

Maharashtra Board Class 10 Mathematics (Geometry) Question Paper with Solutions

Time Allowed :2 Hours

Maximum Marks :40

Total Questions :8

General Instructions:

Read the following instructions very carefully and strictly follow them:

1. All questions are compulsory.
2. Use of a calculator is not allowed.
3. The numbers to the right of the questions indicate full marks.
4. In case of MCQs (Q. No. 1(A)) only the first attempt will be evaluated and will be given credit.
5. Draw proper figures wherever necessary.
6. The marks of construction should be clear. Do not erase them.
7. Diagram is essential for writing the proof of the theorem.

1. (A) Four alternative answers for each of the following sub-questions are given. Choose the correct alternative and write its alphabet: 4 marks

1. Out of the dates given below, which date constitutes a Pythagorean triplet?

- (A) 15/8/17
- (B) 16/8/16
- (C) 3/5/17
- (D) 4/9/15

Correct Answer: (C) 3/5/17

Solution: Step 1: A Pythagorean triplet satisfies the condition $a^2 + b^2 = c^2$. Let us check the given options:

- For 3, 5, 17:

$$3^2 + 5^2 = 9 + 25 = 34 \neq 17^2.$$

This is incorrect.

- For 4, 9, 15:

$$4^2 + 9^2 = 16 + 81 = 97 \neq 15^2.$$

This is incorrect.

Thus, none of the options form a Pythagorean triplet.

For a Pythagorean triplet, always verify $a^2 + b^2 = c^2$. Ensure the largest number is c , the hypotenuse.

2. Find the value of $\sin 0 \times \csc 0$:

- (A) 1
- (B) 0
- (C) $\frac{1}{2}$
- (D) $\sqrt{2}$

Correct Answer: (A) 1

Solution: Step 1: Using the trigonometric identity:

$$\sin x \times \csc x = 1 \quad \text{for all } x \neq 0.$$

Substituting $x = 0$, we get:

$$\sin 0 \times \csc 0 = 1.$$

The reciprocal identity $\sin x \times \csc x = 1$ is always valid for non-zero angles.

3. What is the slope of the X-axis?

- (A) 1
- (B) -1
- (C) 0

(D) Cannot be determined

Correct Answer: (C) 0

Solution: Step 1: The X-axis is a horizontal line. The slope of a horizontal line is given by:

$$\text{slope} = \frac{\Delta y}{\Delta x} = 0.$$

For horizontal lines (e.g., X-axis), the slope is always 0. For vertical lines (e.g., Y-axis), the slope is undefined.

4. A circle having radius 3 cm, then the length of its largest chord is:

- (A) 1.5 cm
- (B) 3 cm
- (C) 6 cm
- (D) 9 cm

Correct Answer: (C) 6 cm

Solution: Step 1: The largest chord of a circle is its diameter. The diameter of a circle is given by:

$$\text{Diameter} = 2 \times \text{Radius}.$$

Step 2: Substituting the given radius 3 cm, we find:

$$\text{Diameter} = 2 \times 3 = 6 \text{ cm}.$$

Hence, the largest chord is 6 cm.

The diameter of a circle is the longest chord, always equal to $2 \times \text{Radius}$.

1. (B) Solve the following sub-questions:

4 marks

1. If $\triangle ABC \sim \triangle PQR$ and $AB : PQ = 2 : 3$, then find the value of $\frac{A(\triangle ABC)}{A(\triangle PQR)}$:

- (A) $\frac{2}{3}$
- (B) $\frac{4}{9}$
- (C) $\frac{8}{27}$
- (D) $\frac{9}{4}$

Correct Answer: (B) $\frac{4}{9}$

Solution: Step 1: For two similar triangles, the ratio of their areas is equal to the square of the ratio of their corresponding sides:

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \left(\frac{AB}{PQ} \right)^2.$$

Step 2: Substituting $AB : PQ = 2 : 3$:

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}.$$

For similar triangles, the area ratio is the square of the ratio of corresponding sides.

2. Two circles of radii 5 cm and 3 cm touch each other externally. Find the distance between their centres.

Correct Answer: 8 cm

Solution: Step 1: If two circles touch externally, the distance between their centres is equal to the sum of their radii:

$$\text{Distance} = R_1 + R_2.$$

Step 2: Substituting $R_1 = 5$ cm and $R_2 = 3$ cm:

$$\text{Distance} = 5 + 3 = 8 \text{ cm.}$$

For externally touching circles, the distance between their centres equals the sum of their radii.

