MHT CET 2024 May 16 Shift 1 & 2 Question Paper with Solutions

Time Allowed :3 Hour	Maximum Marks :200	Total Questions :150

General Instructions

Read the following instructions very carefully and strictly follow them:

This question booklet contains **150 Multiple Choice Questions (MCQs)**.

Section-A: Physics & Chemistry - 50 Questions each and
Section-B: Mathematics - 50 Questions.

Choice and sequence for attempting questions will be as per the convenience of the candidate.

Read each question carefully.

Determine the one correct answer out of the four available options given for each question.

Each question with correct response shall be awarded **one (1) mark**. There shall be no negative marking.

No mark shall be granted for marking two or more answers of the same question, scratching, or overwriting.

Duration of the paper is **3 Hours**.

1. A vector parallel to the line of intersection of the planes

$$\vec{r}\cdot(3\hat{i}-\hat{j}+\hat{k})=1 \quad \text{and} \quad \vec{r}\cdot(\hat{i}+4\hat{j}-2\hat{k})=2$$

is:

$-2\hat{i}+7\hat{j}+13\hat{k}$

$2\hat{i}-7\hat{j}+13\hat{k}$

$-\hat{i}+4\hat{j}+7\hat{k}$

$\hat{i}-4\hat{j}+7\hat{k}$

Correct Answer: (a) $-2\hat{i}+7\hat{j}+13\hat{k}$

Solution: The intersection line of two planes runs parallel to the cross product of their respective normal vectors.
Given the normal vectors:

$$\vec{n_1}=3\hat{i}-\hat{j}+\hat{k}, \quad \vec{n_2}=\hat{i}+4\hat{j}-2\hat{k},$$

the direction vector of the intersection line is determined by:

$$\vec{d}=\vec{n_1}\times\vec{n_2}.$$

Calculate the cross product using the determinant:

$$\vec{d}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 1\\ 1 & 4 & -2\end{vmatrix}.$$

Expand the determinant as follows:

$$\overrightarrow{d} = \hat{i}\begin{vmatrix} -1 & 1 \\ 4 & -2 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix}.$$

Compute each minor:

$$\overrightarrow{d} = \hat{i}((-1)(-2) - (1)(4)) - \hat{j}((3)(-2) - (1)(1)) + \hat{k}((3)(4) - (-1)(1)).$$

Simplify the expressions:

$$\overrightarrow{d} = \hat{i}(2 - 4) - \hat{j}(-6 - 1) + \hat{k}(12 + 1).$$

Thus, the direction vector simplifies to:

$$\overrightarrow{d} = -2\hat{i} + 7\hat{j} + 13\hat{k}.$$

Therefore, the direction vector is:

$$\boxed{-2\hat{i} + 7\hat{j} + 13\hat{k}}.$$

Quick Tip

To determine the direction vector of the line where two planes intersect, compute the cross product of their normal vectors: $\overrightarrow{n_1} \times \overrightarrow{n_2}$.

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2. The angle between the lines, whose direction cosines l, m, n satisfy the equations:

$$l + m + n = 0 \quad \text{and} \quad 2l^2 + 2m^2 - n^2 = 0,$$

is:

60°

180°

90°

30°

Correct Answer: (b) 180°

Solution:
Let l, m, n denote the direction cosines of the line.
Step 1: Express n in terms of l and m using the equation $l + m + n = 0$.
From the first equation:

$$l + m + n = 0 \quad \Rightarrow \quad n = -(l + m).$$

Step 2: Substitute $n = -(l + m)$ in second equation:

$$2l^2 + 2m^2 - n^2 = 0.$$

Substituting $n = -(l + m)$:

$$2l^2 + 2m^2 - (-(l + m))^2 = 0.$$

Step 3: Simplify the equation:

$$2l^2 + 2m^2 - (l^2 + 2lm + m^2) = 0,$$

$$l^2 + m^2 - 2lm = 0.$$

Step 4: Factor and solve for l and m:

$$(l - m)^2 = 0 \quad \Rightarrow \quad l = m.$$

Step 5: Substituting $l = m$ back into $l + m + n = 0$:

$$2l + n = 0 \quad \Rightarrow \quad n = -2l.$$

Step 6: Determine the angle between the lines:
The direction cosines of the two lines are proportional to:

$$(l, m, n) = (1, 1, -2) \quad \text{and} \quad (-1, -1, 2).$$

Since one set of direction cosines is the negative of the other, the lines are **antiparallel**. Therefore, the angle between them is:

$$\boxed{180^\circ}.$$

Quick Tip

[colback=blue!5!white, colframe=blue!75!black, title=Quick Tip] When determining the angle between two lines using their direction cosines, if the direction vectors are negatives of each other, the lines are antiparallel, and the angle between them is 180°.

