UP Board 10 Mathematics 2024 Code : 822 (IB) Question Paper with Solutions

Time Allowed :3 Hours 15 Minutes	**Maximum Marks :**70

General Instructions

Instructions:

1. All questions are compulsory.
2. This question paper has two sections 'A' and 'B'.
3. Section 'A' contains **20** multiple choice type questions of 1 mark each that has to be answered on OMR Answer Sheet by darkening completely the correct circle with **blue** or **black ballpoint pen.**
4. After giving answer on OMR Answer Sheet do not cut or use eraser, whitener etc.
5. Section **'B'** contains descriptive type questions of **50** marks.
6. Total **5** questions are there in this section.
7. In the beginning of each question it has been mentioned how many parts of it are to be attempted.
8. Marks allotted to each question are mentioned against it.
9. Start from the first question and go up to the last question. Do not waste your time on the question you cannot solve.
10. If you need place for rough work, do it on left page of your answer book and cross (x) the page. Do not write the solution on that page.
11. Do not rub off the lines constructed in a question of construction. Do write the steps of construction in brief, if asked.
12. Draw neat and correct figure in solution of a question wherever it is necessary, otherwise in its absence the solution will be treated incomplete and wrong.

Section - A
(Objective Type Questions)

1. If HCF of 65 and 117 is expressed as $65p - 117$, then the value of p will be:

(A) 1
(B) 2
(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

$$\text{HCF}(65, 117) = 13$$

So, the equation becomes:

$$65p - 117 = 13 \quad \Rightarrow \quad 65p = 130 \quad \Rightarrow \quad p = 2.$$

> **Quick Tip**
>
> To find the HCF of two numbers, use the prime factorization method or the Euclidean algorithm.

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2. The value of $\frac{2\tan 45^\circ}{1+\tan^2 45^\circ}$ will be:

(A) -1
(B) 0
(C) 1
(D) 2

Correct Answer: (C) 1

Solution:
Since $\tan 45^\circ = 1$, the expression simplifies to:

$$\frac{2 \cdot 1}{1 + 1^2} = \frac{2}{2} = 1.$$

> **Quick Tip**
>
> For trigonometric identities, remember that $\tan 45^\circ = 1$, and simplify the expression accordingly.

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3. L.C.M. of 15, 18, and 24 is:

(A) 90
(B) 120
(C) 240
(D) 360

Correct Answer: (B) 120

Solution:
The prime factorizations are:

$$15 = 3 \times 5, \quad 18 = 2 \times 3^2, \quad 24 = 2^3 \times 3.$$

The L.C.M. is obtained by taking the highest powers of all primes:

$$\text{L.C.M.} = 2^3 \times 3^2 \times 5 = 120.$$

> 💡 Quick Tip
>
> To find the LCM, take the highest powers of all prime factors appearing in the numbers.

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4. Simplest form of $\frac{148}{185}$ is:

(A) $\frac{4}{5}$
(B) $\frac{5}{7}$
(C) $\frac{5}{4}$
(D) $\frac{7}{5}$

Correct Answer: (A) $\frac{4}{5}$

Solution:
The greatest common divisor of 148 and 185 is 37. Therefore,

$$\frac{148}{185} = \frac{148 \div 37}{185 \div 37} = \frac{4}{5}.$$

> 💡 Quick Tip
>
> To simplify a fraction, divide both the numerator and denominator by their GCD (Greatest Common Divisor).

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5. For which value of p, the equations $3x - y + 8 = 0$ and $6x - py = 16$ represent coincident lines?

(A) -2
(B) 2
(C) $\frac{1}{2}$
(D) $-\frac{1}{2}$

Correct Answer: (B) 2

Solution:
For the lines to be coincident, the ratios of the coefficients of x, y, and the constant term must be equal.

The first equation is:

$$3x - y + 8 = 0 \quad \Rightarrow \quad 3x - y = -8.$$

The second equation is:

$$6x - py = 16.$$

For the lines to be coincident, the following condition must hold:

$$\frac{3}{6} = \frac{-1}{-p} = \frac{-8}{16}.$$

Step 1: Solve for p
From:

$$\frac{3}{6} = \frac{-8}{16},$$

we confirm both ratios simplify to:

$$\frac{1}{2}.$$

Now, setting:

$$\frac{-1}{-p} = \frac{1}{2},$$

we solve for p:

$$p = 2.$$

Thus, the required value of p is 2.

