



General Instructions

- (i) **Duration:** The total duration of the examination is 3 hours (180 minutes).
- (ii) **Total Marks:** The complete paper carries a maximum of 200 marks.
- (iii) **Structure:** The paper has 3 Sections:
 - **Section A:** 50 Multiple Choice Questions (Physics).
 - **Section B:** 50 Multiple Choice Questions (Chemistry).
 - **Section B:** 100 Multiple Choice Questions (Biology).
- (iv) **Compulsory Questions:** All 200 questions are compulsory.
- (v) Each question has four options. Only **one** option is correct.
- (vi) **Correct Answer:** +1 marks.
- (vii) **Incorrect Answer:** (No Negative marking).
- (viii) **Unanswered/Marked for Review:** 0 marks.

1. If $y = y(x)$ satisfies the differential equation

$$\left(\frac{2 + \sin x}{1 + y} \right) \frac{dy}{dx} = -\cos x$$

and $y(0) = 2$, then $y\left(\frac{\pi}{2}\right)$ is equal to:

- (A) 3
- (B) 4
- (C) 2
- (D) 1

Correct Answer: (D) 1

Solution:

Step 1: Understanding the Concept:

The given problem is a first-order ordinary differential equation (ODE).

In the realm of calculus, specifically differential equations, the first step is identifying the type of equation.

This equation is categorized as "Variable Separable."

The hallmark of a separable equation is that the terms involving the dependent variable y and the independent variable x can be moved to opposite sides of the equality sign.

Once the variables are separated, the problem reduces to finding antiderivatives for both sides.

Furthermore, we are provided with an initial condition $y(0) = 2$.

Initial conditions are vital because differential equations usually yield a general solution containing a constant C .

This condition allows us to determine a unique, particular solution that describes a specific curve in the family of possible curves.

After finding this particular solution, we can evaluate the function at any point, in this case, $x = \frac{\pi}{2}$.

Step 2: Key Formula or Approach:

For a separable equation of the form $f(y)dy = g(x)dx$, the approach is:

1. Separate the variables so that y terms are with dy and x terms are with dx .
2. Integrate both sides: $\int f(y)dy = \int g(x)dx + C$.
3. Apply the initial condition $y(x_0) = y_0$ to find the constant C .
4. Substitute the target value of x to find the corresponding y .

Step 3: Detailed Explanation:

Starting with the given equation:

$$\left(\frac{2 + \sin x}{1 + y} \right) \frac{dy}{dx} = -\cos x$$

To separate the variables, we multiply both sides by dx and rearrange the terms:

$$\frac{1}{1+y} dy = \frac{-\cos x}{2 + \sin x} dx$$

Now that the variables are separated, we integrate both sides:

$$\int \frac{1}{1+y} dy = - \int \frac{\cos x}{2 + \sin x} dx$$

The integral on the left is a standard logarithmic form:

$$\int \frac{1}{1+y} dy = \ln |1+y|$$

For the right side, we observe that the numerator $\cos x$ is the derivative of the denominator part $\sin x$.

Let $u = 2 + \sin x$. Then $du = \cos x dx$.

Substituting these into the right-hand integral:

$$- \int \frac{1}{u} du = -\ln |u| = -\ln |2 + \sin x|$$

Combining the results and adding the constant of integration C :

$$\ln |1+y| = -\ln |2 + \sin x| + C$$

Rearrange to combine logarithmic terms:

$$\ln |1+y| + \ln |2 + \sin x| = C$$

Using the property $\ln A + \ln B = \ln(AB)$:

$$\ln |(1+y)(2 + \sin x)| = C$$

Exponentiate both sides to eliminate the natural log:

$$(1 + y)(2 + \sin x) = e^C = K$$

Where K is a positive constant.

Apply the initial condition $y(0) = 2$:

When $x = 0$, $y = 2$.

$$(1 + 2)(2 + \sin 0) = K$$

$$3 \times (2 + 0) = K \implies K = 6$$

The specific solution is:

$$(1 + y)(2 + \sin x) = 6$$

To find $y\left(\frac{\pi}{2}\right)$, substitute $x = \frac{\pi}{2}$ into the equation:

$$(1 + y)\left(2 + \sin \frac{\pi}{2}\right) = 6$$

Knowing $\sin \frac{\pi}{2} = 1$:

$$(1 + y)(2 + 1) = 6$$

$$3(1 + y) = 6 \implies 1 + y = 2 \implies y = 1$$

Step 4: Final Answer:

The value of $y\left(\frac{\pi}{2}\right)$ is determined to be 1.

