MHT CET 2026 May 20 Shift 2

Question Paper with Solutions

Conducted by CET Cell, Maharashtra

General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 200 marks.

(iii) **Structure:** The paper has 3 Sections:

- **Section A:** 50 Multiple Choice Questions (Physics).
- **Section B:** 50 Multiple Choice Questions (Chemistry).
- **Section B:** 100 Multiple Choice Questions (Biology).

(iv) **Compulsory Questions:** All 200 questions are compulsory.

(v) Each question has four options. Only **one** option is correct.

(vi) Correct Answer: +1 marks.

(vii) **Incorrect Answer:** (No Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

1. Given $f(x) = x - 1$, $h(1) = 4$, and $h'(1) = -2$. If $g(x) = (f[4(h(x)) + 3])^2$, find the value of $g'(1)$.

(A) -288

(B) 288

(C) -144

(D) 144

Correct Answer: (A) -288

Solution:

Step 1: Understanding the Concept:

The core of this problem lies in the differentiation of composite functions, often referred to as "function of a function."

In mathematical analysis, when a function depends on another variable that is itself a function of x, we cannot apply standard differentiation rules directly to the innermost variable.

Instead, we must peel back the layers of the function one by one, from the outermost to the innermost.

Here, the function $g(x)$ is built through multiple stages: an innermost function $h(x)$, followed by a linear transformation $4h(x)+3$, then the application of function f, and finally a squaring operation.

The Chain Rule provides the necessary framework to handle these nested dependencies by multiplying the derivatives of each successive layer.

Step 2: Key Formula or Approach:

The primary tool used here is the Chain Rule for differentiation:

$$\frac{d}{dx}[F(G(x))] = F'(G(x)) \cdot G'(x)$$

Additionally, we utilize the power rule for the square:

$$\frac{d}{dx}[u(x)]^n = n \cdot [u(x)]^{n-1} \cdot u'(x)$$

The problem can be simplified significantly by substituting the expression for $f(x)$ into $g(x)$ before differentiating, which reduces the number of "links" in our chain.

Step 3: Detailed Explanation:

Let us first simplify the expression for the function $g(x)$ to make the differentiation process more manageable.

We are given $f(x) = x - 1$. This means whatever argument we provide to f, the output will be that argument minus one.

The argument provided to f in the definition of $g(x)$ is $[4h(x)+3]$.
Applying the definition of f:

$$f[4h(x)+3] = (4h(x)+3) - 1$$

$$f[4h(x)+3] = 4h(x)+2$$

Now, we can rewrite $g(x)$ using this simplified internal term:

$$g(x) = (4h(x)+2)^2$$

To find $g'(x)$, we differentiate this expression with respect to x.
According to the power rule and chain rule, if we have $g(x) = [u(x)]^2$, then $g'(x) = 2 \cdot u(x) \cdot u'(x)$.
In our case, let $u(x) = 4h(x)+2$.
The derivative of $u(x)$ is:

$$u'(x) = \frac{d}{dx}(4h(x)+2) = 4h'(x) + 0 = 4h'(x)$$

Substituting these back into the formula for $g'(x)$:

$$g'(x) = 2 \cdot (4h(x)+2) \cdot (4h'(x))$$

$$g'(x) = 8(4h(x)+2)h'(x)$$

Now, we are required to find the value of the derivative at the point $x = 1$.
Substituting $x = 1$ into the derivative formula:

$$g'(1) = 8(4h(1)+2)h'(1)$$

From the problem statement, we have the values: $h(1) = 4$ and $h'(1) = -2$.
Plugging these values into our expression:

$$g'(1) = 8(4(4) + 2)(-2)$$

First, calculate the value inside the parentheses:

$$4 \times 4 = 16$$

$$16 + 2 = 18$$

Now, multiply the remaining terms:

$$g'(1) = 8 \times 18 \times (-2)$$

$$8 \times 18 = 144$$

$$144 \times (-2) = -288$$

The final numerical result is -288.

Step 4: Final Answer:
By evaluating the nested composition of functions and applying the Chain Rule, we determined that $g'(1) = -288$. This matches Option (A).