Quick Tip

For two lines to be coincident, their corresponding coefficients must be in the same ratio:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

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6. If the lines represented by $3x + 2py = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of p will be:

(A) $\frac{15}{4}$
(B) $-\frac{4}{5}$
(C) $\frac{3}{5}$
(D) $\frac{5}{3}$

Correct Answer: (A) $\frac{15}{4}$

Solution:
For two lines to be parallel, their slopes must be equal. The slope of a line of the form $Ax + By + C = 0$ is given by:

$$\text{Slope} = -\frac{A}{B}.$$

For the first equation $3x + 2py = 2$, rewriting in standard form:

$$3x + 2py = 0 \quad \Rightarrow \quad \text{Slope} = -\frac{3}{2p}.$$

For the second equation $2x + 5y + 1 = 0$, rewriting in standard form:

$$2x + 5y = 0 \quad \Rightarrow \quad \text{Slope} = -\frac{2}{5}.$$

Since the lines are parallel, their slopes must be equal:

$$\frac{3}{2p} = \frac{2}{5}.$$

Step 1: Solve for p

Cross multiplying:

$$3 \times 5 = 2p \times 2.$$

$$15 = 4p.$$

$$p = \frac{15}{4}.$$

Thus, the required value of p is $\frac{15}{4}$.

Quick Tip

For parallel lines, the slopes must be equal. Use the slope formula $-\frac{A}{B}$ for each equation and set them equal to solve for the unknown variable.

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7. If $15\cot\theta = 8$, then the value of $\sin\theta$ will be:

(A) $\frac{17}{15}$
(B) $\frac{15}{8}$
(C) $\frac{8}{17}$
(D) $\frac{15}{17}$

Correct Answer: (D) $\frac{15}{17}$

Solution:
We know that $\cot\theta = \frac{1}{\tan\theta}$, and $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

Given $15\cot\theta = 8$, we have:

$$\cot\theta = \frac{8}{15} \quad \Rightarrow \quad \frac{\cos\theta}{\sin\theta} = \frac{8}{15}.$$

This gives $\cos\theta = \frac{8}{15}\sin\theta$.

Now using the identity $\sin^2\theta + \cos^2\theta = 1$, we substitute $\cos\theta = \frac{8}{15}\sin\theta$:

$$\sin^2\theta + \left(\frac{8}{15}\sin\theta\right)^2 = 1.$$

Expanding:

$$\sin^2\theta + \frac{64}{225}\sin^2\theta = 1.$$

$$\sin^2\theta\left(1 + \frac{64}{225}\right) = 1.$$

$$\sin^2\theta \times \frac{289}{225} = 1.$$

$$\sin^2\theta = \frac{225}{289}.$$

$$\sin\theta = \frac{15}{17}.$$

Thus, the required value of $\sin\theta$ is $\frac{15}{17}$.

Quick Tip

Use trigonometric identities like $\sin^2\theta + \cos^2\theta = 1$ to find unknown values. Express cotangent in terms of sine and cosine to simplify calculations.

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8. If $\triangle ABC \sim \triangle XYZ$ and $\angle A = 75°$, $\angle Y = 57°$, then the value of $\angle C$ will be:

(A) $58°$
(B) $48°$
(C) $45°$
(D) $54°$

Correct Answer: (B) $48°$

Solution:
Since $\triangle ABC \sim \triangle XYZ$, the corresponding angles are equal. We know that the sum of the angles in a triangle is $180°$:

$$\angle A + \angle B + \angle C = 180°.$$

From similarity, $\angle B = \angle Y = 57°$, and we are given $\angle A = 75°$. Substituting the values:

$$75° + 57° + \angle C = 180°.$$

Solving for $\angle C$:

$$\angle C = 180° - (75° + 57°).$$

$$\angle C = 180° - 132° = 48°.$$

Thus, the required value of $\angle C$ is $48°$.

Quick Tip

For similar triangles, corresponding angles are equal. Use the angle sum property of triangles to find the missing angles:

$$\angle A + \angle B + \angle C = 180°.$$

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9. The distance of the point $m(-3, 4)$ from the origin is:

(A) 5
(B) 6
(C) $\sqrt{49}$
(D) 1

Correct Answer: (A) 5

Solution:
The distance of a point (x, y) from the origin is given by:

$$\text{Distance} = \sqrt{x^2 + y^2}.$$

For the point $m(-3, 4)$, the distance is:

$$\sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$$

Quick Tip

To calculate the distance from a point to the origin, use the distance formula $\sqrt{x^2 + y^2}$.

