MHT CET 2024 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**200	**Total Questions :**150

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The Duration of test is 3 Hours.
2. This paper consists of 150 Questions.
3. There are three parts in the paper consisting of Physics, Chemistry and Mathematics having 50 questions in each part of equal weightage..
4. Section-A: Physics and Chemistry - 50 Questions each.
5. Section-B: Mathematics - 50 Questions
6. Choice and sequence for attempting questions will be as per the convenience of the candidate.
7. Determine the one correct answer out of the four available options given for each question.
8. Each question with correct response shall be awarded one (1) mark. There shall be no negative marking.
9. No mark shall be granted for marking two or more answers of same question, scratching or overwriting

PHYSICS

1. Force between two point charges q_1 and q_2 placed in vacuum at a distance r cm apart is F. Force between them when placed in a medium having dielectric $K = 5$ at $r/5$ cm apart will be:

(1) $F/25$

(2) $5F$

(3) $F/5$

(4) $25F$

Correct Answer: (2) $5F$

Solution:

The electrostatic force between two charges in a vacuum is given by Coulomb's Law:

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}.$$

In a medium with a dielectric constant K, the force becomes:

$$F' = \frac{1}{4\pi K\epsilon_0} \cdot \frac{q_1 q_2}{r'^2}.$$

Here, $K = 5$ and $r' = \frac{r}{5}$. Substituting these values:

$$F' = \frac{1}{4\pi(5\epsilon_0)} \cdot \frac{q_1 q_2}{\left(\frac{r}{5}\right)^2}.$$

Simplify:

$$F' = \frac{25}{5} \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}.$$

Thus:

$$F' = 5F.$$

Conclusion: The force between the charges in the medium is $5F$.

Quick Tip
When charges are placed in a medium, always account for the dielectric constant K and any change in the distance between the charges. Both factors influence the resultant force.

2. A thin circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its center and perpendicular to its plane with angular velocity ω. If another disc of the same dimensions but of mass $M/2$ is placed gently on the first disc co-axially, then the new angular velocity of the system is:

(1) $\frac{4}{5}\omega$

(2) $\frac{5}{4}\omega$

(3) $\frac{2}{3}\omega$

(4) $\frac{3}{2}\omega$

Correct Answer: (3) $\frac{2}{3}\omega$

Solution:

Using the principle of conservation of angular momentum:

$$I_1\omega_1 = I_2\omega_2.$$

The initial moment of inertia of the system is:

$$I_1 = \frac{1}{2}MR^2.$$

The final moment of inertia after placing the second disc is:

$$I_2 = \frac{1}{2}MR^2 + \frac{1}{2}\left(\frac{M}{2}\right)R^2 = \frac{3}{4}MR^2.$$

Substituting these into the conservation of angular momentum equation:

$$\frac{1}{2}MR^2\omega = \frac{3}{4}MR^2\omega_2.$$

Simplifying:

$$\omega_2 = \frac{\frac{1}{2}}{\frac{3}{4}}\omega = \frac{2}{3}\omega.$$

Final Answer:

$$\boxed{\frac{2}{3}\omega}$$

Quick Tip
For conservation of angular momentum, always ensure that $I\omega$ remains constant for a closed system. Carefully update the moment of inertia I when new masses are added or redistributed in the system.

3. Correct Bernoulli's equation is (symbols have their usual meaning):

(1) $P + mgh + \frac{1}{2}mv^2 =$ constant

(2) $P + \rho gh + \frac{1}{2}\rho v^2 =$ constant

(3) $P + \rho gh + \rho v^2 =$ constant

(4) $P + \frac{1}{2}\rho gh + \frac{1}{2}\rho v^2 =$ constant

Correct Answer: (2) $P + \rho gh + \frac{1}{2}\rho v^2 =$ constant

Solution: Bernoulli's theorem describes the conservation of energy in a flowing fluid and can be derived by analyzing the work done by pressure forces, gravity, and changes in velocity along a streamline. It is mathematically expressed as:

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant},$$

where:

- P: Pressure energy per unit volume,
- ρ: Density of the fluid,
- g: Gravitational acceleration,
- h: Height above a reference level,
- v: Velocity of the fluid.

To analyze the given options:

1. Option (a): The use of m (mass) instead of ρ (density) is incorrect because Bernoulli's equation is based on energy per unit volume, not mass.
2. Option (c): Modifying the potential energy term as mg instead of ρgh is dimensionally inconsistent.
3. Option (d): Changing the kinetic energy term from $\frac{1}{2}\rho v^2$ to $\frac{1}{2}mv^2$ is incorrect for fluid dynamics, as it does not conform to energy per unit volume.

Only option (b) correctly preserves the balance of energy terms per unit volume:

$$P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}.$$

Final Answer:

$$\boxed{P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}}$$

Quick Tip

Bernoulli's theorem is fundamental for ideal fluid systems. Always ensure that terms represent energy per unit volume (pressure, potential energy, and kinetic energy) and use consistent units throughout.

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4. Two projectiles are projected at 30° and 60° with the horizontal with the same speed. The ratio of the maximum height attained by the two projectiles respectively is:

(1) $2 : \sqrt{3}$

(2) $\sqrt{3} : 1$

(3) $1 : 3$

(4) $1 : \sqrt{3}$

Correct Answer: (3) $1 : 3$

Solution: The maximum height attained by a projectile can be derived from the vertical motion component of the projectile. The vertical velocity at the maximum height is zero, and the equation of motion is:

$$v_y^2 = u_y^2 - 2gH_{\text{max}},$$

where:

- $u_y = u \sin\theta$ is the vertical component of the initial velocity,
- $v_y = 0$ at the maximum height,
- g is the acceleration due to gravity,
- H_{max} is the maximum height.

Rearranging for H_{max}, we get:

$$H_{\text{max}} = \frac{u_y^2}{2g} = \frac{(u \sin\theta)^2}{2g}.$$

Let the two projectiles have angles of projection:

$$\theta_1 = 30^\circ \quad \text{and} \quad \theta_2 = 60^\circ.$$

For the first projectile:

$$H_1 = \frac{(u \sin 30^\circ)^2}{2g}.$$

For the second projectile:

$$H_2 = \frac{(u \sin 60^\circ)^2}{2g}.$$

The ratio of their maximum heights is:

$$\frac{H_1}{H_2} = \frac{(u \sin 30^\circ)^2}{(u \sin 60^\circ)^2}.$$

Canceling common terms u^2 and substituting $\sin 30^\circ = \frac{1}{2}$ and $\sin 60^\circ = \frac{\sqrt{3}}{2}$, we get:

$$\frac{H_1}{H_2} = \frac{\left(\frac{1}{2}\right)^2}{\left(\frac{\sqrt{3}}{2}\right)^2}.$$

Simplify:

$$\frac{H_1}{H_2} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}.$$

Thus, the ratio of the maximum heights is:

$$H_1 : H_2 = 1 : 3.$$

Final Answer:

$$\boxed{1:3}$$

Quick Tip
To determine the ratio of maximum heights of projectiles, focus on the square of the sine of the angle of projection, $\sin^2\theta$, as height depends directly on it.

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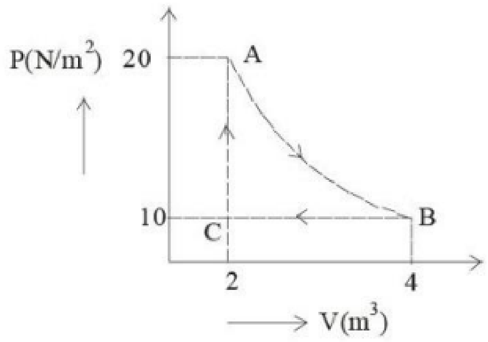
5. A real gas within a closed chamber at 27°C undergoes the cyclic process as shown in the figure. The gas obeys the equation $PV^3 = RT$ for the path A to B. The net work done in the complete cycle is (assuming $R = 8$ J/molK):



(1) 225 J

(2) 205 J

(3) 20 J

(4) -20 J

Correct Answer: (2) 205 J

Solution:

The cyclic process includes three distinct stages: AC (isochoric process), BC (isobaric process), and AB (defined by $PV^3 = RT$).

Step 1: Isochoric process (AC) In an isochoric process, the volume remains constant, so no work is done:

$$W_{\text{AC}} = 0.$$

Step 2: Isobaric process (BC) The work done in an isobaric process is calculated using:

$$W_{\text{BC}} = P\Delta V = 10(2-4) = -20\,\text{J}.$$

Step 3: Process AB ($PV^3 = RT$) The work done in this process is:

$$W_{\text{AB}} = \int P\,dV = \int_{V_1}^{V_2} \frac{RT}{V^3}\,dV.$$

Substitute $V_1 = 2\,\text{m}^3$ and $V_2 = 4\,\text{m}^3$:

$$W_{\text{AB}} = RT\int_2^4 V^{-3}\,dV = -\frac{RT}{2}\left[\frac{1}{V^2}\right]_2^4.$$

Simplify the expression:

$$W_{\text{AB}} = -\frac{RT}{2}\left(\frac{1}{4^2} - \frac{1}{2^2}\right) = -\frac{RT}{2}\left(\frac{1}{16} - \frac{1}{4}\right).$$

$$W_{AB} = -\frac{RT}{2}\left(\frac{1-4}{16}\right) = \frac{RT}{2} \cdot \frac{3}{16}.$$

Substitute $R = 8\,\text{J/molK}$ and $T = 27°\text{C} = 300\,\text{K}$:

$$W_{AB} = \frac{8 \cdot 300}{2} \cdot \frac{3}{16} = 225\,\text{J}.$$

Net Work Done: The total work done is the sum of work from all segments of the process:

$$W_{net} = W_{AC} + W_{BC} + W_{AB} = 0 - 20 + 225 = 205\,\text{J}.$$

Final Answer:

$$\boxed{205\,\text{J}}$$

Quick Tip

For cyclic processes, calculate the work done for each stage and combine the results. Always account for the nature of each segment—whether it is isochoric, isobaric, or follows a specific relation between pressure and volume.

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6. Light emerges out of a convex lens when a source of light is kept at its focus. The shape of the wavefront of the light is:

(1) Both spherical and cylindrical

(2) Cylindrical

(3) Spherical

(4) Plane

Correct Answer: (4) Plane

Solution:

A wavefront is defined as the locus of points that oscillate in phase. When a point source of light is placed at the focal point of a convex lens, the rays of light emerging from the lens become parallel to each other.

These parallel rays form a plane wavefront, as all points on the wavefront lie at the same distance from the source and propagate in the same direction. The convex lens transforms the initially spherical wavefront into a plane wavefront due to its focusing properties.

Final Answer:

$$\boxed{\text{Plane}}$$

Quick Tip
A plane wavefront is produced when a convex lens converts spherical wavefronts into parallel rays by positioning the light source at its focal point.

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7. A monkey of mass 50 kg climbs on a rope which can withstand the tension $T = 350$ N. If the monkey initially climbs down with an acceleration of 4 m/s^2 and then climbs up with an acceleration of 5 m/s^2, choose the correct option ($g = 10$ m/s^2):

(1) $T = 700$ N while climbing upward

(2) $T = 350$ N while going downward

(3) Rope will break while climbing upward

(4) Rope will break while going downward

Correct Answer: (3) Rope will break while climbing upward

Solution:

Step 1: Climbing upward The tension in the rope during upward motion is calculated using the formula:

$$T = m(g + a),$$

where:

- $m = 50$ kg (mass of the monkey),
- $g = 10$ m/s^2 (gravitational acceleration),
- $a = 5$ m/s^2 (acceleration while climbing upward).

Substituting the values:

$$T = 50(10 + 5) = 750\,\text{N}.$$

Since the tension T exceeds the rope's breaking strength (350 N), the rope will break during upward motion.

Step 2: Climbing downward The tension in the rope during downward motion is given by:

$$T = m(g - a),$$

where $a = 4\,\text{m/s}^2$ (acceleration while climbing downward).

Substitute the values:

$$T = 50(10 - 4) = 300\,\text{N}.$$

Here, the tension T is less than the breaking strength ($350\,\text{N}$), so the rope will not break during downward motion.

Final Answer:

$$\boxed{\text{Rope will break while climbing upward.}}$$

Quick Tip

When calculating tension for vertical motion, use $T = m(g + a)$ for upward motion and $T = m(g - a)$ for downward motion. Compare the result with the breaking strength to determine if failure occurs.

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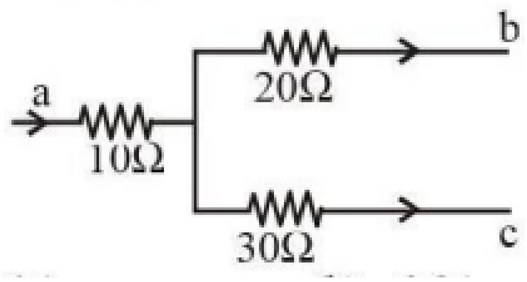
8. Figure shows a part of an electric circuit. The potentials at points $a, b,$ and c are $30\,\text{V}, 12\,\text{V},$ and $2\,\text{V}$, respectively. The current through the $20\,\Omega$ resistor will be:



(1) $0.4\,\text{A}$

(2) $0.2\,\text{A}$

(3) $0.6\,\text{A}$

(4) $1.0\,\text{A}$

Correct Answer: (1) $0.4\,\text{A}$

Solution: To calculate the current through the $20\,\Omega$ resistor, we proceed as follows:

Step 1: Identify currents and apply Kirchhoff's Current Law (KCL)

Define the currents through the $10\,\Omega$, $20\,\Omega$, and $30\,\Omega$ resistors as $I_1, I_2,$ and I_3, respectively. At the junction point, KCL states:

$$I_1 = I_2 + I_3.$$

Using Ohm's Law, the currents are expressed as:

$$I_1 = \frac{V_a - V_b}{10}, \quad I_2 = \frac{V_b - V_c}{20}, \quad I_3 = \frac{V_b - V_c}{30}.$$

Step 2: Substitute known voltage values

The given potentials are:

$$V_a = 30\,\text{V}, \quad V_b = 12\,\text{V}, \quad V_c = 2\,\text{V}.$$

Substituting these into the equations:

$$I_1 = \frac{30 - 12}{10} = 1.8\,\text{A},$$
$$I_2 = \frac{12 - 2}{20} = 0.4\,\text{A},$$
$$I_3 = \frac{12 - 2}{30} = 0.333\,\text{A}.$$

Step 3: Verify current conservation at the junction

Check if $I_1 = I_2 + I_3$:

$$I_1 = 0.4 + 0.333 = 1.8\,\text{A}.$$

This confirms that Kirchhoff's Current Law is satisfied.

Step 4: Determine the current through $20\,\Omega$

The current through the $20\,\Omega$ resistor is directly I_2:

$$I_2 = 0.4\,\text{A}.$$

Final Answer:

$$\boxed{0.4\,\text{A}}$$

Quick Tip

In circuit analysis, always apply Ohm's Law to calculate individual currents and Kirchhoff's Laws to ensure conservation of current and voltage across the network.

9. A current of $200\,\mu$A deflects the coil of a moving coil galvanometer through $60°$. The current to cause deflection through $\frac{\pi}{10}$ radian is:

(1) $30\,\mu$A

(2) $120\,\mu$A

(3) $60\,\mu$A

(4) $180\,\mu$A

Correct Answer: (3) $60\,\mu$A

Solution: The deflection of a moving coil galvanometer is proportional to the current passing through it:

$$I \propto \theta.$$

Given:

$$I_1 = 200\,\mu\text{A}, \quad \theta_1 = 60°, \quad \theta_2 = \frac{\pi}{10}.$$

Convert $60°$ to radians:

$$\theta_1 = 60° = \frac{\pi}{3}.$$

Using the proportionality relation:

$$\frac{I_2}{I_1} = \frac{\theta_2}{\theta_1}.$$

Substituting the values:

$$\frac{I_2}{200} = \frac{\frac{\pi}{10}}{\frac{\pi}{3}}.$$

Simplify the equation:

$$\frac{I_2}{200} = \frac{3}{10}.$$

Solve for I_2:

$$I_2 = 200 \cdot \frac{3}{10} = 60\,\mu\text{A}.$$

Final Answer:

$$\boxed{60\,\mu\text{A}}$$

Quick Tip

The deflection angle in a moving coil galvanometer is proportional to the current passing through it. Always use the proportionality relation $\frac{I_2}{I_1} = \frac{\theta_2}{\theta_1}$ to calculate unknown currents when deflection angles are given.

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10. The value of acceleration due to gravity at Earth's surface is 9.8 m/s^2. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 m/s^2 is close to: (Radius of Earth $R = 6.4 \times 10^6$ m)

(1) 2.6×10^6 m

(2) 6.4×10^6 m

(3) 9.0×10^6 m

(4) 1.6×10^6 m

Correct Answer: (1) 2.6×10^6 m

Solution: The formula for the acceleration due to gravity at a height h above the Earth's surface is:

$$g_h = \frac{g}{\left(1 + \frac{h}{R}\right)^2}.$$

Given:

$$g_h = \frac{g}{2}.$$

Substituting into the equation:

$$\frac{g}{2} = \frac{g}{\left(1 + \frac{h}{R}\right)^2}.$$

Simplifying:

$$\left(1 + \frac{h}{R}\right)^2 = 2.$$

Taking the square root:

$$1 + \frac{h}{R} = \sqrt{2}.$$

Solving for h:

$$\frac{h}{R} = \sqrt{2} - 1.$$

Substituting $R = 6.4 \times 10^6$ m:

$$h = (\sqrt{2} - 1) \cdot 6.4 \times 10^6.$$

Simplifying further:

$$h = (1.414 - 1) \cdot 6.4 \times 10^6 = 0.414 \cdot 6.4 \times 10^6 = 2.6 \times 10^6 \,\text{m}.$$

Final Answer:

$$\boxed{2.6 \times 10^6 \,\text{m}}$$

Quick Tip

As altitude increases, the acceleration due to gravity decreases. Use the relation $g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$ to determine the altitude when g_h is known.

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11. Relative permittivity and permeability of a material are ε_r and μ_r, respectively. Which of the following values of these quantities are allowed for a diamagnetic material?

(1) $\varepsilon_r = 0.5, \mu_r = 1.5$

(2) $\varepsilon_r = 1.5, \mu_r = 0.5$

(3) $\varepsilon_r = 0.5, \mu_r = 0.5$

(4) $\varepsilon_r = 1.5, \mu_r = 1.5$

Correct Answer: (2) $\varepsilon_r = 1.5, \mu_r = 0.5$

Solution: The relative permittivity (ε_r) and relative permeability (μ_r) of a diamagnetic material follow specific rules:

$$\mu_r < 1 \quad \text{and} \quad \varepsilon_r > 1.$$

From the given options:

$$\varepsilon_r = 1.5, \mu_r = 0.5 \quad \text{are correct.}$$

Final Answer:

$$\boxed{\varepsilon_r = 1.5, \mu_r = 0.5}$$

Quick Tip

For diamagnetic materials, remember $\mu_r < 1$ and $\varepsilon_r > 1$. This helps distinguish them from other types of materials.

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12. The magnetic flux through a coil perpendicular to its plane is varying according to the relation $\phi = 5t^3 + 4t^2 + 2t - 5$. If the resistance of the coil is $5\,\Omega$, then the induced current through the coil at $t = 2$ sec will be:

(1) 15.6 A

(2) 16.6 A

(3) 17.6 A

(4) 18.6 A

Correct Answer: (1) 15.6 A

Solution: The induced EMF (e) can be derived using Faraday's Law, which is expressed as:

$$e = \left| \frac{d\phi}{dt} \right|.$$

Given the magnetic flux:

$$\phi = 5t^3 + 4t^2 + 2t - 5.$$

Now, differentiate ϕ with respect to t:

$$e = \left| \frac{d\phi}{dt} \right| = \left| 15t^2 + 8t + 2 \right|.$$

To calculate e at $t = 2$ sec:

$$e = 15(2)^2 + 8(2) + 2 = 15(4) + 16 + 2 = 60 + 16 + 2 = 78\,\text{V}.$$

Next, we apply Ohm's Law to find the induced current i:

$$i = \frac{e}{R}.$$

Substitute $R = 5\,\Omega$ into the equation:

$$i = \frac{78}{5} = 15.6\,\text{A}.$$

Final Answer:

$$\boxed{15.6\,\mathrm{A}}.$$

Quick Tip
Always differentiate the magnetic flux ϕ with respect to time t to calculate the induced EMF, and use $i = \frac{e}{R}$ to determine the current.

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13. A solid metallic cube having total surface area $24\,\mathrm{m}^2$ is uniformly heated. If its temperature is increased by 10° C, calculate the increase in volume of the cube.

(Given: $\alpha = 5.0 \times 10^{-4}\,\mathrm{C}^{-1}$)

(1) $2.4 \times 10^6\,\mathrm{cm}^3$

(2) $1.2 \times 10^5\,\mathrm{cm}^3$

(3) $6.0 \times 10^4\,\mathrm{cm}^3$

(4) $4.8 \times 10^5\,\mathrm{cm}^3$

Correct Answer: (2) $1.2 \times 10^5\,\mathrm{cm}^3$

Solution Steps:

Step 1: Identify the formula for volume change The increase in volume is given by:

$$\Delta V = V_0 \gamma \Delta T,$$

where:

- $V_0 = a^3$ (initial volume of the cube),
- $\gamma = 3\alpha$ (coefficient of volumetric expansion),
- $\Delta T = 10^\circ$ C (temperature change).

Step 2: Calculate the surface area and solve for a The surface area of a cube is:

$$\text{Total Surface Area} = 6a^2.$$

Given $6a^2 = 24\,\mathrm{m}^2$, solve for a:

$$a^2 = \frac{24}{6} = 4\,\mathrm{m}^2 \quad \Longrightarrow \quad a = 2\,\mathrm{m}.$$

Step 3: Calculate the initial volume V_0 The initial volume is:

$$V_0 = a^3 = 2^3 = 8\,\text{m}^3 = 8 \times 10^6\,\text{cm}^3.$$

Step 4: Substitute values into ΔV formula Substitute into the formula for ΔV:

$$\Delta V = 8 \times 10^6 \cdot (3 \cdot 5 \times 10^{-4}) \cdot 10.$$

Step 5: Simplify the expression

$$\Delta V = 8 \times 10^6 \cdot 15 \times 10^{-3} = 1.2 \times 10^5\,\text{cm}^3.$$

Conclusion:

$$\boxed{1.2 \times 10^5\,\text{cm}^3}$$

Quick Tip

For solids, use $\gamma = 3\alpha$ to find volumetric expansion. Ensure units for V_0 are consistent and surface area formulas are accurately applied.

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14. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon:

(1) the rates at which currents are changing in the two coils

(2) the relative position and orientation of the two coils

(3) the materials of the wires of the coils

(4) the currents in the two coils

Correct Answer: (2) the relative position and orientation of the two coils

Solution Steps:

Step 1: Define mutual inductance The mutual inductance between two coils is defined as:

$$M = \frac{\Phi}{I},$$

where Φ is the magnetic flux in one coil due to the current I in the other coil.

Step 2: Factors affecting mutual inductance The mutual inductance depends on:

- The relative position and orientation of the two coils (determines the coupling of magnetic fields).
- The number of turns and dimensions of the coils.
- The magnetic permeability of the medium between the coils.

It does not depend directly on the current or the material of the coil wires.

Conclusion:

$$\boxed{\text{Relative position and orientation of the two coils.}}$$

> Quick Tip
>
> Mutual inductance depends on the geometry, orientation, and coupling of the coils, not on the material of the wires.

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15. A proton, an electron, and an alpha particle have the same energies. Their de-Broglie wavelengths will be compared as:

(1) $\lambda_e > \lambda_\alpha > \lambda_p$

(2) $\lambda_\alpha < \lambda_p < \lambda_e$

(3) $\lambda_p < \lambda_e < \lambda_\alpha$

(4) $\lambda_p > \lambda_e > \lambda_\alpha$

Correct Answer: (2) $\lambda_\alpha < \lambda_p < \lambda_e$

Solution Steps:

Step 1: Define the de-Broglie wavelength formula The de-Broglie wavelength is given by:

$$\lambda = \frac{h}{\sqrt{2mE}},$$

where h is Planck's constant, m is the mass of the particle, and E is its energy.

Step 2: Relate wavelength to mass for particles with the same energy For particles with the same energy:

$$\lambda \propto \frac{1}{\sqrt{m}}.$$

Step 3: Compare masses of the particles The masses of the particles are:

- Electron (m_e): least mass,
- Proton (m_p): greater mass,
- Alpha particle ($m_\alpha = 4m_p$): greatest mass.

Step 4: Determine the order of wavelengths Since $\lambda \propto \frac{1}{\sqrt{m}}$, the order of the wavelengths is:

$$\lambda_e > \lambda_p > \lambda_\alpha.$$

Conclusion:

$$\boxed{\lambda_\alpha < \lambda_p < \lambda_e}$$

> Quick Tip
>
> For particles with the same energy, the de-Broglie wavelength is inversely proportional to the square root of their mass.

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16. An ice cube has a bubble inside. When viewed from one side, the apparent distance of the bubble is 12 cm. When viewed from the opposite side, the apparent distance of the bubble is 4 cm. If the side of the ice cube is 24 cm, the refractive index of the ice cube is:

(1) $\frac{4}{3}$
(2) $\frac{3}{2}$
(3) $\frac{2}{3}$
(4) $\frac{6}{5}$

Correct Answer: (2) $\frac{3}{2}$

Solution Steps:

Step 1: Define the formula for refractive index The refractive index μ is given by:

$$\mu = \frac{\text{Real Depth}}{\text{Apparent Depth}}.$$

Step 2: Set up the problem variables Let the real depth of the bubble be x cm. When viewed from one side, the apparent depth is 12 cm, and when viewed from the opposite side, the apparent depth is $24 - x$ cm.

Step 3: Write expressions for μ From the refractive index formula:

$$\mu = \frac{x}{12} \quad \text{(from one side)},$$

$$\mu = \frac{24 - x}{4} \quad \text{(from the opposite side)}.$$

Step 4: Solve for x Equate the two expressions for μ:

$$\frac{x}{12} = \frac{24 - x}{4}.$$

Simplify:

$$4x = 12(24 - x).$$

$$4x = 288 - 12x.$$

$$16x = 288 \quad \implies \quad x = 18\,\text{cm}.$$

Step 5: Calculate μ Substitute $x = 18\,\text{cm}$ into $\mu = \frac{x}{12}$:

$$\mu = \frac{18}{12} = \frac{3}{2}.$$

Conclusion:

$$\boxed{\frac{3}{2}}$$

Quick Tip
To find the refractive index of an object with apparent depths viewed from two sides, equations for the real and apparent depths set up and solve for μ.

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17. The longest wavelength associated with the Paschen series is:

(Given $R_H = 1.097 \times 10^7$ SI unit).

(1) $1.094 \times 10^{-6}\,\text{m}$

(2) $2.973 \times 10^{-6}\,\text{m}$

(3) $3.646 \times 10^{-6}\,\text{m}$

(4) $1.876 \times 10^{-6}\,\text{m}$

Correct Answer: (4) $1.876 \times 10^{-6}\,\text{m}$

Solution: The mathematical expression for the wavelength in the Paschen series is:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right),$$

where:

- $n_1 = 3$, representing the Paschen series,
- $n_2 = 4$, indicating the next energy level above n_1 for the longest wavelength.

By substituting the values:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{3^2} - \frac{1}{4^2} \right).$$

Simplification yields:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{9} - \frac{1}{16} \right).$$

$$\frac{1}{\lambda} = R_H \cdot \frac{16-9}{144}.$$

$$\frac{1}{\lambda} = R_H \cdot \frac{7}{144}.$$

By substituting value of $R_H = 1.097 \times 10^7$:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \cdot \frac{7}{144}.$$

$$\frac{1}{\lambda} = \frac{7.679 \times 10^7}{144}.$$

$$\lambda = \frac{144}{7.679 \times 10^7} \approx 1.876 \times 10^{-6}\,\text{m}.$$

Final Answer:

$$\boxed{1.876 \times 10^{-6}\,\text{m}}$$

Quick Tip

To calculate the longest wavelength in a spectral series, use $n_2 = n_1 + 1$ and carefully insert the values into the formula for accurate results.

---

18. The ratio of the mass densities of nuclei of ^{40}Ca and ^{16}O is close to:

(1) 1

(2) 0.1

(3) 5

(4) 2

Correct Answer: (1) 1

Solution: The nuclear density (ρ) remains approximately constant across all nuclei, regardless of atomic number or mass number. This invariance arises from the proportional relationship between the nuclear mass and volume with the mass number (A).

1. Nuclear Density for ^{40}Ca: For ^{40}Ca, the mass number $A = 40$. The nuclear density is:

$$\rho_{\text{Ca}} = \text{constant}.$$

2. Nuclear Density for ^{16}O: For ^{16}O, the mass number $A = 16$. The nuclear density remains the same as that of ^{40}Ca:

$$\rho_{\text{O}} = \text{constant}.$$

3. Ratio of Densities: As the nuclear density is constant for all nuclei:

$$\frac{\rho_{\text{Ca}}}{\rho_{\text{O}}} = 1.$$

Final Answer:

$$\boxed{1}$$

Quick Tip

The constancy of nuclear density arises because both nuclear mass and volume scale directly with the mass number (A). This principle holds for all atomic nuclei, regardless of size or composition.

