UP Board Class 12 Physics - 346(JS) - 2025 Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :100	Total Questions :9

General Instructions

Read the following instructions very carefully and strictly follow them:

1. There are in all nine questions in this question paper.
2. All questions are compulsory.
3. In the beginning of each question, the number of parts to be attempted are clearly mentioned.
4. Marks allotted to the questions are indicated against them.
5. Start solving from the first question and proceed to solve till the last one. Do not waste your time over a question you cannot solve.

Section - A

a. The angle between the polarization plane and direction of propagation of electromagnetic waves is :

(A) $0°$
(B) $45°$
(C) $90°$
(D) $180°$

Correct Answer: (A) $0°$

Solution:

Step 1: Understanding the Concept:
This question deals with the basic concepts of the polarization of electromagnetic (EM) waves. In electromagnetic waves, both the electric field ($\vec{E}$) and the magnetic field ($\vec{B}$) oscillate perpendicular to each other and perpendicular to the direction of propagation of the wave.

In a transverse wave like an EM wave, the direction of the electric field oscillation and the magnetic field oscillation are both perpendicular to the direction in which the wave propagates. The concept of the "plane of polarization" relates to the orientation of the electric field vector ($\vec{E}$) in relation to the direction of propagation.

Step 2: Detailed Explanation:
To clarify, the plane of polarization refers to the plane that contains the electric field vector

($\vec{E}$) and the direction of propagation of the EM wave.

Now, let's break down the angle we are looking for: - The direction of propagation of the wave is represented by the wave vector, which points in the direction the wave is moving.
- The plane of polarization, by definition, contains the electric field vector and the propagation direction.

Thus, the electric field vector ($\vec{E}$) lies in the polarization plane, and the direction of wave propagation is parallel to the direction of the wave vector, which also lies within the plane of polarization. Hence, the angle between the polarization plane and the direction of wave propagation is zero.

To visualize this, think of a flat sheet of paper representing the polarization plane, and a straight line representing the direction of wave propagation. The line (direction of propagation) lies entirely within the plane (the sheet of paper), so the angle between them is 0°.

Therefore, the angle between the polarization plane and the direction of propagation is 0°.

Step 3: Final Answer:
The correct answer is (A) 0°. The angle between the polarization plane and the direction of propagation is zero, as they are parallel to each other.

$\boxed{\textbf{(A) 0°}}$.

Quick Tip

Do not confuse the "plane of polarization" with the "plane of vibration". The plane of vibration contains the electric field vector and is perpendicular to the direction of propagation. The plane of polarization contains both the E-field and the direction of propagation.

---

b. The energy is emitted when two nuclei of masses m_1 and m_2 are fused together to make a nucleus of mass m. In this process :

(A) $(m_1 + m_2) < m$
(B) $(m_1 + m_2) > m$
(C) $(m_1 + m_2) = m$
(D) $m_1 m_2 > m^2$

Correct Answer: (B) $(m_1 + m_2) > m$

Solution:

Step 1: Understanding the Concept:
This question involves the principle of mass-energy equivalence in nuclear reactions, specifically nuclear fusion. In nuclear fusion, two light nuclei combine to form a heavier nucleus,

and energy is released in the process. This is because the binding energy per nucleon of the product nucleus is greater than that of the reactant nuclei. This leads to a net release of energy.

Step 2: Key Formula or Approach:
The process follows Einstein's mass-energy equivalence principle, expressed as:

$$E = \Delta m \cdot c^2$$

where E is the energy released, Δm is the change in mass (also called mass defect), and c is the speed of light.

For energy to be emitted (released), there must be a decrease in the total mass of the system. The loss of mass is converted into energy.

Step 3: Detailed Explanation:
Let's define the masses involved in the fusion process: - m_1 and m_2 are the masses of the two initial nuclei. - m is the mass of the resulting nucleus after fusion.
The initial total mass of the system is:

$$m_{\text{initial}} = m_1 + m_2$$

The final mass of the system after fusion is:

$$m_{\text{final}} = m$$

Now, energy is emitted during the fusion process. This means the system loses mass, which is converted into energy. Thus, the initial mass must be greater than the final mass:

$$m_{\text{initial}} > m_{\text{final}}$$

or equivalently:

$$m_1 + m_2 > m$$

This decrease in mass $(m_1 + m_2 - m)$ is the mass defect, and it is directly proportional to the energy released during the fusion process. The energy released is given by:

$$E = (m_1 + m_2 - m)c^2$$

Thus, the correct condition for energy to be emitted during fusion is that the sum of the initial masses must be greater than the final mass, i.e., $(m_1 + m_2) > m$.

Step 4: Final Answer:
For energy to be emitted during fusion, the sum of the masses of the initial nuclei must be greater than the mass of the resulting nucleus. Therefore, the correct answer is (B) $(m_1 + m_2) > m$.

$\boxed{\textbf{(B) } (\mathbf{m_1 + m_2}) > \mathbf{m}}$.

Quick Tip

In any exothermic nuclear reaction (one that releases energy), whether fusion or fission, the total mass of the products is always less than the total mass of the reactants. This "missing" mass is converted into the released energy.

c. The resistance of 8 ohm and inductive reactance of 6 ohm are connected in series in an alternating current circuit. The impedance of the circuit will be :

(A) 2 ohm
(B) 14 ohm
(C) $14\sqrt{2}$ ohm
(D) 10 ohm

Correct Answer: (D) 10 ohm

Solution:

Step 1: Understanding the Concept:
Impedance (Z) is the total opposition to the flow of alternating current (AC) in a circuit. In an AC circuit containing both resistance (R) and inductance, the total opposition is called impedance. In a series R-L circuit, the impedance is the vector sum of the resistance R and the inductive reactance X_L. Impedance combines both resistive and inductive effects, and it is a complex quantity, though we treat it as a magnitude in this case.

The formula to calculate the impedance in a series R-L circuit is:

$$Z = \sqrt{R^2 + X_L^2}$$

where: - Z is the impedance, - R is the resistance, - X_L is the inductive reactance.
This formula gives the magnitude of the impedance, treating the resistance and inductive reactance as orthogonal components.

Step 2: Key Formula or Approach:
To find the impedance of a series R-L circuit, use the following equation:

$$Z = \sqrt{R^2 + X_L^2}$$

where R and X_L are given as the resistance and inductive reactance, respectively.

Step 3: Detailed Explanation:
We are provided with the following values for the circuit:

- Resistance $R = 8\,\Omega$,
- Inductive reactance $X_L = 6\,\Omega$.

Substitute these values into the impedance formula:

$$Z = \sqrt{R^2 + X_L^2}$$

$$Z = \sqrt{8^2 + 6^2}$$

$$Z = \sqrt{64 + 36}$$

$$Z = \sqrt{100}$$
$$Z = 10\,\Omega$$

Thus, the impedance of the circuit is 10 ohms.

Step 4: Final Answer:
The impedance of the circuit is $10\,\Omega$. Therefore, the correct option is (D).

$\boxed{\textbf{(D) 10 ohm}}$.

Quick Tip

This problem uses a 6-8-10 Pythagorean triple, which is a common pattern in physics and math problems. Recognizing this (a multiple of the 3-4-5 triple) can help you find the answer quickly without detailed calculation.

---

d. The unit of permittivity of vacuum is :

(A) Newton m^2/coulomb^2
(B) coulomb^2/Newton m^2
(C) Newton/coulomb
(D) Newton volt/m^2

Correct Answer: (B) coulomb^2/Newton m^2

Solution:

Step 1: Understanding the Concept:
The permittivity of vacuum, denoted by ϵ_0, is a physical constant that represents the ability of a vacuum to permit electric field lines to propagate. It appears in various electromagnetic equations, most notably in Coulomb's Law, which describes the electrostatic force between two point charges. The unit of permittivity can be derived by analyzing this law.

Step 2: Key Formula or Approach:
Coulomb's Law for the electrostatic force F between two point charges q_1 and q_2 separated by a distance r is given by:

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

where: - F is the electrostatic force, - q_1 and q_2 are the charges, - r is the distance between the charges, and - ϵ_0 is the permittivity of vacuum.
We can rearrange this formula to solve for ϵ_0 and then derive its units.

Step 3: Detailed Explanation:
Rearranging Coulomb's Law to solve for ϵ_0:

$$\epsilon_0 = \frac{1}{4\pi} \cdot \frac{F \cdot r^2}{q_1 q_2}$$

Now, let's substitute the SI units for each quantity in the formula: - Force F has the unit of Newton (N), - Charge q_1 and q_2 each have the unit of Coulomb (C), - Distance r has the unit of meter (m).
Substituting these units into the equation:

$$\text{Unit of } \epsilon_0 = \frac{1}{\text{N}} \cdot \frac{\text{C}^2}{\text{m}^2}$$

Simplifying this expression:

$$\text{Unit of } \epsilon_0 = \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}$$

Thus, the unit of permittivity of vacuum is $\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$, which is equivalent to coulomb^2/Newton m^2.

Step 4: Final Answer:
The unit of permittivity of vacuum is coulomb^2/Newton m^2. Therefore, the correct option is (B).

$\boxed{\textbf{(B) coulomb}^2\textbf{/Newton m}^2}$.

Quick Tip

A common mistake is to confuse the units of ϵ_0 with the units of the electrostatic constant $k = 1/(4\pi\epsilon_0)$. The units of k are N m^2/C^2, which is the reciprocal of the units of ϵ_0. Option (A) is the unit for k.

---

e. Electro-magnetic wave of minimum frequency is :

(A) Ultraviolet rays
(B) X-rays
(C) Gamma (γ-) rays
(D) Micro waves

Correct Answer: (D) Micro waves

Solution:

Step 1: Understanding the Concept:
The electromagnetic (EM) spectrum is the range of all types of EM radiation. Radiation is ordered by frequency (or wavelength). We need to identify which of the given options has the lowest frequency.

Step 2: Key Formula or Approach:
Recall the order of the electromagnetic spectrum. The standard order from lowest frequency (longest wavelength) to highest frequency (shortest wavelength) is:
Radio waves $\rightarrow$ Microwaves $\rightarrow$ Infrared $\rightarrow$ Visible Light $\rightarrow$ Ultraviolet $\rightarrow$ X-rays $\rightarrow$ Gamma rays.

Step 3: Detailed Explanation:
We are given the following options:

- (A) Ultraviolet rays
- (B) X-rays
- (C) Gamma rays
- (D) Micro waves

Comparing their positions in the electromagnetic spectrum:

- Gamma rays have the highest frequency.
- X-rays have a lower frequency than gamma rays.
- Ultraviolet rays have a lower frequency than X-rays.
- Microwaves have a lower frequency than ultraviolet rays.

Therefore, among the given choices, microwaves have the minimum frequency.

Step 4: Final Answer:
The electromagnetic wave with the minimum frequency among the options is Micro waves.

Quick Tip

Use a mnemonic to remember the order of the EM spectrum. One popular mnemonic is: "Raging Martians Invaded Venus Using X-ray Guns" (Radio, Microwaves, Infrared, Visible, Ultraviolet, X-ray, Gamma).

---

f. How does the magnetic susceptibility (χ) of paramagnetics change with respect to absolute temperature (T) ?