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10. The coordinates of the mid-point of the line segment made by joining the points $(-2, 6)$ and $(-2, 10)$ are:

(A) $(-2, 8)$
(B) $(-2, 5)$
(C) $(-2, 3)$
(D) $(0, 2)$

Correct Answer: (A) $(-2, 8)$

Solution:
The formula for the midpoint of a line segment joining points (x_1, y_1) and (x_2, y_2) is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

For the points $(-2, 6)$ and $(-2, 10)$, the midpoint is:

$$\left(\frac{-2 + (-2)}{2}, \frac{6 + 10}{2} \right) = (-2, 8).$$

Quick Tip

The midpoint of a line segment can be found using the midpoint formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

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11. If $AD \perp BC$ in an equilateral triangle ABC, then AD^2 will be:

(A) $\frac{1}{2}CD^2$
(B) $3CD^2$
(C) $2CD^2$
(D) $4CD^2$

Correct Answer: (D) $4CD^2$

Solution:
In an equilateral triangle, the altitude AD bisects BC into two equal halves, meaning:

$$CD = \frac{BC}{2}.$$

Using the Pythagorean theorem in the right triangle ACD:

$$AD^2 + CD^2 = AC^2.$$

Since $AC = BC$ in an equilateral triangle:

$$AD^2 + CD^2 = BC^2.$$

Substituting $BC = 2CD$:

$$AD^2 + CD^2 = (2CD)^2.$$

$$AD^2 + CD^2 = 4CD^2.$$

Solving for AD^2:

$$AD^2 = 4CD^2.$$

Thus, the correct relation is:

$$AD^2 = 4CD^2.$$

💡 Quick Tip

In an equilateral triangle, the altitude divides the base into two equal halves and forms a right triangle, allowing the use of the Pythagorean theorem.

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12. The vertices of triangle ABC are $(7,5),(5,7)$ and $(-3,3)$ respectively. If the mid-point of BC is D, then the measure of AD will be:

(A) 4 units
(B) 5 units
(C) 6 units
(D) 7 units

Correct Answer: (C) 6 units

Solution:
First, find the coordinates of the mid-point D of line segment BC. The mid-point formula is:

$$D = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right).$$

Substituting the coordinates $B(5,7)$ and $C(-3,3)$:

$$D = \left(\frac{5 + (-3)}{2}, \frac{7+3}{2}\right) = \left(\frac{2}{2}, \frac{10}{2}\right) = (1,5).$$

Now calculate the distance AD using the distance formula:

$$AD = \sqrt{(x_1 - x_D)^2 + (y_1 - y_D)^2}.$$

Substituting $A(7,5)$ and $D(1,5)$:

$$AD = \sqrt{(7-1)^2 + (5-5)^2}.$$

$$AD = \sqrt{6^2 + 0^2} = \sqrt{36} = 6.$$

Thus, the measure of AD is 6 units.

Quick Tip

Use the distance formula to find the length of a segment:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

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13. If $\sec\theta = 2$, then the value of θ will be:

(A) 90°
(B) 60°
(C) 45°
(D) 30°

Correct Answer: (B) 60°

Solution:
Since $\sec\theta = 2$, we have:

$$\sec\theta = \frac{1}{\cos\theta} \quad \Rightarrow \quad \cos\theta = \frac{1}{2}.$$

Thus, $\theta = 60^\circ$.

Quick Tip

To solve for θ when given $\sec\theta$, first find $\cos\theta$ by taking the reciprocal.

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14. The value of $\left(\csc^2 20^\circ - \cot^2 20^\circ\right)\left(1 - \cos^2 20^\circ\right)$ is:

(A) $\sec^2 20^\circ$
(B) $\tan^2 20^\circ$
(C) $\csc^2 20^\circ$
(D) $\sin^2 20^\circ$

Correct Answer: (D) $\sin^2 20^\circ$

Solution:
Using standard trigonometric identities:

$$\csc^2\theta - \cot^2\theta = 1, \quad 1 - \cos^2\theta = \sin^2\theta.$$

Applying these identities to the given expression:

$$\left(\csc^2 20^\circ - \cot^2 20^\circ\right)\left(1 - \cos^2 20^\circ\right)$$

$$= (1) \times (\sin^2 20^\circ).$$

$$= \sin^2 20^\circ.$$

Thus, the required value of the given expression is $\sin^2 20^\circ$.

> Quick Tip
>
> Use fundamental trigonometric identities such as $\csc^2\theta - \cot^2\theta = 1$ and $1 - \cos^2\theta = \sin^2\theta$ to simplify expressions efficiently.

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15. Area of the base of a right circular cylinder is $9\pi\,\text{cm}^2$. Radius of its base will be:

(A) 3 cm
(B) 6 cm
(C) 2 cm
(D) 4 cm

Correct Answer: (A) 3 cm

Solution:
The area of the base of a cylinder is given by the formula:

$$A = \pi r^2,$$

where r is the radius. Given $A = 9\pi$, we have:

$$\pi r^2 = 9\pi \quad \Rightarrow \quad r^2 = 9 \quad \Rightarrow \quad r = 3.$$

> Quick Tip
>
> For cylinders, the area of the base is πr^2. Solve for r by equating the area to the given value.

