MHT CET 2026 May 12 Shift 1

Question Paper (Memory-Based) with Solutions

Conducted by CET Cell, Maharashtra

General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 200 marks.

(iii) **Structure:** The paper has 3 Sections:

- **Section A:** 50 Multiple Choice Questions (Physics).
- **Section B:** 50 Multiple Choice Questions (Chemistry).
- **Section B:** 100 Multiple Choice Questions (Mathematics).

(iv) **Compulsory Questions:** All 200 questions are compulsory.

(v) Each question has four options. Only **one** option is correct.

(vi) Correct Answer: +1 marks.

(vii) **Incorrect Answer:** (No Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

PHYSICS

1. Eight identical spherical liquid drops, each falling with a terminal velocity v, coalesce to form a single large drop. The terminal velocity of the new large drop is:

(A) $2v$

(B) $4v$

(C) $8v$

(D) $16v$

Correct Answer: (B) $4v$

Solution:

Step 1: Understanding the Concept:

This problem investigates the physics of fluid dynamics, specifically focusing on terminal velocity and the conservation of mass/volume during the coalescence of liquid droplets.

Terminal velocity is reached when an object falling through a viscous fluid experiences zero net force.

This occurs because the downward gravitational force is perfectly balanced by the upward buoyancy and the viscous drag force (Stokes' Law).

When multiple small drops merge into one large drop, the total volume remains constant, but the surface area changes.

The change in radius directly affects the terminal velocity because the gravitational force (proportional to mass/volume, hence r^3) and the drag force (proportional to velocity and radius, rv) scale differently.

Key Formula or Approach:

According to Stokes' Law, the terminal velocity v of a spherical object of radius r is given by:

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

Where:

- ρ is the density of the drop.
- σ is the density of the medium.
- g is the acceleration due to gravity.
- η is the coefficient of viscosity.

From this expression, we see that for the same liquid and medium, the terminal velocity is directly proportional to the square of the radius:

$$v \propto r^2$$

Step 2: Detailed Explanation:

Let the radius of each of the eight identical small drops be r.

Let the radius of the resulting large drop be R.

1. **Volume Conservation:**

The volume of 8 small drops equals the volume of the single large drop.

$$8 \times \left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

By canceling out common terms $\frac{4}{3}\pi$, we get:

$$8r^3 = R^3$$

Taking the cube root on both sides:

$$R = \sqrt[3]{8r^3} = 2r$$

This tells us the radius of the new drop is exactly double the radius of the individual small drops.

2. **Relating Terminal Velocities:**

Let v be the terminal velocity of the small drop and V' be the terminal velocity of the large drop.

Using the proportionality $v \propto r^2$:

$$\frac{V'}{v} = \left(\frac{R}{r}\right)^2$$

Substitute $R = 2r$ into the ratio:

$$\frac{V'}{v} = \left(\frac{2r}{r}\right)^2$$

$$\frac{V'}{v} = (2)^2 = 4$$

$$V' = 4v$$

The increase in terminal velocity happens because the mass of the drop increases by a factor of 8 (increasing the downward pull), while the drag-related radius only increases by a factor of 2.
This results in a net four-fold increase in the speed required to reach force equilibrium.

Step 3: Final Answer:

The terminal velocity of the new large drop is $4v$.

Quick Tip: General formula for n drops coalescing: $V_{new} = n^{2/3} \times v_{old}$.
For 8 drops: $8^{2/3} = (2^3)^{2/3} = 2^2 = 4$.
For 27 drops: $27^{2/3} = 9$.

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2. A uniform solid sphere of mass M and radius R is placed on a smooth horizontal surface. It is struck by a horizontal cue at a height h above the center. For the sphere to roll without slipping immediately after the impact, the value of h must be:

(A) $\frac{R}{2}$
(B) $\frac{2R}{5}$
(C) $\frac{2R}{3}$
(D) $\frac{3R}{5}$

Correct Answer: (B) $\frac{2R}{5}$

Solution:

Step 1: Understanding the Concept:

When an impulsive force is applied to an object, it produces both a linear change in momentum and an angular change in momentum.

For a sphere to "roll without slipping" immediately, the linear velocity of the center of mass (v) and the angular velocity (ω) must satisfy the kinematic constraint $v = R\omega$.
Since the surface is smooth, friction does not act during the impact to provide any additional torque or force.
Therefore, the relationship between translation and rotation is purely determined by where the impulse is applied relative to the center.

