

MHT CET 2025 April 20 Shift 1 Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :200	Total Questions :150
-----------------------	--------------------	----------------------

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 150 questions. The maximum marks are 200.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 50 questions in each part of equal weightage.

Chemistry

1. Solubility of $\text{Ca}_3(\text{PO}_4)_2$ is 'S' mol dm^{-3} . Find solubility product.

- (A) S^5
(B) $108 S^5$
(C) $54 S^5$
(D) $12 S^5$

Correct Answer: (B) $108 S^5$

Solution:

Step 1: Understanding the Concept:

The solubility product constant (K_{sp}) is the equilibrium constant for a solid substance dissolving in an aqueous solution. It represents the product of the equilibrium concentrations of the constituent ions, each raised to the power of its stoichiometric coefficient.

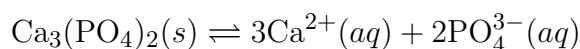
Step 2: Key Formula or Approach:

For a generic salt $A_xB_y \rightleftharpoons xA^{y+} + yB^{x-}$ with solubility S , the K_{sp} expression is:

$$K_{sp} = [A^{y+}]^x [B^{x-}]^y = (xS)^x (yS)^y$$

Step 3: Detailed Explanation:

Write the balanced dissociation equation for calcium phosphate:



Let the solubility of $\text{Ca}_3(\text{PO}_4)_2$ be S . From the stoichiometry of the reaction, the equilibrium concentrations of the respective ions are:

$$[\text{Ca}^{2+}] = 3S$$

$$[\text{PO}_4^{3-}] = 2S$$

Now, write the expression for the solubility product, K_{sp} :

$$K_{sp} = [\text{Ca}^{2+}]^3 [\text{PO}_4^{3-}]^2$$

Substitute the equilibrium concentrations into the expression:

$$K_{sp} = (3S)^3 \cdot (2S)^2$$

$$K_{sp} = 27S^3 \cdot 4S^2$$

$$K_{sp} = 108S^5$$

Step 4: Final Answer:

The solubility product is $108 S^5$.

💡 Quick Tip

A useful shortcut for finding K_{sp} of a salt A_xB_y is the formula $K_{sp} = x^x y^y S^{(x+y)}$. For $\text{Ca}_3(\text{PO}_4)_2$, $x = 3$ and $y = 2$, giving $K_{sp} = 3^3 \cdot 2^2 \cdot S^{(3+2)} = 108S^5$.

2. Which from following reagents is used in Gatterman-Koch formylation of arene?

- (A) $\text{AlH}(\text{i-Bu})_2$
- (B) CO, HCl (anhyd. AlCl_3)
- (C) CrO_2Cl_2 (CS_2)
- (D) DIBAL-H

Correct Answer: (B) CO, HCl (anhyd. AlCl_3)

Solution:

Step 1: Understanding the Concept:

The Gatterman-Koch reaction is an electrophilic aromatic substitution reaction utilized to introduce a formyl group ($-\text{CHO}$) onto an aromatic ring, effectively synthesizing aromatic aldehydes from arenes.

Step 2: Key Formula or Approach:

Approach: Recall the specific reagents and catalyst required for the Gatterman-Koch reaction and distinguish them from reagents used in other named reactions (like Étard or DIBAL-H reductions).

Step 3: Detailed Explanation:

In the Gatterman-Koch reaction, benzene or its derivative is treated with carbon monoxide (CO) and hydrogen chloride (HCl) in the presence of a Lewis acid catalyst such as anhydrous aluminum chloride (AlCl_3) and a trace of cuprous chloride (CuCl). The reagents CO and HCl act together to generate the reactive formyl cation (CHO^+) in situ, which subsequently attacks the aromatic ring. Evaluating the other options: - $\text{AlH}(\text{i-Bu})_2$ and DIBAL-H (options A and D are the same reagent) are reducing agents used to convert esters or nitriles to aldehydes. - CrO_2Cl_2 in CS_2 (option C) is the reagent for the Étard reaction, which oxidizes methyl groups on aromatic rings to aldehydes.

Step 4: Final Answer:

The correct reagent combination is CO, HCl (anhyd. AlCl_3).

💡 Quick Tip

Remember the name association: **Koch** sounds like **CO-Cl** (from $\text{CO} + \text{HCl}$). This helps recall that CO and HCl are the key reagents for the Gatterman-Koch reaction.

3. Which of the following species acts as reducing agent during working of hydrogen-oxygen fuel cell?

- (A) H_2
- (B) O_2
- (C) H^+
- (D) NaOH

Correct Answer: (A) H_2

Solution:**Step 1: Understanding the Concept:**

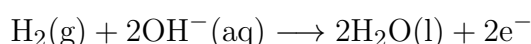
A hydrogen-oxygen fuel cell generates electricity through a continuous redox reaction between hydrogen gas and oxygen gas. A reducing agent is the substance that donates electrons to another substance, thereby undergoing oxidation itself.

Step 2: Key Formula or Approach:

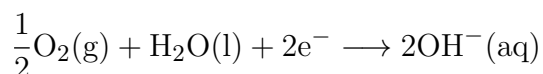
Approach: Write out the electrochemical half-reactions occurring at the anode and cathode to identify which species is oxidized (loses electrons).

Step 3: Detailed Explanation:

Let's examine the half-reactions that occur in a hydrogen-oxygen fuel cell (using an aqueous base like NaOH or KOH as an electrolyte): At the anode (oxidation):



Here, hydrogen gas (H_2) loses electrons (oxidation state changes from 0 to +1) and is oxidized. Therefore, H_2 acts as the reducing agent. At the cathode (reduction):



Oxygen gas (O_2) gains electrons (oxidation state changes from 0 to -2) and is reduced, acting as the oxidizing agent.

Step 4: Final Answer:

H_2 acts as the reducing agent.

💡 Quick Tip

In any combustion process or fuel cell reaction involving oxygen, oxygen is fundamentally the oxidizing agent (it gets reduced). Consequently, the fuel (in this case, H_2) must be the reducing agent.

4. The rate constant for a first order reaction is 0.58 s^{-1} at 300 K and 0.026 s^{-1} at 290 K . What is the energy of activation? ($R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$)

- (A) 124.48 kJ
- (B) 224.55 kJ
- (C) 348.18 kJ
- (D) 513.21 kJ

Correct Answer: (B) 224.55 kJ

Solution:

Step 1: Understanding the Concept:

The dependence of a reaction's rate constant on temperature is mathematically described by the Arrhenius equation. By comparing the rate constants at two different temperatures, we can calculate the activation energy (E_a).

