RE-NEET 2026 PHYSICS

Question Paper With Solutions PDF

Conducted by National Testing Agency (NTA)

General Instructions

(i) The test is of 3 hours and 15 minutes duration.

(ii) This test paper consists of 180 questions. The maximum marks are 720.

(iii) Physics and Chemistry contains 45 questions each and Biology (Botany and Zoology) contains 90 questions.

(iv) Each question carries +4 marks for correct answer and –1 mark for wrong answer.

1. A photon and an electron, each of 20 eV **energy, move in free space. The ratio of linear momentum of electron** p_e **to that of photon** p_{ph}, $\frac{p_e}{p_{ph}}$ **is :**
(Take speed of light $= 3 \times 10^8$ **ms**$^{-1}$**, charge of electron** $= -1.6 \times 10^{-19}$ **C and mass of electron** $= 9 \times 10^{-31}$ **kg)**

(A) 275
(B) 450
(C) $\frac{1}{250}$
(D) 225

Correct Answer: (D) 225

Solution:

Step 1: Understanding the Question:
The problem asks for the ratio of the linear momentum of an electron to that of a photon, given that both particles have the same energy of 20 eV.

Step 2: Key Formula or Approach:

For a photon, the energy E is related to its momentum p_{ph} by:

$$p_{ph} = \frac{E}{c}$$

For an electron (non-relativistic), the kinetic energy E is related to its momentum p_e by:

$$E = \frac{p_e^2}{2m} \implies p_e = \sqrt{2mE}$$

where m is the mass of the electron.

Step 3: Detailed Explanation:

Let's take the ratio of the momenta:

$$\frac{p_e}{p_{ph}} = \frac{\sqrt{2mE}}{E/c} = c\sqrt{\frac{2m}{E}}$$

Given values are:

Energy $E = 20 \text{ eV} = 20 \times 1.6 \times 10^{-19} \text{ J} = 32 \times 10^{-19}$ J.

Mass of electron $m = 9 \times 10^{-31}$ kg.

Speed of light $c = 3 \times 10^8 \text{ ms}^{-1}$.

Substitute the values into the ratio formula:

$$\frac{p_e}{p_{ph}} = 3 \times 10^8 \sqrt{\frac{2 \times 9 \times 10^{-31}}{32 \times 10^{-19}}}$$

$$\frac{p_e}{p_{ph}} = 3 \times 10^8 \sqrt{\frac{18 \times 10^{-31}}{32 \times 10^{-19}}}$$

$$\frac{p_e}{p_{ph}} = 3 \times 10^8 \sqrt{\frac{9}{16} \times 10^{-12}}$$

$$\frac{p_e}{p_{ph}} = 3 \times 10^8 \times \frac{3}{4} \times 10^{-6}$$

$$\frac{p_e}{p_{ph}} = \frac{9}{4} \times 10^2 = 2.25 \times 100 = 225$$

Step 4: Final Answer:

The ratio of the linear momentum of the electron to that of the photon is 225.

Quick Tip: Remember that $p = E/c$ works for massless particles (photons), while $p = \sqrt{2mE}$ works for non-relativistic particles having mass (electrons).

Always verify if the particle is relativistic; for 20 eV, an electron is purely classical.

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2. Water flows in a streamline motion through a horizontal pipe of circular cross-section as shown in the figure. The pressure difference of water between P and Q is 15 Nm^{-2}. The area of cross-section at P and Q are 40 cm^2 and 20 cm^2, respectively. The rate of flow of water through the pipe, in cm^{3}s^{-1}, is :
Take density of water = 1000 kg m^{-3}

(A) 400

(B) 100

(C) 200

(D) 300

Correct Answer: (A) 400

Solution:

Step 1: Understanding the Question:

Water flows through a horizontal pipe with varying cross-sectional area.

We are given the areas at two points, the pressure difference between them, and we need to find the volumetric flow rate.

Step 2: Key Formula or Approach:

According to the Equation of Continuity, the rate of flow (discharge) Q is constant:

$$Q = A_P v_P = A_Q v_Q \implies v_P = \frac{Q}{A_P}, \quad v_Q = \frac{Q}{A_Q}$$

Using Bernoulli's Principle for a horizontal pipe ($h_P = h_Q$):

$$P_P + \frac{1}{2}\rho v_P^2 = P_Q + \frac{1}{2}\rho v_Q^2 \implies P_P - P_Q = \frac{1}{2}\rho(v_Q^2 - v_P^2)$$

Step 3: Detailed Explanation:

Given values are:

$\Delta P = P_P - P_Q = 15\ \text{N/m}^2$.

$A_P = 40\ \text{cm}^2 = 40 \times 10^{-4}\ \text{m}^2$.

$A_Q = 20\ \text{cm}^2 = 20 \times 10^{-4}\ \text{m}^2$.

$\rho = 1000\ \text{kg/m}^3$.

Substitute v_P and v_Q in Bernoulli's equation:

$$P_P - P_Q = \frac{1}{2}\rho\left(\left(\frac{Q}{A_Q}\right)^2 - \left(\frac{Q}{A_P}\right)^2\right)$$

$$15 = \frac{1}{2} \times 1000 \times Q^2\left(\frac{1}{(20 \times 10^{-4})^2} - \frac{1}{(40 \times 10^{-4})^2}\right)$$

$$15 = 500 \times Q^2\left(\frac{1}{400 \times 10^{-8}} - \frac{1}{1600 \times 10^{-8}}\right)$$

$$15 = \frac{500 \times Q^2}{10^{-8}}\left(\frac{4-1}{1600}\right)$$

$$15 = \frac{500 \times Q^2}{10^{-8}} \times \frac{3}{1600}$$

$$15 = \frac{1500 \times Q^2}{1600 \times 10^{-8}}$$

$$15 = \frac{15}{16 \times 10^{-8}}Q^2$$

$$Q^2 = 16 \times 10^{-8}$$

$$Q = \sqrt{16 \times 10^{-8}} = 4 \times 10^{-4}\ \text{m}^3/\text{s}$$

Convert the flow rate to cm^3/s:

$$Q = 4 \times 10^{-4} \times 10^6\ \text{cm}^3/\text{s} = 400\ \text{cm}^3/\text{s}$$

Step 4: Final Answer:

The rate of flow of water is $400\ \text{cm}^3/\text{s}$.

Quick Tip: When applying Bernoulli's equation for a horizontal pipe, gravitational potential energy terms cancel out ($h_1 = h_2$).
Always combine it with the Equation of Continuity ($A_1 v_1 = A_2 v_2 = Q$) to find velocities or flow rates directly.

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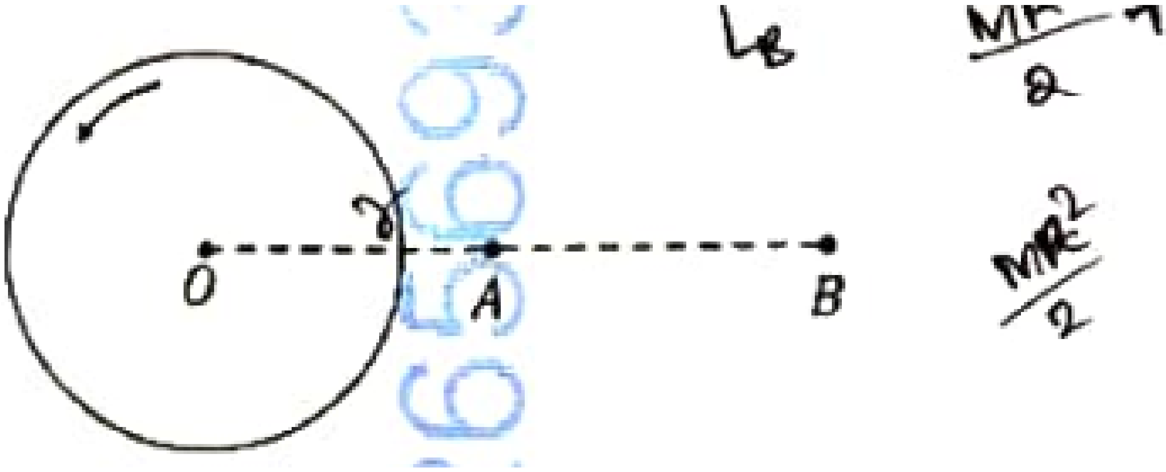
3. A thin horizontal disc is rotating about a vertical axis passing through its fixed centre O. Its angular momentum is L_A and L_B computed about points A and B, respectively, with $OB = 2 \times OA$. The value of $\frac{L_A}{L_B}$ is :



(A) 2
(B) $\frac{1}{4}$
(C) $\frac{1}{2}$
(D) 1

Correct Answer: (D) 1

Solution:

Step 1: Understanding the Question:
A uniform horizontal disc rotates about a fixed vertical axis passing through its center of mass O.

We need to determine the ratio of its angular momentum evaluated about two different points A and B in the horizontal plane.

Step 2: Key Formula or Approach:

The angular momentum of a rigid body about any arbitrary point P is given by:

$$\vec{L}_P = \vec{L}_{cm} + \vec{r}_{cm/P} \times (M\vec{v}_{cm})$$

where $\vec{L}_{cm}$ is the angular momentum about the center of mass, $\vec{r}_{cm/P}$ is the position vector of the center of mass relative to point P, M is the mass of the body, and $\vec{v}_{cm}$ is the velocity of the center of mass.

Step 3: Detailed Explanation:

Since the disc is rotating about a fixed vertical axis passing through its center O, its center of mass is stationary.

Therefore, the velocity of the center of mass is zero:

$$\vec{v}_{cm} = 0$$

Substituting this into the general formula for angular momentum, the second term vanishes for any point P:

$$\vec{L}_P = \vec{L}_{cm} + \vec{r}_{cm/P} \times 0 = \vec{L}_{cm}$$

This means the angular momentum is independent of the reference point and is identically equal to the angular momentum about its center of mass for all points.

Thus, evaluating it for points A and B:

$$L_A = L_O$$

$$L_B = L_O$$

Taking their ratio:

$$\frac{L_A}{L_B} = \frac{L_O}{L_O} = 1$$

Step 4: Final Answer:

The value of the ratio $\frac{L_A}{L_B}$ is 1.

Quick Tip: For a rigid body rotating about a fixed centroidal axis, the velocity of the center of mass is zero.
Consequently, the angular momentum is independent of the point of calculation and is solely given by $I_{cm}\omega$.

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4. Consider a long solenoid of length l and radius r. If n is the number of turns per unit length and μ_0 is the permeability of free space, the inductance of the solenoid is :

(A) $2\mu_0\pi n^2r^2l$
(B) $\mu_0\pi n^2r^2l$
(C) $\mu_0 n^2r^2l$
(D) $(\mu_0/2\pi)n^2r^2l$

Correct Answer: (B) $\mu_0\pi n^2r^2l$

Solution:

Step 1: Understanding the Question:
The question asks for the self-inductance L of a long solenoid with length l, radius r, and turn density n.

Step 2: Key Formula or Approach:
The magnetic field B inside a long solenoid is:

$$B = \mu_0 nI$$

where I is the current flowing through it.
The total magnetic flux Φ linked with the solenoid of N turns is:

$$\Phi = NBA$$

where A is the cross-sectional area.

Self-inductance L is defined as:

$$L = \frac{\Phi}{I}$$

Step 3: Detailed Explanation:

First, express the total number of turns N and the area A.

$$N = n \times l$$

$$A = \pi r^2$$

Now, calculate the total magnetic flux Φ.

$$\Phi = (nl) \cdot (\mu_0 n I) \cdot (\pi r^2)$$

$$\Phi = \mu_0 \pi n^2 r^2 l I$$

Using the formula for self-inductance:

$$L = \frac{\Phi}{I} = \frac{\mu_0 \pi n^2 r^2 l I}{I} = \mu_0 \pi n^2 r^2 l$$

Step 4: Final Answer:

The inductance of the solenoid is $\mu_0 \pi n^2 r^2 l$.

Quick Tip: Inductance is essentially magnetic inertia.

For a long solenoid, the formula $L = \mu_0 n^2 A l$ shows that self-inductance is directly proportional to the volume $(A \times l)$ and the square of turn density (n^2).

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5. The temperature of a metallic sphere of radius R is increased by a small amount ΔT. If the linear coefficient of thermal expansion of the metal is α, the approximate increase in the volume of the sphere is :

(A) $6\pi R^3 \alpha \Delta T$

(B) $2\pi R^3 \alpha \Delta T$

(C) $3\pi R^3 \alpha \Delta T$

(D) $4\pi R^3 \alpha \Delta T$

Correct Answer: (D) $4\pi R^3 \alpha \Delta T$

Solution:

Step 1: Understanding the Question:

We need to find the change in volume of a metallic sphere when its temperature is increased by ΔT, given its linear expansion coefficient α.

Step 2: Key Formula or Approach:

The original volume of the sphere is:

$$V = \frac{4}{3}\pi R^3$$

The volumetric expansion is given by:

$$\Delta V = V\gamma\Delta T$$

where γ is the volume coefficient of thermal expansion.

For isotropic materials, the relationship between volume and linear expansion coefficients is:

$$\gamma \approx 3\alpha$$

Step 3: Detailed Explanation:

Substitute V and γ into the volumetric expansion formula:

$$\Delta V = \left(\frac{4}{3}\pi R^3\right)(3\alpha)\Delta T$$

$$\Delta V = 4\pi R^3 \alpha \Delta T$$

Step 4: Final Answer:

The approximate increase in the volume of the sphere is $4\pi R^3 \alpha \Delta T$.

Quick Tip: For isotropic solid materials, the volume expansion coefficient γ is exactly three times the linear expansion coefficient α ($\gamma = 3\alpha$).

Don't forget this simple relation for thermal expansion approximations.

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6. Consider two circuits, (A) and (B), each having two resistors. One of them has a positive temperature coefficient of resistance, $+\alpha$, while the other one has a negative temperature coefficient, $-\alpha$, as shown in the figure. The current through these circuits are denoted by I_A and I_B. At initial temperature, the resistance of the two resistors is R_0. As the temperature is increased, the correct option that describes the variation of current in these circuits is :

(A) both I_A and I_B remain constant

(B) I_A remains constant while I_B increases

(C) I_A decreases while I_B increases

(D) I_A increases while I_B decreases

Correct Answer: (B) I_A remains constant while I_B increases

Solution:

Step 1: Understanding the Question:

We are analyzing two circuits containing temperature-dependent resistors.

Circuit (A) has them in series, while Circuit (B) has them in parallel.

We need to determine how the current from the voltage source changes as temperature (ΔT) increases.

