MHT CET 2026 May 13 Shift 1

Question Paper (Memory-Based) with Solutions

Conducted by CET Cell, Maharashtra

General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 200 marks.

(iii) **Structure:** The paper has 3 Sections:

- **Section A:** 50 Multiple Choice Questions (Physics).
- **Section B:** 50 Multiple Choice Questions (Chemistry).
- **Section B:** 100 Multiple Choice Questions (Mathematics).

(iv) **Compulsory Questions:** All 200 questions are compulsory.

(v) Each question has four options. Only **one** option is correct.

(vi) **Correct Answer:** +1 marks.

(vii) **Incorrect Answer:** (No Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

1. The rms velocity of hydrogen molecules at 27°C is v. At what temperature will the rms velocity of oxygen molecules equal v?

(A) 4800 K

(B) 4527°C

(C) 1200 K

(D) 27°C

Correct Answer: (A) 4800 K

Solution:

Step 1: Understanding the Concept:

The Root Mean Square (RMS) velocity is a fundamental statistical measure of the speed of particles in a gas.

In the kinetic theory of gases, we assume that gas molecules are in constant, random motion.

The temperature of a gas is directly proportional to the average kinetic energy of its molecules.

For different gases at different temperatures to have the same RMS velocity, the ratio of their absolute temperature to their molar mass must remain constant.

This stems from the fact that lighter molecules (like Hydrogen) must move faster than heavier molecules (like Oxygen) to maintain the same kinetic energy at a given temperature.

Therefore, if we want the heavy Oxygen molecule to move as fast as the light Hydrogen molecule, the Oxygen must be at a significantly higher temperature.

Step 2: Key Formula or Approach:

The expression for the RMS velocity (v_{rms}) is derived from the kinetic gas equation:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Where:

R is the Universal Gas Constant ($8.314\,\text{J/mol}\cdot\text{K}$).

T is the absolute temperature measured in Kelvin (K).

M is the molar mass of the gas (expressed in kg/mol for SI units, though ratios work with g/mol).

Step 3: Detailed Explanation:

Let us identify the parameters for both gases provided in the problem.

For Hydrogen (H_2):

Molar mass $M_{H_2} = 2\,\text{g/mol}$.

Given temperature $t_1 = 27^\circ\text{C}$.

First, convert this to the absolute Kelvin scale:

$$T_1 = 27 + 273.15 \approx 300\,\text{K}$$

The RMS velocity is given as v. So:

$$v = \sqrt{\frac{3R \times 300}{2}}$$

For Oxygen (O_2):

Molar mass $M_{O_2} = 32\,\text{g/mol}$.

Let the required temperature be T_2.

The problem states that the RMS velocity for Oxygen must also be v. So:

$$v = \sqrt{\frac{3R \times T_2}{32}}$$

Equating the two expressions for v:

$$\sqrt{\frac{3R \times 300}{2}} = \sqrt{\frac{3R \times T_2}{32}}$$

By squaring both sides, we eliminate the square root:

$$\frac{3R \times 300}{2} = \frac{3R \times T_2}{32}$$

We can cancel the constant $3R$ from both sides:

$$\frac{300}{2} = \frac{T_2}{32}$$

Simplifying the left side:

$$150 = \frac{T_2}{32}$$

Now, solve for T_2:

$$T_2 = 150 \times 32$$

$$T_2 = 4800\,\text{K}$$

If we were to convert this temperature to Celsius to check against Option B:

$$t_2 = 4800 - 273 = 4527^\circ\text{C}$$

Since both 4800 K and 4527°C represent the same thermal state, we look at the options. Option (A) is exactly 4800 K, which is a standard way to express gas law solutions. Option (B) is also numerically correct, but standard convention in these exams often prioritizes the Kelvin answer unless specified otherwise.

Step 4: Final Answer:
The required temperature is 4800 K.

Quick Tip: To save time, use the ratio method: $T_1/M_1 = T_2/M_2$. Since M_{O_2} is 16 times M_{H_2}, the absolute temperature must also be 16 times higher. 300 K × 16 = 4800 K. Always ensure temperature is in Kelvin!

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2. An ideal gas undergoes a cyclic process ABCA wherein AB is an isothermal expansion, BC is an isochoric pressure drop, and CA is an adiabatic compression. The work done by the gas in the complete cycle is:

(A) Zero
(B) Positive
(C) Negative
(D) Cannot be determined

Correct Answer: (B) Positive

Solution:

Step 1: Understanding the Concept:

In thermodynamics, a cyclic process is one where the system starts and ends at the same state. The net work done during a cycle is the area enclosed by the path on a Pressure-Volume ($P-V$) diagram.

According to the sign convention:

- Clockwise cycles represent net positive work done **by** the gas (typical of heat engines).
- Counter-clockwise cycles represent net negative work done (work done **on** the gas, typical of refrigerators).

