MHT CET 2026 May 13 Shift 2

Question Paper (Memory-Based) with Solutions

Conducted by CET Cell, Maharashtra

General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 200 marks.

(iii) **Structure:** The paper has 3 Sections:

- **Section A:** 50 Multiple Choice Questions (Physics).
- **Section B:** 50 Multiple Choice Questions (Chemistry).
- **Section B:** 100 Multiple Choice Questions (Mathematics).

(iv) **Compulsory Questions:** All 200 questions are compulsory.

(v) Each question has four options. Only **one** option is correct.

(vi) Correct Answer: +1 marks.

(vii) **Incorrect Answer:** (No Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

1. If $I = \int \frac{\sin x + \sin^3 x}{\cos 2x} dx = P\cos x + Q\log\left|\frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}\right| + C$, then the values of P and Q are respectively:

(A) $\frac{1}{2}, \frac{3}{4\sqrt{2}}$
(B) $\frac{1}{2}, \frac{-3}{4\sqrt{2}}$
(C) $\frac{1}{2}, \frac{3}{2\sqrt{2}}$
(D) $\frac{1}{2}, \frac{-3}{2\sqrt{2}}$

Correct Answer: (B) $\frac{1}{2}, \frac{-3}{4\sqrt{2}}$

Solution:

Step 1: Understanding the Concept:

The given problem is an indefinite integral involving trigonometric functions.

To solve this, we must simplify the integrand into a form that allows for a standard substitution.

The presence of $\sin x$ and its higher powers in the numerator suggests that substituting $u = \cos x$ would be effective, as the derivative $-\sin x$ appears naturally.

The denominator $\cos 2x$ is a double-angle term that must be converted to an expression involving $\cos x$ to maintain consistency with our proposed substitution.

Key Formula or Approach:

1. Trigonometric Identity: $\sin^2 x = 1 - \cos^2 x$.
2. Double Angle Formula: $\cos 2x = 2\cos^2 x - 1$.
3. Standard Integral: $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$.

Step 2: Detailed Explanation:

Let us start by rewriting the integral I:

$$I = \int \frac{\sin x + \sin^3 x}{\cos 2x} dx$$

Factor out $\sin x$ from the numerator:

$$I = \int \frac{\sin x(1 + \sin^2 x)}{\cos 2x} dx$$

Now, we apply the identity $\sin^2 x = 1 - \cos^2 x$ in the numerator and $\cos 2x = 2\cos^2 x - 1$ in the denominator:

$$I = \int \frac{\sin x(1 + 1 - \cos^2 x)}{2\cos^2 x - 1} dx = \int \frac{\sin x(2 - \cos^2 x)}{2\cos^2 x - 1} dx$$

Perform the substitution $u = \cos x$.

Differentiating both sides with respect to x, we get $\frac{du}{dx} = -\sin x$, which implies $\sin x dx = -du$.

Substitute these into the integral:

$$I = \int \frac{2 - u^2}{2u^2 - 1}(-du) = \int \frac{u^2 - 2}{2u^2 - 1} du$$

To integrate this rational function, we perform algebraic manipulation (polynomial long division or splitting terms).

The numerator $u^2 - 2$ can be adjusted to match the denominator's leading term $2u^2$:

$$\frac{u^2-2}{2u^2-1} = \frac{1}{2}\left[\frac{2u^2-4}{2u^2-1}\right] = \frac{1}{2}\left[\frac{(2u^2-1)-3}{2u^2-1}\right]$$

Simplify the fraction:

$$I = \frac{1}{2}\int\left(1-\frac{3}{2u^2-1}\right)du = \frac{1}{2}\int 1du - \frac{3}{2}\int\frac{1}{2u^2-1}du$$

Integrating the first part gives $\frac{1}{2}u$.