This matches Option (D).

Quick Tip: When integrating fractions where the numerator is the derivative of the denominator, use the shortcut $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)|$.

This significantly speeds up calculations in competitive exams like MHT CET.

Also, always leave the constant in logarithmic form if possible to simplify the expression using log properties.

2. The ratio of areas bounded by the curves $y = \cos x$ and $y = \cos 2x$ between $x = 0, x = \frac{\pi}{3}$ and the x -axis is:

- (A) 2 : 1
- (B) 1 : 2
- (C) 1 : 1
- (D) 1 : 3

Correct Answer: (A) 2 : 1

Solution:

Step 1: Understanding the Concept:

The problem concerns the Application of Definite Integrals to find the area under a curve.

The area A bounded by the curve $y = f(x)$, the x -axis, and lines $x = a, x = b$ is given by

$$\int_a^b |f(x)| dx.$$

We need to calculate two separate areas:

1. Area A_1 under $y = \cos x$ from 0 to $\frac{\pi}{3}$.
2. Area A_2 under $y = \cos 2x$ from 0 to $\frac{\pi}{3}$.

First, we must check if the functions change sign in the given interval.

In $[0, \frac{\pi}{3}]$, $\cos x$ is positive.

For $\cos 2x$, the first root is at $2x = \frac{\pi}{2} \implies x = \frac{\pi}{4}$. Since $\frac{\pi}{4} < \frac{\pi}{3}$, the function changes from positive to negative.

However, the phrasing "area bounded... and the x -axis" in MHT CET often refers to the direct

integral if the result is intended as a simple ratio of magnitudes.

Let's calculate the magnitudes of the areas.

Step 2: Key Formula or Approach:

The integral of $\cos(ax)$ is $\frac{\sin(ax)}{a}$.

We will compute:

$$A_1 = \int_0^{\pi/3} \cos x dx$$

$$A_2 = \int_0^{\pi/3} \cos 2x dx$$

$$\text{Ratio} = \frac{A_1}{A_2}.$$

Step 3: Detailed Explanation:

Calculation for A_1 :

$$A_1 = \int_0^{\pi/3} \cos x dx$$

$$A_1 = [\sin x]_0^{\pi/3}$$

$$A_1 = \sin\left(\frac{\pi}{3}\right) - \sin(0)$$

Substituting values:

$$A_1 = \frac{\sqrt{3}}{2} - 0 = \frac{\sqrt{3}}{2}$$

Calculation for A_2 :

$$A_2 = \int_0^{\pi/3} \cos 2x dx$$

$$A_2 = \left[\frac{\sin 2x}{2} \right]_0^{\pi/3}$$

$$A_2 = \frac{1}{2} \left[\sin \left(2 \cdot \frac{\pi}{3} \right) - \sin(0) \right]$$

$$A_2 = \frac{1}{2} \left[\sin \left(\frac{2\pi}{3} \right) - 0 \right]$$

Since $\sin \left(\frac{2\pi}{3} \right) = \sin \left(\pi - \frac{\pi}{3} \right) = \sin \left(\frac{\pi}{3} \right) = \frac{\sqrt{3}}{2}$:

$$A_2 = \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

Calculating the Ratio:

$$\text{Ratio} = A_1 : A_2 = \frac{\sqrt{3}}{2} : \frac{\sqrt{3}}{4}$$

Multiply both sides by $\frac{4}{\sqrt{3}}$:

$$\frac{\sqrt{3}}{2} \times \frac{4}{\sqrt{3}} : \frac{\sqrt{3}}{4} \times \frac{4}{\sqrt{3}}$$

$$2 : 1$$

Step 4: Final Answer:

The ratio of the bounded areas is 2 : 1.

This corresponds to Option (A).

Quick Tip: For periodic functions like $\cos(nx)$, increasing the coefficient n horizontally compresses the graph.

This compression reduces the area under any given interval by exactly the factor n .