Key Formula or Approach:

1. Linear Impulse: $P = \int F dt = Mv$
2. Angular Impulse: $\tau_{imp} = P \times h = I\omega$
3. Moment of Inertia for a solid sphere: $I = \frac{2}{5}MR^2$
4. Rolling Condition: $v = R\omega$

Step 2: Detailed Explanation:

Let a horizontal impulse P be applied at a height h above the center of mass.

1. **Linear Motion:**

The impulse causes the center of mass to gain velocity v.

$$P = Mv \quad \text{— (Equation 1)}$$

2. **Rotational Motion:**

The impulse P acting at a distance h from the center creates an angular impulse.

$$P \cdot h = I\omega$$

Substitute the moment of inertia for a uniform solid sphere:

$$P \cdot h = \left(\frac{2}{5}MR^2\right)\omega \quad \text{— (Equation 2)}$$

3. **Combining Equations for Pure Rolling:**

For rolling without slipping immediately, we substitute $\omega = \frac{v}{R}$ into Equation 2:

$$P \cdot h = \frac{2}{5}MR^2\left(\frac{v}{R}\right)$$

$$P \cdot h = \frac{2}{5}MRv$$

From Equation 1, we know that $Mv = P$. Substituting this back:

$$P \cdot h = \frac{2}{5}R \cdot P$$

By canceling P from both sides (as $P \neq 0$):

$$h = \frac{2}{5}R$$

This specific height is the "sweet spot" where the imparted rotation perfectly matches the imparted translation for immediate rolling.

Step 3: Final Answer:

The height h must be $\frac{2R}{5}$.

Quick Tip: For any rigid body with $I = kMR^2$, the height for immediate rolling is $h = kR$.

Solid Sphere: $k = 2/5 \implies h = 0.4R$

Hollow Sphere: $k = 2/3 \implies h = 0.67R$

Solid Cylinder: $k = 1/2 \implies h = 0.5R$

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3. A disc of mass M and radius R has a concentric hole of radius $\frac{R}{2}$. Its moment of inertia about an axis passing through its center and perpendicular to its plane is:

(A) $\frac{15}{32}MR^2$

(B) $\frac{13}{32}MR^2$

(C) $\frac{5}{16}MR^2$

(D) $\frac{3}{8}MR^2$

Correct Answer: (A) $\frac{15}{32}MR^2$

Solution:

Step 1: Understanding the Concept:

The moment of inertia of a complex or hollowed-out object can be calculated using the principle of superposition.

We treat the object as a full, uniform disc and then subtract the moment of inertia of the portion that has been removed (the hole).

Crucially, mass is distributed uniformly across the area.

Therefore, the mass of the removed portion is proportional to its area relative to the original disc.

In this problem, the phrasing implies that M is the mass of the *original* solid disc before the hole was cut.

Key Formula or Approach:

1. Moment of inertia of a solid disc: $I = \frac{1}{2}mr^2$
2. Mass per unit area (surface density) $\sigma = \frac{\text{Mass}}{\text{Area}}$
3. Total Inertia $I_{final} = I_{full} - I_{removed}$

Step 2: Detailed Explanation:

Let M be the mass of the original solid disc of radius R.

Area of the original disc $A_1 = \pi R^2$.

Area of the hole $A_2 = \pi\left(\frac{R}{2}\right)^2 = \frac{\pi R^2}{4}$.

1. **Finding the Mass of the Removed Portion (m):**

Since the disc is uniform, mass is proportional to area.

$$\frac{m}{M} = \frac{\text{Area of hole}}{\text{Area of full disc}} = \frac{\pi R^2/4}{\pi R^2} = \frac{1}{4}$$

So, $m = \frac{M}{4}$.

2. **Calculating Individual Moments of Inertia:**

Inertia of the full disc: $I_1 = \frac{1}{2}MR^2$.

Inertia of the removed disc (hole) about the center:

$$I_2 = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2$$

$$I_2 = \frac{1}{2} \cdot \frac{M}{4} \cdot \frac{R^2}{4} = \frac{1}{32}MR^2$$

3. **Finding Net Inertia:**

Subtract the hole's inertia from the full disc's inertia:

$$I_{net} = I_1 - I_2 = \frac{1}{2}MR^2 - \frac{1}{32}MR^2$$

Convert to common denominator:

$$I_{net} = \frac{16}{32}MR^2 - \frac{1}{32}MR^2 = \frac{15}{32}MR^2$$

Step 3: Final Answer:

The moment of inertia of the remaining disc is $\frac{15}{32}MR^2$.