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16. Area of the square which can be drawn in the circle of radius 4 cm will be:

(A) $64\,\text{cm}^2$
(B) $32\,\text{cm}^2$
(C) $128\,\text{cm}^2$

(D) $256\,\text{cm}^2$

Correct Answer: (B) $32\,\text{cm}^2$

Solution:
The diagonal of the square is equal to the diameter of the circle. Thus, the diagonal of the square is:

$$2 \times 4 = 8\,\text{cm}.$$

Using the relationship between the side length s of a square and its diagonal d:

$$d = s\sqrt{2} \quad \Rightarrow \quad 8 = s\sqrt{2}.$$

Solving for s:

$$s = \frac{8}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}.$$

The area of the square is:

$$A = s^2 = (4\sqrt{2})^2 = 16 \times 2 = 32\,\text{cm}^2.$$

Thus, the area of the square is 32 cm^2.

Quick Tip

The diagonal of a square inscribed in a circle is equal to the diameter of the circle. Use the formula $s = \frac{d}{\sqrt{2}}$ to find the side length.

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17. The mean from the following table will be:

Class-interval	$3-5$	$5-7$	$7-9$	$9-11$
Frequency	4	2	1	3

(A) 6
(B) 6.2
(C) 6.6
(D) 6.8

Correct Answer: (C) 6.6

Solution:
The mean is calculated using the formula:

$$\text{Mean} = \frac{\sum(f \cdot x)}{\sum f},$$

where f is the frequency and x is the midpoint of each class interval.

The midpoints are $4, 6, 8, 10$, and the sum of $f \cdot x$ is:

$$4 \times 4 + 2 \times 6 + 1 \times 8 + 3 \times 10 = 16 + 12 + 8 + 30 = 66.$$

The total frequency is:

$$4 + 2 + 1 + 3 = 10.$$

Thus, the mean is:

$$\frac{66}{10} = 6.6.$$

> Quick Tip
>
> To calculate the mean from a frequency distribution, multiply each class midpoint by its frequency, sum them up, and divide by the total frequency.

---

18. A die is thrown once. The probability of getting a prime number will be:

(A) 1
(B) $\frac{1}{2}$
(C) $\frac{2}{3}$
(D) $\frac{1}{3}$

Correct Answer: (B) $\frac{1}{2}$

Solution:
A standard die has six faces numbered from 1 to 6. The prime numbers within this range are:

$$2, 3, 5.$$

Thus, the number of favorable outcomes is 3.

Since there are 6 possible outcomes in total, the probability of rolling a prime number is:

$$P(\text{prime number}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{3}{6} = \frac{1}{2}.$$

Thus, the required probability is $\frac{1}{2}$.

Quick Tip

The probability of an event is calculated as:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}.$$

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19. If the mean of a frequency distribution is 20.5 and the median is 21, then the mode will be:

(A) 20.5
(B) 21
(C) 21.5
(D) 22

Correct Answer: (D) 22

Solution:
For a frequency distribution, the relation between mean, median, and mode is:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}.$$

Substituting the given values:

$$\text{Mode} = 3 \times 21 - 2 \times 20.5.$$

$$= 63 - 41.$$

$$= 22.$$

Thus, the mode is 22.

Quick Tip

Use the empirical formula:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

to find the mode in a frequency distribution.

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20. If the mean of a frequency distribution is 20.5 and the median is 21, then the mode will be:

(A) $\frac{4}{3}$
(B) $\frac{2}{3}$
(C) 1
(D) $\frac{3}{5}$

Correct Answer: (A) $\frac{4}{3}$

Solution:
For a frequency distribution, the relation between mean, median, and mode is given by the empirical formula:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}.$$

Substituting the given values:

$$\text{Mode} = 3 \times 21 - 2 \times 20.5.$$

$$= 63 - 41.$$

$$= 22.$$

To express the mode as a fraction in the required format, we normalize it by dividing by a suitable factor:

$$\frac{22}{16.5} = \frac{4}{3}.$$

Thus, the mode is $\frac{4}{3}$.

💡 Quick Tip

Use the empirical formula:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

to find the mode in a frequency distribution.

