UP Board Class 12 Physics - 346 (JY) - 2025 Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :100	Total questions :9

General Instructions

1. All questions are compulsory.
2. This question paper has five sections: Section A, Section B, Section C, Section D and Section E.
3. Section A is of multiple choice type and each question carries 1 mark.
4. Section B is of very short answer type and each question carries 1 mark.
5. Section C is of short answer type-I and carries 2 marks each.
6. Section D is of short answer type-II and carries 3 marks each.
7. Section E is of long answer type. Each question carries 5 marks. In all four questions of this section with internal choices have been given. You have to do only one question from the choices given in the question.
8. The symbols used in the question paper have usual meaning.

Section - A

Q.1 (a) If polarising angle is θ_1, and critical angle is θ_2, then:

(A) $\tan\theta_1 = \sin\theta_2$

(B) $\tan\theta_1 = \cos\theta_2$

(C) $\cot\theta_1 = \sin\theta_2$

(D) $\cot\theta_1 = \cos\theta_2$

Correct Answer: (A) $\tan\theta_1 = \sin\theta_2$

Solution:

Step 1: Clarify the Scenario

The problem relates two phenomena that occur at the interface between a rarer medium (refractive index n_1) and a denser medium (refractive index n_2).

- **Critical Angle (θ_2):** This occurs when light travels from the denser medium (n_2) to the rarer one (n_1).
- **Polarising Angle (θ_1):** This is usually defined for light traveling from the rarer to the denser medium. However, to find a direct relationship as given in the options, let's consider the polarising angle for light incident from within the denser medium onto the interface with the rarer medium.

Step 2: Express the Critical Angle

For light going from the denser medium to the rarer medium, the critical angle θ_2 is defined by Snell's Law when the angle of refraction is $90°$:

$$n_2 \sin\theta_2 = n_1 \sin 90°$$

$$\sin\theta_2 = \frac{n_1}{n_2} \quad \text{— (Equation I)}$$

Step 3: Express the Polarising Angle from the Denser Medium

Brewster's Law states that the tangent of the polarising angle is equal to the ratio of the refractive index of the second medium to the first. For light incident from within the denser medium (n_2) onto the interface with the rarer medium (n_1), the polarising angle θ_1 is given by:

$$\tan\theta_1 = \frac{n_1}{n_2} \quad \text{— (Equation II)}$$

Step 4: Combine the Results

We now have two separate expressions for the same ratio, $\frac{n_1}{n_2}$. By equating the left-hand sides of Equation I and Equation II, we get the relationship:

$$\tan\theta_1 = \sin\theta_2$$

This matches option (A).

Quick Tip
Brewster's law states that the polarising angle θ_1 is related to the critical angle θ_2 by $\tan\theta_1 = \sin\theta_2$.

(b) The magnetic susceptibility χ of paramagnetic substance varies with absolute temperature T as:

(A) $\chi \propto T$
(B) $\chi \propto T^{-1}$
(C) $\chi \propto \sqrt{T}$
(D) $\chi \propto T^2$

Correct Answer: (B) $\chi \propto T^{-1}$

Solution:

Step 1: The Physical Basis of Paramagnetism

Paramagnetic materials are composed of atoms or molecules that possess permanent magnetic dipole moments. In the absence of an external magnetic field, these dipoles are randomly oriented due to thermal agitation, and the net magnetization of the material is zero.

Step 2: The Competing Effects of Field and Temperature

When an external magnetic field is applied, it exerts a torque on the atomic dipoles, tending to align them with the field. This alignment creates a net magnetization in the direction of the field. Simultaneously, the thermal energy of the atoms (proportional to the absolute temperature T) causes random collisions and vibrations, which works to disrupt this alignment and randomize the orientation of the dipoles.

Step 3: Analyzing the Relationship

The magnetic susceptibility (χ) is a measure of how easily the material becomes magnetized. This depends on the competition between the aligning effect of the field and the randomizing effect of temperature.

- At **low temperatures**, the randomizing thermal motion is weak. The external magnetic field can more easily align the atomic dipoles, leading to a stronger magnetization and therefore a **high** magnetic susceptibility.
- At **high temperatures**, the randomizing thermal motion is very strong. This thermal agitation vigorously opposes the aligning influence of the magnetic field, making it difficult to magnetize the material. This results in a **low** magnetic susceptibility.

Step 4: Curie's Law

This observed inverse relationship is described by Curie's Law, which states that the magnetic susceptibility (χ) of a paramagnetic material is inversely proportional to the absolute temperature (T).

$$\chi \propto \frac{1}{T}$$

This can also be written as $\chi \propto T^{-1}$. Therefore, the correct answer is option (B).

Quick Tip
For paramagnetic substances, magnetic susceptibility varies inversely with temperature, following Curie's law $\chi \propto \frac{1}{T}$.

(c) The correct relationship of mobility μ of charge carrier with drift velocity v_d and electric field E, is:

(A) $\mu = \frac{v_d}{E}$
(B) $\mu = \frac{E}{v_d}$
(C) $\mu = Ev_d$
(D) $\mu = E + v_d$

Correct Answer: (A) $\mu = \frac{v_d}{E}$

Solution:

Step 1: The Concept of Drift Velocity

When an electric field (E) is applied across a material, the free charge carriers (like electrons) experience a force that accelerates them. However, they continuously collide with the atoms of the material, which means they don't accelerate indefinitely. Instead, they attain an average velocity, superimposed on their random thermal motion. This net average velocity in response to the electric field is called the drift velocity (v_d).

Step 2: The Proportionality between Drift Velocity and Electric Field

For a given material at a constant temperature, it is found that the drift velocity of the charge carriers is directly proportional to the strength of the applied electric field. A stronger field leads to a higher drift velocity. This can be written as:

$$v_d \propto E$$

Step 3: Defining the Constant of Proportionality

To change this proportionality into an equation, we introduce a constant. This constant is a property of the material and the charge carrier, and it is called the **mobility**, denoted by the symbol μ. Mobility is a measure of how easily (or "mobile") a charge carrier can move through the material under the influence of an electric field. The relationship is therefore:

$$v_d = \mu E$$

Step 4: Finding the Expression for Mobility

The question asks for the relationship that defines mobility, μ. We can rearrange the equation from Step 3 by dividing both sides by the electric field, E:

$$\mu = \frac{v_d}{E}$$

This shows that mobility is the drift velocity achieved per unit of electric field strength.

Step 5: Conclusion

The correct relationship is $\mu = \frac{v_d}{E}$, which corresponds to option (A).

Quick Tip
The mobility of charge carriers is the ratio of their drift velocity to the applied electric field, i.e., $\mu = \frac{v_d}{E}$.

(d) 1 GHz frequency corresponds to which region of the electromagnetic spectrum?

(A) Ultraviolet rays

(B) Radio waves

(C) Visible rays

(D) X-rays

Correct Answer: (B) Radio waves

Solution:

Step 1: Identify the Given Frequency

The given frequency is 1 GHz. The prefix ”giga” (G) represents a factor of 10^9. Therefore:

$$1 \text{ GHz} = 1 \times 10^9 \text{ Hz}$$

Step 2: Compare with the Electromagnetic Spectrum Regions

The electromagnetic spectrum is ordered by frequency (or wavelength). Let’s compare our given frequency with the typical ranges for each of the options provided:

- **X-rays:** These are very high-energy waves, with frequencies typically in the range of 10^{16} Hz to 10^{19} Hz. Our frequency of 10^9 Hz is far too low to be in the X-ray region.
- **Ultraviolet (UV) rays:** These have frequencies just above visible light, roughly from 10^{15} Hz to 10^{17} Hz. Again, 10^9 Hz is much lower than this.
- **Visible rays:** The narrow band of light we can see has frequencies approximately between 4×10^{14} Hz and 8×10^{14} Hz. The given frequency is well below the visible spectrum.
- **Radio waves:** This is the lowest-frequency portion of the spectrum. The radio wave band is very broad, officially spanning from about 3 kHz (3×10^3 Hz) up to 300 GHz (3×10^{11} Hz). Our frequency of 1×10^9 Hz falls comfortably within this range.

Step 3: Conclusion

Since 10^9 Hz lies within the radio wave band, 1 GHz corresponds to radio waves.
Frequencies in this specific range (UHF/Microwave) are commonly used for mobile phones, Wi-Fi, and GPS.
Hence, the correct answer is option (B) Radio waves.

Quick Tip

1 GHz frequency lies in the radio wave region of the electromagnetic spectrum, which is used for various communication applications.