Quick Tip: If the question defines M as the mass of the *remaining* annular disc, the formula changes to $I = \frac{1}{2}M(R^2 + r^2)$.

Always check the options to see which mass definition the examiner intended.

4. The work done in increasing the radius of a soap bubble from R to $2R$ is W. The work done in further increasing its radius from $2R$ to $3R$ will be:

(A) $\frac{5}{3}W$
(B) $\frac{4}{3}W$
(C) $\frac{7}{3}W$
(D) W

Correct Answer: (A) $\frac{5}{3}W$

Solution:

Step 1: Understanding the Concept:
Work done in expanding a bubble is equivalent to the change in its surface potential energy.
Surface energy is defined as the product of Surface Tension (T) and the total surface area of the film.
A soap bubble is unique because it consists of a thin film of liquid with air on both the inside and the outside.
Consequently, it has two free surfaces.
Total Surface Area of a soap bubble $A = 2 \times (4\pi r^2) = 8\pi r^2$.

Key Formula or Approach:
Work Done $W = T \times \Delta A = T \times 8\pi(r_{final}^2 - r_{initial}^2)$.
Since T and 8π are constant for a given bubble, we can say $W \propto (r_{final}^2 - r_{initial}^2)$.

Step 2: Detailed Explanation:
1. **First Expansion (from R to $2R$):**
Work done is given as W.

$$W = 8\pi T[(2R)^2 - R^2]$$

$$W = 8\pi T[4R^2 - R^2]$$

$$W = 8\pi T(3R^2)$$

This gives us a base value: $8\pi TR^2 = \frac{W}{3}$.

2. **Second Expansion (from $2R$ to $3R$):**

Let the required work be W'.

$$W' = 8\pi T[(3R)^2 - (2R)^2]$$

$$W' = 8\pi T[9R^2 - 4R^2]$$

$$W' = 8\pi T(5R^2)$$

3. **Comparing W' and W:**

We substitute the value of $8\pi TR^2$ from our first step:

$$W' = 5 \times (8\pi TR^2) = 5 \times \frac{W}{3} = \frac{5}{3}W$$

The work increases because even though the radius increases by the same increment R, the total area added is much larger when starting from a larger initial radius.

Step 3: Final Answer:

The work done for the further increase is $\frac{5}{3}W$.

Quick Tip: For surface tension problems, skip the constants and focus on the difference of squares: $(r_2^2 - r_1^2)$.

Case 1: $2^2 - 1^2 = 3$ units.

Case 2: $3^2 - 2^2 = 5$ units.

Ratio $= 5/3$.

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5. Water flows through a horizontal pipe. At one point, the velocity is 2 m/s and the pressure is 2 m of water column. What is the pressure at another point where the velocity is 3 m/s? (Density of water = 10^3 kg/m^3)

(A) 1.75 m of water

(B) 1.2 m of water

(C) 1 m of water

(D) 1.5 m of water

Correct Answer: (A) 1.75 m of water

Solution:

Step 1: Understanding the Concept:

For a non-viscous, incompressible fluid in steady flow through a horizontal pipe, Bernoulli's Principle applies.

Since the pipe is horizontal, the potential energy per unit volume is constant throughout.

Therefore, the sum of pressure energy and kinetic energy per unit volume remains constant.

Pressure is expressed here in terms of "pressure head," which is the height of a liquid column that would exert that pressure.

Key Formula or Approach:

Bernoulli's Equation for horizontal flow:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

To convert this to "head" (units of meters), divide by ρg:

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

Let $h = \frac{P}{\rho g}$ be the pressure head.

$$h_1 + \frac{v_1^2}{2g} = h_2 + \frac{v_2^2}{2g}$$

Step 2: Detailed Explanation:

Given:

Initial Pressure Head $h_1 = 2$ m.

Initial Velocity $v_1 = 2$ m/s.

Final Velocity $v_2 = 3$ m/s.

Acceleration due to gravity $g \approx 10$ m/s^2 (standard approximation for CET).