For the second part, factor out 2 from the denominator:

$$\frac{3}{2}\int\frac{1}{2(u^2-1/2)}du = \frac{3}{4}\int\frac{1}{u^2-(1/\sqrt{2})^2}du$$

Using the standard integral $\int\frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|$, where $a = 1/\sqrt{2}$:

$$\text{Integral Part} = \frac{3}{4}\cdot\frac{1}{2(1/\sqrt{2})}\log\left|\frac{u-1/\sqrt{2}}{u+1/\sqrt{2}}\right|$$
$$= \frac{3\sqrt{2}}{8}\log\left|\frac{\sqrt{2}u-1}{\sqrt{2}u+1}\right| = \frac{3}{4\sqrt{2}}\log\left|\frac{\sqrt{2}u-1}{\sqrt{2}u+1}\right|$$

Combining the parts:

$$I = \frac{1}{2}u - \frac{3}{4\sqrt{2}}\log\left|\frac{\sqrt{2}u-1}{\sqrt{2}u+1}\right| + C$$

Substituting back $u = \cos x$:

$$I = \frac{1}{2}\cos x - \frac{3}{4\sqrt{2}}\log\left|\frac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}\right| + C$$

Comparing this with the given form $P\cos x + Q\log|\dots| + C$:

We find $P = \frac{1}{2}$ and $Q = -\frac{3}{4\sqrt{2}}$.

Step 3: Final Answer:

Comparing the results, $P = 1/2$ and $Q = -3/(4\sqrt{2})$.

This matches Option (B).

Quick Tip: When integrating trigonometric ratios, if the powers of $\sin x$ are odd and only $\cos x$ appears in the denominator identities, the substitution $u = \cos x$ is almost always the most efficient route. Always convert double angles ($\cos 2x$) to the same single angle variable first.

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2. In a triangle $\triangle ABC$, if a, b, and c are in arithmetic progression, then $\cos A + 2\cos B + \cos C =$

(A) 1
(B) 2
(C) $\frac{3}{2}$
(D) $\sqrt{3}+1$

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Concept:
Arithmetic Progression (A.P.) implies that the middle term is the average of the surrounding terms.
For sides a, b, c, this means $2b = a + c$.
We need to evaluate a trigonometric sum of cosines for a triangle.
Properties of triangles, such as $A + B + C = \pi$, and the Sine Rule ($a = 2R\sin A$) are key here.

Key Formula or Approach:
1. A.P. Condition: $a + c = 2b$.
2. Sine Rule: $a = 2R\sin A$, $b = 2R\sin B$, $c = 2R\sin C$.
3. Sum-to-Product Identity: $\sin A + \sin C = 2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)$.
4. Triple-Cosine Relation: $\cos A + \cos C = 2\cos\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)$.

Step 2: Detailed Explanation:
Given a, b, c are in A.P., we have:

$$a + c = 2b$$

Using the Sine Rule, we substitute the sides with their sine equivalents:

$$2R\sin A + 2R\sin C = 2(2R\sin B)$$

Dividing by $2R$:

$$\sin A + \sin C = 2\sin B$$

Applying the sum-to-product identity:

$$2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) = 2\sin B$$

Since $A+B+C=\pi$, we have $A+C=\pi-B$, thus $\frac{A+C}{2}=\frac{\pi}{2}-\frac{B}{2}$.
This gives $\sin\left(\frac{A+C}{2}\right)=\sin\left(\frac{\pi}{2}-\frac{B}{2}\right)=\cos\left(\frac{B}{2}\right)$.
Also, we use the double angle formula $\sin B = 2\sin\frac{B}{2}\cos\frac{B}{2}$.
Substituting these back:

$$2\cos\frac{B}{2}\cos\frac{A-C}{2} = 2(2\sin\frac{B}{2}\cos\frac{B}{2})$$

Assuming $\cos\frac{B}{2}\neq 0$ (valid as $0<B<\pi$):

$$\cos\frac{A-C}{2} = 2\sin\frac{B}{2}$$

Now we look at the required expression $E=\cos A+2\cos B+\cos C$:

$$E = (\cos A+\cos C)+2\cos B$$

$$E = 2\cos\frac{A+C}{2}\cos\frac{A-C}{2}+2\cos B$$

Substitute $\cos\frac{A+C}{2}=\sin\frac{B}{2}$ and our derived relation $\cos\frac{A-C}{2}=2\sin\frac{B}{2}$:

$$E = 2(\sin\frac{B}{2})(2\sin\frac{B}{2})+2\cos B$$

$$E = 4\sin^2\frac{B}{2}+2\cos B$$

Using the identity $\cos B = 1-2\sin^2\frac{B}{2}$:

$$E = 4\sin^2\frac{B}{2}+2(1-2\sin^2\frac{B}{2})$$

$$E = 4\sin^2\frac{B}{2} + 2 - 4\sin^2\frac{B}{2} = 2$$

Step 3: Final Answer:

The sum $\cos A + 2\cos B + \cos C$ is equal to 2.

This corresponds to Option (B).

Quick Tip: For triangle problems where sides are in A.P., the relation $\sin A + \sin C = 2\sin B$ is the starting point.

A useful shortcut to remember: In any triangle where sides are in A.P., the specific sum $\cos A + 2\cos B + \cos C$ is invariant and always equal to 2.

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3. If the area of triangle ABC is $b^2 - (c-a)^2$, then $\tan B =$

(A) 1

(B) $\frac{13}{15}$

(C) $\frac{1}{4}$

(D) $\frac{8}{15}$

Correct Answer: (D) $\frac{8}{15}$

Solution:

Step 1: Understanding the Concept:

The problem provides an algebraic expression for the area (Δ) of a triangle.

We must relate this algebraic form to the standard trigonometric formula for the area: $\Delta = \frac{1}{2}ac\sin B$.

Additionally, the Cosine Rule relates the sides a, b, c to the cosine of the angle B.

Key Formula or Approach:

1. Area of triangle: $\Delta = \frac{1}{2}ac\sin B$.

2. Law of Cosines: $b^2 = a^2 + c^2 - 2ac\cos B$.

3. Algebraic expansion: $(c-a)^2 = c^2 + a^2 - 2ac$.

Step 2: Detailed Explanation:

The given area is $\Delta = b^2 - (c-a)^2$.

Expanding the square term:

$$\Delta = b^2 - (c^2 + a^2 - 2ac) = b^2 - c^2 - a^2 + 2ac$$

Substitute the Law of Cosines expression $b^2 = a^2 + c^2 - 2ac\cos B$:

$$\Delta = (a^2 + c^2 - 2ac\cos B) - c^2 - a^2 + 2ac$$

The terms a^2 and c^2 cancel out:

$$\Delta = 2ac - 2ac\cos B = 2ac(1-\cos B)$$

We know that the area is also given by $\Delta = \frac{1}{2}ac\sin B$.

Equating the two expressions for Δ:

$$\frac{1}{2}ac\sin B = 2ac(1-\cos B)$$

Assuming $a, c \neq 0$, we divide both sides by ac:

$$\frac{1}{2}\sin B = 2(1-\cos B) \implies \sin B = 4(1-\cos B)$$

Using half-angle identities $\sin B = 2\sin\frac{B}{2}\cos\frac{B}{2}$ and $1-\cos B = 2\sin^2\frac{B}{2}$:

$$2\sin\frac{B}{2}\cos\frac{B}{2} = 4(2\sin^2\frac{B}{2}) = 8\sin^2\frac{B}{2}$$

Divide by $2\sin\frac{B}{2}$ (since $B > 0$, $\sin\frac{B}{2} \neq 0$):

$$\cos\frac{B}{2} = 4\sin\frac{B}{2} \implies \tan\frac{B}{2} = \frac{1}{4}$$

To find $\tan B$, use the double angle formula:

$$\tan B = \frac{2\tan(B/2)}{1-\tan^2(B/2)} = \frac{2(1/4)}{1-(1/4)^2} = \frac{1/2}{1-1/16}$$

$$\tan B = \frac{1/2}{15/16} = \frac{1}{2} \cdot \frac{16}{15} = \frac{8}{15}$$

Step 3: Final Answer:

The value of $\tan B$ is $8/15$.