Quick Tip: Always look for opportunities to simplify the function algebraically before differentiating. By substituting $f(x) = x - 1$ directly, we turned a triple-composite function into a double-composite function, which is much easier to differentiate and less prone to arithmetic errors.

---

2. The area in sq. units bounded by the curve $y = 2\sqrt{1-x^2}$ and the x-axis is :

(A) π

(B) 2π

(C) $\pi/2$

(D) 4π

Correct Answer: (A) π

Solution:

Step 1: Understanding the Concept:

Area calculation in coordinate geometry can be performed using definite integrals, but identifying standard geometric shapes can often provide a more direct path.

The equation $y = 2\sqrt{1-x^2}$ belongs to a family of curves derived from the standard equations of conic sections, specifically ellipses and circles.

Since the square root symbol represents the principal (non-negative) root, the value of y is always greater than or equal to zero for all real values of x.

This indicates that the curve represents only the portion of the geometric figure that lies on or above the x-axis.

The boundaries of the region are the curve itself and the x-axis ($y = 0$).

Step 2: Key Formula or Approach:

The standard equation of an ellipse centered at the origin is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The total area of a complete ellipse is given by:

$$\text{Total Area} = \pi ab$$

Where a and b are the lengths of the semi-major and semi-minor axes.
Since we are dealing with the region bounded by the curve and the x-axis for $y \geq 0$, we are looking for the area of exactly half of the ellipse.

Step 3: Detailed Explanation:
Let's analyze the given equation to identify the parameters of the ellipse.
Given: $y = 2\sqrt{1-x^2}$
To remove the radical sign, we square both sides of the equation:

$$y^2 = (2\sqrt{1-x^2})^2$$

$$y^2 = 4(1-x^2)$$

Now, distribute the 4 into the terms inside the parentheses:

$$y^2 = 4 - 4x^2$$

Move the term involving x to the left side to align with standard conic form:

$$4x^2 + y^2 = 4$$

To obtain the standard ellipse form where the constant on the right is 1, divide every term in the equation by 4:

$$\frac{4x^2}{4} + \frac{y^2}{4} = \frac{4}{4}$$

$$\frac{x^2}{1} + \frac{y^2}{4} = 1$$

Comparing this result to the standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we can identify the semi-axis lengths:

$$a^2 = 1 \implies a = 1$$

$$b^2 = 4 \implies b = 2$$

The total area of the complete ellipse would be:

$$\text{Total Area} = \pi \times a \times b = \pi \times 1 \times 2 = 2\pi$$

Recall that our original function $y = 2\sqrt{1-x^2}$ is only defined for $y \geq 0$.
The region bounded by this curve and the x-axis is the upper half of the ellipse.
Therefore, the required area is:

$$\text{Required Area} = \frac{1}{2} \times \text{Total Area}$$

$$\text{Required Area} = \frac{1}{2} \times 2\pi = \pi$$

The area is π square units.

Step 4: Final Answer:
By recognizing the curve as the upper half of an ellipse with semi-axes 1 and 2, we calculated the total area as 2π and correctly identified the bounded region's area as π. This corresponds to Option (A).

Quick Tip: Recognizing standard curves can save significant time in competitive exams like MHT CET. Whenever you see $y = b\sqrt{1-(x/a)^2}$, it always represents the upper half of an ellipse with area $\frac{\pi ab}{2}$. If $a = b$, it simplifies to a semicircle with area $\frac{\pi a^2}{2}$.

---

3. The objective function is $Z = 3x + 5y$. The difference between the maximum and minimum values of Z is $3a$, subject to constraints: $x + y \geq 1, 5x + 10y \leq 50, x, y \geq 0$. Find the value of a.

(A) 3

(B) 6

(C) 9

(D) 27

Correct Answer: (C) 9

Solution:

Step 1: Understanding the Concept:

Linear Programming Problems (LPP) focus on optimizing a linear objective function within a set of linear constraints.

A fundamental principle of LPP is that the optimal solution (both maximum and minimum) always occurs at one of the vertices (corner points) of the feasible region.

The feasible region is the area on a coordinate plane where all the inequality constraints are satisfied simultaneously.