Step 2: Key Formula or Approach:

The two-point form of the Arrhenius equation is:

$$\log_{10} \left(\frac{k_2}{k_1} \right) = \frac{E_a}{2.303R} \left(\frac{T_2 - T_1}{T_1 T_2} \right)$$

Step 3: Detailed Explanation:

Given values from the problem: $k_1 = 0.026 \text{ s}^{-1}$ at $T_1 = 290 \text{ K}$ $k_2 = 0.58 \text{ s}^{-1}$ at $T_2 = 300 \text{ K}$
 $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$ Substitute these values into the Arrhenius equation:

$$\log_{10} \left(\frac{0.58}{0.026} \right) = \frac{E_a}{2.303 \times 8.314} \left(\frac{300 - 290}{300 \times 290} \right)$$
$$\log_{10}(22.307) = \frac{E_a}{19.147} \left(\frac{10}{87000} \right)$$

The value of $\log_{10}(22.307)$ is approximately 1.3484.

$$1.3484 = \frac{E_a}{19.147} \times 1.1494 \times 10^{-4}$$

Now, isolate and solve for E_a :

$$E_a = \frac{1.3484 \times 19.147}{1.1494 \times 10^{-4}}$$
$$E_a = \frac{25.818}{1.1494 \times 10^{-4}}$$
$$E_a \approx 224621 \text{ J mol}^{-1}$$

Convert the activation energy from Joules to kilojoules:

$$E_a \approx 224.62 \text{ kJ mol}^{-1}$$

Comparing this result with the given options, the closest matched value is 224.55 kJ.

Step 4: Final Answer:

The activation energy is approximately 224.55 kJ.

💡 Quick Tip

To save time during exams, you can use approximations like $\log(22.3) \approx \log(20) \approx \log(10) + \log(2) \approx 1 + 0.3 = 1.3$. This helps estimate the order of magnitude quickly to eliminate incorrect options.

5. The reaction of propane with bromine in presence of UV light predominantly forms

- (A) 1-Bromopropane
- (B) 2-Bromopropane
- (C) 1,2-dibromopropane
- (D) 1,3-dibromopropane

Correct Answer: (B) 2-Bromopropane

Solution:

Step 1: Understanding the Concept:

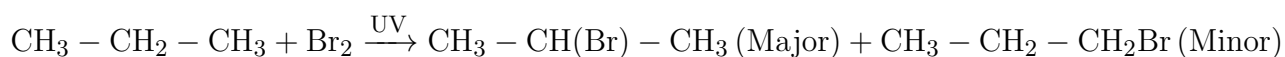
The reaction of an alkane with a halogen in the presence of ultraviolet (UV) light proceeds via a free radical substitution mechanism. Halogenation, and specifically bromination, is highly selective regarding which hydrogen atom is replaced.

Step 2: Key Formula or Approach:

Approach: Analyze the types of hydrogen atoms in the alkane, determine the relative stabilities of the possible free radical intermediates, and use the high selectivity of bromination to predict the major product.

Step 3: Detailed Explanation:

Propane ($\text{CH}_3 - \text{CH}_2 - \text{CH}_3$) has two distinct types of hydrogen atoms: 1. Six primary (1°) hydrogens on the terminal carbon atoms. 2. Two secondary (2°) hydrogens on the middle carbon atom. During the propagation step of free radical substitution, a hydrogen atom is abstracted to form an alkyl radical. - Abstraction of a 1° hydrogen forms a primary free radical (1° radical). - Abstraction of a 2° hydrogen forms a secondary free radical (2° radical). The stability of free radicals follows the order: $3^\circ > 2^\circ > 1^\circ$. Therefore, the secondary isopropyl radical is significantly more stable than the primary n-propyl radical due to hyperconjugation. Bromination is highly selective and regioselective, heavily favoring the pathway that forms the most stable intermediate radical. Consequently, the major product is formed from the more stable secondary radical.



The predominant product is 2-bromopropane.

Step 4: Final Answer:

The predominant product formed is 2-Bromopropane.

 Quick Tip

Bromination is much more selective than chlorination. It almost exclusively substitutes at the most substituted carbon available (tertiary > secondary > primary) because the Br radical is less reactive and thus more discriminating.

6. Which of the following pair of compounds on heating gives butanenitrile?

- (A) Propanol and alcoholic KCN
- (B) Butanol and alcoholic KCN
- (C) n -Butylchloride and alcoholic KCN
- (D) n -Propylchloride and alcoholic KCN

Correct Answer: (D) n -Propylchloride and alcoholic KCN

Solution:

Step 1: Understanding the Concept:

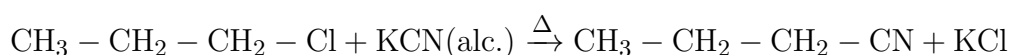
Alkyl nitriles can be prepared through a nucleophilic substitution reaction of alkyl halides with potassium cyanide (KCN) in an alcoholic medium. This reaction replaces the halide ion with the cyanide ($-\text{CN}$) ion, which effectively increases the carbon chain length of the principal molecule by one carbon atom.

Step 2: Key Formula or Approach:

Approach: Identify the total number of carbon atoms in the desired product (butanenitrile). Then, evaluate each reactant pair to find which combination of alkyl group and cyanide group yields that exact carbon count.

Step 3: Detailed Explanation:

The target molecule is butanenitrile, which has the chemical structure $\text{CH}_3 - \text{CH}_2 - \text{CH}_2 - \text{CN}$. This molecule contains a total of four carbon atoms. In the reaction with KCN, the cyanide group provides one carbon atom to the new chain. Therefore, the starting alkyl halide must contain exactly three carbon atoms (a propyl group). Let's evaluate the given options: - (A) & (B) Alcohols (Propanol, Butanol) generally do not react directly with KCN to form nitriles because the hydroxide ion (OH^-) is a poor leaving group. - (C) n-Butylchloride (4 carbons) + alcoholic KCN (1 carbon) $\longrightarrow$ Pentanenitrile (5 carbons). - (D) n-Propylchloride (3 carbons) + alcoholic KCN (1 carbon) $\longrightarrow$ Butanenitrile (4 carbons). Reaction equation for D:



Step 4: Final Answer:

The pair that gives butanenitrile is n-Propylchloride and alcoholic KCN.

 Quick Tip

Reactions with KCN act as a "step-up" or chain-lengthening reaction in organic synthesis. Always count the carbons in your reactants and products to ensure they match!

7. Standard potential (E°) of $\text{Zn}_{(\text{aq})}^{+2} + 2\text{e}^- \longrightarrow \text{Zn}_{(\text{s})}$ is -0.76 V . What is standard potential of reaction $2\text{Zn}_{(\text{s})} \longrightarrow 2\text{Zn}_{(\text{aq})}^{+2} + 4\text{e}^-$?