This is Option (D).

Quick Tip: When an area is expressed as a difference of squares of sides, think of the Law of Cosines. Specifically, $b^2 - (c-a)^2 = 4(s-a)(s-c)$. Comparing this to $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ is another valid path, but comparing to the $\frac{1}{2}ac \sin B$ formula is usually faster.

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4. In a $\Delta ABC, a = 1, b = \sqrt{3}$ and $\angle C = \frac{\pi}{6}$. Then the measure of the third side $c =$

(A) 4

(B) 3

(C) 1

(D) 2

Correct Answer: (C) 1

Solution:

Step 1: Understanding the Concept:

This problem involves solving a triangle when two sides and the included angle (SAS property) are known.

The Law of Cosines is the standard tool to find the third side in such scenarios.

Key Formula or Approach:

1. Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$.
2. Standard Value: $\cos(\pi/6) = \cos 30^\circ = \frac{\sqrt{3}}{2}$.

Step 2: Detailed Explanation:

Given values are $a = 1$, $b = \sqrt{3}$, and $C = \pi/6$.

Plug these values into the formula:

$$c^2 = (1)^2 + (\sqrt{3})^2 - 2(1)(\sqrt{3})\cos(30^\circ)$$
$$c^2 = 1 + 3 - 2\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$$

The factor 2 in the numerator and denominator cancels out:

$$c^2 = 4 - (\sqrt{3} \cdot \sqrt{3})$$

$$c^2 = 4 - 3 = 1$$

Taking the square root (since length must be positive):

$$c = \sqrt{1} = 1$$

Step 3: Final Answer:

The third side c has a measure of 1.

This matches Option (C).

Quick Tip: Always double-check your trigonometric values. For MHT CET, common angles like $30^\circ, 45^\circ, 60^\circ$ appear frequently.

A triangle with sides $1, \sqrt{3}, 1$ and angle 30° is an isosceles triangle because two sides are equal.

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5. The integral $\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$ is equal to: (where C is a constant of integration)

(A) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{1/4} + C$
(B) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{5/4} + C$
(C) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$
(D) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{5/4} + C$

Correct Answer: (C) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$

Solution:

Step 1: Understanding the Concept:

The integrand contains a product of linear factors raised to fractional powers in the denominator.

Specifically, it is of the form $(x-a)^{-m}(x-b)^{-n}$ where $m+n=\frac{3}{4}+\frac{5}{4}=2$.

For integrals where the sum of powers equals 2, a substitution of the ratio $t=\frac{x-a}{x-b}$ often reduces the expression to a basic power integral.

Key Formula or Approach:

1. Factor out a term to create a ratio.
2. Substitution: $t=\frac{x-1}{x+2}$.
3. Derivative: $\frac{dt}{dx}=\frac{(x+2)(1)-(x-1)(1)}{(x+2)^2}=\frac{3}{(x+2)^2}$.

Step 2: Detailed Explanation:

Let $I=\int\frac{1}{(x-1)^{3/4}(x+2)^{5/4}}dx$.

Multiply and divide the denominator by $(x+2)^{3/4}$:

$$I=\int\frac{1}{\left(\frac{x-1}{x+2}\right)^{3/4}(x+2)^{3/4}(x+2)^{5/4}}dx$$

$$I=\int\frac{1}{\left(\frac{x-1}{x+2}\right)^{3/4}(x+2)^{2}}dx$$

Now, let $t=\frac{x-1}{x+2}$.

Differentiating both sides: $dt=\frac{3}{(x+2)^2}dx$, which means $\frac{dx}{(x+2)^2}=\frac{dt}{3}$.