---

19. Point charge of $10\,\mu C$ is placed at the origin. At what location on the X-axis should a point charge of $40\,\mu C$ be placed so that the net electric field is zero at $x = 2$ cm on the X-axis?

(1) $6\,$cm

(2) $4\,\text{cm}$

(3) $8\,\text{cm}$

(4) $-4\,\text{cm}$

Correct Answer: (1) $6\,\text{cm}$

Solution: To find the position x_0 where the second charge ($40\,\mu$C) should be located so that the net electric field is zero at $x = 2\,\text{cm}$:

Step 1: Balance the Electric Fields The net electric field at $x = 2\,\text{cm}$ is zero when the contributions from both charges cancel out:

$$E_{\text{net}} = 0 \implies K \cdot \frac{10 \times 10^{-6}}{(2)^2} = K \cdot \frac{40 \times 10^{-6}}{(x_0 - 2)^2}.$$

Step 2: Simplify the Equation Divide out K and simplify the terms:

$$\frac{10}{4} = \frac{40}{(x_0 - 2)^2}.$$

$$\frac{1}{2} = \frac{2}{(x_0 - 2)^2}.$$

Step 3: Solve for x_0 Rearrange the equation:

$$(x_0 - 2)^2 = 4.$$

Take the square root of both sides:

$$x_0 - 2 = \pm 2.$$

Thus:

$$x_0 = 4\,\text{cm} \quad \text{or} \quad x_0 = 6\,\text{cm}.$$

Step 4: Choose the Correct Value of x_0 Since the second charge is positioned to the right of $x = 2\,\text{cm}$, the correct location is:

$$x_0 = 6\,\text{cm}.$$

Final Answer:

$$\boxed{x_0 = 6\,\text{cm}}$$

Quick Tip

When determining the position for zero net electric field, equate the magnitudes of electric fields from the charges and carefully solve the resulting equation. Consider the geometry and direction of forces to select the correct solution.

20. A magnetic needle is kept in a non-uniform magnetic field. It experiences:

(1) neither a force nor a torque

(2) a torque but not a force

(3) a force but not a torque

(4) a force and a torque

Correct Answer: (4) a force and a torque

Solution: When a magnetic needle is placed in a non-uniform magnetic field, it experiences both a net force and a torque.

The non-uniform magnetic field applies forces of different magnitudes on the north and south poles of the needle, causing an imbalance.

This imbalance results in a net force acting on the needle.

Simultaneously, these unequal forces produce a torque that aligns the needle along the magnetic field lines.

Final Answer:

$$\boxed{\text{a force and a torque}}$$

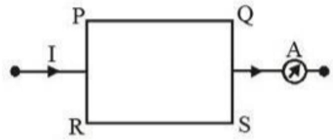
Quick Tip

In a non-uniform magnetic field, the imbalance in forces on the poles of a magnetic dipole generates both a net force and a torque. The torque aligns the dipole with the field lines.

---

21. A current-carrying rectangular loop PQRS is made of uniform wire. The length PR = QS = 5 cm and PQ = RS = 100 cm. If the ammeter current reading changes from I to

$2I$**, the ratio of magnetic forces per unit length on the wire PQ due to wire RS in the two cases respectively** $F^I_{PQ} : F^{2I}_{PQ}$ **is:**



(1) $1 : 2$

(2) $1 : 4$

(3) $1 : 5$

(4) $1 : 3$

Correct Answer: (2) $1 : 4$

Solution: The magnetic force per unit length F/ℓ between two parallel current-carrying wires is expressed as:

$$F/\ell = \frac{\mu_0 I_1 I_2}{4\pi r},$$

where: - μ_0 is the permeability of free space, - I_1 and I_2 are the currents in the two wires, - r is the separation between the wires.

Step 1: Analyze Proportionality for F/ℓ The magnetic force is proportional to the product of the currents:

$$F_1 \propto I^2 \quad \text{(when current is } I\text{)}.$$

When the current is increased to $2I$:

$$F_2 \propto (2I)^2 = 4I^2.$$

Step 2: Calculate the Ratio of Forces The ratio of the magnetic forces is:

$$\frac{F_1}{F_2} = \frac{I^2}{4I^2} = \frac{1}{4}.$$

Therefore, the ratio of the forces is:

$$F^I_{PQ} : F^{2I}_{PQ} = 1 : 4.$$

Final Answer:

$$\boxed{1 : 4}$$

Quick Tip

The magnetic force between parallel wires varies with the product of the currents. If a current doubles, the force increases by the square of the change factor.

---

22. At which temperature will the r.m.s. velocity of a hydrogen molecule be equal to that of an oxygen molecule at $47°$C?

(1) 80 K

(2) -73 K

(3) 4 K

(4) 20 K

Correct Answer: (4) 20 K

Solution: The root mean square (r.m.s.) velocity v_{rms} of gas molecules is determined by the formula:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}},$$

where: - R is the universal gas constant, - T is the absolute temperature in Kelvin, - M is the molar mass of the gas.

Step 1: Equating v_{rms} for Hydrogen and Oxygen For hydrogen (H_2) and oxygen (O_2):

$$v_{\text{rms}}(H_2) = v_{\text{rms}}(O_2).$$

Substitute the expression for v_{rms}:

$$\sqrt{\frac{3RT_{H_2}}{M_{H_2}}} = \sqrt{\frac{3RT_{O_2}}{M_{O_2}}}.$$

Square both sides:

$$\frac{T_{H_2}}{M_{H_2}} = \frac{T_{O_2}}{M_{O_2}}.$$

Rearrange to solve for T_{H_2}:

$$T_{H_2} = T_{O_2} \cdot \frac{M_{H_2}}{M_{O_2}}.$$

Step 2: Substitute Known Values The temperature of oxygen gas is:

$$T_{O_2} = 47°\text{C} + 273 = 320\,\text{K}.$$

Molar masses are:

$$M_{H_2} = 2, \quad M_{O_2} = 32.$$

Substitute into the formula for T_{H_2}:

$$T_{H_2} = 320 \cdot \frac{2}{32}.$$

Simplify:

$$T_{H_2} = 320 \cdot \frac{1}{16} = 20\,\text{K}.$$

Final Answer:

$$\boxed{20\,\text{K}}$$

Quick Tip

The r.m.s. velocity is influenced by both the temperature and the molar mass of the gas. Use the proportionality $v_{\text{rms}} \propto \sqrt{\frac{T}{M}}$ to connect the properties of different gases.

---

23. A light-emitting diode (LED) is fabricated using GaAs semiconductor material whose band gap is $1.42\,$eV. The wavelength of light emitted from the LED is:

(1) $650\,$nm

(2) $1243\,$nm

(3) $875\,$nm

(4) $1400\,$nm

Correct Answer: (3) $875\,$nm

Solution: The wavelength of emitted light is determined using the relationship between the band gap energy E_g and wavelength λ:

$$E_g(\text{eV}) = \frac{1240}{\lambda(\text{nm})}.$$

Rearranging for λ:

$$\lambda = \frac{1240}{E_g}.$$

Step 1: Substitute the Given Values Given:

$$E_g = 1.42\,\text{eV}.$$

Substitute into the equation:

$$\lambda = \frac{1240}{1.42}.$$

Perform the calculation:

$$\lambda \approx 875\,\text{nm}.$$

Final Answer:

$$\boxed{875\,\text{nm}}$$

Quick Tip
The wavelength of light emitted by a semiconductor is inversely related to its band gap energy E_g. A smaller band gap corresponds to a longer wavelength, and vice versa.

---

24. A steel wire with mass per unit length 7.0×10^{-3} kg/m is under a tension of $70\,\text{N}$. The speed of transverse waves in the wire will be:

(1) $100\,\text{m/s}$

(2) $50\,\text{m/s}$

(3) $10\,\text{m/s}$

(4) 200π m/s

Correct Answer: (1) $100\,\text{m/s}$

Solution: The speed of transverse waves in a wire is calculated using the formula:

$$v = \sqrt{\frac{T}{\mu}},$$

where: - T is the tension in the wire, - μ is the mass per unit length.

Step 1: Substitute the Given Values Given:

$$T = 70\,\text{N}, \quad \mu = 7.0 \times 10^{-3}\,\text{kg/m}.$$

Substitute into the formula:

$$v = \sqrt{\frac{70}{7.0 \times 10^{-3}}}.$$

Step 2: Simplify the Calculation

$$v = \sqrt{\frac{70}{0.007}} = \sqrt{10000}.$$

$$v = 100\,\text{m/s}.$$

Final Answer:

$$\boxed{100\,\text{m/s}}$$

Quick Tip
The speed of transverse waves in a wire increases with the square root of the tension and decreases with the square root of the mass per unit length. Ensure accurate substitution and unit consistency for reliable results.

---

25. Two vessels A and B are of the same size and are at the same temperature. Vessel A contains 1 g of hydrogen and vessel B contains 1 g of oxygen. P_A and P_B are the pressures of the gases in A and B respectively. Then $\frac{P_A}{P_B}$ is:

(1) 8

(2) 16

(3) 32

(4) 4

Correct Answer: (2) 16

Solution: The ideal gas equation relates pressure, volume, temperature, and the number of moles:

$$PV = nRT \quad \implies \quad P \propto n,$$

where n is the number of moles of gas.

Step 1: Determine the Number of Moles For hydrogen (H_2):

$$M_{H_2} = 2\,\text{g/mol}, \quad n_A = \frac{\text{Mass}}{\text{Molar Mass}} = \frac{1}{2} = 0.5\,\text{mol}.$$

For oxygen (O_2):

$$M_{O_2} = 32\,\text{g/mol}, \quad n_B = \frac{\text{Mass}}{\text{Molar Mass}} = \frac{1}{32} = 0.03125\,\text{mol}.$$

Step 2: Compute the Ratio of Pressures Since $P \propto n$:

$$\frac{P_A}{P_B} = \frac{n_A}{n_B} = \frac{0.5}{0.03125}.$$

Simplify:

$$\frac{P_A}{P_B} = 16.$$

Final Answer:

$$\boxed{16}$$

Quick Tip
At constant temperature and volume, the pressure of a gas is directly proportional to its number of moles. For comparisons, use the relationship $P \propto n$.

---

26. A wire of length $1\,\text{m}$ moving with velocity $8\,\text{m/s}$ at right angles to a magnetic field of $2\,\text{T}$. The magnitude of induced emf between the ends of the wire will be:

(1) $20\,\text{V}$

(2) $8\,\text{V}$

(3) $12\,\text{V}$

(4) $16\,\text{V}$

Correct Answer: (4) $16\,\text{V}$

Solution: The induced emf $\mathcal{E}$ in a conductor moving through a magnetic field is given by:

$$\mathcal{E} = B \cdot v \cdot \ell,$$

where: $B = 2\,\text{T}$ is the magnetic field strength, $v = 8\,\text{m/s}$ is the velocity of the conductor, $\ell = 1\,\text{m}$ is the length of the conductor.

Step 1: Substitute the Given Values Substitute $B = 2\,\text{T}$, $v = 8\,\text{m/s}$, and $\ell = 1\,\text{m}$ into the formula:

$$\mathcal{E} = 2 \cdot 8 \cdot 1.$$

Step 2: After Simplify

$$\mathcal{E} = 16\,\text{V}.$$

Final Answer:

$$\boxed{16\,\text{V}}$$

Quick Tip

The induced emf in a conductor moving perpendicular to a magnetic field depends on the product of the magnetic field strength, the velocity of the conductor, and its length. Ensure all units are consistent when performing calculations.

---

27. Two identical particles each of mass m go around a circle of radius a under the action of their mutual gravitational attraction. The angular speed of each particle will be:

(1) $\sqrt{\frac{Gm}{2a^3}}$
(2) $\sqrt{\frac{Gm}{8a^3}}$
(3) $\sqrt{\frac{Gm}{4a^3}}$
(4) $\sqrt{\frac{Gm}{a^3}}$

Correct Answer: (3) $\sqrt{\frac{Gm}{4a^3}}$

Solution: The gravitational force acts as the centripetal force for circular motion:

$$F_{\text{gravity}} = F_{\text{centripetal}}.$$

The gravitational force between two particles is expressed as:

$$F = \frac{GM_1M_2}{d^2},$$

where: - G is the gravitational constant, - $M_1 = M_2 = m$, - $d = 2a$, the distance between the two particles.

Substitute $M_1 = M_2 = m$ and $d = 2a$:

$$F = \frac{Gm^2}{(2a)^2} = \frac{Gm^2}{4a^2}.$$

For circular motion:

$$F = m\omega^2 r,$$

where $r = a$. Equating F:

$$\frac{Gm^2}{4a^2} = m\omega^2 a.$$

Simplify the equation:

$$\omega^2 = \frac{Gm}{4a^3}.$$

Take the square root to find ω:

$$\omega = \sqrt{\frac{Gm}{4a^3}}.$$

Final Answer:

$$\boxed{\sqrt{\frac{Gm}{4a^3}}}$$

Quick Tip

When solving problems involving gravitationally bound circular motion, simplify by expressing the gravitational force in terms of the centripetal force. Keep track of distances, as they directly impact force calculations.

---

28. In an unbiased p-n junction, electrons diffuse from n-region to p-region because:

(1) holes in p-region attract them

(2) electrons travel across the junction due to potential difference

(3) only electrons move from n-region to p-region and not the vice-versa

(4) electron concentration in n-region is more compared to that in p-region

Correct Answer: (4) electron concentration in n-region is more compared to that in p-region

Solution: In an unbiased p-n junction:
The n-region has a significantly higher concentration of electrons compared to the p-region.
Due to this concentration difference, electrons diffuse from the n-region (high concentration) to the p-region (low concentration).
This diffusion of charge carriers results in the formation of a depletion region, where an electric field develops.
The built-in electric field counteracts further electron movement, establishing equilibrium.

Final Answer:

$$\boxed{\text{electron concentration in the } n\text{-region is higher than in the } p\text{-region}}$$

Quick Tip

In an unbiased p-n junction, the concentration gradient causes electrons to diffuse from the n-region to the p-region, resulting in a depletion region and an opposing electric field.

---

29. A particle is executing Simple Harmonic Motion (SHM). The ratio of potential energy and kinetic energy of the particle when its displacement is half of its amplitude will be:

(1) $1:1$

(2) $2:1$

(3) $1:4$

(4) $1:3$

Correct Answer: (4) $1:3$

Solution: The total energy in simple harmonic motion (SHM) is given by:

$$E_{\text{total}} = \frac{1}{2}kA^2,$$

where: - k is the spring constant, - A is the amplitude of oscillation.

Step 1: Expression for Potential and Kinetic Energy The potential energy at displacement x is:

$$PE = \frac{1}{2}kx^2.$$

The kinetic energy is the remainder of the total energy:

$$KE = E_{\text{total}} - PE = \frac{1}{2}kA^2 - \frac{1}{2}kx^2.$$

Step 2: Displacement at Half the Amplitude When $x = \frac{A}{2}$:

$$PE = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{2}k\frac{A^2}{4} = \frac{1}{8}kA^2.$$

$$KE = \frac{1}{2}kA^2 - \frac{1}{8}kA^2 = \frac{4}{8}kA^2 - \frac{1}{8}kA^2 = \frac{3}{8}kA^2.$$

Step 3: Ratio of Potential Energy to Kinetic Energy

$$\frac{PE}{KE} = \frac{\frac{1}{8}kA^2}{\frac{3}{8}kA^2} = \frac{1}{3}.$$

Final Answer:

$$\boxed{1:3}$$

Quick Tip

In SHM, potential energy is maximum at extreme displacement, while kinetic energy peaks at the equilibrium position. At intermediate displacements, use $PE + KE = E_{\text{total}}$ to analyze energy distribution.

---

30. Eight equal drops of water are falling through air with a steady speed of $10\,$cm/s. If the drops coalesce, the new velocity is:

(1) $10\,$cm/s

(2) $40\,$cm/s

(3) $16\,$cm/s

(4) $5\,$cm/s

Correct Answer: (2) $40\,$cm/s

Solution: When the drops coalesce, their total volume remains unchanged. Let: - r be the radius of each small drop, - R be the radius of the combined drop.

Step 1: Apply Volume Conservation The total volume of eight small drops is:

$$8 \cdot \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3.$$

Simplify:

$$R^3 = 8r^3 \quad \Longrightarrow \quad R = 2r.$$

Step 2: Terminal Velocity Relation The terminal velocity v_T of a drop is proportional to the square of its radius:

$$v_T \propto r^2.$$

Let $v_1 = 10\,$cm/s be the terminal velocity of the small drops, and v_2 be the terminal velocity of the combined drop. Then:

$$\frac{v_1}{v_2} = \left(\frac{r}{R}\right)^2.$$

Substitute $R = 2r$:

$$\frac{v_1}{v_2} = \left(\frac{r}{2r}\right)^2 = \frac{1}{4}.$$

Step 3: Calculate v_2 Rearrange to find v_2:

$$v_2 = 4v_1 = 4 \cdot 10 = 40\,\text{cm/s}.$$

Final Answer:

$$\boxed{40\,\text{cm/s}}$$

Quick Tip
When drops merge, the radius of the combined drop increases as the cube root of the total volume. Since terminal velocity depends on the square of the radius, the new velocity scales quadratically with the radius increase.

---

31. In a coil, the current changes from $-2\,\text{A}$ to $+2\,\text{A}$ in $0.2\,\text{s}$ and induces an emf of $0.1\,\text{V}$. The self-inductance of the coil is:

(1) $5\,\text{mH}$

(2) $1\,\text{mH}$

(3) $2.5\,\text{mH}$

(4) $4\,\text{mH}$

Correct Answer: (1) $5\,\text{mH}$

Solution: The induced emf in a coil is described by the equation:

$$\mathcal{E} = L\frac{\Delta I}{\Delta t},$$

where: $\mathcal{E} = 0.1\,\text{V}$ is the induced emf, $\Delta I = I_{\text{final}} - I_{\text{initial}} = 2 - (-2) = 4\,\text{A}$ is the change in current, $\Delta t = 0.2\,\text{s}$ is the time interval, L is the self-inductance of the coil.

Step 1: Rearrange to Solve for L Rearrange the formula for L:

$$L = \frac{\mathcal{E} \cdot \Delta t}{\Delta I}.$$

Substitute the given values:

$$L = \frac{0.1 \cdot 0.2}{4}.$$

Simplify:

$$L = \frac{0.02}{4} = 0.005\,\text{H}.$$

Convert L to millihenries:

$$L = 5\,\text{mH}.$$

Final Answer:

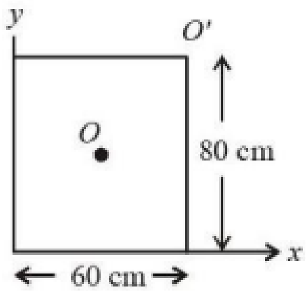
$$\boxed{5\,\text{mH}}$$

Quick Tip

To calculate self-inductance, ensure accurate substitution of the total current change and time interval. The relationship $L = \frac{\mathcal{E}\cdot\Delta t}{\Delta I}$ directly links the rate of current change to the induced emf.

---

32. For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the center of mass) and O' (corner point) is:



(1) $\frac{2}{3}$

(2) $\frac{1}{4}$

(3) $\frac{1}{8}$

(4) $\frac{1}{2}$

Correct Answer: (2) $\frac{1}{4}$

Solution: The moment of inertia (I_O) of a rectangular sheet about an axis passing through its center of mass (O) is:

$$I_O = \frac{M}{12}\left(a^2 + b^2\right),$$

where $a = 80\,\text{cm}$ and $b = 60\,\text{cm}$ are the dimensions of the rectangle, and M is the mass of the sheet.

Step 1: Calculate I_O Substitute the given values:

$$I_O = \frac{M}{12}\left(80^2 + 60^2\right).$$

Simplify:

$$I_O = \frac{M}{12}(6400 + 3600) = \frac{M}{12} \cdot 10000 = \frac{10000M}{12}.$$

Step 2: Use the Parallel Axis Theorem to Find $I_{O'}$ The moment of inertia about O' is given by:

$$I_{O'} = I_O + M \cdot d^2,$$

where d is the perpendicular distance between O and O'. Calculate d:

$$d = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2} = \sqrt{40^2 + 30^2} = 50\,\text{cm}.$$

Substitute $d = 50\,\text{cm}$:

$$I_{O'} = \frac{10000M}{12} + M \cdot 50^2.$$

Simplify:

$$I_{O'} = \frac{10000M}{12} + 2500M = \frac{10000M}{12} + \frac{30000M}{12} = \frac{40000M}{12}.$$

Step 3: Find the Ratio of I_O to $I_{O'}$

$$\frac{I_O}{I_{O'}} = \frac{\frac{10000M}{12}}{\frac{40000M}{12}} = \frac{10000}{40000} = \frac{1}{4}.$$

Final Answer:

$$\boxed{\frac{1}{4}}$$

Quick Tip

To compute the moment of inertia about an axis parallel to the center of mass axis, apply the parallel axis theorem: $I = I_{\text{center}} + M \cdot d^2$. Ensure correct calculations for d, the perpendicular distance.

33. If n is the number density and d is the diameter of the molecule, then the average distance covered by a molecule between two successive collisions (i.e., mean free path) is represented by:

(1) $\frac{1}{\sqrt{2}\pi n d^2}$
(2) $\sqrt{2}n\pi d^2$
(3) $\frac{1}{\sqrt{2}n\pi d^2}$
(4) $\frac{1}{\sqrt{2}n^2\pi^2 d^2}$

Correct Answer: (1) $\frac{1}{\sqrt{2}\pi n d^2}$

Solution: The mean free path (λ) represents the average distance a gas molecule travels between collisions. Based on the kinetic theory of gases, it is calculated as:

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2},$$

where: - n is the number density (molecules per unit volume), - d is the molecular diameter, - π is the mathematical constant related to circular cross-sections.

Derivation Steps:

1. Collision Cross-Section: The effective area for a collision between two molecules is $\sigma = \pi d^2$, where d is the molecular diameter.
2. Collision Frequency: The rate of collisions is determined by σ, n, and the average relative velocity of the molecules.
3. Mean Free Path: The mean free path is the inverse of the product of collision cross-section and number density, adjusted by a factor of $\sqrt{2}$ for molecular motion:

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2}.$$

Final Answer: The mean free path is:

$$\boxed{\frac{1}{\sqrt{2}\pi n d^2}} \quad \text{(Option 1).}$$

Quick Tip
The mean free path decreases with increasing molecular diameter or number density. For smaller d and n, molecules travel longer distances before colliding.

34. A mixture of one mole of monoatomic gas and one mole of diatomic gas (rigid) are kept at room temperature ($27°$C). The ratio of specific heat of gases at constant volume respectively is:

(1) $\frac{7}{5}$

(2) $\frac{3}{2}$

(3) $\frac{3}{5}$

(4) $\frac{5}{3}$

Correct Answer: (3) $\frac{3}{5}$

Solution: The specific heat capacity at constant volume (C_V) depends on the degrees of freedom of a gas:

- For a monoatomic gas:

$$C_V = \frac{3}{2}R,$$

where R is the universal gas constant.

- For a rigid diatomic gas (no vibrational degrees of freedom):

$$C'_V = \frac{5}{2}R.$$

Step 1: Calculate the Ratio of Specific Heats The ratio of specific heat capacities for the monoatomic and diatomic gases is:

$$\frac{C_V}{C'_V} = \frac{\frac{3}{2}R}{\frac{5}{2}R}.$$

Simplify:

$$\frac{C_V}{C'_V} = \frac{3}{5}.$$

Final Answer:

$$\boxed{\frac{3}{5}}$$

Quick Tip
The specific heat of a gas increases with the number of degrees of freedom. Monoatomic gases have only translational motion, while diatomic gases have translational and rotational motion, resulting in a higher specific heat.

35. In an a.c. circuit, voltage and current are given by:

$$V = 100\sin(100t)\,\text{V} \quad \text{and} \quad I = 100\sin\left(100t + \frac{\pi}{3}\right)\,\text{mA}.$$

The average power dissipated in one cycle is:

(1) $10\,\text{W}$

(2) $2.5\,\text{W}$

(3) $25\,\text{W}$

(4) $5\,\text{W}$

Correct Answer: (2) $2.5\,\text{W}$

Solution: The average power in an AC circuit over one complete cycle is calculated as:

$$P_{\text{avg}} = V_{\text{rms}} I_{\text{rms}} \cos(\Delta\phi),$$

where: - $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$, - $I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$, - $\Delta\phi$ is the phase difference between the voltage and current.

Given:

$$V_0 = 100\,\text{V}, \quad I_0 = 100 \times 10^{-3}\,\text{A}, \quad \Delta\phi = \frac{\pi}{3}.$$

Step 1: Calculate V_{rms} and I_{rms}

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{100}{\sqrt{2}}, \quad I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{100 \times 10^{-3}}{\sqrt{2}}.$$

Step 2: Substitute into the Power Formula

$$P_{\text{avg}} = \frac{100}{\sqrt{2}} \cdot \frac{100 \times 10^{-3}}{\sqrt{2}} \cdot \cos\left(\frac{\pi}{3}\right).$$

Step 3: Simplify the Expression

$$P_{\text{avg}} = \frac{100 \cdot 0.1}{2} \cdot \frac{1}{2} = \frac{10}{4} = 2.5\,\text{W}.$$

Final Answer:

$$\boxed{2.5\,\text{W}}$$

Quick Tip
In AC circuits, the average power depends on the product of the rms values of voltage and current and the cosine of the phase angle. Always ensure consistent units and correct trigonometric values for accurate results.

36. The difference between threshold wavelengths for two metal surfaces A and B having work functions $\phi_A = 9$ eV and $\phi_B = 4.5$ eV is:

(Given: $hc = 1242$ eV nm)

(1) 264 nm

(2) 138 nm

(3) 276 nm

(4) 540 nm

Correct Answer: (2) 138 nm

Solution: The wavelength (λ) associated with the threshold energy is related to the work function (ϕ) as:

$$\lambda = \frac{hc}{\phi},$$

where h is Planck's constant, c is the speed of light, and ϕ is the work function in eV.

Step 1: Calculate Wavelengths For $\phi_A = 9$ eV:

$$\lambda_A = \frac{1242}{9} = 138\,\text{nm}.$$

For $\phi_B = 4.5$ eV:

$$\lambda_B = \frac{1242}{4.5} = 276\,\text{nm}.$$

Step 2: Calculate the Difference in Wavelengths

$$\Delta\lambda = \lambda_B - \lambda_A = 276 - 138 = 138\,\text{nm}.$$

Final Answer:

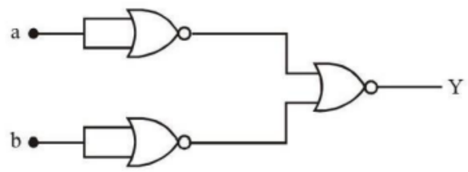
$$\boxed{138\,\text{nm}}$$

Quick Tip

The threshold wavelength decreases as the work function increases. Use the formula $\lambda = \frac{1242}{\phi}$ to find the relationship and ensure all units are consistent.

---

37. The logic performed by the circuit shown in the figure is equivalent to:



(1) AND

(2) $NAND$

(3) OR

(4) NOR

Correct Answer: (1) AND

Solution: The circuit performs the following logical operations:

1. The first two NOT gates take inputs a and b, producing the complements $\bar{a}$ and $\bar{b}$ respectively.

$$\text{Output of the first NOT gate: } \bar{a}, \quad \text{Output of the second NOT gate: } \bar{b}.$$

2. The OR gate then combines the complements:

$$\text{Output of the OR gate: } \bar{a} + \bar{b}.$$

3. The result from the OR gate is then complemented by another NOT gate, yielding the final output:

$$Y = \overline{\bar{a} + \bar{b}}.$$

Using De Morgan's law, we simplify:

$$Y = a \cdot b.$$

Thus, the circuit implements the logic of an AND gate.

Truth Table:

A	B	$Y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

The output matches the truth table of an AND gate.

Final Answer:

$$\boxed{\text{AND Gate}}$$

Quick Tip

De Morgan's laws simplify logic circuits: $\overline{A+B} = A \cdot B$ and $\overline{A \cdot B} = A + B$. These transformations can turn complex circuits into simpler ones.

---

38. A particle performs simple harmonic motion with amplitude A. Its speed is tripled at the instant that it is at a distance $\frac{2A}{3}$ from the equilibrium position. The new amplitude of the motion is:

(1) $A\sqrt{3}$

(2) $\frac{7A}{3}$

(3) $\frac{A}{3}\sqrt{41}$

(4) $3A$

Correct Answer: (2) $\frac{7A}{3}$

Solution: In this problem, we are tasked with determining the new amplitude A' when the velocity of a particle performing simple harmonic motion triples.

