $\triangle OAB$ and $\triangle ODC$ are isosceles.* - In $\triangle OAB$, since $OA = OB$, by the properties of isosceles triangles, we know that the angles opposite these equal sides are also

equal. Hence:

$$\angle OAB = \angle OBA.$$

- Similarly, in $\triangle ODC$, since $OC = OD$, we can conclude:

$$\angle ODC = \angle OCD.$$

Step 2: *Prove that $\angle A = \angle C$ and $\angle B = \angle D$.* - Since $\angle OAB = \angle OBA$ and $\angle ODC = \angle OCD$, and given that the angle between the two triangles at O is the same (i.e., the angles at O in $\triangle OAB$ and $\triangle ODC$ are the same), it follows that the corresponding angles at points A and C are equal. - Therefore:

$$\angle A = \angle C.$$

- Similarly, the angles at points B and D are also equal:

$$\angle B = \angle D.$$

Thus, we have shown that $\angle A = \angle C$ and $\angle B = \angle D$.

> Quick Tip
>
> When solving problems involving isosceles triangles, always look for equal sides and use the fact that angles opposite equal sides are also equal. This can help prove other relationships between the angles.

(e) One card is drawn at random from a well-shuffled deck of 52 cards. Find the probability that the card drawn is:
(i) a king, (ii) not a king.
Solution:
A standard deck of 52 playing cards consists of 4 suits: hearts, diamonds, clubs, and spades. Each suit contains 13 cards, including one king. Therefore, there are 4 kings in a deck.
(i) Probability of drawing a king:
The total number of outcomes (total cards in the deck) is 52, and the favorable outcomes (kings) are 4. The probability $P(\text{King})$ is given by:

$$P(\text{King}) = \frac{\text{Number of kings}}{\text{Total number of cards}} = \frac{4}{52} = \frac{1}{13}.$$

(ii) Probability of drawing a card that is not a king:
The probability of drawing a card that is not a king is the complement of the probability of drawing a king. Therefore, the probability $P(\text{Not a King})$ is:

$$P(\text{Not a King}) = 1 - P(\text{King}) = 1 - \frac{1}{13} = \frac{12}{13}.$$

Thus, the probabilities are: - (i) $P(\text{King}) = \frac{1}{13}$, - (ii) $P(\text{Not a King}) = \frac{12}{13}$.

> Quick Tip
>
> To calculate the probability of an event, divide the number of favorable outcomes by the total number of possible outcomes. For complementary events, subtract the probability from 1.

(f) Find the median of the following frequency distribution:

Class Interval	Frequency
$0 - 10$	5
$10 - 20$	8
$20 - 30$	20
$30 - 40$	15
$40 - 50$	7
$50 - 60$	5

Solution:
To find the median, we first calculate the cumulative frequency and use the formula for the median:

$$\text{Median} = L + \frac{\frac{N}{2} - F}{f} \times h,$$

where: - L is the lower boundary of the median class,
- N is the total frequency,
- F is the cumulative frequency before the median class,
- f is the frequency of the median class,
- h is the class width.

Step 1: Calculate the cumulative frequency.

Class Interval	Frequency	Cumulative Frequency
$0 - 10$	5	5
$10 - 20$	8	13
$20 - 30$	20	33
$30 - 40$	15	48
$40 - 50$	7	55
$50 - 60$	5	60

Step 2: Find the median class.

The total frequency $N = 60$, so $\frac{N}{2} = 30$.

The cumulative frequency just greater than 30 is 33, which corresponds to the class interval $20 - 30$. Therefore, the median class is $20 - 30$.

Step 3: Apply the formula for the median.

From the table: - $L = 20$ (lower boundary of the median class),
- $F = 13$ (cumulative frequency before the median class),
- $f = 20$ (frequency of the median class),

- $h = 10$ (class width, since $30 - 20 = 10$).

Now, substitute these values into the median formula:

$$\text{Median} = 20 + \frac{\frac{60}{2} - 13}{20} \times 10 = 20 + \frac{30 - 13}{20} \times 10 = 20 + \frac{17}{20} \times 10 = 20 + 8.5 = 28.5.$$

Thus, the median is $\boxed{28.5}$.

> **Quick Tip**
>
> To calculate the median from a frequency distribution, find the cumulative frequency, locate the median class, and then apply the median formula using the appropriate values.