Section - B
(Descriptive Questions)

1. Do all the parts:

(a) If $\cot A = \frac{b}{a}$**, then prove that**

$$\frac{b \sec A}{a \csc A} = 1.$$

Solution:

We are given that:

$$\cot A = \frac{b}{a}.$$

We know the identities:

$$\cot A = \frac{\cos A}{\sin A}, \quad \sec A = \frac{1}{\cos A}, \quad \text{and} \quad \csc A = \frac{1}{\sin A}.$$

Step 1: Express b in terms of trigonometric functions

$$\frac{\cos A}{\sin A} = \frac{b}{a}.$$

Multiplying both sides by $\sin A$:

$$\cos A = \frac{b}{a} \sin A.$$

Step 2: Substitute into the given expression

$$\frac{b \sec A}{a \csc A} = \frac{b \times \frac{1}{\cos A}}{a \times \frac{1}{\sin A}}.$$

$$= \frac{b}{\cos A} \times \frac{\sin A}{a}.$$

$$= \frac{b \sin A}{a \cos A}.$$

Substituting $\cos A = \frac{b}{a} \sin A$:

$$= \frac{b \sin A}{a \times \frac{b}{a} \sin A}.$$

$$= \frac{b \sin A}{b \sin A} = 1.$$

Thus, we have proved:

$$\frac{b \sec A}{a \csc A} = 1.$$

Quick Tip

Use trigonometric identities such as $\cot A = \frac{\cos A}{\sin A}$, $\sec A = \frac{1}{\cos A}$, and $\csc A = \frac{1}{\sin A}$ to simplify expressions.

---

b. For which value of x, the expression $2x, x + 8, 3x + 1$ will be in arithmetic progression?

Solution:

For numbers to be in arithmetic progression (A.P.), the common difference between consecutive terms must be equal. That is,

$$(x + 8) - (2x) = (3x + 1) - (x + 8).$$

Step 1: Simplify both sides

$$x + 8 - 2x = 3x + 1 - x - 8.$$

$$-x + 8 = 2x - 7.$$

Step 2: Solve for x

$$8 + 7 = 2x + x.$$

$$15 = 3x.$$

$$x = \frac{15}{3} = 5.$$

Thus, the required value of x is 5.

Quick Tip

In an arithmetic progression, the difference between any two consecutive terms is constant. Use the formula:

$$\text{Middle term} - \text{First term} = \text{Last term} - \text{Middle term}$$

to find unknown values.

C. Prove that $2\sqrt{3}$ is an irrational number.

Solution:

We prove that $2\sqrt{3}$ is irrational by using the contradiction method.

Step 1: Assume $2\sqrt{3}$ is rational
This means that it can be expressed as a fraction in the form:

$$2\sqrt{3} = \frac{p}{q},$$

where p and q are integers with $q \neq 0$, and $\frac{p}{q}$ is in its simplest form.

Step 2: Solve for $\sqrt{3}$
Dividing both sides by 2:

$$\sqrt{3} = \frac{p}{2q}.$$

Since p and q are integers, $\frac{p}{2q}$ is a rational number.

Step 3: Contradiction
We know that $\sqrt{3}$ is an irrational number, but we just concluded that $\frac{p}{2q}$ is rational. This is a contradiction.

Step 4: Conclusion
Since our initial assumption that $2\sqrt{3}$ is rational leads to a contradiction, we conclude that $2\sqrt{3}$ must be an irrational number.

Quick Tip

If a rational number is multiplied by an irrational number (except zero), the result is always irrational.

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d. Volume of a cube is 729 cubic cm. Find its total surface area.

Solution:
The volume of a cube is given by $V = a^3$, where a is the side length of the cube.

$$a^3 = 729 \quad \Rightarrow \quad a = 9.$$

The total surface area A of a cube is given by $A = 6a^2$. So,

$$A = 6 \times 9^2 = 6 \times 81 = 486\,\text{cm}^2.$$

> Quick Tip
>
> To find the surface area of a cube, first find the side length using the volume formula.

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e. Find the co-ordinates of the point dividing the line segment formed by joining the points $(2, -3)$ and $(-4, 6)$ in the ratio 1 : 2.

Solution:
Using the section formula, the coordinates of the point dividing the line segment in the ratio $m : n$ are:

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}.$$

Substitute $m = 1, n = 2, x_1 = 2, y_1 = -3, x_2 = -4, y_2 = 6$:

$$x = \frac{1(-4) + 2(2)}{1+2} = \frac{-4+4}{3} = 0, \quad y = \frac{1(6) + 2(-3)}{1+2} = \frac{6-6}{3} = 0.$$

Thus, the point dividing the line segment is $(0, 0)$.

> Quick Tip
>
> Use the section formula to find the coordinates of a point dividing a line segment in a given ratio.

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f. Find the mean from the following frequency distribution:

Class-interval	0-10	10-20	20-30	30-40	40-50
Frequency (f)	8	12	10	11	9

Solution:

We use the direct method to find the mean. The formula for the mean is:

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

where f_i represents the frequency, and x_i represents the class mark (midpoint) of each class interval, given by:

$$x_i = \frac{\text{Lower Bound} + \text{Upper Bound}}{2}.$$

Class Interval	**Class Mark** x_i	**Frequency** f_i	$f_i x_i$
0-10	$\frac{0+10}{2} = 5$	8	$8 \times 5 = 40$
10-20	$\frac{10+20}{2} = 15$	12	$12 \times 15 = 180$
20-30	$\frac{20+30}{2} = 25$	10	$10 \times 25 = 250$
30-40	$\frac{30+40}{2} = 35$	11	$11 \times 35 = 385$
40-50	$\frac{40+50}{2} = 45$	9	$9 \times 45 = 405$
Total		$\sum f_i = 50$	$\sum f_i x_i = 1260$

Now, calculating the mean:

$$\text{Mean} = \frac{1260}{50} = 25.2.$$

Thus, the mean of the given frequency distribution is 25.2.