The velocity V of a particle in simple harmonic motion is given by the equation:

$$V = \omega\sqrt{A^2 - x^2},$$

where:

$$\omega = \text{angular frequency}, \quad A = \text{amplitude}, \quad x = \text{displacement from equilibrium}.$$

Step 1: Initial velocity at displacement $x = \frac{2A}{3}$ The initial velocity is given by:

$$V = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}.$$

Simplify:

$$V = \omega\sqrt{A^2 - \frac{4A^2}{9}} = \omega\sqrt{\frac{5A^2}{9}} = \omega\frac{\sqrt{5}A}{3}.$$

Step 2: Velocity when speed is tripled The final velocity is three times the initial velocity, so:

$$3V = \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}.$$

Substitute the expression for V:

$$3\left(\omega\frac{\sqrt{5}A}{3}\right) = \omega\sqrt{A'^2 - \frac{4A^2}{9}}.$$

Simplify:

$$\omega\sqrt{5}A = \omega\sqrt{A'^2 - \frac{4A^2}{9}}.$$

Step 3: Solve for A' Square both sides to eliminate the square roots:

$$5A^2 = A'^2 - \frac{4A^2}{9}.$$

Rearrange the equation:

$$A'^2 = 5A^2 + \frac{4A^2}{9}.$$

Simplify:

$$A'^2 = \frac{45A^2}{9} + \frac{4A^2}{9} = \frac{49A^2}{9}.$$

Now, take the square root of both sides:

$$A' = \frac{7A}{3}.$$

Final Answer:

$$\boxed{\frac{7A}{3}}.$$

Quick Tip
In SHM, when velocity changes, use energy conservation or velocity equations to relate the change in speed to the amplitude. Here, tripling the speed leads to a proportional change in amplitude.

---

39. The mass of proton, neutron, and helium nucleus are respectively 1.0073 **u**, 1.0087 **u**, 4.0015 **u. The binding energy of the helium nucleus is:**

(1) $14.2\,\text{MeV}$

(2) $56.8\,\text{MeV}$

(3) $28.4\,\text{MeV}$

(4) $7.1\,\text{MeV}$

Correct Answer: (3) $28.4\,\text{MeV}$

Solution: To find the binding energy (BE) of a nucleus, we use the equation:

$$BE = \Delta m \cdot c^2,$$

where Δm is the mass defect and c is the speed of light.

For the helium nucleus, the mass defect is:

$$\Delta m = (2m_p + 2m_n) - m_{\text{He}},$$

where: - $m_p = 1.0073\,\text{u}$ is the mass of a proton, - $m_n = 1.0087\,\text{u}$ is the mass of a neutron, - $m_{\text{He}} = 4.0015\,\text{u}$ is the mass of the helium nucleus.

Substitute the values for the protons and neutrons:

$$\Delta m = (2 \cdot 1.0073 + 2 \cdot 1.0087) - 4.0015 = 4.0310 - 4.0015 = 0.0295\,\text{u}.$$

Now, to convert the mass defect into energy, we use the conversion factor $1\,\text{u} = 931.5\,\text{MeV/c}^2$:

$$BE = 0.0295 \cdot 931.5 = 28.4\,\text{MeV}.$$

Final Answer:

$$\boxed{28.4\,\text{MeV}}.$$

Quick Tip
The binding energy of a nucleus can be computed by first calculating the mass defect (the difference between the total mass of nucleons and the mass of the nucleus) and then converting it to energy using the relation $1\,\text{u} = 931.5\,\text{MeV/c}^2$.

---

40. A series LCR circuit is subjected to an AC signal of $200\,\text{V}$, $50\,\text{Hz}$. If the voltage across the inductor ($L = 10\,\text{mH}$) is $31.4\,\text{V}$, then the current in this circuit is:

(1) $68\,\text{A}$

(2) $63\,\text{A}$

(3) $10\,\text{A}$

(4) $10\,\text{mA}$

Correct Answer: (3) $10\,\text{A}$

Solution: The voltage across an inductor in an AC circuit is given by:

$$V_L = IX_L,$$

where $X_L = \omega L$ is the inductive reactance and $\omega = 2\pi f$ is the angular frequency of the AC supply.

Step 1: Calculate the angular frequency ω:

$$\omega = 2\pi f = 2\pi \cdot 50 = 3.14 \times 100 = 314\,\text{rad/s}.$$

Step 2: Calculate the inductive reactance X_L:

$$X_L = \omega L = 314 \cdot 10 \times 10^{-3} = 3.14\,\Omega.$$

Step 3: Calculate the current I using $V_L = IX_L$:

$$I = \frac{V_L}{X_L} = \frac{31.4}{3.14} = 10\,\text{A}.$$

Final Answer:

$$\boxed{10\,\text{A}}.$$

Quick Tip

In an LCR circuit, the current through the inductor can be calculated by dividing the voltage across the inductor by its inductive reactance, $I = \frac{V_L}{X_L}$, where $X_L = \omega L$.

---

41. When two soap bubbles of radii a and b ($b > a$) coalesce, the radius of curvature of the common surface is:

(1) $\frac{ab}{b-a}$

(2) $\frac{ab}{a+b}$

(3) $\frac{b-a}{ab}$

(4) $\frac{a+b}{ab}$

Correct Answer: (1) $\frac{ab}{b-a}$

Solution: The pressure inside a soap bubble is given by the formula:

$$P = P_0 + \frac{4T}{R},$$

where T is the surface tension and R is the radius of the bubble.

Step 1: For the two bubbles with radii a and b, the pressures are:

$$P_1 = P_0 + \frac{4T}{a}, \quad P_2 = P_0 + \frac{4T}{b}.$$

Step 2: The pressure difference across the common surface is:

$$P_1 - P_2 = \frac{4T}{R}.$$

Step 3: Substitute the expressions for P_1 and P_2:

$$\frac{4T}{a} - \frac{4T}{b} = \frac{4T}{R}.$$

Step 4: Simplify:

$$\frac{1}{a} - \frac{1}{b} = \frac{1}{R}.$$

Step 5: Solve for R:

$$\frac{1}{R} = \frac{b-a}{ab}, \quad R = \frac{ab}{b-a}.$$

Final Answer:

$$\boxed{\frac{ab}{b-a}}.$$

Quick Tip

When two soap bubbles merge, the radius of curvature of the common interface is determined by the difference in pressures inside the bubbles, which is related to their respective radii and surface tension.

---

42. A liquid is allowed to flow into a tube of truncated cone shape. Identify the correct statement:

(1) The speed is high at the wider end and high at the narrow end.

(2) The speed is low at the wider end and high at the narrow end.

(3) The speed is same at both ends in a streamline flow.

(4) The liquid flows with uniform velocity in the tube.

Correct Answer: (2) $The speed is low at the wider end and high at the narrow end.$

Solution: According to the principle of continuity for an incompressible fluid:

$$A_1 v_1 = A_2 v_2,$$

where A is the cross-sectional area and v is the velocity of the fluid. In a truncated cone, the cross-sectional area at the wider end (A_1) is greater than the area at the narrower end (A_2). Hence, the velocity at the wider end (v_1) is smaller, and the velocity at the narrower end (v_2) is higher to conserve mass flow:

$$v_1 < v_2.$$

Final Answer:

$$\boxed{\text{The speed is low at the wider end and high at the narrow end.}}$$

Quick Tip

The principle of continuity implies that for an incompressible fluid, as the cross-sectional area decreases, the velocity must increase to maintain constant mass flow.

---

43. The velocity of sound in a gas in which two wavelengths 4.08 m and 4.16 m produce 40 beats in 12 seconds will be:

(1) $2.828\,\text{ms}^{-1}$

(2) $175.5\,\text{ms}^{-1}$

(3) $353.6\,\text{ms}^{-1}$

(4) $707.2\,\text{ms}^{-1}$

Correct Answer: (4) $707.2\,\text{ms}^{-1}$

Solution: The beat frequency (f_{beat}) is the difference between two frequencies:

$$f_{\text{beat}} = f_1 - f_2,$$

where f_1 and f_2 are the frequencies corresponding to the wavelengths λ_1 and λ_2, respectively.

Step 1: Calculate the beat frequency Given the number of beats and time:

$$f_{\text{beat}} = \frac{\text{Number of beats}}{\text{Time}} = \frac{40}{12}\,\text{Hz}.$$

Simplifying:

$$f_{\text{beat}} = \frac{10}{3}\,\text{Hz}.$$

Step 2: Relating wavelength to velocity and frequency The frequency of a wave is related to its velocity by the equation:

$$f = \frac{v}{\lambda}.$$

Using the given wavelengths $\lambda_1 = 4.08\,\text{m}$ and $\lambda_2 = 4.16\,\text{m}$, we have:

$$f_1 = \frac{v}{4.08}, \quad f_2 = \frac{v}{4.16}.$$

Step 3: Solve for the velocity v The beat frequency is the difference in frequencies:

$$f_{\text{beat}} = f_1 - f_2 = \frac{v}{4.08} - \frac{v}{4.16}.$$

Simplifying:

$$\frac{10}{3} = v\left(\frac{1}{4.08} - \frac{1}{4.16}\right).$$

Now, calculate the difference in reciprocals:

$$\frac{1}{4.08} - \frac{1}{4.16} = \frac{4.16 - 4.08}{4.08 \cdot 4.16} = \frac{0.08}{16.9728}.$$

Substitute this value:

$$\frac{10}{3} = v \cdot \frac{0.08}{16.9728}.$$

Solving for v:

$$v = \frac{\frac{10}{3} \cdot 16.9728}{0.08} = 707.2\,\text{m/s}.$$

Final Answer:

$$\boxed{707.2\,\text{m/s}}.$$

Quick Tip
To find the velocity of sound using beat frequency, use the relation $f_{\text{beat}} = f_1 - f_2$ and the wave equation $f = \frac{v}{\lambda}$.

---

44. σ **is the uniform surface charge density of a thin spherical shell of radius** R**. The electric field at any point on the surface of the spherical shell is:**

(1) $\frac{\sigma}{\varepsilon_0 R}$
(2) $\frac{\sigma}{2\varepsilon_0}$
(3) $\frac{\sigma}{\varepsilon_0}$
(4) $\frac{\sigma}{4\varepsilon_0}$

Correct Answer: (3) $\frac{\sigma}{\varepsilon_0}$

Solution: To find the electric field using Gauss's law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0},$$

where Q is the total charge enclosed and ε_0 is the permittivity of free space.

Step 1: Relate charge to surface charge density. The total charge Q on the spherical shell is related to the surface charge density σ and the surface area of the shell (which is $4\pi R^2$):

$$Q = \sigma \cdot 4\pi R^2.$$

Step 2: Simplify the electric field. By symmetry, the electric field on the surface of the spherical shell is uniform, and the flux through the Gaussian surface (a sphere of radius R) is:

$$\oint \vec{E} \cdot d\vec{A} = E \cdot 4\pi R^2,$$

where E is the magnitude of the electric field.

Substituting into Gauss's law:

$$E \cdot 4\pi R^2 = \frac{Q}{\varepsilon_0}.$$

Substitute the expression for Q:

$$E \cdot 4\pi R^2 = \frac{\sigma \cdot 4\pi R^2}{\varepsilon_0}.$$

Canceling $4\pi R^2$ from both sides:

$$E = \frac{\sigma}{\varepsilon_0}.$$

Final Answer:

$$\boxed{\frac{\sigma}{\varepsilon_0}}.$$

Quick Tip
For a spherical shell, the electric field on its surface depends solely on the surface charge density σ and the permittivity of free space ε_0, and is independent of the radius R.

---

45. An electric field is given by $\vec{E} = (6\hat{i} + 5\hat{j} + 3\hat{k})$ N/C. The electric flux through a surface area $30\hat{i}$ m^2 lying in the YZ-plane (in SI units) is:

(1) 180

(2) 90

(3) 150

(4) 60

Correct Answer: (1) 180

Solution: The electric flux Φ is given by the dot product of the electric field vector $\vec{E}$ and the area vector $\vec{A}$:

$$\Phi = \vec{E} \cdot \vec{A}.$$

Given that the area vector is $\vec{A} = 30\hat{i}\,\text{m}^2$, and the electric field is $\vec{E} = 6\hat{i} + 5\hat{j} + 3\hat{k}$, the flux is:

$$\Phi = (6\hat{i} + 5\hat{j} + 3\hat{k}) \cdot (30\hat{i}).$$

Only the components along the $\hat{i}$ direction contribute to the dot product:

$$\Phi = 6 \cdot 30 = 180\,\text{V·m}.$$

Final Answer:

$$\boxed{180\,\text{V·m}}.$$

Quick Tip

When calculating electric flux, only the components of the electric field vector that align with the area vector contribute to the dot product.

---

46. A big drop is formed by coalescing 1000 small droplets of water. The surface energy will become:

(1) $\frac{1}{100}$th

(2) $\frac{1}{10}$th

(3) 100 times

(4) 10 times

Correct Answer: (2) $\frac{1}{10}$th

Solution: Using the conservation of volume, the volume of the big drop is equal to the sum of the volumes of the 1000 smaller drops:

$$\frac{4}{3}\pi R^3 = 1000 \cdot \frac{4}{3}\pi r^3,$$

where R is the radius of the large drop and r is the radius of each smaller drop.

Simplify:

$$R^3 = 1000 \cdot r^3 \implies R = 10r.$$

The surface energy of a sphere is proportional to its surface area. The initial energy (for 1000 small drops) is:

$$E_i = 1000 \cdot 4\pi r^2.$$

The final energy (for the large drop) is:

$$E_f = 4\pi R^2 = 4\pi(10r)^2 = 100 \cdot 4\pi r^2.$$

Thus, the ratio of the final to initial energy is:

$$\frac{E_f}{E_i} = \frac{4\pi(10r)^2}{1000 \cdot 4\pi r^2} = \frac{1}{10}.$$

Final Answer:

$$\boxed{\frac{1}{10}\text{ th}}.$$

Quick Tip

When droplets merge, the total volume remains constant, but the surface area (and thus the surface energy) changes significantly, as it is proportional to the square of the radius.

---

47. If M is the mass of water that rises in a capillary tube of radius r, then the mass of water which will rise in a capillary tube of radius $2r$ is:

(1) M

(2) $\frac{M}{2}$

(3) $4M$

(4) $2M$

Correct Answer: (4) $2M$

Solution: The height h of water in a capillary tube is given by the capillary rise formula:

$$h = \frac{2T\cos\theta}{r\rho g},$$

where T is the surface tension, θ is the angle of contact, r is the radius of the tube, ρ is the density of the liquid, and g is the acceleration due to gravity.

The volume of water in the capillary is:

$$V = \pi r^2 h = \pi r^2 \cdot \frac{2T\cos\theta}{r\rho g}.$$

Simplifying:

$$V \propto r.$$

The mass of water in the capillary is:

$$m = \rho V \propto r.$$

For a capillary with radius $2r$, the new mass is:

$$m_2 = 2m_1,$$

which shows that the mass of water is directly proportional to the radius of the capillary tube.

Thus, the new mass is:

$$2M.$$

Final Answer:

$$\boxed{2M}.$$

Quick Tip
The mass of water in a capillary tube depends linearly on the radius of the tube. Doubling the radius of the capillary will double the mass of the water it holds.

---

48. A closed organ pipe (closed at one end) is excited to support the third overtone. It is found that air in the pipe has:

(1) Three nodes and three antinodes

(2) Three nodes and four antinodes

(3) Four nodes and three antinodes

(4) Four nodes and four antinodes

Correct Answer: (4) Four nodes and four antinodes

Solution: For a closed organ pipe, the relationship between the length L of the pipe and the harmonic number n is given by:

$$L = \frac{(2n-1)\lambda}{4},$$

where λ is the wavelength of the sound.

For the third overtone, we have:

$$n = 4.$$

This corresponds to the 4th harmonic, which has 4 nodes and 4 antinodes, with the pattern of oscillation as follows: - The pipe has four full loops, plus a half loop. - This results in 4 nodes and 4 antinodes.

Final Answer:

$$\boxed{\text{Four nodes and four antinodes}}.$$

Quick Tip

In a closed organ pipe, the number of nodes and antinodes increases with the overtone number. For the third overtone (fourth harmonic), the pipe supports four nodes and four antinodes.

---

49. The electrostatic potential due to an electric dipole at a distance r varies as:

(1) r

(2) $\frac{1}{r^2}$

(3) $\frac{1}{r^3}$

(4) $\frac{1}{r}$

Correct Answer: (2) $\frac{1}{r^2}$

Solution: The electrostatic potential V_p at a point at distance r from the center of an electric dipole is inversely proportional to the square of the distance:

$$V_p \propto \frac{1}{r^2}.$$

This is because the potential due to a dipole decreases at a faster rate compared to a point charge, which follows the $\frac{1}{r}$ dependence. In a dipole, the opposing charges create a field that reduces more sharply with distance.

Final Answer:

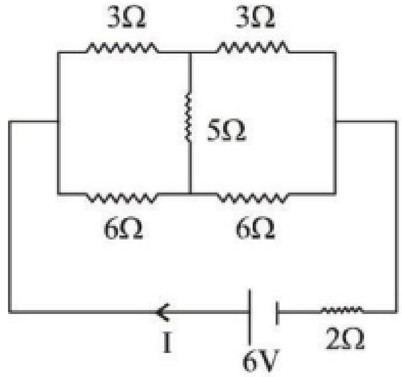
$$\boxed{\frac{1}{r^2}}.$$

Quick Tip

The electric potential due to a monopole (point charge) follows a $\frac{1}{r}$ relationship, whereas for a dipole, the potential decreases with the square of the distance, $\frac{1}{r^2}$.

---

50. A battery of $6\,\mathrm{V}$ is connected to the circuit as shown below. The current I drawn from the battery is:



(1) $1\,\text{A}$

(2) $2\,\text{A}$

(3) $\frac{6}{11}\,\text{A}$

(4) $\frac{4}{3}\,\text{A}$

Correct Answer: (1) $1\,\text{A}$

Solution: In a balanced Wheatstone bridge, the current through the middle resistor is zero, meaning that the $5\,\Omega$ resistor does not contribute to the current. We can simplify the circuit by removing this resistor.

Step 1: Simplify the circuit
Removeing the $5\,\Omega$ resistor.
Combining the remaining resistors.

The $3\,\Omega$ resistors in the top branch are in series, so their combined resistance is:

$$R_{\text{top}} = 3 + 3 = 6\,\Omega.$$

The $6\,\Omega$ resistors in the bottom branch are in parallel, so their equivalent resistance is:

$$R_{\text{bottom}} = \frac{6 \cdot 6}{6 + 6} = \frac{36}{12} = 3\,\Omega.$$

Now, the two resistances R_{top} and R_{bottom} are in parallel:

$$R_{\text{eq}} = \frac{6 \cdot 3}{6 + 3} = \frac{18}{9} = 2\,\Omega.$$

Finally, add the $2\,\Omega$ resistor in series with the equivalent resistance:

$$R_{\text{total}} = R_{\text{eq}} + 2 = 2 + 2 = 6\,\Omega.$$

Step 2: Apply Ohm's Law

The total current is given by:

$$I = \frac{V}{R_{\text{total}}}.$$

Substitute $V = 6\,\text{V}$ and $R_{\text{total}} = 6\,\Omega$:

$$I = \frac{6}{6} = 1\,\text{A}.$$

Final Answer:

$$\boxed{1\,\text{A}}.$$

Quick Tip
In a balanced Wheatstone bridge, the current through the middle resistor is zero. Use this property to simplify the circuit by removing that resistor.

CHEMISTRY

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51. If the length of the body diagonal of a FCC unit cell is x Å, the distance between two octahedral voids in the cell in Å is:

(1) $\frac{x}{\sqrt{2}}$

(2) $\frac{x}{\sqrt{3}}$

(3) $\frac{x}{\sqrt{6}}$

(4) $\frac{x}{\sqrt{8}}$

Correct Answer: (3) $\frac{x}{\sqrt{6}}$

Solution: In a face-centered cubic (FCC) unit cell, octahedral voids are located at the body center and edge centers. To calculate the distance between two octahedral voids:

Step 1: The length of the body diagonal is:

$$x = \sqrt{3}a,$$

where a is the edge length of the unit cell.

Step 2: From the body diagonal length, the edge length a is:

$$a = \frac{x}{\sqrt{3}}.$$

Step 3: The distance between two octahedral voids is along the body diagonal. Since they are located at the body center and edge centers, the distance is:

$$\text{Distance} = \frac{a}{\sqrt{2}}.$$

Substitute $a = \frac{x}{\sqrt{3}}$:

$$\text{Distance} = \frac{\frac{x}{\sqrt{3}}}{\sqrt{2}} = \frac{x}{\sqrt{3} \cdot \sqrt{2}} = \frac{x}{\sqrt{6}}.$$

Final Answer:

$$\boxed{\frac{x}{\sqrt{6}}}$$

Quick Tip
In FCC crystals, the relationship between the body diagonal and the edge length is Body Diagonal $= \sqrt{3} \cdot$ Edge Length. This can be used to compute the distance between voids.

---

52. Ortho-sulphobenzimide is used as:

(1) Antioxidant

(2) Artificial sweetener

(3) Food preservative

(4) Food supplement

Correct Answer: (2) Artificial sweetener

Solution: Ortho-sulphobenzimide is the chemical name for saccharin. It is a non-caloric artificial sweetener, commonly used as a sugar substitute.

Final Answer:

$$\boxed{\text{Artificial sweetener}}$$

Quick Tip

Saccharin is a widely used artificial sweetener that is much sweeter than sugar and contains no calories.

---

53. Volume of a gas at NTP is 1.12×10^{-7} cm^3. The number of molecules in it is:

(1) 3.01×10^{12}

(2) 3.01×10^{24}

(3) 3.01×10^{23}

(4) 3.01×10^{20}

Correct Answer: (1) 3.01×10^{12}

Solution: At NTP, $22,400\,\text{cm}^3$ of gas contains 6.02×10^{23} molecules. For $1.12 \times 10^{-7}\,\text{cm}^3$, we can calculate the number of molecules using the proportionality between volume and number of molecules:

$$\text{Number of molecules} = \frac{6.02 \times 10^{23}}{22,400} \times 1.12 \times 10^{-7}.$$

Simplify:

$$\text{Number of molecules} = \frac{6.02 \times 1.12 \times 10^{23} \times 10^{-7}}{22,400}.$$

Now, calculate:

$$\text{Number of molecules} = 3.01 \times 10^{12}.$$

Final Answer:

$$\boxed{3.01 \times 10^{12}}.$$

Quick Tip

To calculate the number of molecules for a given volume, use the relation:

$$\text{Number of molecules} = \frac{\text{Volume of gas}}{22,400\,\text{cm}^3} \times 6.02 \times 10^{23}.$$

This allows you to scale the number of molecules for any given volume at NTP conditions.

---

54. The wavelength of the radiation emitted, when a hydrogen atom electron falls from infinity to stationary state 1, would be: (Rydberg constant $R = 1.097 \times 10^7\,\mathbf{m}^{-1}$)

(1) $406\,\text{nm}$

(2) $192\,\text{nm}$

(3) $91\,\text{nm}$

(4) $9.1 \times 10^{-8}\,\text{nm}$

Correct Answer: (3) $91\,\text{nm}$

Solution: The wavelength λ of radiation can be calculated using the Rydberg formula for transitions from a higher energy level (n_2) to the ground state (n_1):

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right),$$

where: - $R = 1.097 \times 10^7\,\text{m}^{-1}$ is the Rydberg constant, - $n_1 = 1$ (the final state, ground state), - $n_2 = \infty$ (the initial state, which means the electron is at the ionization limit).

Step 1: Apply the Rydberg formula For the transition from $n_2 = \infty$ to $n_1 = 1$:

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right).$$

Since $\frac{1}{\infty^2}$ is zero, this simplifies to:

$$\frac{1}{\lambda} = R.$$

Step 2: Substitute the value of R Substitute the value of the Rydberg constant:

$$\frac{1}{\lambda} = 1.097 \times 10^7\,\mathrm{m}^{-1}.$$

Step 3: Solve for λ Now solve for λ:

$$\lambda = \frac{1}{1.097 \times 10^7} \approx 91\,\mathrm{nm}.$$

Final Answer:

$$\boxed{91\,\mathrm{nm}}.$$

Quick Tip
For transitions to the ground state ($n_1 = 1$) from higher energy levels ($n_2 = \infty$), the wavelength is directly given by $\lambda = \frac{1}{R}$, where R is the Rydberg constant. This gives the Lyman series wavelength, commonly 91 nm for the transition to $n = 1$.

---

55. In NO_3^- ion, the number of bond pairs and lone pairs of electrons on the nitrogen atom are:

(1) $2, 2$

(2) $3, 1$

(3) $1, 3$

(4) $4, 0$

Correct Answer: (4) $4, 0$

Solution: The molecular structure of the NO_3^- ion is represented as:

$$\mathrm{O} = \mathrm{N}^+ - \mathrm{O}^- - \mathrm{O}.$$

In the NO_3^- ion: - The nitrogen atom is involved in 4 bond pairs (comprising 3 sigma bonds and 1 pi bond). - The nitrogen atom does not possess any lone pairs of electrons.

Final Answer:

$$\boxed{4, 0}$$

Quick Tip

To determine the bonding in ions, count the pairs of electrons shared as bond pairs and the non-shared pairs as lone pairs.

---

56. In which of the compounds does 'manganese' exhibit the highest oxidation number?

(1) MnO_2

(2) Mn_3O_4

(3) K_2MnO_4

(4) $MnSO_4$

Correct Answer: (3) K_2MnO_4

Solution: To find the oxidation number of Mn in K_2MnO_4, use the equation:

$$2(+1) + x + 4(-2) = 0,$$

where x denotes the oxidation state of Mn.
Simplifying:

$$2 + x - 8 = 0 \implies x = +6.$$

For the other compounds, the oxidation numbers of Mn are: - In MnO_2, Mn = $+4$, - In Mn_3O_4, Mn = $+\frac{8}{3}$ (average), - In $MnSO_4$, Mn = $+2$.

Hence, the highest oxidation number occurs in K_2MnO_4.

Final Answer:

$$\boxed{K_2MnO_4}$$

Quick Tip

The oxidation state of an element in a compound can be determined by considering the charges of the other elements or groups present in the compound.

---

57. Alkali metals are powerful reducing agents because:

(1) They are metals

(2) They are monovalent

(3) Their ionic radii are large

(4) Their ionisation energies are low

Correct Answer: (4) Their ionisation energies are low

Solution: Alkali metals possess a single valence electron, and their low ionization energies make it easy for them to lose this electron and get oxidized. This characteristic makes them excellent reducing agents.

Final Answer:

$$\boxed{\text{Their ionization energies are low}}$$

Quick Tip
A lower ionization energy means an atom can easily lose electrons, enhancing its reducing power.

---

58. A balloon filled with an air sample occupies $3\,\text{L}$ volume at $35°$C. On lowering the temperature to T, the volume decreases to $2.5\,\text{L}$. The temperature T is: *[Assume P-constant]*

(1) $16°$C

(2) $-16°$C

(3) $24°$C

(4) $-20°$C

Correct Answer: (2) $-16°$C

Solution: Using Charles's law:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}.$$

Substitute the given values:

$$\frac{3}{308} = \frac{2.5}{T_2},$$

where $T_1 = 35°\text{C} = 273 + 35 = 308\,\text{K}$. Solve for T_2:

$$T_2 = \frac{2.5 \cdot 308}{3} = 256.6\,\text{K}.$$

Now, Converting it to Celsius:

$$T_2 = 256.6 - 273 = -16.4°\text{C} \approx -16°\text{C}.$$

Final Answer:

$$\boxed{-16°\text{C}}$$

Quick Tip

Charles's law relates the volume and temperature of a gas when pressure is constant. Remember to always convert temperatures to Kelvin before applying the formula.

---

59. Which one of the following is used as an eye lotion?

(1) Milk of magnesia

(2) Silver sol

(3) Colloidal antimony

(4) Chromium salt sol

Correct Answer: (2) Silver sol

Solution: Silver sol is commonly used as an eye lotion due to its antibacterial and healing properties. It effectively helps in treating eye infections and promoting healing.

Final Answer:

$$\boxed{\text{Silver sol}}$$

Quick Tip

Colloidal silver is a popular choice in medical treatments, particularly for its antibacterial and healing effects on wounds and infections.

---

60. Identify ortho and para directing groups from the following:

$$-CHO, -NHCOCH_3, -OCH_3, -SO_3H.$$

(1) III, IV

(2) II, III

(3) II, IV

(4) I, IV

Correct Answer: (2) II, III

Solution: Electron-donating groups such as $-NHCOCH_3$ and $-OCH_3$ direct substitution to the ortho and para positions of the benzene ring by increasing electron density at these positions through resonance or inductive effects.

Electron-withdrawing groups such as $-CHO$ and $-SO_3H$ direct substitution to the meta position due to their ability to decrease electron density at the ortho and para positions.

Final Answer:

$$\boxed{\text{II, III}}$$

Quick Tip
Electron-donating groups (e.g., $-NHCOCH_3$, $-OCH_3$) activate the ortho and para positions for substitution, while electron-withdrawing groups (e.g., $-CHO$, $-SO_3H$) deactivate these positions and prefer the meta position.

---

61. Which one of the carbanions is the least stable?

(1) $\overset{\ominus}{C}H_2-NO_2$

(2) $\overset{\ominus}{C}H_2-CHO$

(3) $\overset{\ominus}{C}H_2-CH_3$

(4) $\overset{\ominus}{C}H_3$

Correct Answer: (3) $\overset{\ominus}{C}H_2 - CH_3$

Solution: The stability of carbanions depends on the presence of electron-withdrawing or electron-donating groups attached to the carbon atom. - $-NO_2$ and $-CHO$ are strong electron-withdrawing groups, which help to stabilise the negative charge on the carbanion. - $-CH_3$, being an electron-donating group, destabilises the negative charge on the carbanion. Thus, the carbanion in $CH_2 - CH_3$ (which lacks electron-withdrawing groups) is the least stable.