3. Find the side of a square whose diagonal is $10\sqrt{2}$ cm:

Correct Answer: 10 cm

Solution: Step 1: The diagonal d of a square relates to its side a by:

$$d = a\sqrt{2}.$$

Step 2: Substituting $d = 10\sqrt{2}$ cm:

$$10\sqrt{2} = a\sqrt{2}.$$

Step 3: Dividing both sides by $\sqrt{2}$:

$$a = 10 \text{ cm.}$$

The diagonal of a square is always $\sqrt{2}$ times the length of its side.

4. A line makes an angle of 45° with the positive direction of the X-axis. Find the slope of the line.

Correct Answer: 1

Solution: Step 1: The slope of a line is given by:

$$\text{Slope} = \tan \theta,$$

where θ is the angle made with the positive X-axis.

Step 2: Substituting $\theta = 45^\circ$:

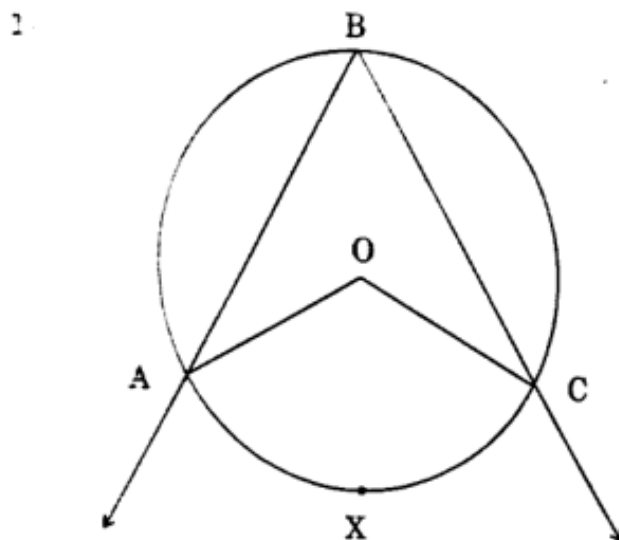
$$\text{Slope} = \tan 45^\circ = 1.$$

The slope of a line at 45° with the X-axis is always 1, as $\tan 45^\circ = 1$.

2. (A) Complete any 2 questions:

4 marks

1. In the above figure, $\triangle ABC$ is inscribed in arc ABC . If $\angle ABC = 60^\circ$, find $m\angle AOC$:



Solution:

Step 1: Using the property of inscribed angles, the measure of an inscribed angle is half the measure of the arc it subtends:

$$\angle ABC = \frac{1}{2}m(\text{arc AXC}).$$

Substituting $\angle ABC = 60^\circ$:

$$60^\circ = \frac{1}{2}m(\text{arc AXC}).$$

Step 2: Solving for $m(\text{arc AXC})$:

$$m(\text{arc AXC}) = 2 \times 60^\circ = 120^\circ.$$

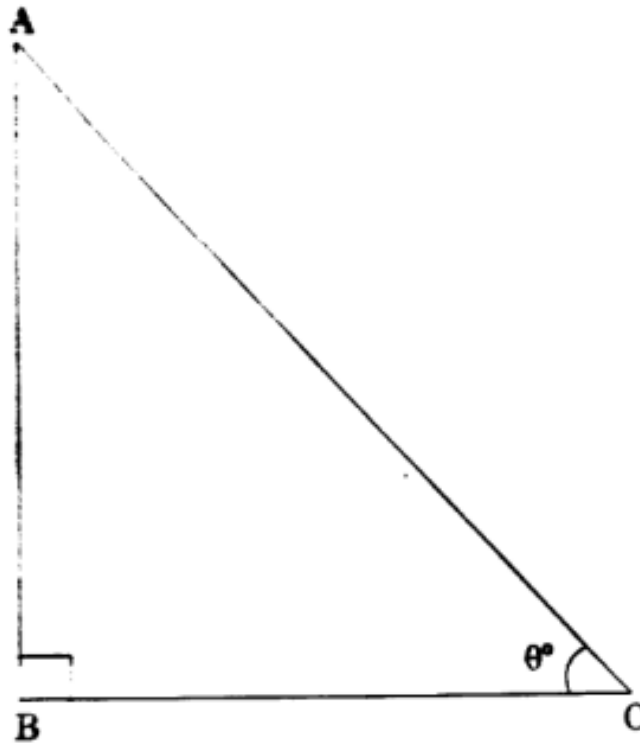
Step 3: Using the property of central angles, the measure of a central angle is equal to the measure of the arc it subtends:

$$m\angle AOC = m(\text{arc AXC}) = 120^\circ.$$

Hence, $m\angle AOC = 120^\circ$.

For inscribed angles, remember that $\text{Inscribed Angle} = \frac{1}{2} \times \text{Intercepted Arc}$. For central angles, $\text{Central Angle} = \text{Intercepted Arc}$.

2. Find the value of $\sin^2 \theta + \cos^2 \theta$:



Solution:

Step 1: In $\triangle ABC$, $\angle ABC = 90^\circ$ and $\angle C = \theta$. From the Pythagoras theorem:

$$AB^2 + BC^2 = AC^2.$$

Step 2: Dividing throughout by AC^2 :

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}.$$

Step 3: Simplifying, we get:

$$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = 1.$$

Step 4: Substituting trigonometric ratios:

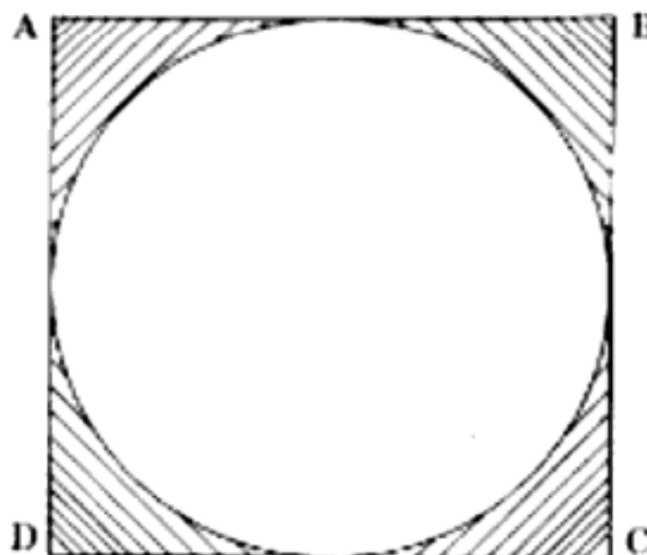
$$\sin \theta = \frac{BC}{AC}, \quad \cos \theta = \frac{AB}{AC}.$$

Thus:

$$\sin^2 \theta + \cos^2 \theta = 1.$$

The Pythagoras theorem is the foundation of the identity $\sin^2 \theta + \cos^2 \theta = 1$. Always check for right-angle triangles when applying this formula.

3. In the figure given above, $ABCD$ is a square, and a circle is inscribed in it. All sides of the square touch the circle. If $AB = 14$ cm, find the area of the shaded region.



Solution:

Step 1: The area of the square is given by:

$$\text{Area of square} = (\text{side})^2.$$

Substituting side = 14 cm:

$$\text{Area of square} = 14^2 = 196 \text{ cm}^2.$$

Step 2: The area of the circle is given by:

$$\text{Area of circle} = \pi r^2.$$

Here, the diameter of the circle is equal to the side of the square, so the radius $r = \frac{14}{2} = 7$ cm.
Substituting $r = 7$:

$$\text{Area of circle} = \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2.$$

Step 3: The area of the shaded region is the difference between the area of the square and the area of the circle:

$$\text{Area of shaded region} = \text{Area of square} - \text{Area of circle}.$$

Substituting the values:

$$\text{Area of shaded region} = 196 - 154 = 42 \text{ cm}^2.$$

When a circle is inscribed in a square, the diameter of the circle equals the side length of the square. Use this relationship to compute the radius of the circle.