(e) The voltage and current of an ac circuit are represented as $V = 100\sin(100t)$ volt and $i = 100\sin(100t + \frac{\pi}{3})$ mA respectively. The power dissipated in the circuit is:

(A) 10^4 watt

(B) 2.5 watt

(C) 0.25 watt

(D) 25 watt

Correct Answer: (B) 2.5 watt

Solution:

The instantaneous power dissipated in an AC circuit is given by the formula:

$$P = V_{\text{max}} I_{\text{max}} \cos(\phi)$$

Where:

- V_{max} is the maximum voltage,
- I_{max} is the maximum current,
- ϕ is the phase difference between the voltage and current.

From the given expressions:

- $V = 100\sin(100t)$, so $V_{\text{max}} = 100$ volts,
- $i = 100\sin(100t + \frac{\pi}{3})$, so $I_{\text{max}} = 100$ mA or 0.1 A,
- The phase difference $\phi = \frac{\pi}{3}$.

Now, substitute these values into the power formula:

$$P = (100)(0.1)\cos\left(\frac{\pi}{3}\right)$$

Since $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$, we get:

$$P = (100)(0.1) \times \frac{1}{2} = 2.5\,\text{watt}$$

Thus, the correct answer is option (B) 2.5 watt.

> **Quick Tip**
>
> The power dissipated in an AC circuit is given by the product of the maximum voltage, maximum current, and the cosine of the phase difference between them.

(f) In hydrogen atom, the kinetic energy of electron in an orbit of radius r, is given by:

(A) $\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r}$
(B) $-\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r}$
(C) $-\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{2r}$
(D) $\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{2r}$

Correct Answer: (B) $-\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r}$

Solution:

In a hydrogen atom, the electron orbits the nucleus (proton) in a circular path under the influence of the electrostatic force between the electron and proton. The Coulomb force provides the centripetal force for the electron's motion.

The electrostatic force between the electron and proton is given by Coulomb's law:

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r^2}$$

Where: - e is the charge of the electron, - ϵ_0 is the permittivity of free space, - r is the radius of the electron's orbit.

The kinetic energy $K.E.$ of the electron is given by the relation:

$$K.E. = \frac{1}{2}mv^2$$

Using the fact that the centripetal force F is equal to $\frac{mv^2}{r}$, and equating the electrostatic force to the centripetal force, we find:

$$K.E. = -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r}$$

Thus, the correct expression for the kinetic energy of the electron in the hydrogen atom is:

$$K.E. = -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r}$$

Hence, the correct answer is option (B) $-\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r}$.

Quick Tip

The kinetic energy of an electron in a hydrogen atom is related to the electrostatic force between the electron and proton and is negative because of the attractive nature of the force.

Section - B

Q.2 (a) Mention any two applications of total internal reflection.

Solution:

Total internal reflection occurs when light hits the boundary between two media at an angle greater than the critical angle and gets entirely reflected back into the denser medium. This phenomenon plays a crucial role in several important optical applications. Two common applications are:

1. Optical Fibers:

Optical fibers use total internal reflection to transmit light signals over long distances. The core of the fiber has a higher refractive index than the surrounding cladding, allowing the light to be continually reflected inside the fiber core. This process minimizes signal loss, making optical fibers ideal for telecommunications, internet data transfer, and medical endoscopy.

2. Prism in Periscopes and Binoculars:
Total internal reflection is used in the design of optical instruments such as periscopes and binoculars. In periscopes, prisms are used to reflect light through 90-degree angles without any loss of light intensity. This allows a viewer to see over obstacles by redirecting light from the scene into their eyes. Similarly, in binoculars, prisms help in the efficient reflection of light to give a clear and bright image.

> Quick Tip
>
> Total internal reflection is not only crucial for optical fibers but is also extensively used in optical devices like periscopes and binoculars to effectively manage light reflection. The condition for total internal reflection is the angle of incidence being greater than the critical angle for the medium.

(b) What is meant by doping in semiconductors?

Solution:
Doping in semiconductors refers to the process of intentionally adding impurity atoms to an intrinsic (pure) semiconductor to modify its electrical properties. This process is vital for controlling the electrical conductivity of semiconductors and making them suitable for use in various electronic devices such as diodes, transistors, and integrated circuits. Doping creates two types of charge carriers in the semiconductor:
1. N-type Doping:
In N-type doping, semiconductor material (like silicon) is doped with elements that have more valence electrons than the semiconductor atoms, such as phosphorus. Phosphorus has five valence electrons, one more than silicon's four. The extra electron is loosely bound and can move freely in the crystal, resulting in the formation of free electrons (negative charge carriers). This increases the electrical conductivity by introducing more free electrons.
2. P-type Doping:
In P-type doping, the semiconductor is doped with elements that have fewer valence electrons than the semiconductor atoms, such as boron. Boron has only three valence electrons, which creates "holes" (missing electrons) in the crystal lattice. These holes can

move through the semiconductor, effectively behaving as positive charge carriers. As a result, the P-type semiconductor has an abundance of holes that carry positive charge and enhance conductivity.
Doping is essential in forming P-N junctions, which are the basic building blocks of semiconductor devices such as diodes and transistors.

Quick Tip

Doping is the process of adding impurities to semiconductors to control their electrical properties. N-type doping introduces free electrons, while P-type doping creates holes. These two types of semiconductors are crucial for making devices like diodes, transistors, and solar cells.

(c) What is the meaning of stopping potential in photoelectric effect?

Solution:

The stopping potential in the photoelectric effect is the minimum negative potential that must be applied to the anode to stop the most energetic photoelectrons from reaching it. When monochromatic light strikes a material, photoelectrons are emitted due to the energy transferred from the photons of light. These electrons are emitted with a certain kinetic energy, which depends on the frequency of the incident light and the work function of the material.
To stop the photoelectrons from reaching the anode, a negative potential (stopping potential) is applied. The stopping potential is the voltage required to reduce the kinetic energy of the emitted electrons to zero. It essentially counters the kinetic energy of the fastest electrons, halting their motion. The relation between the kinetic energy of the emitted photoelectron E_k and the stopping potential V_0 is:

$$E_k = eV_0,$$

where: - E_k is the kinetic energy of the photoelectron,
- e is the charge of the electron,

- V_0 is the stopping potential.

Thus, the stopping potential is a measure of the maximum kinetic energy of the emitted photoelectrons. The photoelectric effect demonstrates that light behaves as particles (photons), as only photons with energy greater than the material's work function can release electrons.

> Quick Tip
>
> The stopping potential is used to measure the kinetic energy of photoelectrons emitted in the photoelectric effect. It is the minimum potential required to stop the most energetic photoelectrons from reaching the anode. This potential is crucial in verifying Einstein's photoelectric equation.

(d) Compute the electric flux linked with a surface $\vec{A} = 2\hat{\jmath}$ m^2, placed in a uniform electric field $\vec{E} = (4\hat{\imath} + 3\hat{\jmath})$ V/m.

Solution:

The electric flux Φ_E linked with a surface is given by the dot product of the electric field $\vec{E}$ and the area vector $\vec{A}$:

$$\Phi_E = \vec{E} \cdot \vec{A}$$

The area vector $\vec{A}$ is given as $\vec{A} = 2\hat{\jmath}\,\mathrm{m}^2$, and the electric field is $\vec{E} = 4\hat{\imath} + 3\hat{\jmath}$ V/m. The dot product of the electric field and the area vector is:

$$\Phi_E = (4\hat{\imath} + 3\hat{\jmath}) \cdot (2\hat{\jmath})$$

Since the dot product of two perpendicular vectors is zero and the dot product of two parallel vectors is the product of their magnitudes, we get:

$$\Phi_E = (4 \times 0) + (3 \times 2) = 6\,\mathrm{V} \cdot \mathrm{m}^2$$

Thus, the electric flux linked with the surface is $\Phi_E = 6\,\mathrm{V} \cdot \mathrm{m}^2$.

Quick Tip

The electric flux is the dot product of the electric field vector and the area vector. If the vectors are perpendicular, the flux is zero; if parallel, the flux is the product of the magnitudes.

(e) How is a galvanometer converted into a voltmeter?

Solution:

A galvanometer can be converted into a voltmeter by connecting a high resistance R_v in series with it. This series resistance is chosen such that it allows only a small current to flow through the galvanometer when a high potential difference is applied. The total resistance of the voltmeter is then the sum of the resistance of the galvanometer and the series resistance:

$$R_\text{total} = R_\text{g} + R_\text{v}$$

This modification ensures that the voltmeter can measure high voltages without damaging the galvanometer or causing excessive current flow.