- (A) -1.52 V
- (B) $+1.52 \text{ V}$
- (C) -0.76 V
- (D) $+0.76 \text{ V}$

Correct Answer: (D) $+0.76 \text{ V}$

Solution:

Step 1: Understanding the Concept:

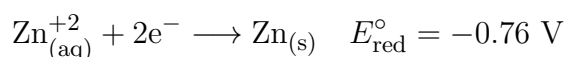
The standard electrode potential (E°) is an intensive thermodynamic property. This means its value does not depend on the amount of substance reacting or how the stoichiometric coefficients in the chemical equation are scaled. However, if a reaction is reversed, the sign of its standard potential must also be reversed.

Step 2: Key Formula or Approach:

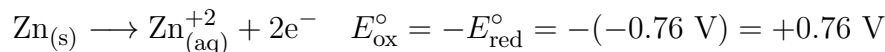
Approach: Identify the relationship between the given half-reaction and the target half-reaction (reversal and stoichiometric multiplication) and apply the rules for manipulating E° values.

Step 3: Detailed Explanation:

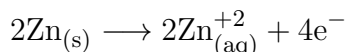
The provided half-reaction is a reduction process:



The target reaction is an oxidation process, which is the exact reverse of the given reaction:



The target reaction is also multiplied by a stoichiometric factor of 2:



Because electrode potential is an intensive property (unlike Gibbs free energy ΔG), multiplying the reaction stoichiometry by any constant does not change the value of E° . Therefore, the standard potential for the manipulated reaction remains $+0.76 \text{ V}$.

Step 4: Final Answer:

The standard potential of the specified reaction is $+0.76 \text{ V}$.

 Quick Tip

Never multiply the voltage E° by stoichiometric coefficients when balancing redox equations. Only flip the mathematical sign if you reverse the direction of the half-reaction.

8. Identify the ligands present in cisplatin.

- (A) Cl^- and CN^-
- (B) NH_3 and Cl^-

(C) NH_3 and H_2O

(D) Cl^- and H_2O

Correct Answer: (B) NH_3 and Cl^-

Solution:

Step 1: Understanding the Concept:

Cisplatin is a very important coordination compound, widely recognized for its application as an anti-cancer chemotherapy drug. To identify its ligands, we must know its IUPAC name or chemical formula.

Step 2: Key Formula or Approach:

Approach: Recall the chemical formula of cisplatin and identify the species coordinately bonded to the central metal atom.

Step 3: Detailed Explanation:

The formal chemical name for cisplatin is cis-diamminedichloridoplatinum(II). Its molecular formula is $[\text{Pt}(\text{NH}_3)_2\text{Cl}_2]$. In this square planar coordination complex, the central metal ion is Platinum (Pt^{2+}). The chemical species that donate electron pairs to form coordinate bonds with the central metal ion are called ligands. From the chemical formula, we can clearly see two distinct types of ligands attached to the platinum center: 1. Ammine ligands (NH_3) 2. Chlorido ligands (Cl^-) The "cis" designation indicates that the identical ligands are positioned adjacent to each other in the square planar geometry.

Step 4: Final Answer:

The ligands present are NH_3 and Cl^- .

 Quick Tip

The common name "cisplatin" itself holds the structural clues: "plat" stands for platinum, "in" comes from ammine, and it contains chloride ions.

9. Identify false statement from the following about fluorine.

(A) It is highly electronegative element.

(B) It exhibits only -1 oxidation state.

(C) It has high bond dissociation enthalpy among all halogens.

(D) It form only one oxoacid.

Correct Answer: (C) It has high bond dissociation enthalpy among all halogens.

Solution:

Step 1: Understanding the Concept:

Fluorine is the first member of the halogen group (Group 17). It exhibits several anomalous properties compared to the rest of the group due to its extremely small atomic size, highest electronegativity, low F-F bond dissociation enthalpy, and the absence of vacant d-orbitals in its valence shell.

Step 2: Key Formula or Approach:

Approach: Evaluate each given statement against the established chemical and physical properties of the element fluorine.

Step 3: Detailed Explanation:

Let's analyze each statement: - **(A) It is highly electronegative element:** True. Fluorine is the most electronegative element in the entire periodic table. - **(B) It exhibits only -1 oxidation state:** True. Because it is the most electronegative element and lacks d-orbitals to expand its octet, it can only attract electrons and thus exclusively shows a -1 oxidation state in all its compounds (and 0 in its elemental form F_2). - **(C) It has high bond dissociation enthalpy among all halogens:** False. The F-F bond dissociation enthalpy is anomalously low, actually being lower than that of Cl-Cl and Br-Br. This occurs because the fluorine atom is very small, leading to strong interelectronic repulsions between the non-bonding electron pairs on the two closely situated fluorine atoms in the F_2 molecule. The correct trend is $Cl_2 > Br_2 > F_2 > I_2$. - **(D) It forms only one oxoacid:** True. Due to its small size and very high electronegativity, it forms only a single oxoacid, which is fluoric(I) acid or hypofluorous acid (HOF).

Step 4: Final Answer:

The false statement is that fluorine has a high bond dissociation enthalpy among all halogens.

 Quick Tip

The exceptionally low bond dissociation energy of F_2 is a frequent exam topic. This property is primarily responsible for fluorine gas being exceptionally reactive compared to other halogens.

10. Which from following process involves zero work done?

- (A) Isobaric expansion of gas.
- (B) Adiabatic compression of gas.
- (C) Isothermal expansion of gas.
- (D) Free expansion of gas.

Correct Answer: (D) Free expansion of gas.

Solution:

Step 1: Understanding the Concept:

In classical thermodynamics, pressure-volume work (W) done by or on a gaseous system is calculated based on the external pressure opposing the expansion or compression.

Step 2: Key Formula or Approach:

The general formula for irreversible pressure-volume work is:

$$W = -p_{\text{ext}}\Delta V$$

where p_{ext} is the external pressure and ΔV is the change in volume.

Step 3: Detailed Explanation:

Let's analyze the work done in each process described in the options: - **(A) Isobaric expansion:** This process occurs at a constant external pressure ($p_{\text{ext}} > 0$). Since it is an expansion, $\Delta V > 0$. Thus, the work done $W = -p_{\text{ext}}\Delta V$ is non-zero (specifically, work is done by the system). - **(B) Adiabatic compression:** This process involves a decrease in volume ($\Delta V < 0$)

against a positive external pressure. Thus, work done is non-zero (work is done on the system, raising its internal energy). - **(C) Isothermal expansion:** The gas expands against an external pressure while maintaining constant temperature. The work done is mathematically $W = -nRT \ln(V_f/V_i)$, which is non-zero. - **(D) Free expansion:** By definition, free expansion refers to the expansion of a gas into a vacuum. In a vacuum, there is no opposing force, so the external pressure is zero ($p_{\text{ext}} = 0$). Substituting this into the work equation gives $W = -0 \times \Delta V = 0$. Therefore, strictly zero work is done during free expansion.