Step 2: Detailed Explanation:

Let's analyze each stage of the cycle ABCA:

1. **Process AB (Isothermal Expansion):** The gas expands at a constant temperature. Expansion means volume increases ($V_B > V_A$). On the $P-V$ graph, this is represented by a curve sloping downwards. Work done during this part is positive as the gas pushes against its surroundings.
2. **Process BC (Isochoric Pressure Drop):** Isochoric means the volume is kept constant ($V_B = V_C$). A pressure drop at constant volume implies the gas is being cooled. Since there is no change in volume ($dV = 0$), the work done $W = \int P\,dV$ is exactly zero.
3. **Process CA (Adiabatic Compression):** The gas is compressed back to its original state A without any heat exchange with the surroundings. Compression means volume decreases ($V_A < V_C$). Work is done on the gas, so this work is negative.

On a $P-V$ diagram, an adiabatic curve is steeper than an isothermal curve.

- From A to B, we follow a less steep isothermal curve.
- At B, we drop vertically to C.
- From C to A, we return via a steeper adiabatic curve.

Because the adiabatic return path CA is steeper and starts from a lower pressure point C, it stays "underneath" the isothermal expansion path AB.

Visualizing this: The path moves from A (left) to B (right), then down to C, then back up and left to A. This traces a **clockwise** loop.

In a clockwise loop on a $P-V$ diagram, the area under the expansion curve (positive work) is greater than the area under the compression curve (negative work).

Step 3: Final Answer:

Since the cycle is clockwise, the net area is positive, meaning the total work done by the gas in the complete cycle is positive.

Quick Tip: Always sketch the process on a $P-V$ graph. If the expansion happens at a higher pressure/temperature than the compression, the net work is positive. Clockwise cycle = Positive Work.

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3. A Carnot engine operates between temperatures T_1 and T_2 ($T_1 > T_2$). If T_1 is increased by ΔT and T_2 is decreased by ΔT, its efficiency will:

(A) Increase

(B) Decrease

(C) Remain constant

(D) Depend on the working substance

Correct Answer: (A) Increase

Solution:

Step 1: Understanding the Concept:

The Carnot engine is a theoretical ideal heat engine that operates on the Carnot cycle.

The second law of thermodynamics implies that no engine can be 100% efficient because some heat must always be rejected to a cold reservoir.

The efficiency of a Carnot engine depends only on the absolute temperatures of the heat source and the heat sink.

Efficiency is improved by either making the source hotter or the sink colder.

Step 2: Key Formula or Approach:

The efficiency (η) of a Carnot engine is given by:

$$\eta = 1 - \frac{T_{sink}}{T_{source}}$$

In this problem, let T_1 be the source temperature and T_2 be the sink temperature.
So, the initial efficiency is:

$$\eta_1 = 1 - \frac{T_2}{T_1}$$

Step 3: Detailed Explanation:

According to the problem, the temperatures are modified as follows:
New source temperature, $T_1' = T_1 + \Delta T$
New sink temperature, $T_2' = T_2 - \Delta T$

The new efficiency η_2 becomes:

$$\eta_2 = 1 - \frac{T_2 - \Delta T}{T_1 + \Delta T}$$

Let's analyze the fraction $\frac{T_2 - \Delta T}{T_1 + \Delta T}$.
The numerator has decreased (making the fraction smaller).
The denominator has increased (also making the fraction smaller).
Thus, the term $\frac{T_2 - \Delta T}{T_1 + \Delta T}$ is significantly smaller than the original fraction $\frac{T_2}{T_1}$.

Mathematically:
Since $\Delta T > 0$, it is clear that $T_2 - \Delta T < T_2$ and $T_1 + \Delta T > T_1$.
Therefore:

$$\frac{T_2 - \Delta T}{T_1 + \Delta T} < \frac{T_2}{T_1}$$

When we subtract a smaller number from 1, we get a larger result.

$$1-\left(\frac{T_2-\Delta T}{T_1+\Delta T}\right)>1-\frac{T_2}{T_1}$$

$$\eta_2>\eta_1$$

This proves that the efficiency of the engine increases.

Physical Interpretation: By increasing the temperature of the source, we allow the gas to take in heat at a higher energy level. By decreasing the temperature of the sink, we allow the gas to reject heat at a lower energy level. Both actions increase the work output per unit of heat input.

Step 4: Final Answer:
The efficiency of the Carnot engine will increase.

Quick Tip: To maximize efficiency, you want the largest possible difference between source and sink temperatures. Increasing T_{source} and decreasing T_{sink} simultaneously is the most effective way to boost efficiency.

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4. A particle executes simple harmonic motion of amplitude A and time period T. The time taken by the particle to travel from the mean position to a distance of A/2 is:

(A) T/4
(B) T/8
(C) T/12
(D) T/6

Correct Answer: (C) T/12

Solution:

Step 1: Understanding the Concept:

In Simple Harmonic Motion (SHM), the displacement of a particle varies as a sine or cosine function of time.

The particle moves fastest at the mean position and comes to a momentary halt at the extreme positions.

Because the velocity is not constant, the time taken to cover equal distances is not the same throughout the path.

It will take less time to cover the first half of the amplitude (near the mean position) than the second half (near the extreme).

