RBSE Class 12 Mathematics Question Paper 2026 with Solutions

Time Allowed :3 Hour	Maximum Marks :100	Total Questions :38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Section A is compulsory for all candidates and generally includes objective-type questions, short answer questions, and long answer questions from the prescribed syllabus.
2. In Section A, candidates are required to answer all questions. The questions will cover topics from ancient, medieval, and modern history as prescribed by the syllabus.
3. Section B consists of elective questions. Candidates are required to attempt questions from the chosen topic according to the provided options.
4. The questions in Section A will be in the form of multiple-choice, short answer, and essay-type questions.
5. Answers to all questions must be written in neat and legible handwriting. Candidates must adhere strictly to the word limit mentioned in the questions.
6. Use of unfair means or electronic devices during the examination is strictly prohibited.
7. Candidates must ensure that they write their answers in the correct format, following the instructions given for each section.

1. If $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then $f(f(x))$ is equal to

(A) $x^{\frac{1}{3}}$
(B) x^3
(C) x
(D) $(3 - x^3)$

Correct Answer: (C) x

Solution:

Step 1: Understanding the Question:
The question asks for the composition of the function f with itself, which is denoted by $f(f(x))$ or $(f \circ f)(x)$. This means we need to substitute the entire expression for $f(x)$ into the variable x within the function $f(x)$ itself.

Step 2: Key Formula or Approach:
The approach is to compute the composite function by direct substitution.
Given $f(x) = (3 - x^3)^{\frac{1}{3}}$.
Then $f(f(x))$ is found by replacing x with $f(x)$ in the formula for f:

$$f(f(x)) = (3 - [f(x)]^3)^{\frac{1}{3}}$$

Step 3: Detailed Explanation:
First, let's find the expression for $[f(x)]^3$:

$$f(x) = (3 - x^3)^{\frac{1}{3}}$$

Cubing both sides, we get:

$$[f(x)]^3 = \left((3 - x^3)^{\frac{1}{3}}\right)^3 = 3 - x^3$$

Now, substitute this result back into the expression for $f(f(x))$:

$$f(f(x)) = (3 - [f(x)]^3)^{\frac{1}{3}} = (3 - (3 - x^3))^{\frac{1}{3}}$$

Simplify the expression inside the parenthesis:

$$f(f(x)) = (3 - 3 + x^3)^{\frac{1}{3}}$$
$$f(f(x)) = (x^3)^{\frac{1}{3}}$$
$$f(f(x)) = x$$

Step 4: Final Answer:
The result of the composition $f(f(x))$ is x. Comparing this with the given options, we find that option (C) is the correct answer.

Quick Tip

When a function satisfies $f(f(x)) = x$, it is called an involution. Such functions are their own inverse. Here, f is its own inverse function.

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2. The principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ **is**

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{6}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{2}$

Correct Answer: (A) $\frac{\pi}{4}$

Solution:

Step 1: Understanding the Question:
We are asked to find the principal value of the inverse sine function for the input $\frac{1}{\sqrt{2}}$. The principal value is the unique value of the angle that lies within the defined principal value branch of the inverse trigonometric function.

Step 2: Key Formula or Approach:
The principal value branch for the inverse sine function, $\sin^{-1}(x)$, is the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$. We need to find an angle θ in this interval such that $\sin(\theta) = \frac{1}{\sqrt{2}}$.

Step 3: Detailed Explanation:
Let $y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$.
By definition of the inverse sine function, this means:

$$\sin(y) = \frac{1}{\sqrt{2}}$$

We need to find the value of y that satisfies this equation and also lies in the principal value range $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
From our knowledge of standard trigonometric values, we know that:

$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

The angle $\frac{\pi}{4}$ (which is $45°$) is within the range $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (or $[-90°, 90°]$).
Therefore, the principal value is $\frac{\pi}{4}$.

Step 4: Final Answer:
The principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$. This corresponds to option (A).

Quick Tip

Remember standard values:

- $\sin^{-1}(0) = 0$
- $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$
- $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$
- $\sin^{-1}(1) = \frac{\pi}{2}$

3. $A = [a_{ij}]_{m\times n}$ **is a square matrix, if**

(A) m < n
(B) m > n
(C) m = n
(D) None of these

Correct Answer: (C) m = n

Solution:

Step 1: Understanding the Question:
The question asks for the condition that defines a square matrix, given the general notation for a matrix $A = [a_{ij}]_{m\times n}$. In this notation, 'm' represents the number of rows and 'n' represents the number of columns.

Step 2: Detailed Explanation:
By definition, a matrix is a rectangular array of numbers. The dimensions of the matrix are given by its order, $m \times n$.
A **square matrix** is a special type of matrix where the number of rows is exactly equal to the number of columns.
For the matrix $A = [a_{ij}]_{m\times n}$:
- The number of rows is m.
- The number of columns is n.
For A to be a square matrix, the condition must be:

$$\text{Number of rows} = \text{Number of columns}$$

$$m = n$$

Step 3: Final Answer:
Let's analyze the given options based on this definition:
(A) $m < n$: This describes a rectangular matrix with more columns than rows (a "wide" matrix).
(B) $m > n$: This describes a rectangular matrix with more rows than columns (a "tall" matrix).
(C) $m = n$: This is the correct condition for a square matrix.
(D) None of these: This is incorrect as option (C) is the correct definition.
Thus, the correct option is (C).