The full-scale deflection of the galvanometer will correspond to the full-scale voltage of the voltmeter, which can be calibrated accordingly.

Quick Tip

To convert a galvanometer into a voltmeter, connect a large resistance in series with the galvanometer to limit the current and enable it to measure high voltages.

(f) What is meant by wattless current?

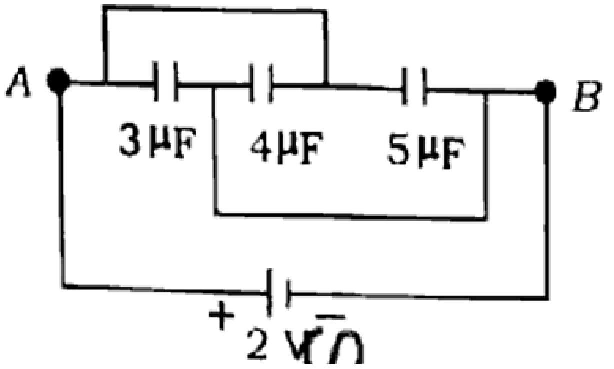
Solution:

Wattless current refers to the current that flows in an AC circuit when the voltage and current are out of phase with each other by 90 degrees. This happens in circuits that contain purely inductive or purely capacitive elements. In such cases, the power consumed by the circuit is zero because the current and voltage are not in phase, meaning that the energy supplied to the circuit is stored and returned (in the case of inductance or capacitance) without being dissipated as heat.

For example, in a purely inductive circuit, the current lags the voltage by 90° and the power factor is zero, meaning no real power is consumed. This results in wattless current, as the average power over a complete cycle is zero.

> Quick Tip
>
> Wattless current occurs in purely inductive or capacitive circuits where the current and voltage are 90 degrees out of phase, leading to zero power consumption.

Q.3 (a) Find out the equivalent capacitance and total energy stored in the given combination of capacitors:



Solution:

Capacitor Combination: The given combination of capacitors is shown below:

3 mu F || (4 F in series with 5 F)

1. Step 1: Calculate the Equivalent Capacitance of the 4 F and 5 F Capacitors in Series:

For capacitors in series, the equivalent capacitance $C_{\text{eq, series}}$ is given by:

$$\frac{1}{C_{\text{eq, series}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Substituting the values:

$$\frac{1}{C_{\text{eq, series}}} = \frac{1}{4} + \frac{1}{5} = \frac{5+4}{20} = \frac{9}{20}$$

Thus:

$$C_{\text{eq, series}} = \frac{20}{9} \approx 2.22\,\mu\text{F}$$

2. Step 2: Find the Equivalent Capacitance of the Entire Combination:

Now, the equivalent capacitance of the 3 F capacitor in parallel with the series combination (2.22 F) is:

$$C_{\text{eq, total}} = C_1 + C_{\text{eq, series}} = 3 + 2.22 = 5.22\,\mu\text{F}$$

So, the total equivalent capacitance of the combination is 5.22 F.

3. Step 3: Calculate the Total Energy Stored:

The energy stored in a capacitor is given by the formula:

$$E = \frac{1}{2}CV^2$$

Substituting the values ($C = 5.22\,\mu$F and $V = 2\,$V):

$$E = \frac{1}{2} \times 5.22 \times 10^{-6} \times (2)^2 = \frac{1}{2} \times 5.22 \times 10^{-6} \times 4 = 1.044 \times 10^{-5}\,\textbf{J} = 10.44\,\mu\text{J}$$

Thus, the total energy stored in the combination is 10.44 J.

Quick Tip

For capacitors in series, use $\frac{1}{C_{\text{eq}}} = \sum \frac{1}{C_i}$. For capacitors in parallel, simply add them: $C_{\text{eq}} = \sum C_i$.

(b) A proton and an α-particle are accelerated by the same potential difference. Find out the ratio of their de Broglie wavelengths.

Solution:

The de Broglie wavelength λ of a particle is given by the formula:

$$\lambda = \frac{h}{p}$$

Where: - h is Planck's constant ($6.626 \times 10^{-34}\,\mathrm{J \cdot s}$), - p is the momentum of the particle.

The momentum p of a particle accelerated by a potential difference V is given by:

$$p = \sqrt{2meV}$$

Where: - m is the mass of the particle,

- e is the charge of the particle ($1.6 \times 10^{-19}\,\mathrm{C}$),

- V is the potential difference.

Now, we calculate the momentum for both the proton and the α-particle.

1. For the proton:

- Mass of proton $m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$,

- Charge of proton $e_p = 1.6 \times 10^{-19}\,\mathrm{C}$.

The momentum of the proton is:

$$p_p = \sqrt{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19} \times V}$$

2. For the α-particle:

- Mass of α-particle $m_\alpha = 4 \times 1.67 \times 10^{-27}\,\mathrm{kg} = 6.68 \times 10^{-27}\,\mathrm{kg}$,

- Charge of α-particle $e_\alpha = 2 \times 1.6 \times 10^{-19}\,\mathrm{C}$.

The momentum of the α-particle is:

$$p_\alpha = \sqrt{2 \times 6.68 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times V}$$

3. Ratio of the de Broglie wavelengths:

Since the de Broglie wavelength is inversely proportional to the momentum, the ratio of the wavelengths is:

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{p_\alpha}{p_p} = \frac{\sqrt{2 \times 6.68 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times V}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19} \times V}}$$

Simplifying the ratio:

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{6.68 \times 2}{1.67}} = \sqrt{8} = 2.83$$

Thus, the ratio of the de Broglie wavelengths is approximately 2.83.

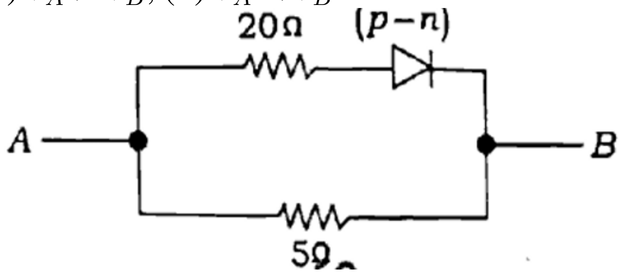
💡 Quick Tip

For de Broglie wavelengths, remember that the momentum p is proportional to $\sqrt{mV}$, so heavier particles or particles with higher charges will have smaller wavelengths.

(c)

In the given circuit, find out the equivalent resistances across the points A and B when:

(i) $V_A > V_B$, (ii) $V_A < V_B$.



Solution:

The given circuit consists of resistors of values $200\ \Omega$, $50\ \Omega$, and $173\ \Omega$, with a voltage difference between points A and B. We need to find the equivalent resistance when: - (i) $V_A > V_B$, - (ii) $V_A < V_B$.

For this circuit, when $V_A > V_B$, the resistors will be connected in series, and the equivalent resistance is the sum of the individual resistances. Thus, the equivalent resistance R_{eq} will be:

$$R_{\text{eq}} = 200 + 50 + 173 = 423\ \Omega$$

For the second case, when $V_A < V_B$, the resistors will be in parallel. The equivalent resistance for parallel resistors is given by the reciprocal sum of the individual resistances:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{200} + \frac{1}{50} + \frac{1}{173}$$

Solving this:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{200} + \frac{1}{50} + \frac{1}{173} = 0.005 + 0.02 + 0.00578 = 0.03078$$

Thus, the equivalent resistance is:

$$R_{\text{eq}} = \frac{1}{0.03078} \approx 32.5\ \Omega$$

Quick Tip

In circuits with series resistors, simply add the resistances. For parallel resistors, use the reciprocal sum to find the equivalent resistance.

(d) If the circumference of the nth orbit of an electron is S and the corresponding de Broglie wavelength of the orbit is λ, then on the basis of Bohr's atom model, prove that $S = n\lambda$.

Solution:

According to Bohr's atomic model, the angular momentum of an electron in orbit n is quantized and given by:

$$L = n\hbar$$

Where: - L is the angular momentum, - n is the principal quantum number, - $\hbar = \frac{h}{2\pi}$ is the reduced Planck's constant, where h is Planck's constant.

The angular momentum is also related to the linear momentum of the electron and its radius r as:

$$L = mvr$$

Where: - m is the mass of the electron, - v is the linear velocity of the electron, - r is the radius of the orbit.