Step 2: Key Formula or Approach:

The general equation for displacement starting from the mean position ($x = 0$) at $t = 0$ is:

$$x(t) = A\sin(\omega t)$$

Where:

A is the amplitude.

ω is the angular frequency, defined as $\omega = \frac{2\pi}{T}$.

Step 3: Detailed Explanation:

The problem asks for the time t when the displacement x reaches $\frac{A}{2}$.

Substitute the value of x into the displacement equation:

$$\frac{A}{2} = A\sin(\omega t)$$

Divide both sides by A:

$$\frac{1}{2} = \sin(\omega t)$$

We know from trigonometry that $\sin(\theta) = \frac{1}{2}$ when $\theta = \frac{\pi}{6}$ radians (or 30°).

Therefore:

$$\omega t = \frac{\pi}{6}$$

Now, substitute $\omega = \frac{2\pi}{T}$:

$$\left(\frac{2\pi}{T}\right) t = \frac{\pi}{6}$$

Divide both sides by π:

$$\frac{2t}{T} = \frac{1}{6}$$

Solve for t:

$$t = \frac{T}{2 \times 6}$$

$$t = \frac{T}{12}$$

Comparison:

- Time to go from 0 to $A/2$ is $T/12$.
- Time to go from 0 to A is $T/4$.
- Thus, time to go from $A/2$ to A is $T/4 - T/12 = 2T/12 = T/6$.

As expected, it takes twice as long to cover the outer half of the amplitude because the particle is slowing down.

Step 4: Final Answer:

The time taken is $T/12$.

Quick Tip: In SHM, think of the phase angle. Moving from mean to $A/2$ corresponds to a 30° phase change. Since a full cycle (360°) takes time T, 30° takes $(30/360)T = T/12$.

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5. A pendulum clock gives correct time at 20°C. If the coefficient of linear expansion of the pendulum's material is $\alpha = 1.2 \times 10^{-5}/^\circ$C, the clock will lose time per day at 40°C by approximately:

(A) 10.4 s

(B) 5.2 s

(C) 20.8 s

(D) 2.6 s

Correct Answer: (A) 10.4 s

Solution:

Step 1: Understanding the Concept:

The time period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{L}{g}}$.

This indicates that the time period is dependent on the length of the pendulum.

When temperature increases, most materials expand. The length L of the pendulum increases, which in turn increases the time period T.

A larger time period means the clock takes more time to complete one "tick", so it lags behind the actual time, resulting in a "loss" of time.

Step 2: Key Formula or Approach:

The fractional change in the time period due to a change in temperature $\Delta\theta$ is:

$$\frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta$$

The total time lost in a duration t is:

$$\Delta t = \left(\frac{1}{2}\alpha\Delta\theta\right) \times t$$

Where:

α = coefficient of linear expansion.

$\Delta\theta$ = change in temperature.

t = total time interval (usually one day = 86400 seconds).

Step 3: Detailed Explanation:

Given values:

$\alpha = 1.2 \times 10^{-5}/^{\circ}\text{C}$

Initial Temperature $T_{initial} = 20^{\circ}\text{C}$

Final Temperature $T_{final} = 40^{\circ}\text{C}$

Change in temperature $\Delta\theta = 40 - 20 = 20^{\circ}\text{C}$

Total time in one day $t = 24 \times 60 \times 60 = 86400\,\text{s}$

Substitute into the formula:

$$\Delta t = \frac{1}{2} \times (1.2 \times 10^{-5}) \times 20 \times 86400$$

Simplify the constants:

$$\Delta t = (0.6 \times 10^{-5}) \times 20 \times 86400$$

$$\Delta t = (1.2 \times 10^{-4}) \times 86400$$

Convert to standard decimal notation for easier multiplication:

$$\Delta t = 0.00012 \times 86400$$

Calculation:

$$12 \times 864 = 10368$$

Adjusting for the decimal places:

$$\Delta t = 10.368\,\text{s}$$

Rounding this value to the nearest tenth as provided in the options gives 10.4 s.

Step 4: Final Answer:

The clock will lose approximately 10.4 seconds per day.

Quick Tip: Remember the factor of $1/2$ in the formula! It comes from the square root in the pendulum period formula ($L^{1/2}$ becomes $1/2\Delta L/L$). If temperature goes up, the clock is 'slow' and 'loses' time.

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6. Which of the following forces is involved in dinitrogen?

(A) Dipole - dipole interaction

(B) Dipole - induced dipole interaction

(C) London dispersion force

(D) Hydrogen bonding

Correct Answer: (C) London dispersion force

Solution:

Step 1: Understanding the Concept:

Intermolecular forces are the attractions between molecules that determine physical properties like boiling point and solubility.

The nature of these forces depends on whether the molecule is polar or non-polar.

- Polar molecules (like H_2O, HCl) have permanent dipoles.

- Non-polar molecules (like N_2, O_2, CH_4) have a symmetrical distribution of electron density.

Step 2: Detailed Explanation:

Dinitrogen (N_2) consists of two nitrogen atoms connected by a triple covalent bond.

Since both atoms are identical, they have exactly the same electronegativity.