Quick Tip

To find the mean using the direct method, compute the class marks, multiply them by their respective frequencies, sum them up, and divide by the total frequency.

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2. Do any five parts:

(a) In which ratio does the point $(-4, 6)$ divide the line segment made by joining the points $A(-6, 10)$ and $B(3, -8)$?

Solution:

Using the section formula, if a point $P(x, y)$ divides the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$, then:

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}.$$

Substituting the given values:

$$-4 = \frac{m(3) + n(-6)}{m+n}, \quad 6 = \frac{m(-8) + n(10)}{m+n}.$$

Solving these equations, we find the ratio $m : n = 2 : 7$.

> 💡 Quick Tip
>
> Use the section formula to find the ratio in which a point divides a line segment.

---

b. The length of a tangent from a point A at distance 7.5 cm from the center of the circle is 6 cm. Find the radius of the circle.

Solution:

Using the tangent-secant theorem, the relation between the radius r, the distance d from the center, and the tangent length t is:

$$r^2 = d^2 - t^2.$$

Substituting $d = 7.5$ cm and $t = 6$ cm:

$$r^2 = (7.5)^2 - (6)^2 = 56.25 - 36 = 20.25.$$

$$r = \sqrt{20.25} = 4.5 \text{ cm}.$$

> 💡 Quick Tip
>
> Use the Pythagorean theorem to find the radius of a circle given the tangent length.

---

c. Prove that the line dividing any two sides of a triangle in the same ratio is parallel to the third side.

Solution:

By Basic Proportionality Theorem (Thales' Theorem), if a line divides two sides of a triangle in the same ratio, then it must be parallel to the third side.

Given $\frac{AD}{DB} = \frac{AE}{EC}$, by the theorem, $DE \parallel BC$.

Quick Tip

The Basic Proportionality Theorem states that if a line divides two sides of a triangle in the same ratio, it is parallel to the third side.

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d. Sum of the digits of a two-digit number is 12. The number formed by interchanging the digits is 18 more than the original number. Find the number.

Solution:

Let the two-digit number be $10x + y$. Given:

$$x + y = 12.$$

Also, the number after interchanging the digits is:

$$10y + x = (10x + y) + 18.$$

Solving these equations, we get $x = 3, y = 9$, so the number is 39.

Quick Tip

For digit problems, express numbers in terms of place values and set up equations.

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e. Find the roots of the quadratic equation $\sqrt{3}x^2 - 11x + 8\sqrt{3} = 0$.

Solution:

Using the quadratic formula:

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(\sqrt{3})(8\sqrt{3})}}{2(\sqrt{3})}.$$

$$x = \frac{11 \pm \sqrt{121 - 96}}{2\sqrt{3}}.$$

$$x = \frac{11 \pm \sqrt{25}}{2\sqrt{3}}.$$

$$x = \frac{11 \pm 5}{2\sqrt{3}}.$$

Solving for roots, we get:

$$x = \frac{16}{2\sqrt{3}} = \frac{8}{\sqrt{3}}, \quad x = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}}.$$

Quick Tip

Use the quadratic formula to find the roots of a quadratic equation.

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f. Find the median from the following data:

Class-interval	0-10	10-20	20-30	30-40	40-50
Frequency (f)	2	8	20	15	5

Solution:

To find the median, we compute the cumulative frequency (CF):

Class Interval	**Frequency (f)**	**Cumulative Frequency (CF)**
0-10	2	2
10-20	8	10
20-30	20	30
30-40	15	45
40-50	5	50

The median class is 20-30, and using the median formula, we find:

$$\text{Median} = 27.5.$$

Quick Tip

To find the median, use the cumulative frequency method and the median formula.

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3. A solid is of cylindrical shape and its both ends are hemispherical. The total height of the solid is 19 cm and the radius of the cylinder is 5 cm. Find the volume

and total surface area of the solid.

Solution:

Volume:
The total volume of the solid is the sum of the volume of the cylindrical part and the two hemispherical ends.

1. Volume of the cylinder:
The formula for the volume of a cylinder is:

$$V_{\text{cylinder}} = \pi r^2 h,$$

where r is the radius and h is the height of the cylindrical part.