2. (B) Solve any four of the following sub-questions:

8 marks

1. Radius of a sector of a circle is 3.5 cm and length of its arc is 2.2 cm. Find the area of the sector.

Solution: Step 1: The area of the sector is given by:

$$\text{Area of sector} = \frac{1}{2} \times r \times \text{arc length}.$$

Step 2: Substituting $r = 3.5$ cm and arc length = 2.2 cm:

$$\text{Area of sector} = \frac{1}{2} \times 3.5 \times 2.2 = 3.85 \text{ cm}^2.$$

To calculate the area of a sector, use $\text{Area} = \frac{1}{2} \times r \times \text{arc length}$ directly if the arc length is given.

2. Find the length of the hypotenuse of a right-angled triangle if the remaining sides are 9 cm and 12 cm.

Solution: Step 1: Using the Pythagoras theorem:

$$\text{Hypotenuse}^2 = \text{Side}_1^2 + \text{Side}_2^2.$$

Step 2: Substituting $\text{Side}_1 = 9$ cm and $\text{Side}_2 = 12$ cm:

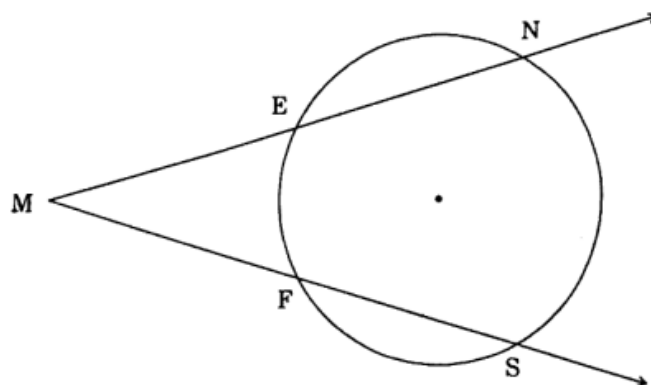
$$\text{Hypotenuse}^2 = 9^2 + 12^2 = 81 + 144 = 225.$$

Step 3: Taking the square root:

$$\text{Hypotenuse} = \sqrt{225} = 15 \text{ cm}.$$

For right triangles, always apply the Pythagoras theorem: $c^2 = a^2 + b^2$.

3. In the figure below, $m(\text{arc NS}) = 125^\circ$, $m(\text{arc EF}) = 37^\circ$. Find the measure of $\angle NMS$.



Solution: Step 1: The measure of the angle $\angle NMS$ is equal to half the difference of the measures of the intercepted arcs:

$$\angle NMS = \frac{1}{2} [m(\text{arc NS}) - m(\text{arc EF})].$$

Step 2: Substituting $m(\text{arc NS}) = 125^\circ$ and $m(\text{arc EF}) = 37^\circ$:

$$\angle NMS = \frac{1}{2} (125^\circ - 37^\circ) = \frac{1}{2} \times 88^\circ = 44^\circ.$$

For angles formed by two secants, use the formula: Angle = $\frac{1}{2}$ (larger arc – smaller arc).

4. Find the slope of the line passing through the points A(2, 3) and B(4, 7).

Solution: Step 1: The slope of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Step 2: Substituting $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (4, 7)$:

$$m = \frac{7 - 3}{4 - 2} = \frac{4}{2} = 2.$$

For slope, remember the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ and ensure you correctly subtract coordinates.

5. Find the surface area of a sphere of radius 7 cm.

Solution: Step 1: The surface area of a sphere is given by:

$$\text{Surface area} = 4\pi r^2.$$

Step 2: Substituting $r = 7$ cm:

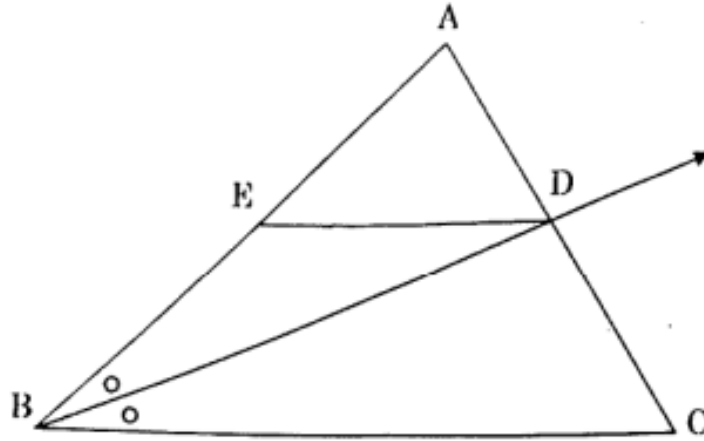
$$\text{Surface area} = 4 \times \frac{22}{7} \times 7^2 = 4 \times \frac{22}{7} \times 49 = 616 \text{ cm}^2.$$

For spheres, the surface area formula is $4\pi r^2$. Substitute $\pi = \frac{22}{7}$ for approximate calculations.