By equating these two expressions for L:

$$mvr = n\hbar$$

Now, using the de Broglie hypothesis, the wavelength λ of the electron is related to its momentum $p = mv$ as:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Thus, the radius r of the nth orbit can be expressed as:

$$r = \frac{n\hbar}{mv}$$

The circumference S of the nth orbit is:

$$S = 2\pi r$$

Substituting the expression for r from above:

$$S = 2\pi \times \frac{n\hbar}{mv}$$

Now, from the de Broglie relation, we can substitute $mv = \frac{h}{\lambda}$ into the equation:

$$S = 2\pi \times \frac{n\hbar}{\frac{h}{\lambda}} = 2\pi \times n\lambda$$

Thus, we obtain the desired result:

$$S = n\lambda$$

Quick Tip

The relationship $S = n\lambda$ is a result of Bohr's quantization condition and de Broglie's wave-particle duality, which ties the electron's orbit and wavelength together.

Section - D

Q.4 (a) Obtain the formula for the width of central maxima from the experiment of diffraction of light through a single slit and draw the diagram of the intensity distribution of the light obtained on the screen.

Solution:

Diffraction through a Single Slit:

In the case of diffraction of light through a single slit, the central maximum is formed on the screen due to constructive and destructive interference. The angular width of the central maximum is given by the angle θ for the first minimum. The condition for the first minimum is:

$$a \sin\theta = m\lambda, \quad m = \pm1, \pm2, \pm3, \ldots$$

Where:

- a is the width of the slit,
- λ is the wavelength of the light,
- m is the order of the minima.

For the central maximum, we consider the case of $m = \pm1$, and the angular width of the central maximum is the difference in angles between the first minima on either side of the central maximum:

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right).$$

The angular width of the central maximum is twice the angle for the first minimum:

$$\Delta\theta = 2\theta_1 = 2\sin^{-1}\left(\frac{\lambda}{a}\right).$$

For small angles (which is usually the case in most practical diffraction experiments), $\sin\theta \approx \theta$, so the formula for the angular width becomes approximately:

$$\Delta\theta \approx \frac{2\lambda}{a}.$$

This formula gives the angular width of the central maximum. The linear width W on the screen at a distance L from the slit is given by:

$$W = L \cdot \Delta\theta = \frac{2L\lambda}{a}.$$

Thus, the width of the central maxima on the screen is $W = \frac{2L\lambda}{a}$.

The intensity distribution of light through a single slit diffraction experiment shows a central bright fringe (central maximum) with diminishing intensity for subsequent maxima and minima.

> 💡 Quick Tip
>
> In single slit diffraction, the angular width of the central maximum is inversely proportional to the slit width. For wider slits, the central maximum becomes narrower.

(b) What is Ampere's circuital law? Using it, obtain the formula for the magnetic field produced due to a straight current-carrying conductor of infinite length.

Solution:

Ampere's Circuital Law:

Ampere's circuital law states that the line integral of the magnetic field B around a closed loop is proportional to the total current I passing through the loop. Mathematically, it is given by:

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}},$$

where:

- $\oint \mathbf{B} \cdot d\mathbf{l}$ is the line integral of the magnetic field around a closed loop,
- μ_0 is the permeability of free space ($\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$),
- I_{enc} is the enclosed current by the loop.

Magnetic Field Due to a Straight Current-Carrying Conductor:

Consider a long, straight conductor carrying a current I. To find the magnetic field at a distance r from the conductor, we apply Ampere's circuital law. We take a circular loop of radius r around the conductor, as the magnetic field produced by a straight conductor is symmetric in nature and forms concentric circles around the conductor.

The magnitude of the magnetic field at a distance r from the wire is constant along the circular loop, and the direction of the magnetic field is tangential to the loop. Therefore, the line integral simplifies to:

$$\oint \mathbf{B} \cdot d\mathbf{l} = B \cdot 2\pi r,$$

where B is the magnetic field at a distance r from the wire. Using Ampere's law:

$$B \cdot 2\pi r = \mu_0 I.$$

Solving for B, we get:

$$B = \frac{\mu_0 I}{2\pi r}.$$

Thus, the magnetic field at a distance r from a long, straight current-carrying conductor is:

$$B = \frac{\mu_0 I}{2\pi r}.$$

Quick Tip

Ampere's circuital law is useful for calculating the magnetic field around symmetrical current distributions. For a straight conductor, the field decreases with the distance from the conductor.

(c) Electric field E is applied across a metallic wire of length l and area of cross-section A. Obtain the formula of the relationship between the drift velocity (v_d) of free electrons of the conductor and electric field E in vector form.

Solution:

When an electric field E is applied across a metallic wire, the free electrons experience a force that causes them to move in the direction opposite to the electric field, resulting in a drift velocity v_d. The relationship between the drift velocity and the electric field can be derived using the following steps:

Step 1: Force on an electron due to electric field.

The force F on an electron of charge e due to the electric field is given by:

$$F = eE$$

Where:

- F is the force on the electron,

- e is the charge of the electron ($e = 1.6 \times 10^{-19}$ C),

- E is the electric field.

Step 2: Acceleration of an electron.

From Newton's second law, the acceleration a of the electron is given by:

$$a = \frac{F}{m_e} = \frac{eE}{m_e}$$

Where:

- m_e is the mass of the electron.

Step 3: Drift velocity.

The drift velocity v_d is the average velocity that the electron acquires due to the electric field. This velocity is reached after a characteristic time called the relaxation time τ, which is the average time between collisions of the electron with the atoms in the wire. The drift velocity is related to the acceleration by:

$$v_d = a \times \tau = \frac{eE}{m_e} \times \tau$$

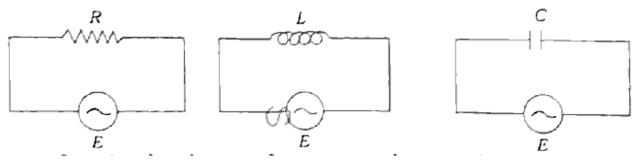
Thus, the drift velocity v_d of the electrons is given by:

$$v_d = \frac{eE\tau}{m_e}$$

This equation gives the relationship between the drift velocity v_d, the electric field E, and the properties of the electron (charge and mass).

> **Quick Tip**
>
> The drift velocity is proportional to the electric field. The stronger the field, the higher the drift velocity of the electrons. The drift velocity is also influenced by the relaxation time, which depends on the material of the conductor.

(d) Three ac circuits are shown in the figures with equal currents. Explain with reason, if the frequency of the voltage E is increased then what will be the effect on the currents in them.



Solution:

The three types of circuits given are:

1. A resistive circuit (R),

2. An inductive circuit (L),

3. A capacitive circuit (C).

Resistive Circuit: In a purely resistive circuit, the current I is related to the applied voltage V by Ohm's law:

$$I = \frac{V}{R}$$

Where:

- V is the voltage across the resistor,
- R is the resistance.

In a resistive circuit, the current is independent of frequency. Therefore, increasing the frequency of the voltage will not affect the current in a purely resistive circuit.

Inductive Circuit: In a purely inductive circuit, the current I lags the voltage by $90°$, and the impedance Z_L is given by:

$$Z_L = \omega L = 2\pi f L$$

Where:

- ω is the angular frequency ($\omega = 2\pi f$),
- L is the inductance,
- f is the frequency of the voltage.

As the frequency increases, the inductive reactance X_L increases, which causes the impedance Z_L to increase. According to Ohm's law for AC circuits, the current decreases as the impedance increases. Therefore, increasing the frequency decreases the current in an inductive circuit.

Capacitive Circuit: In a purely capacitive circuit, the current I leads the voltage by $90°$, and the impedance Z_C is given by:

$$Z_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Where:
- C is the capacitance,
- f is the frequency of the voltage.

As the frequency increases, the capacitive reactance X_C decreases, which causes the impedance Z_C to decrease. According to Ohm's law, as the impedance decreases, the current increases. Therefore, increasing the frequency increases the current in a capacitive circuit.

Thus, the effect of increasing frequency on the current in the three circuits is:
- In the resistive circuit, the current remains unchanged.
- In the inductive circuit, the current decreases.
- In the capacitive circuit, the current increases.

> Quick Tip
>
> The behavior of current in an AC circuit depends on the type of element present. In resistive circuits, current is unaffected by frequency; in inductive circuits, current decreases with increasing frequency; and in capacitive circuits, current increases with increasing frequency.

(e) Show the electromagnetic wave by a diagram and write down its three important properties. Which radiation has the least wavelength in the spectrum of electromagnetic waves?

Solution:

An electromagnetic wave consists of oscillating electric and magnetic fields that are perpendicular to each other and to the direction of propagation of the wave. These waves travel at the speed of light ($c = 3 \times 10^8$ m/s) in a vacuum. The electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ oscillate in phase, and their energy is transferred through the space. The diagram for an electromagnetic wave is shown below:

In the diagram: - The electric field ($\mathbf{E}$) oscillates in one plane (perpendicular to the direction of wave propagation). - The magnetic field ($\mathbf{B}$) oscillates in a plane perpendicular to the electric field, and also perpendicular to the direction of wave propagation. - The wave propagates in the direction perpendicular to both the electric and magnetic fields.