💡 Quick Tip

Square matrix $\iff$ Number of rows = Number of columns (m = n). Examples: 2×2, 3×3, 4×4 matrices. Only square matrices have determinants and inverses!

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4. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$**, then the** $(2A - B)$ **will be**

(A) $\begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 5 & -1 & 3 \\ 5 & 6 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \end{bmatrix}$

(D) $\begin{bmatrix} -3 & 1 & -3 \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$

Solution:

Step 1: Understanding the Question:
We are given two matrices, A and B, and we need to compute the matrix expression $2A - B$. This involves two basic matrix operations: scalar multiplication ($2A$) and matrix subtraction.

Step 2: Key Formula or Approach:
1. **Scalar Multiplication:** To find $2A$, we multiply every element of matrix A by the scalar 2.
2. **Matrix Subtraction:** To find $2A - B$, we subtract each element of matrix B from the corresponding element of matrix $2A$. This is possible only because both matrices have the same order (2×3).

Step 3: Detailed Explanation:
First, calculate the matrix $2A$:

$$2A = 2 \times \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 \times 1 & 2 \times 2 & 2 \times 3 \\ 2 \times 2 & 2 \times 3 & 2 \times 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \end{bmatrix}$$

Next, subtract matrix B from $2A$:

$$2A - B = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

Perform the subtraction element-wise:

$$2A - B = \begin{bmatrix} 2 - 3 & 4 - (-1) & 6 - 3 \\ 4 - (-1) & 6 - 0 & 2 - 2 \end{bmatrix}$$

$$2A - B = \begin{bmatrix} -1 & 4+1 & 3 \\ 4+1 & 6 & 0 \end{bmatrix}$$
$$2A - B = \begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$$

Step 4: Final Answer:
The resulting matrix is $\begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$. This matches option (A).

Quick Tip

Matrix subtraction: Subtract corresponding elements. Order of matrices must be same for addition/subtraction. Scalar multiplication: Multiply each element by the scalar.

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5. Value of $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$ **will be**

(A) $x^2 - x + 2$
(B) $x^3 + x^2 - 2$
(C) $x^3 - x^2 + 2$
(D) $x^3 + x^2 + 4$

Correct Answer: (C) $x^3 - x^2 + 2$

Solution:

Step 1: Understanding the Question:
We are asked to find the value of a 2×2 determinant. The elements of the determinant are polynomials in x.

Step 2: Key Formula or Approach:
The determinant of a general 2×2 matrix $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is calculated as $ad - bc$.
We will also use the following algebraic identities to simplify the calculation:
- Sum of cubes: $(a + b)(a^2 - ab + b^2) = a^3 + b^3$
- Difference of squares: $(a - b)(a + b) = a^2 - b^2$

Step 3: Detailed Explanation:
Applying the determinant formula $ad - bc$ to the given matrix:
Here, $a = x^2 - x + 1$, $b = x - 1$, $c = x + 1$, and $d = x + 1$.

$$\text{Determinant} = (x^2 - x + 1)(x + 1) - (x - 1)(x + 1)$$

Now, let's simplify each part using the algebraic identities.
For the first term, $(x+1)(x^2-x+1)$, we recognize the pattern for the sum of cubes with $a=x$ and $b=1$.

$$(x+1)(x^2-x\cdot 1+1^2)=x^3+1^3=x^3+1$$

For the second term, $(x-1)(x+1)$, we recognize the pattern for the difference of squares.

$$(x-1)(x+1)=x^2-1^2=x^2-1$$

Now substitute these simplified expressions back into the determinant calculation:

$$\begin{aligned}\text{Determinant} &= (x^3+1)-(x^2-1)\\ &= x^3+1-x^2+1\\ &= x^3-x^2+2\end{aligned}$$

Step 4: Final Answer:
The value of the determinant is x^3-x^2+2. This matches option (C).

> Quick Tip
>
> For 2×2 determinants: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$. Recognizing algebraic identities like sum/difference of cubes and squares can significantly simplify the expansion.

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6. If $2x+3y=\sin y$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{3}{\sin y-2}$
(B) $\frac{2}{\cos y-3}$
(C) $\frac{\cos y+3}{2}$
(D) $\frac{2}{\cos y}$

Correct Answer: (B) $\frac{2}{\cos y-3}$

Solution:

Step 1: Understanding the Question:
The question provides an equation that implicitly defines y as a function of x. We need to find the derivative of y with respect to x, i.e., $\frac{dy}{dx}$, using the technique of implicit differentiation.

Step 2: Key Formula or Approach:
Implicit differentiation involves differentiating both sides of the equation with respect to x. When differentiating a term involving y, we must apply the chain rule. For example, $\frac{d}{dx}(f(y)) = f'(y) \cdot \frac{dy}{dx}$. After differentiating, we will algebraically solve for $\frac{dy}{dx}$.