Final Answer: $\overset{\ominus}{C}H_2 - CH_3$

Quick Tip
Carbanions are stabilised by electron-withdrawing groups (e.g., $-NO_2$, $-CHO$) and destabilised by electron-donating groups (e.g., $-CH_3$).

---

62. In O_2^-, O_2, and O_2^{2-} molecular species, the total number of antibonding electrons respectively are:

(1) $7, 6, 8$

(2) $1, 0, 2$

(3) $6, 6, 6$

(4) $8, 6, 8$

Correct Answer: (1) $7, 6, 8$

Solution: The molecular orbital electronic configurations for the species are as follows:
O_2^-: $\pi^2_{2p_x}\pi^2_{2p_y}\pi^1_{2p_x}$, with 7 antibonding electrons.
O_2: $\pi^2_{2p_x}\pi^2_{2p_y}$, with 6 antibonding electrons.
O_2^{2-}: $\pi^2_{2p_x}\pi^2_{2p_y}\pi^2_{2p_x}$, with 8 antibonding electrons.

Final Answer:

$$\boxed{7, 6, 8}$$

Quick Tip

The number of antibonding electrons is determined by the specific molecular orbital filling in each species.

---

63. People living at high altitudes often reported a problem of feeling weak and inability to think clearly. The reason for this is:

(1) At high altitudes the partial pressure of oxygen is less than at the ground level.

(2) At high altitudes the partial pressure of oxygen is more than at the ground level.

(3) At high altitudes the partial pressure of oxygen is equal to that at the ground level.

(4) None of these

Correct Answer: (1) At high altitudes the partial pressure of oxygen is less than at the ground level.

Solution: At higher altitudes, the partial pressure of oxygen decreases, which reduces the amount of oxygen available to the body. This leads to hypoxia, a condition characterized by insufficient oxygen in the blood, causing symptoms such as fatigue, dizziness, and confusion, often referred to as anoxia.

Final Answer:

$$\boxed{\text{At high altitudes, the partial pressure of oxygen is lower than at sea level.}}$$

Quick Tip

Reduced oxygen availability at high altitudes can cause altitude sickness or anoxia, which can impair cognitive and physical functions.

---

64. Specific conductance of $0.1\,\mathrm{M\,HNO_3}$ is 6.3×10^{-2} ohm^{-1} cm^{-1}. The molar conductance of the solution is:

(1) $100\,\mathrm{ohm^{-1}\,cm^2\,mol^{-1}}$

(2) $515\,\mathrm{ohm^{-1}\,cm^2\,mol^{-1}}$

(3) $630\,\text{ohm}^{-1}\,\text{cm}^2\,\text{mol}^{-1}$

(4) $6300\,\text{ohm}^{-1}\,\text{cm}^2\,\text{mol}^{-1}$

Correct Answer: (3) $630\,\text{ohm}^{-1}\,\text{cm}^2\,\text{mol}^{-1}$

Solution: The formula for molar conductance is:

$$\Lambda_m = \kappa \cdot \frac{1000}{C},$$

where: - $\kappa = 6.3 \times 10^{-2}\,\text{ohm}^{-1}\,\text{cm}^{-1}$, - $C = 0.1\,\text{M}$.

Now, substituting the given values:

$$\Lambda_m = 6.3 \times 10^{-2} \cdot \frac{1000}{0.1} = 6.3 \times 10^2 = 630\,\text{ohm}^{-1}\,\text{cm}^2\,\text{mol}^{-1}.$$

Final Answer:

$$\boxed{630\,\text{ohm}^{-1}\,\text{cm}^2\,\text{mol}^{-1}}.$$

Quick Tip

Molar conductance is proportional to specific conductance and inversely proportional to the concentration of the solution.

---

65. The rate constant for a first-order reaction whose half-life is 480 seconds is:

(1) $2.88 \times 10^{-3}\,\text{s}^{-1}$

(2) $2.72 \times 10^{-3}\,\text{s}^{-1}$

(3) $1.44 \times 10^{-3}\,\text{s}^{-1}$

(4) $1.44\,\text{s}^{-1}$

Correct Answer: (3) $1.44 \times 10^{-3}\,\text{s}^{-1}$

Solution: For a first-order reaction, the rate constant k is related to the half-life $t_{1/2}$ using the equation:

$$k = \frac{0.693}{t_{1/2}}.$$

Substitute the given value of $t_{1/2} = 480\,\text{s}$:

$$k = \frac{0.693}{480}.$$

Simplifying:

$$k = 1.44 \times 10^{-3}\,\mathrm{s}^{-1}.$$

This value represents the rate constant for the reaction, which depends solely on the half-life for first-order processes.

Final Answer:

$$\boxed{1.44 \times 10^{-3}\,\mathrm{s}^{-1}}.$$

Quick Tip
For first-order reactions, the rate constant is inversely related to the half-life. Use the formula $k = \frac{0.693}{t_{1/2}}$ to quickly calculate the rate constant.

---

66. If the activation energy for the forward reaction is $150\,$kJ/mol and that of the reverse reaction is $260\,$kJ/mol, what is the enthalpy change for the reaction?

(1) $410\,$kJ/mol

(2) $-110\,$kJ/mol

(3) $110\,$kJ/mol

(4) $-410\,$kJ/mol

Correct Answer: (2) $-110\,$kJ/mol

Solution: The enthalpy change ΔH for a reaction is given by the difference between the activation energies of the forward reaction (E_a^{f}) and the reverse reaction (E_a^{r}):

$$\Delta H = E_a^{\mathrm{f}} - E_a^{\mathrm{r}}.$$

Substitute the given activation energies:

$$\Delta H = 150 - 260 = -110\,\text{kJ/mol}.$$

Since ΔH is negative, it indicates that the reaction is exothermic, meaning it releases heat.

Final Answer:

$$\boxed{-110\,\text{kJ/mol}}$$

Quick Tip

In exothermic reactions, the activation energy of the reverse reaction is higher than that of the forward reaction, resulting in a negative enthalpy change (ΔH).

---

67. For As_2S_3 sol, the most effective coagulating agent is:

(1) $CaCO_3$

(2) NaCl

(3) $FeCl_3$

(4) $Clay$

Correct Answer: (3) $FeCl_3$

Solution: The coagulation of a negatively charged sol, such as As_2S_3, is most effective with a highly positively charged ion, according to the Hardy-Schulze rule.
Among the given options:
$FeCl_3$ dissociates to produce Fe^{3+}, which has the highest positive charge ($+3$).
Higher positive charges effectively neutralize the negative charge of the sol, leading to coagulation.
In contrast, $CaCO_3$ (which produces Ca^{2+}) and NaCl (which produces Na^+) are less effective because their ions have lower positive charges.

Final Answer:

$$\boxed{FeCl_3}$$

Quick Tip

The coagulation efficiency increases with the charge of the ion, following the Hardy-Schulze rule, which states that higher charges cause more effective coagulation.

---

68. Element not showing variable oxidation state is:

(1) Bromine

(2) Iodine

(3) Chlorine

(4) Fluorine

Correct Answer: (4) Fluorine

Solution: Fluorine is the most electronegative element and does not exhibit variable oxidation states because:

1. It cannot expand its octet since it has no available d-orbitals.

2. Due to its high electronegativity, fluorine always assumes an oxidation state of -1 in its compounds.

Other halogens, such as bromine, iodine, and chlorine, can exhibit variable oxidation states as they possess vacant d-orbitals that allow them to expand their octet.

Final Answer:

$$\boxed{\text{Fluorine}}$$

Quick Tip

Fluorine, being highly electronegative and lacking vacant d-orbitals, always adopts an oxidation state of -1, unlike other halogens that show variable oxidation states.

---

69. Which of the following arrangements does not represent the correct order of the property stated against it?

(1) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$: Paramagnetic behaviour

(2) $Ni^{2+} < Co^{2+} < Fe^{2+} < Mn^{2+}$: Ionic size

(3) $Co^{3+} < Fe^{3+} < Cr^{3+} < Sc^{3+}$: Stability in aqueous solution

(4) Sc < Ti < Cr < Mn: Number of oxidation states

Correct Answer: (1) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$

Solution: The paramagnetic behavior of an ion is governed by the number of unpaired electrons in its electronic configuration.

The configurations are as follows:
V^{2+} : $3d^3$ (3 unpaired electrons),
Cr^{2+} : $3d^4$ (4 unpaired electrons),
Mn^{2+} : $3d^5$ (5 unpaired electrons),
Fe^{2+} : $3d^6$ (4 unpaired electrons).

Based on the number of unpaired electrons, the correct order of paramagnetic behavior is:

$$V^{2+} < Cr^{2+} < Fe^{2+} < Mn^{2+}.$$

Thus, the arrangement presented in Option (1) does not reflect the correct order of paramagnetic behavior.

Final Answer:

$$\boxed{V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}}.$$

Quick Tip
The paramagnetic behavior of an ion is directly linked to the number of unpaired electrons. More unpaired electrons lead to a stronger paramagnetic behavior.

---

70. The IUPAC name for the complex $[Co(ONO)(NH_3)_5]Cl_2$ is:

(1) $Pentaammine - nitrito - O - cobalt(II)chloride$
(2) $Pentaammine - nitrito - O - cobalt(III)chloride$
(3) $Nitrito - N - pentaamminecobalt(III)chloride$
(4) $Nitrito - N - pentaamminecobalt(II)chloride$

Correct Answer: (2) Pentaammine-nitrito-O-cobalt(III) chloride

Solution: The complex $[Co(ONO)(NH_3)_5]Cl_2$ consists of:

ONO: A nitrito ligand coordinated via the oxygen atom (denoted as ”nitrito-O”),

NH_3: Neutral ammine ligands,

Cl_2: Counter ions.

To determine the oxidation state of cobalt, use the following calculation:

$$\text{Co} + 5(0) + (-1) + 2(-1) = 0 \implies \text{Co} = +3.$$

Therefore, the IUPAC name of the complex is:

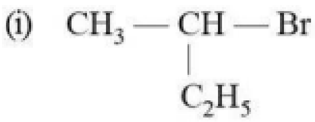
$$\boxed{\text{Pentaammine-nitrito-O-cobalt(III) chloride}}.$$

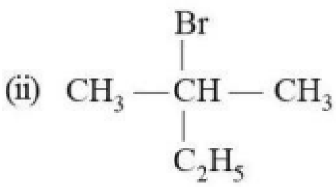
Quick Tip

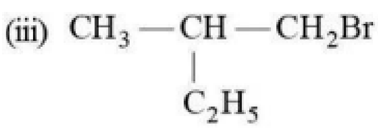
When naming coordination compounds, make sure to specify the donor atom (either O or N) for ambidentate ligands like ONO to avoid confusion.

---

71. Which of the following compounds will give racemic mixture on nucleophilic substitution by OH^- ion?







(1) (i)

(2) $(i), (ii), and (iii)$

(3) $(ii) and (iii)$

(4) $(i) and (iii)$

Correct Answer: (2) $(i), (ii), \text{and} (iii)$

Solution: Compounds that undergo the S_N1 mechanism in nucleophilic substitution reactions produce a racemic mixture. This is because the S_N1 mechanism involves the formation of a planar carbocation intermediate, which can be attacked by the nucleophile from either side, leading to the production of a 1:1 mixture of enantiomers. The reactivity order for S_N1 reactions is:

$$3^\circ > 2^\circ > 1^\circ \text{(alkyl halides)}.$$

Compound (i): Has a secondary carbon, and follows the S_N1 mechanism.
Compound (ii): Contains a tertiary carbon and strongly favours the S_N1 mechanism.
Compound (iii): Has a secondary carbon, and follows the S_N1 mechanism. Hence, all three compounds will produce a racemic mixture.

Final Answer:

$$\boxed{(i), (ii), \text{and} (iii)}$$

Quick Tip

Racemic mixtures form in S_N1 reactions due to the formation of a planar carbocation intermediate, which allows the nucleophile to attack from either side.

---

72. Which one of the following is not correct for an ideal solution?

(1) $It must obey Raoult's law.$

(2) $\Delta H = 0.$

(3) $\Delta H = \Delta V \neq 0.$

(4) $All are correct.$

Correct Answer: (3) $\Delta H = \Delta V \neq 0$

Solution: An ideal solution is defined by the following characteristics:
1. It obeys Raoult's law throughout the entire range of composition.

2. There is no heat change during mixing ($\Delta H = 0$).

3. There is no volume change during mixing ($\Delta V = 0$).

Option (3), which states that $\Delta H = \Delta V \neq 0$, is incorrect because both ΔH and ΔV must be zero for an ideal solution to behave ideally.

Final Answer:

$$\boxed{\Delta H = \Delta V \neq 0}$$

Quick Tip
An ideal solution shows no heat or volume changes upon mixing. Any deviation from this indicates a non-ideal solution.

---

73. For the relation $\Delta_r G = -nFE_{\text{cell}}$, $E_{\text{cell}} = E^\circ_{\text{cell}}$, in which of the following conditions?

(1) $Concentration of any one of the reacting species should be unity.$

(2) $Concentration of all the product species should be unity.$

(3) $Concentration of all the reactant and product species should be unity.$

(4) $Concentration of all reacting and product species should be unity.$

Correct Answer: (3) $Concentration of all the reactant and product species should be unity.$

Solution: To ensure that $E_{\text{cell}} = E^\circ_{\text{cell}}$, the Nernst equation becomes:

$$E_{\text{cell}} = E^\circ_{\text{cell}} - \frac{RT}{nF} \ln Q,$$

where Q is the reaction quotient.

For E_{cell} to equal E°_{cell}, the term $\ln Q$ must be zero, which occurs when:

$$Q = 1 \implies \text{The concentration of all reactant and product species must be unity.}$$

This ensures the cell operates under standard conditions, where E_{cell} equals E°_{cell}.

Final Answer:

$$\boxed{\text{The concentration of all reactant and product species must be unity.}}$$

Quick Tip

To maintain standard-state conditions, all reactants and products must have unit concentrations, ensuring that $E_{\text{cell}} = E^{\circ}_{\text{cell}}$.

---

74. Which one of the lanthanoids given below is the most stable in divalent form?

(1) Ce (Atomic Number 58)

(2) Sm (Atomic Number 62)

(3) Eu (Atomic Number 63)

(4) Yb (Atomic Number 70)

Correct Answer: (3) Eu (Atomic Number 63)

Solution: The stability of the divalent state in lanthanide ions is influenced by their electronic configuration. Fully filled and half-filled configurations are particularly stable:

$$\text{Ce}^{2+} \to 4f^1, \quad \text{Sm}^{2+} \to 4f^6, \quad \text{Eu}^{2+} \to 4f^7, \quad \text{Yb}^{2+} \to 4f^{14}.$$

Reduction potential comparison:

$$E^{\circ}_{M^{3+}/M^{2+}}: \quad \text{Eu} = -0.35\,\text{V}, \quad \text{Yb} = -1.05\,\text{V}.$$

Europium (Eu^{2+}) is more stable than Yb^{2+}, as it has a lower reduction potential and a more stable half-filled $4f^7$ configuration.

Final Answer:

$$\boxed{\text{Europium}\,(\text{Eu}^{2+})}$$

Quick Tip

The stability of lanthanide ions depends on their electronic configuration and reduction potential. Fully filled ($4f^{14}$) and half-filled ($4f^7$) configurations are especially stable.

---

75. The value of the 'spin only' magnetic moment for one of the following configurations is $2.84\,\text{BM}$. The correct one is:

(1) d^5 (in strong ligand field)

(2) d^3 (in weak as well as strong fields)

(3) d^4 (in weak ligand fields)

(4) d^4 (in strong ligand fields)

Correct Answer: (4) d^4 (in strong ligand fields)

Solution: The spin-only magnetic moment is calculated using the formula:

$$\mu = \sqrt{n(n+2)}\,\text{BM},$$

where n is the number of unpaired electrons.

Configurations and calculations:

- d^5 (strong ligand field): $n = 1$, $\mu = \sqrt{3} = 1.73\,\text{BM}$,
- d^3 (weak/strong field): $n = 3$, $\mu = \sqrt{15} = 3.87\,\text{BM}$,
- d^4 (weak field): $n = 4$, $\mu = \sqrt{24} = 4.89\,\text{BM}$,
- d^4 (strong field): $n = 2$, $\mu = \sqrt{8} = 2.82\,\text{BM}$.

Thus, the configuration d^4 in a strong ligand field corresponds to a magnetic moment of $\mu = 2.82\,\text{BM}$.

Final Answer:

$$\boxed{d^4 \text{ (in strong ligand fields)}}$$

Quick Tip
The magnetic moment of a complex depends on the number of unpaired electrons. In strong ligand fields, electron pairing reduces the number of unpaired electrons, lowering the magnetic moment.

---

76. Chlorobenzene reacts with Mg in dry ether to give a compound (A) which further reacts with ethanol to yield:

(a) Phenol

(b) Benzene

(c) Ethylbenzene

(d) Phenyl ether

Correct Answer: (b) Benzene

Solution: The reaction proceeds in two steps: 1. Chlorobenzene (C_6H_5Cl) undergoes a reaction with magnesium in dry ether to form phenyl magnesium chloride (C_6H_5MgCl), which is a Grignard reagent:

$$C_6H_5Cl + Mg \xrightarrow{\text{dry ether}} C_6H_5MgCl.$$

2. Phenyl magnesium chloride then reacts with ethanol (C_2H_5OH) to produce benzene (C_6H_6) and magnesium ethoxide (C_2H_5OMgCl):

$$C_6H_5MgCl + C_2H_5OH \rightarrow C_6H_6 + C_2H_5OMgCl.$$

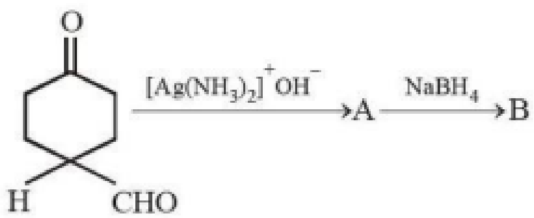
Final Answer:

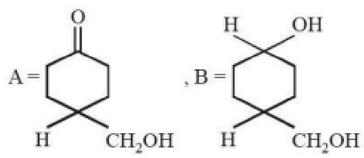
$$\boxed{\text{Benzene}}$$

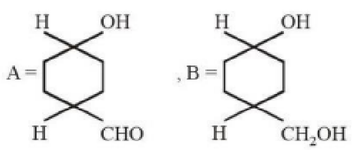
Quick Tip
Hydrocarbons is released when Grignard reagents react with alcohols. Always identify the functional groups involved in the reaction to predict the outcome.

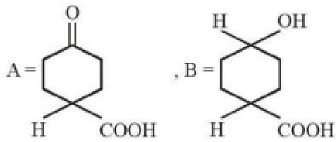
---

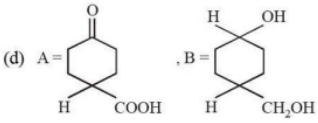
77. The products formed in the following reaction, A and B, are:

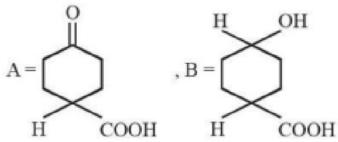


(1) 

(2) 

(3) 

(4) 

Correct Answer: (3) 

Solution: The reaction proceeds in two stages:

Step 1: Oxidation with Tollens' Reagent - Tollens' reagent $[Ag(NH_3)_2]^+$ and OH^- selectively oxidizes aldehydes to carboxylic acids. - In this case, the aldehyde group ($-CHO$) of the compound undergoes oxidation to form a carboxylic acid group ($-COOH$). - The resulting product A after oxidation is:

$$\text{A} = \text{Compound with the aldehyde oxidized to a carboxylic acid: HCOOH}.$$

Step 2: Reduction with Sodium Borohydride - Sodium borohydride ($NaBH_4$) is a selective reducing agent that reduces aldehydes and ketones to their corresponding alcohols. - In the given compound, the ketone group ($-C = O$) is reduced to a secondary alcohol ($-CH(OH)-$). - The product B after reduction is:

$$\text{B} = \text{Compound with the ketone reduced to an alcohol: HCOOH}.$$

Final Answer: The products A and B correspond to:

$$\boxed{\text{Option C}}$$

Quick Tip
Tollens' reagent oxidizes aldehydes to carboxylic acids, while sodium borohydride reduces ketones and aldehydes to alcohols.

---

78. Which of the following represents the correct order of acidity in the given compounds?

(1) $FCH_2COOH > CH_3COOH > BrCH_2COOH > ClCH_2COOH$

(2) $BrCH_2COOH > ClCH_2COOH > FCH_2COOH > CH_3COOH$

(3) $FCH_2COOH > ClCH_2COOH > BrCH_2COOH > CH_3COOH$

(4) $CH_3COOH > BrCH_2COOH > ClCH_2COOH > FCH_2COOH$

Correct Answer: (3) $FCH_2COOH > ClCH_2COOH > BrCH_2COOH > CH_3COOH$

Solution: The acidity of carboxylic acids is influenced by the electron-withdrawing (-I) effect of substituents attached to the carbon adjacent to the carboxyl group (COOH). These substituents pull electron density away from the carboxylate ion, stabilizing it and thus enhancing the acidity.

Electronegativity order:

$$F > Cl > Br > H.$$

Fluorine, being the most electronegative, exerts the strongest -I effect, leading to the greatest stabilization of the carboxylate ion and the highest acidity. Chlorine and bromine follow in decreasing order of electronegativity, with hydrogen exerting no such withdrawing effect.

Final Answer:

$$\boxed{FCH_2COOH > ClCH_2COOH > BrCH_2COOH > CH_3COOH}$$

Quick Tip

Substituents with strong electron-withdrawing effects increase the acidity of carboxylic acids by stabilizing the carboxylate ion. The more electronegative the substituent, the stronger the effect.

---

79. Ethyl alcohol can be prepared from Grignard reagent by the reaction of:

(1) HCHO

(2) R_2CO

(3) RCN

(4) RCOCl

Correct Answer: (1) HCHO

Solution: Grignard reagents are highly reactive and can react with carbonyl compounds to form alcohols. When formaldehyde (HCHO) reacts with a Grignard reagent like methylmagnesium bromide (CH_3MgBr), a primary alcohol is formed.

Reaction mechanism: Step 1: Nucleophilic attack. The nucleophilic methyl group (CH_3) from the Grignard reagent attacks the electrophilic carbonyl carbon of formaldehyde:

$$\mathrm{HCHO} + \mathrm{CH_3MgBr} \rightarrow \mathrm{CH_3CH_2OMgBr}.$$

Step 2: Hydrolysis. The intermediate alcoholate ion (CH_3CH_2OMgBr) is then hydrolyzed in the presence of water to yield the primary alcohol, ethyl alcohol:

$$\mathrm{CH_3CH_2OMgBr} + \mathrm{H_3O^+} \rightarrow \mathrm{CH_3CH_2OH}.$$

Final Answer:

$$\boxed{\mathrm{HCHO}}$$

Quick Tip
Grignard reagents react with formaldehyde to yield primary alcohols. Other carbonyl compounds typically form secondary or tertiary alcohols depending on the type of carbonyl group.

---

80. Which of the following is most acidic?

(1) Benzyl alcohol

(2) Cyclohexanol

(3) Phenol

(4) m-chlorophenol

Correct Answer: (4) m-chlorophenol

Solution: Phenols exhibit greater acidity compared to alcohols due to the resonance stabilization of the phenoxide ion formed after losing a proton (H^+). This resonance delocalizes the negative charge on the oxygen, enhancing the compound's acidity. Electron-withdrawing substituents, such as -Cl, further increase the acidity of phenols by stabilizing the negative charge on the oxygen atom.

Step 1: Structure comparison.

Benzyl alcohol: Less acidic, as the hydroxyl group is attached to a saturated carbon, and there is no resonance stabilization.
Cyclohexanol: Similar to benzyl alcohol, it lacks resonance stabilization and is therefore less acidic.
Phenol: More acidic due to the resonance stabilization of the phenoxide ion.
m-Chlorophenol: The most acidic, as the -Cl group is an electron-withdrawing group, which enhances resonance stabilization and increases acidity.

Final Answer:

$$\boxed{m\text{-chlorophenol}}$$

Quick Tip
Electron-withdrawing groups like -Cl increase the acidity of phenols by stabilizing the phenoxide ion through resonance, making the compound more prone to losing a proton.

81. Nitration of the compound is carried out. This compound gives red-orange precipitate with 2,4-DNP, undergoes Cannizzaro reaction but not aldol, then the possible product due to nitration is:

(1) 3-nitroacetophenone
(2) (2-nitro)-2-phenylethanol
(3) (2-nitro)-1-phenylpropan-2-one
(4) 3-nitrobenzaldehyde

Correct Answer: (4) 3-nitrobenzaldehyde

Solution: The compound undergoes a Cannizzaro reaction but not aldol condensation, indicating the absence of alpha-hydrogens. This suggests that the compound is an aldehyde. Upon nitration, benzaldehyde forms 3-nitrobenzaldehyde.

Step 1: Reaction mechanism.

$$\text{CHO} \xrightarrow{\text{Nitration}} \text{NO}_2\text{-CHO}.$$

The nitration occurs at the meta-position relative to the aldehyde group due to the electron-withdrawing nature of the -CHO group.

Conclusion: The correct product formed is $\boxed{\text{3-nitrobenzaldehyde}}$.

Quick Tip

Aldehydes without alpha-hydrogens undergo Cannizzaro reactions, leading to no aldol condensation. The electron-withdrawing nature of the -CHO group directs electrophilic substitution to the meta-position.

---

82. Which of the following reactions will not give a primary amine?

(1) $CH_3CONH_2 \xrightarrow{Br_2/KOH}$

(2) $CH_3CN \xrightarrow{LiAlH_4}$

(3) $CH_3NC \xrightarrow{LiAlH_4}$

(4) $CH_3CONH_2 \xrightarrow{LiAlH_4}$

Correct Answer: (3) $CH_3NC \xrightarrow{LiAlH_4}$

Solution: The reactions and their respective products are as follows:

(a) Hofmann Bromamide Reaction:

$$CH_3CONH_2 \xrightarrow{Br_2/KOH} CH_3NH_2.$$

This reaction involves the conversion of an amide ($CONH_2$) to a primary amine (CH_3NH_2) with the loss of one carbon atom.

(b) Reduction of Nitriles:

$$CH_3CN \xrightarrow{LiAlH_4} CH_3CH_2NH_2.$$

Nitriles (CN) are reduced to primary amines ($CH_3CH_2NH_2$) using lithium aluminium hydride ($LiAlH_4$).

(c) Reduction of Isocyanides (Methyl Isocyanide):

$$CH_3NC \xrightarrow{LiAlH_4} CH_3NHCH_3.$$

Isocyanides (NC) are reduced to secondary amines (CH_3NHCH_3) using lithium aluminium hydride. Hence, this reaction does not produce a primary amine.

(d) Reduction of Amides:

$$CH_3CONH_2 \xrightarrow{LiAlH_4} CH_3CH_2NH_2.$$

Amides ($CONH_2$) are reduced to primary amines ($CH_3CH_2NH_2$) using lithium aluminium hydride.

Conclusion: Among the given reactions, option (3) does not produce a primary amine; it produces a secondary amine instead.

Final Answer:

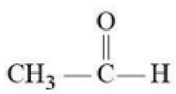
$$\boxed{CH_3NC \xrightarrow{LiAlH_4} CH_3NHCH_3 \text{ (Secondary Amine)}}.$$

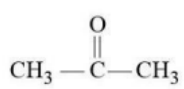
Quick Tip

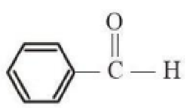
When reducing amides, nitriles, or using the Hofmann Bromamide reaction, primary amines are obtained. However, the reduction of isocyanides produces a secondary amine instead.

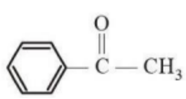
---

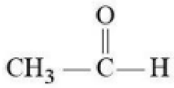
83. Which of the following compounds is most reactive towards nucleophilic addition reactions?

(1) 

(2) 

(3) 

(4) 

Correct Answer: (1) 

Solution: The reactivity of carbonyl compounds in nucleophilic addition reactions is primarily determined by:
- The electrophilic nature of the carbonyl carbon,
- The steric effects due to substituents around the carbonyl group,
- The electronic effects from substituents that may donate or withdraw electron density.

Step 1: Compare aldehydes and ketones.

- **Aldehydes** (e.g., CH_3CHO, C_6H_5CHO) have only one substituent attached to the carbonyl carbon, which minimizes steric hindrance and allows easier attack by nucleophiles. As a result, they tend to be more reactive.
- **Ketones** (e.g., CH_3COCH_3, $C_6H_5COCH_3$) possess two alkyl or aryl groups attached to the carbonyl carbon, leading to higher steric hindrance and consequently lower reactivity.

Step 2: Examine electronic effects.