3. (A) Solve any one of the following sub-questions:

3 marks

1. In $\triangle ABC$, ray BD bisects $\angle ABC$, $A - D - C$, and seg $DE \parallel$ side BC . If $A - E - B$, then for showing $\frac{AB}{BC} = \frac{AE}{EB}$, complete the following activity:



Proof:

Step 1: In $\triangle ABC$, ray BD bisects $\angle B$, so:

$$\frac{AB}{BC} = \frac{AD}{DC} \quad (\text{I}).$$

Step 2: In $\triangle ABC$, $DE \parallel BC$, so:

$$\frac{AE}{EB} = \frac{AD}{DC} \quad (\text{II}).$$

Step 3: From equations (I) and (II), we have:

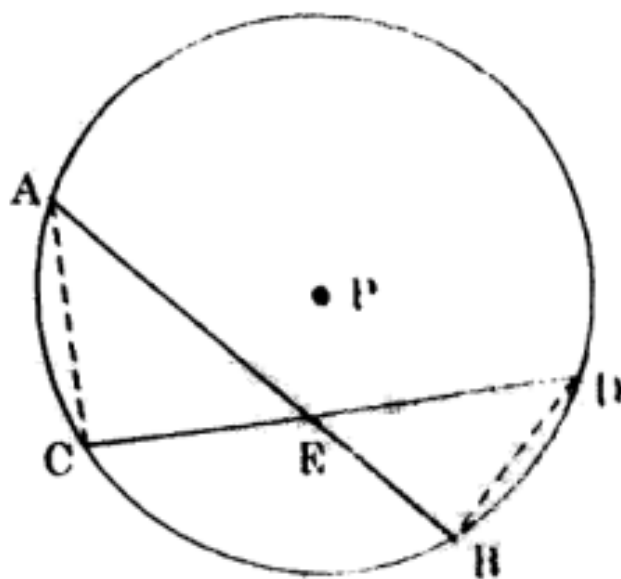
$$\frac{AB}{BC} = \frac{AE}{EB}.$$

Hence proved.

For proving ratios in triangles with parallel lines, use the basic proportionality theorem or the angle bisector theorem effectively.

2. Given: Chords AB and CD of a circle with center P intersect at point E .

To Prove: $AE \times EB = CE \times ED$.



Construction: Draw segment AC and segment BD .

Proof:

Step 1: In $\triangle CAE$ and $\triangle BDE$,

$$\angle AEC = \angle DEB \quad (\text{angles inscribed in the same arc}).$$

Step 2: Similarly,

$$\angle CAE = \angle BDE \quad (\text{angles inscribed in the same arc}).$$

Step 3: By AA similarity criterion, $\triangle CAE \sim \triangle BDE$.

Step 4: From the property of similar triangles, we have:

$$\frac{AE}{CE} = \frac{EB}{ED}.$$

Step 5: Cross-multiplying, we get:

$$AE \times EB = CE \times ED.$$

Hence proved.

For intersecting chords in a circle, use the inscribed angle theorem and properties of similar triangles to establish proportionality and cross-multiplication.

3. (B) Solve any two of the following sub-questions:

6 marks

1. Determine whether the points $A(1, -3)$, $B(2, -5)$, $C(-4, 7)$ are collinear.

Solution:

Step 1: To determine collinearity, check if the slope of AB is equal to the slope of BC .

Step 2: Slope of AB :

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - (-3)}{2 - 1} = \frac{-5 + 3}{1} = -2.$$

Step 3: Slope of BC :

$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-5)}{-4 - 2} = \frac{7 + 5}{-6} = \frac{12}{-6} = -2.$$

Step 4: Since $m_{AB} = m_{BC}$, the points are collinear.

To check for collinearity, verify if the slopes of the line segments between the points are equal.