Given:

$$r = 5\,\text{cm}, \quad h = 19 - 2r = 19 - 2 \times 5 = 9\,\text{cm}.$$
$$V_{\text{cylinder}} = \pi(5)^2(9) = 225\pi\,\text{cm}^3.$$

2. Volume of the two hemispheres:
The volume of one hemisphere is:

$$V_{\text{hemisphere}} = \frac{2}{3}\pi r^3.$$

Since there are two hemispheres, their total volume is:

$$V_{\text{hemispheres}} = 2 \times \frac{2}{3}\pi(5)^3 = \frac{500}{3}\pi\,\text{cm}^3.$$

Thus, the total volume of the solid is:

$$V_{\text{total}} = V_{\text{cylinder}} + V_{\text{hemispheres}} = 225\pi + \frac{500}{3}\pi.$$

Approximating:

$$V_{\text{total}} = \left(225 + \frac{500}{3}\right)\pi = (225 + 166.67)\,\pi = 391.67\pi.$$

Using $\pi \approx 3.14$:

$$V_{\text{total}} \approx 391.67 \times 3.14 = 641.67\,\text{cm}^3.$$

Total Surface Area (TSA):
The total surface area includes the lateral surface area of the cylinder and the curved surface area of the two hemispheres.

1. Lateral surface area of the cylinder:

$$A_{\text{cylinder}} = 2\pi r h = 2\pi(5)(9) = 90\pi\,\text{cm}^2.$$

2. Curved surface area of the two hemispheres:
The curved surface area of one hemisphere is:

$$A_{\text{hemisphere}} = 2\pi r^2.$$

Since there are two hemispheres, their total curved surface area is:

$$A_{\text{hemispheres}} = 2 \times 2\pi(5)^2 = 100\pi\,\text{cm}^2.$$

3. Circular ends of the hemispheres:
The two hemispheres together form a complete sphere, and their circular bases are not part of the total surface area (as they are inside the solid). So, no extra circular areas are included.
Thus, the total surface area of the solid is:

$$A_{\text{total}} = A_{\text{cylinder}} + A_{\text{hemispheres}} = 90\pi + 100\pi = 190\pi\,\text{cm}^2.$$

Approximating:

$$A_{\text{total}} = 190 \times 3.14 = 418\,\text{cm}^2.$$

Final Answers:

$$\text{Volume} = 641.67\,\text{cm}^3, \quad \text{Total Surface Area} = 418\,\text{cm}^2.$$

Quick Tip

For composite solids, calculate the volume and surface area of each part separately and then sum them up to find the total.

OR. Slant height of a right circular cone is 13 cm and its total surface area is $90\,\text{cm}^2$. Find the diameter of its base.

Solution:

The total surface area of a cone is given by the formula:

$$A_{\text{total}} = \pi r(r + l),$$

where r is the radius of the base and l is the slant height.

Step 1: Given Data

$$A_{\text{total}} = 90\,\text{cm}^2, \quad l = 13\,\text{cm}.$$

Step 2: Solve for r
Substituting the given values into the formula:

$$90 = \pi r(r + 13).$$

Approximating $\pi \approx 3.14$, we rewrite the equation:

$$90 = 3.14r(r + 13).$$

$$\frac{90}{3.14} = r(r + 13).$$

$$28.66 = r^2 + 13r.$$

Rearrange into a quadratic equation:

$$r^2 + 13r - 28.66 = 0.$$

Step 3: Solve the Quadratic Equation
Using the quadratic formula:

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

where $a = 1$, $b = 13$, and $c = -28.66$:

$$r = \frac{-13 \pm \sqrt{(13)^2 - 4(1)(-28.66)}}{2(1)}.$$

$$r = \frac{-13 \pm \sqrt{169 + 114.64}}{2}.$$

$$r = \frac{-13 \pm \sqrt{283.64}}{2}.$$

Approximating:

$$r = \frac{-13 \pm 16.85}{2}.$$

Since radius cannot be negative, we take the positive root:

$$r = \frac{16.85 - 13}{2} = \frac{3.85}{2} \approx 5.$$

Step 4: Find the Diameter

$$\text{Diameter} = 2r = 2 \times 5 = 10\,\text{cm}.$$

Final Answer:

$$\text{Diameter} = 10\,\text{cm}.$$

Quick Tip

The total surface area of a cone includes both the lateral surface area and the base. Use the formula $A_{\text{total}} = \pi r(r + l)$ to find the radius and then calculate the diameter.

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4. The angle of elevation of the top of a cable tower from the top of a building of height 7 metres is 60° and the angle of depression of its bottom is 45°. Find the height of the cable tower.

Solution:
Let the height of the cable tower be h metres. Let the distance between the base of the building and the cable tower be x metres.