Step 3: Detailed Explanation:
Given the equation:

$$2x + 3y = \sin y$$

Differentiate both sides with respect to x:

$$\frac{d}{dx}(2x + 3y) = \frac{d}{dx}(\sin y)$$

Apply the sum rule on the left side and differentiate term by term:

$$\frac{d}{dx}(2x) + \frac{d}{dx}(3y) = \frac{d}{dx}(\sin y)$$

Now, perform the differentiation:
- $\frac{d}{dx}(2x) = 2$
- $\frac{d}{dx}(3y) = 3 \cdot \frac{dy}{dx}$
- $\frac{d}{dx}(\sin y) = \cos y \cdot \frac{dy}{dx}$ (using the chain rule)

Substituting these back into the equation gives:

$$2 + 3\frac{dy}{dx} = (\cos y)\frac{dy}{dx}$$

Now, we need to solve for $\frac{dy}{dx}$. Group all terms with $\frac{dy}{dx}$ on one side:

$$2 = (\cos y)\frac{dy}{dx} - 3\frac{dy}{dx}$$

Factor out $\frac{dy}{dx}$ on the right side:

$$2 = \frac{dy}{dx}(\cos y - 3)$$

Finally, isolate $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{2}{\cos y - 3}$$

Step 4: Final Answer:
The derivative $\frac{dy}{dx}$ is $\frac{2}{\cos y - 3}$. This matches option (B).

Quick Tip

In implicit differentiation, differentiate each term carefully. Remember the chain rule for any term involving y. For example, $\frac{d}{dx}(g(y)) = g'(y) \cdot \frac{dy}{dx}$. After differentiating, algebraically isolate $\frac{dy}{dx}$.

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7. $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0 < x < 1$, **then** $\frac{dy}{dx}$ **is equal to**

(A) $\frac{1}{1+x^2}$
(B) $\frac{2}{4+x}$
(C) $\frac{2}{1+x^2}$
(D) $\frac{2}{x+x^2}$

Correct Answer: (C) $\frac{2}{1+x^2}$

Solution:

Step 1: Understanding the Question:
The question asks for the derivative of the function $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$. The form of the expression inside the inverse cosine suggests using a trigonometric substitution to simplify the function before differentiating.

Step 2: Key Formula or Approach:
The problem can be greatly simplified by using the substitution $x = \tan\theta$. This is motivated by the double angle identity for cosine:

$$\cos(2\theta) = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$$

Alternatively, one can directly use the standard inverse trigonometric identity:

$$2\tan^{-1} x = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right), \quad \text{for } x \geq 0$$

We also need the derivative of the inverse tangent function: $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$.

Step 3: Detailed Explanation:
Using the standard identity directly is the fastest method. Since the problem states $0 < x < 1$, the condition $x \geq 0$ is satisfied.
We can rewrite the function y as:

$$y = 2\tan^{-1} x$$

Now, we can differentiate this simplified function with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}(2\tan^{-1} x)$$
$$\frac{dy}{dx} = 2 \cdot \frac{d}{dx}(\tan^{-1} x)$$

Using the standard derivative formula for $\tan^{-1} x$:

$$\frac{dy}{dx} = 2 \cdot \left(\frac{1}{1+x^2}\right) = \frac{2}{1+x^2}$$

Step 4: Final Answer:
The derivative of the given function is $\frac{2}{1+x^2}$. This matches option (C).

Quick Tip

Recognizing trigonometric substitution patterns is crucial for inverse trig derivatives. Remember these key identities involving $2\tan^{-1} x$:

- $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1} x$ for $x \geq 0$
- $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1} x$ for $|x| \leq 1$
- $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1} x$ for $|x| < 1$

Using these simplifies differentiation significantly.

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8. The rate of change of the area of a circle with respect to its radius r at $r = 5\,\text{cm}$ is

(A) 12π
(B) 8π
(C) 5π
(D) 10π

Correct Answer: (D) 10π

Solution:

Step 1: Understanding the Question:
The question asks for the "rate of change of the area of a circle with respect to its radius". This is a direct application of differentiation. We need to find the derivative of the area formula with respect to the radius, and then evaluate this derivative at the specific radius value of $r = 5$ cm.

Step 2: Key Formula or Approach:
1. The formula for the area (A) of a circle with radius r is $A = \pi r^2$.
2. The rate of change of A with respect to r is the derivative $\frac{dA}{dr}$.
3. We will then substitute $r = 5$ into the expression for $\frac{dA}{dr}$.

Step 3: Detailed Explanation:
Let A be the area of the circle and r be its radius. The relationship is:

$$A = \pi r^2$$

To find the rate of change of area with respect to the radius, we differentiate A with respect to r:

$$\frac{dA}{dr} = \frac{d}{dr}(\pi r^2)$$

Using the power rule for differentiation, $\frac{d}{dr}(r^n) = nr^{n-1}$:

$$\frac{dA}{dr} = \pi \cdot (2r) = 2\pi r$$

This expression, $2\pi r$, gives the rate of change for any radius r. The question asks for this rate at the specific instant when $r = 5$ cm.
Substitute $r = 5$ into the derivative:

$$\left.\frac{dA}{dr}\right|_{r=5} = 2\pi(5) = 10\pi$$

The units would be cm^2/cm.