Substitute these into the integral:

$$I=\int\frac{1}{t^{3/4}}\cdot\frac{dt}{3}=\frac{1}{3}\int t^{-3/4}dt$$

Evaluate the integral using the power rule:

$$I=\frac{1}{3}\left[\frac{t^{-3/4+1}}{-3/4+1}\right]+C=\frac{1}{3}\left[\frac{t^{1/4}}{1/4}\right]+C$$

$$I=\frac{1}{3}\cdot4\cdot t^{1/4}+C=\frac{4}{3}t^{1/4}+C$$

Substitute back for t:

$$I = \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$$

Step 3: Final Answer:

The result of the integration is $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$.

This matches Option (C).

Quick Tip: Check the sum of powers in the denominator. If $m+n=2$, use $t = \frac{\text{factor}_1}{\text{factor}_2}$.

This is a standard pattern for MHT CET and JEE exams that eliminates the need for complex partial fractions or trigonometric substitutions.

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6. Calculate the pH of 0.02 M monobasic acid having 2% dissociation.

(A) 3.4

(B) 4.5

(C) 5.1

(D) 5.8

Correct Answer: (A) 3.4

Solution:

Step 1: Understanding the Concept:

A monobasic acid is one that yields one hydrogen ion (H^+) per molecule upon dissociation.

For weak acids, the concentration of H^+ ions depends on the total concentration (C) and the degree of dissociation (α).

Key Formula or Approach:

1. $[H^+] = C \cdot \alpha$.

2. $pH = -\log_{10}[H^+]$.

Step 2: Detailed Explanation:

Given:

Concentration $C = 0.02$ M.

Degree of dissociation $\alpha = 2\% = 0.02$.

First, we find the hydrogen ion concentration:

$$[H^+] = C \cdot \alpha = 0.02 \times 0.02 = 0.0004 \text{ M}$$

$$[H^+] = 4 \times 10^{-4} \text{ M}$$

Now, calculate the pH:

$$pH = -\log(4 \times 10^{-4}) = -(\log 4 + \log 10^{-4})$$

$$pH = -(0.602 - 4) = 4 - 0.602 = 3.398$$

Rounding to one decimal place, we get approximately 3.4.

Step 3: Final Answer:

The pH is 3.4.

This matches Option (A).

Quick Tip: To quickly estimate pH for $n \times 10^{-x}$, the result is $x - \log n$.
Since $\log 4 \approx 0.6$, then $4 - 0.6 = 3.4$.

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7. What is the pH of 2×10^{-3} M solution of monoacidic weak base if it ionises to the extent of 5%?

(A) 14

(B) 6

(C) 4

(D) 2

Correct Answer: (C) 4

Solution:

Step 1: Understanding the Concept:

For a weak base, ionization produces hydroxide ions (OH^-).

We first calculate $[OH^-]$, then find the pOH.

The pH is derived using the relation $pH + pOH = 14$.

Step 2: Detailed Explanation:

Given:

Concentration $C = 2 \times 10^{-3}$ M.

Degree of ionization $\alpha = 5\% = 0.05$.

Calculate $[OH^-]$:

$$[OH^-] = C \cdot \alpha = (2 \times 10^{-3}) \times (0.05) = 1 \times 10^{-4} \text{ M}$$

Calculate pOH:

$$pOH = -\log(10^{-4}) = 4$$

Calculate pH:

$$pH = 14 - pOH = 14 - 4 = 10$$

Looking at the options (A) 14, (B) 6, (C) 4, (D) 2, none match 10. However, Option (C) 4 is exactly the calculated pOH. The question may have intended to ask for pOH.

Step 3: Final Answer:

Based on the calculation, $pOH = 4$. If the question intended pH, the answer is 10. Given the choices, (C) is the most likely intended answer for the numerical result of the ionization log.

Quick Tip: Always distinguish between $[H^+]$ and $[OH^-]$. Bases give OH^-, so you calculate pOH first. If the options don't match the pH, check if they match the pOH.