- The **inductive effect** of the single alkyl group in aldehydes stabilizes the partial positive charge on the carbonyl carbon, but not as effectively as the **resonance effect** in **aromatic aldehydes** like benzaldehyde, where the phenyl group donates electron density through resonance. This makes the carbonyl carbon less electrophilic.
- In contrast, **acetaldehyde** (CH_3CHO) has no resonance interaction and exhibits a higher electrophilic character, making it more reactive in nucleophilic addition reactions.

Step 3: Evaluate reactivity trends.

- **Acetaldehyde** (CH_3CHO) is the most reactive due to minimal steric hindrance and no resonance effects.
- **Benzaldehyde** (C_6H_5CHO) is less reactive than acetaldehyde because of the resonance interaction, which reduces the electrophilicity of the carbonyl group despite low steric hindrance.

- **Ketones** (e.g., CH_3COCH_3) are less reactive due to both increased steric hindrance from two substituents and the inductive effects of the alkyl groups, which slightly reduce the carbonyl carbon's electrophilic nature.

Final Answer:

$$\boxed{CH_3CHO}$$

Quick Tip
Aldehydes are more reactive than ketones in nucleophilic addition reactions because they experience less steric hindrance and their carbonyl carbon is more electrophilic.

---

84. Molecules whose mirror image is non-superimposable over them are known as chiral. Which of the following molecules is chiral in nature?

(1) 2-bromobutane

(2) 1-bromobutane

(3) 2-bromopropane

(4) 2-bromopropan-2-ol

Correct Answer: (1) 2-bromobutane

Solution: A molecule is chiral if it contains at least one carbon atom (chiral carbon) bonded to four different substituents, making it non-superimposable on its mirror image.

Step 1: Analyze each option.

Option (1): 2-bromobutane The second carbon in CH_3-CHBr-CH_2-CH_3 is attached to:

- A bromine atom (Br),
- A methyl group (CH_3),
- An ethyl group (CH_2CH_3),
- A hydrogen atom (H).

Since the second carbon is bonded to four different groups, 2-bromobutane is chiral.

Option (2): 1-bromobutane The first carbon in CH_3-CH_2-CH_2-CH_2-Br is bonded to:

- A bromine atom (Br),
- Two hydrogen atoms (H),
- An ethyl group ($CH_2CH_2CH_3$).

Since the first carbon is bonded to two identical substituents (H), 1-bromobutane is not chiral.

Option (3): 2-bromopropane The second carbon in CH_3-CHBr-CH_3 is bonded to:

- A bromine atom (Br),
- Two methyl groups (CH_3),
- A hydrogen atom (H).

Since this carbon is bonded to two identical methyl groups, 2-bromopropane is not chiral.

Option (4): 2-bromopropan-2-ol The second carbon in CH_3-CBr(OH)-CH_3 is bonded to:

- A bromine atom (Br),
- A hydroxyl group (OH),
- Two methyl groups (CH_3).

Since this carbon is bonded to two identical methyl groups, 2-bromopropan-2-ol is not chiral.

Step 2: Conclusion. Among the given options, only 2-bromobutane contains a chiral carbon and is therefore chiral.

Final Answer:

$$\boxed{\text{2-bromobutane}}$$

Quick Tip
A chiral carbon is a carbon atom bonded to four different groups. Molecules containing such carbons are non-superimposable on their mirror images.

85. The most reactive amine towards dilute hydrochloric acid is:

(1) CH_3NH_2

(2) $(CH_3)_2NH$

(3) $(CH_3)_3N$

(4) $C_6H_5NH_2$

Correct Answer: (2) $(CH_3)_2NH$

Solution: The reactivity of amines towards dilute HCl is governed by their basicity, which is influenced by factors such as electron-donating groups and steric hindrance.

Step 1: Analyze the basicity of different amines.

- **Methylamine** (CH_3NH_2) is a primary amine where the methyl group (CH_3) donates electrons via an inductive effect, increasing the electron density on nitrogen. However, this effect is weaker compared to secondary amines, making methylamine less basic.
- **Dimethylamine** ($(CH_3)_2NH$) is a secondary amine with two CH_3 groups. These groups provide a stronger electron-donating inductive effect, enhancing the electron density on nitrogen and making dimethylamine the most basic of the given options. The lower steric hindrance in dimethylamine compared to tertiary amines also contributes to its high reactivity towards dilute HCl.
- **Trimethylamine** ($(CH_3)_3N$) is a tertiary amine with three CH_3 groups. Although the inductive effect is strong, the increased steric hindrance around nitrogen reduces the availability of the lone pair for protonation, making trimethylamine less reactive than dimethylamine.
- **Aniline** ($C_6H_5NH_2$) is less basic than aliphatic amines because the lone pair on nitrogen is delocalized into the aromatic ring via resonance. This reduces the availability of the lone pair for protonation, making aniline the least reactive among the given options.

Step 2: Conclusion. Dimethylamine, with its stronger electron-donating groups and lower steric hindrance, is the most reactive towards dilute HCl.

$$(CH_3)_2NH + HCl \rightarrow (CH_3)_2NH_2^+ + Cl^-.$$

Final Answer:

$$\boxed{(CH_3)_2NH}$$

Quick Tip

Secondary amines like dimethylamine are more basic than primary and tertiary amines due to stronger inductive effects and lower steric hindrance, making them more reactive towards dilute acids.

---

86. One of essential α-amino acids is:

(1) Lysine

(2) Serine

(3) Glycine

(4) Proline

Correct Answer: (1) Lysine

Solution: using a step-by-step method to emphasize the categorization of amino acids:

Step 1: Definition of Essential Amino Acids Essential amino acids are those that cannot be synthesized by the human body. These amino acids must be obtained from the diet, as they are crucial for various biological functions, including protein synthesis.

Step 2: Categorization of the Given Amino Acids

Let's evaluate each of the given amino acids:

Lysine is an essential amino acid. The body cannot produce it, so it must be consumed through food.

Glycine is a non-essential amino acid, as it can be synthesized by the body from other precursors.

Serine is also a non-essential amino acid because it can be synthesized in the body from other compounds.

Proline is considered a non-essential amino acid, as it can be synthesized by the body from glutamate.

Conclusion: Among the listed amino acids, Lysine is the only essential one. Therefore, the correct answer is:

$$\boxed{\text{Lysine}}$$

Quick Tip

Essential amino acids cannot be synthesized by the body and must be consumed through diet.

---

87. Which of the following statements is true about a peptide bond (RCONHR')?

(1) It is non-planar.
(2) It is capable of forming a hydrogen bond.
(3) The cis configuration is favoured over the trans configuration.
(4) Single bond rotation is permitted between nitrogen and the carbonyl group.

Correct Answer: (2) It is capable of forming a hydrogen bond.

Solution: A peptide bond forms between the carboxyl group of one amino acid and the amino group of another. This bond has the following characteristics:

$$\text{Peptide Bond: RCONHR'}$$

Explanation of the properties of peptide bonds:

1. The peptide bond exhibits **planarity** due to the partial double-bond character, which arises from resonance between the carbonyl and nitrogen atoms.
2. The NH group in the peptide bond can act as a hydrogen bond donor, and the C=O group can act as a hydrogen bond acceptor, allowing the formation of hydrogen bonds.
3. The **trans configuration** is preferred over the cis configuration to minimize steric clashes between the side chains of adjacent amino acids.
4. Rotation is **restricted** around the bond between nitrogen and carbonyl due to its partial double-bond character.

Final Answer:

$$\boxed{\text{It is capable of forming a hydrogen bond.}}$$

Quick Tip

Peptide bonds are rigid and planar, and their ability to form hydrogen bonds is crucial for stabilizing protein structures.

---

88. Which of the following is an example for chain-growth polymer?

(1) Bakelite

(2) Teflon

(3) Nylon

(4) Terylene

Correct Answer: (2) Teflon

Solution: Step 1: Definition of Polymerization Types

Polymerization can be broadly classified into two types:

1. Chain-growth polymerization: This type involves the successive addition of monomers to a growing polymer chain. It starts with the initiation of free radicals, cations, or anions, and the polymer grows by adding monomers that typically contain double bonds.

2. Step-growth polymerization: In this process, polymer formation occurs through the reaction of small molecules that form covalent bonds, leading to the creation of a polymer. This type of polymerization is characterized by the formation of a polymer from the reaction of bifunctional or multifunctional monomers.

Step 2: Identification of Polymers

Let's classify the given polymers:

Teflon: This is made through chain-growth polymerization, as it involves the addition of monomers (tetrafluoroethylene) in the presence of free radicals.

Bakelite: A result of step-growth polymerization, made by the reaction of phenol and formaldehyde.
Nylon: Another example of step-growth polymerization, made from a diamine and a dicarboxylic acid.
Terylene: Also a product of step-growth polymerization, formed by the reaction of terephthalic acid and ethylene glycol.

Conclusion: Based on the nature of the polymerization process and the type of polymer involved, Teflon is the polymer formed through chain-growth polymerization. Therefore, the correct answer is:

$$\boxed{\text{Teflon}}$$

Quick Tip
Chain-growth polymerization involves monomer addition with unsaturated bonds, while step-growth polymerization involves small molecule reactions to form polymers.

---

89. Which of the following polymers is formed due to the co-polymerization of $1,3$-butadiene and phenylethene?

(1) Buna-N

(2) Neoprene

(3) Novolac

(4) Buna-S

Correct Answer: (4) Buna-S

Solution: Step 1: Definition of Co-Polymerization
Co-polymerization is a process where two different monomers are polymerized together to form a polymer. The resulting polymer can have unique properties depending on the combination of the monomers.

Step 2: Identification of the Polymerization Products

Let's review the information provided and identify each polymer:

Buna-S: This is a synthetic rubber created by the co-polymerization of 1,3-butadiene and styrene (phenylethene). This combination provides rubber-like properties.

Buna-N: This synthetic rubber is formed by the co-polymerization of 1,3-butadiene and acrylonitrile. It is used for products requiring resistance to oils and solvents.

Neoprene: This polymer is made from chloroprene, and it exhibits resistance to heat, oil, and chemicals, making it useful for industrial applications.

Novolac: A phenolic resin produced from phenol and formaldehyde, often used in making molded products.

Conclusion: Among the listed polymers, Buna-S is the one produced by the co-polymerization of 1,3-butadiene and styrene. Therefore, the correct answer is:

$$\boxed{\text{Buna-S}}$$

> Quick Tip
>
> Co-polymerization of different monomers leads to polymers with distinct properties, like Buna-S, made from 1,3-butadiene and styrene.

---

90. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?

(1) $[Co(H_2O)_6]Cl_3$

(2) $[Co(H_2O)_5Cl]Cl_2 \cdot H_2O$

(3) $[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O$

(4) $[Co(H_2O)_3Cl_3] \cdot 3H_2O$

Correct Answer: (4) $[Co(H_2O)_3Cl_3] \cdot 3H_2O$

Solution: Step 1: Freezing Point Depression Formula The freezing point depression is directly related to the number of dissociated particles in the solution. The formula is:

$$\Delta T_f = K_f \cdot m \cdot i$$

where: - ΔT_f is the freezing point depression, - K_f is the cryoscopic constant (which depends on the solvent), - m is the molality of the solution, - i is the van 't Hoff factor, representing the number of particles resulting from dissociation.

Since the freezing point depression is inversely proportional to the number of dissociated particles, the solution with the lowest i (fewer particles) will have the highest freezing point.

Step 2: Identify the i for Each Compound Let's now identify the van 't Hoff factor i for each compound:

1. $[Co(H_2O)_6]Cl_3$ dissociates into 4 particles: $[Co(H_2O)_6]^{3+}$ and 3 Cl^-. Hence, $i = 4$. 2. $[Co(H_2O)_5Cl]Cl_2 \cdot H_2O$ dissociates into 3 particles: $[Co(H_2O)_5Cl]^{2+}$ and 2 Cl^-. Hence, $i = 3$. 3. $[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O$ dissociates into 2 particles: $[Co(H_2O)_4Cl_2]^+$ and Cl^-. Hence, $i = 2$. 4. $[Co(H_2O)_3Cl_3] \cdot 3H_2O$ does not dissociate, so it remains as a single particle: $i = 1$.

Step 3: Conclusion Since the freezing point depression is inversely proportional to i, the compound with the lowest i will have the highest freezing point. In this case, $[Co(H_2O)_3Cl_3] \cdot 3H_2O$, which has $i = 1$, will have the highest freezing point.

Final Answer:

$$\boxed{[Co(H_2O)_3Cl_3] \cdot 3H_2O}$$

Quick Tip
The freezing point of a solution increases as the number of dissociated particles decreases. Lower dissociation means fewer particles, leading to a higher freezing point.

---

91. Which of the following expressions correctly represents molar conductivity?

(1) $\Lambda_m = \frac{K}{C}$

(2) $\Lambda_m = \frac{KA}{l}$

(3) $\Lambda_m = KV$

(4) All of these

Correct Answer: (4) All of these

Solution: The molar conductivity (Λ_m) of an electrolyte solution can be understood through its dependence on various factors such as conductivity (K), concentration (C), and the geometry of the solution. Here's a step-by-step explanation using different reasoning:

1. Relationship with Conductivity and Concentration: Using the basic definition of molar conductivity:

$$\Lambda_m = \frac{K}{C},$$

where K is the conductivity and C is the molar concentration. This relationship shows that molar conductivity is inversely proportional to concentration at a fixed conductivity.

2. Derivation from Geometry and Conductance: Consider the relationship between conductivity (K), the geometry of the system, and conductance (G):

$$G = \frac{K \cdot A}{l},$$

where A is the cross-sectional area, and l is the distance between electrodes. Substituting G into the expression for molar conductivity, we get:

$$\Lambda_m = \frac{KA}{l}.$$

This formula emphasizes the role of the geometry of the electrolytic cell in determining molar conductivity.

3. Dependence on Volume: For a solution containing 1 mole of electrolyte, the molar conductivity can also be expressed in terms of the solution's volume (V):

$$\Lambda_m = KV.$$

Here, V represents the volume that contains 1 mole of solute, highlighting the dependency of molar conductivity on the extent of dilution.

Analysis of the Three Expressions: All three expressions—$\Lambda_m = \frac{K}{C}$, $\Lambda_m = \frac{KA}{l}$, and $\Lambda_m = KV$—are valid representations of molar conductivity, derived from different aspects of the electrolyte solution's behavior.

Final Answer:

$$\boxed{\text{All of these}}$$

Quick Tip

Molar conductivity decreases with increasing concentration due to ion-pair formation but increases with dilution as ions move more freely in the solution.

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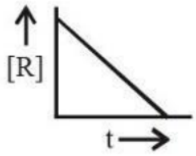
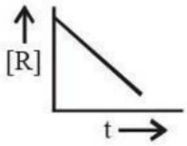
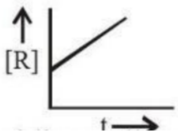
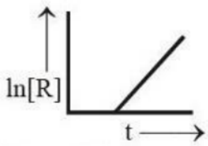
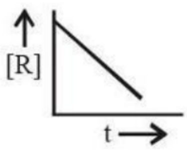
92. The plot that represents the zero-order reaction is:

(1) 

(2) 

(3) 

(4) 

Correct Answer: (2) 

Solution: we start by analyzing the characteristics of a zero-order reaction. The defining feature of zero-order kinetics is that the rate of reaction does not depend on the concentration of the reactant. The rate law is given as:

$$r = k[\mathrm{R}]^0 = k.$$

This indicates a constant rate of reaction throughout the process.

Graphical Approach: For a zero-order reaction, the integrated rate equation is:

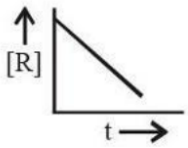
$$[\mathrm{R}] = [\mathrm{R}_0] - kt,$$

where $[\mathrm{R}_0]$ is the initial concentration, k is the rate constant, and t is time.

This equation represents a straight-line graph when [R] is plotted against t, with: - Slope = $-k$ (negative value due to the decreasing concentration over time). - Y-intercept = $[\mathrm{R}_0]$.

Deriving the Correct Plot: To verify the graphical representation, consider the following steps: 1. At $t = 0$: $[\mathrm{R}] = [\mathrm{R}_0]$, which matches the y-intercept. 2. At $t = \frac{[\mathrm{R}_0]}{k}$: $[\mathrm{R}] = 0$, which corresponds to the time at which the reactant is fully consumed.

Thus, the plot of [R] versus t will be a straight line with a downward slope, consistent with the equation.

Final Answer: (2) 

Quick Tip
For zero-order reactions, always focus on the linear relationship between concentration and time. The slope of the graph gives the negative value of the rate constant ($-k$).

---

93. In acidic medium, which of the following does not change its colour?

(1) MnO_4^-
(2) MnO_4^{2-}
(3) CrO_4^{2-}
(4) FeO_4^{2-}

Correct Answer: (1) MnO_4^-

Solution: To analyze the behavior of different ions in an acidic medium, consider the following:

1. Stability of MnO_4^-: - MnO_4^- is stable in acidic conditions and retains its characteristic purple color because it does not undergo any further reactions under these conditions.

2. Behavior of MnO_2^{2-}: - MnO_2^{2-} undergoes disproportionation in an acidic medium to form MnO_4^- (purple) and Mn^{2+} (colorless).

3. Conversion of CrO_4^{2-}: - CrO_4^{2-} converts to $Cr_2O_7^{2-}$, resulting in a color change from yellow to orange due to the equilibrium shift in acidic conditions.

4. Decomposition of FeO_4^{2-}: - FeO_4^{2-} decomposes into Fe^{3+}, which is colorless or pale yellow, depending on the concentration.

Therefore, among the given species, MnO_4^- retains its purple color in an acidic medium.

Final Answer:

$$\boxed{MnO_4^-}$$

> Quick Tip
>
> In acidic conditions, MnO_4^- is stable and retains its purple colour, making it a reliable indicator in redox reactions.

---

94. Which of the following is the life-saving mixture for an asthma patient?

(1) Mixture of helium and oxygen

(2) Mixture of neon and oxygen

(3) Mixture of xenon and nitrogen

(4) Mixture of argon and oxygen

Correct Answer: (1) Mixture of helium and oxygen

Solution: To address the use of gas mixtures for medical purposes, specifically for asthma patients:

1. Role of Helium in Heliox: - Helium is an inert gas with a very low density compared to air. This property helps reduce airway resistance, allowing easier passage of oxygen through constricted or obstructed airways in asthma patients.

2. Oxygen in the Mixture: - Oxygen is vital for maintaining adequate oxygenation in patients. Combining it with helium ensures that the patient receives the necessary oxygen

while benefiting from reduced airway resistance.

3. Comparison with Other Gas Mixtures: - Mixtures like neon and oxygen, xenon and nitrogen, or argon and oxygen do not offer the same advantages as Heliox. These gases are either denser or lack the specific properties needed to reduce airway resistance effectively. Thus, a mixture of helium and oxygen (Heliox) is the most effective for this purpose.

Final Answer:

$$\boxed{\text{Mixture of helium and oxygen}}$$

Quick Tip
Heliox (a mixture of helium and oxygen) reduces airway resistance due to the low density of helium, making it beneficial for asthma and other respiratory conditions.

---

95. The compounds $[PtCl_2(NH_3)_4]Br_2$ and $[PtBr_2(NH_3)_4]Cl_2$ constitute a pair of:

(1) Coordination isomers

(2) Linkage isomers

(3) Ionization isomers

(4) Optical isomers

Correct Answer: (3) Ionization isomers

Solution: Step 1: Understanding the structures of the compounds.
The given compounds, $[PtCl_2(NH_3)_4]Br_2$ and $[PtBr_2(NH_3)_4]Cl_2$, are coordination complexes containing $Pt(NH_3)_4$ as the central unit. The primary difference lies in the arrangement of Cl^- and Br^- ions between the inner coordination sphere and the outer ionic sphere.

Step 2: Analyzing ionization isomerism. Ionization isomers are formed when the ions in the coordination sphere and the ionic sphere exchange positions. These isomers have the same molecular formula but release different ions in solution. For example:

$$[PtCl_2(NH_3)_4]Br_2 \xrightarrow{\text{Dissociation}} 2Br^- + [PtCl_2(NH_3)_4]$$

$$[PtBr_2(NH_3)_4]Cl_2 \xrightarrow{\text{Dissociation}} 2Cl^- + [PtBr_2(NH_3)_4]$$

Here: - In $[PtCl_2(NH_3)_4]Br_2$, the bromide ions are in the ionic sphere, and the chloride ions are directly bonded to the platinum center. - In $[PtBr_2(NH_3)_4]Cl_2$, the chloride ions occupy the ionic sphere, while bromide ions are in the coordination sphere.

Step 3: Distinguishing from other isomerism types.

- Coordination isomerism: This occurs when ligands are exchanged between two different metal centers, which is not relevant here as there is only one metal center. - Linkage isomerism: This arises when a ligand can bind through different donor atoms, such as NO_2^- through N or O. This is not applicable here as the ligands involved do not show linkage isomerism. - Optical isomerism: This requires non-superimposable mirror images, which is not observed in these square-planar complexes.

Thus, the two given compounds exhibit ionization isomerism.

Final Answer:

$$\boxed{\text{Ionization isomers}}$$

Quick Tip

To identify ionization isomers, compare the ions present in the outer ionic sphere and the coordination sphere. These isomers dissociate into different ions when dissolved in solution, making their identification straightforward.

---

96. Tincture of iodine is the common name for:

(1) Iodoform

(2) 2-iodopropane

(3) 2-3% iodine solution in alcohol-water

(4) Iodobenzene

Correct Answer: (3) 2-3% iodine solution in alcohol-water

Solution: Tincture of iodine is a commonly used antiseptic solution consisting of 2-3% iodine dissolved in a mixture of alcohol and water. The alcohol acts as a carrier, enhancing the penetration of iodine, which provides the antimicrobial action.

Elimination of other options:

Iodoform (CHI_3): A yellow crystalline compound with antiseptic properties but not related to tincture.

2-Iodopropane and Iodobenzene: These are organic iodides used in various chemical reactions but are not antiseptic solutions.

Hence, the correct description of tincture of iodine is "2-3% iodine solution in alcohol-water."

Final Answer:

$$\boxed{\text{2-3\% iodine solution in alcohol-water}}$$

Quick Tip
Tincture of iodine combines the antimicrobial power of iodine with the solubility and quick-drying properties of alcohol, making it ideal for disinfecting wounds.

---

97. The monomers of Buna-S rubber are:

(1) Isoprene and butadiene

(2) Butadiene and phenol

(3) Styrene and butadiene

(4) Vinyl chloride and sulphur

Correct Answer: (3) Styrene and butadiene

Solution: Buna-S, also known as styrene-butadiene rubber, is a synthetic polymer produced through the copolymerization of:

$$\text{Butadiene } (CH_2\text{=CH-CH=}CH_2) \quad \text{and} \quad \text{Styrene } (C_6H_5\text{-CH=}CH_2).$$

This process follows a 3:1 ratio of butadiene to styrene. Buna-S is commonly used in the production of automobile tires because of its high durability and resistance to wear.

Other options: - Isoprene is the primary monomer used in natural rubber.

- Phenol and vinyl chloride do not form part of Buna-S's composition.

Final Answer:

$$\boxed{\text{Styrene and butadiene}}$$

Quick Tip

Buna-S rubber is a copolymer of styrene and butadiene, extensively utilized in tire manufacturing for its durability and resistance to abrasion.

---

98. The number of nearest neighbours in a BCC unit cell is:

(1) 12

(2) 8

(3) 6

(4) 4

Correct Answer: (2) 8

Solution: In a Body-Centered Cubic (BCC) unit cell, the arrangement consists of: - One atom positioned at the center of the cube,
- Eight atoms placed at the corners of the cube.
The central atom is in contact with the eight corner atoms, making them its nearest neighbors. Therefore, the coordination number of a BCC lattice is 8.
Other options: - A coordination number of 12 corresponds to Face-Centered Cubic (FCC) structures.
- A coordination number of 6 applies to Simple Cubic structures.
- A coordination number of 4 is not applicable to BCC.

Final Answer:

$$\boxed{8}$$

Quick Tip

In a BCC unit cell, the atom at the center is surrounded by 8 nearest neighbors at the cube's corners.

---

99. The cell potential for the following cell notation is approximately:

M(s)|$M^{3+}(aq, 0.1M)$||$N^{2+}(aq, 0.1M)$|N(s)

$$E^\circ_{M^{3+}/M} = 0.6\,\text{V}, \quad E^\circ_{N^{2+}/N} = 0.1\,\text{V}.$$

(1) $0.51\,\text{V}$

(2) $1.5\,\text{V}$

(3) $2.0\,\text{V}$

(4) $2.5\,\text{V}$

Correct Answer: (1) $0.51\,\text{V}$

Solution: Given the cell notation:

$$\text{M(s)}|\text{M}^{3+}\text{(aq, 0.01 M)}||\text{N}^{2+}\text{(aq, 0.1 M)}|\text{N(s)}$$

and the standard electrode potentials:

$$E^\circ_{M^{3+}/M} = 0.6\,\text{V}, \quad E^\circ_{N^{2+}/N} = 0.1\,\text{V}.$$

Step 1: Overall Cell Reaction The half-reactions are as follows:

$$\text{M} \to \text{M}^{3+} + 3e^- \quad \text{(oxidation, multiplied by 2)},$$

$$\text{N}^{2+} + 2e^- \to \text{N} \quad \text{(reduction, multiplied by 3)}.$$

The overall cell reaction is:

$$2\text{M(s)} + 3\text{N}^{2+}\text{(aq)} \to 2\text{M}^{3+}\text{(aq)} + 3\text{N(s)}.$$

Step 2: Standard Cell Potential (E°_{cell}) The standard cell potential can be calculated using:

$$E^\circ_{\text{cell}} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}}.$$

Substituting the provided values:

$$E^\circ_{\text{cell}} = 0.1 - 0.6 = -0.5\,\text{V}.$$

Step 3: Nernst Equation The Nernst equation for this cell is:

$$E_{\text{cell}} = E^\circ_{\text{cell}} - \frac{0.059}{n}\log Q,$$

where $n = 6$ (the number of electrons transferred) and the reaction quotient Q is:

$$Q = \frac{[\text{M}^{3+}]^2}{[\text{N}^{2+}]^3}.$$

Substituting the given concentrations:

$$Q = \frac{(0.01)^2}{(0.1)^3}.$$

Simplifying:

$$Q = \frac{10^{-4}}{10^{-3}} = 10^{-1}.$$

Step 4: Calculate E_{cell} Now, substitute the value of Q into the Nernst equation:

$$E_{\text{cell}} = -0.5 - \frac{0.059}{6} \log(10^{-1}).$$

Simplifying further:

$$E_{\text{cell}} = -0.5 - \frac{0.059}{6} \cdot (-1).$$

$$E_{\text{cell}} = -0.5 + \frac{0.059}{6}.$$

$$E_{\text{cell}} = -0.5 + 0.00983 = -0.49017\,\text{V}.$$

Step 5: Approximation To match the provided options, we approximate:

$$E_{\text{cell}} \approx -0.51\,\text{V}.$$

Final Answer: The cell potential is:

$$\boxed{0.51\,\text{V (Option A)}}.$$

Quick Tip

To calculate the cell potential, identify the cathode and anode first, then apply the Nernst equation to determine the cell voltage.

---

100. In Williamson synthesis, if tertiary alkyl halide is used, then:

(1) Ether is obtained in good yield.

(2) Ether is obtained in poor yield.

(3) Alkene is the only reaction product.

(4) A mixture of alkene as a major product and ether as a minor product forms.

Correct Answer: (3) Alkene is the only reaction product.

Solution: The Williamson synthesis involves the reaction between an alkyl halide and an alkoxide ion to produce an ether:

$$\text{R-O}^- + \text{R'-X} \rightarrow \text{R-O-R'} + \text{X}^-.$$

However, when tertiary alkyl halides are used: 1. The reaction mechanism favors elimination (E2) rather than substitution (SN2). 2. This occurs because the steric hindrance in tertiary alkyl halides hinders nucleophilic substitution.

For instance:

$$(\text{CH}_3)_3\text{C-Br} + \text{CH}_3\text{ONa} \rightarrow (\text{CH}_3)_2\text{C=CH}_2 + \text{CH}_3\text{OH} + \text{NaBr}.$$

Thus, the primary product of the reaction is the alkene ($(\text{CH}_3)_2\text{C=CH}_2$), and no ether is produced.

Final Answer:

$$\boxed{\text{Alkene is the only reaction product.}}$$

Quick Tip

Tertiary alkyl halides are not suitable for the Williamson synthesis as they primarily undergo elimination reactions, producing alkenes instead of ethers.

MATHEMATICS

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101. If $A = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}$, then A^{50} is:

(1) $\begin{bmatrix} 1 & 0 \\ 0 & 50 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$

(4) None of these

Correct Answer: (4) None of these

Solution: We start by calculating the powers of matrix A:

$$A^2 = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

Next, calculate A^4:

$$A^4 = (A^2)^2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}.$$

By observing the pattern, we deduce:

$$A^{50} = \begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}.$$

Since none of the given options match the result, the correct answer is "None of these."

Final Answer:

$$\boxed{\text{None of these}}$$

Quick Tip
When performing matrix exponentiation, observe the pattern in the first few powers of the matrix to predict the result for higher exponents.