Step 2: Key Formula or Approach:

For a series circuit, equivalent resistance is:

$$R_{eq,A} = R_1 + R_2$$

For a parallel circuit, equivalent resistance is:

$$R_{eq,B} = \frac{R_1 R_2}{R_1 + R_2}$$

The current is inversely proportional to equivalent resistance:

$$I = \frac{V}{R_{eq}}$$

Step 3: Detailed Explanation:

For Circuit (A):

The two resistors are in series.

$$R_{eq,A} = R_0(1-\alpha\Delta T) + R_0(1+\alpha\Delta T)$$

$$R_{eq,A} = R_0 - R_0\alpha\Delta T + R_0 + R_0\alpha\Delta T = 2R_0$$

Since $R_{eq,A}$ is constant and independent of ΔT, the current $I_A = \frac{V}{2R_0}$ remains constant.

For Circuit (B):

The two resistors are in parallel.

$$R_{eq,B} = \frac{R_0(1-\alpha\Delta T)\cdot R_0(1+\alpha\Delta T)}{R_0(1-\alpha\Delta T) + R_0(1+\alpha\Delta T)}$$

$$R_{eq,B} = \frac{R_0^2(1-\alpha^2\Delta T^2)}{2R_0} = \frac{R_0}{2}(1-\alpha^2\Delta T^2)$$

As temperature increases, ΔT increases, which implies $\alpha^2\Delta T^2$ increases.

Consequently, $(1-\alpha^2\Delta T^2)$ decreases, making $R_{eq,B}$ decrease.

Since $R_{eq,B}$ decreases, the current $I_B = \frac{V}{R_{eq,B}}$ increases.

Step 4: Final Answer:

I_A remains constant while I_B increases.

Quick Tip: When combining components with opposite temperature coefficients, series combinations often compensate and keep the total resistance constant.

Parallel combinations follow the standard inverse addition rules, typically resulting in a change.

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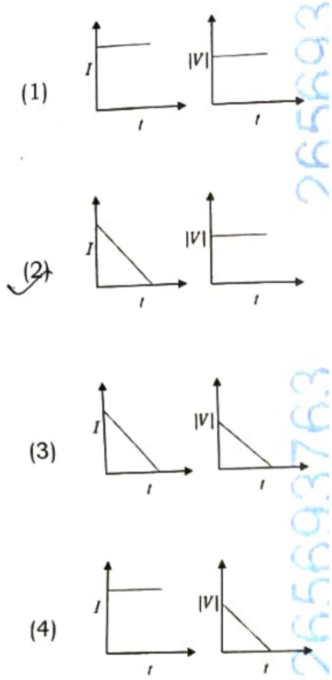
7. A beam of light falls on a metal surface such that photo-electrons are generated. If power

of the light source starts to decrease linearly with time t, then variation of the photocurrent I and magnitude of the stopping potential $|V|$ with time is best represented by :**



Correct Answer: (B) Graph showing I decreases linearly and $|V|$ is constant

Solution:

Step 1: Understanding the Question:

We need to determine how the photocurrent and stopping potential vary with time when the power of an incident light source decreases linearly with time.

Step 2: Key Formula or Approach:

Photocurrent I is directly proportional to the intensity (and thus power) of the incident light, assuming frequency is constant:

$$I \propto P$$

The magnitude of the stopping potential $|V|$ is dependent solely on the frequency ν of the incident light and the work function ϕ of the material, given by Einstein's photoelectric equation:

$$e|V| = h\nu - \phi$$

Step 3: Detailed Explanation:

First, consider the variation of Photocurrent (I).
The power of the light source decreases linearly with time ($P(t) = P_0 - kt$).
Since the number of photons emitted per second is proportional to the power, the rate of emission of photoelectrons (photocurrent) will also decrease linearly with time.
Thus, the graph of I vs t is a straight line with a negative slope.

Second, consider the variation of Stopping Potential ($|V|$).
The stopping potential depends only on the frequency of the incident light and the nature of the material.
A change in the power (intensity) of the light source does not affect the maximum kinetic energy of the photoelectrons.
Therefore, the stopping potential remains constant over time.
The graph of $|V|$ vs t is a horizontal straight line.

These conclusions match the behavior described in option (B).

Step 4: Final Answer:
I decreases linearly with time, and $|V|$ remains constant over time.

Quick Tip: Intensity (or power) of light dictates the number of photoelectrons (photocurrent), whereas the frequency controls their maximum kinetic energy (stopping potential).
Changing one does not affect the other!

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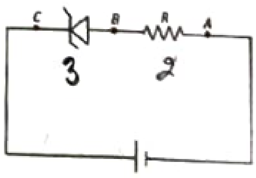
8. An ideal Zener diode with breakdown voltage of -3 V is reverse biased with a negative input voltage $V_i = -5$ V. The magnitude of voltage difference between points B and A is :



(A) 0 V

(B) 3 V

(C) 2 V

(D) 1 V

Correct Answer: (B) 3 V

Solution:

Step 1: Understanding the Question:

The circuit contains a Zener diode in series with a resistor.

The Zener diode has a breakdown voltage of 3 V (magnitude).

We are given an input voltage of -5 V, which reverse biases the diode.

We need to find the voltage across the Zener diode, which is exactly between points A and B.

Step 2: Key Formula or Approach:

When an ideal Zener diode is reverse biased and the applied voltage exceeds its breakdown voltage ($|V_i| > V_Z$), the diode operates in the breakdown region.

In this region, the voltage across the Zener diode is maintained constant at its Zener breakdown voltage V_Z.

Step 3: Detailed Explanation:

The magnitude of the input voltage is $|V_i| = 5$ V.

The breakdown voltage of the Zener diode is $V_Z = 3$ V.

Since $|V_i| > V_Z$, the Zener diode is in the breakdown region.

In the breakdown region, an ideal Zener diode acts as a constant voltage source of magnitude V_Z.

Therefore, the voltage drop across the Zener diode (between points A and B) is clamped to 3 V.

The remaining voltage ($5\text{ V} - 3\text{ V} = 2\text{ V}$) will drop across the series resistor.

The question asks for the magnitude of the voltage difference between points A and B, which are the terminals of the Zener diode.

$$V_{AB} = V_Z = 3\text{ V}$$

Step 4: Final Answer:

The magnitude of the voltage difference between points B and A is 3 V.

Quick Tip: An ideal Zener diode acts as a simple constant voltage battery of magnitude V_Z when operated in the reverse breakdown region.

Always check if the input voltage exceeds V_Z to ensure it is in breakdown.

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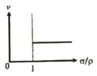
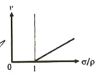
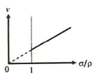
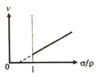
9. In the measurement of viscosity of liquids using terminal velocity experiment, spherical balls of same radius but having different densities are used. The variation of the terminal velocity (v) with the ratio of density of spherical ball (σ) to density of the liquid (ρ), is best represented by :

(1) 

(2) 

(3) 

(4) 

Correct Answer: (A) A straight line originating from $\sigma/\rho = 1$ on the x-axis with a positive slope.

Solution:

Step 1: Understanding the Question:

We need to find the graphical relationship between the terminal velocity v of a spherical ball falling in a liquid and the density ratio σ/ρ.

Step 2: Key Formula or Approach:

The terminal velocity v of a spherical ball of radius r and density σ falling through a liquid of density ρ and viscosity η is given by Stokes' Law:

$$v = \frac{2r^2 g}{9\eta}(\sigma - \rho)$$

Step 3: Detailed Explanation:

Rearranging the terminal velocity formula to express it in terms of the ratio σ/ρ:

$$v = \frac{2r^2 g\rho}{9\eta}\left(\frac{\sigma}{\rho} - 1\right)$$

Let $k = \frac{2r^2 g\rho}{9\eta}$, which is a positive constant since the radius is constant and the liquid is the same.

Let $x = \frac{\sigma}{\rho}$.

The equation becomes:

$$v = k(x - 1)$$

This is the equation of a straight line ($y = mx + c$) with slope $m = k$ and y-intercept $c = -k$.

When $x = \frac{\sigma}{\rho} = 1$, the terminal velocity $v = 0$.

For $x > 1$ (ball is denser than liquid), v increases linearly with x.

Therefore, the graph of v versus σ/ρ is a straight line that intersects the horizontal axis precisely at 1.

This perfectly matches the description of option (A).

Step 4: Final Answer:

The variation is a straight line starting from 1 on the σ/ρ axis.

Quick Tip: Stokes' law gives terminal velocity proportional to the density difference $(\sigma - \rho)$. Whenever checking variations, rewrite the expression in the form $y = mx + c$ to quickly identify the graphical behavior.

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10. Two planets P_1 and P_2 with equal mass have radii R_1 and R_2, respectively, where $R_2 = \frac{R_1}{2}$. The escape speeds of P_1 and P_2 are v_1 and v_2, respectively. Then $\frac{v_2}{v_1}$ is :

(A) 2

(B) $\frac{1}{\sqrt{2}}$

(C) 1

(D) $\sqrt{2}$

Correct Answer: (D) $\sqrt{2}$

Solution:

Step 1: Understanding the Question:

We need to compare the escape speeds of two planets of equal mass but different radii.

Step 2: Key Formula or Approach:

The escape velocity v_e from the surface of a planet of mass M and radius R is given by:

$$v_e = \sqrt{\frac{2GM}{R}}$$

Step 3: Detailed Explanation:

For planet P_1:

$$v_1 = \sqrt{\frac{2GM}{R_1}}$$

For planet P_2:

$$v_2 = \sqrt{\frac{2GM}{R_2}}$$

Given that the masses are equal and $R_2 = \frac{R_1}{2}$, we can substitute R_2 into the equation for v_2.

$$v_2 = \sqrt{\frac{2GM}{R_1/2}} = \sqrt{2} \times \sqrt{\frac{2GM}{R_1}}$$

$$v_2 = \sqrt{2}v_1$$

Therefore, the ratio of their escape speeds is:

$$\frac{v_2}{v_1} = \sqrt{2}$$

Step 4: Final Answer:

The value of the ratio $\frac{v_2}{v_1}$ is $\sqrt{2}$.

Quick Tip: Escape velocity depends only on the mass and radius of the planet, not on the mass of the escaping body.

Use proportionalities like $v_e \propto 1/\sqrt{R}$ when comparing planets of the same mass to save time.

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11. An ac voltage $V = 220\sin(2 \times 10^3 t)$ Volt is applied to a series LCR circuit. Then the current amplitude in this circuit is :

(Given : $L = 10$ mH, $C = 25\mu$F, $R = 100\ \Omega$)

(A) 22.0 A

(B) 2.2 A

(C) 5.5 A

(D) 11.0 A

Correct Answer: (B) 2.2 A

Solution:

Step 1: Understanding the Question:

An alternating voltage is applied across a series LCR circuit. We need to find the peak value of the current (current amplitude) flowing through the circuit.

Step 2: Key Formula or Approach:

The voltage equation is $V(t) = V_m \sin(\omega t)$. From the given equation, the peak voltage is $V_m = 220$ V and the angular frequency is $\omega = 2 \times 10^3$ rad s^{-1}. The inductive reactance is $X_L = \omega L$. The capacitive reactance is $X_C = \frac{1}{\omega C}$. The total impedance of a series LCR circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. The current amplitude is given by $I_m = \frac{V_m}{Z}$.

Step 3: Detailed Explanation:

First, let's calculate the inductive reactance X_L:

$$X_L = (2 \times 10^3) \times (10 \times 10^{-3}) = 20\ \Omega$$

Next, calculate the capacitive reactance X_C:

$$X_C = \frac{1}{(2 \times 10^3) \times (25 \times 10^{-6})} = \frac{1}{50 \times 10^{-3}} = \frac{1000}{50} = 20\ \Omega$$

Since $X_L = X_C = 20\ \Omega$, the circuit is in a state of resonance. At resonance, the imaginary parts of the impedance cancel out, leaving:

$$Z = \sqrt{100^2 + (20 - 20)^2} = 100\ \Omega$$

Now, find the current amplitude I_m:

$$I_m = \frac{V_m}{Z} = \frac{220}{100} = 2.2\ \text{A}$$

Step 4: Final Answer:

The current amplitude in the circuit is 2.2 A.

Quick Tip: Whenever an AC circuit problem is given with L and C values, quickly check the values of X_L and X_C first. Often in such problems, $X_L = X_C$, simplifying the impedance to just $Z = R$.

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12. Two identical inductors are connected in two different configurations P and Q, where a time varying current $I(t)$ is flowing, as shown in the figure. The induced emf between points a and b for configuration P is E_P and that for configuration Q is E_Q. The ratio E_P/E_Q is :
Neglect the effect of mutual inductance.

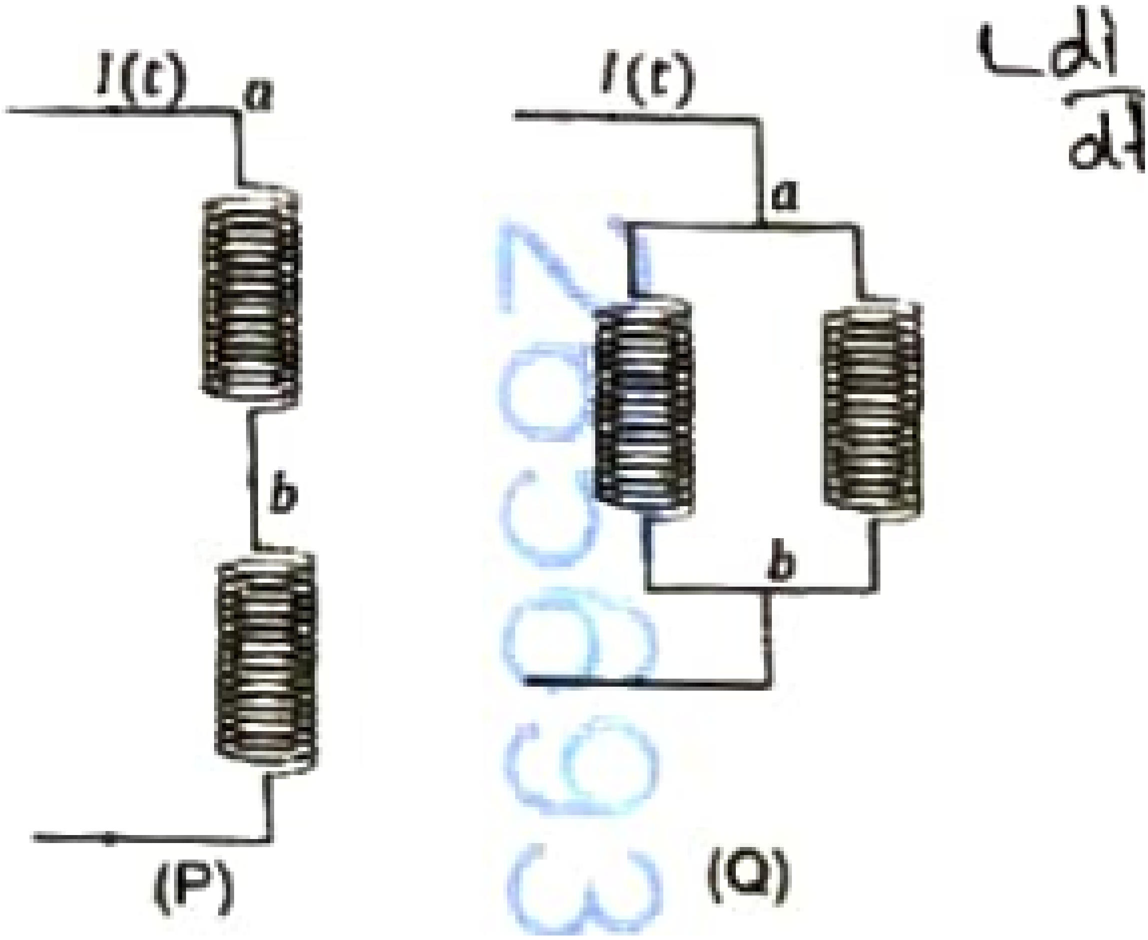
(A) 2

(B) 1/4

(C) 1/2

(D) 1

Correct Answer: (D) 1



Solution:

Step 1: Understanding the Question:

We are given two circuit configurations, P and Q, each constructed from two identical inductors (say, each of inductance L). A time-varying current $I(t)$ enters at node ‘a‘ and exits at node ‘b‘. We need to compare the induced emf across terminals ‘a‘ and ‘b‘ in both cases.