(A) $\chi \propto$ T
(B) $\chi \propto$ T^{-1}
(C) $\chi =$ constant
(D) $\chi \propto$ e^{T}

Correct Answer: (B) $\chi \propto \mathrm{T}^{-1}$

Solution:

Step 1: Understanding the Concept:
This question is about the behavior of paramagnetic materials in a magnetic field at different temperatures. Magnetic susceptibility (χ) is a measure of how much a material becomes magnetized in an applied magnetic field.

Step 2: Key Formula or Approach:
The relationship between magnetic susceptibility and absolute temperature for paramagnetic materials is described by Curie's Law.
Curie's Law states that the magnetization of a paramagnetic material is directly proportional to the applied magnetic field and inversely proportional to the absolute temperature. This implies that the magnetic susceptibility (χ) is inversely proportional to the absolute temperature (T).

$$\chi = \frac{C}{T}$$

where C is the Curie constant.

Step 3: Detailed Explanation:
From Curie's Law, we have the relationship:

$$\chi \propto \frac{1}{T}$$

This can also be written using a negative exponent as:

$$\chi \propto T^{-1}$$

This means that as the temperature of a paramagnetic substance increases, its ability to be magnetized decreases. This is because the random thermal motion of the atoms at higher temperatures opposes the alignment of their magnetic dipoles with the external field.

Step 4: Final Answer:
The magnetic susceptibility (χ) of a paramagnetic substance is inversely proportional to the absolute temperature (T), so $\chi \propto T^{-1}$.

Quick Tip

Remember the temperature dependence for different magnetic materials: - **Paramagnetic:** $\chi \propto 1/T$ (Curie's Law). - **Diamagnetic:** χ is small, negative, and independent of temperature. - **Ferromagnetic:** $\chi \propto 1/(T - T_c)$ for T ¿ T_c (Curie-Weiss Law), where T_c is the Curie temperature.

Section - B

2.

a. What do you mean by total internal reflection ? Show it by drawing ray diagram.

Solution:

Step 1: Understanding the Concept:
Total Internal Reflection (TIR) is the phenomenon where a wave, such as light, traveling from a denser medium to a less dense medium is completely reflected back into the denser medium.

Step 2: Conditions for TIR:
For total internal reflection to occur, two conditions must be met:

1. The light ray must travel from a denser optical medium to a rarer optical medium.
2. The angle of incidence in the denser medium must be greater than a specific angle called the **critical angle** (i_c). The critical angle is the angle of incidence for which the angle of refraction is $90°$.

Step 3: Ray Diagram:

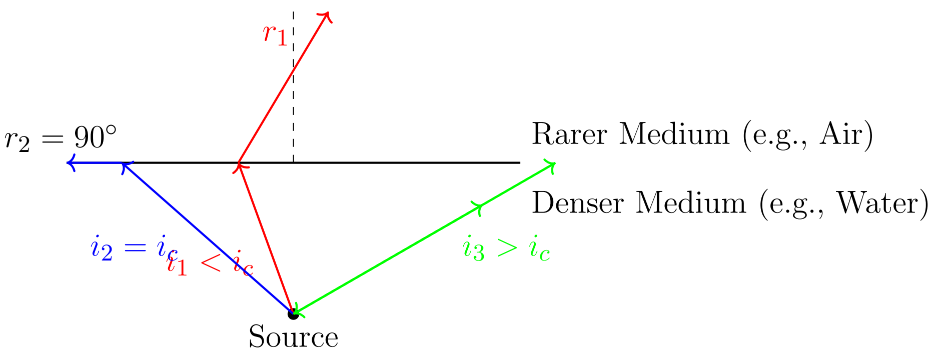


Diagram Explanation:

- **Ray 1:** The angle of incidence i_1 is less than the critical angle i_c. The ray refracts into the rarer medium, bending away from the normal.

- **Ray 2:** The angle of incidence i_2 is equal to the critical angle i_c. The refracted ray travels along the boundary of the two media (angle of refraction $r_2 = 90°$).

- **Ray 3:** The angle of incidence i_3 is greater than the critical angle i_c. The ray does not refract but is completely reflected back into the denser medium, following the laws of reflection ($i_3 = r_3$). This is Total Internal Reflection.

Quick Tip

Remember the key application of TIR is in optical fibers, where light signals are transmitted over long distances with minimal loss of intensity. Other examples include the sparkling of diamonds and mirages.

---

b. Differentiate between isotopes and isobars.

Solution:

Step 1: Understanding the Concept:
Isotopes and isobars are terms used to classify nuclides (a specific type of atomic nucleus). The classification is based on the number of protons (atomic number, Z) and the total number of nucleons (mass number, A).

Step 2: Detailed Comparison:
Here is a point-by-point differentiation between isotopes and isobars:

Property	**Isotopes**	**Isobars**
Definition	Nuclides having the same atomic number (Z) but different mass numbers (A).	Nuclides having the same mass number (A) but different atomic numbers (Z).
Protons (Z)	Same	Different
Neutrons (A-Z)	Different	Different
Nucleons (A)	Different	Same
Element	They are atoms of the same chemical element.	They are atoms of different chemical elements.
Position in Periodic Table	Occupy the same position.	Occupy different positions.
Chemical Properties	Nearly identical chemical properties (as they depend on the electron configuration, which is determined by Z).	Different chemical properties.
Physical Properties	Different physical properties (e.g., mass, density, boiling point).	Different physical properties.
Example	Protium (${}^{1}_{1}H$), Deuterium (${}^{2}_{1}H$), and Tritium (${}^{3}_{1}H$) are isotopes of Hydrogen.	Argon-40 (${}^{40}_{18}Ar$) and Calcium-40 (${}^{40}_{20}Ca$) are isobars.

Quick Tip

Use the names to remember the definitions: - Iso**t**o**p**es have the same number of **p**rotons (same a**t**omic number).
- Isob**a**rs have the same m**a**ss number (A).

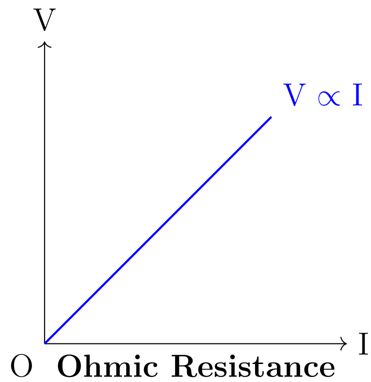
---

c. Draw the graph between Voltage (V) and Current (I) for ohmic and non-ohmic resistances.

Solution:

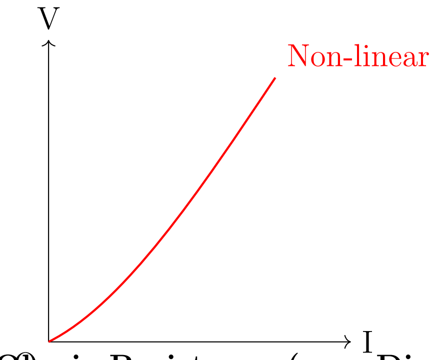
Step 1: Understanding the Concept:
Ohmic resistances (or conductors) are those that obey Ohm's Law, which states that the current (I) through a conductor is directly proportional to the voltage (V) across it, provided the temperature and other physical conditions remain unchanged ($V \propto I$).
Non-ohmic resistances are those that do not obey Ohm's Law. Their V-I relationship is not linear.

Step 2: V-I Graph for Ohmic Resistance:
According to Ohm's Law, $V = IR$. If R is constant, this is the equation of a straight line passing through the origin, with the slope equal to the resistance R (Slope = $\frac{\Delta V}{\Delta I} = R$).



Ohmic Resistance

Step 3: V-I Graph for Non-Ohmic Resistance:
For non-ohmic devices, the resistance is not constant but changes with voltage or current. The V-I graph is a curve, not a straight line. Examples include semiconductor diodes, transistors, and thermistors.



Non-Ohmic Resistance (e.g., Diode)

Graph Explanation:

- **Ohmic:** A straight line through the origin indicates a constant ratio of V to I, meaning constant resistance.

- **Non-Ohmic:** The curve shows that the ratio of V to I (the resistance) is not constant. The slope of the tangent at any point on the curve gives the dynamic resistance.

Quick Tip

Remember that "Ohmic" means a linear V-I relationship (a straight line graph), while "non-ohmic" implies a non-linear relationship (a curved graph).

---

d. A conductor has positive charge of 2.4×10^{-18} coulomb. Find how much electrons are in deficit/excess on the conductor.

Correct Answer: 15 electrons are in deficit.

Solution:

Step 1: Understanding the Concept:
The charge on any object is an integer multiple of the elementary charge (the charge of a single electron). This is the principle of quantization of charge. A positive charge on a conductor indicates a removal of electrons, leading to a deficit.

Step 2: Key Formula or Approach:
The formula for quantization of charge is:

$$Q = ne$$

where:

- Q is the total charge on the object.
- n is the number of electrons in excess or deficit (must be an integer).
- e is the magnitude of the elementary charge, $e = 1.6 \times 10^{-19}$ C.

We need to solve for n.

Step 3: Detailed Explanation:
We are given:

- Total charge $Q = +2.4 \times 10^{-18}$ C.
- Elementary charge $e = 1.6 \times 10^{-19}$ C.

Rearranging the formula to solve for n:

$$n = \frac{Q}{e}$$

Substituting the values:

$$n = \frac{2.4 \times 10^{-18}}{1.6 \times 10^{-19}}$$

$$n = \frac{2.4}{1.6} \times 10^{-18-(-19)} = \frac{24}{16} \times 10^{1} = 1.5 \times 10 = 15$$

Since the charge Q is positive, it means that electrons (which are negatively charged) have been removed from the conductor. Therefore, there is a **deficit** of 15 electrons.

Step 4: Final Answer:
The conductor has a deficit of 15 electrons.

> Quick Tip
>
> Remember the sign convention: A positive net charge means an object has lost electrons (deficit). A negative net charge means an object has gained electrons (excess).

---

e. Voltage (V) equation in an alternating current circuit is represented by $V = 40\sin(100\pi t)$ volt. Here t is in second. Draw the time-voltage graph for one cycle with proper scale.

Solution:

Step 1: Understanding the Concept:
The given equation is for a sinusoidal alternating voltage. We need to identify its key characteristics (amplitude, frequency, time period) to plot its graph accurately for one complete cycle.

Step 2: Key Formula or Approach:
The standard equation for a sinusoidal AC voltage is $V(t) = V_m \sin(\omega t)$, where:

- V_m is the peak voltage or amplitude.
- ω is the angular frequency.

The time period (T) is related to ω by $T = \frac{2\pi}{\omega}$.

Step 3: Detailed Explanation:
Comparing the given equation $V = 40\sin(100\pi t)$ with the standard form:

- Peak Voltage (V_m): $V_m = 40$ V. This means the voltage varies between +40 V and -40 V.
- Angular Frequency (ω): $\omega = 100\pi$ rad/s.