Step 4: Final Answer:

Free expansion of gas involves zero work done.

 Quick Tip

"Free expansion" always implies expansion against zero opposing force ($P_{\text{ext}} = 0$). In this unique thermodynamic case, regardless of whether the boundary is adiabatic or isothermal, the work done (W) is invariably zero.

11. Calculate the constant external pressure required to expand 2 moles of an ideal gas from volume 15dm^3 to 20dm^3 if amount of work done is -600 J .

- (A) 1.2 bar
- (B) 1.5 bar
- (C) 1.8 bar
- (D) 2.1 bar

Correct Answer: (A) 1.2 bar

Solution:

Step 1: Understanding the Concept:

The work done (W) during the expansion or compression of a gas against a constant external pressure (P_{ext}) is given by the formula for irreversible thermodynamic work.

Step 2: Key Formula or Approach:

The formula for pressure-volume work is:

$$W = -P_{\text{ext}}\Delta V = -P_{\text{ext}}(V_2 - V_1)$$

Step 3: Detailed Explanation:

Given data from the problem: Work done (W) = -600 J Initial volume (V_1) = $15\text{dm}^3 = 15\text{ L}$
 Final volume (V_2) = $20\text{dm}^3 = 20\text{ L}$ Change in volume (ΔV) = $V_2 - V_1 = 20 - 15 = 5\text{dm}^3 = 5\text{ L}$
 Substituting the known values into the work formula:

$$-600\text{ J} = -P_{\text{ext}} \times 5\text{ L}$$

Solving for the external pressure P_{ext} :

$$P_{\text{ext}} = \frac{600\text{ J}}{5\text{ L}} = 120\text{ J L}^{-1}$$

To match the given options, we must convert the pressure units from J L^{-1} to bar. We use the standard conversion factor:

$$1 \text{ L bar} = 100 \text{ J} \implies 1 \text{ J L}^{-1} = \frac{1}{100} \text{ bar}$$

Applying the conversion:

$$P_{\text{ext}} = 120 \times \left(\frac{1}{100} \text{ bar} \right) = 1.2 \text{ bar}$$

Step 4: Final Answer:

The required constant external pressure is 1.2 bar.

💡 Quick Tip

Always double-check unit conversions in thermodynamics problems! Remembering that $1 \text{ L} \cdot \text{atm} \approx 101.3 \text{ J}$ and $1 \text{ L} \cdot \text{bar} = 100 \text{ J}$ is absolutely crucial for arriving at the correct numerical answer.

12. Which from following tests confirms presence of aldehydic group in glucose?

- (A) Formation of oxime when reacted with NH_2OH .
- (B) Formation of gluconic acid when reacted with Br_2 water.
- (C) Formation of saccharic acid when glucose and gluconic acid treated with dil. HNO_3 .
- (D) Formation of penta acetate when treated with acetic anhydride.

Correct Answer: (B) Formation of gluconic acid when reacted with Br_2 water.

Solution:

Step 1: Understanding the Concept:

To establish the straight-chain structure of glucose, specific chemical reactions are performed to confirm the presence of its various functional groups. We need to identify the specific reaction that distinguishes an aldehyde group from a general carbonyl or ketone group.

Step 2: Key Formula or Approach:

Approach: Review standard chemical reactions for carbohydrates and identify which reagent specifically oxidizes aldehydes but leaves ketones unaffected.

Step 3: Detailed Explanation:

Let's analyze what each reaction specifically proves about the structure of glucose: - **(A) Reaction with NH_2OH :** Glucose reacts with hydroxylamine to form an oxime. This reaction broadly confirms the presence of a carbonyl group ($> \text{C}=\text{O}$), but it does not distinguish between an aldehyde and a ketone. - **(B) Reaction with Bromine water:** Bromine water ($\text{Br}_2/\text{H}_2\text{O}$) is a mild oxidizing agent. It is strong enough to oxidize aldehydes to carboxylic acids but is not strong enough to oxidize ketones. When glucose is treated with bromine water, it gets oxidized to a six-carbon carboxylic acid named gluconic acid. This specific oxidation confirms that the carbonyl group in glucose must be an aldehydic group. - **(C) Reaction with dil. HNO_3 :** Dilute nitric acid is a stronger oxidizing agent. It oxidizes both the terminal aldehyde and the terminal primary alcohol group of glucose to form a dicarboxylic acid, saccharic acid. This test primarily indicates the presence of a primary alcoholic group. - **(D) Reaction with acetic**

anhydride: Glucose reacts to form a pentaacetate. This confirms the presence of exactly five hydroxyl ($-\text{OH}$) groups attached to different carbon atoms.

Step 4: Final Answer:

The formation of gluconic acid upon reaction with bromine water confirms the aldehydic group.

 Quick Tip

Bromine water ($\text{Br}_2/\text{H}_2\text{O}$) is the standard mild reagent used to distinguish between aldoses (which it oxidizes) and ketoses (which it does not affect) in carbohydrate chemistry.

13. Which metal in following compounds is not present in fractional oxidation state?

- (A) Fe_3O_4
- (B) Mn_3O_4
- (C) Pb_3O_4
- (D) $\text{Na}_2\text{S}_4\text{O}_6$

Correct Answer: (D) $\text{Na}_2\text{S}_4\text{O}_6$

Solution:

Step 1: Understanding the Concept:

A fractional oxidation state is typically an average value that arises when a compound contains multiple atoms of the same element existing in different discrete, integer oxidation states. We need to calculate the oxidation state of the central metal atom in each compound.

Step 2: Key Formula or Approach:

Approach: Use the standard rules for assigning oxidation numbers (e.g., Oxygen is usually -2 , Alkali metals are $+1$) to calculate the average oxidation state of the designated metal in each option.

