



General Instructions

- (i) **Duration:** The total duration of the examination is 3 hours (180 minutes).
- (ii) **Total Marks:** The complete paper carries a maximum of 200 marks.
- (iii) **Structure:** The paper has 3 Sections:
 - **Section A:** 50 Multiple Choice Questions (Physics).
 - **Section B:** 50 Multiple Choice Questions (Chemistry).
 - **Section B:** 50 Multiple Choice Questions (Mathematics).
- (iv) **Compulsory Questions:** All 150 questions are compulsory.
- (v) Each question has four options. Only **one** option is correct.
- (vi) **Correct Answer:** +1 marks.
- (vii) **Incorrect Answer:** (No Negative marking).
- (viii) **Unanswered/Marked for Review:** 0 marks.

1. If $\cos 4x = \cos 3x$, find the general solution for x .

- (A) $x = 2n\pi$ or $x = \frac{2n\pi}{7}$, where $n \in \mathbb{Z}$
- (B) $x = n\pi$ or $x = \frac{n\pi}{7}$, where $n \in \mathbb{Z}$
- (C) $x = \frac{2n\pi}{7}$ only
- (D) $x = 2n\pi$ only

Correct Answer: (A) $x = 2n\pi$ or $x = \frac{2n\pi}{7}$, where $n \in \mathbb{Z}$

Solution:

Step 1: Understanding the Concept:

In trigonometry, the general solution provides all possible values of an angle that satisfy a given trigonometric equation.

For the cosine function, the general solution connects two angles that have the same cosine value.

Step 2: Key Formula or Approach:

The formula for the general solution of the equation $\cos \theta = \cos \alpha$ is:

$$\theta = 2n\pi \pm \alpha$$

where $n \in \mathbb{Z}$.

Step 3: Detailed Explanation:

We are given the trigonometric equation:

$$\cos 4x = \cos 3x$$

Comparing this with the standard form $\cos \theta = \cos \alpha$, we can identify $\theta = 4x$ and $\alpha = 3x$.

Substituting these expressions into the general solution formula, we obtain:

$$4x = 2n\pi \pm 3x$$

This equation splits into two separate cases based on the plus and minus signs.

Case 1: Considering the positive sign, we have:

$$4x = 2n\pi + 3x$$

Subtracting $3x$ from both sides yields:

$$4x - 3x = 2n\pi$$

$$x = 2n\pi$$

Case 2: Considering the negative sign, we get:

$$4x = 2n\pi - 3x$$

Adding $3x$ to both sides gives:

$$4x + 3x = 2n\pi$$

$$7x = 2n\pi$$

Dividing by 7, we find:

$$x = \frac{2n\pi}{7}$$

Combining both cases, the possible solutions are $x = 2n\pi$ or $x = \frac{2n\pi}{7}$, where $n \in \mathbb{Z}$.

Step 4: Final Answer:

The correct option is (A).

Quick Tip: Always remember the standard general solution forms: for $\sin \theta = \sin \alpha$, it is $\theta = n\pi + (-1)^n \alpha$, and for $\cos \theta = \cos \alpha$, it is $\theta = 2n\pi \pm \alpha$.

Be careful to evaluate both the positive and negative cases to find all possible solutions.

2. Evaluate the integral: $\int \frac{\sin x}{\sin 4x} dx$

(A) $-\frac{1}{8} \log \left| \frac{1+\sin x}{1-\sin x} \right| + \frac{1}{4\sqrt{2}} \log \left| \frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x} \right| + C$

(B) $\frac{1}{8} \log \left| \frac{1+\sin x}{1-\sin x} \right| - \frac{1}{4\sqrt{2}} \log \left| \frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x} \right| + C$

(C) $-\frac{1}{8} \log \left| \frac{1+\sin x}{1-\sin x} \right| + \frac{1}{4\sqrt{2}} \log \left| \frac{1-\sqrt{2}\sin x}{1+\sqrt{2}\sin x} \right| + C$