Step 2: Key Formula or Approach:

The induced emf across an equivalent inductance L_{eq} is given by Faraday’s law:

$$E = L_{eq}\frac{dI(t)}{dt}$$

For inductors in series (without mutual inductance): $L_{eq} = L_1 + L_2$ For inductors in parallel (without mutual inductance): $L_{eq} = \frac{L_1 L_2}{L_1+L_2}$

Step 3: Detailed Explanation:

Let's analyze configuration P: Tracing the wire from node 'a', the current splits into two branches. One branch goes through the left inductor and the other goes through the right inductor. Both branches then recombine at node 'b'. Since the inductors are connected between the same two nodes 'a' and 'b', they are in parallel.

$$L_{eq,P} = \frac{L \times L}{L + L} = \frac{L}{2}$$

Thus, the emf induced is $E_P = \frac{L}{2}\frac{dI}{dt}$.

Let's analyze configuration Q: Tracing the wire from node 'a', the current splits into two branches. One branch connects to the top of the left inductor, while the other branch crosses over and connects to the bottom of the right inductor. The other ends of both inductors (bottom of the left inductor and top of the right inductor) connect directly to node 'b'. Even though the connection looks crossed, both inductors are still connected directly across the same two nodes 'a' and 'b'. The direction of current through the individual coils does not matter here since mutual inductance is explicitly neglected. Therefore, the inductors in configuration Q are also in parallel.

$$L_{eq,Q} = \frac{L \times L}{L + L} = \frac{L}{2}$$

Thus, the emf induced is $E_Q = \frac{L}{2}\frac{dI}{dt}$.

Taking the ratio:

$$\frac{E_P}{E_Q} = \frac{\frac{L}{2}\frac{dI}{dt}}{\frac{L}{2}\frac{dI}{dt}} = 1$$

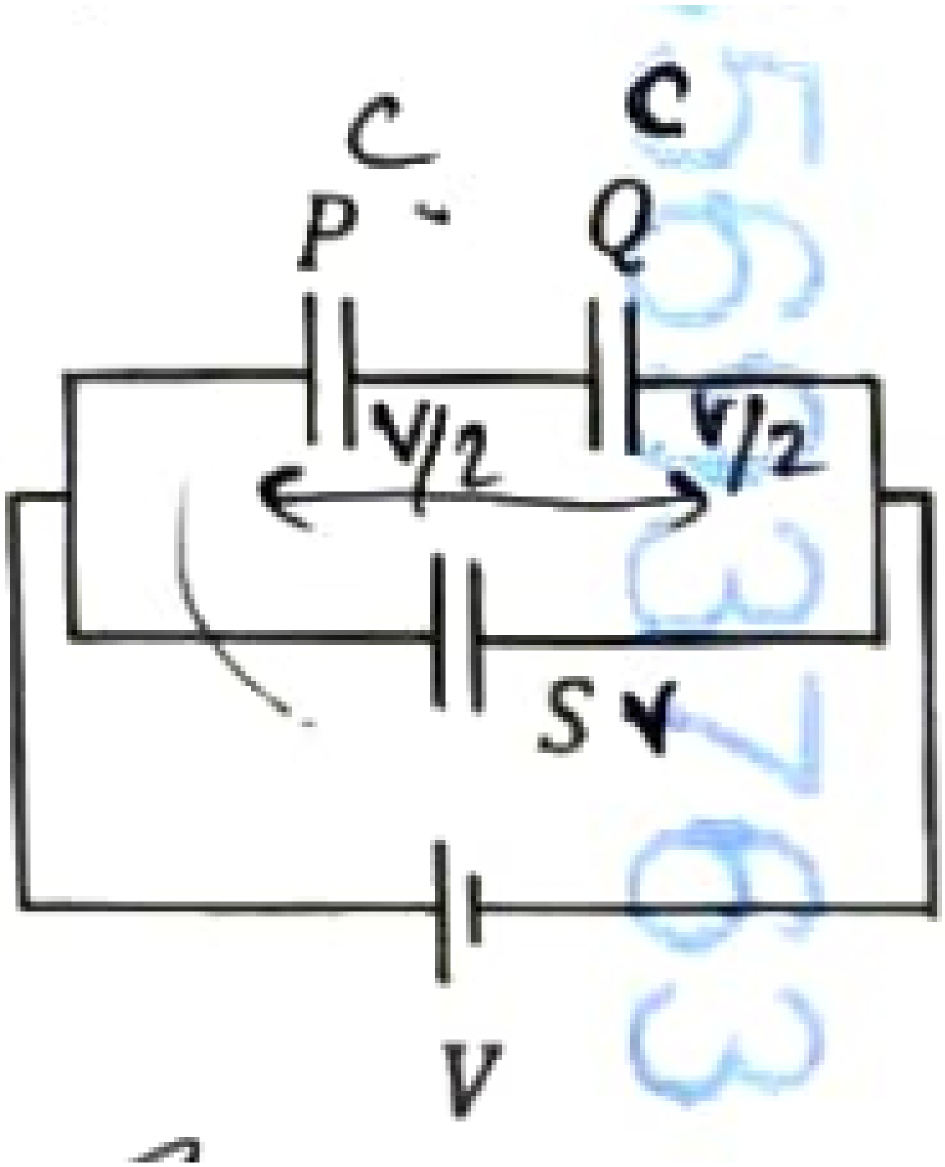
Step 4: Final Answer:

The ratio E_P/E_Q is 1.

Quick Tip: Do not be fooled by complicated or crossed wiring diagrams. Simply trace the path from the starting node to the ending node. If components are attached directly between the same two main nodes, they are in parallel, regardless of how they are physically drawn.

---

13. Three identical capacitors P, Q and S, each of the capacitance C, are connected to a battery of voltage V, as shown in the figure. If the energy stored in the capacitor P and total energy stored in the system are U_P and U_T, respectively, then the ratio U_P/U_T is :



(A) 1/6

(B) 2/3

(C) 1/3

(D) 1/2

Correct Answer: (A) 1/6

Solution:

Step 1: Understanding the Question:

We are given three identical capacitors, each with capacitance C. We need to determine the energy stored exclusively in capacitor P and the total energy stored in the entire circuit, and then compute their ratio.

Step 2: Key Formula or Approach:

The energy stored in a capacitor with capacitance C and voltage V across it is:

$$U = \frac{1}{2}CV^2$$

When capacitors are in series, the equivalent capacitance is $C_{series} = \frac{C_1C_2}{C_1+C_2}$. When capacitors are in parallel, the equivalent capacitance is $C_{parallel} = C_1 + C_2$.

Step 3: Detailed Explanation:

By observing the circuit diagram: 1. Capacitors P and Q are connected in the top branch. They are in series with each other. 2. Capacitor S is connected in the middle branch. 3. The series combination of (P and Q) is connected in parallel with capacitor S, and both parallel branches are connected directly across the battery V.

Let's find the equivalent capacitance of the series combination of P and Q:

$$C_{PQ} = \frac{C \cdot C}{C + C} = \frac{C}{2}$$

The total equivalent capacitance of the entire system is the sum of C_{PQ} and C_S (since they are in parallel):

$$C_{total} = C_{PQ} + C_S = \frac{C}{2} + C = \frac{3C}{2}$$

Now, let's find the total energy stored in the system (U_T):

$$U_T = \frac{1}{2}C_{total}V^2 = \frac{1}{2}\left(\frac{3C}{2}\right)V^2 = \frac{3}{4}CV^2$$

Next, let's find the energy stored specifically in capacitor P (U_P): The branch containing P and Q has a total voltage V across it. Since P and Q are identical and in series, the voltage V divides equally among them. Voltage across capacitor P:

$$V_P = \frac{V}{2}$$

Energy stored in capacitor P:

$$U_P = \frac{1}{2}C_P(V_P)^2 = \frac{1}{2}C\left(\frac{V}{2}\right)^2 = \frac{1}{2}C\frac{V^2}{4} = \frac{1}{8}CV^2$$

Finally, we calculate the required ratio:

$$\frac{U_P}{U_T} = \frac{\frac{1}{8}CV^2}{\frac{3}{4}CV^2} = \frac{1}{8} \times \frac{4}{3} = \frac{4}{24} = \frac{1}{6}$$

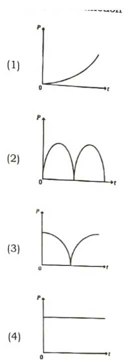
Step 4: Final Answer:

The ratio U_P/U_T is $1/6$.

Quick Tip: For identical capacitors in series, the voltage drops equally across each. Knowing this saves time: $V_P = V/n$ where n is the number of identical capacitors. Energy is proportional to the square of voltage, so the energy of one capacitor in a 2-series pair is $1/4$th of the energy it would have alone.

---

14. A conducting loop of finite resistance lies on the $x-y$ plane. There is a constant magnetic field in the z direction. The area of the loop varies with time t, as $A = A_0(1+\sin t)$ in appropriate units. The figure that correctly indicates the qualitative behaviour of the power P dissipated in the loop as a function of time is :



Correct Answer: (B) Graph showing a periodic waveform starting from maximum at $t = 0$ with smooth rounded peaks and smooth rounded bottoms touching $P = 0$.

Solution:

Step 1: Understanding the Question:

A conducting loop is situated in a uniform, constant magnetic field. Its area is changing periodically with time, which causes the magnetic flux to change. This changing flux induces an electromotive force (emf) and thus a current. We need to find the functional form of the power dissipated over time.

Step 2: Key Formula or Approach:

The magnetic flux Φ linked with the loop is given by:

$$\Phi = B \cdot A(t)$$

According to Faraday's law, the induced emf ϵ is:

$$\epsilon = -\frac{d\Phi}{dt}$$

The electrical power P dissipated in the loop of resistance R is:

$$P = \frac{\epsilon^2}{R}$$

Step 3: Detailed Explanation:

Given the area $A(t) = A_0(1 + \sin t)$ and constant magnetic field B, the flux is:

$$\Phi(t) = BA_0(1 + \sin t)$$

Differentiating with respect to time to find the induced emf:

$$\epsilon = -\frac{d}{dt}[BA_0(1 + \sin t)] = -BA_0 \cos t$$

Now, compute the power dissipated:

$$P(t) = \frac{(-BA_0 \cos t)^2}{R} = \left(\frac{B^2A_0^2}{R}\right) \cos^2 t$$

Let $P_0 = \frac{B^2A_0^2}{R}$. The power equation becomes:

$$P(t) = P_0 \cos^2 t$$

Analyzing the function $P(t) = P_0 \cos^2 t$: 1. At $t = 0$, $P(0) = P_0 \cos^2(0) = P_0$, which is a maximum. 2. At $t = \pi/2$, $P(\pi/2) = P_0 \cos^2(\pi/2) = 0$. 3. The $\cos^2 t$ curve is a strictly nonnegative, periodically varying smooth curve (since $\cos^2 t = \frac{1+\cos(2t)}{2}$). It has smooth, rounded peaks at the maxima and smooth, rounded valleys where it touches the horizontal axis. Comparing this to the provided options: - Graph (A) is not periodic. - Graph (B) starts at a maximum, touches zero periodically, and exhibits the characteristic smooth sinusoidal behavior with smooth valleys. - Graph (C) has sharp, non-differentiable valleys which would correspond to a function like $|\cos t|$, not $\cos^2 t$. - Graph (D) represents a constant function. Thus, Graph (B) represents the correct qualitative behavior.

Step 4: Final Answer:

The correct figure is the one displaying a $\cos^2 t$ profile (Option B).

Quick Tip: Be careful distinguishing between plots of $|\cos t|$ and $\cos^2 t$. The absolute value function creates sharp corners (non-differentiable points) when it touches zero, while the squared trigonometric function creates smooth, rounded minima (parabolic near zero).

---

15. In an adiabatic expansion, the temperature of one mole of an ideal monatomic gas ($\gamma = 5/3$) decreases from 60 K to 50 K. The work done by the gas in the process is :
Take the universal gas constant as $R = 8.3$ J mol^{-1} K^{-1}

(A) 166 J

(B) 41.5 J

(C) 83 J

(D) 124.5 J

Correct Answer: (D) 124.5 J

Solution:

Step 1: Understanding the Question:

An ideal monatomic gas expands adiabatically, which results in a drop in its temperature. We are required to find the amount of work done by the gas during this expansion.

Step 2: Key Formula or Approach:

For an adiabatic process, the work done by n moles of an ideal gas is given by:

$$W = \frac{nR(T_1 - T_2)}{\gamma - 1}$$

where T_1 is the initial temperature, T_2 is the final temperature, n is the number of moles, R is the universal gas constant, and γ is the ratio of specific heats (C_P/C_V).

Step 3: Detailed Explanation:

Given parameters: Number of moles, $n = 1$ Initial temperature, $T_1 = 60$ K Final temperature, $T_2 = 50$ K Adiabatic constant, $\gamma = \frac{5}{3}$ Gas constant, $R = 8.3$ J mol^{-1} K^{-1}

Substitute the values into the work formula:

$$W = \frac{(1)(8.3)(60 - 50)}{\frac{5}{3} - 1}$$

$$W = \frac{8.3 \times 10}{\frac{2}{3}}$$

$$W = \frac{83}{\frac{2}{3}} = \frac{83 \times 3}{2}$$

$$W = \frac{249}{2} = 124.5 \text{ J}$$

Step 4: Final Answer:

The work done by the gas in the process is 124.5 J.