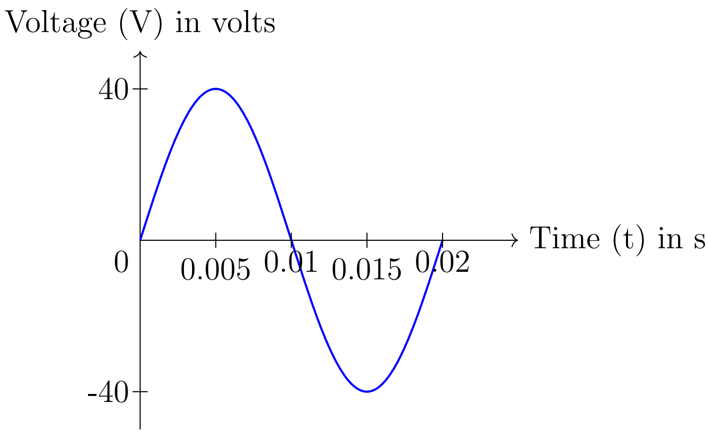
Now, we calculate the time period (T) for one complete cycle:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = \frac{2}{100} = 0.02\,\text{seconds}$$

To draw the graph, we need some key points in the first cycle (from t=0 to t=0.02 s):

- At $t = 0$, $V = 40\sin(0) = 0$ V.
- At $t = T/4 = 0.005$ s, $V = 40\sin(100\pi \cdot 0.005) = 40\sin(\pi/2) = 40$ V (Peak).
- At $t = T/2 = 0.01$ s, $V = 40\sin(100\pi \cdot 0.01) = 40\sin(\pi) = 0$ V.
- At $t = 3T/4 = 0.015$ s, $V = 40\sin(100\pi \cdot 0.015) = 40\sin(3\pi/2) = -40$ V (Trough).
- At $t = T = 0.02$ s, $V = 40\sin(100\pi \cdot 0.02) = 40\sin(2\pi) = 0$ V.

Step 4: Graph:



Quick Tip
When asked to draw a sinusoidal graph, always calculate the amplitude and the time period first. Then, mark the key points at t=0, T/4, T/2, 3T/4, and T to ensure the shape and scale of the sine wave are correct.

---

f. Electron in the hydrogen atom is moving round the nucleus with 6.0×10^{15} cycle per second. What will be the value of current at a point on circular path ?

Correct Answer: 0.96×10^{-3} A or 0.96 mA

Solution:

Step 1: Understanding the Concept:
An electron revolving in a circular orbit constitutes an electric current. The current is defined as the rate of flow of charge. For a revolving electron, the current at any point on the orbit is the total charge that passes that point per unit time.

Step 2: Key Formula or Approach:
The electric current (I) is given by:

$$I = \frac{Q}{t}$$

In this case, Q is the charge of the electron (e), and the time taken for one revolution is the time period (T). So, $I = \frac{e}{T}$.
The frequency (f) is the number of cycles per second, which is the reciprocal of the time period ($f = 1/T$). Therefore, the formula can be written as:

$$I = e \cdot f$$

Step 3: Detailed Explanation:
We are given:

- Frequency of revolution $f = 6.0 \times 10^{15}$ cycles/second (or Hz).
- The charge of an electron $e = 1.6 \times 10^{-19}$ C.

Now, we can calculate the current using the formula $I = ef$:

$$I = (1.6 \times 10^{-19}\,\text{C}) \times (6.0 \times 10^{15}\,\text{s}^{-1})$$
$$I = (1.6 \times 6.0) \times 10^{-19+15}\,\text{C/s}$$
$$I = 9.6 \times 10^{-4}\,\text{A}$$

This can also be expressed as:

$$I = 0.96 \times 10^{-3}\,\text{A} = 0.96\,\text{mA}$$

Step 4: Final Answer:
The value of the current at a point on the circular path is 9.6×10^{-4} A or 0.96 mA.

Quick Tip

Remember that "cycles per second" is just another way of stating the frequency in Hertz (Hz). The concept of an orbiting charge creating a current is fundamental in understanding the magnetic properties of atoms (orbital magnetic moment).

Section - C

3.

a. What are the demerits of Rutherford model of an atom ?

Solution:

Step 1: Understanding the Concept:
Rutherford's nuclear model of the atom, proposed after the gold foil experiment, successfully described the atom as having a small, dense, positively charged nucleus with negatively charged electrons orbiting it. However, this model had significant shortcomings when analyzed with the principles of classical physics.

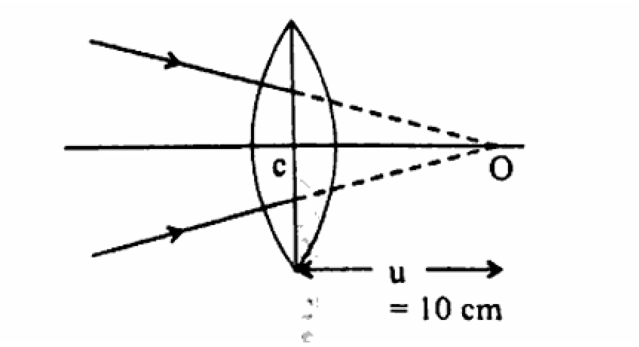
Step 2: Detailed Explanation of Demerits:
The two main demerits (drawbacks) of the Rutherford model are as follows:

1. Instability of the Atom:
According to Maxwell's theory of classical electromagnetism, any charged particle undergoing acceleration must radiate energy continuously in the form of electromagnetic waves.
In Rutherford's model, the electrons are revolving around the nucleus. This circular motion is an accelerated motion.
Therefore, an orbiting electron should continuously lose energy by radiation.
This loss of energy would cause its orbit to shrink, and the electron would spiral into the nucleus in a very short time (calculated to be about 10^{-8} seconds).
This would make the atom highly unstable. However, we know that atoms are stable. Rutherford's model could not explain this stability.

2. Inability to Explain the Line Spectrum:
As the electron spirals inwards towards the nucleus, its speed and frequency of revolution would increase continuously.
According to classical physics, the frequency of the emitted electromagnetic radiation should be equal to the frequency of revolution.
Since the electron's frequency is changing continuously, it should emit a continuous spectrum of radiation.
However, experimental observations show that atoms, like hydrogen, emit a discrete line spectrum, i.e., radiation of only specific frequencies or wavelengths.
Rutherford's model failed to explain the origin of these discrete spectral lines.

> Quick Tip
>
> The failures of Rutherford's model were critical in the development of modern atomic theory. These demerits were later addressed by Niels Bohr, who incorporated the concept of quantization of energy levels for electrons, leading to the Bohr model of the atom.

b. The light rays are incident on a convex lens as in figure. If focal length of lens is 20 cm then find the position of image. Show ray diagram as well.



Solution:

Step 1: Understanding the Concept:
The given figure shows a beam of light that is already converging towards a point O. When this beam passes through the convex lens, the point O acts as a **virtual object**. We need to use the lens formula to find the position of the final image.

Step 2: Key Formula or Approach:
We will use the lens formula:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

And the standard sign convention:

- Light travels from left to right.
- Distances measured in the direction of light are positive.
- Distances measured opposite to the direction of light are negative.
- The focal length (f) of a convex lens is positive.

Step 3: Detailed Explanation:
From the problem and the diagram:

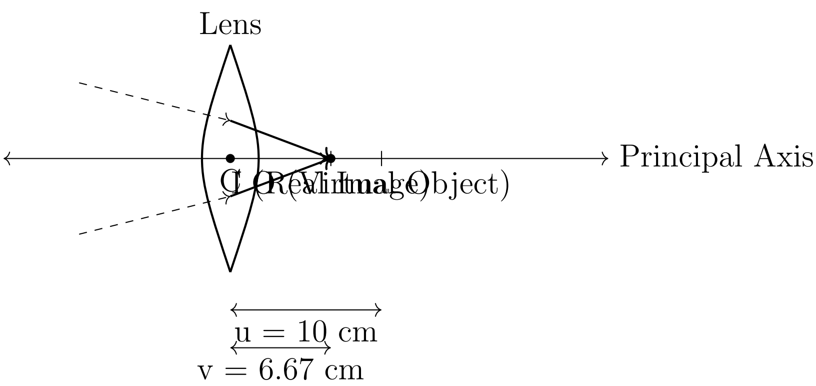
- **Focal length (f):** The lens is convex, so $f = +20$ cm.
- **Object distance (u):** The incident rays are converging to a point O which is 10 cm to the right of the lens. Since this point is where the rays would meet if the lens were not present, it acts as a virtual object. Its distance is measured from the optical center in the direction of light, so it is positive. $u = +10$ cm.

Now, we substitute these values into the lens formula to find the image distance (v):

$$\frac{1}{v} - \frac{1}{+10} = \frac{1}{+20}$$
$$\frac{1}{v} = \frac{1}{20} + \frac{1}{10}$$
$$\frac{1}{v} = \frac{1+2}{20} = \frac{3}{20}$$
$$v = \frac{20}{3} \approx +6.67\,\text{cm}$$

The positive sign for v indicates that the image is formed on the right side of the lens. Since the light rays actually converge at this point, the image is **real**.

Step 4: Ray Diagram:



The diagram shows the converging incident rays (dashed) heading towards the virtual object O. The convex lens converges them further, causing them to meet at the real image I, which is formed closer to the lens.

> **Quick Tip**
>
> Remember that a converging beam incident on a lens or mirror corresponds to a virtual object, and its distance (u) is taken as positive according to the Cartesian sign convention.

---

c. What are electromagnetic waves ? By drawing its propagation diagram, show the electric field and magnetic field in it.

Solution:

Step 1: Definition of Electromagnetic Waves:
Electromagnetic (EM) waves are disturbances consisting of time-varying, sinusoidal electric and magnetic fields that oscillate perpendicular to each other and also perpendicular to the direction of wave propagation. Key properties include:

- They are **transverse** in nature.
- They are produced by **accelerating electric charges**.
- They do **not require a material medium** for their propagation and can travel through a vacuum.
- In a vacuum, they travel at a constant speed, the **speed of light**, $c \approx 3 \times 10^8$ m/s.
- The electric field ($\vec{E}$) and magnetic field ($\vec{B}$) are always in the same phase.

Step 2: Propagation Diagram:
The diagram below shows the orientation of the electric field, magnetic field, and the direction of propagation for a plane-polarized electromagnetic wave.

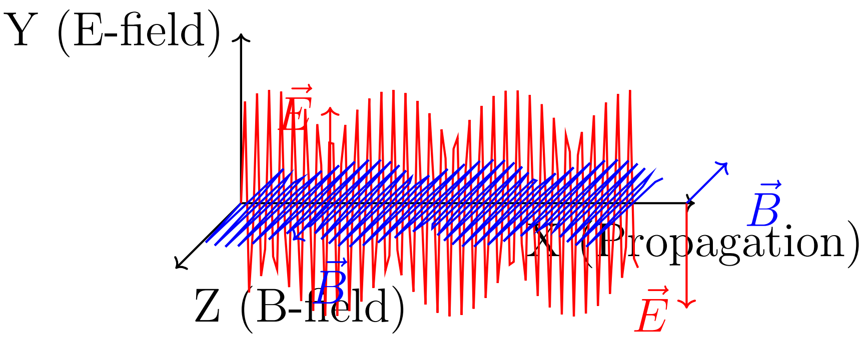


Diagram Explanation:

- The wave is propagating along the positive X-axis.
- The electric field vector $\vec{E}$ (in red) oscillates along the Y-axis.
- The magnetic field vector $\vec{B}$ (in blue) oscillates along the Z-axis.
- At every point and every instant, $\vec{E}$ is perpendicular to $\vec{B}$, and both are perpendicular to the direction of propagation.
- The vectors $\vec{E}$, $\vec{B}$, and the propagation vector $\vec{k}$ form a right-handed system ($\vec{E} \times \vec{B}$ points in the direction of propagation).