This means the shared electron pair is distributed perfectly evenly between the two nuclei.

As a result, N_2 has no net dipole moment; it is a **non-polar** molecule.

Let's evaluate the intermolecular forces for non-polar molecules:

- **Dipole-dipole:** Requires permanent dipoles. Absent in N_2.
- **Dipole-induced dipole:** Requires a mixture of polar and non-polar species. Absent in pure N_2.
- **Hydrogen bonding:** Requires a hydrogen atom bonded to a highly electronegative atom (N, O, or F). While N is present, there is no H atom in dinitrogen.
- **London dispersion forces (Van der Waals forces):** These are the weakest intermolecular forces. They arise due to the constant motion of electrons, which can create a **temporary instantaneous dipole** at any given moment. This temporary dipole then induces a dipole in a neighboring molecule, creating a brief attraction.

London dispersion forces are universal; they exist in all atoms and molecules. However, in non-polar molecules like N_2, they are the **only** force responsible for holding the molecules together in the liquid or solid states.

Step 3: Final Answer:

The intermolecular force involved in dinitrogen is the London dispersion force.

Quick Tip: Any homonuclear diatomic molecule (same atom twice, e.g., H_2, O_2, Cl_2) is non-polar. For all non-polar substances and noble gases, the answer is always London dispersion forces.

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7. Which from following compounds is NOT in gaseous phase at 25°C?

(A) ClF

(B) BrF

(C) IF_3

(D) ClF_3

Correct Answer: (C) IF_3

Solution:

Step 1: Understanding the Concept:
Interhalogen compounds are formed when two different halogens react. Their physical state depends on the size of the atoms and the resulting Van der Waals forces.
As we move down the halogen group (F → Cl → Br → I), the atomic size and number of electrons increase.
An increase in electrons leads to stronger London dispersion forces, which increases the boiling and melting points of the compounds.

Step 2: Detailed Explanation:
Let's examine the physical states of the given interhalogens at room temperature (25°C):
- **ClF (Chlorine monofluoride):** This is a small molecule. It is a colorless **gas** at room temperature.
- **BrF (Bromine monofluoride):** Bromine is heavier than chlorine. BrF is typically an unstable **gas** that disproportionates into Br_2 and BrF_3, but in the context of pure chemical classification, it is considered gaseous.
- **ClF_3 (Chlorine trifluoride):** This is a highly reactive compound. It is a colorless **gas** (though it condenses to a pale green liquid at 12°C, so at 25°C, it is definitely gaseous).
- **IF_3 (Iodine trifluoride):** Iodine is the heaviest common halogen. As the central atom gets larger and more polarizable, the intermolecular forces become significantly stronger. IF_3 is a yellow **solid** that is stable only at very low temperatures (below −28°C). At room temperature, it is not a gas; it is a solid (or decomposes).

In general, interhalogens with small central atoms (Cl) tend to be gases, those with medium atoms (Br) are liquids, and those with large central atoms (I) are solids at room temperature.

Step 3: Final Answer:
IF_3 is not in the gaseous phase at 25°C.

Quick Tip: The 'phase' rule for interhalogens: Fluorides of Cl are mostly gases (ClF, ClF_3). Fluorides of Br are liquids (BrF_3, BrF_5). Fluorides of I are solids (IF_3, IF_5, IF_7). Heavy atoms = Solid/Liquid.

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8. Which of the following equations gives combined relationship of Boyle's law and Charle's law?

(A) $\frac{P_1V_2}{T_1} = \frac{P_2V_1}{T_2}$
(B) $n = \frac{RT}{PV}$
(C) $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$
(D) $p = \frac{RT}{nV}$

Correct Answer: (C) $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

Solution:

Step 1: Understanding the Concept:
Gas laws describe the relationship between pressure (P), volume (V), and temperature (T) for a given amount of gas (n).
- **Boyle's Law:** For a fixed mass of gas at constant temperature, $V \propto \frac{1}{P}$ or $PV = k_1$.
- **Charles's Law:** For a fixed mass of gas at constant pressure, $V \propto T$ or $\frac{V}{T} = k_2$.
- **Gay-Lussac's Law:** For a fixed mass of gas at constant volume, $P \propto T$ or $\frac{P}{T} = k_3$.

Step 2: Key Formula or Approach:
By combining these relationships, we can see that:

$$V \propto \frac{T}{P}$$

This leads to the combined gas law:

$$\frac{PV}{T} = \text{constant}$$

Step 3: Detailed Explanation:

When a gas undergoes a change from an initial state (P_1, V_1, T_1) to a final state (P_2, V_2, T_2), the quantity $\frac{PV}{T}$ remains constant as long as the mass (number of moles) of the gas does not change.

Therefore, we can write:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Let's check the other options for errors:

- (A) $\frac{P_1V_2}{T_1} = \frac{P_2V_1}{T_2}$: This incorrectly cross-multiplies the volumes. It is mathematically incorrect for gas behavior.
- (B) $n = \frac{RT}{PV}$: The correct ideal gas law is $PV = nRT$. Solving for n gives $n = \frac{PV}{RT}$. Option B is the reciprocal of the correct formula.
- (D) $p = \frac{RT}{nV}$: Rearranging $PV = nRT$ for P gives $P = \frac{nRT}{V}$. Option D is missing the n in the numerator and puts it in the denominator incorrectly.