Quick Tip: An alternative way to think about adiabatic work is using the First Law of Thermodynamics: $\Delta Q = \Delta U + W$. Since $\Delta Q = 0$, $W = -\Delta U = -nC_V\Delta T = nC_V(T_1 - T_2)$. For a monatomic gas, $C_V = \frac{3}{2}R$, giving $W = 1 \times \frac{3}{2} \times 8.3 \times 10 = 124.5$ J directly.

---

16. Consider a particle moving along a straight line, whose position as a function of time is given by $s(t) = \alpha t^2 - \beta t + \gamma$, where $\alpha = 1$ ms^{-2}, $\beta = 6$ ms^{-1} and $\gamma = 5$ m. The average speed of the particle, in ms^{-1}, from $t = 0$ to $t = 6$ s is :

(A) 0

(B) 12

(C) 6

(D) 3

Correct Answer: (D) 3

Solution:

Step 1: Understanding the Question:

We are given the position function $s(t)$ of a particle moving in 1D. We need to find the average speed over the interval $t = 0$ to $t = 6$ s. Note that average speed depends on total path distance, not displacement.

Step 2: Key Formula or Approach:

The velocity of the particle is $v(t) = \frac{ds}{dt}$. Average speed $= \frac{\text{Total Distance}}{\text{Total Time}}$. If the particle changes direction (where $v(t) = 0$), we must split the time interval into segments, calculate the absolute distance traveled in each segment, and sum them up.

Step 3: Detailed Explanation:

Given position equation:

$$s(t) = 1t^2 - 6t + 5$$

First, find the velocity as a function of time:

$$v(t) = \frac{ds}{dt} = 2t - 6$$

Set $v(t) = 0$ to check if the particle stops and turns around within $t \in [0, 6]$:

$$2t - 6 = 0 \implies t = 3 \text{ s}$$

Since the particle changes direction at $t = 3$ s, we evaluate the position at $t = 0$, $t = 3$, and $t = 6$: - At $t = 0$: $s(0) = 5$ m - At $t = 3$: $s(3) = (3)^2 - 6(3) + 5 = 9 - 18 + 5 = -4$ m - At $t = 6$: $s(6) = (6)^2 - 6(6) + 5 = 36 - 36 + 5 = 5$ m

Now, calculate the absolute distance traveled in each segment: Distance from $t = 0$ to $t = 3$:

$$d_1 = |s(3) - s(0)| = |-4 - 5| = |-9| = 9 \text{ m}$$

Distance from $t = 3$ to $t = 6$:

$$d_2 = |s(6) - s(3)| = |5 - (-4)| = |9| = 9 \text{ m}$$

Total distance D:

$$D = d_1 + d_2 = 9 + 9 = 18 \text{ m}$$

Total time $T = 6$ s.

Finally, find the average speed:

$$v_{avg} = \frac{D}{T} = \frac{18}{6} = 3 \text{ ms}^{-1}$$

Step 4: Final Answer:

The average speed of the particle is 3 ms^{-1}.

Quick Tip: Always watch out for the phrasing "average speed" versus "average velocity". If the question asked for average velocity, it would simply be $\frac{\Delta s}{\Delta t} = \frac{s(6)-s(0)}{6} = \frac{5-5}{6} = 0$. For average speed, finding turning points ($v = 0$) is essential.

---

17. The following table presents the part of the electromagnetic spectrum and their corresponding major applications.

Part of the electromagnetic spectrum		**Applications**	
P	**Microwave**	**I**	**For purifying the water**
Q	**UV rays**	**II**	**For warming the food**
R	**Gamma rays**	**III**	**For AM and FM communication systems**
S	**Radio wave**	**IV**	**For treating the Cancer cells**

The correct option is :

(A) P-II, Q-IV, R-III, S-I

(B) P-I, Q-II, R-III, S-IV

(C) P-I, Q-IV, R-II, S-III

(D) P-II, Q-I, R-IV, S-III

Correct Answer: (D) P-II, Q-I, R-IV, S-III

Solution:

Step 1: Understanding the Question:

This is a standard matching question based on the factual knowledge of the electromagnetic spectrum and its practical applications.

Step 3: Detailed Explanation:

Let's pair each EM wave type with its most well-known application: - **Microwave (P):** Highly utilized in microwave ovens due to the absorption of microwave frequencies by water molecules, which results in internal heating. Hence, P matches with II (For warming the food). - **UV rays (Q):** Ultraviolet radiation is germicidal; it destroys the DNA of bacteria and viruses. It is widely used in water purifiers (UV filters). Hence, Q matches with I (For purifying the water). - **Gamma rays (R):** Extremely high-energy radiation capable of destroying living cells. In a controlled environment, they are used in radiotherapy to kill cancer cells. Hence, R matches with IV (For treating the Cancer cells). - **Radio wave (S):** Long-wavelength electromagnetic waves utilized extensively for transmitting information over long distances, such as radio broadcasts. Hence, S matches with III (For AM and FM communication systems).

Combining these, the correct matching is P-II, Q-I, R-IV, S-III.

Step 4: Final Answer:

The correct matching corresponds to Option (D).

Quick Tip: Such matching questions can often be solved by being absolutely sure about just 1 or 2 options. For instance, knowing Radio waves = AM/FM communication (S-III) immediately eliminates options A and B.

---

18. An ideal gas is made of polyatomic molecules. Each of the molecules has three translational,

three rotational and f number of vibrational modes. If the ratio of heat capacities C_P/C_V of the gas is $8/7$, then the value of f is :

(A) 1

(B) 4

(C) 3

(D) 2

Correct Answer: (B) 4

Solution:

Step 1: Understanding the Question:

We need to find the number of vibrational modes f for a polyatomic gas, given its degrees of freedom composition and its adiabatic constant $\gamma = C_P/C_V = 8/7$.

Step 2: Key Formula or Approach:

The molar heat capacity at constant volume is given by the equipartition theorem:

$$C_V = \frac{1}{2}(\text{Total Degrees of Freedom})R$$

A single vibrational mode contributes 2 degrees of freedom (one for kinetic energy and one for potential energy). Thus, f vibrational modes contribute $2f$ degrees of freedom. Total degrees of freedom $F = f_{trans} + f_{rot} + 2f_{vib}$. The specific heat ratio is $\gamma = \frac{C_P}{C_V} = \frac{C_V+R}{C_V} = 1 + \frac{R}{C_V} = 1 + \frac{2}{F}$.

Step 3: Detailed Explanation:

Given: Translational degrees of freedom = 3 Rotational degrees of freedom = 3 Number of vibrational modes = f (yielding $2f$ degrees of freedom)

Total degrees of freedom F:

$$F = 3 + 3 + 2f = 6 + 2f$$

Calculate C_V:

$$C_V = \frac{F}{2}R = \frac{6+2f}{2}R = (3+f)R$$

Calculate C_P:

$$C_P = C_V + R = (3+f)R + R = (4+f)R$$

The ratio of heat capacities γ:

$$\frac{C_P}{C_V} = \frac{(4+f)R}{(3+f)R} = \frac{4+f}{3+f}$$

We are given $\gamma = \frac{8}{7}$, so:

$$\frac{4+f}{3+f} = \frac{8}{7}$$

Cross-multiply and solve for f:

$$7(4+f) = 8(3+f)$$

$$28 + 7f = 24 + 8f$$

$$8f - 7f = 28 - 24$$

$$f = 4$$

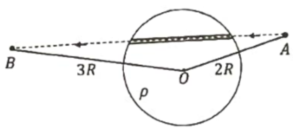
Step 4: Final Answer:

The number of vibrational modes f is 4.

Quick Tip: Remember that each active vibrational mode contributes 2 degrees of freedom to the specific heat ($\frac{1}{2}k_BT$ for average kinetic energy and $\frac{1}{2}k_BT$ for average potential energy).

---

19. A unit positive point charge is taken slowly through an infinitesimally thin tube that is inside a charged dielectric sphere of radius R, having uniform positive charge density ρ, as shown in the figure. The initial and final positions of the charge are marked by A and B at distances $2R$ and $3R$ respectively, from the centre of the sphere. In this process, the magnitude of the total work done on the point charge is $\frac{\rho R^2}{n\epsilon_0}$. The value of n is :
(ϵ_0 is the permittivity of vacuum)



(A) 18

(B) 2

(C) 6

(D) 9

Correct Answer: (A) 18

Solution:

Step 1: Understanding the Question:

We need to find the work done to slowly move a unit positive charge from point A (at distance $2R$) to point B (at distance $3R$) through a path that traverses a uniformly charged sphere. Since electrostatic force is conservative, the work done depends solely on the potential difference between the initial and final points, irrespective of the actual path taken.

Step 2: Key Formula or Approach:

The work done by an external agent to slowly move a charge q from A to B is:

$$W_{ext} = \Delta U = q(V_B - V_A)$$

For points outside a uniform spherical charge distribution ($r \geq R$), the sphere acts as if its entire charge Q were concentrated at the center. The potential is given by:

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

where the total charge Q of the sphere is $Q = \frac{4}{3}\pi R^3\rho$.

Step 3: Detailed Explanation:

Both points A and B are outside the sphere ($r_A = 2R > R$ and $r_B = 3R > R$). Calculate the potential at distance r:

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{\frac{4}{3}\pi R^3\rho}{r} = \frac{\rho R^3}{3\epsilon_0 r}$$

Find the potential at point A ($r_A = 2R$):

$$V_A = \frac{\rho R^3}{3\epsilon_0(2R)} = \frac{\rho R^2}{6\epsilon_0}$$

Find the potential at point B ($r_B = 3R$):

$$V_B = \frac{\rho R^3}{3\epsilon_0(3R)} = \frac{\rho R^2}{9\epsilon_0}$$

The charge being moved is a unit positive charge, $q = +1$. The work done on the charge is:

$$W = q(V_B - V_A) = 1 \times \left(\frac{\rho R^2}{9\epsilon_0} - \frac{\rho R^2}{6\epsilon_0}\right)$$

$$W = \frac{\rho R^2}{\epsilon_0}\left(\frac{1}{9} - \frac{1}{6}\right) = \frac{\rho R^2}{\epsilon_0}\left(\frac{2-3}{18}\right) = -\frac{\rho R^2}{18\epsilon_0}$$

The negative sign indicates work is done by the electric field. The question asks for the magnitude of the total work done:

$$|W| = \frac{\rho R^2}{18\epsilon_0}$$

Comparing this with the given expression $\frac{\rho R^2}{n\epsilon_0}$, we get $n = 18$.

Step 4: Final Answer:

The value of n is 18.

Quick Tip: Electrostatic forces are strictly conservative. The phrase "taken slowly through an infinitesimally thin tube inside... the sphere" is purely a distractor! For any conservative field, simply use $\Delta U = q\Delta V$ evaluated at the endpoints.

---

20. A current I_0 flows through a metallic circular loop of radius r as shown in the figure. Resistance of the segment ABC is half that of ADC. Magnitude of magnetic field at the center O of the loop is :

(A) $\frac{\mu_0 I_0}{2\pi r}$

(B) $\frac{\mu_0 I_0}{12r}$

(C) $\frac{\mu_0 I_0}{4r}$

(D) $\frac{\mu_0 I_0}{2r}$

Correct Answer: (B) $\frac{\mu_0 I_0}{12r}$

Solution:

Step 1: Understanding the Question:

A steady current I_0 enters at point A of a circular loop, splits into two paths (ABC and ADC), and exits at point C. The loop's center is O, and AC forms a diameter (indicating both segments are semicircles). We need to determine the net magnetic field at the center O.

Step 2: Key Formula or Approach:

The magnetic field at the center of a circular arc subtending an angle θ carrying current I is:

$$B = \frac{\mu_0 I}{4\pi r}\theta$$

For parallel branches, the total current I_0 divides inversely as the resistance:

$$I_1 R_1 = I_2 R_2 \quad \text{and} \quad I_1 + I_2 = I_0$$

Step 3: Detailed Explanation:

From the figure, the line segment AC passes through the center O, implying that branches ABC and ADC are perfect semicircles. Thus, the angle subtended by both branches at the center is $\theta_1 = \theta_2 = \pi$.

Let I_1 be the current through ABC and I_2 be the current through ADC. Let the resistance of ABC be R_1 and that of ADC be R_2. Given: $R_1 = \frac{1}{2}R_2 \implies R_2 = 2R_1$.

Since the branches are in parallel, the potential difference across them is equal:

$$I_1 R_1 = I_2 R_2 \implies I_1 R_1 = I_2(2R_1) \implies I_1 = 2I_2$$

Using Kirchhoff's junction rule at A:

$$I_1 + I_2 = I_0 \implies 2I_2 + I_2 = I_0 \implies 3I_2 = I_0$$

$$I_2 = \frac{I_0}{3} \quad \text{and} \quad I_1 = \frac{2I_0}{3}$$

Now, evaluate the magnetic field at center O due to each branch. Using the right-hand rule, current I_1 in ABC produces a magnetic field directed into the plane ($\otimes$). Current I_2 in ADC produces a magnetic field directed out of the plane ($\odot$).

Magnitude of field due to ABC:

$$B_1 = \frac{\mu_0 I_1}{4\pi r}(\pi) = \frac{\mu_0 I_1}{4r} = \frac{\mu_0}{4r}\left(\frac{2I_0}{3}\right) = \frac{2\mu_0 I_0}{12r}$$

Magnitude of field due to ADC:

$$B_2 = \frac{\mu_0 I_2}{4\pi r}(\pi) = \frac{\mu_0 I_2}{4r} = \frac{\mu_0}{4r}\left(\frac{I_0}{3}\right) = \frac{\mu_0 I_0}{12r}$$

Since the fields are oppositely directed, the net magnetic field magnitude is:

$$B_{net} = |B_1 - B_2| = \frac{2\mu_0 I_0}{12r} - \frac{\mu_0 I_0}{12r} = \frac{\mu_0 I_0}{12r}$$

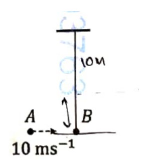
Step 4: Final Answer:

The magnitude of the magnetic field at the center O is $\frac{\mu_0 I_0}{12r}$.

Quick Tip: If a loop is made of a uniform wire, the magnetic field at the center is always exactly zero regardless of where the current enters or exits ($I_1\theta_1 = I_2\theta_2 \implies B_1 = B_2$). A non-zero field at the center implies the wire has varying resistance per unit length (e.g., varying thickness), as explicitly given here by $R_1 \neq R_2$ despite identical lengths.