Quick Tip

When drawing the propagation diagram, ensure that the electric and magnetic fields are shown in phase, meaning they reach their maximum and zero values at the same points along the propagation axis.

---

d. Establish the formula of magnetic dipole moment.

Solution:

Step 1: Understanding the Concept:
The magnetic dipole moment is a quantity that describes the magnetic strength of a magnet or a current-carrying loop. We can establish its formula by analyzing the torque experienced by a current loop in a uniform magnetic field and drawing an analogy with the torque on an electric dipole.

Step 2: Torque on a Current Loop in a Magnetic Field:
Consider a rectangular loop PQRS with length l and breadth b, carrying a current I. Let this loop be placed in a uniform magnetic field $\vec{B}$ such that the normal to the plane of the loop makes an angle θ with $\vec{B}$.

- The magnetic forces on the arms QR and SP are equal, opposite, and collinear, so they cancel each other out and produce no torque.

- The force on arm PQ, using the formula $\vec{F} = I(\vec{l} \times \vec{B})$, is $F_{PQ} = IlB$ (directed into the page).

- The force on arm RS is $F_{RS} = IlB$ (directed out of the page).

These two forces, F_{PQ} and F_{RS}, are equal and opposite, forming a couple that exerts a torque on the loop.
The torque (τ) is given by the product of one of the forces and the perpendicular distance between their lines of action.

$$\tau = \text{Force} \times \text{Perpendicular distance}$$

The perpendicular distance between the forces is $b \sin\theta$.

$$\tau = (IlB)(b \sin\theta)$$

Since the area of the loop is $A = l \times b$, we can write:

$$\tau = IAB \sin\theta$$

If the loop has N turns, the torque is multiplied by N:

$$\tau = NIAB \sin\theta$$

Step 3: Establishing the Formula for Magnetic Dipole Moment:
This expression for torque on a current loop is analogous to the torque experienced by an electric dipole (with dipole moment $\vec{p}$) in a uniform electric field ($\vec{E}$):

$$\tau_e = pE \sin\theta$$

By comparing the two torque equations, we can define a magnetic analogue to the electric dipole moment. This quantity is called the **magnetic dipole moment**, denoted by m (or μ). Comparing $\tau = (NIA)B\sin\theta$ with $\tau_e = pE\sin\theta$, we can establish the formula for the magnitude of the magnetic dipole moment as:

$$m = NIA$$

The magnetic dipole moment is a vector quantity, $\vec{m}$. Its direction is perpendicular to the plane of the current loop, given by the right-hand thumb rule (if the fingers curl in the direction of the current, the thumb points in the direction of $\vec{m}$).
Using this vector definition, the torque equation can be written in a more general vector form:

$$\vec{\tau} = \vec{m} \times \vec{B}$$

Step 4: Final Answer:
The formula for the magnetic dipole moment of a planar current loop with N turns, area A, and carrying current I is established as $m = NIA$.

Quick Tip

The analogy between electrostatics and magnetism is very powerful. Remembering that torque $\vec{\tau} = \vec{p} \times \vec{E}$ helps in recalling and understanding the magnetic equivalent $\vec{\tau} = \vec{m} \times \vec{B}$ and the definition of the magnetic dipole moment.

Section - D

4.

a. Derive the formula for electric field due to a uniformly charged straight wire of infinite length using Gauss's law.

Solution:

Step 1: Understanding the Concept and Choosing a Gaussian Surface:
We want to find the electric field (E) at a distance r from an infinitely long straight wire with a uniform linear charge density λ (charge per unit length). Gauss's law is effective for symmetric charge distributions. Due to the cylindrical symmetry of the infinite wire, the electric field must

be directed radially outwards (for $\lambda > 0$) and its magnitude can only depend on the radial distance r. We choose a cylindrical Gaussian surface of radius r and length l, coaxial with the wire.

Step 2: Applying Gauss's Law:
Gauss's law states that the total electric flux (Φ_E) through a closed surface is equal to the net charge enclosed (q_{in}) divided by the permittivity of free space (ϵ_0).

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

The total flux through our cylindrical surface can be split into the flux through the two flat end caps (top and bottom) and the flux through the curved side wall.

$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{top}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom}} \vec{E} \cdot d\vec{A} + \int_{\text{curved}} \vec{E} \cdot d\vec{A}$$

Step 3: Calculating the Electric Flux:

- **Flux through the end caps:** On the top and bottom circular surfaces, the electric field vector $\vec{E}$ is radial and perpendicular to the wire, while the area vector $d\vec{A}$ is directed along the axis of the cylinder (upwards for the top cap, downwards for the bottom). Thus, $\vec{E}$ is perpendicular to $d\vec{A}$, and the angle between them is 90°.

  $$\vec{E} \cdot d\vec{A} = E\,dA\,\cos(90^\circ) = 0$$

  So, the flux through the top and bottom caps is zero.

- **Flux through the curved surface:** On the curved side wall, the electric field vector $\vec{E}$ is always parallel to the area vector $d\vec{A}$ (both point radially outwards). The angle between them is 0°.

  $$\int_{\text{curved}} \vec{E} \cdot d\vec{A} = \int_{\text{curved}} E\,dA\,\cos(0^\circ) = \int_{\text{curved}} E\,dA$$

  Since the magnitude of the electric field E is constant at a fixed distance r from the wire, we can take it out of the integral.

  $$\Phi_E = E \int_{\text{curved}} dA = E \times (\text{Area of curved surface}) = E(2\pi r l)$$

Step 4: Calculating the Enclosed Charge and Final Formula:
The charge enclosed by the Gaussian surface of length l is the linear charge density λ multiplied by the length l.

$$q_{in} = \lambda l$$

Now, we substitute the flux and enclosed charge back into Gauss's law:

$$E(2\pi r l) = \frac{\lambda l}{\epsilon_0}$$

The length l cancels out from both sides, which is expected as the wire is infinitely long.

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

This is the formula for the electric field due to an infinitely long charged wire. The field is inversely proportional to the distance from the wire.

> **Quick Tip**
>
> The key to using Gauss's law is choosing a "Gaussian surface" that matches the symmetry of the charge distribution. For a line of charge, a cylinder is the ideal choice. For a point charge, a sphere. For a plane of charge, a cylinder or a cuboid (pillbox).

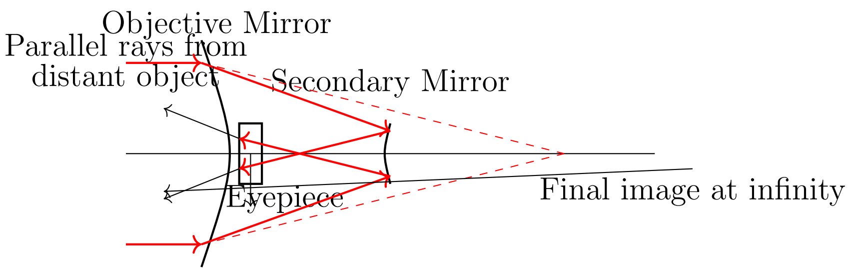
---

b. Draw a ray diagram of reflecting telescope. Explain its working.

Solution:

Step 1: Understanding the Concept:
A reflecting telescope is an optical instrument used to view distant objects. Unlike a refracting telescope which uses lenses, a reflecting telescope uses a large concave mirror as its primary light-gathering element (the objective). This design avoids chromatic aberration and allows for the construction of very large diameter objectives. The Cassegrain design is a common type.

Step 2: Ray Diagram (Cassegrain Telescope):



Step 3: Working Principle:

1. **Objective Mirror:** A large concave parabolic mirror serves as the objective. It collects parallel rays of light coming from a distant astronomical object (like a star or a planet).
2. **Formation of Intermediate Image:** The objective mirror reflects these parallel rays, causing them to converge towards its principal focus.
3. **Secondary Mirror:** Before the rays can converge to form an image, they are intercepted by a smaller, convex secondary mirror. This secondary mirror is positioned coaxially with the primary mirror.
4. **Redirection of Light:** The secondary mirror reflects the light rays back through a hole in the center of the primary objective mirror.
5. **Eyepiece and Final Image:** The rays then pass through an eyepiece (a convex lens or a system of lenses) located behind the primary mirror. The eyepiece acts as a magnifier, producing a final, highly magnified virtual image, which is then viewed by the observer.

Advantages: Reflecting telescopes are free from chromatic aberration (color distortion) because mirrors reflect all wavelengths of light in the same way. They can also be built with much larger apertures than refracting telescopes, giving them greater light-gathering power and better resolution.

> Quick Tip
>
> The key feature of a reflecting telescope is the use of a concave mirror as the objective to collect and focus light. The different types (Newtonian, Cassegrain, etc.) mainly differ in how the light is redirected by the secondary mirror to the eyepiece.

---

c. Differentiate between paramagnetic, diamagnetic and ferromagnetic substances by giving one example of each.

Solution:

Step 1: Understanding the Concept:
Materials are classified into three main types based on their behavior when placed in an external magnetic field. This behavior is determined by their atomic structure and is quantified by their magnetic susceptibility (χ).

Step 2: Detailed Comparison:
The key differences between diamagnetic, paramagnetic, and ferromagnetic substances are summarized in the table below.

Property	Diamagnetic	Paramagnetic	Ferromagnetic
Behavior in Field	Feebly repelled by a magnetic field.	Feebly attracted by a magnetic field.	Strongly attracted by a magnetic field.
Origin of Magnetism	Arises from the orbital motion of electrons, which induces a magnetic moment opposite to the applied field.	Arises from the presence of permanent atomic magnetic dipoles due to unpaired electron spins.	Arises from the formation of large domains where the permanent atomic dipoles are spontaneously aligned by strong exchange interactions.
Magnetic Susceptibility (χ)	Small and negative ($\chi \approx -10^{-5}$).	Small and positive ($\chi \approx +10^{-5}$).	Very large and positive ($\chi \approx 10^3$ to 10^5).
Temperature Dependence	Independent of temperature.	Inversely proportional to absolute temperature ($\chi \propto 1/T$), according to Curie's Law.	Susceptibility decreases with temperature. Above the Curie temperature (T_c), they become paramagnetic.
Hysteresis	Does not exhibit hysteresis.	Does not exhibit hysteresis.	Exhibits hysteresis (retains magnetization after the external field is removed).
Example	Bismuth, Copper, Water, Gold, Mercury.	Aluminum, Platinum, Oxygen, Sodium, Calcium.	Iron, Cobalt, Nickel, and their alloys like Alnico.