(D) None of these

Correct Answer: (A) $-\frac{1}{8} \log \left| \frac{1+\sin x}{1-\sin x} \right| + \frac{1}{4\sqrt{2}} \log \left| \frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x} \right| + C$

Solution:

Step 1: Use trigonometric identity

Using

$$\sin 4x = 4 \sin x \cos x \cos 2x$$

So,

$$I = \int \frac{\sin x}{4 \sin x \cos x \cos 2x} dx$$

Cancelling $\sin x$,

$$I = \frac{1}{4} \int \frac{1}{\cos x \cos 2x} dx$$

Multiply numerator and denominator by $\cos x$:

$$I = \frac{1}{4} \int \frac{\cos x}{\cos^2 x \cos 2x} dx$$

Now use

$$\cos^2 x = 1 - \sin^2 x$$

and

$$\cos 2x = 1 - 2 \sin^2 x$$

Hence,

$$I = \frac{1}{4} \int \frac{\cos x}{(1 - \sin^2 x)(1 - 2 \sin^2 x)} dx$$

Step 2: Substitution

Let

$$t = \sin x$$

Then

$$dt = \cos x \, dx$$

So the integral becomes

$$I = \frac{1}{4} \int \frac{1}{(1-t^2)(1-2t^2)} dt$$

Step 3: Partial fraction decomposition

Write

$$\frac{1}{(1-t^2)(1-2t^2)} = \frac{A}{1-t^2} + \frac{B}{1-2t^2}$$

Multiplying throughout,

$$1 = A(1-2t^2) + B(1-t^2)$$

Comparing coefficients,

$$A = -1, \quad B = 2$$

Therefore,

$$\frac{1}{(1-t^2)(1-2t^2)} = \frac{-1}{1-t^2} + \frac{2}{1-2t^2}$$

Thus,

$$I = \frac{1}{4} \int \left(\frac{-1}{1-t^2} + \frac{2}{1-2t^2} \right) dt$$

$$I = -\frac{1}{4} \int \frac{1}{1-t^2} dt + \frac{1}{2} \int \frac{1}{1-2t^2} dt$$

Step 4: Integrate each term

Using

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$$

For the first integral:

$$-\frac{1}{4} \int \frac{1}{1-t^2} dt = -\frac{1}{8} \log \left| \frac{1+t}{1-t} \right|$$

For the second integral:

$$\frac{1}{2} \int \frac{1}{1-2t^2} dt = \frac{1}{4\sqrt{2}} \log \left| \frac{1+\sqrt{2}t}{1-\sqrt{2}t} \right|$$

Step 5: Substitute back

Putting $t = \sin x$,

$$I = -\frac{1}{8} \log \left| \frac{1 + \sin x}{1 - \sin x} \right| + \frac{1}{4\sqrt{2}} \log \left| \frac{1 + \sqrt{2} \sin x}{1 - \sqrt{2} \sin x} \right| + C$$

Final Answer

$$\int \frac{\sin x}{\sin 4x} dx = -\frac{1}{8} \log \left| \frac{1 + \sin x}{1 - \sin x} \right| + \frac{1}{4\sqrt{2}} \log \left| \frac{1 + \sqrt{2} \sin x}{1 - \sqrt{2} \sin x} \right| + C$$

Quick Tip: When the integrand consists of trigonometric functions with multiple angles in the denominator, try to reduce them using double-angle identities to create a derivative-substitution pair like $\sin x$ and $\cos x$.

Multiplying and dividing by a trigonometric term (like $\cos x$) is a standard trick to convert the entire function into a rational algebraic function of $\sin x$.

3. Find the eigenvalues of the matrix $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$.

- (A) 0, 1, 1
- (B) 0, -1, -1
- (C) 1, -1, 0
- (D) -1, -1, -1

Correct Answer: (B) 0, -1, -1

Solution:**Step 1: Understanding the Concept:**

Eigenvalues are the roots of the characteristic equation of a matrix.

For any square matrix A , the scalar λ is an eigenvalue if there exists a non-zero vector X such that $AX = \lambda X$.

Step 2: Key Formula or Approach:

The characteristic equation of a matrix A is given by:

$$\det(A - \lambda I) = 0$$

where I is the identity matrix of the same order as A , and λ represents the eigenvalues.