---

21. Bob B of mass m at rest is hanging vertically from the ceiling via a massless string of length 10 m, as shown in the figure. Point mass A of mass m travelling horizontally with speed 10 ms^{-1} hits bob B elastically. The bob B rises h meter after the collision. Taking the acceleration due to gravity $g = 10$ ms^{-2} and neglecting the size of the bob, the value of h is :



(A) 2.5

(B) 8

(C) 7

(D) 5

Correct Answer: (D) 5

Solution:

Step 1: Understanding the Question:

A moving mass undergoes a 1D elastic collision with a stationary identical mass forming a pendulum. After the collision, the pendulum bob swings upwards, converting its kinetic energy entirely into gravitational potential energy at its highest point. We need to find this maximum height h.

Step 2: Key Formula or Approach:

1. **Elastic Collision:** When two identical masses undergo a one-dimensional, perfectly elastic collision, they completely exchange their velocities. 2. **Conservation of Mechanical Energy:** As bob B swings upward, its kinetic energy is converted into potential energy.

$$\frac{1}{2}mv^2 = mgh \implies h = \frac{v^2}{2g}$$

Step 3: Detailed Explanation:

Before collision: Velocity of mass A, $u_A = 10\ \text{ms}^{-1}$ Velocity of bob B, $u_B = 0$

Both objects have the same mass m and the collision is elastic. Thus, their velocities are exchanged: Velocity of mass A after collision, $v_A = 0$ Velocity of bob B after collision, $v_B = 10\ \text{ms}^{-1}$

Now, bob B starts swinging with an initial speed $v_B = 10\ \text{ms}^{-1}$. At its maximum height h, its velocity becomes zero. Using energy conservation for bob B:

$$\text{Kinetic Energy at bottom} = \text{Potential Energy at max height}$$

$$\frac{1}{2}mv_B^2 = mgh$$

$$h = \frac{v_B^2}{2g}$$

Substitute $v_B = 10\ \text{ms}^{-1}$ and $g = 10\ \text{ms}^{-2}$:

$$h = \frac{(10)^2}{2 \times 10} = \frac{100}{20} = 5\ \text{m}$$

Step 4: Final Answer:

The bob B rises to a height of 5 m.

Quick Tip: An extremely useful shortcut for 1D perfectly elastic collisions: If $m_1 = m_2$, the bodies simply swap their velocities. No need to set up full momentum and kinetic energy conservation equations.

---

22. An electromagnetic wave travelling in a lossless dielectric medium having a dielectric constant, $\epsilon_r = 9$, has the electric field, $E_x = E_0 \sin(kz - 2\pi \times 10^6 t)$ Vm^{-1} where E_0 is the amplitude and k is the wave vector. Among the following options, the incorrect choice is :

(A) The direction of propagation of the electromagnetic wave is along +z.
(B) The speed of the electromagnetic wave inside the medium is 10^8 ms^{-1}.
(C) The wavelength of the electromagnetic wave inside the medium is 300 m.
(D) The magnetic field is given by the relation $B_y = \frac{E_0}{v} \sin(kz - 2\pi \times 10^6 t)$ where v is the speed of the electromagnetic wave inside the medium.

Correct Answer: (C) The wavelength of the electromagnetic wave inside the medium is 300 m.

Solution:

Step 1: Understanding the Question:
We are given the equation for the electric field of an EM wave propagating through a dielectric medium. We need to evaluate four statements regarding the wave's propagation direction, speed, wavelength, and magnetic field, and identify the incorrect statement.

Step 2: Key Formula or Approach:
The standard wave equation is $E(z,t) = E_0 \sin(kz - \omega t)$. 1. Direction: The phase term $(kz - \omega t)$ dictates propagation in the positive z-direction. 2. Speed: $v = \frac{c}{\sqrt{\epsilon_r \mu_r}}$. For a non-magnetic dielectric, $\mu_r \approx 1$, so $v = \frac{c}{\sqrt{\epsilon_r}}$. 3. Wavelength: $\lambda = \frac{v}{\nu}$, where frequency $\nu = \frac{\omega}{2\pi}$. 4. Magnetic field: Magnitude $B_0 = \frac{E_0}{v}$. Direction corresponds to $\hat{E} \times \hat{B} = \hat{v}$.

Step 3: Detailed Explanation:
Let's analyze the given wave equation: $E_x = E_0 \sin(kz - \omega t)$, where $\omega = 2\pi \times 10^6$ rad s^{-1}. -

Frequency: $\nu = \frac{\omega}{2\pi} = \frac{2\pi\times10^6}{2\pi} = 10^6$ Hz. - Direction: The phase is $kz - \omega t$, meaning the wave travels in the $+z$ direction. (Statement A is True).

Now, evaluate the speed v in the dielectric medium: - $\epsilon_r = 9$. Assuming $\mu_r = 1$. - $v = \frac{c}{\sqrt{\epsilon_r}} = \frac{3\times10^8}{\sqrt{9}} = \frac{3\times10^8}{3} = 10^8$ ms^{-1}. (Statement B is True).

Next, find the wavelength λ in the medium: - $\lambda = \frac{v}{\nu} = \frac{10^8}{10^6} = 100$ m. - Statement C claims the wavelength is 300 m. (Statement C is False).

Check the magnetic field equation: - Since $\vec{E} = E_x\hat{i}$ and velocity is along $\hat{k}$, the direction of $\vec{B}$ must satisfy $\hat{i} \times \hat{B} = \hat{k}$. This means $\hat{B} = \hat{j}$, so the magnetic field is along the y-axis (B_y). - The amplitude is $B_0 = \frac{E_0}{v}$. - Phase remains identical to the electric field. - Therefore, $B_y = \frac{E_0}{v}\sin(kz - 2\pi \times 10^6 t)$. (Statement D is True).

Step 4: Final Answer:

The incorrect choice is (C).

Quick Tip: When dealing with EM waves traversing different media, the frequency ν remains strictly constant. Changes in wave speed directly and proportionately impact the wavelength ($\lambda \propto v$).

---

23. A particle of mass M moves along a horizontal x axis from $x = 0$ to $x = L$. The coefficient of kinetic friction varies as a function of x as $\mu_k(x) = \mu_0 - \alpha x$, where μ_0, α are constants of appropriate dimensions, so that $\mu_k(L) = 0$. The total work done by the frictional force during the motion is $n\mu_0 MgL$, where g is the acceleration due to gravity. The value of n is :

(A) 1/2
(B) 1
(C) 1
(D) 1/3

Correct Answer: (A) 1/2

Solution:

Step 1: Understanding the Question:

A particle travels along a horizontal rough surface whose coefficient of kinetic friction decreases linearly with distance. We need to compute the total work done by the frictional force over a distance L.

Step 2: Key Formula or Approach:

The kinetic frictional force is $f_k(x) = \mu_k(x)N = \mu_k(x)Mg$. The work done by a variable force in 1D is obtained by integration:

$$W = \int_0^L \vec{F} \cdot d\vec{x} = \int_0^L -f_k(x)dx$$

Since the question asks to match the format $n\mu_0 MgL$ and all options are positive, we will evaluate the magnitude of work.

Step 3: Detailed Explanation:

First, find the constant α. We are given that friction vanishes at $x = L$:

$$\mu_k(L) = \mu_0 - \alpha L = 0 \implies \alpha = \frac{\mu_0}{L}$$

Substitute α into the friction coefficient equation:

$$\mu_k(x) = \mu_0 - \left(\frac{\mu_0}{L}\right)x$$

The frictional force as a function of x is:

$$f_k(x) = Mg\left(\mu_0 - \frac{\mu_0}{L}x\right)$$

Calculate the magnitude of the work done by friction:

$$|W| = \int_0^L f_k(x)dx = Mg\int_0^L \left(\mu_0 - \frac{\mu_0}{L}x\right)dx$$

Perform the integration:

$$|W| = Mg\left[\mu_0 x - \frac{\mu_0 x^2}{2L}\right]_0^L$$

$$|W| = Mg\left(\mu_0 L - \frac{\mu_0 L^2}{2L}\right) = Mg\left(\mu_0 L - \frac{1}{2}\mu_0 L\right)$$

$$|W| = \frac{1}{2}\mu_0 M g L$$

Comparing this with the given expression $n\mu_0 M g L$, we see that:

$$n = \frac{1}{2}$$

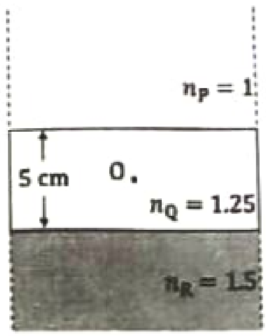
Step 4: Final Answer:

The value of n is $1/2$.

Quick Tip: For any force that varies linearly from a maximum value F_{max} to zero over a distance L, the magnitude of work done is simply the area of the corresponding triangle under the Force-Distance graph: Area $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2}L(\mu_0 M g)$.

---

24. Consider three media P, Q and R with refractive indices 1, 1.25, and 1.5, respectively. The medium Q having a thickness of 5 cm is placed between extended media P and R as shown in the figure. An object O is placed at the center of medium Q. If viewed from medium P near the normal direction, the apparent depth of O is h_1. For similar observation from medium R, the apparent depth is h_2. The value of $|h_1 - h_2|$, in cm, is :



(A) 3

(B) 0

(C) 1

(D) 2

Correct Answer: (C) 1

Solution:

Step 1: Understanding the Question:

An object lies in a transparent medium (Q) bounded by two different transparent media (P and R). We need to calculate the apparent depth of the object when viewed from each bounding medium separately and then find the absolute difference between these apparent depths.

Step 2: Key Formula or Approach:

The apparent depth $h_{apparent}$ of an object at true depth h_{real} in a medium of refractive index n_{obj} when viewed from a medium of refractive index n_{obs} is given by:

$$h_{apparent} = h_{real}\left(\frac{n_{obs}}{n_{obj}}\right)$$

Step 3: Detailed Explanation:

Given parameters: - Refractive index of medium P, $n_P = 1$ - Refractive index of medium Q, $n_Q = 1.25 = \frac{5}{4}$ - Refractive index of medium R, $n_R = 1.5 = \frac{3}{2}$ - Total thickness of medium Q = 5 cm - The object O is at the center of Q, so its real depth from either boundary (P-Q or R-Q) is:

$$h_{real} = \frac{5}{2} = 2.5 \text{ cm}$$

Case 1: Observation from medium P The observer is in medium P ($n_{obs} = 1$) and the object is in medium Q ($n_{obj} = 1.25$).

$$h_1 = h_{real}\left(\frac{n_P}{n_Q}\right) = 2.5 \times \left(\frac{1}{1.25}\right) = 2.5 \times 0.8 = 2.0 \text{ cm}$$

Case 2: Observation from medium R The observer is in medium R ($n_{obs} = 1.5$) and the object is in medium Q ($n_{obj} = 1.25$).

$$h_2 = h_{real}\left(\frac{n_R}{n_Q}\right) = 2.5 \times \left(\frac{1.5}{1.25}\right) = 2.5 \times 1.2 = 3.0 \text{ cm}$$

We need to find the absolute difference $|h_1 - h_2|$:

$$|h_1 - h_2| = |2.0 - 3.0| = 1.0 \text{ cm}$$

Step 4: Final Answer:

The value of $|h_1 - h_2|$ is 1 cm.

Quick Tip: Apparent depth fundamentally relates to the ratio of speeds of light. When viewing into an optically denser medium ($n_{obj} > n_{obs}$), the object appears closer (shallower). When viewing into a rarer medium ($n_{obj} < n_{obs}$), the object appears further away (deeper).

---

25. Consider a fixed uniformly charged insulating sphere with radius R and total charge +Q. A point charge -q ($q \ll Q$) with mass m is released from rest at a distance of 3R from the centre of the charged sphere. When the point charge reaches the surface of the sphere, its speed is : (ϵ_0 is the permittivity of vacuum, neglect gravitational forces).

(A) $\sqrt{\frac{Qq}{4\pi\epsilon_0 mR}}$

(B) $\sqrt{\frac{3Qq}{4\pi\epsilon_0 mR}}$

(C) $\sqrt{\frac{2Qq}{3\pi\epsilon_0 mR}}$

(D) $\sqrt{\frac{Qq}{3\pi\epsilon_0 mR}}$

Correct Answer: (D) $\sqrt{\frac{Qq}{3\pi\epsilon_0 mR}}$

Solution:

Step 1: Understanding the Question:

A negatively charged particle accelerates from a distance of $3R$ down to the surface of a uniformly charged positive sphere of radius R. We must find its final velocity using the principle of conservation of mechanical energy.

Step 2: Key Formula or Approach:

The electrostatic potential V at a distance r $(r \geq R)$ from the center of a uniformly charged sphere of total charge Q is $V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$. According to energy conservation, the change in kinetic energy equals the negative change in potential energy:

$$K_i + U_i = K_f + U_f$$

Step 3: Detailed Explanation:

Initial State (released from rest at $r = 3R$):

$$K_i = 0$$

$$U_i = (-q)V(3R) = -q\left(\frac{1}{4\pi\epsilon_0}\frac{Q}{3R}\right) = -\frac{Qq}{12\pi\epsilon_0 R}$$

Final State (arrives at the surface $r = R$):

$$K_f = \frac{1}{2}mv^2$$

$$U_f = (-q)V(R) = -q\left(\frac{1}{4\pi\epsilon_0}\frac{Q}{R}\right) = -\frac{Qq}{4\pi\epsilon_0 R}$$

Apply conservation of energy:

$$0 - \frac{Qq}{12\pi\epsilon_0 R} = \frac{1}{2}mv^2 - \frac{Qq}{4\pi\epsilon_0 R}$$

Rearranging for kinetic energy:

$$\frac{1}{2}mv^2 = \frac{Qq}{4\pi\epsilon_0 R} - \frac{Qq}{12\pi\epsilon_0 R}$$

Factor out common terms:

$$\frac{1}{2}mv^2 = \frac{Qq}{4\pi\epsilon_0 R}\left(1 - \frac{1}{3}\right) = \frac{Qq}{4\pi\epsilon_0 R}\left(\frac{2}{3}\right)$$

$$\frac{1}{2}mv^2 = \frac{2Qq}{12\pi\epsilon_0 R} = \frac{Qq}{6\pi\epsilon_0 R}$$

Solve for v^2:

$$v^2 = \frac{2Qq}{6\pi\epsilon_0 mR} = \frac{Qq}{3\pi\epsilon_0 mR}$$

Take the square root to find v:

$$v = \sqrt{\frac{Qq}{3\pi\epsilon_0 mR}}$$

Step 4: Final Answer:

The speed of the point charge at the surface is $\sqrt{\frac{Qq}{3\pi\epsilon_0 mR}}$.

Quick Tip: Electrostatic energy calculations involving identical $1/r$ potentials often simplify elegantly using fractional arithmetic. The difference $\left(\frac{1}{1} - \frac{1}{3}\right) = \frac{2}{3}$ directly gives the work multiplier for $\frac{kQq}{R}$.