Quick Tip

A simple way to remember the basic behavior: - **Dia**magnetic: "Di" for diverge, they are repelled. - **Para**magnetic: "Para" for parallel, they are attracted. - **Ferro**magnetic: "Ferro" from iron, they are very strongly attracted.

---

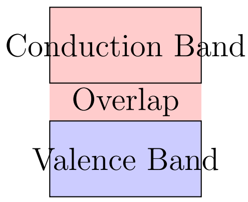
d. Explain the classification of conductors, insulators and semiconductors on the basis of energy bands.

Solution:

Step 1: Understanding Energy Band Theory:
In solids, due to the interaction between atoms, the discrete energy levels of isolated atoms broaden into continuous bands of allowed energy levels, separated by forbidden energy gaps. The highest energy band that is completely filled with electrons at absolute zero temperature (0 K) is called the **Valence Band (VB)**. The next higher permitted energy band, which may be empty or partially filled, is called the **Conduction Band (CB)**. The energy difference between the top of the valence band and the bottom of the conduction band is the **Forbidden Energy Gap (E_g)**. The electrical conductivity of a solid is determined by the size of this energy gap.

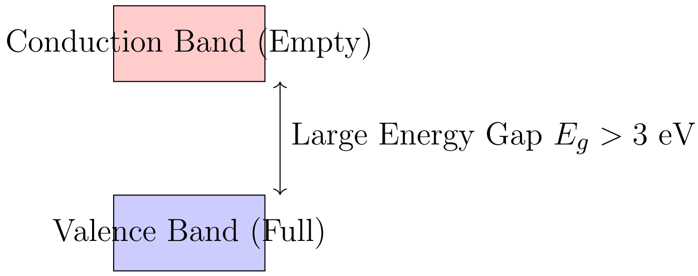
Step 2: Classification and Energy Band Diagrams:

1. Conductors (Metals):
In conductors, the valence band and the conduction band overlap. There is no forbidden energy gap between them ($E_g \approx 0$). Due to this overlap, a large number of free electrons are readily available in the conduction band to move freely throughout the material, even with a small applied electric field. This results in high electrical conductivity.



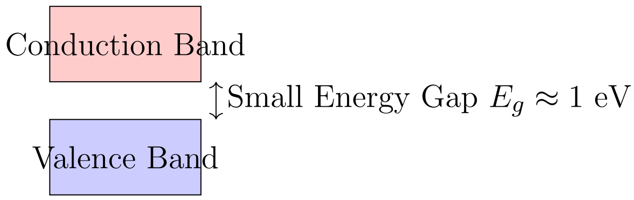
Conductor

2. Insulators:
In insulators, the valence band is completely filled with electrons, and the conduction band is completely empty. The forbidden energy gap (E_g) between the valence and conduction bands is very large (typically $E_g > 3$ eV). A very high amount of energy is required to excite an electron from the valence band to the conduction band. Therefore, at room temperature, there are virtually no free electrons in the conduction band, and the material has very low electrical conductivity.



Insulator

3. Semiconductors:
Semiconductors have an energy band structure similar to insulators, but with a much smaller forbidden energy gap (typically $0.2\,\text{eV} < E_g < 3\,\text{eV}$). For example, $E_g \approx 1.1$ eV for Silicon and $E_g \approx 0.7$ eV for Germanium. At absolute zero (0 K), the valence band is full and the conduction band is empty, so they behave as insulators. However, at room temperature, some electrons gain enough thermal energy to jump across the small gap into the conduction band, leaving behind vacancies called "holes" in the valence band. Both the electrons in the CB and the holes in the VB contribute to electrical conductivity.



Semiconductor

> Quick Tip
>
> The key distinguishing factor is the size of the forbidden energy gap (E_g): - **Conductors:** $E_g \approx 0$ (overlapping bands). - **Semiconductors:** E_g is small (around 1 eV). - **Insulators:** E_g is large (greater than 3 eV).

---

e. What is meant by self-inductance ? The self-inductance of a coil is 0.4 m Henry. The value of current flowing in it changes by 1 ampere in 0.1 second. Calculate the induced electro motive force.

Correct Answer: The induced e.m.f. is 4 mV.

Solution:

Step 1: Definition of Self-Inductance:
Self-inductance is the property of an electrical conductor or coil by which a change in the current flowing through it induces an electromotive force (e.m.f.) in the conductor itself. This induced e.m.f., often called a "back e.m.f.", always opposes the change in current that produces it (Lenz's Law). The magnetic flux (Φ_B) linked with a coil is directly proportional to the

current (I) flowing through it, so $\Phi_B \propto I$. This can be written as $\Phi_B = LI$, where L is the constant of proportionality called the self-inductance of the coil. The SI unit of self-inductance is the Henry (H).

Step 2: Key Formula for Calculation:
According to Faraday's law of electromagnetic induction, the induced e.m.f. (ϵ) in a coil is equal to the negative rate of change of magnetic flux linked with it.

$$\epsilon = -\frac{d\Phi_B}{dt}$$

Substituting $\Phi_B = LI$, we get:

$$\epsilon = -\frac{d(LI)}{dt}$$

Since L is a constant for a given coil, the formula becomes:

$$\epsilon = -L\frac{dI}{dt}$$

where $\frac{dI}{dt}$ is the rate of change of current.

Step 3: Detailed Calculation:
We are given the following values:

- Self-inductance, $L = 0.4\,\text{mH} = 0.4 \times 10^{-3}\,\text{H}$
- Change in current, $\Delta I = 1\,\text{A}$
- Time interval, $\Delta t = 0.1\,\text{s}$

First, we find the rate of change of current:

$$\frac{dI}{dt} \approx \frac{\Delta I}{\Delta t} = \frac{1\,\text{A}}{0.1\,\text{s}} = 10\,\text{A/s}$$

Now, we can calculate the magnitude of the induced e.m.f. using the formula. The negative sign indicates the direction of the e.m.f. (opposition to the change), but the question asks for the value.

$$|\epsilon| = L\left|\frac{dI}{dt}\right|$$

$$|\epsilon| = (0.4 \times 10^{-3}\,\text{H}) \times (10\,\text{A/s})$$

$$|\epsilon| = 4.0 \times 10^{-3}\,\text{V}$$

This is equal to 4 millivolts (mV).

Step 4: Final Answer:
The induced electromotive force in the coil is 4×10^{-3} volts or 4 mV.

💡 Quick Tip

Pay close attention to units. Self-inductance is often given in millihenries (mH) or microhenries (μH). Always convert these to the base unit of Henry (H) before performing calculations to ensure the final answer is in Volts.

5.

a. What is the principle of transformer ? Explain the working of step-up transformer by drawing circuit diagram.

Solution:

Step 1: Principle of a Transformer:
A transformer works on the principle of **mutual induction**. This principle states that when a changing current flows through one coil (the primary coil), it produces a changing magnetic flux in the space around it. If another coil (the secondary coil) is placed in this changing magnetic field, an electromotive force (e.m.f.) is induced across the secondary coil.

Step 2: Working of a Step-Up Transformer:
A step-up transformer is a device used to convert a low alternating voltage into a high alternating voltage.

1. **Construction:** It consists of two coils, a primary coil (P) and a secondary coil (S), wound on a common laminated soft iron core. For a step-up transformer, the number of turns in the secondary coil (N_s) is greater than the number of turns in the primary coil (N_p).

2. **Working:**

   - An alternating voltage source (V_p) is connected across the primary coil. This causes an alternating current to flow through it.

   - This alternating current produces a continuously changing magnetic flux in the soft iron core. The soft iron core concentrates and guides this magnetic flux.

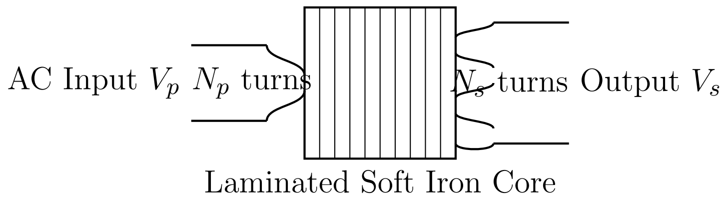
   - The changing magnetic flux gets linked with the secondary coil. According to Faraday's law of electromagnetic induction, this induces an alternating e.m.f. (V_s) across the secondary coil.

   - The magnitude of the induced e.m.f. in each coil is proportional to the number of turns in that coil. For an ideal transformer (assuming no flux leakage):

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

- Since for a step-up transformer $N_s > N_p$, it follows that $V_s > V_p$. Thus, the output voltage is increased. In an ideal transformer, to conserve power ($P_{in} = P_{out}$), the current is stepped down ($I_s < I_p$).

Step 3: Circuit Diagram:



> Quick Tip
>
> The key relationship for an ideal transformer is $\frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$. For a step-up transformer, $N_s > N_p$, so voltage is increased and current is decreased. For a step-down transformer, $N_s < N_p$, so voltage is decreased and current is increased.

---

b. What is the effect on the capacitance of a parallel plate capacitor when
(i) distance between the plates is doubled.
(ii) area of the plates is halved.
(iii) a dielectric medium is filled between the plates.

Correct Answer: (i) Halved, (ii) Halved, (iii) Increased.

Solution:

Step 1: Key Formula:
The capacitance (C) of a parallel plate capacitor with plate area A, distance between plates d, and vacuum (or air) between the plates is given by the formula:

$$C = \frac{\epsilon_0 A}{d}$$

where ϵ_0 is the permittivity of free space. We will use this formula as the baseline for all three cases.

Step 2: Analysis of each case:

(i) Distance between the plates is doubled.
The initial capacitance is $C = \frac{\epsilon_0 A}{d}$.
The new distance is $d' = 2d$.
The new capacitance, C', will be:

$$C' = \frac{\epsilon_0 A}{d'} = \frac{\epsilon_0 A}{2d} = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right) = \frac{1}{2}C$$

Effect: The capacitance is halved.

(ii) Area of the plates is halved.
The initial capacitance is $C = \frac{\epsilon_0 A}{d}$.
The new area is $A' = A/2$.
The new capacitance, C', will be:

$$C' = \frac{\epsilon_0 A'}{d} = \frac{\epsilon_0 (A/2)}{d} = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right) = \frac{1}{2}C$$

Effect: The capacitance is halved.

(iii) A dielectric medium is filled between the plates.
When a dielectric medium of dielectric constant K is completely filled between the plates, the capacitance formula becomes:

$$C' = \frac{K\epsilon_0 A}{d}$$

Since the original capacitance was $C = \frac{\epsilon_0 A}{d}$, we can write:

$$C' = K \cdot C$$

For any dielectric medium, the dielectric constant K is always greater than 1 ($K > 1$).
Effect: The capacitance increases by a factor of K.

Quick Tip

Remember the dependencies for a parallel plate capacitor: Capacitance is directly proportional to the plate area (A) and the dielectric constant (K), and inversely proportional to the distance between the plates (d).

---

c. What is meant by interference and diffraction of light ? Write the conditions of interference.

Solution:

Step 1: Definition of Interference and Diffraction:

Interference of Light:
Interference is the phenomenon in which two or more light waves, originating from coherent sources, superimpose to produce a resultant wave of greater, lower, or the same amplitude. This superposition results in a modification of the distribution of light energy in the region of superposition, creating a pattern of alternating bright and dark fringes.