Step 3: Detailed Explanation:

Let's evaluate the average oxidation states in each given compound: - **(A) Fe_3O_4 :** This is a mixed oxide physically consisting of FeO and Fe_2O_3 . The average oxidation state of the metal iron (Fe) is calculated as: $3x + 4(-2) = 0 \implies 3x = 8 \implies x = +8/3$ (Fractional). - **(B) Mn_3O_4 :** Similar to the iron compound, this is a mixed oxide ($\text{MnO} \cdot \text{Mn}_2\text{O}_3$). The average oxidation state of the metal manganese (Mn) is $3x + 4(-2) = 0 \implies x = +8/3$ (Fractional). - **(C) Pb_3O_4 :** Commonly known as red lead, this is also a mixed oxide ($2\text{PbO} \cdot \text{PbO}_2$). The average oxidation state of the metal lead (Pb) is calculated as $x = +8/3$ (Fractional). - **(D) $\text{Na}_2\text{S}_4\text{O}_6$:** In sodium tetrathionate, the central non-metal sulfur atoms have different integer oxidation states (two are $+5$ and two are 0), giving an average fractional oxidation state for sulfur of $10/4 = +2.5$. However, the question specifically asks about the **metal**. The only metal in this compound is Sodium (Na). Alkali metals invariably have an oxidation state of $+1$ in all their compounds. Thus, the oxidation state of the metal Na is exactly $+1$, which is a whole integer.

Step 4: Final Answer:

The metal sodium in $\text{Na}_2\text{S}_4\text{O}_6$ is present in a $+1$ (non-fractional) oxidation state.

 Quick Tip

Read the question wording extremely carefully! It explicitly asked for the oxidation state of the *metal*, not just any element in the compound. While Sulfur (a non-metal) in $\text{Na}_2\text{S}_4\text{O}_6$ has a fractional average, Sodium does not.

14. Which among the following is NOT dicarboxylic acid?

- (A) Adipic acid
- (B) Glutaric acid
- (C) Valeric acid
- (D) Malonic acid

Correct Answer: (C) Valeric acid

Solution:

Step 1: Understanding the Concept:

A dicarboxylic acid is an organic compound that contains exactly two carboxyl ($-\text{COOH}$) functional groups. We must map the common names provided in the options to their chemical structures to identify the exception.

Step 2: Key Formula or Approach:

Approach: Recall the chemical structures or IUPAC names for the common carboxylic acids listed in the options.

Step 3: Detailed Explanation:

Let's analyze the structure of each acid: - **(A) Adipic acid:** Its systematic IUPAC name is hexanedioic acid. Chemical Formula: $\text{HOOC} - (\text{CH}_2)_4 - \text{COOH}$. It clearly contains two carboxyl groups, making it a dicarboxylic acid. - **(B) Glutaric acid:** Its systematic IUPAC name is pentanedioic acid. Chemical Formula: $\text{HOOC} - (\text{CH}_2)_3 - \text{COOH}$. It contains two carboxyl groups and is a dicarboxylic acid. - **(C) Valeric acid:** Its systematic IUPAC name is pentanoic acid. Chemical Formula: $\text{CH}_3 - (\text{CH}_2)_3 - \text{COOH}$. It contains only one carboxyl group at the terminal end, making it a monocarboxylic acid. - **(D) Malonic acid:** Its systematic IUPAC name is propanedioic acid. Chemical Formula: $\text{HOOC} - \text{CH}_2 - \text{COOH}$. It contains two carboxyl groups and is a dicarboxylic acid.

Step 4: Final Answer:

Valeric acid is a monocarboxylic acid and is NOT a dicarboxylic acid.

 Quick Tip

Use the classic mnemonic "Oh My Such Good Apple" to remember the first five straight-chain dicarboxylic acids by carbon count: **O**xalic (2C), **M**alonic (3C), **S**uccinic (4C), **G**lutaric (5C), **A**dipic (6C).

15. Identify the substrate 'X' in the following reaction.
$$\text{X} + \text{O}_2 \xrightarrow[\text{(air)}]{\text{i) Co-naphenate 423 K, ii) dil HCl } \Delta}$$

Phenol + Acetone

- (A) Chlorobenzene
- (B) Benzene sulphonic acid
- (C) Benzenamine
- (D) Isopropyl benzene

Correct Answer: (D) Isopropyl benzene

Solution:

Step 1: Understanding the Concept:

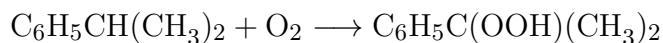
The given reaction scheme depicts the major commercial method for the large-scale preparation of phenol and acetone. This specific industrial process is universally known as the Cumene process.

Step 2: Key Formula or Approach:

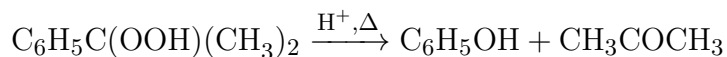
Approach: Recognize the final products (phenol and acetone) and the specific oxidizing conditions to identify the starting material of the Cumene process.

Step 3: Detailed Explanation:

In the Cumene process, the starting substrate 'X' is cumene. The systematic IUPAC name for cumene is isopropyl benzene. The reaction proceeds in two fundamental steps: 1. **Oxidation Step:** Isopropyl benzene (cumene) undergoes aerial oxidation in the presence of a catalyst like cobalt naphthenate at 423 K to form cumene hydroperoxide.



2. **Acid Hydrolysis Step:** The intermediate cumene hydroperoxide is then treated with dilute acid (such as HCl or H₂SO₄) and heated. It undergoes a rearrangement and cleavage to yield phenol and acetone.



This synthetic method is highly favored industrially because it simultaneously yields two highly valuable chemical products from a relatively cheap starting material.

Step 4: Final Answer:

The starting substrate X is isopropyl benzene.

💡 Quick Tip

The "cumene process" is a very high-yield exam topic. Always rigidly associate "Isopropyl benzene (cumene)", "Phenol", and "Acetone" together for this specific reaction pathway.

16. Identify the monomers use to prepare glyptal.

- (A) β -Hydroxy butyric acid and β -hydroxy valeric acid
- (B) Ethylene glycol and phenol

- (C) Ethylene glycol and terephthalic acid
(D) Ethylene glycol and phthalic acid

Correct Answer: (D) Ethylene glycol and phthalic acid

Solution:

Step 1: Understanding the Concept:

Glyptal is a well-known type of polyester resin. Polyesters are synthesized by the step-growth condensation polymerization of dicarboxylic acids and diols. We must correctly identify the specific constituent monomers for glyptal.

Step 2: Key Formula or Approach:

Approach: Recall the specific monomer combinations that form common commercial polymers like PHBV, Bakelite, Terylene, and Glyptal.

Step 3: Detailed Explanation:

Let's carefully analyze the polymers formed by the monomer pairs given in the options: - **(A) β -Hydroxy butyric acid and β -hydroxy valeric acid:** These two monomers copolymerize to form PHBV, which is a notable biodegradable polyester. - **(B) Ethylene glycol and phenol:** These monomers do not standardly undergo condensation polymerization with each other. Phenol typically polymerizes with formaldehyde to form Bakelite. - **(C) Ethylene glycol and terephthalic acid:** The condensation polymerization of these two monomers yields Terylene (also widely known as Dacron). Terephthalic acid is the para-isomer of phthalic acid. - **(D) Ethylene glycol and phthalic acid (or phthalic anhydride):** The condensation of ethylene glycol (ethane-1,2-diol) and phthalic acid (benzene-1,2-dicarboxylic acid, the ortho-isomer) yields the cross-linked polyester known as Glyptal. It forms an alkyd resin utilized primarily in the manufacture of paints and lacquers.

