UP Board Class 12 Mathematics Question Paper 2026 with Solutions

Time Allowed :3 Hours	**Maximum Marks :**100	**Total Questions :**38

General Instructions

1. The paper is divided into two sections – Section A (Compulsory) and Section B (Elective).
2. Section A is compulsory for all candidates and generally includes objective-type questions, short answer questions, and long answer questions from the prescribed syllabus.
3. In Section A, candidates are required to answer all questions. The questions will cover topics from ancient, medieval, and modern history as prescribed by the syllabus.
4. Section B consists of elective questions. Candidates are required to attempt questions from the chosen topic according to the provided options.
5. The questions in Section A will be in the form of multiple-choice, short answer, and essay-type questions.
6. Answers to all questions must be written in neat and legible handwriting. Candidates must adhere strictly to the word limit mentioned in the questions.
7. Use of unfair means or electronic devices during the examination is strictly prohibited.
8. Candidates must ensure that they write their answers in the correct format, following the instructions given for each section.

1. The value of $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}}$ will be

(A) 0
(B) $\frac{\pi}{2}$
(C) $\frac{\pi}{4}$
(D) $\frac{\pi}{8}$

Correct Answer: (C) $\frac{\pi}{4}$

Solution:

Step 1: Understanding the Concept:
This problem uses the property of definite integrals: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.
By applying this property, we can transform the integrand and add the two forms to simplify the expression.

Step 2: Key Formula or Approach:
1. Convert $\tan x$ into $\frac{\sin x}{\cos x}$ to simplify the radical expression.

2. Use the property: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

Step 3: Detailed Explanation:
Let $I = \int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}}\,dx$.
Rewriting $\sqrt{\tan x}$ as $\frac{\sqrt{\sin x}}{\sqrt{\cos x}}$, we get:

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\,dx \quad \ldots(i)$$

Using the property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$:

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos(\pi/2 - x)}}{\sqrt{\cos(\pi/2 - x)} + \sqrt{\sin(\pi/2 - x)}}\,dx$$

Since $\cos(\pi/2 - x) = \sin x$ and $\sin(\pi/2 - x) = \cos x$:

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx \quad \ldots(ii)$$

Adding equations (i) and (ii):

$$2I = \int_0^{\pi/2} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}\,dx$$

$$2I = \int_0^{\pi/2} 1\,dx$$

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0$$

$$I = \frac{\pi}{4}$$

Step 4: Final Answer:
The value of the integral is $\frac{\pi}{4}$.

> **Quick Tip**
>
> For integrals of the type $\int_0^{\pi/2} \frac{1}{1+\tan^n x}\,dx$ or $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+f(\cos x)}\,dx$, the answer is always half of the upper limit, which is $\frac{\pi}{4}$.

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2. The degree of differential equation $9\frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{3}}$ is

(A) 1
(B) 6
(C) 3
(D) 2

Correct Answer: (C) 3

Solution:

Step 1: Understanding the Concept:
The degree of a differential equation is the power of the highest order derivative when the equation is expressed as a polynomial in derivatives.
Before finding the degree, all fractional powers or radicals involving derivatives must be removed.

Step 2: Key Formula or Approach:
Identify the highest order derivative and raise the entire equation to a power that clears the fraction 1/3.

Step 3: Detailed Explanation:
The given equation is:

$$9\frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{3}}$$

The highest order derivative present is $\frac{d^2y}{dx^2}$ (Order = 2).
To find the degree, we eliminate the fractional power by cubing both sides:

$$\left(9\frac{d^2y}{dx^2}\right)^3 = \left[\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{3}}\right]^3$$

$$729\left(\frac{d^2y}{dx^2}\right)^3 = 1 + \left(\frac{dy}{dx}\right)^2$$

The highest order derivative is $\frac{d^2y}{dx^2}$, and its power in this polynomial form is 3.

Step 4: Final Answer:
The degree of the differential equation is 3.

> Quick Tip
>
> Always simplify the equation into polynomial form with respect to derivatives before determining the degree. Radical signs or fractional exponents must be eliminated.