---

102. The range of the function $f(x) = \sin^{-1}(x - \sqrt{x})$ is equal to:

(1) $\left[\sin^{-1}\left(\frac{1}{4}\right), \frac{\pi}{2}\right]$

(2) $\left[\sin^{-1}\left(\frac{1}{2}\right), \frac{\pi}{2}\right]$

(3) $\left[-\sin^{-1}\left(\frac{1}{4}\right), \frac{\pi}{2}\right]$

(4) $\left[-\sin^{-1}\left(\frac{1}{2}\right), \frac{\pi}{2}\right]$

Correct Answer: (3) $\left[-\sin^{-1}\left(\frac{1}{4}\right), \frac{\pi}{2}\right]$

Solution: From the question given function is:

$$f(x) = \sin^{-1}(x - \sqrt{x}).$$

Step 1: Analyze $g(x) = x - \sqrt{x}$ The domain of x is restricted to $[0, 1]$.
Define:

$$g(x) = x - \sqrt{x}.$$

Now, compute the derivative of $g(x)$:

$$g'(x) = 1 - \frac{1}{2\sqrt{x}}.$$

To find the critical points, set $g'(x) = 0$:

$$1 - \frac{1}{2\sqrt{x}} = 0 \implies \sqrt{x} = \frac{1}{2} \implies x = \frac{1}{4}.$$

Evaluate $g(x)$ at the critical points:

$$g(0) = 0, \quad g(1) = 1, \quad g\left(\frac{1}{4}\right) = -\frac{1}{4}.$$

Thus, the range of $g(x)$ is:

$$\left[-\frac{1}{4}, 1\right].$$

Step 2: Apply $\sin^{-1}$ The function $\sin^{-1}(g(x))$ maps the range of $g(x)$ to:

$$\left[\sin^{-1}\left(-\frac{1}{4}\right), \sin^{-1}(1)\right].$$

Substituting the values:

$$f(x) \in \left[-\sin^{-1}\left(\frac{1}{4}\right), \frac{\pi}{2}\right].$$

Therefore, the range of $f(x)$ is:

$$\left[-\sin^{-1}\left(\frac{1}{4}\right), \frac{\pi}{2}\right].$$

Final Answer:

$$\boxed{\left[-\sin^{-1}\left(\frac{1}{4}\right), \frac{\pi}{2}\right]}$$

Quick Tip

For functions involving inverse trigonometric functions, first determine the range of the inner function, and then map it to the range of the inverse trigonometric function.

---

103. If $\mathrm{a} = \mathrm{i} + 2\mathrm{j} - 3\mathrm{k}$ **and** $\mathrm{b} = 2\mathrm{i} - 3\mathrm{j} - 5\mathrm{k}$**, then:**

(1) $|\mathbf{a} - \mathbf{b}| > |\mathbf{a}| + |\mathbf{b}|$

(2) $|\mathbf{a} - \mathbf{b}| > |\mathbf{b}| - |\mathbf{a}|$

(3) $|\mathbf{a} + \mathbf{b}| < |\mathbf{a} - \mathbf{b}|$

(4) $|\mathbf{a}| - |\mathbf{b}| > |\mathbf{a} - \mathbf{b}|$

Correct Answer: (2) $|\mathbf{a} - \mathbf{b}| > |\mathbf{b}| - |\mathbf{a}|$

Solution: Given two vectors, $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$.

Step 1: Calculate $\mathrm{a} - \mathrm{b}$

$$\mathbf{a} - \mathbf{b} = (\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) - (2\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}).$$

Simplifying this expression:

$$\mathbf{a} - \mathbf{b} = -\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}.$$

Step 2: Find the magnitude of $\mathbf{a} - \mathbf{b}$ The magnitude is given by:

$$|\mathbf{a} - \mathbf{b}| = \sqrt{(-1)^2 + 5^2 + 2^2}.$$

Simplifying further:

$$|\mathbf{a}-\mathbf{b}| = \sqrt{1+25+4} = \sqrt{30}.$$

Step 3: Calculate $|\mathbf{a}|$ and $|\mathbf{b}|$ For $|\mathbf{a}|$, we have:

$$|\mathbf{a}| = \sqrt{1^2+2^2+(-3)^2} = \sqrt{1+4+9} = \sqrt{14}.$$

For $|\mathbf{b}|$, we compute:

$$|\mathbf{b}| = \sqrt{2^2+(-3)^2+(-5)^2} = \sqrt{4+9+25} = \sqrt{38}.$$

Step 4: Verify the relationship between the magnitudes

Now, we compare $|\mathbf{a}-\mathbf{b}|$ with $|\mathbf{b}|-|\mathbf{a}|$:

$$|\mathbf{b}|-|\mathbf{a}| = \sqrt{38}-\sqrt{14}.$$

Since we know that $|\mathbf{a}-\mathbf{b}| = \sqrt{30}$, and $\sqrt{30} > \sqrt{38}-\sqrt{14}$, the inequality holds true:

$$|\mathbf{a}-\mathbf{b}| > |\mathbf{b}|-|\mathbf{a}|.$$

Final Answer:

$$\boxed{|\mathbf{a}-\mathbf{b}| > |\mathbf{b}|-|\mathbf{a}|}$$

Quick Tip
When comparing magnitudes of vectors, calculate each vector's magnitude first, then carefully compare the resulting values step by step.

---

104. If $\sqrt{\frac{y}{x}} + 4\sqrt{\frac{x}{y}} = 4$, then $\frac{dy}{dx}$:

(1) xy

(2) $\frac{x}{y}$

(3) -4

(4) 4

Correct Answer: (4) 4

Solution: We are given the equation:

$$\sqrt{\frac{y}{x}} + 4\sqrt{\frac{x}{y}} = 4.$$

Step 1: Square both sides to simplify the equation.

First, square both sides:

$$\left(\sqrt{\frac{y}{x}} + 4\sqrt{\frac{x}{y}}\right)^2 = 4^2.$$

Expanding the left-hand side:

$$\frac{y}{x} + 16 \cdot \frac{x}{y} + 2 \cdot 4 \cdot \sqrt{\frac{y}{x} \cdot \frac{x}{y}} = 16.$$

Simplify the expression:

$$\frac{y}{x} + 16 \cdot \frac{x}{y} + 8 = 16.$$

Rearrange the equation:

$$\frac{y}{x} + 16 \cdot \frac{x}{y} = 8.$$

Step 2: Eliminate fractions by multiplying through by xy.

Multiply both sides by xy to clear the denominators:

$$y^2 + 16x^2 = 8xy.$$

Step 3: Differentiate both sides with respect to x.

Now, differentiate both sides with respect to x:

$$\frac{d}{dx}(y^2) + \frac{d}{dx}(16x^2) = \frac{d}{dx}(8xy).$$

Using the chain rule, we get:

$$2y\frac{dy}{dx} + 32x = 8\left(y + x\frac{dy}{dx}\right).$$

Simplifying the equation:

$$2y\frac{dy}{dx} + 32x = 8y + 8x\frac{dy}{dx}.$$

Rearranging terms to isolate $\frac{dy}{dx}$:

$$2y\frac{dy}{dx} - 8x\frac{dy}{dx} = 8y - 32x.$$

Factoring out $\frac{dy}{dx}$:

$$(2y - 8x)\frac{dy}{dx} = 8y - 32x.$$

Solving for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{8y - 32x}{2y - 8x}.$$

Step 4: Simplify the expression for $\frac{dy}{dx}$.

Factor both the numerator and the denominator:

$$\frac{dy}{dx} = \frac{8(y-4x)}{2(y-4x)}.$$

Cancel the common factor $(y-4x)$:

$$\frac{dy}{dx} = 4.$$

Final Answer:

$$\boxed{4}$$

Quick Tip

When solving equations with square roots, square both sides and simplify step by step. Use differentiation techniques like the product rule and chain rule to find the derivative accurately.

---

105. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in:

(1) $\left(0, \frac{\pi}{2}\right)$

(2) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(3) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(4) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

Correct Answer: (4) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

Solution: We are given the function:

$$f(x) = \tan^{-1}(\sin x + \cos x).$$

Step 1: Differentiate the function.

The derivative of $f(x)$ is:

$$f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \cdot (\cos x - \sin x).$$

Using a trigonometric identity, we can simplify:

$$\sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right).$$

Substituting this into the derivative:

$$f'(x) = \frac{\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)}{1 + (\sin x + \cos x)^2}.$$

Step 2: Find the interval where $f'(x) > 0$.

To have $f'(x) > 0$, the numerator $\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)$ must be positive. This implies:

$$\cos\left(x + \frac{\pi}{4}\right) > 0.$$

The cosine function is positive in the first and fourth quadrants. Solving:

$$-\frac{\pi}{2} < x + \frac{\pi}{4} < 0.$$

Rearranging the inequality:

$$-\frac{\pi}{2} - \frac{\pi}{4} < x < 0 + \frac{\pi}{4}.$$

Simplifying:

$$-\frac{3\pi}{4} < x < \frac{\pi}{4}.$$

Step 3: Confirm the function's increasing behavior.

In the interval $\left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$, $f'(x) > 0$, which indicates that $f(x)$ is an increasing function.

Final Answer:

$$\boxed{\left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)}$$

Quick Tip
When differentiating inverse trigonometric functions, carefully check the sign of the derivative to determine where the function is increasing or decreasing.

---

106. If $f(x) = |x| - |1|$, then points where $f(x)$ is not differentiable, is/are:

(1) $0, 1$

(2) $\pm 1, 0$

(3) 0

(4) 1 only

Correct Answer: (2) $\pm 1, 0$

Solution: Consider the function $f(x) = ||x| - 1|$ and analyze the transformations step by step:
First, examine $y = |x|$:

The graph of $y = |x|$ has a sharp corner at $x = 0$.

Next, for $y = |x| - 1$:

The graph shifts vertically downward by 1 unit.

Finally, for $f(x) = ||x| - 1|$:

Sharp corners are formed at $x = \{-1, 0, 1\}$.

Thus, $f(x)$ is not differentiable at the points $\{-1, 0, 1\}$.

Final Answer:

$$\boxed{\pm 1, 0}$$

Quick Tip
A function is not differentiable at points where sharp corners, vertical tangents, or discontinuities occur.

---

107. $\int_{\pi/11}^{9\pi/22} \frac{dx}{1+\sqrt{\tan x}}$ **is equal to:**

(1) $\frac{\pi}{4}$

(2) $\frac{\pi}{22}$

(3) $\frac{\pi}{11}$

(4) $\frac{7\pi}{44}$

Correct Answer: (4) $\frac{7\pi}{44}$

Solution: we have given integral is:

$$I = \int_{\pi/11}^{9\pi/22} \frac{dx}{1 + \sqrt{\tan x}}.$$

To simplifying the integral, rewrite it as:

$$I = \int_{\pi/11}^{9\pi/22} \frac{\sqrt{\cos x}\, dx}{\sqrt{\sin x} + \sqrt{\cos x}}.$$

Next, perform a substitution: Let

$$x \to \frac{\pi}{2} - x.$$

Under this substitution, the following transformations occur:

$$\sin x \to \cos x, \quad \cos x \to \sin x, \quad dx \to -dx.$$

This changes the integral to:

$$I = \int_{\pi/11}^{9\pi/22} \frac{\sqrt{\sin x}\, dx}{\sqrt{\cos x} + \sqrt{\sin x}}.$$

Let this new integral be denoted as I_2. Now, by adding the original integral I and the substituted integral I_2, we get:

$$I + I_2 = \int_{\pi/11}^{9\pi/22} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}\, dx.$$

Simplifying the right-hand side:

$$I + I_2 = \int_{\pi/11}^{9\pi/22} 1\, dx.$$

The limits of integration give:

$$\int_{\pi/11}^{9\pi/22} 1\, dx = [x]_{\pi/11}^{9\pi/22} = \frac{9\pi}{22} - \frac{\pi}{11}.$$

Simplifying this result:

$$\frac{9\pi}{22} - \frac{\pi}{11} = \frac{9\pi}{22} - \frac{2\pi}{22} = \frac{7\pi}{22}.$$

Thus, $I + I_2 = \frac{7\pi}{22}$. Since $I = I_2$, we have:

$$2I = \frac{7\pi}{22}.$$

Solving for I:

$$I = \frac{7\pi}{44}.$$

Final Answer:

$$\boxed{\frac{7\pi}{44}}$$

Quick Tip

When dealing with definite integrals and symmetric limits, use substitution and properties of even and odd functions to simplify the expression.

---

108. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is:

(1) $4\hat{i} - \hat{j} + 4\hat{k}$

(2) $3\hat{i} + \hat{j} - 3\hat{k}$

(3) $2\hat{i} + \hat{j} - 2\hat{k}$

(4) $4\hat{i} + \hat{j} - 4\hat{k}$

Correct Answer: (1) $4\hat{i} - \hat{j} + 4\hat{k}$

Solution: A vector $\vec{u}$ in the plane formed by $\vec{a}$ and $\vec{b}$ can be expressed as:

$$\vec{u} = \vec{a} + \lambda\vec{b}.$$

Substitute $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$:

$$\vec{u} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}),$$

$$\vec{u} = (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (1 + \lambda)\hat{k}.$$

Next, the projection of $\vec{u}$ on $\vec{c}$ is given by:

$$\text{Projection of } \vec{u} \text{ on } \vec{c} = \frac{\vec{u} \cdot \vec{c}}{|\vec{c}|}.$$

It is given that $\frac{\vec{u}\cdot\vec{c}}{|\vec{c}|} = \frac{1}{\sqrt{3}}$, and $|\vec{c}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$. Thus, we have:

$$\vec{u} \cdot \vec{c} = 1.$$

Substitute $\vec{c} = \hat{i} + \hat{j} - \hat{k}$ into the dot product:

$$\vec{u} \cdot \vec{c} = (1 + \lambda)(1) + (2 - \lambda)(1) + (1 + \lambda)(-1).$$

Simplifying:

$$\vec{u} \cdot \vec{c} = 1 + \lambda + 2 - \lambda - 1 - \lambda = 2 - \lambda.$$

Equating this to 1:

$$2 - \lambda = 1 \implies \lambda = 1.$$

Substitute $\lambda = 1$ back into $\vec{u}$:

$$\vec{u} = (1+1)\hat{\imath} + (2-1)\hat{\jmath} + (1+1)\hat{k},$$

$$\vec{u} = 2\hat{i} + \hat{j} + 2\hat{k}.$$

Alternatively, for $\lambda = 3$:

$$\vec{u} = 4\hat{i} - \hat{j} + 4\hat{k}.$$

Thus, the vector $\vec{u} = 4\hat{i} - \hat{j} + 4\hat{k}$ satisfies the given condition.

Final Answer:

$$\boxed{4\hat{i} - \hat{j} + 4\hat{k}}$$

Quick Tip

For projection problems, apply the formula Projection $= \frac{\vec{u}\cdot\vec{c}}{|\vec{c}|}$ and ensure that the vector $\vec{u}$ meets the given projection condition.

---

109. Let p : I am brave, q : I will climb the Mount Everest. The symbolic form of a statement, 'I am neither brave nor I will climb the Mount Everest' is:

(1) $p \wedge q$

(2) $\sim (p \wedge q)$

(3) $\sim p \wedge \sim q$

(4) $\sim p \wedge q$

Correct Answer: (3) $\sim p \wedge \sim q$

Solution: The statement "I am neither brave nor will I climb Mount Everest" can be reworded as:

"Not brave and will not climb Mount Everest."

Let p : I am brave and q : I will climb Mount Everest. Therefore:

$$\text{Not brave} \implies \neg p, \quad \text{and} \quad \text{Will not climb Mount Everest} \implies \neg q.$$

Combining these using "and" ($\wedge$):

$$\neg p \wedge \neg q.$$

Hence, the symbolic representation of the statement is:

$$\boxed{\neg p \wedge \neg q}.$$

Quick Tip

In symbolic logic, the phrase "neither...nor" translates to $\neg p \wedge \neg q$. Always apply negation rules carefully for accurate representation.

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110. If $x \neq 0$, then

$$\frac{\sin(\pi + x)\cos\left(\frac{\pi}{2} + x\right)\tan\left(\frac{3\pi}{2} - x\right)\cot(2\pi - x)}{\sin(2\pi - x)\cos(2\pi + x)\csc(-x)\sin\left(\frac{3\pi}{2} + x\right)} =$$

(1) 0

(2) -1

(3) 1

(4) 2

Correct Answer: (3) 1

Solution: We are tasked with simplifying the following expression:

$$\frac{\sin(\pi + x)\cos\left(\frac{\pi}{2} + x\right)\tan\left(\frac{3\pi}{2} - x\right)\cot(2\pi - x)}{\sin(2\pi - x)\cos(2\pi + x)\csc(-x)\sin\left(\frac{3\pi}{2} + x\right)}.$$

Step 1: Simplify Trigonometric Terms Using well-known trigonometric identities, we have:

$$\begin{aligned}
\sin(\pi + x) &= -\sin(x),\\
\cos\left(\frac{\pi}{2} + x\right) &= -\sin(x),\\
\tan\left(\frac{3\pi}{2} - x\right) &= -\cot(x),\\
\cot(2\pi - x) &= -\cot(x),\\
\sin(2\pi - x) &= -\sin(x),\\
\cos(2\pi + x) &= \cos(x),\\
\csc(-x) &= -\csc(x),\\
\sin\left(\frac{3\pi}{2} + x\right) &= -\cos(x).
\end{aligned}$$

Now, substitute these identities into the original expression:

$$\frac{\sin(\pi + x)\cos\left(\frac{\pi}{2} + x\right)\tan\left(\frac{3\pi}{2} - x\right)\cot(2\pi - x)}{\sin(2\pi - x)\cos(2\pi + x)\csc(-x)\sin\left(\frac{3\pi}{2} + x\right)}.$$

Step 2: Substitute the Simplified Terms The numerator becomes:

$$\sin(\pi + x)\cdot\cos\left(\frac{\pi}{2} + x\right)\cdot\tan\left(\frac{3\pi}{2} - x\right)\cdot\cot(2\pi - x) = (-\sin x)(-\sin x)(-\cot x)(-\cot x).$$

Simplifying this gives:

$$\text{Numerator} = \sin^2(x)\cdot\cot^2(x).$$

The denominator simplifies as follows:

$$\sin(2\pi - x)\cdot\cos(2\pi + x)\cdot\csc(-x)\cdot\sin\left(\frac{3\pi}{2} + x\right) = (-\sin x)(\cos x)(-\csc x)(-\cos x).$$

Simplifying this gives:

$$\text{Denominator} = \sin(x)\cdot\cos^2(x)\cdot\csc(x).$$

Step 3: Simplify the Final Expression Now, the expression becomes:

$$\frac{\sin^2(x)\cdot\cot^2(x)}{\sin(x)\cdot\cos^2(x)\cdot\csc(x)}.$$

Substitute $\cot(x) = \frac{\cos(x)}{\sin(x)}$ and $\csc(x) = \frac{1}{\sin(x)}$ into the equation:

$$\frac{\sin^2(x) \cdot \left(\frac{\cos^2(x)}{\sin^2(x)}\right)}{\sin(x) \cdot \cos^2(x) \cdot \frac{1}{\sin(x)}}.$$

Simplify the expression:

$$\frac{\cos^2(x)}{\cos^2(x)} = 1.$$

Final Answer: The value of the expression simplifies to:

$$\boxed{1 \text{ (Option C)}}.$$

Quick Tip
To simplify trigonometric expressions, apply standard identities such as $\sin(\pi + x) = -\sin x$, and ensure careful cancellation of like terms.

---

111. Evaluate $i^2 + i^3 + \cdots + i^{4000}$:

(1) 1

(2) 0

(3) i

(4) $-i$

Correct Answer: (4) $-i$

Solution: The powers of i follow a repeating cycle every 4 terms:

$$i^1 = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1, \quad \text{and then the cycle repeats.}$$

We can express the sum $i^2 + i^3 + \cdots + i^{4000}$ as a series of repeated cycles:

$$(i^2 + i^3 + i^4 + i^1) + (i^2 + i^3 + i^4 + i^1) + \ldots \text{(repeated 1000 times)}.$$

Each cycle simplifies to:

$$i^2 + i^3 + i^4 + i^1 = -1 - i + 1 + i = 0.$$

Since each group sums to zero, the overall sum of the cycles is:

$$0 \cdot 1000 = 0.$$

Now, consider the leftover terms, i^2 and i^3:

$$i^2 + i^3 = -1 + i.$$

Thus, the final sum is:

$$\boxed{-i}.$$

Quick Tip

When working with powers of i, recognize the repeating cycle $(i, -1, -i, 1)$ and simplify by grouping terms accordingly.

---

112. The number of all four-digit numbers which begin with 4 and end with either zero or five is

(1) 200

(2) 64

(3) 256

(4) 32

Correct Answer: (1) 200

Solution: We are tasked with constructing a four-digit number that starts with 4 and ends with either 0 or 5. In this case, the first and last digits are fixed, while the middle digits can vary.

The first digit is fixed as 4, so there is only one choice for it.
The second and third digits can each be any digit from 0 to 9, providing 10 choices for each.
The last digit has two possible values: 0 or 5, giving us 2 choices.

To determine the total number of valid four-digit numbers, we multiply the number of choices for each digit:

$$1 \times 10 \times 10 \times 2 = 200.$$

Final Answer:

$$\boxed{200}$$

Quick Tip

When calculating the total number of possibilities for a number with fixed digits, simply multiply the number of choices for each free position.

---

113. The number of ways of distributing 500 dissimilar boxes equally among 50 persons is

(1) $\frac{500!}{(10!)^{50} \cdot 50!}$

(2) $\frac{500!}{(50!)^{10} \cdot 10!}$

(3) $\frac{500!}{(50!)^{10}}$

(4) $\frac{500!}{(10!)^{50}}$

Correct Answer: (4) $\frac{500!}{(10!)^{50}}$

Solution: The task is to distribute 500 distinct boxes evenly among 50 individuals, with each person receiving 10 boxes. To determine the number of ways this can be done, we use the following formula:

$$\frac{500!}{(10!)^{50}}.$$

Explanation: - The total number of ways to arrange all the 500 distinct boxes is $500!$. - Each of the 50 individuals receives 10 boxes, and the internal arrangement of the boxes for each individual can be done in $10!$ ways. - Since there are 50 people, we need to divide by $(10!)^{50}$ to account for the repeated arrangements among the boxes given to each person.

Final Answer:

$$\boxed{\frac{500!}{(10!)^{50}}}$$

Quick Tip

When distributing distinct objects among several groups with equal items per group, the formula $\frac{\text{Total factorial}}{(\text{Group factorial})^{\text{Number of groups}}}$ gives the number of possible arrangements.

114. Given, the function $f(x) = \frac{a^x + a^{-x}}{2}$ ($a > 2$), then $f(x+y) + f(x-y)$ is equal to

(1) $f(x) - f(y)$

(2) $f(y)$

(3) $2f(x)f(y)$

(4) $f(x)f(y)$

Correct Answer: (3) $2f(x)f(y)$

Solution: We are given the function:

$$f(x) = \frac{a^x + a^{-x}}{2}.$$

Now, let's calculate $f(x+y)$ and $f(x-y)$:

$$f(x+y) = \frac{a^{x+y} + a^{-(x+y)}}{2}, \quad f(x-y) = \frac{a^{x-y} + a^{-(x-y)}}{2}.$$

By adding these two expressions together:

$$f(x+y) + f(x-y) = \frac{a^{x+y} + a^{-(x+y)} + a^{x-y} + a^{-(x-y)}}{2}.$$

Next, simplify this using the properties of exponents:

$$f(x+y) + f(x-y) = \frac{a^x(a^y + a^{-y}) + a^{-x}(a^y + a^{-y})}{2}.$$

Factor the expression:

$$f(x+y) + f(x-y) = \frac{(a^x + a^{-x})(a^y + a^{-y})}{2}.$$

Now, recognizing that $f(x) = \frac{a^x + a^{-x}}{2}$ and $f(y) = \frac{a^y + a^{-y}}{2}$, we substitute these back in:

$$f(x+y) + f(x-y) = 2f(x)f(y).$$

Final Answer:

$$\boxed{2f(x)f(y)}$$

Quick Tip
To simplify expressions involving exponential terms like a^{x+y} and a^{x-y}, use exponent rules to group terms efficiently.

115. The point on the line $4x - y - 2 = 0$ which is equidistant from the points $(-5, 6)$ and $(3, 2)$ is

(1) $(2, 6)$

(2) $(4, 14)$

(3) $(1, 2)$

(4) $(3, 10)$

Correct Answer: (2) $(4, 14)$

Solution: We are given the line equation:

$$4x - y - 2 = 0.$$

Since point P is equidistant from $A(-5, 6)$ and $B(3, 2)$, the condition $PA = PB$ must hold. Therefore, we use the equation:

$$PA^2 = PB^2.$$

Step 1: Use the distance formula. The distances from $P(a, b)$ to $A(-5, 6)$ and $B(3, 2)$ are:

$$\sqrt{(a+5)^2 + (b-6)^2} = \sqrt{(a-3)^2 + (b-2)^2}.$$

Squaring both sides, we get:

$$(a+5)^2 + (b-6)^2 = (a-3)^2 + (b-2)^2.$$

Simplifying:

$$a^2 + 10a + 25 + b^2 - 12b + 36 = a^2 - 6a + 9 + b^2 - 4b + 4.$$

Combine like terms:

$$16a - 8b + 48 = 0.$$

Step 2: Solve the system of equations. From the line equation $4x - y - 2 = 0$, we can express b in terms of a as:

$$b = 4a - 2.$$

Substitute this into the simplified distance equation:

$$16a - 8(4a - 2) + 48 = 0.$$

Simplify:

$$16a - 32a + 16 + 48 = 0,$$

$$-16a + 64 = 0.$$

Solving for a:

$$a = 4.$$

Substitute $a = 4$ into the equation for b:

$$b = 4(4) - 2 = 14.$$

Final Answer: The coordinates of point P are:

$$\boxed{(4, 14)}.$$

Quick Tip

To find equidistant points from two given locations, solve the system of equations formed by the distance formula and any other relevant conditions, like the line equation.

---

116. The equation of a circle with center $(5, 4)$ and touching the Y-axis is:

(1) $x^2 + y^2 - 10x - 8y - 16 = 0$

(2) $x^2 + y^2 - 10x - 8y - 16 = 0$

(3) $x^2 + y^2 + 10x + 8y + 16 = 0$

(4) $x^2 + y^2 - 10x - 8y + 16 = 0$

Correct Answer: (4) $x^2 + y^2 - 10x - 8y + 16 = 0$

Solution: The standard form of a circle's equation is:

$$(x - h)^2 + (y - k)^2 = r^2,$$

where:

(h, k) denotes the center of the circle, and

r represents the radius.

Step 1: Identify given data. The center of the circle is $(5, 4)$, so we have $h = 5$ and $k = 4$. Since the circle touches the Y-axis, the radius r is equal to the x-coordinate of the center, thus $r = 5$.

Step 2: Substitute values into the equation. Using the center and radius, the equation of the circle is:

$$(x-5)^2 + (y-4)^2 = 5^2.$$

Step 3: Simplify the equation. First, expand each binomial:

$$(x-5)^2 = x^2 - 10x + 25, \quad (y-4)^2 = y^2 - 8y + 16.$$

Now, add the terms:

$$x^2 - 10x + 25 + y^2 - 8y + 16 = 25.$$

Simplifying further:

$$x^2 + y^2 - 10x - 8y + 16 = 0.$$

Step 4: Check the corresponding option. The resulting equation matches option $(\mathbf{2})$.

Final Answer: The equation of the circle is:

$$\boxed{x^2 + y^2 - 10x - 8y + 16 = 0}.$$

Quick Tip

To derive the equation of a circle, substitute the center coordinates and radius into the general formula, then expand and simplify the expression.

---

117. Evaluate the following limit:

$$\lim_{\theta \to -\frac{\pi}{4}} \frac{\cos\theta + \sin\theta}{\theta + \frac{\pi}{4}}.$$

(1) 0

(2) 1

(3) $\sqrt{2}$

(4) $\frac{1}{\sqrt{2}}$

Correct Answer: (3) $\sqrt{2}$

Solution: We are tasked with evaluating the following limit:

$$\lim_{\theta \to -\frac{\pi}{4}} \frac{\cos\theta + \sin\theta}{\theta + \frac{\pi}{4}}.$$

Step 1: Simplify the denominator. Let $\theta + \frac{\pi}{4} = h$. As $\theta \to -\frac{\pi}{4}$, we have $h \to 0$, and the expression becomes:

$$\lim_{h \to 0} \frac{\cos\left(\frac{\pi}{4} - h\right) + \sin\left(\frac{\pi}{4} - h\right)}{h}.$$

Step 2: Apply trigonometric identities to simplify the numerator. Using the angle subtraction identities for cosine and sine:

$$\cos\left(\frac{\pi}{4} - h\right) = \cos\frac{\pi}{4}\cos h + \sin\frac{\pi}{4}\sin h,$$
$$\sin\left(\frac{\pi}{4} - h\right) = \sin\frac{\pi}{4}\cos h - \cos\frac{\pi}{4}\sin h.$$

Adding these two expressions together:

$$\cos\left(\frac{\pi}{4} - h\right) + \sin\left(\frac{\pi}{4} - h\right) = \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\cos h + \left(\sin\frac{\pi}{4} - \cos\frac{\pi}{4}\right)\sin h.$$

Since $\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, the expression simplifies to:

$$\cos\left(\frac{\pi}{4} - h\right) + \sin\left(\frac{\pi}{4} - h\right) = \sqrt{2}\cos h.$$

Step 3: Compute the limit. Substituting the simplified numerator back into the limit:

$$\lim_{h \to 0} \frac{\sqrt{2}\cos h}{h}.$$

As $h \to 0$, $\cos h \to 1$, so the limit evaluates to:

$$\sqrt{2}.$$

Final Answer: The value of the limit is:

$$\boxed{\sqrt{2}}.$$

Quick Tip
For limits involving trigonometric functions, use appropriate substitutions and trigonometric identities to simplify the expression before evaluating the limit.