In this problem, we need to define the boundary lines, find their intersections to identify the corner points, evaluate the objective function Z at each point, and then use the difference to solve for a.

Step 2: Key Formula or Approach:

The boundary lines are obtained by converting the inequalities into equations:

Line 1: $x + y = 1$

Line 2: $5x + 10y = 50$ (which can be simplified by dividing by 5: $x + 2y = 10$)

The constraints $x, y \geq 0$ restrict the feasible region to the first quadrant.

We will find the x and y intercepts for each line to plot them and find the vertices.

Step 3: Detailed Explanation:

Let's find the intercepts for the boundary lines.

For Line 1: $x + y = 1$

If $x = 0$, then $y = 1$. Point is $(0, 1)$.

If $y = 0$, then $x = 1$. Point is $(1, 0)$.

For Line 2: $x + 2y = 10$

If $x = 0$, then $2y = 10 \implies y = 5$. Point is $(0, 5)$.

If $y = 0$, then $x = 10$. Point is $(10, 0)$.

The region is defined by $x + y \geq 1$ (outside the line relative to origin) and $x + 2y \leq 10$ (inside the line relative to origin).

Combined with $x, y \geq 0$, the vertices of the feasible region are:

$A(1, 0), B(10, 0), C(0, 5)$, and $D(0, 1)$.

Now, we evaluate the objective function $Z = 3x + 5y$ at each of these four corner points:

1. At $A(1, 0)$: $Z = 3(1) + 5(0) = 3$
2. At $B(10, 0)$: $Z = 3(10) + 5(0) = 30$
3. At $C(0, 5)$: $Z = 3(0) + 5(5) = 25$
4. At $D(0, 1)$: $Z = 3(0) + 5(1) = 5$

From these evaluations, we identify the extremes:

Maximum value $Z_{\text{max}} = 30$ at point B.

Minimum value $Z_{\text{min}} = 3$ at point A.

The problem states that the difference between the maximum and minimum values is $3a$.

Calculation of the difference:

$$Z_{\text{max}} - Z_{\text{min}} = 30 - 3 = 27$$

Setting this equal to the given expression:

$$3a = 27$$

Dividing both sides by 3:

$$a = 9$$

Thus, the value of a is 9.

Step 4: Final Answer:
By identifying the vertices of the feasible region and calculating the objective function's values at those points, we found the range to be 27. Equating this to $3a$ gave $a = 9$. This matches Option (C).

Quick Tip: In LPP, you don't need to test every point in the region, only the corner points.
If the region is bounded (like a polygon), the max and min values are guaranteed to exist at vertices.
Always check for simplified versions of inequalities (like dividing $5x + 10y \leq 50$ by 5) to make intercept calculation effortless.

---

4. The surrounding temperature is 20°C. A body cools from 100°C to 60°C in 20 minutes. Find the time taken for the body to cool down to 30°C.

(A) 40 minutes
(B) 50 minutes
(C) 60 minutes
(D) 80 minutes

Correct Answer: (C) 60 minutes

Solution:

Step 1: Understanding the Concept:
This problem is an application of Newton's Law of Cooling, which describes how the temperature of an object changes over time when it is placed in an environment with a different temperature.
The law states that the rate of change of temperature ($\frac{dT}{dt}$) is directly proportional to the difference between the object's current temperature (T) and the constant surrounding

temperature (T_s).

Mathematically, this relationship leads to an exponential decay of the temperature difference over time.

The colder the object gets, the slower it continues to cool because the temperature gradient decreases.

Step 2: Key Formula or Approach:

The differential equation is $\frac{dT}{dt} = -k(T - T_s)$.

The solution to this equation in terms of logarithms is:

$$\ln\left(\frac{T - T_s}{T_0 - T_s}\right) = -kt$$

Where:

T_s = surrounding temperature

T_0 = initial temperature

T = temperature after time t

k = cooling constant

Alternatively, we can use the property that the ratios of temperature differences over equal intervals of time follow a geometric progression.

Step 3: Detailed Explanation:

Given constants: $T_s = 20°\text{C}$.

Initial condition: At $t = 0$, $T_0 = 100°\text{C}$.

First cooling stage: At $t_1 = 20$ minutes, $T = 60°\text{C}$.