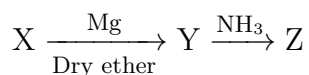
Step 4: Final Answer:

The correct monomers are ethylene glycol and phthalic acid.

 Quick Tip

Notice the spelling similarity in the names to help you remember the ingredients:
Glycol + Phthalic acid $\longrightarrow$ Glyptal.

17. Identify the product ' Z ' in the following series of reactions. Ethanol $\xrightarrow[\Delta]{\text{SOCl}_2}$



- (A) Ethyl chloride
(B) Ethyl magnesium chloride
(C) Ethyl amine
(D) Ethane

Correct Answer: (D) Ethane

Solution:

Step 1: Understanding the Concept:

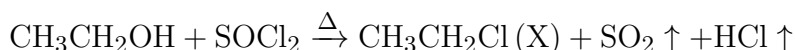
This synthetic sequence involves three classical organic transformations: conversion of a primary alcohol to an alkyl halide, generation of a Grignard reagent, and finally, the acid-base reaction of the Grignard reagent with a compound containing an acidic hydrogen.

Step 2: Key Formula or Approach:

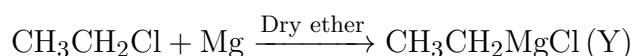
Approach: Systematically trace the functional group transformations step-by-step through the given reagents to determine structures X, Y, and Z.

Step 3: Detailed Explanation:

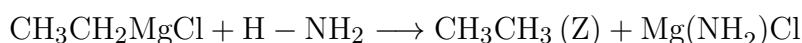
Let's break down the reaction sequence step-by-step: 1. **Formation of X:** Ethanol ($\text{CH}_3\text{CH}_2\text{OH}$) reacts with thionyl chloride (SOCl_2) upon heating. This is a highly efficient method (Darzens procedure) for preparing alkyl chlorides because the byproducts are gases.



Intermediate X is ethyl chloride. 2. **Formation of Y:** Ethyl chloride (X) is treated with magnesium metal (Mg) in an anhydrous solvent (dry ether) to form a Grignard reagent.



Intermediate Y is ethyl magnesium chloride. 3. **Formation of Z:** Grignard reagents behave as strong bases and strong nucleophiles. When exposed to compounds containing an active (acidic) hydrogen—such as water, alcohols, or ammonia (NH_3)—they rapidly abstract a proton to form the corresponding alkane. Here, ammonia provides the acidic proton.



The final product Z is ethane (CH_3CH_3).

Step 4: Final Answer:

The final product Z in the sequence is Ethane.

 Quick Tip

Grignard reagents (RMgX) are exceptionally reactive towards any source of protons (moisture, alcohols, amines, carboxylic acids) and will invariably undergo an acid-base reaction to form the alkane (RH).

18. Calculate the enthalpy of vaporisation of ethanol if 11.5 g of ethanol is completely vaporised by supplying 11.8 kJ of heat.

- (A) 21.7 kJ mol^{-1}
- (B) 47.2 kJ mol^{-1}
- (C) 65.1 kJ mol^{-1}
- (D) 39.0 kJ mol^{-1}

Correct Answer: (B) 47.2 kJ mol^{-1}

Solution:

Step 1: Understanding the Concept:

Enthalpy of vaporization (ΔH_{vap}) is defined as the amount of heat energy required to completely vaporize exactly one mole of a liquid substance at constant temperature and pressure.

Step 2: Key Formula or Approach:

The formula connecting heat, moles, and enthalpy of vaporization is:

$$\Delta H_{\text{vap}} = \frac{q}{n}$$

where q is the total heat supplied and n is the number of moles of the substance.

Step 3: Detailed Explanation:

1. First, we must calculate the molar mass of ethanol ($\text{C}_2\text{H}_5\text{OH}$). Using standard atomic masses: C = 12 g/mol, H = 1 g/mol, O = 16 g/mol. Molar mass of $\text{C}_2\text{H}_5\text{OH} = (2 \times 12) + (6 \times 1) + 16 = 24 + 6 + 16 = 46$ g/mol. 2. Next, calculate the number of moles (n) of ethanol present in the given sample. Given mass = 11.5 g

$$n = \frac{\text{Mass}}{\text{Molar mass}} = \frac{11.5 \text{ g}}{46 \text{ g/mol}} = 0.25 \text{ mol}$$

3. Finally, calculate the enthalpy of vaporization per mole. Heat supplied (q) for 0.25 mol = 11.8 kJ

$$\Delta H_{\text{vap}} = \frac{11.8 \text{ kJ}}{0.25 \text{ mol}}$$

$$\Delta H_{\text{vap}} = 11.8 \times 4 \text{ kJ mol}^{-1}$$

$$\Delta H_{\text{vap}} = 47.2 \text{ kJ mol}^{-1}$$

Step 4: Final Answer:

The calculated enthalpy of vaporization is 47.2 kJ mol^{-1} .

 Quick Tip

To perform calculations involving 0.25 quickly without a calculator, recognize that 0.25 is the fraction $1/4$. Dividing any number by a fraction is identical to multiplying by its reciprocal. So, $11.8/0.25 = 11.8 \times 4$.

19. Identify the reagent involved in Sandmeyer reaction.

- (A) CuCN/KCN
- (B) Cu (Powder) /HBr
- (C) H_3PO_3
- (D) $\text{CH}_3\text{CH}_2\text{OH}$

Correct Answer: (A) CuCN/KCN

Solution:

Step 1: Understanding the Concept:

The Sandmeyer reaction is a prominent chemical reaction utilized to synthesize aryl halides or aryl nitriles from reactive aryl diazonium salts. The critical distinguishing feature of this specific reaction is the necessary use of cuprous (Cu^{I}) salts as reagents/catalysts.

Step 2: Key Formula or Approach:

Approach: Identify the characteristic copper-based salt reagents used specifically in the Sandmeyer reaction and distinguish them from variants like the Gattermann reaction.