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3. The value of expression $\hat{i} \cdot \hat{i} - \hat{j} \cdot \hat{j} + \hat{k} \times \hat{k}$ is

(A) 0
(B) 1

(C) 2
(D) 3

Correct Answer: (A) 0

Solution:

Step 1: Understanding the Concept:
This question evaluates the properties of unit vector products.
Dot products of identical unit vectors result in scalar values, whereas cross products of identical vectors result in a zero vector.

Step 2: Key Formula or Approach:
1. $\hat{a} \cdot \hat{a} = |\hat{a}||\hat{a}| \cos(0^\circ) = 1$.
2. $\vec{a} \times \vec{a} = \vec{0}$ because the angle between them is 0° and $\sin(0^\circ) = 0$.

Step 3: Detailed Explanation:
Given expression: $\hat{i} \cdot \hat{i} - \hat{j} \cdot \hat{j} + \hat{k} \times \hat{k}$.
- The dot product $\hat{i} \cdot \hat{i} = 1$.
- The dot product $\hat{j} \cdot \hat{j} = 1$.
- The cross product $\hat{k} \times \hat{k} = \vec{0}$.
Substituting these into the expression:

$$1 - 1 + \vec{0} = 0$$

Step 4: Final Answer:
The numerical value of the expression is 0.

> Quick Tip
>
> Remember: $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$ and $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$.

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4. The modulus function $f : \mathbb{R} \to \mathbb{R}^+$ given by $f(x) = |x|$ is

(A) one-one and onto
(B) many-one and onto
(C) one-one but not onto
(D) neither one-one nor onto

Correct Answer: (B) many-one and onto

Solution:

Step 1: Understanding the Concept:
A function is one-one if unique inputs yield unique outputs. It is onto if the range equals the

codomain.
The modulus function $f(x) = |x|$ maps all real numbers to non-negative real numbers.

Step 2: Detailed Explanation:
1. **Check for One-one:**
Let $x_1 = 1$ and $x_2 = -1$. Both are real numbers in domain $\mathbb{R}$.
$f(1) = |1| = 1$
$f(-1) = |-1| = 1$
Since $f(1) = f(-1)$ but $1 \neq -1$, the function is **many-one**.
2. **Check for Onto:**
The codomain is given as $\mathbb{R}^+$, which represents non-negative real numbers $[0, \infty)$.
The range of $f(x) = |x|$ is also $[0, \infty)$.
Since Range = Codomain, the function is **onto**.

Step 3: Final Answer:
The function is many-one and onto.

Quick Tip

The nature of a function (one-one/onto) depends heavily on the defined Domain and Codomain. If the codomain was $\mathbb{R}$, this function would be "not onto".

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5. A relation $R = \{(a, b) : a = b - 1, b \geq 3\}$ is defined on set N, then

(A) $(2, 4) \in R$
(B) $(4, 5) \in R$
(C) $(4, 6) \in R$
(D) $(1, 3) \in R$

Correct Answer: (B) $(4, 5) \in R$

Solution:

Step 1: Understanding the Concept:
An ordered pair (a, b) belongs to the relation R if it satisfies both conditions: $a = b-1$ and $b \geq 3$.

Step 2: Detailed Explanation:
Let's test each option:
(A) $(2, 4)$: Here $b = 4$. Condition $b \geq 3$ is satisfied. But $a = 4 - 1 = 3$. Given $a = 2$, so $(2, 4) \notin R$.
(B) $(4, 5)$: Here $b = 5$. Condition $b \geq 3$ is satisfied. Check $a = 5 - 1 = 4$. This matches. So, $(4, 5) \in R$.
(C) $(4, 6)$: Here $b = 6$. Condition $b \geq 3$ is satisfied. Check $a = 6 - 1 = 5$. Given $a = 4$, so $(4, 6) \notin R$.

(D) $(1, 3)$: Here $b = 3$. Condition $b \geq 3$ is satisfied. Check $a = 3 - 1 = 2$. Given $a = 1$, so $(1, 3) \notin R$.