Let's find the constant k using the first set of data.

Using the formula:

$$\ln\left(\frac{60 - 20}{100 - 20}\right) = -k(20)$$

$$\ln\left(\frac{40}{80}\right) = -20k$$

$$\ln\left(\frac{1}{2}\right) = -20k$$

Since $\ln(1/2) = -\ln 2$, we have:

$$-\ln 2 = -20k \implies k = \frac{\ln 2}{20}$$

Now, we need to find the total time t when the temperature reaches 30°C.

Using the formula again from the starting point:

$$\ln\left(\frac{30-20}{100-20}\right) = -kt$$

$$\ln\left(\frac{10}{80}\right) = -kt$$

$$\ln\left(\frac{1}{8}\right) = -kt$$

We know that $\frac{1}{8} = \left(\frac{1}{2}\right)^3$, so:

$$\ln\left(\frac{1}{2}\right)^3 = -kt$$

$$3\ln\left(\frac{1}{2}\right) = -kt$$

Substitute the value of $\ln(1/2) = -20k$ back into the equation:

$$3(-20k) = -kt$$

$$-60k = -kt$$

Dividing both sides by $-k$:

$$t = 60 \text{ minutes}$$

This means the total time elapsed from the beginning is 60 minutes.

Step 4: Final Answer:

By applying the exponential decay model of Newton's Law of Cooling and recognizing the logarithmic ratio between successive temperature differences, we calculated that the body takes 60 minutes to reach 30°C. This corresponds to Option (C).

Quick Tip: Always work with "Temperature Difference" $(T - T_s)$ rather than raw temperatures.
Notice that the difference goes from 80 $(100 - 20)$ to 40 $(60 - 20)$ in 20 mins. The difference halved.
To reach 30°C, the difference must be 10 $(30 - 20)$.
Halving 80 three times gives $80 \to 40 \to 20 \to 10$.
If it takes 20 mins for one halving, it takes $20 \times 3 = 60$ mins for three.

---

5. For the parabola $y^2 = 16x$, find the distance between the focus and the directrix.

(A) 2
(B) 4
(C) 8
(D) 16

Correct Answer: (C) 8

Solution:

Step 1: Understanding the Concept:

A parabola is defined geometrically as the set of all points that are equidistant from a fixed point called the focus and a fixed straight line called the directrix.

For a standard parabola of the form $y^2 = 4ax$, the vertex is at the origin $(0, 0)$.

The focus is located on the axis of symmetry (in this case, the x-axis) at a distance of 'a' units from the vertex in the direction the parabola opens.

The directrix is a line perpendicular to the axis of symmetry, located at the same distance 'a' from the vertex but in the opposite direction.

Consequently, the distance between the focus and the directrix is exactly twice the distance from the vertex to either one.

Step 2: Key Formula or Approach:

1. Compare the given equation $y^2 = 16x$ with the standard form $y^2 = 4ax$.
2. Identify the value of the parameter a.
3. Use the focus-directrix distance formula: $d = 2a$.

Step 3: Detailed Explanation:

The given equation of the parabola is:

$$y^2 = 16x$$

Comparing this with the standard form:

$$y^2 = 4ax$$

We equate the coefficients of x:

$$4a = 16$$

Dividing both sides by 4:

$$a = 4$$

Based on the properties of a standard parabola $y^2 = 4ax$:
- The coordinates of the focus S are $(a, 0) = (4, 0)$.
- The equation of the directrix line is $x = -a \implies x = -4$.

We want to find the distance between the point $(4, 0)$ and the line $x = -4$.
Since the focus is a point and the directrix is a vertical line, the distance is simply the absolute difference between the x-coordinates:

$$\text{Distance} = |x_{\text{focus}} - x_{\text{directrix}}|$$

$$\text{Distance} = |4 - (-4)|$$

$$\text{Distance} = |4 + 4| = 8$$

Alternatively, we can use the direct shortcut:

$$\text{Distance between focus and directrix} = 2a$$

Substituting $a = 4$:

$$\text{Distance} = 2 \times 4 = 8 \text{ units}$$

This result matches Option (C).