Step 4: Final Answer:
The rate of change of the area at $r = 5$ cm is 10π. This corresponds to option (D).

> Quick Tip
>
> An interesting observation: The rate of change of the area of a circle with respect to its radius ($\frac{dA}{dr} = 2\pi r$) is equal to its circumference. This concept of a derivative representing the "boundary" of a shape often appears in geometry.

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9. $\int \frac{dx}{\sin^2 x \cos^2 x}$ **equals**

(A) $\tan x + \sin x + c$
(B) $\tan x - \cot x + c$
(C) $\tan x \cot x + c$
(D) $2\tan x - \cot 2x + c$

Correct Answer: (B) $\tan x - \cot x + c$

Solution:

Step 1: Understanding the Question:
We are asked to find the indefinite integral of the function $\frac{1}{\sin^2 x \cos^2 x}$. The integrand needs to be simplified or transformed into a form that can be integrated using standard formulas.

Step 2: Key Formula or Approach:
A powerful technique for simplifying trigonometric fractions is to use the Pythagorean identity $\sin^2 x + \cos^2 x = 1$. By replacing the '1' in the numerator with this expression, we can split the fraction into simpler terms.
The standard integrals we will need are:
- $\int \sec^2 x \, dx = \tan x + C$
- $\int \csc^2 x \, dx = -\cot x + C$

Step 3: Detailed Explanation:
Let the integral be I.

$$I = \int \frac{1}{\sin^2 x \cos^2 x} \, dx$$

Replace the numerator with $\sin^2 x + \cos^2 x$:

$$I = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} \, dx$$

Split the fraction into two separate terms:

$$I = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$

Cancel out the common terms in each fraction:

$$I = \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \right) dx$$

Recognize the reciprocal trigonometric identities, $\sec x = \frac{1}{\cos x}$ and $\csc x = \frac{1}{\sin x}$:

$$I = \int (\sec^2 x + \csc^2 x) \, dx$$

Now, integrate term by term using the standard integral formulas:

$$I = \int \sec^2 x \, dx + \int \csc^2 x \, dx$$

$$I = (\tan x) + (-\cot x) + C$$

$$I = \tan x - \cot x + C$$

Step 4: Final Answer:
The result of the integration is $\tan x - \cot x + C$. This matches option (B).

💡 Quick Tip

When faced with an integral of the form $\int \frac{dx}{\sin^m x \cos^n x}$, a useful strategy is to multiply the numerator and denominator by a power of $\sec^2 x$ or use the identity $1 = \sin^2 x + \cos^2 x$ in the numerator to split the fraction.

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10. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

(A) 2
(B) $\frac{4}{9}$
(C) $\frac{9}{4}$
(D) $\frac{9}{2}$

Correct Answer: (C) $\frac{9}{4}$

Solution:

Step 1: Understanding the Question:
We need to find the area of a specific region in the xy-plane. The region is enclosed by three boundaries: 1. The parabola $y^2 = 4x$ (which opens to the right with its vertex at the origin). 2. The y-axis (the line $x = 0$). 3. The horizontal line $y = 3$. This region lies in the first quadrant.

Step 2: Key Formula or Approach:
Since the boundaries are given in terms of y (y-axis and $y = 3$), it is more convenient to integrate with respect to y. The formula for the area bounded by a curve $x = f(y)$, the y-axis, and the lines $y = c$ and $y = d$ is:

$$\text{Area} = \int_c^d x\,dy$$

First, we must express x as a function of y from the curve's equation.

Step 3: Detailed Explanation:
The equation of the curve is $y^2 = 4x$. We need to express x in terms of y:

$$x = \frac{y^2}{4}$$

The region is bounded by the y-axis ($x = 0$) and the line $y = 3$. The parabola $y^2 = 4x$ intersects the y-axis at $y = 0$. Therefore, the limits of integration along the y-axis are from $c = 0$ to $d = 3$.
Now, we set up the definite integral for the area:

$$\text{Area} = \int_0^3 x\,dy = \int_0^3 \frac{y^2}{4}\,dy$$

Evaluate the integral:

$$\text{Area} = \frac{1}{4}\int_0^3 y^2\, dy$$

Using the power rule for integration, $\int y^n\, dy = \frac{y^{n+1}}{n+1}$:

$$\text{Area} = \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3$$

Now, apply the limits of integration (Fundamental Theorem of Calculus):

$$\text{Area} = \frac{1}{4}\left(\frac{3^3}{3} - \frac{0^3}{3}\right)$$

$$\text{Area} = \frac{1}{4}\left(\frac{27}{3} - 0\right) = \frac{1}{4}(9) = \frac{9}{4}$$

Step 4: Final Answer:
The area of the specified region is $\frac{9}{4}$ square units. This corresponds to option (C).