Three Important Properties of Electromagnetic Waves:

1. Transverse Nature: Electromagnetic waves are transverse waves, meaning the oscillations of the electric and magnetic fields are perpendicular to the direction of wave propagation.
2. Speed of Light: In a vacuum, electromagnetic waves travel at a constant speed of $c = 3 \times 10^8$ m/s.
3. Electromagnetic Spectrum: Electromagnetic waves encompass a wide range of wavelengths and frequencies, forming the electromagnetic spectrum, which includes radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays, and gamma rays. The frequency and wavelength of an electromagnetic wave are related by:

$$c = \lambda \times \nu$$

Where: - c is the speed of light (3×10^8 m/s), - λ is the wavelength, - ν is the frequency.

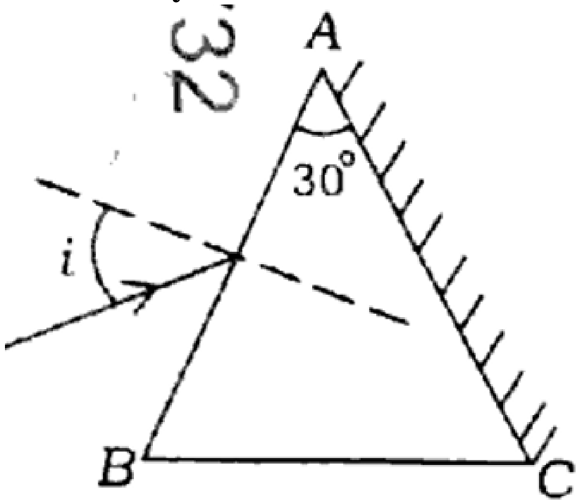
Which Radiation Has the Least Wavelength?

Gamma rays have the least wavelength in the electromagnetic spectrum. They have the highest frequency and the most energy compared to other types of electromagnetic radiation. The wavelength of gamma rays can be as small as 10^{-12} m, making them the most penetrating form of electromagnetic radiation.

> Quick Tip
>
> Remember, in the electromagnetic spectrum, the wavelength decreases and frequency increases as we move from radio waves to gamma rays. Gamma rays have the least wavelength and the highest frequency.

Q.5 (a) What are the conditions for the minimum deviation by a prism? One face (AC) of a prism of refracting angle 30° and refractive index $\sqrt{2}$, is silvered. What should be the angle of incidence i on the surface AB, so that after refraction into the prism, the

reflected ray from the silvered surface retraces its path? See the figure.



Solution:

To achieve the condition of minimum deviation in a prism, the following conditions must be met:

1. The incident ray and the refracted ray inside the prism must make equal angles with the prism surface. This means that the angle of incidence at the surface AB must be equal to the angle of refraction inside the prism.
2. The angle of deviation is minimized when the ray passes symmetrically through the prism. This implies that the path of the light through the prism should be symmetric in nature with respect to the prism's surface.
3. The angle of incidence at the silvered surface (the reflecting surface) must allow total internal reflection. The light should reflect off the silvered surface in such a way that the ray retraces its original path inside the prism. This occurs when the angle of incidence on the surface is greater than the critical angle for total internal reflection.

The relationship between the angles is given by:

$$i + r = A,$$

where: - i is the angle of incidence on surface AB, - r is the angle of refraction inside the prism, - A is the refracting angle of the prism.

Also, the angle of deviation D is given by:

$$D = i + r - A.$$

For minimum deviation, the deviation is at its smallest value when the ray enters and exits the prism symmetrically. This means that the angle of incidence and the angle of refraction must be equal, i.e., $i = r$. Thus, for minimum deviation:

$$D_{\text{min}} = 2i - A.$$

Now, considering the silvered surface at the other end of the prism, the ray must undergo total internal reflection. The critical angle θ_c for total internal reflection is given by:

$$\sin\theta_c = \frac{1}{n},$$

where $n = \sqrt{2}$ is the refractive index of the prism. Therefore, the critical angle is:

$$\theta_c = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ.$$

For total internal reflection to occur, the angle of incidence at the silvered surface must be greater than this critical angle. Thus, we set the angle of incidence i such that the reflected ray retraces its path. The angle i must satisfy:

$$i = 45^\circ.$$

Thus, the angle of incidence i on the surface AB is 45° for the reflected ray to retrace its path after total internal reflection at the silvered surface.

Quick Tip

The key for minimum deviation in a prism is that the incident and refracted rays are symmetric. The condition of total internal reflection at the silvered surface is crucial to ensure that the reflected ray retraces its path.

(b) What is the phenomenon of mutual induction? What is meant by 1 henry mutual inductance? If a current of 4 A is reduced to zero in 10 μs in the primary coil of a transformer, then 40 kV of induced e.m.f. is produced in the secondary coil. Find out the mutual inductance between the primary and the secondary coils.

Solution:

Mutual Induction is the phenomenon in which a change in current in one coil induces a voltage in another coil that is placed nearby. The amount of voltage induced in the secondary coil depends on the rate of change of current in the primary coil. Mutual induction occurs when two coils are magnetically coupled, and the changing magnetic field from the primary coil induces an electromotive force (e.m.f.) in the secondary coil.
The mutual inductance M between two coils is defined as the ratio of the induced e.m.f. in one coil to the rate of change of current in the other coil. The formula for mutual inductance is:

$$M = \frac{\text{Induced e.m.f.}}{\frac{dI}{dt}}$$

Where:
- M is the mutual inductance in henries (H),
- Induced e.m.f. is the voltage induced in the secondary coil,
- $\frac{dI}{dt}$ is the rate of change of current in the primary coil.
Given:
- Induced e.m.f. = 40 kV = 40×10^3 V,
- Initial current $I = 4$ A,
- Final current = 0 (current reduces to zero),
- Time interval $dt = 10\,\mu\text{s} = 10 \times 10^{-6}$ s.
First, calculate the rate of change of current:

$$\frac{dI}{dt} = \frac{I - 0}{dt} = \frac{4}{10 \times 10^{-6}} = 4 \times 10^5 \text{ A/s}$$

Now, substitute the values into the formula for mutual inductance:

$$M = \frac{40 \times 10^3}{4 \times 10^5} = 0.1\,\text{H}$$

Thus, the mutual inductance between the primary and secondary coils is $M = 0.1\,\text{H}$.

Quick Tip

The mutual inductance M measures the ability of two coils to induce e.m.f. in each other. It depends on the rate of change of current in one coil and the induced voltage in the other.

OR What is the working principle of a transformer? The ratio of the number of turns in the primary and secondary coils in an ideal step-down transformer is 20 : 1. When input voltage of 250 V is applied, then the output current is 8 A. Calculate:
i) **Current in the primary coil**
ii) **Output power**

Solution:
Working Principle of a Transformer:
A transformer works on the principle of mutual induction. It consists of two coils: the primary coil and the secondary coil, wound on a common iron core. When an alternating current is passed through the primary coil, it creates a changing magnetic field, which induces a voltage in the secondary coil. The voltage ratio between the primary and secondary coils is proportional to the ratio of the number of turns in each coil. For an ideal transformer, the input power is equal to the output power. The voltage and current are related by the following equations:

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

$$\frac{I_p}{I_s} = \frac{N_s}{N_p}$$

Where:
- V_p and V_s are the voltages in the primary and secondary coils,
- N_p and N_s are the number of turns in the primary and secondary coils,
- I_p and I_s are the currents in the primary and secondary coils.
Given: - Number of turns ratio $\frac{N_p}{N_s} = 20 : 1$,
- Input voltage $V_p = 250\,\text{V}$,

- Output current $I_s = 8\,\text{A}$.

Step 1: Calculate the current in the primary coil.

Using the current ratio equation:

$$\frac{I_p}{I_s} = \frac{N_s}{N_p} = \frac{1}{20}$$

Thus, the current in the primary coil is:

$$I_p = \frac{I_s}{20} = \frac{8}{20} = 0.4\,\text{A}$$

So, the current in the primary coil is $0.4\,\text{A}$.

Step 2: Calculate the output power.

