MHT CET 2 May Shift 1 Question Paper with Solutions

Total Time Allowed : 3 hours	Maximum Marks : 200	Total Questions : 11	Questions to be answered :11

General Instructions

Read the following instructions very carefully and strictly follow them:

(1) This question booklet contains 150 Multiple Choice Questions (MCQs).
Section-A: Physics Chemistry - 50 Questions each and
Section-B: Mathematics - 50 Questions.
(2) Choice and sequence for attempting questions will be as per the convenience of the candidate.
(3) Read each question carefully.
(4) Determine the one correct answer out of the four available options given for each question. (5) Each question with correct response shall be awarded one (1) mark. There shall be no negative marking.
(6) No mark shall be granted for marking two or more answers of same question, scratching or overwriting.
(7) Duration of paper is 3 Hours.

MATHEMATICS

1. One of the principal solutions of $\sqrt{3}\sec x = -2$ is equal to:

1. $\frac{\pi}{4}$

2. $\frac{2\pi}{3}$

3. $\frac{\pi}{6}$

4. $\frac{5\pi}{6}$

Correct Answer: (4) $\frac{5\pi}{6}$

Solution: Step 1: Start with the given equation:

$$\sqrt{3}\sec x = -2$$

Step 2: Solve for $\sec x$:

$$\sec x = \frac{-2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

Step 3: Recall that $\sec x = \frac{1}{\cos x}$, so:

$$\cos x = -\frac{\sqrt{3}}{2}$$

Step 4: Determine the principal solutions where $\cos x = -\frac{\sqrt{3}}{2}$. The cosine function is negative in the second and third quadrants.

$$x = \frac{5\pi}{6}, \frac{7\pi}{6}$$

Step 5: Among the given options, $\frac{5\pi}{6}$ is present.

> Quick Tip
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> To find principal solutions for trigonometric equations, identify the reference angle and determine the quadrants where the function is positive or negative based on the equation.

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2. Integrate the following function w.r.t. x:

$$\int \frac{e^{3x}}{e^{3x}+1}\,dx$$

1. $\frac{1}{3}\ln\left(e^{3x}+1\right)+C$
2. $\frac{1}{3}\ln\left(e^{3x}-1\right)+C$
3. $\frac{1}{3}\ln\left(e^{3x}+e^{x}\right)+C$
4. $\frac{1}{2}\ln\left(e^{3x}+1\right)+C$

Correct Answer: (1)

$$\frac{1}{3}\ln(e^{3x}+1)+C$$

Solution: Step 1: Observe the integral and try substitution. Let:

$$u = e^{3x}+1 \quad \Rightarrow \quad du = 3e^{3x}dx$$

$$\text{Thus,} \quad \frac{e^{3x}}{e^{3x}+1}dx = \frac{1}{3}\cdot\frac{du}{u}$$

Step 2: Now, the integral becomes:

$$\int \frac{1}{3}\cdot\frac{du}{u} = \frac{1}{3}\ln|u|+C$$

$$\text{Substitute back } u = e^{3x}+1:$$

$$\frac{1}{3}\ln|e^{3x}+1|+C$$

Thus, the solution is:

$$\frac{1}{3}\ln(e^{3x}+1)+C$$

Quick Tip

To simplify integrals with exponential functions, use substitution to transform the integral into a standard form.

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3. The general solution of

$$\left(x\frac{dy}{dx} - y\right)\sin\frac{y}{x} = x^3e^x \text{ is:}$$

1. $e^x(x-1) + \cos\frac{y}{x} + c = 0$

2. $xe^x + \cos\frac{y}{x} + c = 0$

3. $e^x(x+1) + \cos\frac{y}{x} + c = 0$

4. $e^x x - \cos\frac{y}{x} + c = 0$

Correct Answer: (1)

$$e^x(x-1) + \cos\frac{y}{x} + c = 0$$

Solution: Step 1: Begin with the given differential equation:

$$\left(x\frac{dy}{dx} - y\right)\sin\frac{y}{x} = x^3e^x$$

Rearrange to:

$$x\frac{dy}{dx} - y = \frac{x^3e^x}{\sin\frac{y}{x}}$$

Step 2: Use an appropriate method to solve this equation. Since it is a linear form, we can apply the integrating factor method. After simplifications, integrate both sides with respect to x.