- **Constructive Interference:** Occurs when waves meet in phase, resulting in a bright fringe (maximum intensity).

- **Destructive Interference:** Occurs when waves meet out of phase, resulting in a dark fringe (minimum intensity).

Diffraction of Light:
Diffraction is the phenomenon of the bending of light waves as they pass around the edges of an obstacle or through a narrow aperture. This bending causes the light to spread into the regions of the geometrical shadow. Diffraction is a characteristic property of all waves and becomes more pronounced when the size of the obstacle or aperture is comparable to the wavelength of the light.

Step 2: Conditions for Sustained Interference:
For interference to be observed and for the fringe pattern to be stable (sustained), the following conditions must be met:

1. **Coherent Sources:** The two sources of light must be coherent. This means they must emit light waves of the same frequency (or wavelength) and must have a constant phase difference between them. This is the most crucial condition.
2. **Monochromatic Light:** The light from the sources should be monochromatic (of a single wavelength) to obtain a clear and distinct interference pattern. If white light is used, the fringes will be colored and will overlap.
3. **Equal Amplitudes:** The amplitudes of the interfering waves should be equal or nearly equal. This ensures that the intensity at the dark fringes is close to zero, providing good contrast between the bright and dark fringes.
4. **Sources Close Together:** The two coherent sources must be very close to each other. This is necessary to produce interference fringes of sufficient width to be observable.

Quick Tip

The main difference to remember is that interference is due to the superposition of waves from two (or more) different coherent sources, while diffraction is due to the superposition of secondary wavelets from different parts of the same wavefront.

---

d. Establish the relation between the resistances of arms of wheatstone bridge in balance conditions.

OR

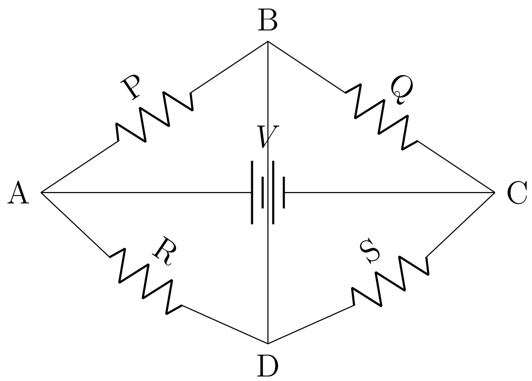
The ratio of lengths and masses of three wires of same metal are 3 : 2 : 1 and 1 : 2 : 3 respectively. Find the ratio of resistances of those wires.

Correct Answer: For Wheatstone Bridge: $\frac{P}{Q} = \frac{R}{S}$. For the OR part: 27 : 6 : 1.

Solution:

Part I: Wheatstone Bridge

Step 1: Principle and Diagram:
A Wheatstone bridge is an electrical circuit used to measure an unknown electrical resistance by balancing two legs of a bridge circuit. The circuit consists of four resistors P, Q, R, and S arranged in a quadrilateral shape. A galvanometer G is connected between two opposite junctions, and a voltage source is connected to the other two junctions.



Step 2: Balanced Condition and Derivation:
The bridge is said to be **balanced** when no current flows through the galvanometer ($I_g = 0$).
This occurs when the potential at point B is equal to the potential at point D ($V_B = V_D$).
Let the current from the source split at A into I_1 (through P) and I_2 (through R).
When the bridge is balanced ($I_g = 0$), the current I_1 also flows through Q, and the current I_2 also flows through S.
Since $V_B = V_D$, the potential drop across P must be equal to the potential drop across R.

$$V_A - V_B = V_A - V_D \implies I_1 P = I_2 R \quad \cdots (1)$$

Similarly, the potential drop across Q must be equal to the potential drop across S.

$$V_B - V_C = V_D - V_C \implies I_1 Q = I_2 S \quad \cdots (2)$$

Dividing equation (1) by equation (2), we get:

$$\frac{I_1 P}{I_1 Q} = \frac{I_2 R}{I_2 S}$$

$$\frac{P}{Q} = \frac{R}{S}$$

This is the required relation for a balanced Wheatstone bridge.

Part II: OR Question

Step 1: Key Formulas:
The resistance R of a wire is given by $R = \rho\frac{L}{A}$, where ρ is the resistivity, L is the length, and A is the cross-sectional area.
The mass m of a wire is given by $m = \text{Volume} \times \text{Density} = (A \cdot L) \cdot d$.

Step 2: Deriving the Resistance Ratio Formula:
From the mass formula, we can express the area A as $A = \frac{m}{Ld}$.
Substituting this into the resistance formula:

$$R = \rho\frac{L}{A} = \rho\frac{L}{(m/Ld)} = \frac{\rho d L^2}{m}$$

Since the three wires are made of the "same metal", their resistivity (ρ) and density (d) are the same. Therefore, the resistance is proportional to L^2/m.

$$R \propto \frac{L^2}{m}$$

So, the ratio of their resistances will be:

$$R_1 : R_2 : R_3 = \frac{L_1^2}{m_1} : \frac{L_2^2}{m_2} : \frac{L_3^2}{m_3}$$

Step 3: Calculation:
We are given the ratios:

- Ratio of lengths, $L_1 : L_2 : L_3 = 3 : 2 : 1$
- Ratio of masses, $m_1 : m_2 : m_3 = 1 : 2 : 3$

Substituting these ratios into our derived proportion:

$$R_1 : R_2 : R_3 = \frac{(3)^2}{1} : \frac{(2)^2}{2} : \frac{(1)^2}{3}$$

$$R_1 : R_2 : R_3 = \frac{9}{1} : \frac{4}{2} : \frac{1}{3}$$

$$R_1 : R_2 : R_3 = 9 : 2 : \frac{1}{3}$$

To express this ratio in integers, we multiply all parts by 3:

$$R_1 : R_2 : R_3 = 9 \times 3 : 2 \times 3 : \frac{1}{3} \times 3$$

$$R_1 : R_2 : R_3 = 27 : 6 : 1$$

Step 4: Final Answer:
The ratio of the resistances of the three wires is $27 : 6 : 1$.

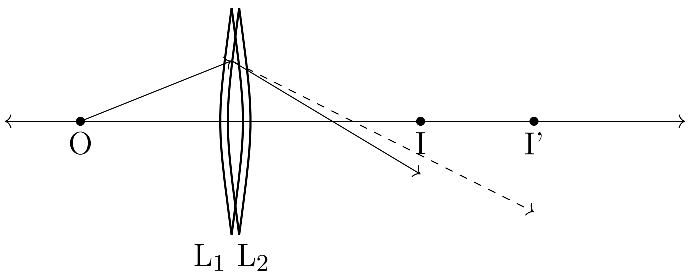
Quick Tip

For the OR part, the key is to combine the formulas for resistance and mass to find how resistance depends on length and mass. The relation $R \propto L^2/m$ is very useful for such comparison problems.

e. Establish the formula for focal length of combination of two thin lenses placed in contact.

Solution:

Step 1: Concept and Diagram:
Consider two thin convex lenses L_1 and L_2 with focal lengths f_1 and f_2 respectively, placed in contact with each other. We want to find the focal length, F, of the combination, which behaves like a single equivalent lens.



An object O is placed at a distance u from the optical center of the combination. The first lens L_1 forms a real image I' at a distance v'. This image I' acts as a virtual object for the second lens L_2, which forms the final real image I at a distance v.

Step 2: Derivation using the Lens Formula:
We apply the thin lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, to each lens separately.

For the first lens L_1:
Object distance = u
Image distance = v'
The lens formula is:

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1} \quad \cdots (1)$$

For the second lens L_2:
The image I' formed by L_1 acts as the object for L_2. Since the lenses are thin and in contact, the object distance for L_2 is v'.
Object distance = v'
Final image distance = v
The lens formula is:

$$\frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2} \quad \cdots (2)$$

Step 3: Combining the Equations:
Now, we add equation (1) and equation (2):

$$\left(\frac{1}{v'} - \frac{1}{u}\right) + \left(\frac{1}{v} - \frac{1}{v'}\right) = \frac{1}{f_1} + \frac{1}{f_2}$$

The terms $\frac{1}{v'}$ and $-\frac{1}{v'}$ cancel out:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \quad \cdots (3)$$

Now, if we replace the combination of two lenses with a single equivalent lens of focal length F, it should form the same image I at distance v for the same object O at distance u. The lens formula for this equivalent lens would be:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F} \quad \cdots (4)$$

Step 4: Final Formula:
Comparing equation (3) and equation (4), we get the formula for the focal length of the combination:

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

The power of the combination is the sum of the individual powers: $P = P_1 + P_2$.

> **Quick Tip**
>
> This formula can be extended to any number of thin lenses in contact: $\frac{1}{F_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots$. Remember to use the correct sign convention for the focal lengths of concave (negative) and convex (positive) lenses.

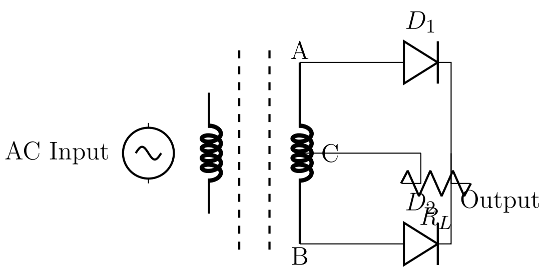
Section - E

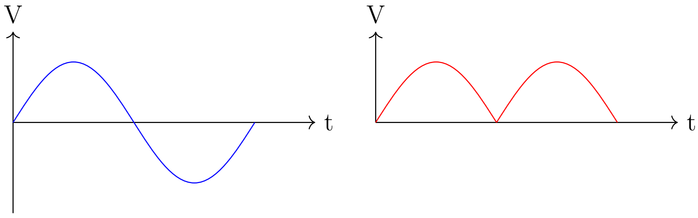
6. How is the p-n junction diode used as the full wave rectifier ? Explain its working by drawing simple circuit.

Solution:

Step 1: Understanding the Concept:
A full-wave rectifier is an electronic circuit that converts both halves (positive and negative) of an alternating current (AC) input into a pulsating direct current (DC) output. It uses two p-n junction diodes and a center-tapped transformer. The basic principle is that a p-n junction diode allows current to flow only when it is forward-biased.

Step 2: Circuit Diagram and Waveforms:





Input AC Waveform   Output DC Waveform

Step 3: Working Principle:

1. **During the positive half-cycle of the AC input:** The end A of the secondary coil is positive with respect to the center tap C, and the end B is negative. This makes the diode D_1 forward-biased and the diode D_2 reverse-biased.
2. As a result, current flows through diode D_1 and the load resistor R_L from top to bottom. No current flows through D_2.
3. **During the negative half-cycle of the AC input:** The polarity reverses. The end A becomes negative, and the end B becomes positive with respect to the center tap C.
4. This makes diode D_2 forward-biased and diode D_1 reverse-biased.
5. Current now flows through diode D_2 and the load resistor R_L. Importantly, the direction of current through R_L is again from top to bottom.