2. $\triangle ABC \sim \triangle LMN$. In $\triangle ABC$, $AB = 5.5$ cm, $BC = 6$ cm, $CA = 4.5$ cm. Construct $\triangle ABC$ and $\triangle LMN$ such that $\frac{BC}{MN} = \frac{5}{4}$.

Correct Answer: Construction of $\triangle ABC$ and $\triangle LMN$

Solution:

Step 1: Construct $\triangle ABC$:

- Draw a base $BC = 6$ cm.
- At point B , construct $\angle ABC$ using a protractor such that $AB = 5.5$ cm.
- At point C , construct $\angle ACB$ using a protractor such that $CA = 4.5$ cm.
- Mark the point of intersection of the two arcs as A , and join AB and AC to complete $\triangle ABC$.

Step 2: Construct $\triangle LMN$ similar to $\triangle ABC$ with a scale factor of $\frac{4}{5}$:

- Draw a base MN such that $MN = \frac{4}{5} \times BC = \frac{4}{5} \times 6 = 4.8$ cm.
- At point M , construct an angle equal to $\angle ABC$.
- At point N , construct an angle equal to $\angle ACB$.
- Mark the point of intersection of the two arcs as L , and join LM and LN to complete $\triangle LMN$.

Step 3: Verify similarity:

$$\frac{AB}{LM} = \frac{BC}{MN} = \frac{CA}{LN} = \frac{5}{4}.$$

For constructing similar triangles, ensure that the scale factor is applied consistently to all sides, and corresponding angles are preserved.

3. Segment PM is a median of $\triangle PQR$, $PM = 9$, and $PQ^2 + PR^2 = 290$. Find QR .

Solution:

Step 1: Using the Apollonius theorem for medians in a triangle:

$$PQ^2 + PR^2 = 2PM^2 + \frac{1}{2}QR^2.$$

Step 2: Substituting the given values $PQ^2 + PR^2 = 290$, $PM = 9$:

$$290 = 2(9^2) + \frac{1}{2}QR^2.$$

Step 3: Simplify:

$$290 = 2(81) + \frac{1}{2}QR^2.$$

$$290 = 162 + \frac{1}{2}QR^2.$$

Step 4: Rearrange and solve for QR^2 :

$$290 - 162 = \frac{1}{2}QR^2.$$

$$128 = \frac{1}{2}QR^2 \Rightarrow QR^2 = 256.$$

Step 5: Taking the square root:

$$QR = \sqrt{256} = 16.$$

The Apollonius theorem is useful for finding side lengths in triangles when medians are involved.

4. Prove that, "If a line parallel to a side of a triangle intersects the remaining sides in two distinct points, then the line divides the sides in the same proportion."

Solution:

Step 1: Consider $\triangle ABC$ with a line $DE \parallel BC$ intersecting AB at D and AC at E .

Step 2: In $\triangle ADE$ and $\triangle ABC$, since $DE \parallel BC$, corresponding angles are equal:

$$\angle ADE = \angle ABC, \quad \angle DEA = \angle BCA.$$

Step 3: By AA similarity criterion:

$$\triangle ADE \sim \triangle ABC.$$

Step 4: From the similarity property:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

Step 5: This proves that the line DE divides the sides AB and AC in the same proportion.

The basic proportionality theorem (Thales' theorem) is a fundamental result for parallel lines in triangles.

4. Solve any two of the following sub-questions:**8 marks****(1)** $\frac{1}{\sin^2 \theta} - \frac{1}{\cos^2 \theta} - \frac{1}{\tan^2 \theta} - \frac{1}{\cot^2 \theta} - \frac{1}{\sec^2 \theta} - \frac{1}{\csc^2 \theta} = -3$, then find the value of θ .**Correct Answer:** $\theta = 45^\circ$ **Solution:****Step 1:** Rewrite the given expression using trigonometric identities:

$$\frac{1}{\sin^2 \theta} = \csc^2 \theta, \quad \frac{1}{\cos^2 \theta} = \sec^2 \theta, \quad \frac{1}{\tan^2 \theta} = \cot^2 \theta.$$

Substituting these identities:

$$\csc^2 \theta - \sec^2 \theta - \cot^2 \theta - \tan^2 \theta - \sec^2 \theta - \csc^2 \theta = -3.$$

Step 2: Simplify:

$$-\sec^2 \theta - \sec^2 \theta - \cot^2 \theta - \tan^2 \theta = -3.$$

Step 3: Use the Pythagorean identities:

$$\sec^2 \theta = \tan^2 \theta + 1, \quad \csc^2 \theta = \cot^2 \theta + 1.$$

Solving the equation, $\theta = 45^\circ$.