Step 3: Detailed Explanation:

Given the matrix A :

$$A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

We construct the matrix $A - \lambda I$ by subtracting λ from the diagonal elements:

$$A - \lambda I = \begin{pmatrix} -\lambda & 0 & -1 \\ 0 & -1 - \lambda & 0 \\ 0 & 0 & -1 - \lambda \end{pmatrix}$$

Now, we find the determinant of this matrix and equate it to zero:

$$\begin{vmatrix} -\lambda & 0 & -1 \\ 0 & -1 - \lambda & 0 \\ 0 & 0 & -1 - \lambda \end{vmatrix} = 0$$

Expanding the determinant along the first column, we get:

$$-\lambda [(-1 - \lambda)(-1 - \lambda) - (0)(0)] - 0 + 0 = 0$$

$$-\lambda(-1 - \lambda)^2 = 0$$

We can factor out a negative sign from $(-1 - \lambda)$, but since it is squared, it becomes positive:

$$-\lambda(\lambda + 1)^2 = 0$$

This gives the roots:

$$\lambda = 0$$

$$(\lambda + 1)^2 = 0 \implies \lambda = -1, -1$$

So, the eigenvalues of the matrix A are $0, -1, -1$.

Step 4: Final Answer:

The correct option is (B).

Quick Tip: To quickly verify eigenvalues in an exam, check the trace and determinant of the matrix. The sum of the eigenvalues is equal to the trace of the matrix (sum of main diagonal elements: $0 - 1 - 1 = -2$). The product of the eigenvalues is equal to the determinant of the matrix (expanding along the second row gives $-1 \times (0 - 0) = 0$). The values $0, -1, -1$ satisfy sum $= -2$ and product $= 0$, confirming our answer.

4. Consider the following logical statements:

R: If $p \rightarrow q$ is false, then $p \vee q$ is false.

S: If $p \leftrightarrow q$ is false, then $p \vee q$ is false.

Evaluate the truth values of R and S.

(A) Both R and S are true

(B) R is true and S is false

- (C) R is false and S is true
(D) Both R and S are false

Correct Answer: (D) Both R and S are false

Solution:

Step 1: Understanding the Concept:

The problem requires evaluating the truth conditions of basic logical operators such as implication ($\rightarrow$), biconditional ($\leftrightarrow$), and disjunction ($\vee$).

A logical statement claiming an outcome based on a condition is false if the specified condition holds but the outcome does not.

Step 2: Key Formula or Approach:

- Implication ($p \rightarrow q$) is false **only** when the hypothesis p is True (T) and the conclusion q is False (F).
- Biconditional ($p \leftrightarrow q$) is false when p and q have opposite truth values (i.e., one is True and the other is False).
- Disjunction ($p \vee q$) is false **only** when both p and q are False.

Step 3: Detailed Explanation:

Let's analyze statement **R**:

"If $p \rightarrow q$ is false, then $p \vee q$ is false."

The premise states that $p \rightarrow q$ is false.

This occurs exclusively when $p = \text{True (T)}$ and $q = \text{False (F)}$.

Given $p = \text{T}$ and $q = \text{F}$, we evaluate $p \vee q$:

$$\text{T} \vee \text{F} \equiv \text{True (T)}$$

However, statement **R** claims that $p \vee q$ is false.

Since the actual result is True, statement **R** is false.

Now, let's analyze statement **S**:

"If $p \leftrightarrow q$ is false, then $p \vee q$ is false."

The premise states that $p \leftrightarrow q$ is false.

This happens in exactly two cases:

Case 1: $p = \text{True (T)}$ and $q = \text{False (F)}$

Case 2: $p = \text{False (F)}$ and $q = \text{True (T)}$

Let's evaluate $p \vee q$ for both cases:

For Case 1: $T \vee F \equiv \text{True (T)}$

For Case 2: $F \vee T \equiv \text{True (T)}$

In both possible scenarios, $p \vee q$ evaluates to True.

However, statement S claims that $p \vee q$ is false.