The output power P_{out} is given by:

$$P_{\text{out}} = V_s \times I_s$$

From the voltage ratio, we can find the secondary voltage:

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = 20 \quad \Rightarrow \quad V_s = \frac{V_p}{20} = \frac{250}{20} = 12.5\,\text{V}$$

Thus, the output power is:

$$P_{\text{out}} = 12.5\,\text{V} \times 8\,\text{A} = 100\,\text{W}$$

Thus, the output power is $100\,\text{W}$.

Quick Tip

In an ideal transformer, the power input to the primary coil is equal to the power output from the secondary coil. The voltage and current ratios depend on the turns ratio of the coils.

(c) Write down Einstein's photoelectric equation. Explain with the help of this equation that what is the effect on the maximum kinetic energy of the emitted electrons, if

frequency of the incident light (photons) is increased by n times. What is the relationship between the work function of the metal surface and threshold wavelength?

Solution:

Einstein's photoelectric equation relates the energy of the incident photons to the kinetic energy of the emitted photoelectrons. The equation is:

$$K_{\text{max}} = h\nu - \Phi$$

Where: - K_{max} is the maximum kinetic energy of the emitted electrons,
- h is Planck's constant (6.626×10^{-34} J $\cdot$ s),
- ν is the frequency of the incident light (photons),
- Φ is the work function of the metal, which is the minimum energy required to remove an electron from the metal surface.

The equation shows that the maximum kinetic energy of the emitted electrons is the difference between the energy of the incident photons and the work function of the material.

Effect of Increasing Frequency by n Times:

If the frequency of the incident light is increased by a factor of n, the energy of the photons increases by the same factor, i.e., the energy of the photons becomes $n \cdot h\nu$. The new maximum kinetic energy K'_{max} is given by:

$$K'_{\text{max}} = n \cdot h\nu - \Phi$$

Thus, if the frequency is increased by n times, the maximum kinetic energy of the emitted electrons also increases by a factor of n.

Threshold Wavelength and Work Function:

The threshold wavelength λ_{th} is the wavelength of the incident light below which no photoelectron is emitted, even if the intensity of the light is increased. The threshold wavelength is related to the work function Φ by the equation:

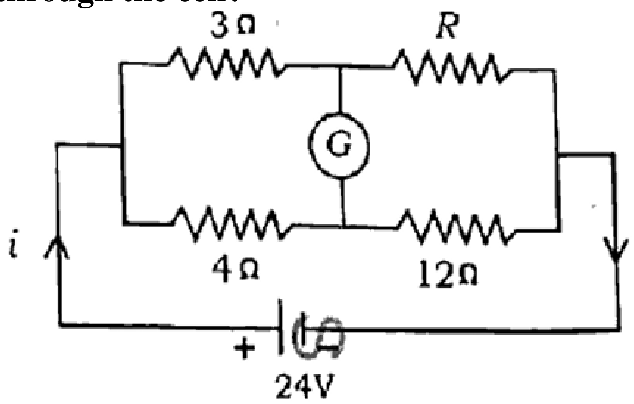
$$\Phi = \frac{hc}{\lambda_{\text{th}}}$$

Where: - c is the speed of light,
- λ_{th} is the threshold wavelength.

This equation shows that a larger work function corresponds to a shorter threshold wavelength, i.e., the more energy required to release electrons from the metal, the shorter the wavelength of light needed to release those electrons.

> Quick Tip
>
> When frequency increases, the energy of the incident light increases. The kinetic energy of emitted electrons increases accordingly, provided the frequency exceeds the threshold frequency.

(d) Explain the principle of Wheatstone's bridge by Kirchhoff's law. In the given circuit, there is no deflection in the galvanometer G. What is the current flowing through the cell?



Solution:

The Wheatstone bridge is a circuit used to measure an unknown resistance by balancing two legs of a bridge circuit. The principle of the Wheatstone bridge states that when the bridge is balanced, the ratio of resistances in one leg equals the ratio of resistances in the other leg, which is derived from Kirchhoff's law. The balance condition of the Wheatstone bridge is given by:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Where: - R_1, R_2, R_3, R_4 are the resistors in the four arms of the bridge,

- G is the galvanometer, which detects the balance condition by indicating zero deflection when the bridge is balanced.

In the given circuit:

$$R_1 = 3\,\Omega,\ R_2 = 4\,\Omega,\ R_3 = 12\,\Omega$$

We need to find the value of R_4 (which is unknown). Since there is no deflection in the galvanometer, the bridge is balanced. Therefore:

$$\frac{3}{4} = \frac{12}{R_4}$$

Solving for R_4:

$$R_4 = \frac{4 \times 12}{3} = 16\,\Omega$$

Now, using Kirchhoff's law, we can find the current flowing through the cell. The total resistance in the circuit is the sum of the resistances in series. The total resistance R_{total} is:

$$R_{\text{total}} = R_1 + R_2 + R_3 + R_4 = 3 + 4 + 12 + 16 = 35\,\Omega$$

The current flowing through the cell is given by Ohm's law:

$$I = \frac{V}{R_{\text{total}}} = \frac{24}{35} = 0.686\,\text{A}$$

Thus, the current flowing through the cell is 0.686 A.

Quick Tip

In a Wheatstone bridge, the bridge is balanced when the ratio of resistances in one leg is equal to the ratio of resistances in the other leg. Use Kirchhoff's law to solve for unknown values in the circuit.

(e) Obtain the formula for the magnetic force between two parallel long current carrying conductors and define 1 A of current with its help.

Solution:

The magnetic force between two parallel current-carrying conductors is derived using Ampère's law. The magnetic field generated by one conductor creates a force on the other conductor. The force per unit length between two parallel conductors is given by the formula:

$$F/L = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Where: - F is the force between the conductors,
- L is the length of the conductors,
- μ_0 is the permeability of free space ($\mu_0 = 4\pi \times 10^{-7}$ N/A^2),
- I_1 and I_2 are the currents in the two conductors,
- r is the distance between the two conductors.

The direction of the force is determined by the direction of the currents in the conductors. If the currents flow in the same direction, the force is attractive. If the currents flow in opposite directions, the force is repulsive.

This formula expresses the force per unit length between two parallel conductors due to the magnetic field produced by one conductor that acts on the other conductor.

Definition of 1 Ampere:

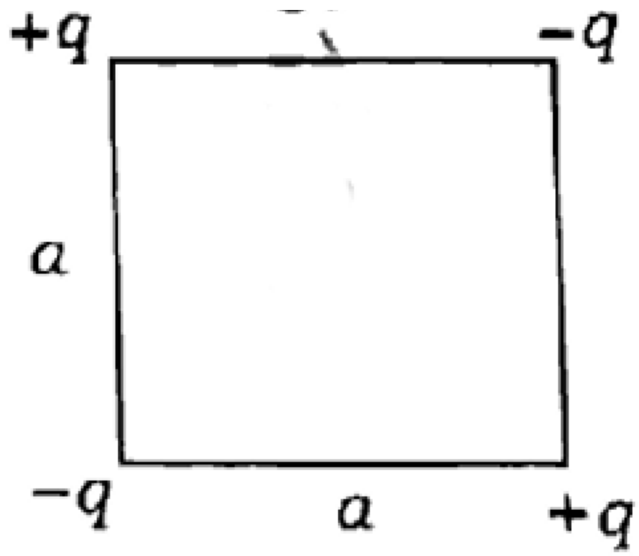
1 Ampere (1 A) is defined as the current that, when flowing through two parallel conductors placed 1 meter apart in a vacuum, produces a force of 2×10^{-7} N/m between the conductors. This definition links the concept of current with the magnetic force between conductors, providing a direct method for measuring electric current.

Quick Tip

The magnetic force between two current-carrying conductors is directly proportional to the product of the currents and inversely proportional to the distance between them. The definition of 1 Ampere is based on this fundamental principle.

Section - E

Q.6 (a) What is meant by the electric potential energy of the system of point charges? Compute the total electric potential energy of the system of charges given in the figure.



Solution:

The electric potential energy of a system of point charges is the work done to assemble the system of charges from infinity. It is given by the formula:

$$U = \sum_{i<j} \frac{k_e q_i q_j}{r_{ij}},$$

where:

- U is the total electric potential energy of the system,
- $k_e = 9 \times 10^9\,\mathrm{N \cdot m^2/C^2}$ is Coulomb's constant,
- q_i and q_j are the charges,
- r_{ij} is the distance between charges i and j.

In the given figure, we have a system of four point charges arranged at the corners of a square. The charges are:

- $+q$ at $(0, 0)$,
- $-q$ at $(a, 0)$,
- $-q$ at (a, a),
- $+q$ at $(0, a)$.

The distances between each pair of charges are either a (side of the square) or $\sqrt{2}a$ (diagonal of the square).