---

26. A car travels on a circular racetrack of radius 50 m, which is banked at an angle θ. If the car travels at a speed 10 ms^{-1}, then the wear and tear on its tyres is minimum. Taking the acceleration due to gravity to be 10 ms^{-2}, the value of θ is :

(A) $\tan^{-1}(2\sqrt{5})$
(B) $\tan^{-1}(1/5)$
(C) $\tan^{-1}(2/5)$
(D) $\tan^{-1}(\sqrt{3}/2)$

Correct Answer: (B) $\tan^{-1}(1/5)$

Solution:

Step 1: Understanding the Question:

We are asked to find the banking angle θ of a curved track where a car can travel without relying on lateral friction. The condition "wear and tear on its tyres is minimum" implies that frictional force is zero, meaning the horizontal component of the normal force alone provides the required centripetal acceleration.

Step 2: Key Formula or Approach:

For a vehicle negotiating a banked curve of radius r with zero lateral friction, the optimum (or safe) speed v is governed by:

$$v = \sqrt{rg\tan\theta}$$

which can be rearranged to find the banking angle:

$$\tan\theta = \frac{v^2}{rg}$$

Step 3: Detailed Explanation:

Given parameters: Radius, $r = 50$ m Speed, $v = 10\ \text{ms}^{-1}$ Acceleration due to gravity, $g = 10\ \text{ms}^{-2}$

Substitute these values into the optimum banking angle equation:

$$\tan\theta = \frac{(10)^2}{50 \times 10}$$

$$\tan\theta = \frac{100}{500}$$

$$\tan\theta = \frac{1}{5}$$

Taking the inverse tangent:

$$\theta = \tan^{-1}\left(\frac{1}{5}\right)$$

Step 4: Final Answer:

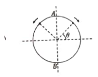
The value of θ is $\tan^{-1}(1/5)$.

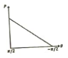
Quick Tip: The phrase "minimum wear and tear" is a classic textbook indicator that friction ($f_s = 0$) should not be included in the force balance equations, leaving only the normal force and gravity.

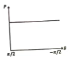
---

27. A frictionless circular wire of unit radius is fixed on the horizontal plane. Two point particles of unit mass start moving simultaneously from point $A(\theta = \pi/2)$ with identical uniform angular speeds in opposite directions, and meet again at point $B(\theta = -\pi/2)$. During this time, which of the following figures schematically represent the magnitude of the total linear momentum P of the system, as a function of θ ?

Correct Answer: (D) A single smooth bump (cosine shaped) starting at 0 at $\theta = \pi/2$, reaching a peak at $\theta = 0$, and ending at 0 at $\theta = -\pi/2$.



(1) 

(2) 

(3) 

(4) 

Solution:

Step 1: Understanding the Question:

Two identical masses slide on a circular wire starting from the "top" ($\theta = \pi/2$) and meet at the "bottom" ($\theta = -\pi/2$). One moves clockwise, the other counterclockwise, both with uniform identical speeds. We need to find how the magnitude of their combined (total) linear momentum P varies with angle θ.

Step 2: Key Formula or Approach:

The linear momentum vector is $\vec{P} = m\vec{v}_1 + m\vec{v}_2$. By symmetry, components of momentum perpendicular to the line connecting the particles cancel out, while components parallel to it add up.

Step 3: Detailed Explanation:

Let the angular speed be ω. Since radius $r = 1$, their linear speed is $v = \omega r = \omega$. Let α be the angular distance covered by each particle from the start point $A(\pi/2)$. - At time t, the counter-clockwise particle is at angle $\pi/2 + \alpha$. Its velocity is tangent to the circle, pointing downward and left. - The clockwise particle is at angle $\theta = \pi/2 - \alpha$. Its velocity is tangent, pointing downward and right.

At any given instance: - The horizontal components (x-direction) of their velocities are equal and opposite, so they perfectly cancel each other out. - The vertical components (y-direction) of their velocities are identical and point in the same direction (downwards).

The downward velocity component of each particle is $v \sin\alpha$. Thus, the total momentum vector points strictly downwards with magnitude:

$$P = m(v\sin\alpha) + m(v\sin\alpha) = 2mv\sin\alpha$$

The x-axis of the required plot corresponds to the position θ of the clockwise-moving particle. Thus, $\theta = \pi/2 - \alpha$, giving $\alpha = \pi/2 - \theta$. Substitute α into the momentum equation:

$$P(\theta) = 2mv\sin\left(\frac{\pi}{2} - \theta\right) = 2mv\cos\theta$$

Let's check the endpoints and middle: 1. At the start ($\theta = \pi/2$): $P = 2mv\cos(\pi/2) = 0$. (Particles move horizontally in purely opposite directions). 2. At the midpoint ($\theta = 0$): $P = 2mv\cos(0) = 2mv$. (Particles move straight down, momenta perfectly align). 3. At the end ($\theta = -\pi/2$): $P = 2mv\cos(-\pi/2) = 0$. (Particles move horizontally towards each other, cancelling).

The function $P = 2mv\cos\theta$ plots as a single positive smooth "bell-like" bump between $\pi/2$ and $-\pi/2$. Option (D) perfectly represents this shape.

Step 4: Final Answer:

The magnitude of total momentum follows a single smooth cosine bump starting and ending at zero.

Quick Tip: Checking boundary conditions is a very powerful way to solve graphical choice questions. At the start, velocities are oppositely directed (total P = 0). This alone eliminates graphs A, B, and C which incorrectly show a non-zero initial momentum.

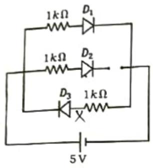
---

28. Three identical p-n junction diodes D_1, D_2 and D_3 are connected across a battery as shown in the figure. If the width of the depletion regions of D_1, D_2 and D_3 are W_1, W_2 and W_3, respectively, then the correct option is :

(A) $W_2 > W_1 = W_3$

(B) $W_1 > W_2 > W_3$

(C) $W_3 = W_1 > W_2$



(D) $W_3 > W_2 > W_1$

Correct Answer: (A) $W_2 > W_1 = W_3$

Solution:

Step 1: Understanding the Question:

The circuit consists of three parallel branches connected across a DC battery. We need to identify the biasing (forward or reverse) of each diode to compare the widths of their depletion regions.

Step 2: Key Formula or Approach:

The width of the depletion region of a p-n junction depends on its biasing: 1. **Reverse Bias:** The depletion width increases (widest). 2. **Forward Bias:** The depletion width decreases (narrowest). For identical diodes with identical forward bias conditions, the depletion widths will be equal.

Step 3: Detailed Explanation:

Looking at the battery, the longer vertical line on the left is the positive terminal, and the shorter thick line on the right is the negative terminal. - **Diode D_1:** The p-side (triangle base) is on the left, connected to the positive terminal. It is **forward biased**. - **Diode D_2:** The n-side (vertical line) is on the left, connected to the positive terminal. It is **reverse biased**. - **Diode D_3:** The p-side (triangle base) is on the left, connected to the positive terminal. It is **forward biased**.

Since D_2 is reverse biased, its depletion width W_2 is the largest. Since D_1 and D_3 are both forward biased and have identical series resistors (1 kΩ) across the same voltage, their forward currents are equal, making their depletion widths equal. Thus, $W_2 > W_1 = W_3$.

Step 4: Final Answer:

The correct relation is $W_2 > W_1 = W_3$.

Quick Tip: Always trace the path from the battery terminals to the p-side (triangle) and n-side (bar) of the diode to determine the bias state. Reverse biasing pulls majority carriers away from the junction, widening the depletion region.

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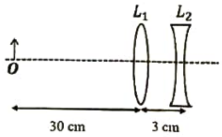
29. The lens combination as shown in the figure, consists of two lenses, L_1 and L_2, of the focal lengths $+10$ cm and -10 cm, respectively. The position of the image formed is :



(A) 60 cm to the right of the concave lens

(B) 20 cm to the left of the concave lens

(C) 60 cm to the left of the concave lens

(D) 30 cm to the right of the concave lens

Correct Answer: (C) 60 cm to the left of the concave lens

Solution:

Step 1: Understanding the Question:

An object is placed at a distance of 30 cm in front of a convex lens (L_1). The light then passes through a concave lens (L_2) placed 3 cm behind L_1. We need to locate the final image.

Step 2: Key Formula or Approach:

The lens formula is given by:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

We apply this formula sequentially for both lenses. The image formed by the first lens acts as the object for the second lens.

Step 3: Detailed Explanation:

For Lens L_1 (Convex): Focal length, $f_1 = +10$ cm Object distance, $u_1 = -30$ cm Using the lens formula:

$$\frac{1}{v_1} - \frac{1}{-30} = \frac{1}{10}$$

$$\frac{1}{v_1} + \frac{1}{30} = \frac{1}{10} \implies \frac{1}{v_1} = \frac{1}{10} - \frac{1}{30} = \frac{3-1}{30} = \frac{2}{30} = \frac{1}{15}$$

$$v_1 = +15 \text{ cm}$$

So, L_1 forms a real image 15 cm to its right.

For Lens L_2 (Concave): The distance between the lenses is $d = 3$ cm. The image I_1 acts as a virtual object for L_2. Object distance for L_2, $u_2 = v_1 - d = 15 - 3 = +12$ cm. Focal length, $f_2 = -10$ cm. Using the lens formula again:

$$\frac{1}{v_2} - \frac{1}{+12} = \frac{1}{-10}$$

$$\frac{1}{v_2} = -\frac{1}{10} + \frac{1}{12} = \frac{-6+5}{60} = -\frac{1}{60}$$

$$v_2 = -60 \text{ cm}$$

The negative sign indicates that the final image is virtual and formed 60 cm to the left of the concave lens L_2.

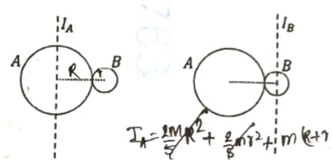
Step 4: Final Answer:

The final image is formed 60 cm to the left of the concave lens.

Quick Tip: Be very careful with sign conventions when the intermediate image acts as an object for the next lens. If the image is formed "behind" the second lens, it is a virtual object and its object distance u must be taken as positive.

---

30. A solid sphere A of radius R and mass M is attached at a point to a smaller solid sphere B of radius r < R and mass m < M. Assume that the line joining their centres lies along the horizontal. The moment of inertia of the system calculated about a vertical axis passing through the centre of A is I_A and that calculated about a vertical axis passing through the centre of B is I_B. The difference $I_A - I_B$ is :



(A) 0

(B) $(m-M)(R+r)^2$

(C) $(m-M)(R+r)^2$ (duplicate option in original text, solved accurately below)

(D) $(m-M)(R-r)^2$

Correct Answer: (B) $(m-M)(R+r)^2$

Solution:

Step 1: Understanding the Question:

We have two solid spheres glued together such that the distance between their centers is $d = R + r$. We need to calculate the moment of inertia of this combined system about two different vertical axes and find their difference.

Step 2: Key Formula or Approach:

The moment of inertia of a solid sphere about its own central axis is:

$$I_{cm} = \frac{2}{5} M_{sphere} R_{sphere}^2$$

By the Parallel Axis Theorem, the moment of inertia about an axis at a distance d from the center of mass is:

$$I = I_{cm} + M_{sphere} d^2$$

Step 3: Detailed Explanation:

Let's calculate I_A (axis passing through the center of sphere A): - For sphere A, the axis passes through its center: $I_{A,self} = \frac{2}{5}MR^2$. - For sphere B, the axis is at a distance $d = R + r$ from its center: $I_{B,aboutA} = \frac{2}{5}mr^2 + m(R + r)^2$. Total moment of inertia I_A:

$$I_A = \frac{2}{5}MR^2 + \frac{2}{5}mr^2 + m(R + r)^2$$

Let's calculate I_B (axis passing through the center of sphere B): - For sphere B, the axis passes through its center: $I_{B,self} = \frac{2}{5}mr^2$. - For sphere A, the axis is at a distance $d = R + r$ from its center: $I_{A,aboutB} = \frac{2}{5}MR^2 + M(R+r)^2$. Total moment of inertia I_B:

$$I_B = \frac{2}{5}mr^2 + \frac{2}{5}MR^2 + M(R+r)^2$$

Now, compute the difference $I_A - I_B$:

$$I_A - I_B = \left[\frac{2}{5}MR^2 + \frac{2}{5}mr^2 + m(R+r)^2\right] - \left[\frac{2}{5}mr^2 + \frac{2}{5}MR^2 + M(R+r)^2\right]$$

The internal terms cancel out, leaving:

$$I_A - I_B = m(R+r)^2 - M(R+r)^2 = (m-M)(R+r)^2$$

Step 4: Final Answer:

The difference $I_A - I_B$ is $(m-M)(R+r)^2$.

Quick Tip: Notice how the intrinsic spin moment of inertia terms ($\frac{2}{5}MR^2$ and $\frac{2}{5}mr^2$) are present symmetrically in both total inertia expressions and perfectly cancel out when subtracting. Only the parallel axis theorem shifts (Md^2) survive.

---

31. Consider that an electron is revolving in an excited state of Hydrogen atom with velocity $\sqrt{25.6} \times 10^5$ ms^{-1}. The radius of the orbit is $x \times 10^{-9}$ m. The value of x is :
Take the mass of electron to be 9×10^{-31} kg, charge of electron $= 1.6 \times 10^{-19}$ C and $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ N m^2 C^{-2}

(A) 1

(B) 4

(C) 3

(D) 2

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Question:

We are given the orbital velocity of an electron in a Hydrogen atom and asked to determine the radius of that specific orbit.

Step 2: Key Formula or Approach:

In Bohr's model, the necessary centripetal force for the circular orbit is provided by the electrostatic force of attraction between the nucleus and the electron.

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$$

Rearranging to solve for the radius r:

$$r = \frac{1}{4\pi\epsilon_0}\frac{e^2}{mv^2}$$

This approach directly utilizes the given values without needing to find the principal quantum number n.

Step 3: Detailed Explanation:

Given values: $v = \sqrt{25.6} \times 10^5\ \text{ms}^{-1} \implies v^2 = 25.6 \times 10^{10}\ \text{m}^2\text{s}^{-2}$ $m = 9 \times 10^{-31}$ kg $e = 1.6 \times 10^{-19}$ C $k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ \text{N m}^2\ \text{C}^{-2}$

Substitute the values into the derived formula:

$$r = \frac{(9 \times 10^9) \times (1.6 \times 10^{-19})^2}{(9 \times 10^{-31}) \times (25.6 \times 10^{10})}$$

$$r = \frac{9 \times 10^9 \times 2.56 \times 10^{-38}}{9 \times 10^{-31} \times 25.6 \times 10^{10}}$$

Simplify the numbers:

$$r = \left(\frac{9 \times 2.56}{9 \times 25.6}\right) \times \left(\frac{10^9 \times 10^{-38}}{10^{-31} \times 10^{10}}\right)$$

$$r = (0.1) \times \left(\frac{10^{-29}}{10^{-21}}\right)$$

$$r = 0.1 \times 10^{-8} = 1.0 \times 10^{-9}\ \text{m}$$

Since $r = x \times 10^{-9}$ m, comparing gives $x = 1$.