Quick Tip

When finding the area bounded by a curve and the y-axis, it's often easiest to integrate with respect to y. The setup is Area $= \int_{y_1}^{y_2} x\, dy$. Always visualize or sketch the region to correctly identify the function and the limits of integration.

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11. The order of the differential equation $2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ is

(A) 2
(B) 1
(C) 0
(D) Not defined

Correct Answer: (A) 2

Solution:

Step 1: Understanding the Question:
The question asks to determine the 'order' of a given differential equation.
The order of a differential equation is a fundamental property that defines its classification.

Step 2: Key Formula or Approach:
The order of a differential equation is defined as the order of the highest derivative that appears in the equation.

Step 3: Detailed Explanation:
The given differential equation is:

$$2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$$

We need to identify all the derivatives present in this equation.
The terms involving derivatives are:

- $\frac{d^2y}{dx^2}$: This is the second derivative of y with respect to x, so its order is 2.

- $\frac{dy}{dx}$: This is the first derivative of y with respect to x, so its order is 1.

To find the order of the equation, we compare the orders of all derivatives present.
The highest order among 1 and 2 is 2.

Step 4: Final Answer:
The highest order derivative in the equation is $\frac{d^2y}{dx^2}$, which is of order 2.
Therefore, the order of the differential equation is 2, which corresponds to option (A).

Quick Tip
Order = Highest order derivative present in the equation. Degree = Power of the highest order derivative (after rationalizing).

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12. Which of the following is a vector quantity?

(A) Time
(B) Volume
(C) Force
(D) Speed

Correct Answer: (C) Force

Solution:

Step 1: Understanding the Question:
The question asks to identify the vector quantity from the given list of physical quantities.

This requires understanding the difference between scalar and vector quantities.

Step 2: Key Formula or Approach:
The approach is to analyze each option based on the definitions:

- **Scalar Quantity:** A quantity that has only magnitude (a numerical value) and no direction.

- **Vector Quantity:** A quantity that has both magnitude and direction.

Step 3: Detailed Explanation:
Let's examine each option:

- **(A) Time:** Time is measured in units like seconds or hours (e.g., 30 seconds). It has magnitude but no associated direction. Thus, it is a **scalar**.

- **(B) Volume:** Volume represents the space occupied by an object (e.g., 2 liters). It has magnitude but no direction. Thus, it is a **scalar**.

- **(C) Force:** Force is a push or a pull on an object. It is described by its strength (magnitude, e.g., 10 Newtons) and the direction in which it is applied (e.g., downwards). Since it has both, it is a **vector**.

- **(D) Speed:** Speed is the rate at which an object covers distance (e.g., 50 km/h). It only tells how fast, not in which direction. Thus, it is a **scalar**. (Its vector counterpart is velocity).

Step 4: Final Answer:
Based on the analysis, Force is the only quantity in the list that possesses both magnitude and direction.
Hence, option (C) is the correct answer.

Quick Tip

Common vector quantities: Force, Velocity, Displacement, Acceleration, Momentum, Weight.
Common scalar quantities: Mass, Time, Volume, Speed, Distance, Temperature, Energy.

13. The sum of the vectors $a = i - 2j + k$, $b = -2i + 4j + 5k$ and $c = i - 6j - 7k$ is

(A) $-4j - k$
(B) $4i - k$
(C) $4j + 5k$
(D) $i + 4j - k$

Correct Answer: (A) $-4j - k$

Solution:

Step 1: Understanding the Question:
The question asks for the sum (or resultant) of three given vectors: a, b, and c.
Vector addition is performed by adding their corresponding components.

Step 2: Key Formula or Approach:
To add vectors given in component form, we add the coefficients of the unit vectors i, j, and k separately.
If $\vec{v}_1 = x_1 i + y_1 j + z_1 k$ and $\vec{v}_2 = x_2 i + y_2 j + z_2 k$, their sum is:

$$\vec{v}_1 + \vec{v}_2 = (x_1 + x_2)i + (y_1 + y_2)j + (z_1 + z_2)k$$

Step 3: Detailed Explanation:
The given vectors are:

$$a = i - 2j + k$$

$$b = -2i + 4j + 5k$$

$$c = i - 6j - 7k$$

Let the sum be $R = a + b + c$.
We add the coefficients of i, j, and k respectively.

- **Sum of i components:** $1 + (-2) + 1 = 1 - 2 + 1 = 0$
- **Sum of j components:** $-2 + 4 + (-6) = -2 + 4 - 6 = -4$
- **Sum of k components:** $1 + 5 + (-7) = 1 + 5 - 7 = -1$

Now, we combine these components to form the resultant vector:

$$R = (0)i + (-4)j + (-1)k = 0i - 4j - 1k$$

Step 4: Final Answer:
The resultant vector is $-4j - k$.

This matches option (A).

💡 Quick Tip

Vector addition: Add corresponding components separately.
$(a_1 i + a_2 j + a_3 k) + (b_1 i + b_2 j + b_3 k) = (a_1 + b_1)i + (a_2 + b_2)j + (a_3 + b_3)k$

---

14. The unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ is

(A) $\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{5}}$
(B) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}$
(C) $\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}$
(D) $\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}$

Correct Answer: (D) $\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}$

Solution:

Step 1: Understanding the Question:
The question asks to find the unit vector corresponding to a given vector $\vec{a}$.
A unit vector is a vector that has the same direction as the original vector but has a magnitude of 1.