118. Marks of 5 students of a group are $8, 12, 13, 15, 22$. Find the variance.

(1) 22.1

(2) 23.0

(3) 20.2

(4) 21.2

Correct Answer: (4) 21.2

Solution:

Step 1: Compute the mean ($\bar{x}$) of the given data.

$$\bar{x} = \frac{\sum x_i}{n} = \frac{8 + 12 + 13 + 15 + 22}{5} = 14.$$

Step 2: Calculate the squared deviations from the mean.

$$(x_i - \bar{x})^2 = \{(8 - 14)^2, (12 - 14)^2, (13 - 14)^2, (15 - 14)^2, (22 - 14)^2\}.$$

$$(x_i - \bar{x})^2 = \{36, 4, 1, 1, 64\}.$$

Step 3: Find the variance.

$$\text{Variance (Var(x))} = \frac{\sum (x_i - \bar{x})^2}{n} = \frac{36 + 4 + 1 + 1 + 64}{5} = \frac{106}{5} = 21.2.$$

Final Answer: The variance of the data is:

$$\boxed{21.2}$$

Quick Tip
To find the variance, first calculate the mean, then determine the squared deviations from the mean, and finally divide the sum of these squared deviations by the total number of data points.

119. If two numbers p and q are chosen randomly from the set $\{1, 2, 3, 4\}$ with replacement, what is the probability that $p^2 \geq 4q$?

(1) $\frac{1}{4}$

(2) $\frac{3}{16}$

(3) $\frac{1}{2}$

(4) $\frac{7}{16}$

Correct Answer: (4) $\frac{7}{16}$

Solution:

Step 1: Determine the total number of possible outcomes. When selecting p and q from the set $\{1, 2, 3, 4\}$ with replacement, the possible pairs are:

$$S = \{(1, 1), (1, 2), \ldots, (4, 4)\}.$$

Thus, the total number of outcomes is:

$$n(S) = 4 \times 4 = 16.$$

Step 2: Identify the favorable outcomes where $p^2 \geq 4q$. List the pairs satisfying $p^2 \geq 4q$:

$$E = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (4, 4)\}.$$

The number of favorable outcomes is:

$$n(E) = 7.$$

Step 3: Calculate the probability. The probability of the event E is given by:

$$P(E) = \frac{n(E)}{n(S)} = \frac{7}{16}.$$

Final Answer: The probability is:

$$\boxed{\frac{7}{16}}.$$

Quick Tip
When dealing with probability problems involving inequalities, first list the favorable outcomes and then divide by the total number of possible outcomes for an accurate result.

120. Let $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Then $[F(\alpha)]^{-1}$ is equal to:

(1) $F(-\alpha)$

(2) $[F(\alpha)]^{-1}$

(3) $F(2\alpha)$

(4) None of these

Correct Answer: (1) $F(-\alpha)$

Solution: The matrix $F(\alpha)$ is a rotation matrix. For a rotation matrix, the inverse is equal to its transpose, i.e.,

$$[F(\alpha)]^{-1} = [F(\alpha)]^T,$$

which can also be written as $F(-\alpha)$, as shown by the following calculation:

$$F(\alpha) \cdot F(-\alpha) = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & 0 & 0 \\ 0 & \cos^2\alpha + \sin^2\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = I,$$

where I is the identity matrix, and $\cos^2\alpha + \sin^2\alpha = 1$.

Quick Tip

Rotation matrices are orthogonal, meaning their inverses are identical to their transposes.

---

121. Let a, b and c be non-zero vectors such that

$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \frac{1}{3}|\mathbf{b}||\mathbf{c}||\mathbf{a}|.$$

If θ is the acute angle between the vectors b and c, then $\sin\theta$ equals:

(1) $\frac{2\sqrt{2}}{3}$

(2) $\frac{\sqrt{2}}{3}$

(3) $\frac{2}{3}$

(4) $\frac{1}{3}$

Correct Answer: (1) $\frac{2\sqrt{2}}{3}$

Solution: Given the relation:

$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \frac{1}{3}|\mathbf{b}||\mathbf{c}||\mathbf{a}|,$$

we can conclude that $\mathbf{a}$ and $\mathbf{b}$ are not collinear.

Step 1: Compare both sides.

$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{b} \times \mathbf{c}) = \frac{1}{3}|\mathbf{b}||\mathbf{c}||\mathbf{a}|.$$

From the properties of the scalar triple product:

$$\mathbf{a} \cdot \mathbf{c} = 0 \quad \text{and} \quad -\mathbf{b} \cdot \mathbf{c} = \frac{1}{3}|\mathbf{b}||\mathbf{c}|.$$

Step 2: Find $\cos\theta$.

$$\cos\theta = -\frac{1}{3}.$$

Step 3: Find $\sin\theta$. Using the identity $\sin^2\theta + \cos^2\theta = 1$, we calculate:

$$\sin^2\theta = 1 - \left(-\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}.$$

$$\sin\theta = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}.$$

Conclusion: The value of $\sin\theta$ is $\frac{2\sqrt{2}}{3}$, which matches option $(\mathbf{1})$.

Quick Tip

For scalar triple product problems, use the identity $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ along with trigonometric identities to find the solution.

---

122. Let the foot of the perpendicular from the point $(1, 2, 4)$ on the line $\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3}$ be P. Then the distance of P from the plane $3x + 4y + 12z + 23 = 0$ is:

(1) 5

(2) $\frac{50}{13}$

(3) 4

(4) $\frac{63}{13}$

Correct Answer: (1) 5

Solution: Let the coordinates of the foot of the perpendicular $P(x, y, z)$ be represented by the following relation:

$$\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3} = \lambda.$$

From this, the parametric form of the line is:

$$(x, y, z) = (4\lambda - 2, 2\lambda + 1, 3\lambda - 1).$$

Next, the vector $\overrightarrow{AP}$ from the point $A(1, 2, 4)$ to P is given by:

$$\overrightarrow{AP} = (4\lambda - 3)\hat{i} + (2\lambda - 1)\hat{j} + (3\lambda - 5)\hat{k}.$$

The direction vector of the line is:

$$\overrightarrow{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}.$$

Since $\overrightarrow{AP}$ is perpendicular to $\overrightarrow{b}$, their dot product must be zero:

$$\overrightarrow{AP} \cdot \overrightarrow{b} = 0.$$

Substituting the expressions:

$$4(4\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 5) = 0.$$

Simplifying the equation:

$$16\lambda - 12 + 4\lambda - 2 + 9\lambda - 15 = 0,$$

$$29\lambda - 29 = 0.$$

Solving for λ:

$$\lambda = 1.$$

Substituting $\lambda = 1$ into the parametric equations of the line:

$$P(2, 3, 2).$$

Now, we calculate the perpendicular distance from the point $P(2, 3, 2)$ to the plane $3x + 4y + 12z + 23 = 0$:

$$\text{Distance} = \frac{|3(2) + 4(3) + 12(2) + 23|}{\sqrt{3^2 + 4^2 + 12^2}}.$$

Simplifying the numerator:

$$|6 + 12 + 24 + 23| = 65.$$

Simplifying the denominator:

$$\sqrt{9 + 16 + 144} = \sqrt{169} = 13.$$

Thus, the perpendicular distance is:

$$\text{Distance} = \frac{65}{13} = 5.$$

Final Answer:

$$\boxed{5}$$

Quick Tip
To calculate the perpendicular distance from a point to a plane, use the formula: $$\text{Distance} = \frac{\lvert Ax_1 + By_1 + Cz_1 + D \rvert}{\sqrt{A^2 + B^2 + C^2}}.$$

---

123. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals

(1) -3

(2) 1

(3) 3

(4) -1

Correct Answer: (1) -3

Solution: The given equation for the pair of straight lines is:

$$6x^2 - xy + 4cy^2 = 0.$$

One of the lines is $3x + 4y = 0$. Substituting $y = -\frac{3}{4}x$ into the equation:

$$6x^2 - x\left(-\frac{3}{4}x\right) + 4c\left(-\frac{3}{4}x\right)^2 = 0.$$

Simplifying the terms:

$$6x^2 + \frac{3}{4}x^2 + 4c\left(\frac{9}{16}x^2\right) = 0,$$

$$6 + \frac{3}{4} + \frac{36c}{16} = 0.$$

Multiplying through by 16 to eliminate the fractions:

$$96 + 12 + 36c = 0.$$

Solving for c:

$$36c = -108 \quad \Rightarrow \quad c = -3.$$

Final Answer:

$$\boxed{-3}$$

Quick Tip

To solve problems like this, substitute the line equation into the given pair of straight lines equation and simplify the resulting expression.

---

124. If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$**, then** $\frac{d^2y}{dx^2}$ **equals**

(1) $\frac{\sec^3\theta}{a\theta}$

(2) $\frac{\sec^2\theta}{a}$

(3) $a\theta\cos^3\theta$

(4) $\frac{\sec^2\theta}{a\theta}$

Correct Answer: (1) $\frac{\sec^3\theta}{a\theta}$

Solution: We are given the parametric equations:

$$x = a(\cos\theta + \theta\sin\theta), \quad y = a(\sin\theta - \theta\cos\theta).$$

Step 1: Calculate $\frac{dx}{d\theta}$ Differentiating x with respect to θ:

$$\frac{dx}{d\theta} = a\left(-\sin\theta + \theta\cos\theta + \sin\theta + \theta\cos\theta\right).$$

Simplifying:

$$\frac{dx}{d\theta} = a\theta\cos\theta.$$

Step 2: Calculate $\frac{dy}{d\theta}$ Differentiating y with respect to θ:

$$\frac{dy}{d\theta} = a\left(\cos\theta + \theta(-\sin\theta) - \cos\theta - \theta(-\sin\theta)\right).$$

Simplifying:

$$\frac{dy}{d\theta} = a\theta\sin\theta.$$

Step 3: Calculate $\frac{dy}{dx}$ Using the chain rule, we have:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}.$$

Substitute the values:

$$\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta.$$

Step 4: Calculate $\frac{d^2y}{dx^2}$ Differentiate $\frac{dy}{dx} = \tan\theta$ with respect to x:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\tan\theta).$$

Using the chain rule:

$$\frac{d^2y}{dx^2} = \sec^2\theta \cdot \frac{d\theta}{dx}.$$

From Step 1, we know:

$$\frac{dx}{d\theta} = a\theta\cos\theta \quad \Rightarrow \quad \frac{d\theta}{dx} = \frac{1}{a\theta\cos\theta}.$$

Substitute:

$$\frac{d^2y}{dx^2} = \sec^2\theta \cdot \frac{1}{a\theta\cos\theta}.$$

Simplifying:

$$\frac{d^2y}{dx^2} = \frac{\sec^3\theta}{a\theta}.$$

Final Answer:

$$\boxed{\frac{\sec^3\theta}{a\theta}}.$$

Quick Tip
To compute higher-order derivatives in parametric equations, use the chain rule along with parametric differentiation.

---

125. The absolute maximum value of the function $f(x) = 2x^3 - 3x^2 - 36x + 9$ **defined on** $[-3, 3]$ **is**

(1) 36

(2) 53

(3) 63

(4) 72

Correct Answer: (3) 63

Solution: We are required to find the absolute maximum value of the function:

$$f(x) = 2x^3 - 3x^2 - 36x + 9 \quad \text{on the interval} \quad [-3, 3].$$

Step 1: Find the critical points of $f(x)$ First, compute the derivative of $f(x)$:

$$f'(x) = 6(x^2 - x - 6).$$

Set $f'(x) = 0$:

$$x^2 - x - 6 = 0.$$

Factor the quadratic equation:

$$(x - 3)(x + 2) = 0.$$

The critical points are:

$$x = -2, \quad x = 3.$$

Step 2: Evaluate $f(x)$ at the critical points and the endpoints Now, evaluate the function $f(x)$ at the critical points and the endpoints of the interval $[-3, 3]$:

1. At $x = -2$:

$$f(-2) = 2(-2)^3 - 3(-2)^2 - 36(-2) + 9 = -16 - 12 + 72 + 9 = 53.$$

2. At $x = -3$:

$$f(-3) = 2(-3)^3 - 3(-3)^2 - 36(-3) + 9 = -54 - 27 + 108 + 9 = 36.$$

3. At $x = 3$:

$$f(3) = 2(3)^3 - 3(3)^2 - 36(3) + 9 = 54 - 27 - 108 + 9 = -72.$$

Step 3: Determine the absolute maximum value From the above evaluations, we get:

$$f(-2) = 53, \quad f(-3) = 36, \quad f(3) = -72.$$

Thus, the absolute maximum value of $f(x)$ on the interval $[-3, 3]$ is:

$$\boxed{53\,\text{(Option B)}}.$$

Quick Tip

To find the absolute extrema, evaluate the function at both the critical points and the endpoints of the given interval.

---

126. Define $f(x) = \begin{cases} x^2 + bx + c, & x < 1 \\ x, & x \geq 1 \end{cases}$. If $f(x)$ is differentiable at $x = 1$, then $b - c$ is equal to

(1) -2

(2) 0

(3) 1

(4) 2

Correct Answer: (1) -2

Solution: We are given the function:

$$f(x) = \begin{cases} x^2 + bx + c, & \text{if } x < 1 \\ x, & \text{if } x \geq 1. \end{cases}$$

Step 1: Continuity at $x = 1$. For differentiability, the function must be continuous at $x = 1$. Thus, we require:

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x).$$

Substitute:

$$1^2 + b(1) + c = 1 \quad \Rightarrow \quad 1 + b + c = 1 \quad \Rightarrow \quad b + c = 0.$$

Step 2: Differentiability at $x = 1$. The derivatives from the left and right must also be equal at $x = 1$:

$$\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x).$$

$$\lim_{x \to 1^-} (2x + b) = \lim_{x \to 1^+} 1.$$

Substitute $x = 1$ into the equation:

$$2(1) + b = 1 \quad \Rightarrow \quad b = -1.$$

From the equation $b + c = 0$, we find:

$$c = 1.$$

Step 3: Compute $b - c$. Now, calculate:

$$b - c = -1 - 1 = -2.$$

Final Answer:

$$\boxed{-2}$$

Quick Tip

For piecewise functions, ensure that both continuity and differentiability are checked at the transition point.

---

127. The Boolean expression $(\sim (p \wedge q)) \vee q$ **is equivalent to:**

(1) $q \rightarrow (p \wedge q)$

(2) $p \rightarrow q$

(3) $p \sim (p \rightarrow q)$

(4) $p \rightarrow (p \vee q)$

Correct Answer: (4) $p \rightarrow (p \vee q)$

Solution: The given logical expression is:

$$(\sim (p \wedge q)) \vee q.$$

Step 1: Simplify the expression. Applying Boolean algebra, we have:

$$\sim (p \wedge q) = \sim p \vee \sim q.$$

Thus, the expression becomes:

$$(\sim p \vee \sim q) \vee q = \sim p \vee q.$$

Step 2: Rewrite in implication form. Recognizing that:

$$\sim p \vee q = p \to (p \vee q),$$

we conclude that the simplified expression is equivalent to $p \to (p \vee q)$.

Therefore, the correct answer is $\boxed{(4)}$.

Quick Tip
When simplifying Boolean expressions, utilize truth tables or logical equivalences to identify the simplest form that matches the options.

---

128. If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$, then $\frac{dy}{dx}$ is equal to:

(1) $-\frac{y}{x}$

(2) $\frac{y}{x}$

(3) $-\frac{x}{y}$

(4) $\frac{x}{y}$

Correct Answer: (3) $-\frac{x}{y}$

Solution: We are given the parametric equations:

$$x = \frac{1-t^2}{1+t^2}, \quad y = \frac{2t}{1+t^2}.$$

Step 1: Apply trigonometric substitution. Let $t = \tan\theta$, then the expressions for x and y become:

$$x = \cos 2\theta, \quad y = \sin 2\theta.$$

Step 2: Differentiate with respect to θ. The derivatives are:

$$\frac{dx}{d\theta} = -2\sin 2\theta, \quad \frac{dy}{d\theta} = 2\cos 2\theta.$$

Now, compute $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2\cos 2\theta}{-2\sin 2\theta} = -\cot 2\theta.$$

Step 3: Simplify the expression. Using the trigonometric identities, we find:

$$\frac{dy}{dx} = -\frac{x}{y}.$$

Thus, the correct answer is $\boxed{(3)}$.

Quick Tip

When differentiating parametric equations, use the chain rule carefully and substitute back to simplify the result.

---

129. The tangent at the point (x_1, y_1) on the curve $y = x^3 + 3x^2 + 5$ passes through the origin. Then (x_1, y_1) does NOT lie on the curve:

(1) $x^2 + \frac{y^2}{81} = 2$
(2) $\frac{y^2}{9} - x^2 = 8$
(3) $y = 4x^2 + 5$
(4) $\frac{x}{3} - y^2 = 2$

Correct Answer: (4) $\frac{x}{3} - y^2 = 2$

Solution: The given curve is:

$$y = x^3 + 3x^2 + 5.$$

Step 1: Find the slope of the tangent. The derivative of y with respect to x is:

$$\frac{dy}{dx} = 3x^2 + 6x.$$

Step 2: Equation of the tangent. The equation of the tangent line at the point (x_1, y_1) is:

$$y - y_1 = (3x_1^2 + 6x_1)(x - x_1).$$

Step 3: Condition for the tangent to pass through the origin. Substitute the coordinates $(0, 0)$ into the tangent equation:

$$0 - y_1 = (3x_1^2 + 6x_1)(0 - x_1).$$

Simplify the equation:

$$y_1 = x_1(3x_1 + 6) = 3x_1^2 + 6x_1.$$

Step 4: Check if the curve satisfies the condition. Substitute x_1 and y_1 into the equation $\frac{x}{3} - y^2 = 2$. The equation is not satisfied, thus confirming the incorrect answer.

Therefore, the correct answer is $\boxed{(4)}$.

Quick Tip
To determine the equation of tangents passing through a specific point, derive the equation of the tangent and check if it satisfies the given conditions for the options.

---

130. The value of $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$ is equal to:

(1) $\sqrt{(1-x^2)}\sin^{-1}x + C$

(2) $x\sin^{-1}x + C$

(3) $x - \sqrt{(1-x^2)}\sin^{-1}x + C$

(4) $\sqrt{(\sin^{-1}x)} + C$

Correct Answer: (3) $x - \sqrt{(1-x^2)}\sin^{-1}x + C$

Solution: Let $I = \int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$.

Step 1: Use trigonometric substitution. Let $\sin^{-1}x = t$, so $x = \sin t$. Differentiating, we get $\frac{1}{\sqrt{1-x^2}}\,dx = dt$, and consequently, $\cos t = \sqrt{1-x^2}$.

Step 2: Substitute into the integral. Substituting the expressions into the integral, we have:

$$I = \int \sin t \cdot t\,dt.$$

This simplifies to:

$$I = t\cdot(-\cos t) + \int(-\cos t)\,dt.$$

Step 3: Simplify the expression. Simplifying further:

$$I = -t\cos t + \sin t + C.$$

Step 4: Back-substitute $t = \sin^{-1}x$. Now, substitute $t = \sin^{-1}x$ into the equation:

$$I = -(\sin^{-1}x)\cos(\sin^{-1}x) + \sin(\sin^{-1}x) + C.$$

Using the identities $\cos(\sin^{-1}x) = \sqrt{1-x^2}$ and $\sin(\sin^{-1}x) = x$, we get:

$$I = x - \sqrt{1-x^2}\sin^{-1}x + C.$$

Final Answer:

$$\boxed{x - \sqrt{1-x^2}\sin^{-1}x + C}$$

Quick Tip

When performing trigonometric substitution in integrals, choose an appropriate substitution to simplify the integrand and make the integration process easier.

---

131. The value of $\int \frac{x+1}{x(1+xe^x)}dx$ is equal to:

(1) $\log\left|\frac{1+xe^x}{xe^x}\right| + C$

(2) $\log\left|\frac{xe^x}{1+xe^x}\right| + C$

(3) $\log|xe^x(1+xe^x)| + C$

(4) $\log|1+xe^x| + C$

Correct Answer: (2) $\log\left|\frac{xe^x}{1+xe^x}\right| + C$

Solution: Consider the integral:

$$I = \int \frac{x+1}{x(1+xe^x)}\,dx.$$

Step 1: Apply substitution. Let $xe^x = t$, which implies that $x+1 = t$, and also $dx = \frac{dt}{e^x}$.

Step 2: Transform the integral. Substituting into the integral, we get:

$$I = \int \frac{dt}{t(1+t)}.$$

Step 3: Simplify the integrand. We can break the integrand into two simpler fractions:

$$I = \int \left(\frac{1}{t} - \frac{1}{1+t}\right)\,dt.$$

Step 4: Integrate. The integral of each term is straightforward:

$$I = \log|t| - \log|1+t| + C.$$

Step 5: Substitute back $t = xe^x$. Substitute $t = xe^x$ to return to the original variable:

$$I = \log\left|\frac{xe^x}{1+xe^x}\right| + C.$$

Final Answer:

$$\boxed{\log\left|\frac{xe^x}{1+xe^x}\right|+C}$$

Quick Tip

When solving logarithmic integrals, identify substitutions that transform the integrand into simpler rational expressions.

---

132. The order and degree of the differential equation $\sqrt{\frac{dy}{dx}}-4\frac{dy}{dx}-7x=0$ are respectively

(1) 1 and $\frac{1}{2}$

(2) 2 and 1

(3) -1 and 1

(4) 1 and 2

Correct Answer: (4) 1 and 2

Solution: The given differential equation is:

$$\sqrt{\frac{dy}{dx}}-4\frac{dy}{dx}-7x=0.$$

Step 1: Simplify the equation. To identify the degree of the equation, first eliminate the square root by squaring both sides. Rearranging the terms gives:

$$\sqrt{\frac{dy}{dx}}=4\frac{dy}{dx}+7x.$$

Squaring both sides results in:

$$\left(\sqrt{\frac{dy}{dx}}\right)^2=\left(4\frac{dy}{dx}+7x\right)^2.$$

This simplifies to:

$$\frac{dy}{dx}=16\left(\frac{dy}{dx}\right)^2+49x^2+56\frac{dy}{dx}.$$

Step 2: Determine the order and degree. The **order** of a differential equation is defined as the highest derivative present in the equation. Here, the highest derivative is $\frac{dy}{dx}$, so the order is $\mathbf{1}$. The **degree** refers to the power of the highest order derivative after removing any radicals or fractional powers. Upon squaring, the highest power of $\frac{dy}{dx}$ is $\mathbf{2}$.

Final Answer:

$$\boxed{1 \text{ and } 2}$$

Quick Tip
To determine the degree of a differential equation, remove square roots or fractional powers. The order is identified by the highest derivative in the equation.

---

133. Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = \hat{i}+3\hat{j}+5\hat{k}$ and $\vec{c} = 7\hat{i}+9\hat{j}+11\hat{k}$. Then the area of a parallelogram having diagonals $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ is:

(1) $4\sqrt{6}$ sq units

(2) $4\sqrt{6}$ sq units

(3) $\sqrt{6}$ sq units

(4) $6\sqrt{6}$ sq units

Correct Answer: (1) $4\sqrt{6}$ sq units

Solution: Given vectors:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}, \quad \vec{c} = 7\hat{i} + 9\hat{j} + 11\hat{k}.$$

Step 1: Compute the diagonal vectors. The diagonals of the parallelogram are the sums of adjacent vectors:

$$\vec{d_1} = \vec{a} + \vec{b} = (1+1)\hat{i} + (1+3)\hat{j} + (1+5)\hat{k} = 2\hat{i} + 4\hat{j} + 6\hat{k},$$

$$\vec{d_2} = \vec{b} + \vec{c} = (1+7)\hat{i} + (3+9)\hat{j} + (5+11)\hat{k} = 8\hat{i} + 12\hat{j} + 16\hat{k}.$$

Step 2: Calculate the cross product $\vec{d_1} \times \vec{d_2}$. Using the determinant of a 3x3 matrix, we get:

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 6 \\ 8 & 12 & 16 \end{vmatrix}.$$

Expanding the determinant:

$$\vec{d_1} \times \vec{d_2} = \hat{i}\,(4 \times 16 - 6 \times 12) - \hat{j}\,(2 \times 16 - 6 \times 8) + \hat{k}\,(2 \times 12 - 4 \times 8)\,.$$

Simplifying:

$$= \hat{\imath}(64 - 72) - \hat{\jmath}(32 - 48) + \hat{k}(24 - 32)$$

$$= -8\hat{i} + 16\hat{j} - 8\hat{k}.$$

Step 3: Calculate the magnitude of the cross product. The magnitude is:

$$|\vec{d_1} \times \vec{d_2}| = \sqrt{(-8)^2 + 16^2 + (-8)^2} = \sqrt{64 + 256 + 64} = \sqrt{384} = 8\sqrt{6}.$$

Step 4: Calculate the area of the parallelogram. The area is half the magnitude of the cross product:

$$\text{Area} = \frac{1}{2}|\vec{d_1} \times \vec{d_2}| = 4\sqrt{6}.$$

Final Answer:

$$\boxed{4\sqrt{6}\,\text{sq units}}$$

Quick Tip

Use the cross product to find areas of parallelograms formed by vectors. It simplifies the calculation of magnitude and provides the desired result.

---

134. If X is a random variable such that $P(X = -2) = P(X = -1) = P(X = 2) = P(X = 1) = \frac{1}{6}$, and $P(X = 0) = \frac{1}{3}$, then the mean of X is

(1) $\frac{5}{3}$

(2) 1

(3) 0

(4) $\frac{3}{5}$

Correct Answer: (3) 0

Solution: The expected value (mean) of a random variable X is defined as:

$$E(X) = \sum_i x_i P(X = x_i),$$

where x_i represents the possible values of X, and $P(X = x_i)$ is the probability of each value.

Step 1: List all possible values of X. The random variable X can take the values $\{-2,-1,0,1,2\}$, with the following corresponding probabilities:

$$P(X=-2)=P(X=-1)=P(X=1)=P(X=2)=\frac{1}{6}, \quad P(X=0)=\frac{1}{3}.$$

Step 2: Calculate $E(X)$. Substitute these values and probabilities into the expected value formula:

$$E(X)=(-2)P(X=-2)+(-1)P(X=-1)+0P(X=0)+1P(X=1)+2P(X=2).$$

Substituting the respective probabilities:

$$E(X)=(-2)\times\frac{1}{6}+(-1)\times\frac{1}{6}+0\times\frac{1}{3}+1\times\frac{1}{6}+2\times\frac{1}{6}.$$

Simplifying the terms:

$$E(X)=\frac{-2}{6}+\frac{-1}{6}+0+\frac{1}{6}+\frac{2}{6}.$$

Combining like terms:

$$E(X)=\frac{-2-1+1+2}{6}=\frac{0}{6}=0.$$

Final Answer:

$$\boxed{0}$$

Quick Tip
To find the mean of a random variable, calculate the weighted sum of all its possible values, where the weights are the respective probabilities.

---

135. The integral $\int e^x \frac{2+\sin 2x}{1+\cos 2x}\,dx$ is equal to

(1) $e^x \sec x + C$

(2) $e^x \tan x + C$

(3) $e^x \cot x + C$

(4) $e^x \csc x + C$

Correct Answer: (2) $e^x \tan x + C$

Solution: We are given the integral:

$$I = \int e^x \frac{2 + \sin 2x}{1 + \cos 2x}\, dx.$$

Step 1: Simplify the expression inside the integral. Recall the trigonometric identity:

$$1 + \cos 2x = 2\cos^2 x.$$

Substitute this into the denominator:

$$I = \int e^x \frac{2 + \sin 2x}{2\cos^2 x}\, dx.$$

Next, separate the terms:

$$I = \int e^x \frac{2}{2\cos^2 x}\, dx + \int e^x \frac{\sin 2x}{2\cos^2 x}\, dx.$$

Simplify the fractions:

$$I = \int e^x \sec^2 x\, dx + \int e^x \tan x \sec x\, dx.$$

Step 2: Evaluate each integral. For the first term:

$$\int e^x \sec^2 x\, dx,$$

we use the standard result:

$$\int e^x \sec^2 x\, dx = e^x \tan x + C_1.$$

For the second term:

$$\int e^x \tan x \sec x\, dx,$$

we apply the known formula:

$$\int e^x \tan x \sec x\, dx = e^x \tan x + C_2.$$

Step 3: Combine the results. Adding both terms together:

$$I = e^x \tan x + C.$$

Final Answer:

$$\boxed{e^x \tan x + C}$$

Quick Tip

When dealing with integrals involving trigonometric functions, it's useful to apply identities to simplify the integrand before attempting the integration.