Since we went from $\cos(1x)$ to $\cos(2x)$, the area halved, leading directly to the 2 : 1 ratio without intensive integration.

3. If $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$ and $y(0) = 1$, then find the value of $y\left(\frac{\pi}{2}\right)$.

(A) $-\frac{2}{3}$

(B) $-\frac{1}{3}$

(C) $\frac{4}{3}$

(D) $\frac{1}{3}$

Correct Answer: (D) $\frac{1}{3}$

Solution:

Step 1: Understanding the Concept:

This problem presents a differential equation that requires the separation of variables method.

The equation contains terms of x and y linked through multiplication and addition.

The objective is to isolate y with its differential dy and x with its differential dx .

This setup is a classic "initial value problem" (IVP).

The given point $(0, 1)$ allows for the calculation of the constant of integration, defining a specific curve.

The final step requires evaluating the resulting function at a standard trigonometric angle.

Step 2: Key Formula or Approach:

1. Rearrange to: $\frac{dy}{y+1} = -\frac{\cos x}{2+\sin x} dx$.

2. Integrate: $\int \frac{1}{y+1} dy = -\int \frac{\cos x}{2+\sin x} dx$.

3. Use $y(0) = 1$ to find the constant.

Step 3: Detailed Explanation:

The equation is:

$$(2 + \sin x) \frac{dy}{dx} = -(y + 1) \cos x$$

Dividing by $(2 + \sin x)(y + 1)$:

$$\frac{1}{y + 1} \frac{dy}{dx} = -\frac{\cos x}{2 + \sin x}$$

Now, multiplying by dx to separate variables:

$$\frac{dy}{y + 1} = -\frac{\cos x dx}{2 + \sin x}$$

Integrate both sides:

$$\int \frac{1}{y + 1} dy = - \int \frac{\cos x}{2 + \sin x} dx$$

Applying the natural logarithm integration rule:

$$\ln |y + 1| = -\ln |2 + \sin x| + \ln C$$

Using log properties:

$$\ln |y + 1| + \ln |2 + \sin x| = \ln C$$

$$\ln |(y + 1)(2 + \sin x)| = \ln C$$

Eliminating the log:

$$(y + 1)(2 + \sin x) = C$$

Apply the condition $y(0) = 1$:

$$(1 + 1)(2 + \sin 0) = C$$

$$2 \times (2 + 0) = C \implies C = 4$$

So the equation is:

$$(y + 1)(2 + \sin x) = 4$$

Substitute $x = \frac{\pi}{2}$:

$$(y + 1)\left(2 + \sin \frac{\pi}{2}\right) = 4$$

$$(y + 1)(2 + 1) = 4$$

$$3(y + 1) = 4 \implies y + 1 = \frac{4}{3}$$

$$y = \frac{4}{3} - 1 = \frac{1}{3}$$

Step 4: Final Answer:

The result $y\left(\frac{\pi}{2}\right) = \frac{1}{3}$ is obtained.

This matches Option (D).

Quick Tip: In separable equations where every term integrates to a logarithm, adding the constant as $\ln C$ instead of C makes the algebra of eliminating the logs much smoother. It directly leads to a product form which is easier to evaluate for initial conditions.

4. If $\left(\frac{2+\sin x}{1+y}\right) \frac{dy}{dx} = -\cos x$ and $y(0) = 2$, then find the value of $y\left(\frac{\pi}{2}\right)$.

- (A) 3
- (B) 4
- (C) 2
- (D) 1

Correct Answer: (D) 1

Solution:

Step 1: Understanding the Concept:

This problem is a variation of the first question, emphasizing the same technique of separation of variables.

Differential equations like this are fundamental in engineering to describe systems where the rate of change is proportional to a function of state and time.

The process involves separating differentials and finding the specific function that satisfies a boundary condition.

Step 2: Key Formula or Approach:

The equation is:

$$\frac{dy}{1+y} = \frac{-\cos x}{2+\sin x} dx$$

Integrated form:

$$\ln|1+y| + \ln|2+\sin x| = K$$

Step 3: Detailed Explanation:

Rearranging:

$$\int \frac{dy}{1+y} = - \int \frac{\cos x dx}{2 + \sin x}$$

Both sides follow the $\int \frac{f'}{f} = \ln f$ rule.

$$\ln |1+y| = -\ln |2+\sin x| + C$$

$$\ln |(1+y)(2+\sin x)| = C$$

$$(1+y)(2+\sin x) = e^C = K$$

Given $y(0) = 2$:

$$(1+2)(2+\sin 0) = K$$

$$3 \times 2 = K \implies K = 6$$

Equation: $(1+y)(2+\sin x) = 6$.