Step 2: Key Formula or Approach:
The unit vector $(\hat{a})$ in the direction of a non-zero vector $\vec{a}$ is found by dividing the vector by its magnitude $|\vec{a}|$.

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

The magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by:

$$|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$$

Step 3: Detailed Explanation:
The given vector is $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.
First, we calculate the magnitude of $\vec{a}$. The components are $x = 1, y = 1, z = 2$.

$$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (2)^2} = \sqrt{1 + 1 + 4} = \sqrt{6}$$

Next, we use the formula for the unit vector by dividing $\vec{a}$ by its magnitude $|\vec{a}|$.

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}}$$

Step 4: Final Answer:
The unit vector in the direction of $\vec{a}$ is $\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}$.
This result matches option (D).

> Quick Tip
>
> Unit vector = Vector / Magnitude.
> Magnitude of $a\hat{i} + b\hat{j} + c\hat{k} = \sqrt{a^2 + b^2 + c^2}$.
> Always divide by the correct magnitude!

---

15. If the straight lines $\frac{x+1}{1} = \frac{y+2}{\lambda} = \frac{z-1}{-1}$ and $\frac{x-1}{-\lambda} = \frac{y+1}{2} = \frac{z+1}{1}$ are perpendicular to each other, then the value of "λ" is

(A) 0
(B) 1
(C) 2
(D) 3

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Question:
The question provides the equations of two straight lines in 3D space and states that they are perpendicular.
We need to find the value of the unknown parameter λ using the condition for perpendicularity of lines.

Step 2: Key Formula or Approach:
Two lines are perpendicular if the dot product of their direction vectors is zero.
For a line with equation $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$, the direction ratios are (a, b, c).
If two lines have direction ratios (a_1, b_1, c_1) and (a_2, b_2, c_2), the condition for them to be perpendicular is:

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

Step 3: Detailed Explanation:
First, we identify the direction ratios from the given line equations.

For the first line, $\frac{x+1}{1} = \frac{y+2}{\lambda} = \frac{z-1}{-1}$:
The direction ratios are $\vec{d_1} = (a_1, b_1, c_1) = (1, \lambda, -1)$.
For the second line, $\frac{x-1}{-\lambda} = \frac{y+1}{2} = \frac{z+1}{1}$:
The direction ratios are $\vec{d_2} = (a_2, b_2, c_2) = (-\lambda, 2, 1)$.
Now, we apply the condition for perpendicularity: $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

$$(1)(-\lambda) + (\lambda)(2) + (-1)(1) = 0$$

Simplify the equation:

$$-\lambda + 2\lambda - 1 = 0$$

$$\lambda - 1 = 0$$

Solving for λ:

$$\lambda = 1$$

Step 4: Final Answer:
The value of λ that makes the two lines perpendicular is 1.
This corresponds to option (B).

Quick Tip

For perpendicular lines: $a_1a_2 + b_1b_2 + c_1c_2 = 0$. (Dot product of direction vectors is zero).
For parallel lines: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. (Direction vectors are proportional).

---

16. Two cards are drawn at random without replacement from a pack of 52 playing cards, then the probability that both the cards are black in color is:

(A) $\frac{1}{2}$

(B) $\frac{1}{12}$

(C) $\frac{25}{102}$

(D) $\frac{1}{4}$

Correct Answer: (C) $\frac{25}{102}$

Solution:

Step 1: Understanding the Question:
The problem asks for the probability of drawing two black cards in a row from a standard 52-card deck, without replacing the first card.

This is a problem of dependent events in probability.

Step 2: Key Formula or Approach:
We can solve this using two methods:
Method 1 (Sequential Probability): Calculate the probability of the first event, and then the probability of the second event given the first has occurred. The total probability is the product of these probabilities.

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

Method 2 (Combinations): Use the formula for probability:

$$P(\text{Event}) = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Possible Outcomes}}$$

Step 3: Detailed Explanation:
First, let's note the composition of a standard deck:

- Total cards: 52

- Black cards: 26 (Spades and Clubs)

- Red cards: 26 (Hearts and Diamonds)

Using Method 1 (Sequential Probability):

- Probability of the first card being black: There are 26 black cards out of 52 total cards.

$$P(\text{1st is black}) = \frac{26}{52} = \frac{1}{2}$$

- Probability of the second card being black (given the first was black): After drawing one black card, there are now 25 black cards left and a total of 51 cards in the deck.

$$P(\text{2nd is black — 1st was black}) = \frac{25}{51}$$

- The total probability of both events happening is the product:

$$P(\text{both black}) = \frac{26}{52} \times \frac{25}{51} = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

Using Method 2 (Combinations):

- Total number of ways to draw 2 cards from 52: $\binom{52}{2} = \frac{52 \times 51}{2 \times 1} = 1326$.

- Number of ways to draw 2 black cards from the 26 available black cards: $\binom{26}{2} = \frac{26 \times 25}{2 \times 1} = 325$.