Step 4: Final Answer:
By identifying that $a = 4$ for the parabola $y^2 = 16x$, we applied the formula for the focus-directrix distance $(2a)$ to find the result is 8. This corresponds to Option (C).

Quick Tip: For any standard parabola $y^2 = 4ax$ or $x^2 = 4ay$:

- Distance from vertex to focus $= a$

- Distance from vertex to directrix $= a$

- Distance from focus to directrix $= 2a$

- Length of latus rectum $= 4a$

The distance between focus and directrix is always exactly half the length of the latus rectum.

---

6. The lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ intersect at a point, where $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{i} - \hat{k}$. Find the point of intersection.

(A) $(2, 1, -1)$
(B) $(1, 1, -1)$
(C) $(2, 0, -1)$
(D) $(1, 0, -1)$

Correct Answer: (A) $(2, 1, -1)$

Solution:

Step 1: Understanding the Concept:

Vector product equations can often be simplified into standard line equations in parametric form.

The equation $\vec{r} \times \vec{m} = \vec{c} \times \vec{m}$ can be rewritten as $(\vec{r} - \vec{c}) \times \vec{m} = 0$.

This implies that the vector $(\vec{r} - \vec{c})$ is parallel to the vector $\vec{m}$.

Therefore, $\vec{r} - \vec{c} = \lambda \vec{m}$, or $\vec{r} = \vec{c} + \lambda \vec{m}$.

This is the parametric form of a line passing through point $\vec{c}$ and parallel to direction vector $\vec{m}$.

We have two such lines, and we need to find the specific values of the parameters for each line that yield the same position vector $\vec{r}$.

Step 2: Key Formula or Approach:

For Line 1 (L_1): $\vec{r} \times \vec{a} = \vec{b} \times \vec{a} \implies \vec{r} = \vec{b} + \lambda\vec{a}$.

For Line 2 (L_2): $\vec{r} \times \vec{b} = \vec{a} \times \vec{b} \implies \vec{r} = \vec{a} + \mu\vec{b}$.

We convert vectors $\vec{a}$ and $\vec{b}$ into coordinate form:

$\vec{a} = (1,1,0)$

$\vec{b} = (1,0,-1)$

Step 3: Detailed Explanation:

Write the parametric equations for both lines.

For L_1:

$\vec{r} = (1,0,-1) + \lambda(1,1,0)$

$\vec{r} = (1+\lambda, \lambda, -1)$

For L_2:

$\vec{r} = (1,1,0) + \mu(1,0,-1)$

$\vec{r} = (1+\mu, 1, -\mu)$

At the point of intersection, the coordinates of L_1 and L_2 must be identical.

Comparing corresponding components:

x-components: $1+\lambda = 1+\mu \implies \lambda = \mu$

y-components: $\lambda = 1$

z-components: $-1 = -\mu \implies \mu = 1$

All three equations lead to a consistent solution: $\lambda = 1$ and $\mu = 1$.

Now, substitute $\lambda = 1$ into the equation for L_1 (or $\mu = 1$ into L_2) to find the intersection point:

$\vec{r} = (1+1, 1, -1) = (2,1,-1)$

The point of intersection is $(2,1,-1)$.

This matches Option (A).

Step 4: Final Answer:

By converting the cross-product line equations into parametric form, we found that both lines contain the point $(2,1,-1)$ when their respective parameters are set to 1. This corresponds to Option (A).

Quick Tip: If you see $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$, the intersection point is always simply the sum of the direction vectors: $\vec{r} = \vec{a} + \vec{b}$.

Calculation: $(1, 1, 0) + (1, 0, -1) = (2, 1, -1)$.

This shortcut works because the intersection occurs at $\lambda = 1, \mu = 1$ in the simplified parametric forms.

---

7. The statement $\neg(p \leftrightarrow q)$ is logically equivalent to:

(A) $\neg p \leftrightarrow \neg q$

(B) $\neg p \rightarrow q$

(C) $\neg(p \rightarrow \neg q)$

(D) $p \leftrightarrow \neg q$

Correct Answer: (D) $p \leftrightarrow \neg q$

Solution:

Step 1: Understanding the Concept:

In mathematical logic, the biconditional statement $p \leftrightarrow q$ (if and only if) is true only when both p and q share the same truth value.