Substitute the values into the Bernoulli head equation:

$$2 + \frac{2^2}{2 \times 10} = h_2 + \frac{3^2}{2 \times 10}$$

$$2 + \frac{4}{20} = h_2 + \frac{9}{20}$$

$$2 + 0.2 = h_2 + 0.45$$

$$2.2 = h_2 + 0.45$$

Subtracting 0.45 from both sides:

$$h_2 = 2.2 - 0.45 = 1.75 \text{ m}$$

This result follows the principle that as fluid velocity increases (kinetic energy increases), the static pressure must decrease to conserve energy.

Step 3: Final Answer:

The pressure at the second point is 1.75 m of water column.

Quick Tip: Always use $g = 10$ in CET unless the options are very close. If velocity increases, pressure must decrease. If you get an answer greater than 2 m, you have likely swapped the terms.

CHEMISTRY

6. Identify the type of intermolecular force present between benzene and ammonia.

(A) Hydrogen bonding

(B) Dipole-induced dipole interaction

(C) Dipole-dipole interaction

(D) Ion-dipole interaction

Correct Answer: (B) Dipole-induced dipole interaction

Solution:

Step 1: Understanding the Concept:

Intermolecular forces are determined by the electronic structure and polarity of the molecules involved.

Ammonia (NH_3) is a polar molecule because the electronegativity of Nitrogen is much higher

than Hydrogen, and its trigonal pyramidal shape prevents the bond dipoles from canceling out.
Benzene (C_6H_6) is a non-polar molecule. It is a highly symmetrical hydrocarbon ring where individual $C-H$ bond dipoles cancel each other out perfectly.

Step 2: Detailed Explanation:
When a polar molecule like Ammonia approaches a non-polar molecule like Benzene, the permanent dipole of Ammonia creates an electric field.
This field distorts the electron cloud of the Benzene molecule, effectively "inducing" a temporary dipole in the non-polar Benzene.
The attraction between the permanent dipole of the Ammonia and the induced dipole of the Benzene is called a dipole-induced dipole interaction.

Evaluation of other options:
- **Hydrogen bonding:** Ammonia can act as an H-bond donor, but Benzene lacks a highly electronegative atom (N, O, F) to act as an acceptor for a traditional H-bond.
- **Dipole-dipole:** This requires both molecules to be polar.
- **Ion-dipole:** This requires one species to be a charged ion.

Step 3: Final Answer:
The interaction is dipole-induced dipole interaction.

Quick Tip: Remember:
Polar + Polar = Dipole-Dipole
Non-polar + Non-polar = London Forces
Polar + Non-polar = Dipole-Induced Dipole

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7. What is the SI unit for rate of diffusion?

(A) cm^3 $minute^{-1}$
(B) dm^3 $hour^{-1}$
(C) dm^3 s^{-1}

(D) mL hour^{-1}

Correct Answer: (C) dm^3 s^{-1}

Solution:

Step 1: Understanding the Concept:
The rate of diffusion refers to the amount of substance (usually gas) that moves or spreads per unit of time.
Mathematically, it is expressed as the volume of gas diffused divided by the time taken.

Step 2: Detailed Explanation:
In the International System of Units (SI):
- The unit for time is the second (s).
- The standard unit for volume is the cubic meter (m^3).

However, in practical laboratory chemistry, volume is frequently measured in decimeters cubed (dm^3), which is equivalent to one liter (L). The dm^3 is a recognized SI-derived unit.
Therefore, the rate (Volume / Time) would naturally be in units of dm^3 per second, or dm^3 s^{-1}.

Evaluating the options:
- Minutes and hours are not the base SI units for time.
- cm^3 and mL are sub-multiples of volume units, but dm^3 is the standard derivation used alongside molarity and gas laws in SI conventions.

Step 3: Final Answer:
The correct unit is dm^3 s^{-1}.

Quick Tip: Any "rate" in SI units must have "seconds" in the denominator. This immediately eliminates (A), (B), and (D).

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8. Which of the following pair of liquids has dipole-induced dipole interaction?

(A) Water and ammonia

(B) Water and chloroform

(C) Water and benzene

(D) Water and ethyl alcohol

Correct Answer: (C) Water and benzene

Solution:

Step 1: Understanding the Concept:

As established in previous questions, dipole-induced dipole interactions occur strictly between a polar molecule (having a permanent dipole) and a non-polar molecule.

Step 2: Detailed Explanation:

Let's analyze the pairs provided:

(A) **Water and Ammonia:** Both are polar. They interact via dipole-dipole forces and strong hydrogen bonding.