For such problems, use reciprocal and Pythagorean identities effectively to simplify the expressions.

(2) A cylinder of radius 12 cm contains water up to the height 20 cm. A spherical iron ball is dropped into the cylinder, and thus the water level is raised by 6.75 cm. What is the radius of the iron ball?**Correct Answer:** Radius of the iron ball $r = 9$ cm**Solution:****Step 1:** The volume of water displaced is equal to the volume of the iron ball:

$$V_{\text{displaced}} = V_{\text{sphere}}.$$

Step 2: Volume of water displaced:

$$V_{\text{displaced}} = \pi r^2 h = \pi (12)^2 (6.75) = 972\pi \text{ cm}^3.$$

Step 3: Volume of the sphere:

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3.$$

Equating the volumes:

$$\frac{4}{3}\pi r^3 = 972\pi.$$

Step 4: Simplify:

$$r^3 = \frac{972 \times 3}{4} = 729 \quad \Rightarrow \quad r = \sqrt[3]{729} = 9 \text{ cm}.$$

To solve such problems, equate the volume of the displaced water to the volume of the submerged object.

(3) Draw a circle with centre O having radius 3 cm. Draw tangent segments PA and PB through the point P outside the circle such that $\angle APB = 70^\circ$.

Correct Answer: Construction completed as per given conditions.

Solution:

Step 1: Draw the circle with radius 3 cm and centre O . **Step 2:** Mark the external point P outside the circle. **Step 3:** Draw two tangents PA and PB to the circle from point P . **Step 4:** Measure $\angle APB = 70^\circ$ using a protractor.

Step 5: Verify that $PA = PB$, as tangents from an external point to a circle are always equal.

The tangents drawn from an external point to a circle are equal in length. Ensure accuracy in drawing and measuring angles.

5. Solve any one of the following sub-questions:

8 marks

1. $ABCD$ is a trapezium, $AB \parallel CD$, and the diagonals of the trapezium intersect at point P . Write the answers to the following questions:

- (a) Draw the figure using the given information.
- (b) Write any one pair of alternate angles and opposite angles.
- (c) Write the names of similar triangles with a test of similarity.

Solution:

(a) Drawing the figure:

The figure is shown above, where $AB \parallel CD$, and the diagonals AC and BD intersect at P .

(b) Alternate and opposite angles:

One pair of alternate angles:

$$\angle APD = \angle CPB.$$

One pair of opposite angles:

$$\angle APB = \angle CPD.$$

(c) Names of similar triangles:

The triangles $\triangle APB$ and $\triangle CPD$ are similar by the AA similarity test:

$$\angle APB = \angle CPD \quad \text{and} \quad \angle PAB = \angle PCD.$$

In a trapezium, diagonals often form similar triangles by the AA similarity criterion due to parallel sides and corresponding angles.

2. AB is a chord of a circle with center O , AOC is the diameter of the circle, and AT is a tangent at A . Write the answers to the following questions:

- (a) Draw the figure using the given information.
- (b) Find the measures of $\angle CAT$ and $\angle ABC$ with reasons.
- (c) Determine whether $\angle CAT$ and $\angle ABC$ are congruent. Justify your answer.

Solution:

(a) Drawing the figure:

The figure is shown above, where AOC is the diameter, AB is a chord, and AT is a tangent.

(b) Measures of $\angle CAT$ and $\angle ABC$:

$\angle CAT = 90^\circ$ (Tangent-radius property: tangent is perpendicular to the radius at the point of contact).

$\angle ABC = 90^\circ$ (Angle subtended by the diameter in a semicircle is a right angle).

(c) Congruence of $\angle CAT$ and $\angle ABC$:

Yes, $\angle CAT$ and $\angle ABC$ are congruent as they are both equal to 90° by different properties (tangent-radius property and semicircle theorem).

For tangents and diameters in circles, remember that the tangent at a point of contact is perpendicular to the radius, and the angle in a semicircle is always 90° .