---

23. Solve the following pair of equations:

$$3x - 5y - 4 = 0 \quad \text{and} \quad 9x = 2y + 7.$$

Solution:
Step1. We are given the system of linear equations: 1. $3x - 5y - 4 = 0 \quad$ (Equation 1) 2. $9x = 2y + 7 \quad$ (Equation 2)
Step 2. First, solve Equation 1 for x:

$$3x = 5y + 4 \quad \Rightarrow \quad x = \frac{5y + 4}{3}.$$

Step 3. Substitute this value of x into Equation 2:

$$9\left(\frac{5y + 4}{3}\right) = 2y + 7.$$

Simplifying:

$$3(5y + 4) = 2y + 7 \quad \Rightarrow \quad 15y + 12 = 2y + 7.$$

Solve for y:

$$15y - 2y = 7 - 12 \quad \Rightarrow \quad 13y = -5 \quad \Rightarrow \quad y = \frac{-5}{13}.$$

Substitute $y = \frac{-5}{13}$ into $x = \frac{5y+4}{3}$:

$$x = \frac{5 \times \frac{-5}{13} + 4}{3} = \frac{-\frac{25}{13} + 4}{3} = \frac{-\frac{25}{13} + \frac{52}{13}}{3} = \frac{\frac{27}{13}}{3} = \frac{27}{39} = \frac{9}{13}.$$

Thus, the solution to the system of equations is:

$$x = \frac{9}{13}, \quad y = \frac{-5}{13}.$$

> **Quick Tip**
>
> When solving a system of equations, first isolate one variable and substitute into the other equation to simplify the system.

OR The sum of a two-digit number and the number obtained by reversing the order of its digits is 66. If the digits differ by 2, find the number.

Solution:
Let the two-digit number be $10a + b$, where: - a is the tens digit, - b is the ones digit.
The number obtained by reversing the digits is $10b + a$.
We are given two conditions: **Step 1.** The sum of the number and the reversed number is 66:

$$(10a + b) + (10b + a) = 66.$$

Simplifying:

$$11a + 11b = 66 \quad \Rightarrow \quad a + b = 6 \quad \text{(Equation 1)}.$$

Step 2. The digits differ by 2:

$$a - b = 2 \quad \text{(Equation 2)}.$$

Now, solve the system of equations: - From Equation 1: $a+b = 6$, - From Equation 2: $a-b = 2$.
Add the two equations:

$$(a + b) + (a - b) = 6 + 2,$$

$$2a = 8 \quad \Rightarrow \quad a = 4.$$

Substitute $a = 4$ into Equation 1:

$$4 + b = 6 \quad \Rightarrow \quad b = 2.$$

Thus, the number is:

$$10a + b = 10(4) + 2 = 42.$$

Therefore, the number is $\boxed{42}$.

> Quick Tip
>
> When solving problems involving two-digit numbers, express the number in terms of its digits and use the given conditions to form a system of equations.

---

24. The height of a temple is 15 metres. From the top of the temple, the angle of elevation of the top of a building on the opposite side of the road is 30° and the angle of depression of the foot of the building is 45°. Prove that the height of the building is $15\sqrt{3} + 15\sqrt{3}$ metres.

Solution:
Let the height of the temple be $AB = 15$ m, and let the height of the building be $CD = h$. Let the distance from the base of the temple to the base of the building be x.
Step 1. Using the angle of elevation:
From the top of the temple A, the angle of elevation of the top of the building C is 30°. Thus, in the triangle ABC, we have:

$$\tan 30^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{h - 15}{x}.$$

Since $\tan 30^\circ = \frac{1}{\sqrt{3}}$, we get:

$$\frac{1}{\sqrt{3}} = \frac{h-15}{x}.$$

Hence:

$$x = \sqrt{3}(h-15). \quad \text{(Equation 1)}$$

Step 2. Using the angle of depression:
From the top of the temple A, the angle of depression of the foot of the building D is $45°$. Thus, in the triangle ABD, we have:

$$\tan 45^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{15}{x}.$$

Since $\tan 45^\circ = 1$, we get:

$$1 = \frac{15}{x}.$$

Hence:

$$x = 15. \quad \text{(Equation 2)}$$

Step 3. Substitute Equation 2 into Equation 1:
From Equation 2, we have $x = 15$. Substituting this into Equation 1:

$$15 = \sqrt{3}(h-15).$$

Solving for h:

$$\frac{15}{\sqrt{3}} = h - 15 \quad \Rightarrow \quad 5\sqrt{3} = h - 15 \quad \Rightarrow \quad h = 15 + 5\sqrt{3}.$$

Thus, the height of the building is $h = 15 + 5\sqrt{3}$ metres.

> Quick Tip
>
> When dealing with angles of elevation and depression, use the tangent function to relate the height and distance, and then solve the resulting equations.

OR From the top of a building, the angle of elevation of the top of a tower is 60°. From the top of the building, the angle of depression of the foot of the tower is 45°. If the height of the tower is 40 metres, then prove that the height of the building is $20\left(\sqrt{3}-1\right)$ metres.