The negation of this statement, $\neg(p \leftrightarrow q)$, therefore denotes that p and q must have different truth values.

This logical structure corresponds to the "Exclusive OR" (XOR) operation, which returns True only if exactly one of the inputs is True.

We can evaluate equivalence by constructing truth tables or by applying established laws of propositional logic.

Step 2: Key Formula or Approach:

The truth table for the base statement $p \leftrightarrow q$ is:

- $T \leftrightarrow T = T$
- $T \leftrightarrow F = F$
- $F \leftrightarrow T = F$
- $F \leftrightarrow F = T$

Negating these results gives:

- $\neg(T \leftrightarrow T) = F$
- $\neg(T \leftrightarrow F) = T$
- $\neg(F \leftrightarrow T) = T$
- $\neg(F \leftrightarrow F) = F$

We seek an option that produces this exact sequence of results.

Step 3: Detailed Explanation:

Let's analyze Option (D): $p \leftrightarrow \neg q$.

This statement asserts that p is true if and only if q is false.

Let's check its truth values:

1. If $p = T, q = T$: then $\neg q = F$. Statement becomes $T \leftrightarrow F$, which is F.
2. If $p = T, q = F$: then $\neg q = T$. Statement becomes $T \leftrightarrow T$, which is T.
3. If $p = F, q = T$: then $\neg q = F$. Statement becomes $F \leftrightarrow F$, which is T.
4. If $p = F, q = F$: then $\neg q = T$. Statement becomes $F \leftrightarrow T$, which is F.

The truth values for Option (D) are F, T, T, F.

Comparing these to our negated biconditional truth values (F, T, T, F), we see they are an exact match.

Alternatively, using logical properties:

$\neg(p \leftrightarrow q) \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$

$p \leftrightarrow \neg q \equiv (p \rightarrow \neg q) \wedge (\neg q \rightarrow p) \equiv (\neg p \vee \neg q) \wedge (q \vee p) \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$

The equivalence is proven.

Step 4: Final Answer:

By evaluating the truth tables, we confirmed that the negation of the biconditional requires p and q to have opposite truth values. Option (D) expresses this same condition. This corresponds to Option (D).

Quick Tip: A handy rule for biconditionals: negating the whole statement is the same as negating exactly one side of the statement.

$\neg(p \leftrightarrow q) \equiv (\neg p \leftrightarrow q) \equiv (p \leftrightarrow \neg q)$.

Negating both sides, as in Option (A), results in the original statement: $\neg p \leftrightarrow \neg q \equiv p \leftrightarrow q$.

8. If $A = \begin{bmatrix} \sec\theta & -\tan\theta \\ -\tan\theta & \sec\theta \end{bmatrix}$ and $A + \text{adj}\, A = 4I$, then find the value of θ.

(A) $\pi/6$

(B) $\pi/4$

(C) 0

(D) $\pi/3$

Correct Answer: (D) $\pi/3$

Solution:

Step 1: Understanding the Concept:

The adjoint (or adjugate) of a square matrix is the transpose of its cofactor matrix.

For a 2×2 matrix, there is a simple pattern: swap the diagonal elements and multiply the off-diagonal elements by -1.

Matrix addition is performed by adding the corresponding elements of the matrices.

The identity matrix I of order 2 is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

The scalar multiplication of a matrix by a constant (like 4) multiplies every element of the matrix by that constant.

By equating the resulting matrix on the left to the scalar identity matrix on the right, we can solve for the unknown parameter θ.

Step 2: Key Formula or Approach:

If $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then adj $M = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

The equation to solve is $A + \text{adj}\, A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$.

Step 3: Detailed Explanation:

Let's find the adjoint of matrix $A = \begin{bmatrix} \sec\theta & -\tan\theta \\ -\tan\theta & \sec\theta \end{bmatrix}$.

Following the rule:

- Swap diagonal elements ($\sec\theta$ with $\sec\theta$, no change).

- Multiply off-diagonal elements by -1: $-(-\tan\theta) = \tan\theta$.

So, $\text{adj } A = \begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{bmatrix}$.