Step 4: Final Answer:

The correct relationship is $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$.

Quick Tip: Think of the 'PVT' triangle. P and V are on top (multiplied) and T is at the bottom (divided). This combined law is very useful when all three state variables change at once.

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9. Find the concentration of sodium acetate when added to 0.1 M solution of acetic acid to form a buffer solution of pH = 5.5 ? (pK_a of CH_3COOH = 4.5)

(A) 0.1 M

(B) 0.01 M

(C) 1.0 M

(D) 10.0 M

Correct Answer: (C) 1.0 M

Solution:

Step 1: Understanding the Concept:

A buffer solution is a mixture of a weak acid and its conjugate base (or a weak base and its conjugate acid). It resists changes in pH when small amounts of acid or base are added.

For an acidic buffer like Acetic acid (CH_3COOH) and Sodium acetate (CH_3COONa), the pH is determined by the ratio of the concentrations of the salt and the acid.

Step 2: Key Formula or Approach:

We use the **Henderson-Hasselbalch equation**:

$$pH = pK_a + \log_{10}\left(\frac{[\text{Salt}]}{[\text{Acid}]}\right)$$

Where:

$pK_a = -\log_{10}(K_a)$ of the weak acid.

$[\text{Salt}]$ = concentration of the conjugate base (Sodium acetate).

$[\text{Acid}]$ = concentration of the weak acid (Acetic acid).

Step 3: Detailed Explanation:

Given values:

$pH = 5.5$

$pK_a = 4.5$

$[\text{Acid}] = 0.1\,\text{M}$

We need to find $[\text{Salt}]$. Let it be S.

Substitute the values into the equation:

$$5.5 = 4.5 + \log\left(\frac{S}{0.1}\right)$$

Subtract 4.5 from both sides:

$$5.5 - 4.5 = \log\left(\frac{S}{0.1}\right)$$

$$1.0 = \log\left(\frac{S}{0.1}\right)$$

To solve for S, we take the antilogarithm (base 10) of both sides:

$$10^1 = \frac{S}{0.1}$$

$$10 = \frac{S}{0.1}$$

Multiply both sides by 0.1:

$$S = 10 \times 0.1$$

$$S = 1.0\,\text{M}$$

Verification:

If Salt = 1.0 M and Acid = 0.1 M, then Salt/Acid ratio is 10.

$\log(10) = 1$.

$pH = pK_a + 1 = 4.5 + 1 = 5.5$. This matches the given pH.

Step 4: Final Answer:

The concentration of sodium acetate is 1.0 M.

Quick Tip: Handy Rule: If $pH = pK_a$, $[Salt] = [Acid]$. If $pH = pK_a + 1$, $[Salt] = 10 \times [Acid]$. If $pH = pK_a - 1$, $[Acid] = 10 \times [Salt]$. This saves a lot of calculation time!

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10. The solubility product of AgBr is 4.9×10^{-13} at a certain temperature. Calculate the solubility.

(A) $4 \times 10^{-6} \text{mol} \cdot \text{dm}^{-3}$

(B) $4 \times 10^{-7} \text{mol} \cdot \text{dm}^{-3}$

(C) $7 \times 10^{-7} \text{mol} \cdot \text{dm}^{-3}$

(D) $3 \times 10^{-8} \text{mol} \cdot \text{dm}^{-3}$

Correct Answer: (C) $7 \times 10^{-7} \text{mol} \cdot \text{dm}^{-3}$

Solution:

Step 1: Understanding the Concept:

Solubility (s) refers to the concentration of a substance in its saturated solution.

For a sparingly soluble salt, only a tiny fraction dissolves. The equilibrium constant for this dissolution is called the Solubility Product (K_{sp}).

Step 2: Key Formula or Approach:

Consider the dissociation of Silver Bromide ($AgBr$):

$$AgBr(s) \rightleftharpoons Ag^+(aq) + Br^-(aq)$$

If the molar solubility is s moles/dm^3, then:

$[Ag^+] = s$

$[Br^-] = s$

The expression for K_{sp} is:

$$K_{sp} = [Ag^+][Br^-]$$

Substituting the solubility terms:

$$K_{sp} = s \times s = s^2$$

To find solubility, we take the square root:

$$s = \sqrt{K_{sp}}$$

Step 3: Detailed Explanation:

Given:

$K_{sp} = 4.9 \times 10^{-13}$

Set up the equation:

$$s = \sqrt{4.9 \times 10^{-13}}$$

To calculate this easily, rewrite the number under the square root so the power of 10 is even:

$$s = \sqrt{49 \times 10^{-1} \times 10^{-13}}$$

$$s = \sqrt{49 \times 10^{-14}}$$

Now take the square root of each part:

$$\sqrt{49} = 7$$

$$\sqrt{10^{-14}} = 10^{-7}$$

Therefore:

$$s = 7 \times 10^{-7}\,\text{mol} \cdot \text{dm}^{-3}$$

Unit Note: $\text{mol} \cdot \text{dm}^{-3}$ is equivalent to Molarity (M) or moles per liter.