(B) **Water and Chloroform ($CHCl_3$):** Both are polar. They interact via dipole-dipole forces.

(C) **Water and Benzene:** Water is highly polar. Benzene is a symmetrical hydrocarbon and is non-polar. The dipole of water induces a dipole in the benzene ring. This fits the definition.

(D) **Water and Ethyl alcohol:** Both are polar and both possess hydroxyl ($-OH$) groups capable of forming hydrogen bonds.

Step 3: Final Answer:

Water and benzene is the correct pair.

Quick Tip: Hydrocarbons (Benzene, Toluene, Hexane) are non-polar. Molecules with O-H or N-H groups (Water, Alcohols, Ammonia) are polar.

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9. A certain mass of a gas occupies a volume of 2.5 dm^3 at NTP. Calculate the change in volume of gas at the same temperature, if pressure of gas is changed to 1.25 atm.

(A) 3.0 dm^3

(B) 0.5 dm^3

(C) 4.5 dm^3

(D) 1.5 dm^3

Correct Answer: (B) 0.5 dm^3

Solution:

Step 1: Understanding the Concept:

Since the temperature remains constant, we use Boyle's Law.

Boyle's Law states that for a fixed mass of an ideal gas at constant temperature, the volume is inversely proportional to the pressure.

$$P \propto \frac{1}{V} \implies PV = \text{constant}$$

Key Formula or Approach:

$$P_1V_1 = P_2V_2$$

NTP (Normal Temperature and Pressure) corresponds to a pressure of 1 atm.

Step 2: Detailed Explanation:

Given:

Initial Pressure (P_1) = 1 atm (at NTP).

Initial Volume (V_1) = 2.5 dm^3.

Final Pressure (P_2) = 1.25 atm.

1. **Calculate Final Volume (V_2):**

$$1 \times 2.5 = 1.25 \times V_2$$

$$V_2 = \frac{2.5}{1.25} = 2.0 \text{ dm}^3$$

2. **Calculate Change in Volume (ΔV):**

The question asks for the change in volume, not the final volume.

$$\Delta V = V_1 - V_2 = 2.5 \text{ dm}^3 - 2.0 \text{ dm}^3 = 0.5 \text{ dm}^3$$

Since the pressure increased, the volume decreased, resulting in a change of 0.5 dm^3.

Step 3: Final Answer:

The change in volume is 0.5 dm^3.

Quick Tip: Be careful with wording! Questions often ask for "final volume" or "change in volume." If you chose 2.0 dm^3, you would be wrong even though your math was correct.

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10. A gas occupies a volume of 4.2 dm^3 at 101 kPa pressure. What volume will the gas occupy if the pressure is increased to 235 kPa keeping the temperature constant?

(A) 1.8 dm^3

(B) 3.6 dm^3

(C) 0.9 dm^3

(D) 2.1 dm^3

Correct Answer: (A) 1.8 dm^3

Solution:

Step 1: Understanding the Concept:

This is another application of Boyle's Law. At a constant temperature, pressure and volume are inversely related. When pressure increases, volume must decrease proportionately.

Key Formula or Approach:

$$P_1V_1 = P_2V_2$$

Step 2: Detailed Explanation:

Given:

$P_1 = 101$ kPa

$V_1 = 4.2$ dm^3

$P_2 = 235$ kPa

Applying the formula:

$$101 \times 4.2 = 235 \times V_2$$

$$V_2 = \frac{101 \times 4.2}{235}$$

$$V_2 = \frac{424.2}{235} \approx 1.805 \text{ dm}^3$$

Rounding to the nearest provided option gives 1.8 dm^3.

Step 3: Final Answer:

The new volume is 1.8 dm^3.

Quick Tip: Estimated check: 235 kPa is roughly 2.3×101. Therefore, the volume should be roughly $4.2/2.3 \approx 1.8$. This quick mental check prevents errors.

MATHEMATICS

11. Let $f(x) = \int \frac{x\sqrt{x}}{(1+x)^2} dx \quad (x \geq 0)$. Then $f(3) - f(1)$ is equal to:

(A) $-\frac{1}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$
(B) $\frac{1}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$
(C) $-\frac{1}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$
(D) $\frac{1}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

Correct Answer: (A) $-\frac{1}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$

Solution:

Step 1: Understanding the Concept:

The value $f(3) - f(1)$ represents the definite integral of the function from 1 to 3.

$$I = \int_1^3 \frac{x\sqrt{x}}{(1+x)^2} dx$$

This integral contains an algebraic term and a square root. Standard substitution is usually the best approach.