Solution:
Let the height of the building be h and the height of the tower be 40 metres. Let the distance between the foot of the building and the foot of the tower be x.
Step 1. Using the angle of elevation:
From the top of the building, the angle of elevation of the top of the tower is $60°$. In the triangle formed by the building, the tower, and the ground, we have:

$$\tan 60^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{40-h}{x}.$$

Since $\tan 60^\circ = \sqrt{3}$, we get:

$$\sqrt{3} = \frac{40-h}{x}.$$

Hence:

$$x = \frac{40-h}{\sqrt{3}}. \quad \text{(Equation 1)}$$

Step 2. Using the angle of depression:
From the top of the building, the angle of depression of the foot of the tower is 45°. In the triangle formed by the building and the foot of the tower, we have:

$$\tan 45^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{h}{x}.$$

Since $\tan 45^\circ = 1$, we get:

$$1 = \frac{h}{x}.$$

Hence:

$$x = h. \quad \text{(Equation 2)}$$

Step 3. Substitute Equation 2 into Equation 1:
From Equation 2, $x = h$. Substituting this into Equation 1:

$$h = \frac{40-h}{\sqrt{3}}.$$

Multiply both sides by $\sqrt{3}$:

$$h\sqrt{3} = 40 - h.$$

Solve for h:

$$h\sqrt{3} + h = 40 \quad \Rightarrow \quad h(\sqrt{3}+1) = 40.$$

Hence:

$$h = \frac{40}{\sqrt{3}+1}.$$

Multiply numerator and denominator by $\sqrt{3}-1$ to rationalize the denominator:

$$h = \frac{40(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{40(\sqrt{3}-1)}{3-1} = \frac{40(\sqrt{3}-1)}{2}.$$

Thus:

$$h = 20(\sqrt{3}-1).$$

Therefore, the height of the building is $\boxed{20(\sqrt{3}-1)}$ metres.

> **Quick Tip**
>
> To solve problems involving angles of elevation and depression, use the tangent function and employ algebraic manipulation to isolate the unknown.

---

25. A solid is in the shape of a cone which is surmounted on a hemisphere of the same base radius. If the curved surfaces of the hemisphere and the cone are equal,

find the ratio of radius and height of the cone.

Solution:
Let the radius of the base of the cone and the hemisphere be r and the height of the cone be h.
Step 1. The curved surface area of the hemisphere is:

$$A_{\text{hemisphere}} = 2\pi r^2.$$

Step 2. The curved surface area of the cone is:

$$A_{\text{cone}} = \pi r l,$$

where l is the slant height of the cone. Using the Pythagorean theorem, the slant height l is given by:

$$l = \sqrt{r^2 + h^2}.$$

Step 3. Since the curved surfaces of the hemisphere and the cone are equal, we have:

$$A_{\text{hemisphere}} = A_{\text{cone}}.$$

Thus:

$$2\pi r^2 = \pi r\sqrt{r^2 + h^2}.$$

Divide both sides by πr:

$$2r = \sqrt{r^2 + h^2}.$$

Square both sides:

$$4r^2 = r^2 + h^2.$$

Simplify:

$$3r^2 = h^2.$$

Taking the square root of both sides:

$$h = \sqrt{3}r.$$

Thus, the ratio of the radius to the height of the cone is:

$$\frac{r}{h} = \frac{r}{\sqrt{3}r} = \frac{1}{\sqrt{3}}.$$

Thus, the ratio of the radius to the height of the cone is $\boxed{\frac{1}{\sqrt{3}}}$.

> Quick Tip
>
> When dealing with problems involving surface areas, make sure to set the areas equal and use the properties of geometric shapes to solve for the unknowns.

OR

An arc of a circle of radius 21 cm subtends an angle of 60° at the centre. Find the area of the sector formed by the arc.

Solution:

Step 1. The formula for the area of a sector of a circle is given by:

$$A_{\text{sector}} = \frac{\theta}{360^\circ} \times \pi r^2,$$

where: - θ is the angle subtended by the arc at the center, - r is the radius of the circle.
We are given: - $r = 21$ cm, - $\theta = 60^\circ$.

Step 2. Substituting these values into the formula:

$$A_{\text{sector}} = \frac{60^\circ}{360^\circ} \times \pi(21)^2 = \frac{1}{6} \times \pi \times 441 = \frac{441\pi}{6} = 73.5\pi.$$

Thus, the area of the sector is $\boxed{73.5\pi}\,\text{cm}^2$, or approximately $230.91\,\text{cm}^2$ when using $\pi \approx 3.1416$.

Quick Tip

When calculating the area of a sector, use the angle in degrees and apply the formula $A_{\text{sector}} = \frac{\theta}{360^\circ} \times \pi r^2$.