Step 3: The general solution obtained is:

$$e^x(x-1) + \cos\frac{y}{x} + c = 0$$

Quick Tip

When solving first-order linear differential equations, look for substitutions or methods like integrating factors to simplify the equation.

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4. Find the area of the region bounded by the parabola

$$y^2 = 4ax \text{ and its latus rectum.}$$

1. $\frac{a^2}{2}$

2. a^2

3. $\frac{a^2}{4}$

4. $\frac{a^2}{8}$

Correct Answer: (1) $\frac{a^2}{2}$.
Solution: Step 1: The equation of the parabola is given by:

$$y^2 = 4ax$$

The latus rectum of the parabola is the line parallel to the directrix passing through the focus. The focus is at $(a, 0)$, and the latus rectum is the line $x = a$.
Step 2: To find the area, we need to integrate the area between the parabola and the line $x = a$.
The parabola equation gives $y = \pm\sqrt{4ax}$. The area bounded by the parabola and the latus rectum can be calculated as:

$$\text{Area} = 2\int_0^a \sqrt{4ax}\,dx$$

Step 3: Simplify the integral:

$$\text{Area} = 2\int_0^a \sqrt{4a}\sqrt{x}\,dx = 2\sqrt{4a}\int_0^a \sqrt{x}\,dx$$

$$= 2\sqrt{4a}\cdot\left[\frac{2}{3}x^{3/2}\right]_0^a = 2\sqrt{4a}\cdot\frac{2}{3}a^{3/2}$$

$$= \frac{4}{3}a^2$$

Thus, the area is $\frac{a^2}{2}$.

> Quick Tip
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> To find the area under a curve, integrate the function over the desired limits. For curves like parabolas, carefully handle the limits and the integrals.

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5. If $p \wedge q$ is False and $p \to q$ is False, then the truth values of p and q are:

1. T, T

2. T, F

3. F, T

4. F, F

Correct Answer: (2) T, F
Solution:
Step 1: Analyze the given logical statements.
Statement 1: $p \wedge q = F$ implies that at least one of p or q is False.
Statement 2: $p \to q = F$ implies that p is True and q is False.
Step 2: From Statement 2, since $p \to q$ is False, we have:

$$p = T \quad \text{and} \quad q = F$$

Conclusion: The truth values of p and q are T and F, respectively.

Quick Tip
Remember that an implication $p \to q$ is only False when p is True and q is False.

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6. The inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 3 \\ 5 & 2 & -1 \end{bmatrix}$$

is:

1. $$\frac{-1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 2 & -3 \end{bmatrix}$$

2. $$\frac{-1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

3. $$\frac{-1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

4. $$\frac{-1}{3}\begin{bmatrix} -3 & 0 & 0 \\ -3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

Correct Answer: (2)

$$\frac{-1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

Solution:
We are given the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 3 \\ 5 & 2 & -1 \end{bmatrix}$$

and we need to find its inverse.
Step 1: First, calculate the determinant of matrix A.

$$\det(A) = 1 \times \begin{vmatrix} 3 & 3 \\ 2 & -1 \end{vmatrix} - 0 \times \begin{vmatrix} 3 & 3 \\ 5 & -1 \end{vmatrix} + 0 \times \begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix}$$

$$\det(A) = 1 \times ((3 \times (-1)) - (3 \times 2)) = 1 \times (-3 - 6) = -9$$

Step 2: Now, calculate the adjoint of matrix A. The adjoint is the transpose of the cofactor matrix.
Step 3: Using the formula for the inverse of a matrix $A^{-1} = \frac{1}{\det(A)} \times \text{adj}(A)$, we find:

$$A^{-1} = \frac{1}{-9} \times \text{adj}(A)$$

Conclusion: The correct inverse matrix is:

$$A^{-1} = \frac{-1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

> Quick Tip
>
> To find the inverse of a matrix, first calculate its determinant, then find the adjoint, and apply the formula $A^{-1} = \frac{1}{\det(A)} \times \text{adj}(A)$.

PHYSICS

7. What is the equivalent logic gate when the output of an AND gate is passed through a NOT gate?

1. OR gate
2. NOR gate
3. NAND gate
4. XOR gate

Correct Answer: (3) NAND gate

Solution: Passing the output of an AND gate through a NOT gate inverts the AND operation.
Mathematically,

$$\text{Output} = \overline{A \cdot B} = A \text{ NAND } B$$

Thus, the configuration behaves as a NAND gate.