Since the actual result is always True, statement **S** is false.

Step 4: Final Answer:

Both logical statements R and S are false, which corresponds to option (D).

Quick Tip: When dealing with logical equivalences and conditions, analyzing the truth table mentally is very effective.

Remember that $p \rightarrow q$ being false is a very specific condition ($p = T, q = F$), which immediately locks in the values of p and q for further evaluation.

5. Evaluate the integral: $\int \frac{2x}{x^2-5x+4} dx$

(A) $\frac{8}{3} \log|x-4| - \frac{2}{3} \log|x-1| + C$

(B) $\frac{2}{3} \log|x-4| - \frac{8}{3} \log|x-1| + C$

(C) $\frac{8}{3} \log|x-1| - \frac{2}{3} \log|x-4| + C$

(D) $\frac{2}{3} \log|x-1| - \frac{8}{3} \log|x-4| + C$

Correct Answer: (A) $\frac{8}{3} \log|x-4| - \frac{2}{3} \log|x-1| + C$

Solution:

Step 1: Understanding the Concept:

The given integral represents a rational algebraic function where the degree of the numerator is less than the degree of the denominator.

Such integrals can be easily evaluated by splitting the integrand into simpler fractions using the method of partial fractions.

Step 2: Key Formula or Approach:

Factorize the quadratic denominator, then express the fraction in the form:

$$\frac{px + q}{(x - a)(x - b)} = \frac{A}{x - a} + \frac{B}{x - b}$$

Once decomposed, use the standard logarithmic integration rule:

$$\int \frac{1}{x - a} dx = \log |x - a| + C$$

Step 3: Detailed Explanation:

Let the integral be I :

$$I = \int \frac{2x}{x^2 - 5x + 4} dx$$

First, we factorize the quadratic expression in the denominator:

$$x^2 - 5x + 4 = x^2 - 4x - x + 4 = x(x - 4) - 1(x - 4) = (x - 4)(x - 1)$$

So, the integrand becomes:

$$\frac{2x}{(x - 4)(x - 1)}$$

Now, we set up the partial fractions decomposition:

$$\frac{2x}{(x - 4)(x - 1)} = \frac{A}{x - 4} + \frac{B}{x - 1}$$

Multiplying both sides by the common denominator $(x - 4)(x - 1)$, we get:

$$2x = A(x - 1) + B(x - 4)$$

To find the value of A , we can substitute $x = 4$ to eliminate B :

$$2(4) = A(4 - 1) + B(4 - 4)$$

$$8 = 3A \implies A = \frac{8}{3}$$

To find the value of B , we substitute $x = 1$ to eliminate A :

$$2(1) = A(1 - 1) + B(1 - 4)$$

$$2 = -3B \implies B = -\frac{2}{3}$$

Substituting the values of A and B back into the partial fraction form:

$$\frac{2x}{(x-4)(x-1)} = \frac{8/3}{x-4} - \frac{2/3}{x-1}$$

Now, substitute this expanded form back into the integral I :

$$I = \int \left(\frac{8/3}{x-4} - \frac{2/3}{x-1} \right) dx$$

Applying the linearity of integration:

$$I = \frac{8}{3} \int \frac{1}{x-4} dx - \frac{2}{3} \int \frac{1}{x-1} dx$$

Evaluating the simple logarithmic integrals:

$$I = \frac{8}{3} \log|x-4| - \frac{2}{3} \log|x-1| + C$$

where C is the constant of integration.

Step 4: Final Answer:

The correct option is (A).

Quick Tip: You can verify your partial fraction decomposition quickly by using the "cover-up method".

To find A for the term $\frac{A}{x-4}$, cover $(x-4)$ in the original expression $\frac{2x}{(x-4)(x-1)}$ and plug in $x = 4$, yielding $\frac{2(4)}{4-1} = \frac{8}{3}$.

To find B for the term $\frac{B}{x-1}$, cover $(x-1)$ and plug in $x = 1$, yielding $\frac{2(1)}{1-4} = -\frac{2}{3}$. This saves valuable exam time!