Step 3: Detailed Explanation:

Let's analyze the given reagent options: - **(A) CuCN/KCN:** This is a classic, defining reagent for the Sandmeyer reaction. When a diazonium salt is treated with cuprous cyanide (CuCN) dissolved in potassium cyanide (KCN), the diazonium group is nucleophilically replaced by a cyano ($-\text{CN}$) group, forming an aryl nitrile. Similarly, the use of CuCl/HCl and CuBr/HBr defines the Sandmeyer reaction for introducing $-\text{Cl}$ and $-\text{Br}$ respectively. - **(B) Cu (Powder) /HBr:** When finely divided copper powder is used with a halogen acid (HBr or HCl) instead of a cuprous salt to convert diazonium salts to aryl halides, the reaction is known as the Gattermann reaction, which is a modified variation of the Sandmeyer reaction. - **(C) H_3PO_3 :** Usually, hypophosphorous acid (H_3PO_2) is used as a mild reducing agent to replace the diazonium group with a hydrogen atom ($-\text{H}$), converting the diazonium salt back to a simple arene (deamination). - **(D) $\text{CH}_3\text{CH}_2\text{OH}$:** Ethanol is also used as a mild reducing agent for deamination, replacing the diazonium group with hydrogen while being oxidized to acetaldehyde.

Step 4: Final Answer:

CuCN/KCN is the correct reagent for the Sandmeyer reaction.

💡 Quick Tip

To distinguish between Sandmeyer and Gattermann reactions easily: Sandmeyer requires the whole **Salt** ($\text{Salt} \rightarrow \text{Sandmeyer}$, e.g., CuCl), while Gattermann uses metallic copper powder.

20. The solubility of AgBr is $7.1 \times 10^{-7} \text{ mol dm}^{-3}$. Calculate its solubility product at the same temperature.

- (A) 7.08×10^{-13}
- (B) 3.67×10^{-13}
- (C) 5.89×10^{-13}
- (D) 5.04×10^{-13}

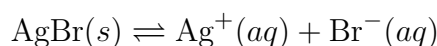
Correct Answer: (D) 5.04×10^{-13}

Solution:**Step 1: Understanding the Concept:**

The solubility product (K_{sp}) is the equilibrium constant representing the dissolution of a sparingly soluble salt. For a simple binary salt that dissociates into a 1 : 1 ratio of ions like AgBr, K_{sp} relates directly to the square of its molar solubility.

Step 2: Key Formula or Approach:

Write the dissociation reaction:



If the molar solubility is S , then at equilibrium: $[\text{Ag}^+] = S$ and $[\text{Br}^-] = S$. The formula for

the solubility product is:

$$K_{sp} = [\text{Ag}^+][\text{Br}^-] = (S) \cdot (S) = S^2$$

Step 3: Detailed Explanation:

Given the molar solubility $S = 7.1 \times 10^{-7} \text{ mol dm}^{-3}$. Substitute this numerical value into the derived K_{sp} equation:

$$K_{sp} = (7.1 \times 10^{-7})^2$$

$$K_{sp} = (7.1)^2 \times (10^{-7})^2$$

$$K_{sp} = 50.41 \times 10^{-14}$$

To match standard scientific notation format (one non-zero digit before the decimal) found in the options, adjust the decimal point:

$$K_{sp} = 5.041 \times 10^{-13}$$

This calculated value matches option (D).

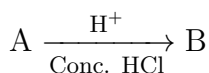
Step 4: Final Answer:

The solubility product is 5.04×10^{-13} .

💡 Quick Tip

For sparingly soluble salts with a simple 1:1 stoichiometry (like AgCl, AgBr, BaSO₄, CaSO₄), the K_{sp} formula is always simply S^2 .

21. Identify the product 'B' in the following sequence of reactions. Methylpropanoate $\xrightarrow[\text{dil. NaOH}]{\Delta}$



- (A) sodium propanoate
- (B) propanone
- (C) propanal
- (D) propanoic acid

Correct Answer: (D) propanoic acid

Solution:

Step 1: Understanding the Concept:

The chemical reaction sequence presented describes the classic alkaline hydrolysis of an ester molecule, followed by a subsequent acidification step to recover the neutral free carboxylic acid.

Step 2: Key Formula or Approach:

Approach: Determine the intermediate salt 'A' formed during ester saponification, and then predict the final product 'B' when this salt is protonated by a strong acid.

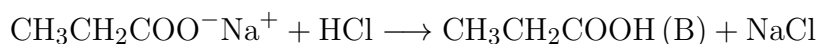
Step 3: Detailed Explanation:

1. **Reaction 1 (Alkaline Hydrolysis):** The starting material is methyl propanoate, an ester with the structure $\text{CH}_3\text{CH}_2\text{COOCH}_3$. When an ester is heated with an aqueous strong base

like dilute NaOH, it undergoes saponification to form the sodium salt of the parent carboxylic acid and an alcohol byproduct.



The major organic intermediate 'A' containing the acyl group is sodium propanoate. **2. Reaction 2 (Acidification):** The basic sodium salt of the acid (A) is then treated with an excess of strong acid, here concentrated HCl. This step protonates the carboxylate anion, liberating the neutral free carboxylic acid.



The final isolated product 'B' is propanoic acid.

Step 4: Final Answer:

The final product B is propanoic acid.

 Quick Tip

The base-catalyzed hydrolysis (saponification) of an ester always yields a carboxylate salt initially. A subsequent mandatory acidification step (H^+ workup) is required to obtain the neutral carboxylic acid.

22. Which from following functional groups has highest priority order for naming the poly functional compound?

- (A) -CN
- (B) $-\text{CONH}_2$
- (C) $-\text{C} \equiv \text{C}-$
- (D) $-\text{NH}_2$

Correct Answer: (B) $-\text{CONH}_2$

Solution:

Step 1: Understanding the Concept:

In IUPAC organic nomenclature, when a compound contains multiple functional groups, a rigid priority sequence is followed to determine the principal functional group. This principal group dictates the main suffix of the parent name, while all other groups are treated as substituents with prefixes.

Step 2: Key Formula or Approach:

Approach: Retrieve the standard IUPAC priority sequence for functional groups from memory and compare the given options to find the highest-ranking group.

Step 3: Detailed Explanation:

The universally accepted IUPAC priority order for principal functional groups (from highest to lowest) is as follows: Carboxylic acids ($-\text{COOH}$) > Sulfonic acids ($-\text{SO}_3\text{H}$) > Esters ($-\text{COOR}$) > Acid halides ($-\text{COX}$) > **Amides ($-\text{CONH}_2$)** > Nitriles ($-\text{CN}$) > Aldehydes ($-\text{CHO}$) > Ketones ($> \text{C}=\text{O}$) > Alcohols ($-\text{OH}$) > Thiols ($-\text{SH}$) > Amines ($-\text{NH}_2$) > Alkenes ($> \text{C}=\text{C} <$) > Alkynes ($-\text{C} \equiv \text{C}-$). Comparing the specific options provided: - (A) $-\text{CN}$

(Nitrile) - (B) $-\text{CONH}_2$ (Amide) - (C) $-\text{C} \equiv \text{C}-$ (Alkyne) - (D) $-\text{NH}_2$ (Amine) According to the sequence, Amides have a higher priority than Nitriles, which in turn rank higher than Amines and Alkynes. Therefore, $-\text{CONH}_2$ holds the highest priority among the choices.