8. Identify base$_2$ for the following equation according to Brønsted–Lowry theory:

$$HCl_{(aq)} + H_2O_{(l)} \rightleftharpoons H_3O^+_{(aq)} + Cl^-_{(aq)}$$

(A) $H_3O^+_{(aq)}$

(B) $H_2O_{(l)}$

(C) $Cl^-_{(aq)}$

(D) $HCl_{(aq)}$

Correct Answer: (B) $H_2O_{(l)}$

Solution:

Step 1: Understanding the Concept:

According to Brønsted–Lowry theory, an acid is a proton (H^+) donor and a base is a proton acceptor.

A reaction involves two conjugate acid-base pairs.

Step 2: Detailed Explanation:

In the given reaction: $HCl + H_2O \rightarrow H_3O^+ + Cl^-$.

1. HCl gives away a proton to become Cl^-. Thus, HCl is Acid$_1$ and Cl^- is Base$_1$.

2. H_2O accepts a proton to become H_3O^+. Thus, H_2O is Base$_2$ and H_3O^+ is Acid$_2$.

The question asks for Base$_2$.

Step 3: Final Answer:

Base$_2$ is $H_2O_{(l)}$.

This matches Option (B).

Quick Tip: The species that gains a proton in the forward reaction is the base.

Water is amphoteric; in the presence of strong acids like HCl, it always acts as the base.

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9. Calculate the pH of 0.01 M sulphuric acid.

(A) 1.699

(B) 2.00

(C) 0.699

(D) 3.398

Correct Answer: (A) 1.699

Solution:

Step 1: Understanding the Concept:

Sulphuric acid (H_2SO_4) is a strong dibasic acid. It dissociates completely in water.

Because it is dibasic, one mole of H_2SO_4 produces two moles of H^+ ions.

Key Formula or Approach:

1. $[H^+] = 2 \times [Acid]$ for strong dibasic acids.

2. $pH = -\log_{10}[H^+]$.

Step 2: Detailed Explanation:

Given molarity of $H_2SO_4 = 0.01$ M.

Since it fully dissociates:

$$[H^+] = 2 \times 0.01 = 0.02 \text{ M} = 2 \times 10^{-2} \text{ M}$$

Now, calculate the pH:

$$pH = -\log(2 \times 10^{-2}) = 2 - \log 2$$

Using the value $\log 2 \approx 0.301$:

$$pH = 2 - 0.301 = 1.699$$

Step 3: Final Answer:

The pH is 1.699.

This matches Option (A).

Quick Tip: For H_2SO_4, always multiply the molarity by 2.

A common mistake is taking $pH = -\log(0.01) = 2.0$, which ignores the dibasic nature of the acid.

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10. What is the pH of a weak dibasic acid that is 2% dissociated in its M/100 solution at 298 K?

(A) 1.6990

(B) 2.3979

(C) 3.3970

(D) 4.6990

Correct Answer: (C) 3.3970

Solution:

Step 1: Understanding the Concept:

A weak dibasic acid dissociates partially.

The total $[H^+]$ concentration depends on the concentration and the degree of dissociation (α).

Step 2: Detailed Explanation:

Given:

Concentration $C = M/100 = 0.01$ M.

Dissociation $\alpha = 2\% = 0.02$.

For a dibasic acid:

$$[H^+] = 2 \times C \times \alpha = 2 \times 0.01 \times 0.02 = 0.0004 \text{ M}$$

$$[H^+] = 4 \times 10^{-4} \text{ M}$$

Calculate pH:

$$pH = -\log(4 \times 10^{-4}) = 4 - \log 4$$

$$pH = 4 - 0.6020 = 3.3980$$

Closest option is 3.3970.

Step 3: Final Answer:

The pH is approximately 3.3970.

This matches Option (C).

Quick Tip: For polyprotic weak acids, unless otherwise specified, we assume both protons dissociate with the same degree of dissociation for basic level calculation.

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11. An open pipe resonates with a tuning fork of frequency 500 Hz. It is observed that two successive nodes are separated by 34 cm. The velocity of sound in air is:

(A) 330 m/s
(B) 340 m/s
(C) 350 m/s
(D) 360 m/s

Correct Answer: (B) 340 m/s

Solution:

Step 1: Understanding the Concept:

In stationary waves, the distance between any two consecutive nodes (or consecutive antinodes) is half the wavelength ($\lambda/2$).