Step 1: Calculate the potential energy due to the interactions between charges on adjacent sides (distance a):

- Interaction between $+q$ and $-q$ at $(0, 0)$ and $(a, 0)$:

$$U_1 = \frac{k_e(+q)(-q)}{a} = -\frac{k_e q^2}{a}.$$

- Interaction between $-q$ and $+q$ at $(a, 0)$ and (a, a):

$$U_2 = \frac{k_e(-q)(+q)}{a} = -\frac{k_e q^2}{a}.$$

- Interaction between $+q$ and $-q$ at (a, a) and $(0, a)$:

$$U_3 = \frac{k_e(+q)(-q)}{a} = -\frac{k_e q^2}{a}.$$

- Interaction between $-q$ and $+q$ at $(0, a)$ and $(0, 0)$:

$$U_4 = \frac{k_e(-q)(+q)}{a} = -\frac{k_e q^2}{a}.$$

Thus, the total potential energy due to adjacent charges is:

$$U_{\text{adj}} = 4\left(-\frac{k_e q^2}{a}\right) = -\frac{4k_e q^2}{a}.$$

Step 2: Calculate the potential energy due to the interactions between charges on the diagonals (distance $\sqrt{2}a$):

- Interaction between $+q$ at $(0, 0)$ and $-q$ at (a, a):

$$U_5 = \frac{k_e(+q)(-q)}{\sqrt{2}a} = -\frac{k_e q^2}{\sqrt{2}a}.$$

- Interaction between $-q$ at $(a, 0)$ and $+q$ at $(0, a)$:

$$U_6 = \frac{k_e(-q)(+q)}{\sqrt{2}a} = -\frac{k_e q^2}{\sqrt{2}a}.$$

Thus, the total potential energy due to diagonal charges is:

$$U_{\text{diag}} = 2\left(-\frac{k_e q^2}{\sqrt{2}a}\right) = -\frac{2k_e q^2}{\sqrt{2}a}.$$

Step 3: Total electric potential energy of the system:

Now, the total electric potential energy of the system is the sum of the contributions from adjacent and diagonal interactions:

$$U_{\text{total}} = U_{\text{adj}} + U_{\text{diag}} = -\frac{4k_e q^2}{a} - \frac{2k_e q^2}{\sqrt{2}a}.$$

Thus, the total electric potential energy of the system is:

$$U_{\text{total}} = -\frac{4k_e q^2}{a} - \frac{2k_e q^2}{\sqrt{2}a}.$$

This is the total electric potential energy of the system of point charges arranged in a square.

Quick Tip

The total electric potential energy of a system of point charges is calculated by summing the potential energies of all pairs of charges, where each pair's potential energy is given by $\frac{k_e q_i q_j}{r_{ij}}$. For symmetric charge configurations, the distances can be easily calculated using geometry.

OR

Write down the formula and unit of surface charge density. A charge Q is distributed over two concentric hollow spheres of radii r_1 and $r_2 (r_1 > r_2)$. If their surface charge densities are equal, find the electric potential at their common centre.

Solution:

The surface charge density σ is the charge per unit area on the surface of a conductor. It is given by the formula:

$$\sigma = \frac{Q}{A}$$

Where:

- Q is the charge distributed over the surface,

- A is the surface area of the sphere.

For a sphere, the surface area is given by:

$$A = 4\pi r^2$$

Thus, the surface charge density for a sphere of radius r is:

$$\sigma = \frac{Q}{4\pi r^2}$$

The unit of surface charge density is:

$$\text{Unit of } \sigma = \frac{\text{Coulomb}}{\text{meter}^2} = \text{C/m}^2$$

Now, we are given two concentric hollow spheres with radii r_1 and r_2, and their surface charge densities are equal. This implies:

$$\sigma_1 = \sigma_2$$

Thus, we have the following equations for the surface charge densities of both spheres:

$$\sigma_1 = \frac{Q}{4\pi r_1^2}, \quad \sigma_2 = \frac{Q}{4\pi r_2^2}$$

Since $\sigma_1 = \sigma_2$, we can equate these expressions:

$$\frac{Q}{4\pi r_1^2} = \frac{Q}{4\pi r_2^2}$$

This simplifies to:

$$r_1^2 = r_2^2$$

However, this is not possible if $r_1 \neq r_2$, implying that the surface charge densities cannot be the same unless $r_1 = r_2$.

Electric Potential at the Common Centre Now, let's calculate the electric potential at the common centre (which is the centre of both spheres). The electric potential at the centre of a spherical shell due to a charge Q distributed uniformly on its surface is given by:

$$V = \frac{1}{4\pi\epsilon_0} \times \frac{Q}{r}$$

For a concentric configuration of two spheres, the potential at the common centre will be the sum of the potentials due to both spheres. Therefore, the total potential at the common centre is:

$$V_{\text{total}} = V_1 + V_2 = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r_1} + \frac{Q}{r_2}\right)$$

Thus, the electric potential at the common centre of the spheres is:

$$V_{\text{total}} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Where: - Q is the charge distributed on the spheres,
- r_1 and r_2 are the radii of the spheres,
- ϵ_0 is the permittivity of free space.

> Quick Tip
>
> The electric potential at the centre of a spherical shell depends only on the charge on the shell and the distance from the centre. For concentric spheres, the total potential at the centre is the sum of the potentials from each sphere.

Q.7 Explain the working method of a reverse biased (p-n) junction diode by making its circuit diagram and explain avalanche breakdown with the help of its V-I characteristic graph.

Solution:

Working of a Reverse Bias (p-n) Junction Diode:

A p-n junction diode works by allowing current to flow in only one direction. When the diode is reverse biased, the p-side is connected to the negative terminal, and the n-side is connected to the positive terminal of the battery. This increases the width of the depletion region, and as a result, no current flows under normal conditions.

The current through the diode in reverse bias remains negligibly small (very small leakage current), and the reverse bias voltage increases the width of the depletion region, thereby preventing the flow of majority charge carriers. The diode essentially acts as an insulator.

The reverse bias is applied in such a way that the n-type material (negative side) is connected to the positive terminal of the battery and the p-type material (positive side) is connected to the negative terminal of the battery. This results in a very small reverse current, mainly due to minority charge carriers.

Avalanche Breakdown:

Avalanche breakdown occurs when the reverse bias voltage exceeds a critical value. When the reverse bias increases, the minority carriers gain more energy and collide with the atoms of the semiconductor, creating more electron-hole pairs. This process multiplies and results in a large current, which is known as avalanche breakdown. The current increases rapidly with an increase in reverse voltage after the breakdown.

V-I Characteristic Graph:

The V-I characteristic graph for a reverse biased diode shows a small leakage current initially. As the reverse voltage increases, the current remains nearly constant until the breakdown voltage is reached. After this point, the current increases rapidly, representing avalanche breakdown.

Explanation of the Graph:

- The current remains negligible in the reverse region until the breakdown voltage is reached.
- Once the breakdown voltage is exceeded, the current increases exponentially, signifying avalanche breakdown.

Quick Tip

Avalanche breakdown is the result of the acceleration of minority carriers due to a high reverse bias, causing a chain reaction of electron-hole pair generation. Always be cautious about exceeding the breakdown voltage of a diode, as it can lead to permanent damage.

OR

What is the rectifying process? Explain the half-wave rectifying action of a (p-n) junction diode with the help of a circuit diagram.

Solution:

Rectifying Process:

The rectifying process refers to the conversion of alternating current (AC) into direct current (DC) by using a diode. A diode only allows current to flow in one direction (forward direction), and thus, when an alternating current is passed through it, only the positive half-cycle or negative half-cycle of the current is allowed to pass. This results in the transformation of AC to a pulsating DC.

Half-Wave Rectifier:

In a half-wave rectifier, a single diode is used to allow current to flow during only one half-cycle (positive or negative) of the AC supply. The negative half-cycle of the AC supply is blocked by the diode. Thus, only the positive half of the input signal is passed to the output.

In this diagram, the AC supply is connected to the anode of the p-n junction diode. The cathode is connected to the load resistor R_L. During the positive half of the AC cycle, the diode is forward biased and allows current to pass, while during the negative half of the cycle, the diode is reverse biased and blocks current, resulting in zero current flow.

Waveform of a Half-Wave Rectifier:

The output waveform of the half-wave rectifier is a pulsating DC signal that follows the positive half of the input AC signal. The negative half-cycle is blocked. The output voltage is non-zero only during the positive half-cycle of the AC input.

Quick Tip

A half-wave rectifier only allows one half of the AC cycle to pass, meaning it is inefficient and provides a pulsating DC output. Full-wave rectifiers are used when a smoother DC output is required.