Step 3: Final Answer:
The correct pair is $(4, 5)$.

> Quick Tip
>
> For relation questions, substitute the given values back into the algebraic conditions provided in the set definition.

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6. Prove that the function $f(x) = |x|$, is continuous at $x = 0$.

Correct Answer:

Solution:

Step 1: Understanding the Concept:
A function is continuous at $x = c$ if $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$.

Step 2: Detailed Explanation:
The modulus function is defined as:

$$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

1. **Value at $x = 0$:**
$f(0) = |0| = 0$.
2. **Left Hand Limit (LHL):**

$$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} |-h| = \lim_{h \to 0} h = 0$$

3. **Right Hand Limit (RHL):**

$$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} |h| = \lim_{h \to 0} h = 0$$

Since LHL = RHL = $f(0) = 0$, the function is continuous at $x = 0$.

Step 3: Final Answer:
The function is proven to be continuous at $x = 0$.

Quick Tip

Graphically, a function is continuous if its graph can be drawn without lifting the pen. The graph of $|x|$ has a V-shape with no breaks at $x = 0$.

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7. Find the degree of the differential equation $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\left(\frac{dy}{dx}\right) = 2$

Correct Answer: 1

Solution:

Step 1: Understanding the Concept:
The degree is the highest power of the highest order derivative in a differential equation, provided it is a polynomial in its derivatives.

Step 2: Detailed Explanation:
The given equation is $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\left(\frac{dy}{dx}\right) = 2$.
1. Identifying the highest order derivative: $\frac{d^2y}{dx^2}$ (Order 2).
2. The entire expression is a polynomial in derivatives $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$.
3. Look at the power of the highest order derivative $\frac{d^2y}{dx^2}$. It is raised to the power 1.

Step 5: Final Answer:
The degree of the differential equation is 1.

Quick Tip

Always identify the order first. The degree is specifically linked to the term containing the highest order derivative.

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8. If $P(A) = 0.12, P(B) = 0.15$ **and** $P(B/A) = 0.18$**, then find the value of** $P(A \cap B)$**.**

Correct Answer: 0.0216

Solution:

Step 1: Understanding the Concept:
Conditional probability $P(B/A)$ is the probability of event B occurring given that event A has already occurred.

Step 2: Key Formula or Approach:

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \implies P(A \cap B) = P(B/A) \cdot P(A)$$

Step 3: Detailed Explanation:
Given values:
$P(A) = 0.12$
$P(B) = 0.15$ (Extra information for this specific calculation)
$P(B/A) = 0.18$
Using the formula for intersection:

$$P(A \cap B) = 0.18 \times 0.12$$
$$P(A \cap B) = 0.0216$$

Step 4: Final Answer:
The value of $P(A \cap B)$ is 0.0216.

Quick Tip

Be careful with decimals during multiplication. Multiplying 18×12 gives 216; placing 4 decimal points yields 0.0216.

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9. Find the angle between the vectors $-2\hat{i} + \hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

Correct Answer: $\cos^{-1}\left(-\frac{5}{14}\right)$

Solution:

Step 1: Understanding the Concept:
The angle θ between two vectors $\vec{a}$ and $\vec{b}$ can be found using the dot product formula.

Step 2: Key Formula or Approach:

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

Step 3: Detailed Explanation:
Let $\vec{a} = -2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$.
1. **Dot Product:**

$$\vec{a} \cdot \vec{b} = (-2)(3) + (1)(-2) + (3)(1) = -6 - 2 + 3 = -5$$

2. **Magnitude of $\vec{a}$:**

$$|\vec{a}| = \sqrt{(-2)^2 + 1^2 + 3^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$$

3. **Magnitude of $\vec{b}$:**

$$|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9+4+1} = \sqrt{14}$$

4. **Angle calculation:**

$$\cos\theta = \frac{-5}{\sqrt{14}\cdot\sqrt{14}} = -\frac{5}{14}$$
$$\theta = \cos^{-1}\left(-\frac{5}{14}\right)$$

Step 4: Final Answer:
The angle is $\cos^{-1}\left(-\frac{5}{14}\right)$.