Key Formula or Approach:

1. Distance between nodes $= \lambda/2$.
2. Speed of sound $v = f \cdot \lambda$.

Step 2: Detailed Explanation:

Given:

Distance between successive nodes = 34 cm = 0.34 m.

Frequency $f = 500$ Hz.

Wavelength λ:

$$\frac{\lambda}{2} = 0.34 \implies \lambda = 0.68 \text{ m}$$

Now, calculate velocity v:

$$v = f \cdot \lambda = 500 \times 0.68 = 340 \text{ m/s}$$

Step 3: Final Answer:

The velocity of sound is 340 m/s.

This matches Option (B).

Quick Tip: Distance between a Node and an Antinode is $\lambda/4$.

Distance between two Nodes is $\lambda/2$.

Always convert cm to meters before calculating velocity.

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12. A train moving at 20 m/s approaches a stationary observer. The frequency of the whistle emitted by the train is 640 Hz. If the velocity of sound is 340 m/s, the apparent frequency heard by the observer is:

(A) 680 Hz

(B) 600 Hz

(C) 720 Hz

(D) 640 Hz

Correct Answer: (A) 680 Hz

Solution:

Step 1: Understanding the Concept:

This problem uses the Doppler Effect. Since the source is moving toward a stationary observer, the frequency heard will be higher than the actual frequency.

Key Formula or Approach:

Doppler Effect formula: $f_{app} = f\left(\frac{v \pm v_o}{v \mp v_s}\right)$.

For a stationary observer ($v_o = 0$) and source approaching ($v - v_s$ in denominator):

$$f' = f\left(\frac{v}{v - v_s}\right)$$

Step 2: Detailed Explanation:

Given:

Actual frequency $f = 640$ Hz.

Velocity of sound $v = 340$ m/s.

Velocity of source (train) $v_s = 20$ m/s.

Apply the formula:

$$f' = 640\left(\frac{340}{340 - 20}\right)$$

$$f' = 640\left(\frac{340}{320}\right)$$

$$f' = 640 \times 1.0625 = 2 \times 340 = 680 \text{ Hz}$$

(Alternatively, $640/320 = 2$, then $2 \times 340 = 680$).

Step 3: Final Answer:

The apparent frequency is 680 Hz.

This matches Option (A).

Quick Tip: Remember: "Approaching" means the denominator must be smaller than the numerator to increase the frequency (use subtraction).

"Receding" means the denominator must be larger (use addition).

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13. Unpolarized light of intensity I_0 is incident on two polaroids placed coaxially. The transmission axis of the second polaroid is at an angle of 60° to the first. The intensity of emerging

light is:

(A) $I_0/2$

(B) $I_0/4$

(C) $I_0/8$

(D) $3I_0/8$

Correct Answer: (C) $I_0/8$

Solution:

Step 1: Understanding the Concept:

When unpolarized light passes through a polaroid, its intensity is reduced by half regardless of the orientation.

Subsequent passage through another polaroid (the analyzer) follows Malus' Law.

Key Formula or Approach:

1. Intensity after 1st polaroid: $I_1 = I_0/2$.
2. Malus' Law: $I_2 = I_1 \cos^2\theta$.

Step 2: Detailed Explanation:

Initial intensity $= I_0$.

After the first polaroid, light becomes plane polarized with intensity:

$$I_1 = \frac{I_0}{2}$$

This polarized light then hits the second polaroid oriented at $\theta = 60°$ to the first.

Using Malus' Law:

$$I_{out} = I_1 \cos^2(60°)$$

Since $\cos 60° = 1/2$, then $\cos^2 60° = 1/4$.

$$I_{out} = \left(\frac{I_0}{2}\right)\left(\frac{1}{4}\right) = \frac{I_0}{8}$$

Step 3: Final Answer:

The final intensity is $I_0/8$.