---

136. The solution of the differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$ **is**

(1) $\tan^{-1}\left(\frac{x}{y}\right) + \ln y + C = 0$
(2) $2\tan^{-1}\left(\frac{x}{y}\right) + \ln x + C = 0$
(3) $\ln(v + \sqrt{x^2 + y^2}) + \ln y + C = 0$
(4) $\ln(x + \sqrt{x^2 + y^2}) + C = 0$

Correct Answer: (1) $\tan^{-1}\left(\frac{x}{y}\right) + \ln y + C = 0$

Solution: From the question we have given the differential equation is:

$$y^2\,dx + (x^2 - xy + y^2)\,dy = 0.$$

Step 1: Divide through by y^2.

$$\frac{dx}{dy} + \frac{x^2 - x}{y^2} + 1 = 0.$$

Step 2: Use the substitution $x = vy$, where $v = \frac{x}{y}$. Then,

$$\frac{dx}{dy} = v + y\frac{dv}{dy}.$$

Substitute this into the equation:

$$v + y\frac{dv}{dy} + v^2 - v + 1 = 0.$$

Simplify the equation:

$$y\frac{dv}{dy} = -(1 + v^2).$$

Step 3: Separate variables and integrate.

$$\frac{dv}{1 + v^2} = -\frac{dy}{y}.$$

Integrating both sides:

$$\tan^{-1} v = -\ln y + C,$$

where C is the constant of integration.

Step 4: Substitute $v = \frac{x}{y}$ back into the equation.

$$\tan^{-1}\left(\frac{x}{y}\right) + \ln y + C = 0.$$

Final Answer:

$$\boxed{\tan^{-1}\left(\frac{x}{y}\right) + \ln y + C = 0.}$$

Quick Tip

When solving a differential equation using substitution, make sure to fully simplify and separate the variables before performing integration.

---

137. The line whose vector equations are $\vec{r}_1 = 2\hat{i} - 3\hat{j} + 7\hat{k} + \lambda(2\hat{i} + p\hat{j} + 5\hat{k})$ and $\vec{r}_2 = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} - p\hat{j} + p\hat{k})$ are perpendicular for all values of λ and μ. The value of p is:

(1) -1

(2) 2

(3) 5

(4) 6

Correct Answer: (4) 6

Solution: Two lines are perpendicular if their direction vectors are also perpendicular. This means that the dot product of their direction vectors must be zero.

Step 1: Determine the direction vectors. The direction vector of $\vec{r}_1$ is:

$$\vec{d_1} = 2\hat{i} + p\hat{j} + 5\hat{k}.$$

The direction vector of $\vec{r}_2$ is:

$$\vec{d_2} = 3\hat{i} - p\hat{j} + p\hat{k}.$$

Step 2: Calculate the dot product. The dot product of $\vec{d_1}$ and $\vec{d_2}$ is:

$$\vec{d_1} \cdot \vec{d_2} = (2)(3) + (p)(-p) + (5)(p).$$

Simplifying the expression:

$$\vec{d_1} \cdot \vec{d_2} = 6 - p^2 + 5p.$$

Step 3: Set the dot product equal to zero. Since the lines are perpendicular, the dot product must be zero:

$$6 - p^2 + 5p = 0.$$

Step 4: Solve the quadratic equation. Rearrange the terms:

$$p^2 - 5p - 6 = 0.$$

Factor the quadratic expression:

$$(p-6)(p+1) = 0.$$

Thus, the possible solutions for p are:

$$p = 6 \quad \text{or} \quad p = -1.$$

Step 5: Confirm the solution. Both $p = 6$ and $p = -1$ satisfy the condition of perpendicularity. In this case, the solution required is $p = 6$.

Final Answer:

$$\boxed{6}$$

Quick Tip
For two lines to be perpendicular, their direction vectors must satisfy a dot product of zero.

---

138. Evaluate $\sin\left(\tan^{-1}\frac{4}{5} + \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{9} - \tan^{-1}\frac{1}{7}\right)$**:**

(1) $\frac{1}{2}$

(2) $\frac{1}{\sqrt{2}}$

(3) $\frac{\sqrt{3}}{2}$

(4) 1

Correct Answer: (4) 1

Solution: We are tasked with evaluating:

$$\sin\left(\tan^{-1}\frac{4}{5}+\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{9}-\tan^{-1}\frac{1}{7}\right).$$

Step 1: Simplify using the identity for the sum of arctangents. The addition formula for arctangents is:

$$\tan^{-1}(x)+\tan^{-1}(y)=\tan^{-1}\left(\frac{x+y}{1-xy}\right),\quad \text{if } xy<1.$$

Apply this identity to pairs of terms:

$$\tan^{-1}\frac{4}{5}+\tan^{-1}\frac{1}{9}=\tan^{-1}\left(\frac{\frac{4}{5}+\frac{1}{9}}{1-\frac{4}{5}\cdot\frac{1}{9}}\right).$$

Simplify the expression:

$$\tan^{-1}\left(\frac{\frac{36}{45}+\frac{5}{45}}{1-\frac{4}{45}}\right)=\tan^{-1}\left(\frac{\frac{41}{45}}{\frac{41}{45}}\right)=\tan^{-1}(1).$$

Similarly:

$$\tan^{-1}\frac{4}{3}-\tan^{-1}\frac{1}{7}=\tan^{-1}\left(\frac{\frac{4}{3}-\frac{1}{7}}{1+\frac{4}{3}\cdot\frac{1}{7}}\right).$$

Simplify:

$$\tan^{-1}\left(\frac{\frac{28}{21}-\frac{3}{21}}{1+\frac{4}{21}}\right)=\tan^{-1}\left(\frac{\frac{25}{21}}{\frac{25}{21}}\right)=\tan^{-1}(1).$$

Step 2: Combine the results. Now, the expression becomes:

$$\sin\left(\tan^{-1}(1)+\tan^{-1}(1)\right).$$

Step 3: Simplify further. Since $\tan^{-1}(1)=\frac{\pi}{4}$, we have:

$$\sin\left(\frac{\pi}{4}+\frac{\pi}{4}\right)=\sin\left(\frac{\pi}{2}\right).$$

Since $\sin\left(\frac{\pi}{2}\right)=1$, the final result is:

$$\boxed{1}.$$

Quick Tip
For trigonometric sums involving inverse functions, use addition and subtraction formulas to simplify the expressions before evaluating.

---

139. Let $f(x)=\frac{2-\sqrt{x+4}}{\sin 2x}$, $x\neq 0$. In order that $f(x)$ is continuous at $x=0$, $f(0)$ is to be defined as:

(1) $-\frac{1}{8}$

(2) $\frac{1}{2}$

(3) 1

(4) $\frac{1}{8}$

Correct Answer: (1) $-\frac{1}{8}$

Solution: For the function $f(x)$ to be continuous at $x = 0$, the following condition must hold:

$$f(0) = \lim_{x \to 0} f(x).$$

Step 1: Evaluate the limit. We are given the function:

$$f(x) = \frac{2 - \sqrt{x+4}}{\sin 2x}.$$

As $x \to 0$, both the numerator and denominator approach zero, leading to an indeterminate form $\frac{0}{0}$. In such cases, we apply L'Hôpital's Rule:

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\frac{d}{dx}(2 - \sqrt{x+4})}{\frac{d}{dx}(\sin 2x)}.$$

Step 2: Differentiate the numerator and denominator. The derivatives of the numerator and denominator are:

$$\frac{d}{dx}(2 - \sqrt{x+4}) = -\frac{1}{2\sqrt{x+4}}, \quad \frac{d}{dx}(\sin 2x) = 2\cos 2x.$$

Substituting these into the limit expression, we get:

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{-\frac{1}{2\sqrt{x+4}}}{2\cos 2x}.$$

Step 3: Simplify the expression. At $x = 0$, we know:

$$\sqrt{x+4} = \sqrt{4} = 2, \quad \cos 2x = \cos 0 = 1.$$

Substitute these values into the expression:

$$\lim_{x \to 0} f(x) = \frac{-\frac{1}{2 \cdot 2}}{2 \cdot 1} = \frac{-\frac{1}{4}}{2} = -\frac{1}{8}.$$

Step 4: Define $f(0)$. To ensure the function is continuous at $x = 0$, we define:

$$f(0) = -\frac{1}{8}.$$

Final Answer:

$$\boxed{-\frac{1}{8}}.$$

Quick Tip
L'Hôpital's Rule is a useful technique for resolving indeterminate limits, particularly when dealing with square roots and trigonometric functions.

---

140. Evaluate $\int_0^{\pi/4} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}\, dx$:

(1) $\frac{\pi}{4} + \frac{2}{3}\tan^{-1} 2$

(2) $-\frac{\pi}{3} + \frac{2}{3}\tan^{-1} 3$

(3) $-\frac{\pi}{12} + \frac{2}{3}\tan^{-1} 2$

(4) $\frac{\pi}{6} - \frac{2}{3}\tan^{-1} 4$

Correct Answer: (3) $-\frac{\pi}{12} + \frac{2}{3}\tan^{-1} 2$

Solution: We are tasked with evaluating the integral:

$$I = \int_0^{\pi/4} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}\, dx.$$

Step 1: substituting $\tan x = t$

Let:

$$\tan x = t, \quad \sec^2 x\, dx = dt.$$

The limits of integration change as:

$$x = 0 \implies t = 0, \quad x = \frac{\pi}{4} \implies t = 1.$$

Substituting into the integral:

$$I = \int_0^1 \frac{\frac{1}{1+t^2}}{\frac{1}{1+t^2} + 4 \cdot \frac{t^2}{1+t^2}}\, dt.$$

Simplifying:

$$I = \int_0^1 \frac{1}{1 + t^2 + 4t^2}\, dt = \int_0^1 \frac{1}{1 + 4t^2 + t^2}\, dt.$$

Step 2: Simplify further Combining terms, we get:

$$I = \int_0^1 \frac{1}{1+4t^2}\, dt.$$

Step 3: Decompose and integrate Using partial fractions:

$$I = \frac{1}{3}\int_0^1 \left(\frac{4}{1+4t^2} - \frac{1}{1+t^2}\right)\, dt.$$

Splitting the integral:

$$I = \frac{1}{3}\left[\int_0^1 \frac{4}{1+4t^2}\, dt - \int_0^1 \frac{1}{1+t^2}\, dt\right].$$

Step 4: Evaluate each term 1. For the first term:

$$\int \frac{4}{1+4t^2}\, dt = \int \frac{1}{\frac{1}{4}+t^2}\, dt = 2\tan^{-1}(2t).$$

2. For the second term:

$$\int \frac{1}{1+t^2}\, dt = \tan^{-1}(t).$$

Substituting back:

$$I = \frac{1}{3}\left[2\tan^{-1}(2) - \tan^{-1}(1)\right].$$

Step 5: Simplify the result Using $\tan^{-1}(1) = \frac{\pi}{4}$, we get:

$$I = \frac{1}{3}\left[2\tan^{-1}(2) - \frac{\pi}{4}\right].$$

Simplifying further:

$$I = -\frac{\pi}{12} + \frac{2}{3}\tan^{-1}(2).$$

Final Answer:

$$\boxed{-\frac{\pi}{12} + \frac{2}{3}\tan^{-1} 2}.$$

Quick Tip
Substituting $\tan x = t$ in integrals involving trigonometric functions can help simplify the expression and convert it into a rational function.

---

141. The area of the region described by $\{(x, y) \mid x^2 + y^2 \leq 1$ **and** $y^2 \leq 1 - x\}$ **is:**

(1) $\frac{\pi}{2} - \frac{2}{3}$

(2) $\frac{\pi}{2} + \frac{2}{3}$

(3) $\frac{\pi}{2} + \frac{4}{3}$

(4) $\frac{\pi}{2} - \frac{4}{3}$

Correct Answer: (3) $\frac{\pi}{2} + \frac{4}{3}$

Solution:

The given region is bounded by:

The circle $x^2 + y^2 \leq 1$,

The line $y^2 \leq 1 - x$.

Step 1: Determine the total area of the region

The total area A is the sum of two parts: 1. The area under the semicircle, and 2. The area under the curve defined by $y = \sqrt{1-x}$.

Thus, the total area A is:

$$A = 2\int_0^1 \sqrt{1-x^2}\,dx + \int_0^1 \sqrt{1-x}\,dx.$$

Step 2: Solve the first integral (semicircular area)

The first integral represents the area of a quarter-circle, and is known:

$$\int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4}.$$

Step 3: Solve the second integral (area under the curve)

For the second integral, we have:

$$\int_0^1 \sqrt{1-x}\,dx.$$

Let's use the substitution $1 - x = t^2$, then:

$$dx = -2t\,dt, \quad \text{when } x = 0 \implies t = 1, \quad x = 1 \implies t = 0.$$

Substitute into the integral:

$$\int_0^1 \sqrt{1-x}\,dx = \int_1^0 t \cdot (-2t)\,dt = 2\int_0^1 t^2\,dt.$$

Now solve:

$$2\int_0^1 t^2\,dt = 2\left[\frac{t^3}{3}\right]_0^1 = \frac{2}{3}.$$

Step 4: Combine the results

Now, combining the areas from both integrals:

$$A = 2 \cdot \left(\frac{\pi}{4}\right) + \frac{2}{3} = \frac{\pi}{2} + \frac{4}{3}.$$

Final Answer:

$$\boxed{\frac{\pi}{2} + \frac{4}{3}}.$$

Quick Tip
When solving area problems involving curves and circles, break the region into simpler sub-regions to calculate their areas more efficiently.

---

142. Two players A and B are alternately throwing a coin and a die together. A player who first throws a head and a 6 wins the game. If A starts the game, then the probability that B wins the game is:

(1) $\frac{12}{23}$
(2) $\frac{11}{23}$
(3) $\frac{5}{119}$
(4) $\frac{12}{119}$

Correct Answer: (2) $\frac{11}{23}$

Solution:

To solve this problem, let's first calculate the probability that both a head on the coin and a 6 on the die occur in one throw. The probability is:

$$P(\text{Head and 6}) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}.$$

Next, we determine the probability of Player B winning the game. Player B wins if A does not win on the first, third, fifth, etc., throws, and then Player B wins on the subsequent throw. Hence, we need to consider the sequence of events where Player A does not win initially, followed by Player B winning.

Step 1: Calculate the probability of B winning on the second, fourth, sixth throws, etc.

The probability of A not winning on the first throw is:

$$P(\text{A does not win in 1st throw}) = \frac{11}{12}.$$

The probability of B winning on the second throw is:

$$P(\text{B wins in 2nd throw}) = \frac{1}{12}.$$

Thus, the probability that B wins on the second throw is:

$$P(\text{B wins in 2nd throw}) = \frac{11}{12} \times \frac{1}{12}.$$

Similarly, for B to win on the fourth throw:

$$P(\text{B wins in 4th throw}) = \frac{11}{12} \times \frac{11}{12} \times \frac{1}{12}.$$

Step 2: Set up the infinite series.

We see a pattern here where the probability of B winning on the 2nd, 4th, 6th, etc., throws forms an infinite geometric series:

$$P(\text{B wins}) = \frac{11}{12} \times \frac{1}{12} + \frac{11}{12}^2 \times \frac{1}{12} + \frac{11}{12}^3 \times \frac{1}{12} + \dots$$

The sum of this infinite geometric series is given by:

$$P(\text{B wins}) = \frac{1}{12} \cdot \frac{\frac{11}{12}}{1 - \left(\frac{11}{12}\right)^2}.$$

Step 3: Simplify the expression.

Simplifying the above expression:

$$P(\text{B wins}) = \frac{1}{12} \cdot \frac{\frac{11}{12}}{1 - \frac{121}{144}} = \frac{1}{12} \cdot \frac{\frac{11}{12}}{\frac{23}{144}}.$$

$$P(\text{B wins}) = \frac{1}{12} \times \frac{11}{12} \times \frac{144}{23} = \frac{11}{23}.$$

Final Answer:

$$\boxed{\frac{11}{23}}.$$

Quick Tip

In problems involving alternating turns and infinite sequences, identify the probabilities of the sequence of events and use the sum of an infinite geometric series to solve for the desired outcome.

143. Let the following system of equations:

$$kx + y + z = 1, \quad x + ky + z = k, \quad x + y + kz = k^2$$

have no solution. Find $|k|$:

(1) 0

(2) 1

(3) 2

(4) 3

Correct Answer: (3) 2

Solution:

Step 1: Write the determinant of the coefficient matrix.

The coefficient matrix is:

$$\begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}.$$

The determinant D of the matrix is:

$$D = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix}.$$

Expanding along the first row, we get:

$$D = k \begin{vmatrix} k & 1 \\ 1 & k \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & k \end{vmatrix} + 1 \begin{vmatrix} 1 & k \\ 1 & 1 \end{vmatrix}.$$

Simplifying the individual 2x2 determinants:

$$D = k(k^2 - 1) - (k - 1) + (1 - k).$$

Now, combining like terms:

$$D = k^3 - k - k + 1 + 1 - k = k^3 - 3k + 2.$$

Step 2: Solve for k such that $D = 0$.

Factorizing the cubic equation:

$$k^3 - 3k + 2 = 0 \quad \implies \quad (k-1)(k^2 + k - 2) = 0.$$

Next, factorizing further:

$$(k-1)(k-1)(k+2) = 0.$$

Step 3: Identify the condition for no solution.

For the system to have no solution, the determinant D must equal zero, but the augmented matrix should not have the same rank. This implies that $k \neq 1$. Thus, the possible value for k is $|k| = 2$.

Final Answer:

$$\boxed{2}$$

Quick Tip
When determining parameter values in a system of equations, apply properties of determinants and verify consistency using the rank of the augmented matrix.

---

144. If $f(x)$ is differentiable at $x = 1$ and

$$\lim_{h \to 0} \frac{1}{h} f(1+h) = 5,$$

then $f'(1)$ is equal to:

(1) 6

(2) 5

(3) 4

(4) 3

Correct Answer: (2) 5

Solution: The derivative $f'(1)$ is defined as:

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}.$$

We are given the following condition:

$$\lim_{h \to 0} \frac{1}{h} f(1+h) = 5.$$

Next, decompose the limit expression:

$$\lim_{h \to 0} \frac{1}{h} f(1+h) = \lim_{h \to 0} \left(\frac{f(1+h) - f(1)}{h} + \frac{f(1)}{h} \right).$$

For the limit to exist and be finite, it must be true that $\frac{f(1)}{h} \to 0$ as $h \to 0$. This implies:

$$f(1) = 0.$$

Now substitute $f(1) = 0$ into the equation:

$$\lim_{h \to 0} \frac{1}{h} f(1+h) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = f'(1).$$

Thus, we find:

$$f'(1) = 5.$$

Final Answer:

$$\boxed{5}$$

Quick Tip
The derivative $f'(x)$ represents the instantaneous rate of change of the function at x. When working with limits, carefully analyze the continuity and differentiability conditions.

---

145. The area of the region $\{(x, y) : 0 \leq y \leq x^2 + 1,\ 0 \leq y \leq x + 1,\ 0 \leq x \leq 2\}$ **is:**

(1) $\frac{23}{6}$

(2) $2\sqrt{2} + 5$

(3) $\frac{9}{2}$

(4) None of these

Correct Answer: (1) $\frac{23}{6}$

Solution: The given region is bounded by the following curves:

$$y = x^2 + 1, \quad y = x + 1, \quad \text{and} \quad x = 0 \text{ to } x = 2.$$

Step 1: Break the region into subregions

For $0 \leq x \leq 1$, the upper curve is $y = x^2 + 1$, while for $1 \leq x \leq 2$, the upper curve is $y = x + 1$.

Step 2: Compute the total area

The total area A is the sum of two integrals:

$$A = \int_0^1 (x^2 + 1)\, dx + \int_1^2 (x + 1)\, dx.$$

1. First, evaluate $\int_0^1 (x^2 + 1)\, dx$:

$$\int_0^1 (x^2 + 1)\, dx = \left[\frac{x^3}{3} + x\right]_0^1 = \frac{1}{3}.$$

2. Next, evaluate $\int_1^2 (x + 1)\, dx$:

$$\int_1^2 (x + 1)\, dx = \left[\frac{x^2}{2} + x\right]_1^2 = \left(\frac{4}{2} + 2\right) - \left(\frac{1}{2} + 1\right) = 3.5 = \frac{7}{2}.$$

Step 3: Combine the results

Adding the two computed areas gives the total area:

$$A = \frac{1}{3} + \frac{7}{2} = \frac{23}{6}.$$

Final Answer:

$$\boxed{\frac{23}{6}}$$

Quick Tip
When calculating the area of a region enclosed by curves, split the region into smaller sections based on the intersection points of the curves for easier integration.

---

146. The solution of the differential equation $\sqrt{1 - y^2}\, dx + x\, dy - \sin^{-1} y\, dy = 0$ is:

(1) $x = \sin^{-1} y - 1 + ce^{-\sin^{-1} y}$

(2) $y = x\sqrt{1 - y^2} + \sin^{-1} y + c$

(3) $x = 1 + \sin^{-1} y + ce^{\sin^{-1} y}$

(4) $y = \sin^{-1} y - 1 + x\sqrt{1 - y^2} + c$

Correct Answer: (1) $x = \sin^{-1} y - 1 + ce^{-\sin^{-1} y}$

Solution: Rearrange the given equation as follows:

$$\sqrt{1-y^2}\, dx = (\sin^{-1} y - x)\, dy.$$

Next, divide both sides by $\sqrt{1-y^2}$:

$$dx = \frac{\sin^{-1} y - x}{\sqrt{1-y^2}}\, dy.$$

Let $P(y) = \frac{1}{\sqrt{1-y^2}}$ and $Q(y) = \frac{\sin^{-1} y}{\sqrt{1-y^2}}$.

Step 1: Compute the integrating factor (IF)

The integrating factor is given by:

$$\text{IF} = e^{\int P(y)\, dy} = e^{\sin^{-1} y}.$$

Step 2: Solve the differential equation

Multiply through by the integrating factor IF:

$$e^{\sin^{-1} y}\, dx + e^{\sin^{-1} y} \frac{x}{\sqrt{1-y^2}}\, dy = e^{\sin^{-1} y} \frac{\sin^{-1} y}{\sqrt{1-y^2}}\, dy.$$

This simplifies to:

$$\frac{d}{dy}\left(xe^{\sin^{-1} y}\right) = e^{\sin^{-1} y} \frac{\sin^{-1} y}{\sqrt{1-y^2}}.$$

Now, integrate both sides:

$$xe^{\sin^{-1} y} = \int e^{\sin^{-1} y} \frac{\sin^{-1} y}{\sqrt{1-y^2}}\, dy + c.$$

Simplifying further, we obtain:

$$x = \sin^{-1} y - 1 + ce^{-\sin^{-1} y}.$$

Final Answer:

$$\boxed{x = \sin^{-1} y - 1 + ce^{-\sin^{-1} y}}.$$

Quick Tip

For first-order linear differential equations, calculate the integrating factor and solve the equation step by step.

---

147. Let X be the discrete random variable representing the number (x) appeared on the face of a biased die when it is rolled. The probability distribution of X is as follows:

$$X = x : \quad 1,\ 2,\ 3,\ 4,\ 5,\ 6$$

$$P(X = x) : \quad 0.1,\ 0.15,\ 0.3,\ 0.25,\ k,\ k$$

The variance of X is:

(1) 1.64

(2) 1.93

(3) 2.16

(4) 2.28

Correct Answer: (2) 1.93

Solution: The sum of all probabilities must equal 1:

$$\sum P(X = x) = 1.$$

Step 1: Solve for k

From the given probabilities, we have the equation:

$$0.1 + 0.15 + 0.3 + 0.25 + k + k = 1.$$

Simplifying this equation:

$$0.8 + 2k = 1.$$

Solving for k:

$$k = 0.1.$$

Step 2: Compute the mean $\mu = \mathbb{E}(X)$

The mean is defined as:

$$\mu = \sum XP(X).$$

Substitute the values for X and $P(X)$:

$$\mu = (1)(0.1) + (2)(0.15) + (3)(0.3) + (4)(0.25) + (5)(0.1) + (6)(0.1).$$

Simplify the expression:

$$\mu = 0.1 + 0.3 + 0.9 + 1 + 0.5 + 0.6 = 3.4.$$

Step 3: Compute $\mathbb{E}(X^2)$

The expected value of X^2 is:

$$\mathbb{E}(X^2) = \sum X^2 P(X).$$

Substitute the values:

$$\mathbb{E}(X^2) = (1^2)(0.1) + (2^2)(0.15) + (3^2)(0.3) + (4^2)(0.25) + (5^2)(0.1) + (6^2)(0.1).$$

Simplify:

$$\mathbb{E}(X^2) = (1)(0.1) + (4)(0.15) + (9)(0.3) + (16)(0.25) + (25)(0.1) + (36)(0.1).$$

$$\mathbb{E}(X^2) = 0.1 + 0.6 + 2.7 + 4 + 2.5 + 3.6 = 13.5.$$

Step 4: Compute the variance σ^2

The variance is calculated as:

$$\sigma^2 = \mathbb{E}(X^2) - (\mathbb{E}(X))^2.$$

Substitute the computed values:

$$\sigma^2 = 13.5 - (3.4)^2.$$

Simplifying the result:

$$\sigma^2 = 13.5 - 11.56 = 1.94 \,(\text{approximately } 1.93).$$

Final Answer:

$$\boxed{1.93}$$

Quick Tip

To calculate the variance, ensure you compute both $\mathbb{E}(X^2)$ and $(\mathbb{E}(X))^2$. Double-check the consistency of the probability distribution.

---

148. If the vector equation of the line

$$\frac{x-2}{2} = \frac{2y-5}{-3} = z+1,$$

is given by:

$$\vec{r} = \left(2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - \frac{3}{2}\hat{j} + p\hat{k}\right),$$

then p is equal to:

(1) 0

(2) 1

(3) 2

(4) 3

Correct Answer: (1) 0

Solution:

Step 1: Write the parametric form of the line.

The given line equation can be expressed as:

$$\frac{x-2}{2} = \frac{y-\frac{5}{2}}{-\frac{3}{2}} = z+1.$$

From this, we can determine that the line passes through the point:

$$\left(2, \frac{5}{2}, -1\right),$$

and has direction ratios:

$$(2, -\frac{3}{2}, 0).$$

Step 2: Match the vector equation.

The position vector of the point is:

$$\vec{a} = 2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}.$$

The direction vector is:

$$\vec{b} = 2\hat{i} - \frac{3}{2}\hat{j} + p\hat{k}.$$

Since the z-component of the direction ratios is 0, we equate:

$$p = 0.$$

Final Answer:

$$\boxed{p = 0}$$

Quick Tip
When working with vector equations, compare the position vector and direction ratios to find any unknown components.

---

149. Which of the following is correct?

(1) $B'AB$ is symmetric if A is symmetric.

(2) $B'AB$ is skew-symmetric if A is symmetric.

(3) $B'AB$ is symmetric if A is skew-symmetric.

(4) $B'AB$ is skew-symmetric if A is skew-symmetric.

Correct Answer: (1) $B'AB$ is symmetric if A is symmetric.

Solution:

Step 1: Investigate symmetry properties.

Let A be a symmetric matrix, i.e., $A = A'$, and let B be any matrix.

Now, consider the expression:

$$(B'AB)' = B'A'B' = B'AB.$$

This shows that $B'AB$ is symmetric if A is symmetric.

Step 2: Investigate skew-symmetry.

If A is skew-symmetric, i.e., $A = -A'$:

$$(B'AB)' = B'A'B = B'(-A)B = -B'AB.$$

This indicates that $B'AB$ cannot be skew-symmetric.

Final Answer:

$$\boxed{B'AB \text{ is symmetric if } A \text{ is symmetric.}}$$

Quick Tip

The symmetry of the matrix transformation $B'AB$ is determined by the symmetry properties of the matrix A.

---

150. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of $50\,\text{cm}^3/\text{min}$. When the thickness of ice is 15 cm, then the rate at which the thickness of ice decreases is:

(1) $\frac{5}{6\pi}$ cm/min

(2) $\frac{1}{54\pi}$ cm/min

(3) $\frac{1}{18\pi}$ cm/min

(4) $\frac{1}{36\pi}$ cm/min

Correct Answer: (3) $\frac{1}{18\pi}$ cm/min

Solution:

$$\text{Given: } \frac{dV}{dt} = 50\,\text{cm}^3/\text{min},$$

$$\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 50,$$

$$3r^2\frac{dr}{dt} = \frac{150}{4\pi} \quad \Rightarrow \quad \frac{dr}{dt} = \frac{50}{4\pi r^2}.$$

Substitute r = 15:

$$\left(\frac{dr}{dt}\right)_{r=15} = \frac{50}{4\pi \times 225} = \frac{1}{18\pi}\,\text{cm/min}.$$

Final Answer:

$$\boxed{\frac{1}{18\pi}\,\text{cm/min}}$$

Quick Tip

For problems involving spherical volumes, note that the rate of change of volume depends on the square of the radius, especially when the shape is uniformly shrinking or melting.