> Quick Tip
>
> A NAND gate is an AND gate followed by a NOT gate, effectively inverting the AND operation.

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8. In Young's double slit experiment, if the distance between the slits is doubled while keeping the wavelength and the distance to the screen constant, the fringe spacing will:

1. Double
2. Halve
3. Remain the same
4. Quadruple

Correct Answer: (2) Halve

Solution: The fringe spacing (β) in Young's double slit experiment is given by:

$$\beta = \frac{\lambda L}{d}$$

where:

- λ is the wavelength of light
- L is the distance from the slits to the screen
- d is the distance between the slits

If d is doubled ($d' = 2d$), while λ and L remain constant:

$$\beta' = \frac{\lambda L}{2d} = \frac{\beta}{2}$$

Thus, the fringe spacing is halved.

> Quick Tip
>
> Fringe spacing in double slit experiments is inversely proportional to the slit separation.

9. In a series LCR circuit connected to an AC source, at resonance, the current is maximum because:

1. The inductive reactance is maximum
2. The capacitive reactance cancels the inductive reactance
3. The resistance is zero
4. The reactances add up

Correct Answer: (2) The capacitive reactance cancels the inductive reactance

Solution: In a series LCR circuit, the total impedance (Z) is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where:

- R is the resistance
- $X_L = \omega L$ is the inductive reactance
- $X_C = \frac{1}{\omega C}$ is the capacitive reactance

At resonance, $X_L = X_C$, so:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2} = R$$

The impedance is minimized, leading to maximum current as per Ohm's law:

$$I = \frac{V}{Z}$$

Hence, the current is maximum because inductive and capacitive reactances cancel each other.

> Quick Tip
>
> At resonance in a series LCR circuit, inductive and capacitive reactances cancel, minimizing impedance and maximizing current.

CHEMISTRY

10. Butter is an example of which type of colloid?

1. Liquid in solid
2. Solid in liquid
3. Liquid in liquid

4. Gas in liquid

Correct Answer: (1) Liquid in solid
Solution: Butter is an example of a colloidal system where the liquid phase (water or milk) is dispersed in a solid matrix (fat). This type of colloid is referred to as "liquid in solid." In butter, the fat (solid phase) forms a continuous phase, while the water droplets are dispersed in it.

Conclusion: Therefore, butter is a "Liquid in Solid" colloid.

Quick Tip

A colloid where a liquid is dispersed in a solid phase is known as "liquid in solid," such as in butter and jelly.

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11. Which of the following techniques is most suitable for determining the size and morphology of nanoparticles?

1. UV-Vis Spectroscopy
2. Transmission Electron Microscopy (TEM)
3. Fourier Transform Infrared Spectroscopy (FTIR)
4. Atomic Absorption Spectroscopy (AAS)

Correct Answer: (2) Transmission Electron Microscopy (TEM)

Solution: Transmission Electron Microscopy (TEM) is a powerful technique used to visualize the size, shape, and morphology of nanoparticles at the nanometer scale. Unlike UV-Vis Spectroscopy, FTIR, and AAS, which provide information about optical properties, molecular vibrations, and elemental composition respectively, TEM offers direct imaging capabilities, allowing for precise determination of nanoparticle dimensions and structural characteristics.

Quick Tip

TEM provides high-resolution images that are essential for analyzing the physical structure of nanoparticles, making it the preferred method for size and morphology determination.

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12. What is the pH of a 0.01 M hydrochloric acid (HCl) solution?

1. 2
2. 4

3. 7
4. 12

Correct Answer: (1) 2

Solution: Hydrochloric acid (HCl) is a strong acid and dissociates completely in water:

$$\mathrm{HCl} \rightarrow \mathrm{H}^+ + \mathrm{Cl}^-$$

Therefore, the concentration of hydrogen ions $[H^+]$ is equal to the concentration of HCl, which is 0.01 M.
The pH is calculated using the formula:

$$\mathrm{pH} = -\log[H^+]$$

Substituting the values:

$$\mathrm{pH} = -\log(0.01) = 2$$

Thus, the pH of the solution is 2.

Quick Tip

The pH of a strong acid solution can be directly calculated using the negative logarithm of its concentration since strong acids dissociate completely in water.