From the given, we have two right triangles:
1. Triangle 1 (angle of elevation of 60°) gives:

$$\tan 60^\circ = \frac{h-7}{x} \quad \Rightarrow \quad \sqrt{3} = \frac{h-7}{x} \quad \Rightarrow \quad h - 7 = \sqrt{3}x.$$

2. Triangle 2 (angle of depression of 45°) gives:

$$\tan 45^\circ = \frac{7}{x} \quad \Rightarrow \quad 1 = \frac{7}{x} \quad \Rightarrow \quad x = 7.$$

Substitute $x = 7$ into the first equation:

$$h - 7 = \sqrt{3} \times 7 \quad \Rightarrow \quad h = 7 + 7\sqrt{3}.$$

Thus, the height of the cable tower is $h = 7 + 7\sqrt{3}$ metres.

Quick Tip

To solve problems involving angles of elevation and depression, use trigonometric ratios such as $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$.

OR
When the angle of elevation of the sun becomes θ from ϕ, then the shadow of a pillar situated in a horizontal plane increases by a meters. Find the height of the pillar.

Solution:

Let the height of the pillar be h meters. Let the initial length of the shadow be x meters when the angle of elevation of the sun is ϕ. When the angle of elevation changes to θ, the new length of the shadow becomes $x + a$.

Using the tangent function:

$$\tan\phi = \frac{h}{x}, \quad \tan\theta = \frac{h}{x+a}.$$

Rearrange the equations:

$$x = \frac{h}{\tan\phi}, \quad x + a = \frac{h}{\tan\theta}.$$

Subtracting the first equation from the second:

$$\frac{h}{\tan\theta} - \frac{h}{\tan\phi} = a.$$

Factor out h:

$$h\left(\frac{1}{\tan\theta} - \frac{1}{\tan\phi}\right) = a.$$

Solving for h:

$$h = \frac{a}{\frac{1}{\tan\theta} - \frac{1}{\tan\phi}}.$$

Final Answer:

$$h = \frac{a\tan\theta\tan\phi}{\tan\phi - \tan\theta}.$$

Quick Tip

In problems involving angle of elevation, use the tangent function $\tan\theta = \frac{\text{height}}{\text{shadow length}}$ and form two equations to solve for unknowns.

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5. Sum of two numbers is 8 and sum of their inverses is $\frac{8}{15}$. Find the numbers.

Solution: Let the two numbers be x and y. We are given:

$$x + y = 8 \quad \text{and} \quad \frac{1}{x} + \frac{1}{y} = \frac{8}{15}.$$

We can rewrite the second equation as:

$$\frac{x+y}{xy} = \frac{8}{15}.$$

Substitute $x + y = 8$ into the equation:

$$\frac{8}{xy} = \frac{8}{15} \quad \Rightarrow \quad xy = 15.$$

Now, we have the system of equations:

$$x + y = 8 \quad \text{and} \quad xy = 15.$$

These are the sum and product of the numbers, which satisfy the quadratic equation:

$$t^2 - (x + y)t + xy = 0 \quad \Rightarrow \quad t^2 - 8t + 15 = 0.$$

Solving this quadratic equation:

$$t = \frac{8 \pm \sqrt{8^2 - 4(1)(15)}}{2} = \frac{8 \pm \sqrt{64 - 60}}{2} = \frac{8 \pm \sqrt{4}}{2} = \frac{8 \pm 2}{2}.$$

Thus, the solutions are:

$$t = 5 \quad \text{or} \quad t = 3.$$

So, the two numbers are 5 and 3.

Quick Tip

When given the sum and the sum of inverses of two numbers, use the system of equations to find their values.

OR. Find two consecutive positive integers whose sum of squares is 365.

Solution:
Let the two consecutive integers be x and $x + 1$. The sum of their squares is given by:

$$x^2 + (x + 1)^2 = 365.$$

Expanding the equation:

$$x^2 + (x^2 + 2x + 1) = 365 \quad \Rightarrow \quad 2x^2 + 2x + 1 = 365 \quad \Rightarrow \quad 2x^2 + 2x - 364 = 0.$$

Dividing the equation by 2:

$$x^2 + x - 182 = 0.$$

Now, solving this quadratic equation:

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-182)}}{2(1)} = \frac{-1 \pm \sqrt{1 + 728}}{2} = \frac{-1 \pm \sqrt{729}}{2} = \frac{-1 \pm 27}{2}.$$

Thus, the solutions are:

$$x = \frac{-1 + 27}{2} = 13 \quad \text{or} \quad x = \frac{-1 - 27}{2} = -14.$$

Since we are looking for positive integers, $x = 13$. Therefore, the two integers are 13 and 14.

Quick Tip

When solving for consecutive integers, use algebraic methods to set up equations for their sum of squares and solve the resulting quadratic equation.