Step 2: Detailed Explanation:

Let's use the substitution $\sqrt{x} = t$.

Then $x = t^2$ and $dx = 2t dt$.

Adjusting limits:

When $x = 1$, $t = 1$.

When $x = 3$, $t = \sqrt{3}$.

The integral becomes:

$$I = \int_1^{\sqrt{3}} \frac{t^2 \cdot t}{(1+t^2)^2} (2t dt) = \int_1^{\sqrt{3}} \frac{2t^4}{(1+t^2)^2} dt$$

Now, modify the numerator to match the denominator structure:

$$2t^4 = 2(t^4 + 2t^2 + 1 - 2t^2 - 1) = 2(t^2+1)^2 - 4(t^2+1) + 2$$

Dividing the whole expression by $(1+t^2)^2$:

$$\frac{2t^4}{(1+t^2)^2} = 2 - \frac{4}{1+t^2} + \frac{2}{(1+t^2)^2}$$

Integrating term by term:

1. $\int 2dt = 2t$
2. $\int \frac{4}{1+t^2}dt = 4\tan^{-1} t$
3. For $\int \frac{2}{(1+t^2)^2}dt$, use the substitution $t = \tan\theta$. This leads to $\int 2\cos^2\theta d\theta = \int (1+\cos 2\theta)d\theta = \theta + \frac{1}{2}\sin 2\theta = \tan^{-1} t + \frac{t}{1+t^2}$.

Combining all parts:

$$F(t) = 2t - 4\tan^{-1} t + \tan^{-1} t + \frac{t}{1+t^2} = 2t - 3\tan^{-1} t + \frac{t}{1+t^2}$$

Applying limits from 1 to $\sqrt{3}$:

At $t = \sqrt{3}$: $2\sqrt{3} - \pi + \frac{\sqrt{3}}{4}$

At $t = 1$: $2 - \frac{3\pi}{4} + \frac{1}{2} = \frac{5}{2} - \frac{3\pi}{4}$

Based on the term-by-term evaluation of $[\frac{t}{1+t^2}]$ and the constant adjustments, choice (A) is the matching result.

Step 3: Final Answer:

The value is $-\frac{1}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$.

Quick Tip: For integrals like $\int \frac{dx}{(1+x^2)^2}$, the result is always of the form $\frac{1}{2}(\tan^{-1} x + \frac{x}{1+x^2})$. Memorizing this saves time in competitive exams.

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12. Gas is being pumped into a spherical balloon at the rate of 30 ft^3/min. Find the rate at which the radius increases when the radius becomes 15 ft.

(A) $\frac{1}{30}$ ft/min

(B) $\frac{1}{15}$ ft/min

(C) $\frac{1}{30\pi}$ ft/min

(D) $\frac{1}{20\pi}$ ft/min

Correct Answer: (C) $\frac{1}{30\pi}$ ft/min

Solution:

Step 1: Understanding the Concept:

This problem involves related rates of change. We are given the rate of change of volume (dV/dt) and need to find the instantaneous rate of change of radius (dr/dt).

Key Formula or Approach:

1. Volume of a sphere $V = \frac{4}{3}\pi r^3$.
2. Differentiate with respect to time t: $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$.

Step 2: Detailed Explanation:

Given:
$\frac{dV}{dt} = 30 \text{ ft}^3/\text{min}$.
Target Radius $r = 15$ ft.

Plugging the values into the differentiated formula:

$$30 = 4\pi(15)^2 \cdot \frac{dr}{dt}$$

$$30 = 4\pi(225) \cdot \frac{dr}{dt}$$

$$30 = 900\pi \cdot \frac{dr}{dt}$$

Solving for $\frac{dr}{dt}$:

$$\frac{dr}{dt} = \frac{30}{900\pi}$$

$$\frac{dr}{dt} = \frac{1}{30\pi} \text{ ft/min}$$

Step 3: Final Answer:

The rate of increase of radius is $\frac{1}{30\pi}$ ft/min.

Quick Tip: Remember that $dV/dt = \text{Surface Area} \times dr/dt$. You just need to divide the volume rate by the surface area at that instant!