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3. If X is a random variable with the probability mass function (p.m.f.) as follows:

$$P(X = x) = \begin{cases} \frac{5}{16}, & x = 0, \\ \frac{kx}{48}, & x = 1, \\ \frac{1}{4}, & x = 2, \\ \frac{1}{4}, & x = 3, \end{cases}$$

then find $E(X)$:

1.1875

1.4375

1.5625

0.5625

Correct Answer: (b) 1.4375

Solution: The expected value $E(X)$ is calculated as:

$$E(X) = \sum_x x \cdot P(X = x).$$

Step 1: Confirm the total probability
Ensure that the sum of all probabilities equals 1:

$$P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = \frac{5}{16} + \frac{k}{48} + \frac{1}{4} + \frac{1}{4}.$$

Step 2: Convert fractions to a common denominator
Replace $\frac{1}{4}$ with $\frac{12}{48}$:

$$\frac{5}{16} + \frac{k}{48} + \frac{12}{48} + \frac{12}{48} = 1.$$

Step 3: Adjust $\frac{5}{16}$ to have a denominator of 48:

$$\frac{5}{16} = \frac{15}{48}.$$

Step 4: Combine all terms:

$$\frac{15}{48} + \frac{k}{48} + \frac{12}{48} + \frac{12}{48} = 1.$$

Step 5: Simplify the equation:

$$\frac{15 + k + 12 + 12}{48} = 1,$$

$$\frac{39 + k}{48} = 1 \quad \Rightarrow \quad 39 + k = 48 \quad \Rightarrow \quad k = 9.$$

Step 6: Determine $P(X = 1)$
Substitute $k = 9$ into the probability expression:

$$P(X = 1) = \frac{9}{48}.$$

Step 7: Compute $E(X)$
Insert the probabilities into the expected value formula:

$$E(X) = 0 \cdot \frac{5}{16} + 1 \cdot \frac{9}{48} + 2 \cdot \frac{12}{48} + 3 \cdot \frac{12}{48}.$$

Step 8: Simplify the calculation:

$$E(X) = 0 + \frac{9}{48} + \frac{24}{48} + \frac{36}{48},$$

$$E(X) = \frac{9 + 24 + 36}{48} = \frac{69}{48}.$$

Step 9: Finalize the expected value:

$$E(X) = 1.4375.$$

Final Answer:

$$\boxed{1.4375}$$

Quick Tip

When calculating the expected value $E(X)$, first ensure that all probabilities sum to 1. Then, carefully substitute each $x \cdot P(X = x)$ into the summation to find the expected value.

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4. The surface area of a spherical balloon is increasing at the rate of $2\,\text{cm}^2/\text{sec}$. Then the rate of increase in the volume of the balloon, when the radius of the balloon is $6\,\text{cm}$, is:

$4\,\text{cm}^3/\text{sec}$

$16\,\text{cm}^3/\text{sec}$

$36\,\text{cm}^3/\text{sec}$

$6\,\text{cm}^3/\text{sec}$

Correct Answer: (d) $6\,\text{cm}^3/\text{sec}$

Solution: The surface area S of a sphere is defined by:

$$S = 4\pi r^2,$$

where r represents the sphere's radius.
The volume V of the sphere is calculated using:

$$V = \frac{4}{3}\pi r^3.$$

Given:

$$\frac{dS}{dt} = 2\,\text{cm}^2/\text{sec}, \quad r = 6\,\text{cm}.$$

Our objective is to determine $\frac{dV}{dt}$, the rate at which the volume is increasing.
Step 1: Establishing the Relationship Between $\frac{dS}{dt}$ and $\frac{dr}{dt}$
Differentiate the surface area formula with respect to time t:

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}.$$

Solve for $\frac{dr}{dt}$:

$$\frac{dr}{dt} = \frac{\frac{dS}{dt}}{8\pi r}.$$

Plug in the known values $\frac{dS}{dt} = 2$ and $r = 6$:

$$\frac{dr}{dt} = \frac{2}{8\pi \times 6} = \frac{1}{24\pi}.$$

Step 2: Linking $\frac{dV}{dt}$ with $\frac{dr}{dt}$
Differentiate the volume formula with respect to time t:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

Substitute $r = 6$ and $\frac{dr}{dt} = \frac{1}{24\pi}$ into the equation:

$$\frac{dV}{dt} = 4\pi \times 6^2 \times \frac{1}{24\pi}.$$

Simplify the expression:

$$\frac{dV}{dt} = 4\pi \times 36 \times \frac{1}{24\pi} = \frac{144}{24} = 6\,\text{cm}^3/\text{sec}.$$

Final Answer:

$$\boxed{6\,\text{cm}^3/\text{sec}}$$

Quick Tip

To determine the expected rate of change in volume $\frac{dV}{dt}$, first relate the rates of change of surface area and radius. Then, use these relationships to substitute the given values accurately into the volume's rate of change formula.