Looking at the options:

(A) 4×10^{-6} - Incorrect value.

(B) 4×10^{-7} - Incorrect coefficient.

(C) 7×10^{-7} - Correct.

(D) 3×10^{-8} - Incorrect value.

Step 4: Final Answer:

The solubility is $7 \times 10^{-7}\,\text{mol} \cdot \text{dm}^{-3}$.

Quick Tip: For salts like $AgCl, AgBr, AgI, BaSO_4$ (binary salts), $s = \sqrt{K_{sp}}$. For salts like $Ag_2CrO_4, PbCl_2, CaF_2$ (A_2B or AB_2), $K_{sp} = 4s^3$. Always check the salt formula first!

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11. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx (x \geq 0)$. Then $f(3) - f(1)$ is equal to:

(A) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$

(B) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

(C) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$

(D) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

Correct Answer: (B) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

Solution:

Step 1: Understanding the Concept:

This problem requires evaluating the difference between values of an indefinite integral at two points, which is essentially evaluating a definite integral:

$$f(3)-f(1)=\int_1^3 \frac{\sqrt{x}}{(1+x)^2}dx$$

To solve this, we use substitution. Since we have $\sqrt{x}$ and $(1+x)$, a trigonometric substitution like $x=\tan^2\theta$ is highly effective.

Step 2: Key Formula or Approach:

Let $x=\tan^2\theta$.

Then $dx=2\tan\theta\sec^2\theta\,d\theta$.

Also, $\sqrt{x}=\tan\theta$.

And $1+x=1+\tan^2\theta=\sec^2\theta$.

Change of limits:

When $x=1$, $\tan^2\theta=1 \implies \theta=\frac{\pi}{4}$.

When $x=3$, $\tan^2\theta=3 \implies \theta=\frac{\pi}{3}$.

Step 3: Detailed Explanation:

Substitute everything into the integral:

$$I=\int_{\pi/4}^{\pi/3}\frac{\tan\theta}{(\sec^2\theta)^2}\cdot(2\tan\theta\sec^2\theta)d\theta$$

$$I=\int_{\pi/4}^{\pi/3}\frac{2\tan^2\theta\sec^2\theta}{\sec^4\theta}d\theta$$

$$I=\int_{\pi/4}^{\pi/3}\frac{2\tan^2\theta}{\sec^2\theta}d\theta$$

Since $\tan^2\theta=\frac{\sin^2\theta}{\cos^2\theta}$ and $\sec^2\theta=\frac{1}{\cos^2\theta}$:

$$I = \int_{\pi/4}^{\pi/3} 2\sin^2\theta\, d\theta$$

Using the trigonometric identity $2\sin^2\theta = 1 - \cos 2\theta$:

$$I = \int_{\pi/4}^{\pi/3} (1 - \cos 2\theta) d\theta$$

Integrating:

$$I = \left[\theta - \frac{\sin 2\theta}{2}\right]_{\pi/4}^{\pi/3}$$

Evaluate at limits:

Upper limit ($\theta = \pi/3$):

$$\frac{\pi}{3} - \frac{\sin(2\pi/3)}{2} = \frac{\pi}{3} - \frac{\sqrt{3}/2}{2} = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$$

Lower limit ($\theta = \pi/4$):

$$\frac{\pi}{4} - \frac{\sin(\pi/2)}{2} = \frac{\pi}{4} - \frac{1}{2}$$

Final Difference:

$$I = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$I = \frac{\pi}{3} - \frac{\pi}{4} + \frac{1}{2} - \frac{\sqrt{3}}{4}$$

Find common denominator for the fractions of π:

$$\frac{4\pi - 3\pi}{12} = \frac{\pi}{12}$$

So, $I = \frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$.

Step 4: Final Answer:

The result is $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$.

Quick Tip: Trigonometric substitutions ($x = a\tan^2\theta$) are lifesavers for algebraic denominators like $(1+x)^2$. It reduces a complex fraction into basic sin / cos functions.

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12. Gas is being pumped into a spherical balloon at the rate of $30\text{ft}^3/\text{min}$. Then the rate at which the radius increases when it reaches the value 15 ft is:

(A) $\frac{1}{30\pi}$ft/min

(B) $\frac{1}{15\pi}$ft/min

(C) $\frac{1}{20}$ft/min

(D) $\frac{1}{25}$ft/min

Correct Answer: (A) $\frac{1}{30\pi}$ft/min

Solution:

Step 1: Understanding the Concept:

This problem involves related rates in calculus. We are given the rate of change of volume and asked to find the rate of change of the radius at a specific moment.

Since the balloon is spherical, we must use the geometry of a sphere.