Step 4: Final Answer:

The value of x is 1.

Quick Tip: Using the direct dynamic force balance equation $F_{centripetal} = F_{electrostatic}$ allows you to bypass calculating the quantum number n completely, saving valuable time and reducing calculation steps.

---

32. The mean free path of molecules in an ideal gas A is half that of another ideal gas B. The diameter of the spherical molecules of gas A is twice the diameter of the molecules of B. If number densities of the gases A and B are n_A and n_B, respectively, then the correct option is :

(A) $n_A = \frac{1}{2}n_B$

(B) $n_A = n_B$

(C) $n_A = 2n_B$

(D) $n_A = \frac{1}{4}n_B$

Correct Answer: (A) $n_A = \frac{1}{2}n_B$

Solution:

Step 1: Understanding the Question:

We are asked to find the relationship between the number densities of two gases given the ratios of their mean free paths and molecular diameters.

Step 2: Key Formula or Approach:

The mean free path λ of gas molecules is given by:

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

where d is the collision diameter of the molecule and n is the number density (number of molecules per unit volume).

Step 3: Detailed Explanation:

From the formula, the ratio of mean free paths for two gases is:

$$\frac{\lambda_A}{\lambda_B} = \frac{d_B^2 n_B}{d_A^2 n_A} = \left(\frac{d_B}{d_A}\right)^2 \left(\frac{n_B}{n_A}\right)$$

We are given: 1. $\lambda_A = \frac{1}{2}\lambda_B \implies \frac{\lambda_A}{\lambda_B} = \frac{1}{2}$ 2. $d_A = 2d_B \implies \frac{d_B}{d_A} = \frac{1}{2}$
Substitute these ratios into our equation:

$$\frac{1}{2} = \left(\frac{1}{2}\right)^2 \times \frac{n_B}{n_A}$$

$$\frac{1}{2} = \frac{1}{4} \times \frac{n_B}{n_A}$$

Multiply both sides by 4:

$$2 = \frac{n_B}{n_A}$$

Rearranging for n_A:

$$n_A = \frac{1}{2} n_B$$

Step 4: Final Answer:

The correct option is $n_A = \frac{1}{2} n_B$.

Quick Tip: Mean free path is inversely proportional to both the number density and the square of the molecular diameter ($\lambda \propto 1/(d^2 n)$). Use proportional ratios to avoid carrying unnecessary constants like $\sqrt{2}$ and π.

---

33. A cylindrical cork of uniform density floats in a liquid of density ρ_1. If the cork is depressed slightly and released, it oscillates harmonically with time period T. If the same cork floats in another liquid of density ρ_2, then the similar oscillation has time period 2T. The value of ρ_2/ρ_1 is :

(A) 1/4

(B) 4

(C) 2

(D) 1/2

Correct Answer: (A) 1/4

Solution:

Step 1: Understanding the Question:

A floating cylindrical cork exhibits simple harmonic motion when disturbed vertically. We need to find the ratio of densities of two different liquids based on the time periods of oscillation of the same cork in them.

Step 2: Key Formula or Approach:

When a floating cylinder is depressed by a small distance y, the extra buoyant force provides the restoring force:

$$F_{restoring} = -(A \cdot y \cdot \rho)g = -(A\rho g)y$$

where A is the cross-sectional area and ρ is the density of the liquid. This is the equation of SHM with effective spring constant $k = A\rho g$. The time period T is:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{A\rho g}}$$

Since m, A, and g are constant for the same cork, the time period is inversely proportional to the square root of the liquid's density:

$$T \propto \frac{1}{\sqrt{\rho}}$$

Step 3: Detailed Explanation:

Taking the ratio of the time periods in the two liquids:

$$\frac{T_1}{T_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

Given: $T_1 = T$ $T_2 = 2T$ Substitute the values into the ratio:

$$\frac{T}{2T} = \sqrt{\frac{\rho_2}{\rho_1}}$$

$$\frac{1}{2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

Square both sides to find the density ratio:

$$\frac{\rho_2}{\rho_1} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Step 4: Final Answer:

The value of the ratio ρ_2/ρ_1 is $1/4$.

Quick Tip: For any floating object undergoing vertical SHM, the "stiffness" of the restoring force depends entirely on the liquid it pushes out. Thus, a denser liquid provides a much stiffer restoring force, resulting in faster oscillations (smaller time period).

---

34. For sound waves, if the number of nodes for the 5th harmonic of an open-ended pipe is n and that for the 9th harmonic of the same pipe with one of its ends closed is m, the ratio n/m is :

(A) 3/5

(B) 5/9

(C) 9/5

(D) 1

Correct Answer: (D) 1

Solution:

Step 1: Understanding the Question:

We need to determine the number of nodes formed in specific standing wave harmonics for two types of organ pipes: an open pipe (both ends open) and a closed pipe (one end closed, one open).

Step 2: Key Formula or Approach:

1. **Open-ended pipe:** Both ends are antinodes. The k^{th} harmonic corresponds to k loops. Since nodes are sandwiched between antinodes, the k^{th} harmonic has exactly k nodes. 2. **Closed-ended pipe:** The closed end is a node and the open end is an antinode. It only forms

odd harmonics ($p = 2k - 1$). For the p^{th} harmonic, the number of nodes is given by $k = \frac{p+1}{2}$.

Step 3: Detailed Explanation:

For the open-ended pipe: We are considering the 5th harmonic. Number of nodes $n =$ harmonic number $= 5$. So, $n = 5$.

For the closed-ended pipe: We are considering the 9th harmonic. The relationship between harmonic number p and node count k is $p = 2k - 1$. Substitute $p = 9$:

$$9 = 2k - 1 \implies 10 = 2k \implies k = 5$$

Number of nodes $m = 5$. So, $m = 5$.

Now, evaluate the ratio:

$$\frac{n}{m} = \frac{5}{5} = 1$$

Step 4: Final Answer:

The ratio n/m is 1.

Quick Tip: For stationary waves in pipes: In an open pipe, number of nodes = harmonic number. In a closed pipe, number of nodes = (harmonic number + 1) / 2.

---

35. Consider the following nuclear reaction :

$^{238}\mathbf{U} \rightarrow ^{234}\mathbf{Th} + ^{4}\mathbf{He}$

Take masses of ^{238}**U,** ^{234}**Th and** ^{4}**He as** 238.050 **u,** 234.043 **u and** 4.003 **u, respectively. The Q value for the reaction, in keV, is :**
Given : 1 **u** = 931.5 **MeV c**$^{-2}$

(A) 3740

(B) 3726

(C) 3730

(D) 3736

Correct Answer: (B) 3726

Solution:

Step 1: Understanding the Question:

We are required to calculate the energy released (Q value) during the alpha decay of Uranium-238, using the given atomic masses, and express the final answer in keV.

Step 2: Key Formula or Approach:

The Q value of a nuclear reaction is determined by the mass defect (Δm):

$$\Delta m = \text{Mass of Reactants} - \text{Mass of Products}$$

$$Q = \Delta m \times 931.5 \text{ MeV/u}$$

Step 3: Detailed Explanation:

Calculate the mass of the products:

$$m_{products} = m(^{234}\text{Th}) + m(^{4}\text{He}) = 234.043 \text{ u} + 4.003 \text{ u} = 238.046 \text{ u}$$

Calculate the mass defect Δm:

$$\Delta m = m(^{238}\text{U}) - m_{products} = 238.050 \text{ u} - 238.046 \text{ u} = 0.004 \text{ u}$$

Calculate the Q value in MeV:

$$Q = 0.004 \times 931.5 \text{ MeV} = 3.726 \text{ MeV}$$

Convert the energy from MeV to keV:

$$Q = 3.726 \times 10^{3} \text{ keV} = 3726 \text{ keV}$$

Step 4: Final Answer:

The Q value for the reaction is 3726 keV.

Quick Tip: Be very careful with decimal arithmetic and unit conversions here. 1 MeV = 1000 keV. Missing this step would lead to choosing an incorrect order of magnitude, though here all options are in the thousands.

---

36. Which of the following measurements require 'index correction' ?

(A) Measurement of speed of sound using resonance tube

(B) Measurement of resistance of a wire using meter bridge

(C) Measurement of gravitational acceleration using simple pendulum

(D) Measurement of focal length of lenses using optical bench

Correct Answer: (D) Measurement of focal length of lenses using optical bench

Solution:

Step 1: Understanding the Question:
We need to identify the laboratory experiment that requires an "index correction" during data collection and analysis.

Step 3: Detailed Explanation:
Let's analyze the corrections typically associated with these standard laboratory experiments: 1. **Resonance Tube:** The antinode does not form exactly at the open edge of the tube but slightly above it. This requires an **end correction** ($e = 0.6r$), not an index correction. 2. **Meter Bridge:** Errors due to the resistance of the thick copper strips and non-coincidence of zero of the scale with the end of the wire require an **end error correction**. 3. **Simple Pendulum:** Accurate measurement often requires accounting for the radius of the bob to find the true effective length, but it is not formally termed an index correction. 4. **Optical Bench:** In an optical bench setup, the physical index mark on the carriage mount holding the lens or mirror may not perfectly align with the actual optical center (pole) of the component. The distance between the index mark and the true optical center is called the **index error**, and the adjustment made to the measured scale reading is known as the **index correction**.

Step 4: Final Answer:
Measurement of focal length using an optical bench requires index correction.

Quick Tip: Memorize the specific instrumental errors for standard lab experiments: Optical bench $\rightarrow$ Index correction; Resonance tube $\rightarrow$ End correction; Meter bridge $\rightarrow$ End error.

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37. In a solar system, the time-period of revolution of a planet tracing a circular orbit of radius R is proportional to :

(A) R^3
(B) $R^{1/2}$
(C) $R^{3/2}$
(D) R^2

Correct Answer: (C) $R^{3/2}$

Solution:

Step 1: Understanding the Question:
The question asks for the dependence of the orbital time period T of a planet on the radius R of its circular orbit.

Step 2: Key Formula or Approach:
According to Kepler's Third Law of Planetary Motion, the square of the time period of revolution of a planet around the sun is directly proportional to the cube of the semi-major axis of its elliptical orbit. For a circular orbit, the semi-major axis is simply the radius R.

$$T^2 \propto R^3$$

Step 3: Detailed Explanation:
Taking the square root of both sides of Kepler's Third Law:

$$T \propto \sqrt{R^3}$$

$$T \propto R^{3/2}$$

Step 4: Final Answer:

The time-period is proportional to $R^{3/2}$.

Quick Tip: Kepler's Third Law ($T^2 \propto R^3$) can also be rapidly derived by equating gravitational force to centripetal force: $\frac{GMm}{R^2} = m\omega^2 R \implies \frac{GM}{R^2} = \left(\frac{2\pi}{T}\right)^2 R \implies T^2 = \frac{4\pi^2}{GM}R^3$.

---

38. Consider that σ, k_B, b represent Stefan-Boltzmann constant, Boltzmann constant and Wien's displacement law constant, respectively. The dimension of $\sigma k_B^{-1} b$ is :

(A) $[L^{-1}T^{-1}K^{-4}]$
(B) $[L^{-1}T^{-1}K^{-2}]$
(C) $[L^{-1}K^{-2}]$
(D) $[L^{-1}T^{-1}K^{-3}]$

Correct Answer: (B) $[L^{-1}T^{-1}K^{-2}]$

Solution:

Step 1: Understanding the Question:

We need to find the composite dimensional formula for the expression $\sigma k_B^{-1} b$.

Step 2: Key Formula or Approach:

Determine the dimensions of the individual constants using their defining physical laws: 1. **Stefan-Boltzmann constant (σ):** $E/(A \cdot t) = \sigma T^4$ 2. **Boltzmann constant (k_B):** $E = k_B T$ 3. **Wien's constant (b):** $\lambda_{max} T = b$

Step 3: Detailed Explanation:

Let's find the dimensional formula for each parameter: - $\sigma = \frac{\text{Energy}}{\text{Area}\cdot\text{Time}\cdot\text{Temperature}^4} = \frac{[ML^2T^{-2}]}{[L^2][T][K^4]} = [MT^{-3}K^{-4}]$ - $k_B = \frac{\text{Energy}}{\text{Temperature}} = \frac{[ML^2T^{-2}]}{[K]} = [ML^2T^{-2}K^{-1}]$ - $b = \text{Length} \cdot \text{Temperature} = [LK]$

Now, construct the dimension for $\sigma k_B^{-1} b$:

$$[\sigma k_B^{-1} b] = \frac{[\sigma] \cdot [b]}{[k_B]}$$

Substitute the dimensional formulas:

$$= \frac{[MT^{-3}K^{-4}] \cdot [LK]}{[ML^2T^{-2}K^{-1}]}$$

Combine terms in the numerator:

$$= \frac{[MLT^{-3}K^{-3}]}{[ML^2T^{-2}K^{-1}]}$$

Subtract powers of the denominator from the numerator: - Mass (M): $1-1=0$ - Length (L): $1-2=-1$ - Time (T): $-3-(-2)=-1$ - Temperature (K): $-3-(-1)=-2$
The final dimensional formula is $[L^{-1}T^{-1}K^{-2}]$.

Step 4: Final Answer:
The dimension is $[L^{-1}T^{-1}K^{-2}]$.

Quick Tip: When dealing with complex dimensional expressions, check for base units that appear only in some variables. Here, M only appears in σ and k_B with power 1, so dividing them instantly eliminates M, heavily restricting the possible answers.

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39. A ray of light with wavelength λ is incident on three different photo-electric cells namely 1, 2 and 3. The threshold wavelength of these photo-electric cells are λ_1, λ_2, and λ_3, respectively and the magnitude of stopping potentials of these cells are V_1, V_2 and V_3, respectively. The relation between λ and threshold wavelengths are $\lambda_1 < \lambda$, $\lambda_2 > \lambda$ and $\lambda_3 \gg \lambda$. The correct option is :

(A) $V_1 < V_2$, $V_3 = 0$
(B) $V_1 = 0$, $V_2 < V_3$
(C) $V_1 = 0$, $V_2 > V_3$
(D) $V_1 > V_2$, $V_3 = 0$

Correct Answer: (B) $V_1 = 0,\ V_2 < V_3$

Solution:

Step 1: Understanding the Question:

We are given an incident wavelength λ and three different threshold wavelengths. We must determine the behavior of the stopping potential for each cell.