Now, calculate the sum $A + \text{adj } A$:

$$\begin{bmatrix} \sec\theta & -\tan\theta \\ -\tan\theta & \sec\theta \end{bmatrix} + \begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{bmatrix} = \begin{bmatrix} \sec\theta + \sec\theta & -\tan\theta + \tan\theta \\ -\tan\theta + \tan\theta & \sec\theta + \sec\theta \end{bmatrix}$$

$$A + \text{adj } A = \begin{bmatrix} 2\sec\theta & 0 \\ 0 & 2\sec\theta \end{bmatrix}$$

We are given that this sum equals $4I$:

$$\begin{bmatrix} 2\sec\theta & 0 \\ 0 & 2\sec\theta \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

Equating the corresponding diagonal elements:

$$2\sec\theta = 4$$

$$\sec\theta = 2$$

In terms of cosine, this is:

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{2}$$

For the standard principal values, we know that:

$$\cos\theta = \frac{1}{2} \implies \theta = \frac{\pi}{3} \text{ (or } 60^\circ)$$

This matches Option (D).

Step 4: Final Answer:

By summing matrix A and its adjoint, the off-diagonal terms cancelled out, and the diagonal terms became $2\sec\theta$. Equating this to the diagonal of $4I$ led to $\theta = \pi/3$. This corresponds to Option (D).

Quick Tip: For a matrix with equal diagonal elements ($a = d$) and negative-symmetric off-diagonal elements ($b = c$), the sum $A + \text{adj}\,A$ will always result in a diagonal matrix where the entries are $2a$. This happens because the off-diagonal terms in A ($-\tan\theta$) are exactly canceled by the off-diagonal terms in $\text{adj}\,A$ ($\tan\theta$).

---

9. Let $f(x) = \sqrt{x^2+1}$, $g(x) = \frac{x+1}{x^2+1}$, $h(x) = 2x - 3$. Then, $f'(h'(g'(x))) = ?$

(A) $2/\sqrt{5}$
(B) $-2/\sqrt{5}$
(C) 0
(D) $1/\sqrt{5}$

Correct Answer: (A) $2/\sqrt{5}$

Solution:

Step 1: Understanding the Concept:

This question tests the ability to evaluate derivatives of multiple nested functions.
The notation $f'(h'(g'(x)))$ means we must find the derivative of f, and evaluate it at a value given by the derivative of h.
The argument of h' is itself the derivative of $g(x)$.
In such nested expressions, it is critical to observe whether any of the intermediate derivatives are constant functions.

If a derivative $\phi'(x)$ is a constant, its value does not change regardless of what is passed into it. This often significantly simplifies "stacked" functional problems.

Step 2: Key Formula or Approach:

We evaluate from the inside out:

1. Find $g'(x)$.
2. Find $h'(g'(x))$.
3. Find $f'(h'(g'(x)))$.

Derivative of $\sqrt{u}$ is $\frac{1}{2\sqrt{u}} \cdot \frac{du}{dx}$.

Derivative of $ax+b$ is a.

Step 3: Detailed Explanation:

Let's start by calculating $h'(x)$ because $h(x)$ is a simple linear function.

Given: $h(x) = 2x - 3$.

Differentiating with respect to x:

$$h'(x) = 2$$

Note that $h'(x)$ is a constant. Its value is always 2, regardless of the input.

Therefore, $h'(g'(x)) = 2$, no matter how complex the expression for $g'(x)$ might be.

We effectively don't even need to calculate $g'(x)$ to solve this problem.

The expression we need to evaluate now is:

$$f'(h'(g'(x))) = f'(2)$$

Now, let's find the derivative of $f(x)$.

Given: $f(x) = \sqrt{x^2+1}$.

Rewrite as: $f(x) = (x^2+1)^{1/2}$.

Using the chain rule:

$$f'(x) = \frac{1}{2}(x^2+1)^{-1/2} \cdot \frac{d}{dx}(x^2+1)$$

$$f'(x) = \frac{1}{2\sqrt{x^2+1}} \cdot (2x)$$

$$f'(x) = \frac{x}{\sqrt{x^2+1}}$$

Finally, evaluate $f'(x)$ at the input we found earlier, which is 2:

$$f'(2) = \frac{2}{\sqrt{2^2+1}}$$

$$f'(2) = \frac{2}{\sqrt{4+1}}$$

$$f'(2) = \frac{2}{\sqrt{5}}$$

This matches Option (A).