Quick Tip
If $\vec{a}\cdot\vec{b}$ is negative, the angle θ will be obtuse ($90^\circ < \theta < 180^\circ$).

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10. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be functions defined by $f(x) = \cos x$ and $g(x) = 3x^2$ respectively, then prove that $g \circ f \neq f \circ g$.

Correct Answer:

Solution:

Step 1: Understanding the Concept:
Function composition is generally not commutative, meaning $g(f(x))$ is usually not equal to $f(g(x))$.

Step 2: Detailed Explanation:
Given $f(x) = \cos x$ and $g(x) = 3x^2$.
1. Calculate $(g \circ f)(x)$:

$$(g \circ f)(x) = g(f(x)) = g(\cos x) = 3(\cos x)^2 = 3\cos^2 x$$

2. Calculate $(f \circ g)(x)$:

$$(f \circ g)(x) = f(g(x)) = f(3x^2) = \cos(3x^2)$$

Comparing the results:

$$3\cos^2 x \neq \cos(3x^2)$$

For instance, at $x = 0$:
$(g \circ f)(0) = 3(\cos 0)^2 = 3(1) = 3$

$(f \circ g)(0) = \cos(3(0)^2) = \cos 0 = 1$
Since $3 \neq 1$, $g \circ f \neq f \circ g$.

Step 3: Final Answer:
It is proven that $g \circ f \neq f \circ g$.

Quick Tip

To disprove equality of functions, it is sufficient to find one value of x where the outputs differ.

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11. Find the general solution of differential equation $ydx + (x - y^2)dy = 0$.

Correct Answer: $xy = \frac{y^3}{3} + C$

Solution:

Step 1: Understanding the Concept:
This equation can be rearranged into the form of a linear differential equation of the type $\frac{dx}{dy} + Px = Q$, where P and Q are functions of y only.

Step 2: Detailed Explanation:
Given: $ydx + (x - y^2)dy = 0$.
Divide by dy and rearrange:

$$y\frac{dx}{dy} + x - y^2 = 0$$

$$y\frac{dx}{dy} + x = y^2$$

Divide by y:

$$\frac{dx}{dy} + \frac{1}{y}x = y$$

This is a linear differential equation in x.
- $P(y) = \frac{1}{y}$, $Q(y) = y$.
- **Integrating Factor (IF):**

$$\text{IF} = e^{\int P(y)dy} = e^{\int \frac{1}{y}dy} = e^{\ln y} = y$$

- **General Solution:**

$$x \cdot (\text{IF}) = \int Q(y) \cdot (\text{IF})dy + C$$

$$x \cdot y = \int y \cdot y dy + C$$

$$xy = \int y^2 dy + C$$

$$xy = \frac{y^3}{3} + C$$

Step 3: Final Answer:
The general solution is $xy = \frac{y^3}{3} + C$.

Quick Tip

If an equation is not linear in y, try checking if it is linear in x by evaluating $\frac{dx}{dy}$.

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12. Prove that (4, 4, 2), (3, 5, 2) and (-1, -1, 2) are vertices of a right angle triangle.

Correct Answer:

Solution:

Step 1: Understanding the Concept:
A triangle is right-angled if the squares of its side lengths satisfy the Pythagoras theorem $(a^2 + b^2 = c^2)$.

Step 2: Detailed Explanation:
Let $A = (4, 4, 2)$, $B = (3, 5, 2)$, and $C = (-1, -1, 2)$.
Calculate the square of lengths of sides using distance formula $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$.
1. $AB^2 = (3 - 4)^2 + (5 - 4)^2 + (2 - 2)^2 = (-1)^2 + (1)^2 + 0^2 = 1 + 1 = 2$.
2. $BC^2 = (-1 - 3)^2 + (-1 - 5)^2 + (2 - 2)^2 = (-4)^2 + (-6)^2 + 0^2 = 16 + 36 = 52$.
3. $AC^2 = (-1 - 4)^2 + (-1 - 4)^2 + (2 - 2)^2 = (-5)^2 + (-5)^2 + 0^2 = 25 + 25 = 50$.
Observe that:

$$AB^2 + AC^2 = 2 + 50 = 52$$

$$BC^2 = 52$$

Since $AB^2 + AC^2 = BC^2$, the triangle is right-angled at A.