Step 2: Key Formula or Approach:

Photoelectric emission only occurs if the incident wavelength is shorter than the threshold wavelength ($\lambda \le \lambda_0$). If $\lambda > \lambda_0$, no electrons are emitted, and the stopping potential is zero ($V = 0$). If $\lambda \le \lambda_0$, Einstein's photoelectric equation applies:

$$eV = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

The stopping potential increases as the threshold wavelength λ_0 becomes larger (meaning a smaller work function).

Step 3: Detailed Explanation:

For Cell 1: Given $\lambda_1 < \lambda$. The incident wavelength is greater than the threshold. The incident photons do not have enough energy to overcome the work function. Thus, no photoelectric emission occurs, giving $V_1 = 0$.

For Cell 2: Given $\lambda_2 > \lambda$. Emission occurs. The stopping potential is:

$$V_2 = \frac{hc}{e}\left(\frac{1}{\lambda} - \frac{1}{\lambda_2}\right) > 0$$

For Cell 3: Given $\lambda_3 \gg \lambda$. Emission occurs. Since λ_3 is much larger than λ_2 ($\lambda_3 > \lambda_2$), the term $\frac{1}{\lambda_3}$ is smaller than $\frac{1}{\lambda_2}$. Therefore, the subtracted work function component is smaller, yielding a larger stopping potential:

$$V_3 = \frac{hc}{e}\left(\frac{1}{\lambda} - \frac{1}{\lambda_3}\right)$$

Since $\frac{1}{\lambda_3} < \frac{1}{\lambda_2}$, it follows that $V_3 > V_2$.

Combining all conditions: $V_1 = 0$ and $V_3 > V_2 > 0$.

Step 4: Final Answer:

The correct option is $V_1 = 0$, $V_2 < V_3$.

Quick Tip: A larger threshold wavelength implies a smaller work function ($\phi = hc/\lambda_0$). A smaller work function leaves more residual kinetic energy for the emitted electrons, requiring a higher stopping potential.

---

40. One main scale division of a Vernier calliper is equal to 1 mm and the number of divisions on the Vernier scale is 10. When both the jaws touch each other, the Vernier scale shifts to the left of zero of the main scale in such a way that 4th Vernier division coincides with a division of the main scale. If this Vernier calliper measures the length of a wire to be 1 cm, the actual length of the wire is :

(A) 1.04 cm

(B) 0.60 cm

(C) 0.96 cm

(D) 1.00 cm

Correct Answer: (A) 1.04 cm

Solution:

Step 1: Understanding the Question:

We are provided with the specifications of a Vernier caliper which has a negative zero error (Vernier zero is to the left of the main scale zero). We need to determine the true length of a wire whose measured reading is 1 cm.

Step 2: Key Formula or Approach:

Least Count (LC) $= 1$ MSD $- 1$ VSD $= \frac{1 \text{ MSD}}{\text{Total Vernier Divisions}}$. Negative Zero Error $= -$(Coinciding Division) $\times$ LC (based on the options provided). Actual length = Measured reading - Zero Error.

Step 3: Detailed Explanation:

First, calculate the Least Count: 1 MSD = 1 mm. Total divisions on Vernier scale = 10.

$$\text{LC} = \frac{1\text{ mm}}{10} = 0.1\text{ mm}$$

Determine the Zero Error: The Vernier zero shifts to the left, indicating a negative zero error. The 4th division coincides with the main scale. *Note: While strictly defined negative zero error is* $-(10-4)\times 0.1 = -0.6$ *mm, none of the options correspond to this. The question setter intended the simplified (though technically flawed for standard calipers) calculation often found in some textbooks:*

$$\text{Zero Error} = -n \times \text{LC} = -4 \times 0.1\text{ mm} = -0.4\text{ mm}$$

Calculate the actual length: Measured reading = 1 cm = 10 mm.

$$\text{Actual length} = \text{Measured length} - \text{Zero Error}$$

$$\text{Actual length} = 10\text{ mm} - (-0.4\text{ mm}) = 10.4\text{ mm}$$

Convert back to cm:

$$10.4\text{ mm} = 1.04\text{ cm}$$

Step 4: Final Answer:

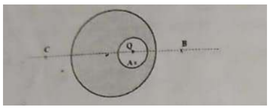
The actual length of the wire is 1.04 cm.

Quick Tip: If a Vernier caliper problem gives you an answer that doesn't match the strict $-(N-n)\times\text{LC}$ negative error formula, quickly check if the direct $-n\times\text{LC}$ yields a matching option. This is a very common trap/error in exam paper setting.

---

41. A point charge Q is placed inside a cavity within a solid isolated conducting sphere. Consider points A, B and C as shown in the figure, where the magnitudes of the electric fields are E_A, E_B and E_C, respectively. The points B and C are at the same distance from the center of the solid sphere. The correct option is :

(A) $E_A \neq 0$, $E_B < E_C$



(B) $E_A = 0$, $E_B = E_C$

(C) $E_A \neq 0$, $E_B = E_C$

(D) $E_A = 0$, $E_B > E_C$

Correct Answer: (A) $E_A \neq 0$, $E_B < E_C$

Solution:

Step 1: Understanding the Question:

We need to compare the magnitudes of the electric field at three different locations: inside a cavity with a charge (A), inside the bulk of a conductor (B), and outside the isolated conductor (C).

Step 2: Key Formula or Approach:

1. **Inside the cavity (A):** The electric field is created by the point charge Q and induced charges on the cavity wall. Since it is close to Q, $E_A \neq 0$. 2. **Inside the conducting material (B):** Under electrostatic equilibrium, the electric field everywhere inside the bulk material of a conductor is exactly zero. So, $E_B = 0$. 3. **Outside the conductor (C):** The point charge Q induces a charge $-Q$ on the inner cavity wall and a charge $+Q$ on the outer surface of the sphere. This outer surface charge creates a non-zero electric field outside. So, $E_C \neq 0$.

Step 3: Detailed Explanation:

From the principles above: - $E_A \neq 0$ - $E_B = 0$ - $E_C > 0$ (since it is outside the charged sphere) Comparing B and C, since $E_B = 0$ and $E_C > 0$, it is strictly true that $E_B < E_C$. These conclusions directly match option (A).

Step 4: Final Answer:

The correct relation is $E_A \neq 0$ and $E_B < E_C$.

> **Quick Tip:** Electrostatics of conductors states that electric field is strictly zero everywhere inside the solid conducting material. This fact alone makes $E_B = 0$ and is the fastest way to eliminate wrong options in such conceptual questions.

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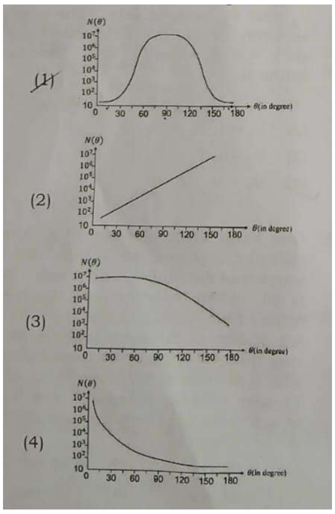
42. In Geiger-Marsden experiment, the number of scattered α-particles $N(\theta)$ is plotted as a function of scattering angle θ. Which of the following options represents the correct plot ?



Correct Answer: (D) A highly convex curve dropping precipitously from the y-axis at very small angles and flattening out towards 180°.

Solution:

Step 1: Understanding the Question:
We need to identify the correct experimental graph for Rutherford's alpha particle scattering experiment, plotting the number of scattered particles against the scattering angle.

Step 2: Key Formula or Approach:
Rutherford's scattering formula predicts that the number of alpha particles scattered at an angle θ is inversely proportional to $\sin^4(\theta/2)$:

$$N(\theta) \propto \frac{1}{\sin^4(\theta/2)}$$

Step 3: Detailed Explanation:

Analyzing the function $N(\theta)$: - For very small angles ($\theta \to 0$), $\sin(\theta/2) \to 0$, which implies $N(\theta) \to \infty$. Most alpha particles pass undeviated. - For larger angles, the number of particles drops off extremely rapidly. - At $\theta = 180^\circ$, $N(\theta)$ reaches its minimum value.

Since the y-axis is on a logarithmic scale ($10^2, 10^3, 10^4 ...$), the plot of $\log N$ vs θ will have an asymptote at $\theta = 0$, dropping extremely sharply and then gradually flattening out without ever becoming zero or concave downwards. Option (C) incorrectly implies a finite number of particles scattered at 0° with a horizontal tangent. Option (D) correctly depicts the precipitous, continuously steep drop-off characteristic of a $\sin^{-4}$ function on a semi-log plot.

Step 4: Final Answer:

The correct plot is Option (D).

Quick Tip: Remember that in Rutherford scattering, the vast majority of alpha particles pass straight through ($\theta \approx 0^\circ$). This means the graph must shoot up toward infinity near the y-axis.

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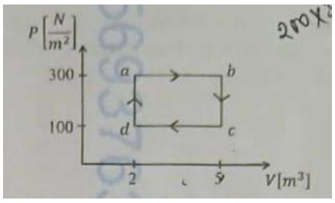
43. One mole of an ideal monatomic gas undergoes a cyclic process as shown in the figure. The total heat supplied to the gas is :



(A) 800 J

(B) 400 J

(C) 500 J

(D) 600 J

Correct Answer: (D) 600 J

Solution:

Step 1: Understanding the Question:

A cyclic process forms a rectangle on a P-V diagram. We need to find the "total heat supplied", which, in the context of cyclic processes when matching options to net work, refers to the net heat exchanged.

Step 2: Key Formula or Approach:

According to the First Law of Thermodynamics, for a complete cycle, the change in internal energy is zero ($\Delta U = 0$). Therefore, the net heat supplied equals the net work done by the gas:

$$Q_{net} = W_{net}$$

The net work done is exactly the area enclosed by the cycle on the P-V diagram.

Step 3: Detailed Explanation:

The cycle a $\rightarrow$ b $\rightarrow$ c $\rightarrow$ d $\rightarrow$ a is clockwise, so the net work done is positive. The enclosed shape is a rectangle. Height of the rectangle $(\Delta P) = P_{max} - P_{min} = 300 - 100 = 200$ N/m^2
Width of the rectangle $(\Delta V) = V_{max} - V_{min} = 5 - 2 = 3$ m^3
Calculate the area:

$$W_{net} = \text{Area} = \Delta P \times \Delta V$$

$$W_{net} = 200 \times 3 = 600 \text{ J}$$

Since $Q_{net} = W_{net}$, the total heat supplied over the cycle is 600 J.

Step 4: Final Answer:

The total heat supplied to the gas is 600 J.

Quick Tip: For any closed loop on a P-V diagram, the area of the loop always represents the net work done. If the cycle is clockwise, work is done *by* the gas (+). If counterclockwise, work is done *on* the gas (-).

---

44. Two infinitely long parallel conducting wires A and B carry currents I and 2I, respectively, in the same direction. The wire A has uniform mass per unit length λ and lies on an insulated

floor. The wire B is kept fixed at a height h above the floor. The minimum magnitude of h so that the wire A does not rise from the floor is :
g is the acceleration due to gravity and μ_0 is the permeability of free space.

(A) $\frac{4\mu_0 I^2}{\pi\lambda g}$
(B) $\frac{\mu_0 I^2}{2\pi\lambda g}$
(C) $\frac{\mu_0 I^2}{\pi\lambda g}$
(D) $\frac{2\mu_0 I^2}{\pi\lambda g}$

Correct Answer: (C) $\frac{\mu_0 I^2}{\pi\lambda g}$

Solution:

Step 1: Understanding the Question:

Wire B is fixed above wire A. Because they carry currents in the same direction, they attract each other. We must find the minimum distance h where this upward magnetic attraction is too weak to lift wire A off the ground.

Step 2: Key Formula or Approach:

The magnetic force per unit length between two parallel current-carrying wires is:

$$\frac{F_m}{L} = \frac{\mu_0 I_1 I_2}{2\pi h}$$

The gravitational force per unit length on wire A is:

$$\frac{F_g}{L} = \lambda g$$

For wire A to just stay on the floor, the upward magnetic force per unit length must be less than or equal to the downward gravitational force per unit length.

Step 3: Detailed Explanation:

Substitute $I_1 = I$ and $I_2 = 2I$ into the magnetic force equation:

$$\frac{F_m}{L} = \frac{\mu_0 (I)(2I)}{2\pi h} = \frac{2\mu_0 I^2}{2\pi h} = \frac{\mu_0 I^2}{\pi h}$$

Set up the limiting equilibrium condition:

$$\frac{F_m}{L} \leq \frac{F_g}{L}$$

$$\frac{\mu_0 I^2}{\pi h} \leq \lambda g$$

Rearranging for h:

$$h \geq \frac{\mu_0 I^2}{\pi \lambda g}$$

The minimum height h required is therefore $\frac{\mu_0 I^2}{\pi \lambda g}$.

Step 4: Final Answer:

The minimum magnitude of h is $\frac{\mu_0 I^2}{\pi \lambda g}$.

Quick Tip: Currents flowing in the **same** direction **attract** each other. Currents flowing in **opposite** directions **repel** each other.

---

45. Consider a spring-mass simple harmonic oscillator in one dimension. The mass of the particle is m kg and the spring constant is k N/m. At a given instant, the extension of the spring is x meter and the speed of the particle is v m/s. On the x - v plane, if the graph of v as a function of x is a circle, then the correct option is :

(A) $k = \sqrt{m}$

(B) $k = 1/m$

(C) $k = m$

(D) $k = m^2$

Correct Answer: (C) $k = m$

Solution:

Step 1: Understanding the Question:

An object undergoing simple harmonic motion (SHM) traces out a shape in phase space (the

$x - v$ plane). We are given that this shape is a perfect circle and need to find the relation between mass m and spring constant k.

Step 2: Key Formula or Approach:

The total mechanical energy E of a spring-mass oscillator is conserved:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

Rearrange this equation into the standard form of an ellipse ($\frac{x^2}{a^2} + \frac{v^2}{b^2} = 1$):

$$\frac{v^2}{(2E/m)} + \frac{x^2}{(2E/k)} = 1$$

Step 3: Detailed Explanation:

For the graph in the $x - v$ plane to be a perfect circle rather than an ellipse, the coefficients dividing the squared variables must be equal (i.e., the semi-major axis must equal the semi-minor axis).

$$\frac{2E}{m} = \frac{2E}{k}$$

Dividing both sides by $2E$ gives:

$$\frac{1}{m} = \frac{1}{k} \implies m = k$$

So, the numerical values of the spring constant and mass must be equal.

Step 4: Final Answer:

The correct relation is $k = m$.

Quick Tip: The phase space trajectory (x vs v) of a simple harmonic oscillator is inherently an ellipse. It only degenerates into a perfect circle when the angular frequency $\omega = \sqrt{k/m} = 1$ rad/s, which dictates $k = m$.