Step 4: Final Answer:

Because the derivative of $h(x)$ was a constant 2, the complex inner function $g(x)$ became irrelevant. Evaluating the derivative of $f(x)$ at 2 yielded $2/\sqrt{5}$. This corresponds to Option (A).

Quick Tip: In composite derivative problems, look for linear functions.

The derivative of a linear function $y = mx + c$ is a constant m.

Any function nested inside a derivative of a linear function effectively "disappears" because the output is always that constant m.

---

10. Evaluate: $\int \frac{x+1}{x(1+xe^x)^2} dx$

(A) $\ln\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + C$
(B) $\ln|xe^x| - \ln|1+xe^x| - \frac{1}{1+xe^x} + C$
(C) $\ln\left|\frac{1+xe^x}{xe^x}\right| + \frac{1}{1+xe^x} + C$
(D) $\frac{1}{1+xe^x} + C$

Correct Answer: (A) $\ln\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + C$

Solution:

Step 1: Understanding the Concept:
This is an indefinite integration problem involving transcendental functions.
The integrand contains a mixture of polynomials and exponentials, specifically the term xe^x.
A common technique for such integrals is to find a substitution that simplifies the expression into a rational function.
We notice that the derivative of xe^x, using the product rule, is $e^x(x+1)$.
The numerator contains $x+1$. If we can introduce e^x into the numerator, we can use the substitution $t = xe^x$.

Step 2: Key Formula or Approach:
1. Multiply numerator and denominator by e^x.
2. Let $t = xe^x$, then $dt = (x+1)e^x dx$.
3. Use partial fraction decomposition for the resulting integral $\int \frac{dt}{t(1+t)^2}$.

Step 3: Detailed Explanation:
Let the integral be I:

$$I = \int \frac{x+1}{x(1+xe^x)^2} dx$$

Multiply both top and bottom by e^x:

$$I = \int \frac{(x+1)e^x}{xe^x(1+xe^x)^2} dx$$

Now, put $t = xe^x$. Differentiating:

$$dt = (x \cdot e^x + e^x \cdot 1)dx = e^x(x+1)dx$$

The integral becomes:

$$I = \int \frac{dt}{t(1+t)^2}$$

We decompose the fraction using partial fractions:

$$\frac{1}{t(1+t)^2} = \frac{A}{t} + \frac{B}{1+t} + \frac{C}{(1+t)^2}$$

Multiply by $t(1+t)^2$:

$$1 = A(1+t)^2 + Bt(1+t) + Ct$$

- If $t = 0$: $1 = A(1)^2 \implies A = 1$.
- If $t = -1$: $1 = C(-1) \implies C = -1$.
- Compare coefficients of t^2: $0 = A + B \implies B = -A = -1$.

The partial fraction decomposition is:

$$\frac{1}{t(1+t)^2} = \frac{1}{t} - \frac{1}{1+t} - \frac{1}{(1+t)^2}$$

Integrating this:

$$I = \int \left(\frac{1}{t} - \frac{1}{1+t} - \frac{1}{(1+t)^2}\right) dt$$

$$I = \ln|t| - \ln|1+t| - \left(-\frac{1}{1+t}\right) + C$$

$$I = \ln\left|\frac{t}{1+t}\right| + \frac{1}{1+t} + C$$

Substitute $t = xe^x$ back into the expression:

$$I = \ln\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + C$$

This matches Option (A).

Step 4: Final Answer:

Through a strategic substitution and partial fraction decomposition, we converted the transcendental integral into a form involving natural logarithms and a simple rational term. This corresponds to Option (A).

Quick Tip: Whenever an integral involves the term xe^x, always check if the numerator has $x+1$. This is a standard "trick" in calculus. You will often need to multiply and divide by e^x to "unlock" the substitution $t = xe^x$.

Also, remember that $\int \frac{1}{(1+t)^2} dt = -\frac{1}{1+t}$.