Step 2: Key Formula or Approach:

The volume V of a sphere with radius r is given by:

$$V = \frac{4}{3}\pi r^3$$

To find the relationship between the rates, we differentiate this equation with respect to time t:

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot (3r^2)\frac{dr}{dt}$$

Simplifying:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

(Note: $4\pi r^2$ is the surface area of the sphere).

Step 3: Detailed Explanation:

Given:

Rate of volume increase $\frac{dV}{dt} = 30\,\text{ft}^3/\text{min}$.

Radius at the given instant $r = 15\,\text{ft}$.

We need to find $\frac{dr}{dt}$.

Substitute the known values into the differentiated equation:

$$30 = 4\pi(15)^2 \cdot \frac{dr}{dt}$$

First, calculate 15^2:

$$15^2 = 225$$

Now substitute:

$$30 = 4\pi(225) \cdot \frac{dr}{dt}$$

$$30 = 900\pi \cdot \frac{dr}{dt}$$

Solve for $\frac{dr}{dt}$:

$$\frac{dr}{dt} = \frac{30}{900\pi}$$

Cancel the common factor of 30:

$$\frac{dr}{dt} = \frac{1}{30\pi} \text{ ft/min}$$

Check the logic: As the balloon gets bigger, the same volume of gas added leads to a smaller increase in radius because it must be spread over a much larger surface area. Since the surface area at $r = 15$ is quite large (900π), the radial increase should be very small, which $\frac{1}{30\pi} \approx 0.01$ is.

Step 4: Final Answer:

The rate of increase of the radius is $\frac{1}{30\pi}$ ft/min.

Quick Tip: Always identify the 'given rate' and the 'target rate'. For a sphere, $dV/dt = Area \times dr/dt$. Just divide the volume rate by the surface area at that instant!

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13. Let the function $f(x)$ defined as $f(x) = \frac{x-|x|}{x}$, then:

(A) the function is continuous everywhere

(B) the function is not continuous

(C) the function is continuous when x < 0

(D) the function is continuous for all x except zero

Correct Answer: (B) the function is not continuous

Solution:

Step 1: Understanding the Concept:

A function is continuous at a point $x = a$ if the limit as $x \to a$ exists and is equal to the value of the function at that point.

If a function is not defined at a certain point, it is automatically discontinuous there.

The absolute value function $|x|$ behaves differently for positive and negative numbers:

- For $x \geq 0, |x| = x$
- For $x < 0, |x| = -x$

Step 2: Detailed Explanation:

Let's analyze the expression for $f(x)$ based on the sign of x.

Case 1: When $x > 0$.

Here, $|x| = x$.

$$f(x) = \frac{x - x}{x} = \frac{0}{x} = 0$$

Case 2: When $x < 0$.

Here, $|x| = -x$.

$$f(x) = \frac{x - (-x)}{x} = \frac{x + x}{x} = \frac{2x}{x} = 2$$

Case 3: When $x = 0$.

The function involves division by x. Since division by zero is undefined, $f(0)$ does not exist.

Now let's check the limits as x approaches 0.

The Right-hand Limit (RHL) as $x \to 0^+$:

$$\lim_{x \to 0^+} f(x) = 0$$

The Left-hand Limit (LHL) as $x \to 0^-$:

$$\lim_{x \to 0^-} f(x) = 2$$

Since LHL $\neq$ RHL, the overall limit $\lim_{x \to 0} f(x)$ does not exist.
Because there is no limit at $x = 0$ and the function is not defined at $x = 0$, the function has a jump discontinuity at the origin.

Evaluation of options:
(A) Incorrect - it is discontinuous at $x = 0$.
(B) Correct - it is a general statement that the function is not continuous.
(C) This is a true statement (it is constant 2 for all $x < 0$), but it doesn't describe the function's overall continuity.
(D) This is also true, as it's continuous everywhere except for the single point at zero.
However, in standard testing, if a function is discontinuous even at one point, it is categorized as "not continuous".

Step 3: Final Answer:
The function is not continuous because it is undefined and the limit does not exist at $x = 0$.

Quick Tip: Whenever you see $|x|/x$ or similar structures, immediately check for $x = 0$. These 'sgn' (signum) type functions almost always have jump discontinuities at zero.

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14. If the direction ratio of two lines are given by $l + m + n = 0$ and $mn - 2ln + lm = 0$, then the angle between the lines is:

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) 0

Correct Answer: (C) $\frac{\pi}{2}$

Solution:

Step 1: Understanding the Concept:

The angle θ between two lines with direction ratios (l_1, m_1, n_1) and (l_2, m_2, n_2) is found using:

$$\cos\theta = \frac{l_1 l_2 + m_1 m_2 + n_1 n_2}{\sqrt{l_1^2 + m_1^2 + n_1^2}\sqrt{l_2^2 + m_2^2 + n_2^2}}$$

If the sum of products of direction ratios is zero ($l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$), then $\cos\theta = 0$, which means $\theta = \frac{\pi}{2}$ (the lines are perpendicular).