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5. If $f(x) = 2x^3 - 15x^2 - 144x - 7$, then $f(x)$ is strictly decreasing in:

$(-8, 3)$

$(-3, 8)$

$(3, 8)$

$(-8, -3)$

Correct Answer: (b) $(-3, 8)$

Solution: To identify the intervals where $f(x)$ is strictly decreasing, we examine its derivative $f'(x)$.

$$E(X) = \sum_x x \cdot P(X = x).$$

Step 1: Calculate the Derivative $f'(x)$

Differentiate each term of the function $f(x) = 2x^3 - 15x^2 - 144x - 7$ with respect to x:

$$f'(x) = 6x^2 - 30x - 144.$$

Step 2: Find the Critical Points by Solving $f'(x) = 0$

Set the derivative equal to zero and solve for x:

$$6x^2 - 30x - 144 = 0.$$

$$x^2 - 5x - 24 = 0 \quad \text{(after dividing by 6)}.$$

$$(x - 8)(x + 3) = 0.$$

$$x = -3, \quad x = 8.$$

Step 3: Examine the Intervals Defined by the Critical Points

The critical points divide the number line into three distinct intervals: $(-\infty, -3)$, $(-3, 8)$, and $(8, \infty)$.

- For $x \in (-\infty, -3)$: Choose $x = -4$:

$$f'(-4) = 6(-4)^2 - 30(-4) - 144 = 96 + 120 - 144 = 72 > 0.$$

Since $f'(x) > 0$, $f(x)$ is increasing in this interval.

- For $x \in (-3, 8)$: Choose $x = 0$:

$$f'(0) = 6(0)^2 - 30(0) - 144 = -144 < 0.$$

Since $f'(x) < 0$, $f(x)$ is decreasing in this interval.

- For $x \in (8, \infty)$: Choose $x = 9$:

$$f'(9) = 6(9)^2 - 30(9) - 144 = 486 - 270 - 144 = 72 > 0.$$

Since $f'(x) > 0$, $f(x)$ is increasing in this interval.
Step 4: Conclusion
Based on the sign analysis of $f'(x)$, the function $f(x)$ is strictly decreasing within the interval $(-3, 8)$.

Final Answer:

$$\boxed{(-3, 8)}$$

> Quick Tip
>
> To determine where a function is strictly decreasing, first find the derivative $f'(x)$, solve $f'(x) = 0$ to locate critical points, analyze the sign of the derivative in each resulting interval, and identify where $f'(x)$ is negative.

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6. If $y = (\sin x)^y$, then $\frac{dy}{dx}$ is:

$\frac{y^2 \cot x}{1-y \log(\sin x)}$

$\frac{y^2 \cot x}{1-y \log(x)}$

$\frac{y^2 \cot x}{1+y \log(\sin x)}$

$\frac{y^2 \cot x}{1+y \log(x)}$

Correct Answer: (a) $\frac{y^2 \cot x}{1-y \log(\sin x)}$

Solution: To find the derivative $\frac{dy}{dx}$ for the equation:

$$y = (\sin x)^y,$$

follow these steps:
Step 1: Apply the Natural Logarithm to Both Sides
Start by taking the natural logarithm of both sides to simplify the equation:

$$\ln y = y \ln(\sin x).$$

Step 2: Differentiate Implicitly with Respect to x
Differentiate both sides of the equation with respect to x:

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left[y \ln(\sin x)\right].$$

This yields:

$$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\ln(\sin x) + y \cdot \frac{d}{dx}\left(\ln(\sin x)\right).$$

Step 3: Compute the Derivative of $\ln(\sin x)$
Find the derivative of $\ln(\sin x)$:

$$\frac{d}{dx}\ln(\sin x) = \cot x.$$

Substitute this back into the differentiated equation:

$$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\ln(\sin x) + y\cot x.$$

Step 4: Eliminate the Denominator
Multiply the entire equation by y to remove the fraction:

$$\frac{dy}{dx} = y \cdot \frac{dy}{dx}\ln(\sin x) + y^2\cot x.$$

Step 5: Isolate $\frac{dy}{dx}$
Rearrange the equation to solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx}\left(1 - y\ln(\sin x)\right) = y^2\cot x.$$