This matches Option (C).

Quick Tip: Always apply the factor of $1/2$ for the *first* polaroid if the incident light is unpolarized. If the incident light was already polarized, you would just use Malus' Law directly with $I_{incident}$.

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14. A parallel plate capacitor has capacitance C. If a dielectric slab of dielectric constant K and thickness equal to half the plate separation is introduced parallel to the plates, the new capacitance is:

(A) $\frac{2K}{K+1}C$
(B) $\frac{K+1}{2K}C$
(C) $\frac{K}{K+1}C$
(D) KC

Correct Answer: (A) $\frac{2K}{K+1}C$

Solution:

Step 1: Understanding the Concept:

Introducing a dielectric slab of thickness t into a capacitor of gap d creates two capacitors in series: one with air and one with dielectric.

Key Formula or Approach:

Capacitance with slab: $C' = \frac{\epsilon_0 A}{d-t+t/K}$.

Original capacitance: $C = \frac{\epsilon_0 A}{d}$.

Step 2: Detailed Explanation:

Given thickness $t = d/2$.

Substitute t into the formula:

$$C' = \frac{\epsilon_0 A}{d - d/2 + (d/2)/K}$$

$$C' = \frac{\epsilon_0 A}{d/2 + d/2K} = \frac{\epsilon_0 A}{\frac{dK+d}{2K}}$$

$$C' = \frac{2K\epsilon_0 A}{d(K+1)}$$

Since $\frac{\epsilon_0 A}{d} = C$:

$$C' = \frac{2K}{K+1}C$$

Step 3: Final Answer:

The new capacitance is $\frac{2K}{K+1}C$.

This matches Option (A).

Quick Tip: For equal thicknesses ($d/2$ each), the new capacitance is the harmonic mean of an air capacitor and a full-dielectric capacitor.

Shortcut: $C_{new} = \frac{2KC}{K+1}$.

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15. An infinite line charge of uniform linear charge density λ is placed along the z-axis. The work done in moving a charge q from $(a, 0, 0)$ to $(2a, 0, 0)$ is:

(A) $\frac{q\lambda}{2\pi\epsilon_0}\ln 2$
(B) $\frac{q\lambda}{4\pi\epsilon_0}\ln 2$
(C) $\frac{q\lambda}{2\pi\epsilon_0}\ln(1/2)$
(D) zero

Correct Answer: (A) $\frac{q\lambda}{2\pi\epsilon_0}\ln 2$

Solution:

Step 1: Understanding the Concept:

Work done is related to the potential difference: $W = q(V_2 - V_1)$.

For a line charge, the electric field is radial and varies as $1/r$.

Key Formula or Approach:

1. Electric field: $E = \frac{\lambda}{2\pi\epsilon_0 r}$.
2. Work: $W = \int F \cdot dr = \int qE dr$.

Step 2: Detailed Explanation:

The work done by an external agent against the field:

$$W = \int_a^{2a} qE dr = q \int_a^{2a} \frac{\lambda}{2\pi\epsilon_0 r} dr$$

Factor out constants:

$$W = \frac{q\lambda}{2\pi\epsilon_0} \int_a^{2a} \frac{1}{r} dr$$

Integrating $1/r$ gives $\ln r$:

$$W = \frac{q\lambda}{2\pi\epsilon_0}[\ln r]_a^{2a} = \frac{q\lambda}{2\pi\epsilon_0}(\ln 2a - \ln a)$$

$$W = \frac{q\lambda}{2\pi\epsilon_0}\ln(2a/a) = \frac{q\lambda}{2\pi\epsilon_0}\ln 2$$

Step 3: Final Answer:

The work done is $\frac{q\lambda}{2\pi\epsilon_0}\ln 2$.

This matches Option (A).

Quick Tip: For point charges, potential varies as $1/r$. For line charges, it varies as $\ln r$.
Moving away from a line charge of same sign requires work against the attraction (if negative) or work done by the field (if positive). The logarithmic result is a signature of cylinder-like symmetry.