In both half-cycles, the current flows through the load resistor in the same direction. Therefore, we get a continuous, pulsating DC voltage across the load, as shown in the output waveform diagram.

Step 4: Final Answer:
A full-wave rectifier uses a center-tapped transformer and two diodes to convert both halves of the AC cycle into a unidirectional (DC) output, making it more efficient than a half-wave rectifier.

Quick Tip

The key to the full-wave rectifier is the center-tapped transformer, which effectively splits the AC input into two out-of-phase sources, allowing one diode to conduct during the positive half and the other to conduct during the negative half, with both driving the current in the same direction through the load.

---

6. (OR) Describe in brief, the α-scattering experiment. Write down about the atomic-structure from the observation obtained from the experiment.

Solution:

Step 1: Description of the α-Scattering Experiment (Geiger-Marsden Experiment):
The α-scattering experiment was performed by Hans Geiger and Ernest Marsden under the direction of Ernest Rutherford around 1909.

- **Setup:** A narrow beam of high-energy alpha particles (which are doubly-ionized helium nuclei, He^{2+}) from a radioactive source (like Radium) was directed at a very thin gold foil (about 10^{-7} m thick).

- **Detection:** The scattered alpha particles were detected using a movable detector consisting of a rotatable zinc sulfide (ZnS) screen and a microscope. When an alpha particle strikes the screen, it produces a tiny flash of light (scintillation) which can be observed.

- **Objective:** The experiment aimed to study the distribution of mass and charge within an atom by observing the deflection (scattering) of alpha particles as they passed through the gold foil.

Step 2: Observations from the Experiment:
The experiment yielded three key observations, which were contrary to the predictions of the then-prevalent Thomson's "plum pudding" model:

1. **Most particles passed undeviated:** The vast majority of the alpha particles passed straight through the gold foil without any deflection.

2. **Small deflections:** A small fraction of the alpha particles were deflected from their original path by small angles.

3. **Large deflections:** A very small number of alpha particles (about 1 in 8000) were deflected by large angles (greater than $90°$), with some even bouncing back along their incident path (a deflection of nearly $180°$).

Step 3: Conclusions about Atomic Structure:
Based on these observations, Rutherford proposed his nuclear model of the atom with the following conclusions:

1. **Most of the atom is empty space:** Since most alpha particles passed through undeflected, Rutherford concluded that the atom must be mostly empty.
2. **Existence of a Nucleus:** The fact that some positively charged alpha particles were deflected means there must be a region of concentrated positive charge within the atom that repels them. Rutherford called this central region the **nucleus**.
3. **The nucleus is small and dense:** The observation that only a very few alpha particles were deflected by large angles indicated that the nucleus must be extremely small in size compared to the atom, and that almost the entire mass of the atom is concentrated in this tiny nucleus.

4. **Electrons orbit the nucleus:** To account for the overall neutrality of the atom, he concluded that negatively charged electrons must be revolving around the positively charged nucleus, much like planets orbiting the sun.

This experiment was monumental as it disproved the Thomson model and established the modern concept of a nuclear atom.

> Quick Tip
>
> Rutherford famously said of the large-angle scattering, "It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you." This highlights how surprising and significant the discovery of the nucleus was.

---

7. What is the solenoid ? Obtain the expression for magnetic field inside a long current carrying solenoid.

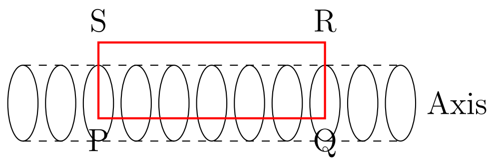
Solution:

Step 1: Definition of a Solenoid:
A **solenoid** is a long coil of wire consisting of many closely wound turns, usually in the shape of a helix. When an electric current flows through the wire, the solenoid acts as an electromagnet. A "long" or "ideal" solenoid is one whose length is much greater than its diameter. Inside a long solenoid, the magnetic field is strong, uniform, and directed along the axis of the solenoid. The magnetic field outside is negligibly weak.

Step 2: Derivation using Ampere's Circuital Law:
We can derive the expression for the magnetic field ($\vec{B}$) inside a long solenoid using Ampere's Circuital Law, which states:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

Consider a long solenoid with n turns per unit length carrying a current I. We choose a rectangular Amperian loop PQRS of length L as shown in the diagram, with side PQ inside the solenoid and parallel to its axis.



Step 3: Evaluating the Line Integral:
The line integral of the magnetic field around the closed loop PQRS is the sum of the integrals over its four segments:

$$\oint_{PQRS} \vec{B} \cdot d\vec{l} = \int_P^Q \vec{B} \cdot d\vec{l} + \int_Q^R \vec{B} \cdot d\vec{l} + \int_R^S \vec{B} \cdot d\vec{l} + \int_S^P \vec{B} \cdot d\vec{l}$$

- **Along PQ:** The magnetic field $\vec{B}$ is uniform and parallel to the path element $d\vec{l}$. So, $\int_P^Q \vec{B} \cdot d\vec{l} = \int_P^Q B\,dl\cos(0°) = B\int_P^Q dl = BL$.
- **Along QR and SP:** The magnetic field $\vec{B}$ is perpendicular to the path elements $d\vec{l}$. So, $\vec{B} \cdot d\vec{l} = B\,dl\cos(90°) = 0$. Thus, the integrals are zero.
- **Along RS:** This segment is outside the long solenoid where the magnetic field is considered to be zero ($B \approx 0$). So, the integral is zero.

Therefore, the total line integral is:

$$\oint \vec{B} \cdot d\vec{l} = BL + 0 + 0 + 0 = BL$$

Step 4: Finding the Final Expression:
The total current enclosed by the loop, I_{enclosed}, is the number of turns within the loop multiplied by the current in each turn. Number of turns in length L = nL.

$$I_{\text{enclosed}} = (nL)I$$

Applying Ampere's Law:

$$BL = \mu_0(nLI)$$

Canceling L from both sides, we get the expression for the magnetic field inside a long solenoid:

$$B = \mu_0 nI$$

Quick Tip

The formula $B = \mu_0 nI$ is for the field deep inside a long solenoid. Near the ends of a finite solenoid, the field is weaker and non-uniform. The field at the very end of a long solenoid is approximately half the value of the field at its center, i.e., $B_{end} \approx \frac{1}{2}\mu_0 nI$.

---

7. (OR) The work function of photoelectric surface is 4.0 eV. Radiation of frequency 3×10^{15} Hz is incident on it. Calculate the maximum velocity of emitted photo-electron.

Correct Answer: 1.72×10^6 m/s

Solution:

Step 1: Understanding the Concept and Key Formula:
This problem involves the photoelectric effect. We need to use Einstein's photoelectric equation to find the maximum kinetic energy of the emitted electrons (photoelectrons) and then relate this kinetic energy to their maximum velocity.
Einstein's Photoelectric Equation:

$$K_{max} = hf - \phi$$

where:

- K_{max} is the maximum kinetic energy of the photoelectron.

- h is Planck's constant (6.63×10^{-34} J·s).

- f is the frequency of the incident radiation.

- ϕ is the work function of the material.

The kinetic energy is related to velocity by $K_{max} = \frac{1}{2}m_e v_{max}^2$, where m_e is the mass of the electron (9.1×10^{-31} kg).

Step 2: Calculating the Energy of the Incident Photon:
We are given:

- Frequency, $f = 3 \times 10^{15}$ Hz.

- Work function, $\phi = 4.0$ eV.

First, calculate the energy of the incident photon (E = hf) in Joules:

$$E = (6.63 \times 10^{-34}\,\text{J·s}) \times (3 \times 10^{15}\,\text{s}^{-1})$$

$$E = 19.89 \times 10^{-19}\,\text{J}$$

To make the calculation easier, let's convert this energy to electron-volts (eV) by dividing by the elementary charge ($e = 1.6 \times 10^{-19}$ C):

$$E(\text{in eV}) = \frac{19.89 \times 10^{-19}\,\text{J}}{1.6 \times 10^{-19}\,\text{J/eV}} \approx 12.43\,\text{eV}$$

Step 3: Calculating the Maximum Kinetic Energy:
Now, use the photoelectric equation to find K_{max} in eV:

$$K_{max} = E - \phi = 12.43\,\text{eV} - 4.0\,\text{eV} = 8.43\,\text{eV}$$

Step 4: Calculating the Maximum Velocity:
To find the velocity, we must convert the kinetic energy back to Joules:

$$K_{max}(\text{in J}) = 8.43\,\text{eV} \times (1.6 \times 10^{-19}\,\text{J/eV}) = 13.488 \times 10^{-19}\,\text{J}$$

Now, use the kinetic energy formula to solve for v_{max}:

$$\frac{1}{2}m_e v_{max}^2 = 13.488 \times 10^{-19}\,\text{J}$$

$$v_{max}^2 = \frac{2 \times 13.488 \times 10^{-19}\,\text{J}}{9.1 \times 10^{-31}\,\text{kg}}$$

$$v_{max}^2 \approx 2.964 \times 10^{12}\,\text{m}^2/\text{s}^2$$

$$v_{max} = \sqrt{2.964 \times 10^{12}}\,\text{m/s} \approx 1.72 \times 10^6\,\text{m/s}$$

Step 5: Final Answer:
The maximum velocity of the emitted photoelectron is approximately 1.72×10^6 m/s.

> **Quick Tip**
>
> When dealing with photoelectric effect problems, it's often convenient to work with energies in electron-volts (eV). Remember the conversion: 1 eV = 1.6×10^{-19} J. Only convert to SI units (Joules) when you need to calculate quantities like velocity or momentum that involve mass in kilograms.

---

8. What do you understand by polarized light ? When a third polaroid is rotated between two crossed polaroids, then discuss the change in the intensity of the transmitted light.

Solution:

Step 1: Understanding Polarized Light:
An ordinary light wave is an electromagnetic wave where the electric field vector vibrates in all possible directions perpendicular to the direction of propagation. This is called **unpolarized light**.
Polarized light is light in which the electric field vector is restricted to vibrate in only a single plane. This plane is called the plane of polarization. The process of restricting the vibrations of light to a single plane is called polarization. A device used to produce polarized light, such as a polaroid, has a specific pass-axis, and it only allows the component of the electric field vector parallel to this axis to pass through.

Step 2: Two Crossed Polaroids:
Let the first polaroid be the **polarizer** and the second be the **analyzer**. When they are **crossed**, their pass-axes are perpendicular to each other (the angle between them is 90°). If unpolarized light of intensity I_0 is incident on the polarizer, the intensity of light transmitted through it is $I_1 = I_0/2$. This polarized light of intensity I_1 then falls on the analyzer. According to **Malus's Law**, the intensity of light transmitted through the analyzer (I_2) is given by $I_2 = I_1 \cos^2\theta$, where θ is the angle between the pass-axes of the polarizer and analyzer. Since the polaroids are crossed, $\theta = 90°$.

$$I_2 = I_1 \cos^2(90°) = I_1 \times 0 = 0$$

Thus, no light is transmitted through two crossed polaroids.

Step 3: A Third Polaroid Between Two Crossed Polaroids:
Now, let's introduce a third polaroid between the polarizer (P1) and the analyzer (P2). Let the pass-axis of this third polaroid (P3) make an angle θ with the pass-axis of the first polaroid (P1).