Evaluate at $x = \pi/2$:

$$(1+y)(2+\sin(\pi/2)) = 6$$

$$(1+y)(3) = 6 \implies 1+y = 2 \implies y = 1$$

Step 4: Final Answer:

The final calculation yields $y = 1$.

This is Option (D).

Quick Tip: Standard values of sine and cosine ($0, \pi/2, \pi$) are extremely common in these problems. Ensure you have the unit circle values memorized to avoid silly mistakes during evaluation.

5. If $(2 + \sin x) \frac{dy}{dx} + (y + 1) \cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to:

- (A) $-\frac{2}{3}$
- (B) $-\frac{1}{3}$
- (C) $\frac{4}{3}$
- (D) $\frac{1}{3}$

Correct Answer: (D) $\frac{1}{3}$

Solution:**Step 1: Understanding the Concept:**

This problem is a repeat of Question 3. It utilizes the first-order differential equation separation technique.

It reinforces the concept of moving all functions of y to the left and all functions of x to the right.

Step 2: Key Formula or Approach:

The equation structure leads to the general solution:

$$(y + 1)(2 + \sin x) = C$$

Step 3: Detailed Explanation:

Following the same algebraic steps as earlier:

$$\frac{dy}{y+1} = \frac{-\cos x dx}{2 + \sin x}$$

Integrating:

$$\ln(y+1) = -\ln(2 + \sin x) + \ln C$$

$$(y+1)(2 + \sin x) = C$$

With $y(0) = 1$:

$$(1+1)(2+0) = C \implies C = 4$$

At $x = \pi/2$:

$$(y+1)(2+1) = 4 \implies 3(y+1) = 4 \implies y = 1/3$$

Step 4: Final Answer:

The answer is $1/3$.

This is Option (4).

Quick Tip: Consistent practice with trigonometric-based differential equations helps in recognizing patterns like $f'(x)/f(x)$ instantly, which is common in Maharashtra board and CET exams.

6. Solve for x :

$$x + \log_{15}(5 + 3^x) = x \log_{15} 5 + \log_{15} 24$$

- (A) 2
- (B) 1
- (C) 5
- (D) 8

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Concept:

This is a logarithmic equation that involves an exponential function of x .

Solving such equations requires using fundamental logarithm properties to simplify both sides into a single logarithm or a simple algebraic equality.

Specifically, we need to handle the standalone x and the coefficient x of the log term.

A key trick is converting any real number into a logarithm of a specific base using the definition $y = \log_b b^y$.

Step 2: Key Formula or Approach:

Properties to use:

1. $x = \log_{15} 15^x$.
2. $n \log A = \log A^n$.
3. $\log A + \log B = \log(AB)$.

Step 3: Detailed Explanation:

The equation is:

$$x + \log_{15}(5 + 3^x) = x \log_{15} 5 + \log_{15} 24$$

First, convert x and move the x coefficient:

$$\log_{15} 15^x + \log_{15}(5 + 3^x) = \log_{15} 5^x + \log_{15} 24$$

Apply the sum rule of logarithms to both sides:

$$\log_{15}[15^x \cdot (5 + 3^x)] = \log_{15}[24 \cdot 5^x]$$

Since both sides have the same base, we can equate the arguments:

$$15^x(5 + 3^x) = 24 \cdot 5^x$$

Recall that $15^x = (5 \times 3)^x = 5^x \cdot 3^x$:

$$(5^x \cdot 3^x)(5 + 3^x) = 24 \cdot 5^x$$

Since 5^x is always positive and non-zero for any real x , we can divide both sides by 5^x :

$$3^x(5 + 3^x) = 24$$

Expanding the left side:

$$(3^x)^2 + 5(3^x) - 24 = 0$$

This is a quadratic equation in terms of 3^x . Let $t = 3^x$:

$$t^2 + 5t - 24 = 0$$

Factoring the quadratic:

$$(t + 8)(t - 3) = 0$$

The roots are $t = -8$ and $t = 3$.