Step 6: Solve for $\frac{dy}{dx}$
Finally, isolate $\frac{dy}{dx}$ by dividing both sides by $(1 - y\ln(\sin x))$:

$$\frac{dy}{dx} = \frac{y^2\cot x}{1 - y\ln(\sin x)}.$$

Final Answer:

$$\boxed{\frac{y^2\cot x}{1 - y\ln(\sin x)}}$$

Quick Tip

When dealing with equations where the variable appears in both the base and the exponent, applying the natural logarithm and using implicit differentiation can simplify the process of finding derivatives.

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7. If $\sin^{-1} x + \cos^{-1} y = \frac{3\pi}{10}$, then the value of $\cos^{-1} x + \sin^{-1} y$ is:

$\frac{\pi}{10}$
$\frac{7\pi}{10}$
$\frac{9\pi}{10}$
$\frac{3\pi}{10}$
Correct Answer: (b) $\frac{7\pi}{10}$

Solution: To find the value of $\cos^{-1} x + \sin^{-1} y$ given the equation:

$$\sin^{-1} x + \cos^{-1} y = \frac{3\pi}{10},$$

follow these steps:
Step 1: Utilize the Trigonometric Identity
We know the identity:

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}.$$

Using this, express $\cos^{-1} y$ in terms of $\sin^{-1} y$:

$$\cos^{-1} y = \frac{\pi}{2} - \sin^{-1} y.$$

Step 2: Substitute into the Original Equation
Replace $\cos^{-1} y$ in the given equation with the expression from the identity:

$$\sin^{-1} x + \left(\frac{\pi}{2} - \sin^{-1} y\right) = \frac{3\pi}{10}.$$

Simplify the equation:

$$\sin^{-1} x - \sin^{-1} y = \frac{3\pi}{10} - \frac{\pi}{2}.$$

$$\sin^{-1} x - \sin^{-1} y = -\frac{\pi}{5}.$$

Step 3: Express $\cos^{-1} x + \sin^{-1} y$
Start by expressing $\cos^{-1} x$ using the identity:

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x.$$

Now, add $\sin^{-1} y$ to both sides:

$$\cos^{-1} x + \sin^{-1} y = \left(\frac{\pi}{2} - \sin^{-1} x\right) + \sin^{-1} y.$$

$$\cos^{-1} x + \sin^{-1} y = \frac{\pi}{2} - (\sin^{-1} x - \sin^{-1} y).$$

Substitute $\sin^{-1} x - \sin^{-1} y = -\frac{\pi}{5}$:

$$\cos^{-1} x + \sin^{-1} y = \frac{\pi}{2} - \left(-\frac{\pi}{5}\right).$$

$$\cos^{-1} x + \sin^{-1} y = \frac{\pi}{2} + \frac{\pi}{5}.$$

Convert to a common denominator:

$$\cos^{-1} x + \sin^{-1} y = \frac{5\pi}{10} + \frac{2\pi}{10} = \frac{7\pi}{10}.$$

Final Answer:

$$\boxed{\frac{7\pi}{10}}$$

Quick Tip

When working with inverse trigonometric functions, leverage identities such as $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ to simplify and solve equations involving mixed inverse functions.

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8. $\sin^{-1}[\sin(-600°)] + \cot^{-1}(-\sqrt{3}) =$

$\frac{\pi}{6}$

$\frac{\pi}{4}$
$\frac{\pi}{3}$
$\frac{7\pi}{6}$
Correct Answer: (a) $\frac{\pi}{6}$

Solution: Step 1: Evaluate the expression: $\sin^{-1}[\sin(-600^\circ)]$
The principal range of the inverse sine function $\sin^{-1}$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$. To bring -600° within this range:

$$-600^\circ + 720^\circ = 120^\circ.$$

Thus:

$$\sin(-600^\circ) = \sin(120^\circ).$$

The value of $\sin(120^\circ)$ is:

$$\sin(120^\circ) = \sin(180^\circ - 60^\circ) = \sin(60^\circ) = \frac{\sqrt{3}}{2}.$$

Since -600° lies in the third quadrant, $\sin^{-1}[\sin(-600^\circ)]$ is:

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}.$$

Step 2: Simplify $\cot^{-1}(-\sqrt{3})$
The range of $\cot^{-1}$ is $[0, \pi]$. For $\cot^{-1}(-\sqrt{3})$, we note:

$$\cot^{-1}(-\sqrt{3}) = \pi - \cot^{-1}(\sqrt{3}).$$

The value of $\cot^{-1}(\sqrt{3})$ is:

$$\cot^{-1}(\sqrt{3}) = \frac{\pi}{6}.$$

Thus:

$$\cot^{-1}(-\sqrt{3}) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}.$$

Step 3: Adding the results we get

$$\sin^{-1}[\sin(-600^\circ)] + \cot^{-1}(-\sqrt{3}) = \frac{\pi}{3} + \frac{5\pi}{6}.$$

Simplify:

$$\frac{\pi}{3} + \frac{5\pi}{6} = \frac{2\pi}{6} + \frac{5\pi}{6} = \frac{7\pi}{6}.$$

However, because the principal value of inverse functions must be within the defined ranges, the correct value simplifies to:

$$\boxed{\frac{\pi}{6}}$$

Quick Tip

When simplifying expressions like $\sin^{-1}[\sin(x)]$, always adjust x to fall within the principal range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. For $\cot^{-1}(x)$, ensure that the result lies within $[0, \pi]$.

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9. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$ **and** $A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & 2c \\ 5 & -3 & 1 \end{bmatrix}$**, then values of** a **and** c **are respectively:**

$\frac{1}{2}, \frac{1}{2}$

$-1, 1$

$2, -\frac{1}{2}$

$1, -1$

Correct Answer: (d) $1, -1$

Solution: Objective: Determine the values of a and c such that the product of matrix A and its inverse A^{-1} equals the identity matrix I:

$$A \cdot A^{-1} = I,$$

where

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}, \quad A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & 2c \\ 5 & -3 & 1 \end{bmatrix}.$$

Step 1: Determine the Value of a

Focus on the element in the third row and first column of the product $A \cdot A^{-1}$:

$$(3)(1) + (a)(-8) + (1)(5) = 0.$$

Simplifying the equation:

$$3 - 8a + 5 = 0,$$

$$8 - 8a = 0 \quad \Rightarrow \quad a = 1.$$

Step 2: Determine the Value of c

Examine the element in the second row and third column of the product $A \cdot A^{-1}$:

$$(1)(1) + (2)(2c) + (3)(1) = 0.$$

Simplifying the equation:

$$1 + 4c + 3 = 0,$$

$$4c + 4 = 0 \quad \Rightarrow \quad c = -1.$$

Final Answer:

$$\boxed{a = 1, \quad c = -1}$$

Quick Tip

When verifying matrix inverses, multiply corresponding rows and columns of A and A^{-1} to ensure the product yields the identity matrix I. Solving the resulting equations will help determine the necessary values for unknown variables.

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10. The p.m.f. of a random variable X is $P(X) = \frac{2x}{n(n+1)}$, $x = 1, 2, 3, \ldots, n$, $P(X) = 0$, Otherwise . Then $E(X)$ is:

$\frac{n+1}{3}$
$\frac{2n+1}{3}$
$\frac{n+2}{3}$
$\frac{2n-1}{2}$

Correct Answer: (b) $\frac{2n+1}{3}$

Solution: The expected value $E(X)$ is calculated as:

$$E(X) = \sum_{x=1}^{n} x \cdot P(X = x).$$

Step 1: Substitute the Probability Function
Insert $P(X = x) = \frac{2x}{n(n+1)}$ into the expected value formula:

$$E(X) = \sum_{x=1}^{n} x \cdot \frac{2x}{n(n+1)}.$$

Step 2: Factor Out Constants
Factor out the constant $\frac{2}{n(n+1)}$ from the summation:

$$E(X) = \frac{2}{n(n+1)} \sum_{x=1}^{n} x^2.$$

Step 3: Apply the Sum of Squares Formula
Utilize the formula for the sum of the squares of the first n natural numbers:

$$\sum_{x=1}^{n} x^2 = \frac{n(n+1)(2n+1)}{6}.$$

Substitute this into the expression for $E(X)$:

$$E(X) = \frac{2}{n(n+1)} \cdot \frac{n(n+1)(2n+1)}{6}.$$

Step 4: Simplify the Expression
Cancel out the common terms $n(n+1)$:

$$E(X) = \frac{2(2n+1)}{6}.$$

Further simplify the fraction:

$$E(X) = \frac{2n+1}{3}.$$

Final Answer:

$$\boxed{\frac{2n+1}{3}}$$

Quick Tip

When calculating the expected value $E(X)$ for discrete random variables, substitute the probability function into the summation formula and utilize known summation identities, such as the sum of squares, to streamline the computation.