- The intensity of light after passing through the first polaroid P1 is $I_1 = I_0/2$.

- This polarized light now falls on the third polaroid P3. The angle between the pass-axes of P1 and P3 is θ. The intensity of light transmitted through P3 is I_3, given by Malus's Law:
$$I_3 = I_1 \cos^2\theta = \frac{I_0}{2}\cos^2\theta$$

- The light emerging from P3 is now polarized at an angle θ with respect to the original polarization axis. This light then falls on the analyzer P2. The pass-axis of P2 is at 90° to P1. Therefore, the angle between the pass-axes of P3 and P2 is $(90° - \theta)$.

- The final intensity of light transmitted through the analyzer P2 is I_{final}, given by Malus's Law again:
$$I_{final} = I_3 \cos^2(90° - \theta) = \left(\frac{I_0}{2}\cos^2\theta\right)(\sin^2\theta)$$

Step 4: Discussion of the Change in Intensity:
The final transmitted intensity is $I_{final} = \frac{I_0}{2}\cos^2\theta\sin^2\theta$. Using the trigonometric identity $\sin(2\theta) = 2\sin\theta\cos\theta$, we can rewrite this as:
$$I_{final} = \frac{I_0}{2}\left(\frac{\sin(2\theta)}{2}\right)^2 = \frac{I_0}{8}\sin^2(2\theta)$$

- When the third polaroid is rotated, the angle θ changes.

- The transmitted intensity is zero when $\sin(2\theta) = 0$, which occurs at $2\theta = 0°$ or $180°$, so $\theta = 0°$ or $90°$. This means if the middle polaroid is aligned with either the first or the second polaroid, no light gets through.

- The transmitted intensity is maximum when $\sin^2(2\theta) = 1$, which occurs at $2\theta = 90°$, so $\theta = 45°$.

- The maximum intensity is $I_{max} = I_0/8$.

Conclusion: As the third polaroid is rotated between the crossed polaroids, light is now transmitted. The intensity of the transmitted light varies from zero, increases to a maximum value of $I_0/8$ when the third polaroid is oriented at 45° to the first, and then decreases back to zero.

> Quick Tip
>
> Malus's Law, $I = I_{initial}\cos^2\theta$, is the fundamental principle for problems involving a series of polarizers. Always apply it sequentially, where the output intensity and polarization of one polaroid become the input for the next.

8. (OR) A lamp "50 watt and 100 volt" is to be connected to AC mains of 200 volt 50 Hz. Calculate the capacity of condenser required in series of lamp.

Correct Answer: 9.2×10^{-6} F or 9.2 μF

Solution:

Step 1: Understanding the Problem and Calculating Lamp Parameters:
The lamp is designed to work at 100 V, but the AC source is 200 V. To operate the lamp safely, we must connect a component in series to drop the excess voltage. Here, a capacitor (condenser) is used. The lamp itself behaves as a resistor.
First, let's find the current required by the lamp and its resistance from its ratings.

- Power of the lamp, $P = 50$ W
- Voltage rating of the lamp, $V_R = 100$ V

The current (RMS) that must flow through the lamp for it to work properly is:

$$I = \frac{P}{V_R} = \frac{50\,\text{W}}{100\,\text{V}} = 0.5\,\text{A}$$

This will be the current flowing through the entire series RC circuit.
The resistance of the lamp is:

$$R = \frac{V_R}{I} = \frac{100\,\text{V}}{0.5\,\text{A}} = 200\,\Omega$$

Step 2: Analyzing the Series RC Circuit:
We now have a circuit with a resistor (the lamp, R=200 Ω) and a capacitor (C) connected in series to an AC source.

- Source voltage, $V_S = 200$ V
- Circuit current, $I = 0.5$ A
- Frequency, $f = 50$ Hz

The total impedance (Z) of the circuit can be calculated as:

$$Z = \frac{V_S}{I} = \frac{200\,\text{V}}{0.5\,\text{A}} = 400\,\Omega$$

Step 3: Calculating Capacitive Reactance (X_C):
The impedance of a series RC circuit is given by the formula:

$$Z = \sqrt{R^2 + X_C^2}$$

where X_C is the capacitive reactance.
We can solve for X_C:

$$Z^2 = R^2 + X_C^2$$
$$(400)^2 = (200)^2 + X_C^2$$
$$160000 = 40000 + X_C^2$$
$$X_C^2 = 120000$$
$$X_C = \sqrt{120000} = \sqrt{40000 \times 3} = 200\sqrt{3}\,\Omega \approx 346.4\,\Omega$$

Step 4: Calculating Capacitance (C):
The capacitive reactance is related to the capacitance and frequency by the formula:

$$X_C = \frac{1}{2\pi f C}$$

Rearranging to solve for C:

$$C = \frac{1}{2\pi f X_C}$$

Substituting the values:

$$C = \frac{1}{2\pi(50)(200\sqrt{3})} = \frac{1}{20000\pi\sqrt{3}}$$
$$C \approx \frac{1}{20000 \times 3.14 \times 1.732} \approx \frac{1}{108828} \approx 9.2 \times 10^{-6}\,\mathrm{F}$$

This is equal to 9.2 microfarads (μF).

Step 5: Final Answer:
The required capacity of the condenser is 9.2×10^{-6} F or 9.2 μF.

> Quick Tip
>
> In series AC circuits, voltages across different components do not add up arithmetically. They add as vectors (phasors). The relation is $V_S^2 = V_R^2 + V_C^2$. You could also solve this problem by finding $V_C = \sqrt{V_S^2 - V_R^2}$, then $X_C = V_C/I$, and finally C from X_C.

---

9. What is meant by the concept of matter-wave of de Broglie ? Establish the relation for de Broglie wavelength in terms of kinetic energy.

Correct Answer: The relation is $\lambda = \frac{h}{\sqrt{2mK}}$. The explanation is given below.

Solution:

Step 1: The Concept of Matter-Wave of de Broglie:
In 1924, Louis de Broglie proposed a revolutionary hypothesis about the dual nature of matter. He suggested that, just as radiation (like light) exhibits both wave-like and particle-like

properties, matter should also exhibit this duality.
The concept of **matter-waves** (or de Broglie waves) states that every moving particle, regardless of its size (from an electron to a macroscopic object), has a wave associated with it. The properties of the particle (like mass and velocity) are related to the properties of its associated wave (like wavelength). This idea unified the physics of matter and energy, suggesting that the wave-particle duality is a universal principle of nature.

Step 2: Establishing the de Broglie Wavelength Relation:
De Broglie combined two fundamental equations from physics:

1. Planck's quantum theory for the energy of a photon: $E = hf$, where E is energy, h is Planck's constant, and f is frequency.
2. Einstein's mass-energy equivalence from the theory of relativity: $E = mc^2$, where m is mass and c is the speed of light.

For a photon, which has momentum $p = E/c$, combining these gives $p = hf/c$. Since $c = f\lambda$, this becomes $p = h/\lambda$.
De Broglie postulated that this relationship between momentum and wavelength should hold true for material particles as well. For a particle of mass m moving with velocity v, the momentum is $p = mv$. Thus, the de Broglie wavelength (λ) associated with a moving particle is given by:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Step 3: Relation in Terms of Kinetic Energy:
We now need to express the momentum (p) of the particle in terms of its kinetic energy (K). The kinetic energy of a non-relativistic particle is given by:

$$K = \frac{1}{2}mv^2$$

The momentum is given by:

$$p = mv$$

We can write the kinetic energy as:

$$K = \frac{1}{2m}(m^2v^2) = \frac{p^2}{2m}$$

Rearranging this equation to solve for momentum p:

$$p^2 = 2mK$$

$$p = \sqrt{2mK}$$

Now, we substitute this expression for momentum back into the de Broglie wavelength formula:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

Step 4: Final Answer:
This is the required relation, showing that the de Broglie wavelength of a particle is inversely proportional to the square root of its kinetic energy.

> 💡 Quick Tip
>
> This formula $\lambda = h/\sqrt{2mK}$ is extremely useful. It can be further adapted for a charged particle (charge q) accelerated through a potential difference V, where its kinetic energy is $K = qV$. The formula then becomes $\lambda = h/\sqrt{2mqV}$.

---

9. (OR) Explain binding energy. If 0.3% be the mass defect in a nuclear fusion reaction then how much energy will be released in fusion reaction of mass of 1 kg ?

Correct Answer: 2.7×10^{14} J of energy will be released.

Solution:

Step 1: Explanation of Binding Energy:

- **Mass Defect:** It is observed that the mass of a stable nucleus is always slightly less than the sum of the masses of its constituent protons and neutrons (nucleons) when they are free. This difference in mass is called the **mass defect (Δm)**.

$$\Delta m = [Zm_p + (A - Z)m_n] - M_{nucleus}$$

  where Z is the atomic number, A is the mass number, m_p is the mass of a proton, m_n is the mass of a neutron, and $M_{nucleus}$ is the actual mass of the nucleus.

- **Binding Energy (B.E.):** According to Einstein's mass-energy equivalence principle ($E = mc^2$), this "lost" mass (Δm) is converted into an equivalent amount of energy. This energy holds the nucleons together inside the nucleus and is known as the **binding energy** of the nucleus.

$$B.E. = \Delta m \cdot c^2$$

  Therefore, binding energy can be defined as the energy required to break a nucleus apart into its individual protons and neutrons. A higher binding energy per nucleon indicates a more stable nucleus.

Step 2: Understanding the Numerical Problem:
In a nuclear reaction (like fusion), energy is released if the total mass of the products is less than the total mass of the reactants. This loss of mass, which is the mass defect of the reaction, is converted into the released energy. We are given the percentage of the initial mass that is converted into energy.

Step 3: Calculating the Mass Defect:

- Mass of the substance undergoing fusion, $m = 1$ kg.
- The mass defect is given as 0.3% of this mass.

We calculate the actual mass defect, Δm:

$$\Delta m = 0.3\% \text{ of } 1\,\text{kg} = \frac{0.3}{100} \times 1\,\text{kg}$$

$$\Delta m = 0.003\,\text{kg}$$

Step 4: Calculating the Energy Released:
Now, we use Einstein's mass-energy equivalence formula to find the energy (E) released:

$$E = \Delta m \cdot c^2$$

where c is the speed of light in vacuum, $c = 3 \times 10^8$ m/s. Substituting the values:

$$E = (0.003\,\text{kg}) \times (3 \times 10^8\,\text{m/s})^2$$

$$E = 0.003 \times (9 \times 10^{16})\,\text{J}$$

$$E = 0.027 \times 10^{16}\,\text{J}$$

Writing this in standard scientific notation:

$$E = 2.7 \times 10^{14}\,\text{J}$$

Step 5: Final Answer:
The energy released in the fusion reaction of 1 kg of mass is 2.7×10^{14} Joules.

Quick Tip

The formula $E = mc^2$ shows that even a very small amount of mass can be converted into a tremendous amount of energy because the conversion factor, c^2, is extremely large (9×10^{16}). This is the principle behind nuclear power and nuclear weapons.