Q.8 Describe briefly Rutherford's α-particle scattering experiment. What are the shortcomings of this model? How are they rectified in Bohr's model?

Solution:

Rutherford's α-Particle Scattering Experiment:

In Rutherford's experiment, a beam of α-particles was directed at a thin gold foil. By observing the scattering angles of the α-particles, Rutherford concluded that most of the mass of an atom is concentrated in a tiny, dense nucleus at the centre, and that the majority of the atom's volume is empty space. The α-particles were deflected at large angles by the positive charge concentrated in the nucleus. This led to the discovery of the nuclear model of the atom, where electrons orbit around a dense nucleus.

Shortcomings of Rutherford's Model:

Rutherford's model, while groundbreaking, had certain flaws:

- It did not explain the stability of the atom. According to classical electromagnetism, the revolving electrons would emit radiation, causing them to lose energy and spiral into the nucleus, leading to atomic collapse.
- It could not explain the observed spectral lines of hydrogen, as predicted by experimental data, where the electron should continuously radiate energy and not remain in discrete energy levels.

Rectification by Bohr's Model:

Niels Bohr improved upon Rutherford's model by introducing the idea that electrons exist in discrete orbits or energy levels. The electrons can only occupy these stable orbits without radiating energy. Radiation occurs only when an electron jumps from one orbit to another. The energy associated with these orbits is quantized, and Bohr used this idea to explain the discrete spectral lines observed in hydrogen.

Quick Tip
Bohr's model introduced quantized orbits to explain atomic stability and the spectral lines of hydrogen, something Rutherford's model could not account for.

OR

What is the difference between nuclear fusion and nuclear fission? Find the value of the energy Q released with the help of the given nuclear fusion reaction:

$$1\,{}^1\mathrm{H}_2 + {}^1\mathrm{H}_3 \rightarrow {}^2\mathrm{He}_4 + {}^1\mathrm{n}^1 + Q$$

Given:

Mass of ${}^1\mathrm{H}_2 = 2.0141$ amu

Mass of ${}^1\mathrm{H}_3 = 3.0160$ amu

Mass of ${}^2\mathrm{He}_4 = 4.0026$ amu

Mass of ${}^1\mathrm{n}^1 = 1.0087$ amu

1 amu = 931 MeV

Solution:

Nuclear Fusion and Nuclear Fission are two types of nuclear reactions:

- Nuclear Fusion is the process in which two light nuclei combine to form a heavier nucleus, releasing a large amount of energy. For example, in stars, hydrogen nuclei fuse to form helium, releasing energy.
- Nuclear Fission is the process in which a heavy nucleus splits into two or more lighter nuclei, accompanied by the release of energy. Fission is commonly used in nuclear reactors.

Energy Released in Nuclear Fusion Reaction: The energy released in a nuclear reaction is given by the difference in mass between the reactants and products, multiplied by c^2 (where c is the speed of light). The mass defect (Δm) is the difference between the total mass of the reactants and the total mass of the products.

Step 1: Calculate the mass defect.

The total mass of the reactants is the sum of the masses of $^1\text{H}_2$ and $^1\text{H}_3$:

$$m_{\text{reactants}} = m_{^1\text{H}_2} + m_{^1\text{H}_3} = 2.0141 + 3.0160 = 5.0301\,\text{amu}$$

The total mass of the products is the sum of the masses of $^2\text{He}_4$ and $^1\text{n}^1$:

$$m_{\text{products}} = m_{^2\text{He}_4} + m_{^1\text{n}^1} = 4.0026 + 1.0087 = 5.0113\,\text{amu}$$

The mass defect Δm is:

$$\Delta m = m_{\text{reactants}} - m_{\text{products}} = 5.0301 - 5.0113 = 0.0188\,\text{amu}$$

Step 2: Convert mass defect to energy.

Using the equivalence of mass and energy ($E = \Delta m \times c^2$), and knowing that $1\,\text{amu} = 931\,\text{MeV}/c^2$, the energy released Q is:

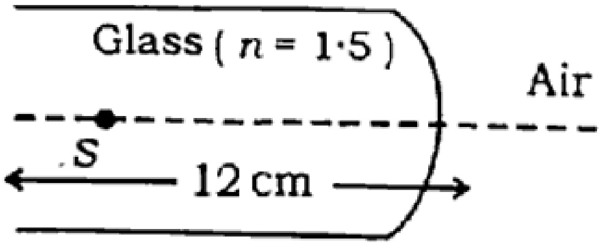
$$Q = \Delta m \times 931 = 0.0188 \times 931 = 17.5\,\text{MeV}$$

Thus, the energy released in this nuclear fusion reaction is $Q = 17.5\,\text{MeV}$.

Quick Tip

The energy released in a nuclear reaction is directly related to the mass defect. For fusion and fission, the energy can be calculated by finding the mass difference between the reactants and products and multiplying by 931 MeV per amu.

Q.9 Write the formula of refraction of light through a single spherical surface. The radius of curvature of one end of a cylindrical glass (n = 1.5) rod is 2 cm. Find the position and nature of the image of the point source S. See the figure and also draw the ray diagram.



Solution:

The refraction of light through a single spherical surface is governed by the formula for refraction at a spherical surface, given by:

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Where:

- n_1 and n_2 are the refractive indices of the media (in this case, $n_1 = 1$ for air and $n_2 = 1.5$ for glass),
- u is the object distance (the distance of the point source S from the spherical surface),
- v is the image distance (the distance of the image formed),
- R is the radius of curvature of the spherical surface.

Given: - Radius of curvature $R = 2\,\text{cm}$,
- The object is placed in air, so $n_1 = 1$,
- The refractive index of glass $n_2 = 1.5$.

Substituting these values into the formula:

$$\frac{1.5}{v} - \frac{1}{u} = \frac{1.5 - 1}{2}$$

$$\frac{1.5}{v} - \frac{1}{u} = \frac{0.5}{2} = 0.25$$

Thus, the image will be formed at a distance v from the spherical surface. To find the position and nature of the image, the value of u (object distance) must be specified, as it determines the final image location.
The ray diagram would show light rays entering the spherical surface from the object at a distance u, refracting and converging at the image point v.

Quick Tip
For a spherical surface, the image formed depends on the refractive indices of the media and the radius of curvature. If the object is in air, the refractive index of air is considered as 1.

OR

Write down the formula for the distances of nth bright and nth dark fringes, from the central fringe in Young's double slit experiment. A monochromatic source of light with wavelength 480 nm, is used in this experiment, in which distance between the double slits is 3 mm. Distance between the slits and screen is 2 m. Find the distance between the 8th bright and the 3rd dark fringes.

Solution:

In Young's double slit experiment, the formula for the distance of the nth bright fringe (denoted as y_n) from the central maximum is given by:

$$y_n = \frac{n\lambda L}{d}$$

Where: - n is the fringe number (for bright fringes, $n = 1, 2, 3, \ldots$),
- λ is the wavelength of light (given as $480\,\text{nm}$ or $480 \times 10^{-9}\,\text{m}$),
- L is the distance between the slits and the screen (given as $2\,\text{m}$),

- d is the distance between the slits (given as $3\,\text{mm}$ or $3 \times 10^{-3}\,\text{m}$).

For the nth dark fringe, the distance from the central fringe is given by the formula:

$$y'_n = \frac{(n - 1/2)\lambda L}{d}$$

To find the distance between the 8th bright and the 3rd dark fringes, we calculate the distance for the 8th bright fringe (y_8) and the 3rd dark fringe (y'_3).

1. Distance for the 8th bright fringe:

$$y_8 = \frac{8 \times 480 \times 10^{-9} \times 2}{3 \times 10^{-3}} = 0.00384\,\text{m} = 3.84\,\text{mm}$$

2. Distance for the 3rd dark fringe:

$$y'_3 = \frac{(3 - 1/2) \times 480 \times 10^{-9} \times 2}{3 \times 10^{-3}} = 0.00480\,\text{m} = 4.80\,\text{mm}$$

Now, to find the distance between the 8th bright and the 3rd dark fringes:

$$\text{Distance} = y'_3 - y_8 = 4.80\,\text{mm} - 3.84\,\text{mm} = 0.96\,\text{mm}$$

Thus, the distance between the 8th bright and the 3rd dark fringes is $0.96\,\text{mm}$.

Quick Tip

In Young's double slit experiment, the distance between bright or dark fringes depends on the wavelength of light, the slit separation, and the distance to the screen. For dark fringes, the fringe position is slightly shifted by $\frac{1}{2}\lambda$.