Step 3: Final Answer:
The vertices form a right-angled triangle.

Quick Tip

Check if the dot product of two vectors (e.g., $\vec{AB} \cdot \vec{AC}$) is zero. This is a faster way to prove perpendicularity.

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13. If three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfying the condition $\vec{a} + \vec{b} + \vec{c} = 0$. If $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 2$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

Correct Answer: -14.5

Solution:

Step 1: Understanding the Concept:
This involves the property $|\vec{a}+\vec{b}+\vec{c}|^2$. Squaring a sum of three vectors leads to terms involving magnitudes and pairwise dot products.

Step 2: Key Formula or Approach:

$$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

Step 3: Detailed Explanation:
Given: $\vec{a} + \vec{b} + \vec{c} = 0$.
Take the square of the magnitude on both sides:

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 0^2$$
$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

Substitute the magnitudes $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 2$:

$$3^2 + 4^2 + 2^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$
$$9 + 16 + 4 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$
$$29 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$
$$2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -29$$
$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -14.5$$

Step 4: Final Answer:
The required value is -14.5.

Quick Tip

Whenever the sum of vectors is zero, squaring the expression is usually the key to finding dot product relations.

14. The radius of an air bubble is increasing at the rate of $\frac{1}{2}$ cm/s. At what rate is the volume of the bubble increasing while the radius is 1 cm?

Correct Answer: 2π cm^3/s

Solution:

Step 1: Understanding the Concept:
This is an application of derivatives problem concerning rates of change. The bubble is spherical.

Step 2: Key Formula or Approach:
Volume of a sphere $V = \frac{4}{3}\pi r^3$.
Differentiate with respect to time t: $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$.

Step 3: Detailed Explanation:
Given: $\frac{dr}{dt} = \frac{1}{2}$ cm/s and current radius $r = 1$ cm.
We know, $V = \frac{4}{3}\pi r^3$.
Differentiating both sides with respect to t:

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot (3r^2) \cdot \frac{dr}{dt}$$
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Substitute the values $r = 1$ and $\frac{dr}{dt} = 0.5$:

$$\frac{dV}{dt} = 4\pi(1)^2 \cdot \left(\frac{1}{2}\right)$$
$$\frac{dV}{dt} = 2\pi$$

Step 4: Final Answer:
The rate of increase of volume is 2π cm^3/s.

Quick Tip

Ensure units are consistent. If radius is in cm/s, volume rate must be in cm^3/s.

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15. Show that the function $f(x) = 7x^2 - 3$ is an increasing function when $x > 0$.

Correct Answer:

Solution:

Step 1: Understanding the Concept:
A function is increasing in an interval if its first derivative $f'(x)$ is positive throughout that interval.

Step 2: Detailed Explanation:
The function is $f(x) = 7x^2 - 3$.
1. Find the first derivative $f'(x)$:

$$f'(x) = \frac{d}{dx}(7x^2 - 3) = 14x$$

2. Test the sign of $f'(x)$ for $x > 0$:
For any value of x such that $x > 0$, the product $14 \cdot x$ will also be greater than 0.
Since $f'(x) > 0$ for all $x \in (0, \infty)$, the function $f(x)$ is strictly increasing in this range.

Step 3: Final Answer:
The function $f(x)$ is confirmed to be increasing for $x > 0$.

> Quick Tip
>
> For quadratic functions $ax^2 + bx + c$, the function is increasing on one side of its vertex and decreasing on the other. For $7x^2 - 3$, the vertex is at $x = 0$.