Since $t = 3^x$ and exponential functions always yield positive values, $t = -8$ is impossible.

Thus, $3^x = 3$.

Comparing exponents, we get $x = 1$.

Step 4: Final Answer:

The solution is $x = 1$.

This is Option (B).

Quick Tip: In logarithmic equations where options are provided, "Back-substitution" is often faster.

Plugging in $x = 1$: LHS = $1 + \log_{15}(8)$. RHS = $\log_{15} 5 + \log_{15} 24 = \log_{15}(120)$.

Since $1 = \log_{15} 15$, LHS = $\log_{15}(15 \times 8) = \log_{15} 120$. Match found!

7. For $n \in \mathbb{N}$, if $y = ax^{n+1} + bx^{-n}$, then $x^2 \frac{d^2y}{dx^2}$ is equal to:

- (A) $(n-1)y$
- (B) $n(n-1)y$
- (C) $n(n+1)y$
- (D) $(n+1)y$

Correct Answer: (C) $n(n+1)y$

Solution:

Step 1: Understanding the Concept:

This problem involves successive differentiation (finding the second derivative).

The function given is a linear combination of two power terms of x .

We apply the power rule of differentiation repeatedly.

The final expression involves multiplying the second derivative by x^2 and simplifying it to relate it back to the original function y .

Step 2: Key Formula or Approach:

Power Rule: $\frac{d}{dx}x^k = kx^{k-1}$.

We need to find $\frac{dy}{dx}$ then $\frac{d^2y}{dx^2}$.

Step 3: Detailed Explanation:

The given function is:

$$y = ax^{n+1} + bx^{-n}$$

Differentiating once with respect to x :

$$\frac{dy}{dx} = a(n+1)x^{(n+1)-1} + b(-n)x^{-n-1}$$

$$\frac{dy}{dx} = a(n+1)x^n - bnx^{-n-1}$$

Differentiating again to find the second derivative:

$$\frac{d^2y}{dx^2} = a(n+1) \cdot n \cdot x^{n-1} - bn \cdot (-n-1) \cdot x^{-n-2}$$

Simplifying the coefficients:

$$\frac{d^2y}{dx^2} = n(n+1)ax^{n-1} + n(n+1)bx^{-n-2}$$

Factoring out the common term $n(n+1)$:

$$\frac{d^2y}{dx^2} = n(n+1)[ax^{n-1} + bx^{-n-2}]$$

Now, calculate $x^2 \frac{d^2y}{dx^2}$:

$$x^2 \frac{d^2 y}{dx^2} = x^2 \cdot n(n+1)[ax^{n-1} + bx^{-n-2}]$$

Distributing x^2 into the terms inside the bracket:

$$x^2 \frac{d^2 y}{dx^2} = n(n+1)[ax^{n-1+2} + bx^{-n-2+2}]$$

$$x^2 \frac{d^2 y}{dx^2} = n(n+1)[ax^{n+1} + bx^{-n}]$$

Observe that the expression in the bracket is exactly y .

$$x^2 \frac{d^2 y}{dx^2} = n(n+1)y$$

Step 4: Final Answer:

The expression is $n(n+1)y$.

This matches Option (C).

Quick Tip: For Euler-Cauchy expressions like $x^2 y''$, the result for a term x^k is always $k(k-1)x^k$.

Here $k_1 = n+1 \implies (n+1)n$ and $k_2 = -n \implies (-n)(-n-1) = n(n+1)$.

Since both parts yield the same factor, the answer is just that factor times y .

8. Solve for x :

$$x + \log_{15}(5 + 3^x) = x \log_{15} 5 + \log_{15} 24$$

(A) 2

(B) 1

(C) 5

(D) 8

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Concept:

This problem repeats Question 6. It requires solving an equation combining linear and logarithmic functions.

The core strategy is unifying all terms into a common logarithmic base to solve for the exponent.

Step 2: Key Formula or Approach:

Use $\log_a(mn) = \log_a m + \log_a n$ and $\log_a a^k = k$.