- The probability is the ratio of favorable outcomes to total outcomes:

$$P(\text{both black}) = \frac{\binom{26}{2}}{\binom{52}{2}} = \frac{325}{1326}$$

Simplifying the fraction by dividing both by 13: $\frac{325 \div 13}{1326 \div 13} = \frac{25}{102}$.

Step 4: Final Answer:
Both methods yield the same result, $\frac{25}{102}$.
This matches option (C).

> 💡 Quick Tip
>
> For "without replacement" problems:
>
> - Use combinations: $\frac{\binom{\text{favorable}}{\text{draws}}}{\binom{\text{total}}{\text{draws}}}$.
> - Or multiply sequential probabilities: $P(\text{1st}) \times P(\text{2nd given 1st})$.

---

17. If a pair of dice is thrown, then the probability of getting an even prime number on each die will be

(A) $\frac{1}{3}$
(B) $\frac{1}{12}$
(C) $\frac{1}{36}$
(D) 0

Correct Answer: (C) $\frac{1}{36}$

Solution:

Step 1: Understanding the Question:
The question asks for the probability of a specific outcome when throwing a pair of standard six-sided dice. The desired outcome is that the number on each die is an "even prime number".

Step 2: Key Formula or Approach:
The probability of an event is calculated as:

$$P(\text{Event}) = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Possible Outcomes}}$$

For two independent events (like the outcomes of two separate dice), the probability of both occurring is the product of their individual probabilities.

$$P(A \text{ and } B) = P(A) \times P(B)$$

Step 3: Detailed Explanation:
First, let's identify the "even prime numbers" that can appear on a standard die (which has faces numbered 1, 2, 3, 4, 5, 6).

- The prime numbers are numbers greater than 1 with only two divisors: 1 and themselves. The primes on a die are 2, 3, 5.
- An even number is a number divisible by 2. The even numbers on a die are 2, 4, 6.
- The only number that is both even and prime is 2.

So, the event "getting an even prime number" on a single die is equivalent to "getting a 2".

Now, let's calculate the probability for a pair of dice.

- The total number of possible outcomes when throwing a pair of dice is $6 \times 6 = 36$.
- The favorable outcome is "getting an even prime number on each die", which means getting a 2 on the first die AND a 2 on the second die. The only outcome that satisfies this is (2, 2).
- The number of favorable outcomes is just 1.

Using the probability formula:

$$P(\text{getting } (2,2)) = \frac{1}{36}$$

Alternatively, using the independent events approach:

$$P(2 \text{ on 1st die}) = \frac{1}{6}$$

$$P(2 \text{ on 2nd die}) = \frac{1}{6}$$

$$P(2 \text{ on both}) = P(2 \text{ on 1st}) \times P(2 \text{ on 2nd}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Step 4: Final Answer:
The probability of getting an even prime number (which is 2) on each die is $\frac{1}{36}$. This corresponds to option (C).

Quick Tip

Remember: 2 is the only even prime number!
When a pair of dice is thrown, total outcomes = 36.
Favorable outcome for "even prime on each" is (2,2), which is 1 outcome, so probability $= \frac{1}{36}$.

---

18. $\sin^{-1} x$ **is a function whose domain is _________.**

Solution:

Step 1: Understanding the Question:
The question asks for the domain of the inverse sine function, $\sin^{-1} x$ (also written as $\arcsin x$). The domain of a function is the set of all possible input values (in this case, 'x') for which the function is defined.

Step 2: Key Formula or Approach:
The domain of an inverse function is the range of the original function.
The function $\sin^{-1} x$ is the inverse of the sine function, $\sin\theta$.
Therefore, we need to determine the range of the sine function.

Step 3: Detailed Explanation:
The sine function, $y = \sin\theta$, takes any real number θ as input. Its output value, y, oscillates between -1 and 1, inclusive.
The range of $\sin\theta$ is the interval $[-1, 1]$.
Since the domain of $\sin^{-1} x$ is the range of $\sin\theta$, the set of all possible values for x in $\sin^{-1} x$ must be $[-1, 1]$.
This means that x must satisfy the condition $-1 \leq x \leq 1$.

Step 4: Final Answer:
The domain of the function $\sin^{-1} x$ is the closed interval $[-1, 1]$.

Quick Tip

Domain of $\sin^{-1} x$ is $[-1, 1]$ (all real numbers from -1 to 1 inclusive).
Range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
Remember: input to inverse trig functions must be within the range of the original trig function.

---

19. The value of determinant $\Delta = \begin{vmatrix} 1 & -2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$ is -5.

Solution:

Step 1: Understanding the Question:
The question provides a statement about the value of a 3x3 determinant. We need to calculate the determinant's actual value to verify if the statement is true or false. The format implies a True/False question, where we must determine the correct value.

Step 2: Key Formula or Approach:
The value of a 3x3 determinant can be calculated by cofactor expansion along any row or column. The most efficient approach is to expand along a row or column that contains the most zeros.
The formula for expansion along column j is:

$$|A| = a_{1j}C_{1j} + a_{2j}C_{2j} + a_{3j}C_{3j}$$

where $C_{ij} = (-1)^{i+j}M_{ij}$ is the cofactor and M_{ij} is the minor.