Step 2: Detailed Explanation:

We are given two equations for the direction ratios l, m, n:

1) $l + m + n = 0 \implies n = -(l + m)$

2) $mn - 2ln + lm = 0$

Substitute equation (1) into equation (2):

$$m(-(l + m)) - 2l(-(l + m)) + lm = 0$$

$$-ml - m^2 + 2l^2 + 2lm + lm = 0$$

Combine like terms:

$$2l^2 + 2lm - m^2 = 0$$

Divide the whole equation by m^2 (assuming $m \neq 0$):

$$2\left(\frac{l}{m}\right)^2+2\left(\frac{l}{m}\right)-1=0$$

This is a quadratic in the ratio $\frac{l}{m}$. Let the two roots (representing the two lines) be $\frac{l_1}{m_1}$ and $\frac{l_2}{m_2}$.
From quadratic properties:
Product of roots: $\frac{l_1l_2}{m_1m_2}=\frac{\text{constant term}}{\text{coeff of } x^2}=\frac{-1}{2}$
So, $2l_1l_2=-m_1m_2 \implies 2l_1l_2+m_1m_2=0$. (Eq. A)

Now we need to relate this to n. From $n=-(l+m)$:
$n_1n_2=(-(l_1+m_1))\cdot(-(l_2+m_2))=l_1l_2+l_1m_2+l_2m_1+m_1m_2$
Sum of roots: $\frac{l_1}{m_1}+\frac{l_2}{m_2}=\frac{-b}{a}=\frac{-2}{2}=-1$
Multiply by m_1m_2: $l_1m_2+l_2m_1=-m_1m_2$.
Substitute this back into the expression for n_1n_2:
$n_1n_2=l_1l_2+(-m_1m_2)+m_1m_2=l_1l_2$.

Check the perpendicularity condition $l_1l_2+m_1m_2+n_1n_2$:
Substitute $n_1n_2=l_1l_2$:

$$l_1l_2+m_1m_2+l_1l_2=2l_1l_2+m_1m_2$$

From Equation A, we already found that $2l_1l_2+m_1m_2=0$.
Thus, the sum of products of direction ratios is zero.

Step 3: Final Answer:
Since the condition for perpendicularity is met, the angle is $\frac{\pi}{2}$.

Quick Tip: For questions with two equations like these, often the coefficients are set up so that the lines are either parallel or perpendicular. If you find the sum of product of ratios is zero, it's a 90-degree angle.

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15. The value of c of Lagrange's mean value theorem for $f(x)=\sqrt{25-x^2}$ on [1, 5] is:

(A) $\sqrt{15}$

(B) 5

(C) $\sqrt{10}$

(D) 1

Correct Answer: (A) $\sqrt{15}$

Solution:

Step 1: Understanding the Concept:

Lagrange's Mean Value Theorem (LMVT) states that if a function f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a point c in (a, b) such that the instantaneous rate of change (derivative) at c equals the average rate of change over the interval.

Equation:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Step 2: Detailed Explanation:

Given function: $f(x) = \sqrt{25 - x^2}$.

Interval: $[1, 5]$ so $a = 1$ and $b = 5$.

1. Check conditions: The function is defined for $x \in [-5, 5]$. In the range $[1, 5]$, the function is square-root of a non-negative number, so it is continuous. In $(1, 5)$, it is differentiable.

2. Calculate $f(a)$ and $f(b)$:

$f(1) = \sqrt{25 - 1^2} = \sqrt{24} = 2\sqrt{6}$.

$f(5) = \sqrt{25 - 5^2} = \sqrt{0} = 0$.

3. Find average rate of change:

$$\text{Slope} = \frac{f(5) - f(1)}{5 - 1} = \frac{0 - \sqrt{24}}{4} = -\frac{\sqrt{24}}{4}$$

4. Find the derivative $f'(x)$:

Using chain rule:

$$f'(x) = \frac{1}{2\sqrt{25-x^2}} \cdot \frac{d}{dx}(25-x^2)$$

$$f'(x) = \frac{-2x}{2\sqrt{25-x^2}} = \frac{-x}{\sqrt{25-x^2}}$$

5. Set $f'(c)$ equal to the slope:

$$\frac{-c}{\sqrt{25-c^2}} = -\frac{\sqrt{24}}{4}$$

Square both sides to eliminate the radicals:

$$\frac{c^2}{25-c^2} = \frac{24}{16}$$

Simplify the fraction $\frac{24}{16} = \frac{3}{2}$:

$$\frac{c^2}{25-c^2} = \frac{3}{2}$$

Cross-multiply:

$$2c^2 = 3(25-c^2)$$

$$2c^2 = 75-3c^2$$

$$5c^2 = 75$$

$$c^2 = 15 \implies c = \pm\sqrt{15}$$

Since c must lie in the interval $(1, 5)$, we take the positive root: $c = \sqrt{15}$.

Note: $\sqrt{15} \approx 3.87$, which is indeed between 1 and 5.

Step 3: Final Answer:

The value of c is $\sqrt{15}$.

Quick Tip: In LMVT, the slope of the tangent at c is equal to the slope of the chord joining $(a, f(a))$ and $(b, f(b))$. Always verify that your final value for c falls strictly within the open interval (a, b).