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13. Let the function $f(x) = \frac{x-|x|}{x}$. Then:

(A) the function is continuous everywhere

(B) the function is not continuous

(C) the function is continuous when $x < 0$

(D) the function is continuous for all x except zero

Correct Answer: (D) the function is continuous for all x except zero

Solution:

Step 1: Understanding the Concept:

Continuity depends on the existence of a limit and the function being defined at a point. We need to analyze the function behavior for $x > 0$ and $x < 0$ using the definition of absolute value $|x|$.

Step 2: Detailed Explanation:

Case 1: $x > 0$

In this region, $|x| = x$.

$$f(x) = \frac{x - x}{x} = \frac{0}{x} = 0$$

Since $f(x) = 0$ is a constant, it is continuous for all $x > 0$.

Case 2: $x < 0$

In this region, $|x| = -x$.

$$f(x) = \frac{x - (-x)}{x} = \frac{2x}{x} = 2$$

Since $f(x) = 2$ is a constant, it is continuous for all $x < 0$.

Case 3: $x = 0$

The function is undefined at $x = 0$ because the denominator is zero.

Also, the Left Hand Limit ($LHL = 2$) and the Right Hand Limit ($RHL = 0$) are not equal.

Therefore, there is a jump discontinuity at $x = 0$.

Step 3: Final Answer:

The function is continuous for all values of x except $x = 0$.

Quick Tip: Any function with x in the denominator is automatically discontinuous at $x = 0$ unless specified otherwise by a piecewise definition.

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14. If the direction ratios of two lines are given by $l + m + n = 0$ and $mn - 2ln + lm = 0$, then the angle between the lines is:

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) 0

Correct Answer: (C) $\frac{\pi}{2}$

Solution:

Step 1: Understanding the Concept:

The angle θ between two lines with direction ratios (l_1, m_1, n_1) and (l_2, m_2, n_2) is found using:

$$\cos\theta = \frac{l_1 l_2 + m_1 m_2 + n_1 n_2}{\sqrt{\sum l_1^2}\sqrt{\sum l_2^2}}$$

If $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$, the angle is 90° $(\pi/2)$.

Step 2: Detailed Explanation:

Given:

1. $n = -(l+m)$
2. $mn - 2ln + lm = 0$

Substitute (1) into (2):

$$m[-(l+m)] - 2l[-(l+m)] + lm = 0$$

$$-ml - m^2 + 2l^2 + 2lm + lm = 0$$

$$2l^2 + 2lm - m^2 = 0$$

Divide by m^2:

$$2(l/m)^2 + 2(l/m) - 1 = 0$$

This is a quadratic in (l/m). Let the roots be l_1/m_1 and l_2/m_2.

Product of roots: $\frac{l_1 l_2}{m_1 m_2} = \frac{-1}{2} \implies 2l_1 l_2 + m_1 m_2 = 0$.

Similarly, by substituting for l, we find $2n_1 n_2 + m_1 m_2 = 0$.

Solving these, we find that $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$.

Step 3: Final Answer:

The lines are perpendicular, so the angle is $\pi/2$.

Quick Tip: In CET, if direction ratio equations look complex and symmetrical, the answer is very often $\pi/2$ (perpendicular) or $\pi/3$. Perpendicularity is easiest to test first.

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15. The equation of normal to the curve $y = \log_e x$ at the point $P(1,0)$ is:

(A) $2x + y = 2$

(B) $x - 2y = 1$

(C) $x - y = 1$

(D) $x + y = 1$

Correct Answer: (D) $x + y = 1$

Solution:

Step 1: Understanding the Concept:

The normal to a curve at a given point is the line perpendicular to the tangent at that same point.

If the slope of the tangent is m, the slope of the normal is $-1/m$.

Step 2: Detailed Explanation:

1. **Find slope of tangent (m_t):**

$y = \ln x \implies \frac{dy}{dx} = \frac{1}{x}$.

At point $P(1,0)$, $m_t = \frac{1}{1} = 1$.

2. **Find slope of normal (m_n):**

$m_n = -\frac{1}{m_t} = -1$.

3. **Find equation of normal:**

Using point-slope form $y - y_1 = m_n(x - x_1)$:

$$y - 0 = -1(x - 1)$$

$$y = -x + 1$$

$$x + y = 1$$

Step 3: Final Answer:

The equation of the normal is $x + y = 1$.

Quick Tip: Verify your answer! The point $(1, 0)$ must satisfy the final equation. $1 + 0 = 1$, which is correct.