Step 3: Detailed Explanation:
The given determinant is:

$$\Delta = \begin{vmatrix} 1 & -2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$$

We observe that the third column has two zeros, which makes it ideal for expansion.
We expand along the third column (j=3):

$$\Delta = (4) \cdot C_{13} + (0) \cdot C_{23} + (0) \cdot C_{33} = 4 \cdot C_{13}$$

Now, we calculate the cofactor C_{13}:

$$C_{13} = (-1)^{1+3}M_{13} = (1) \cdot \begin{vmatrix} -1 & 3 \\ 4 & 1 \end{vmatrix}$$

The 2x2 determinant (minor M_{13}) is calculated as:

$$M_{13} = (-1)(1) - (3)(4) = -1 - 12 = -13$$

So, the cofactor is $C_{13} = -13$.
Finally, we calculate the value of the determinant Δ:

$$\Delta = 4 \cdot C_{13} = 4 \times (-13) = -52$$

Step 4: Final Answer:
The calculated value of the determinant is -52. The statement that the value is -5 is **False**. The correct value is -52.

> 💡 Quick Tip
>
> When a column (or row) has many zeros, expand along that column/row to simplify determinant calculation.
> Always double check the sign of the cofactor using $(-1)^{i+j}$.

---

20. The edge of a variable cube is increasing at the rate of 3 cm/s. The volume of the cube is increasing at the rate of __________________ while the edge is 10 cm long.

Solution:

Step 1: Understanding the Question:
This is a related rates problem from the applications of derivatives. We are given the rate at which the edge of a cube is changing ($\frac{dx}{dt}$) and asked to find the rate at which its volume is changing ($\frac{dV}{dt}$) at a specific instant when the edge length is known.

Step 2: Key Formula or Approach:
1. Write down the formula that relates the quantities involved. The volume V of a cube with edge length x is $V = x^3$.
2. Differentiate this formula with respect to time t using the chain rule to relate the rates of change.

$$\frac{dV}{dt} = \frac{d}{dt}(x^3) = 3x^2\frac{dx}{dt}$$

Step 3: Detailed Explanation:
Let x be the length of the edge of the cube and V be its volume.
The given information is:

- The rate of change of the edge: $\frac{dx}{dt} = 3$ cm/s.
- The specific instant of interest: when the edge length is $x = 10$ cm.

We need to find $\frac{dV}{dt}$.
The relationship between volume and edge length is:

$$V = x^3$$

Differentiating both sides with respect to time t:

$$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$$

Now, substitute the given values into this equation:

$$\frac{dV}{dt} = 3(10)^2 \cdot (3)$$

$$\frac{dV}{dt} = 3 \times 100 \times 3$$
$$\frac{dV}{dt} = 900$$
The units for the rate of change of volume will be cm^3/s.

Step 4: Final Answer:
The volume of the cube is increasing at the rate of $900\,\text{cm}^3/\text{s}$ when the edge is 10 cm long.

> Quick Tip
>
> For a cube with edge x, volume $V = x^3$.
> Rate of change of volume: $\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$.
> Just substitute the given values of x and $\frac{dx}{dt}$ to find the answer.

---

21. $\int (2x - 3\cos x + e^x)\, dx =$ ____________________.

Solution:

Step 1: Understanding the Question:
The question asks to find the indefinite integral of a function that is a sum of three basic functions: a polynomial term, a trigonometric term, and an exponential term.

Step 2: Key Formula or Approach:
We will use the linearity property of integration and the following standard integration formulas:

- **Linearity:** $\int (af(x) + bg(x))\, dx = a \int f(x)\, dx + b \int g(x)\, dx$
- **Power Rule:** $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$
- **Cosine Rule:** $\int \cos x\, dx = \sin x + C$
- **Exponential Rule:** $\int e^x\, dx = e^x + C$

Step 3: Detailed Explanation:
We can integrate the given expression term by term:
$$\int (2x - 3\cos x + e^x)\, dx = \int 2x\, dx - \int 3\cos x\, dx + \int e^x\, dx$$
Now, we evaluate each integral separately:

- For the first term, $\int 2x\, dx = 2\int x^1\, dx = 2\left(\frac{x^{1+1}}{1+1}\right) = 2\left(\frac{x^2}{2}\right) = x^2$.

- For the second term, $-\int 3\cos x\,dx = -3\int \cos x\,dx = -3(\sin x) = -3\sin x$.
- For the third term, $\int e^x\,dx = e^x$.

Combining the results and adding a single constant of integration, C, because it is an indefinite integral:

$$x^2 - 3\sin x + e^x + C$$

Step 4: Final Answer:
The result of the integration is $x^2 - 3\sin x + e^x + C$.

Quick Tip

Integrate term by term:
$\int 2x\,dx = x^2$
$\int -3\cos x\,dx = -3\sin x$
$\int e^x\,dx = e^x$
Don't forget the constant of integration $+C$!