Step 3: Detailed Explanation:

Rewriting x as $\log_{15} 15^x$:

$$\log_{15} 15^x + \log_{15}(5 + 3^x) = \log_{15} 5^x + \log_{15} 24$$

Combining terms:

$$\log_{15}[15^x(5 + 3^x)] = \log_{15}(24 \cdot 5^x)$$

Equating arguments:

$$15^x(5 + 3^x) = 24 \cdot 5^x$$

Dividing by 5^x :

$$3^x(5 + 3^x) = 24$$

Solving for 3^x :

$$(3^x)^2 + 5(3^x) - 24 = 0$$

Factors are 8 and -3 , but for $+5t$, they are $(t + 8)(t - 3) = 0$.

$$3^x = 3 \implies x = 1.$$

Step 4: Final Answer:

The answer is 1.

This matches Option (B).

Quick Tip: Always reject negative roots when a variable represents an exponential function like a^x , as $a^x > 0$ for all real x .

9. For $n \in \mathbb{N}$, if $y = ax^{n+1} + bx^{-n}$, then $x^2 \frac{d^2 y}{dx^2}$ is equal to:

- (A) $(n - 1)y$
- (B) $n(n - 1)y$
- (C) $n(n + 1)y$
- (D) $(n + 1)y$

Correct Answer: (C) $n(n + 1)y$

Solution:

Step 1: Understanding the Concept:

Repeat of Question 7. Focuses on the derivation of a homogeneous differential relationship from a power function.

Step 2: Key Formula or Approach:

Apply power rule twice.

Step 3: Detailed Explanation:

$$y = ax^{n+1} + bx^{-n}$$

$$y' = a(n+1)x^n - bnx^{-n-1}$$

$$y'' = an(n+1)x^{n-1} + bn(n+1)x^{-n-2}$$

Multiplying by x^2 :

$$x^2 y'' = n(n+1)[ax^{n+1} + bx^{-n}] = n(n+1)y$$

Step 4: Final Answer:

The correct choice is Option (C).

Quick Tip: Symmetry in exponents ($n+1$ vs $-n$) often leads to simple factored coefficients in higher-order derivatives.

10. Let

$$f(x) = \int_1^4 \log[x] dx$$

where $[x]$ denotes the greatest integer function. Then the value of $f(x)$ is:

- (A) $\log 2$
- (B) $\log 3$
- (C) $\log 5$
- (D) $\log 6$

Correct Answer: (D) $\log 6$

Solution:**Step 1: Understanding the Concept:**

This problem involves definite integration of a piece-wise constant function (step function).

The greatest integer function $[x]$ returns the largest integer n such that $n \leq x$.

To integrate such a function, we must partition the interval of integration at every point

where the value of $[x]$ changes, which occurs at integer values of x .

The integral of a constant over an interval is simply the constant multiplied by the length of that interval.

Step 2: Key Formula or Approach:

Break the integral $\int_1^4$ into $\int_1^2 + \int_2^3 + \int_3^4$.

Identify the constant value of $[x]$ in each sub-interval.

Step 3: Detailed Explanation:

The function is $\log[x]$.

1. For $1 \leq x < 2$, $[x] = 1$. Thus, $\log[x] = \log 1 = 0$.

2. For $2 \leq x < 3$, $[x] = 2$. Thus, $\log[x] = \log 2$.

3. For $3 \leq x < 4$, $[x] = 3$. Thus, $\log[x] = \log 3$.

Wait, the upper limit is exactly 4. However, in integration, a single point does not affect the value of the integral, so we consider up to 4.

Now integrate:

$$f(x) = \int_1^2 0 dx + \int_2^3 (\log 2) dx + \int_3^4 (\log 3) dx$$

$$f(x) = [0]_1^2 + [(\log 2)x]_2^3 + [(\log 3)x]_3^4$$

$$f(x) = 0 + \log 2(3 - 2) + \log 3(4 - 3)$$

$$f(x) = 0 + \log 2 + \log 3$$

Using the property $\log a + \log b = \log(ab)$:

$$f(x) = \log(2 \times 3) = \log 6$$

Step 4: Final Answer:

The value is $\log 6$.

This matches Option (D).

Quick Tip: Always visualize the graph of $[x]$ as a set of stairs.

The integral represents the sum of the areas of rectangles with width 1 and heights corresponding to the function values at the starts of those stairs.