Step 4: Final Answer:

The functional group with the highest priority is $-\text{CONH}_2$.

💡 Quick Tip

A handy mnemonic for remembering the top functional groups priority is
Carboxylic acids, **E**sters, **A**mides, **N**itriles, **A**ldehydes, **K**etones, **A**lcohols, **A**mines
(CEANAKAA).

23. Identify the product 'X' formed in the following reaction, Sodium ethoxide + Isopropyl chloride $\longrightarrow$ X + Ethanol + Sodium chloride

- (A) 1-Ethoxypropane
- (B) 2-Ethoxypropane
- (C) Propene
- (D) Propane

Correct Answer: (C) Propene

Solution:

Step 1: Understanding the Concept:

The reaction between an alkyl halide and an alkoxide ion can proceed via nucleophilic substitution (S_N2) to form an ether (Williamson ether synthesis) or via elimination (E2) to form an alkene. The dominant reaction pathway depends heavily on the steric nature of the alkyl halide (primary, secondary, or tertiary) and the strength of the base.

Step 2: Key Formula or Approach:

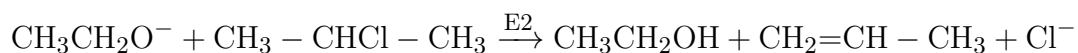
Approach: Analyze the steric hindrance of the secondary alkyl halide reacting with a strong base, and specifically note the byproducts explicitly given in the reaction equation to deduce the mechanism.

Step 3: Detailed Explanation:

Let's examine the reactants: - Sodium ethoxide ($\text{CH}_3\text{CH}_2\text{O}^-\text{Na}^+$): Acts as both a strong base and a strong nucleophile. - Isopropyl chloride ($\text{CH}_3\text{CH}(\text{Cl})\text{CH}_3$): A secondary (2°) alkyl halide. With secondary alkyl halides reacting with strong bases like alkoxides, the elimination (E2) pathway actively competes with, and often dominates, the substitution (S_N2) pathway. The products explicitly stated in the question provide a definitive clue:



The formation of ethanol ($\text{CH}_3\text{CH}_2\text{OH}$) dictates that the ethoxide ion ($\text{CH}_3\text{CH}_2\text{O}^-$) acted purely as a base, not as a nucleophile. It abstracted a proton (H^+) from a β -carbon of the isopropyl chloride, leading to the concerted elimination of HCl (which neutralizes with the sodium ethoxide) and the formation of a carbon-carbon double bond. Reaction details:



The organic product 'X' formed alongside ethanol and NaCl is an alkene, specifically propene. If substitution had occurred instead, the product would have been an ether (2-ethoxypropane), and the byproduct would only be NaCl, with no ethanol produced.

Step 4: Final Answer:

The product 'X' is propene.

💡 Quick Tip

To successfully synthesize ethers using the Williamson synthesis with 2° or 3° alkyl groups, you must use the alkoxide of the bulky branched group and react it with a primary alkyl halide to prevent elimination reactions.

24. Which among the following is NOT benzylic halide?

- (A) Bromophenylmethane
- (B) 1-Bromo-1-phenylethane
- (C) 2-Bromo-2-phenylpropane
- (D) 1-Bromo-2-phenylbutane

Correct Answer: (D) 1-Bromo-2-phenylbutane

Solution:

Step 1: Understanding the Concept:

A benzylic halide is a specific class of organic compound where the halogen atom is directly attached to an sp^3 hybridized carbon atom, which in turn is directly attached to an aromatic benzene ring. This specific bridging carbon is termed the benzylic carbon.

Step 2: Key Formula or Approach:

Approach: Examine the IUPAC name or structural formula of each compound to precisely locate the position of the halogen atom relative to the phenyl group.

Step 3: Detailed Explanation:

Let's draw and analyze the structures to check the attachment point of the bromine atom: -

- **(A) Bromophenylmethane:** Commonly known as benzyl bromide. Structure: $\text{Ph-CH}_2\text{-Br}$. The bromine is attached to the $-\text{CH}_2-$ carbon, which is directly attached to the phenyl (Ph) ring. This is a primary benzylic halide. - **(B) 1-Bromo-1-phenylethane:** Structure: Ph-CH(Br)-CH_3 . The bromine is attached to Carbon-1. Because Carbon-1 is also directly attached to the phenyl ring, this is a secondary benzylic halide. - **(C) 2-Bromo-2-phenylpropane:** Structure: $\text{Ph-C(CH}_3\text{)(Br)-CH}_3$. The bromine is on Carbon-2. The phenyl ring is also attached to Carbon-2. Thus, this is a tertiary benzylic halide. - **(D) 1-Bromo-2-phenylbutane:** Structure: $\text{Ph-CH(CH}_2\text{CH}_3\text{)-CH}_2\text{-Br}$. Numbering starts from the carbon with the primary functional group (bromine). The bromine is on Carbon-1. The phenyl ring is attached to Carbon-2. Because the bromine is attached to an alkyl carbon that is one carbon atom further away from the benzylic position, this is a standard primary alkyl halide, not a benzylic halide.

Step 4: Final Answer:

1-Bromo-2-phenylbutane is not a benzylic halide.

 Quick Tip

In IUPAC nomenclature, carefully look for the numbers. If the phenyl group and the halogen are located on the **exact same** carbon number (e.g., 1-bromo-1-phenyl... or 2-bromo-2-phenyl...), it is a benzylic halide. If they reside on different numbers, it is not.

25. If compressibility factor of real gas is 1.05 at STP. What is molar volume of real gas?

- (A) 22.40dm³
- (B) 21.33dm³
- (C) 23.52dm³
- (D) 23.05dm³

Correct Answer: (C) 23.52dm³

Solution:

Step 1: Understanding the Concept:

The compressibility factor (Z) is a useful correction factor that quantifies the deviation of a real gas from perfect ideal gas behavior. It is mathematically defined as the ratio of the actual molar volume of a gas to the theoretical molar volume of an ideal gas at the same temperature and pressure conditions.

Step 2: Key Formula or Approach:

The fundamental formula for the compressibility factor is:

$$Z = \frac{V_{\text{real}}}{V_{\text{ideal}}}$$

where V_{real} is the actual molar volume of the real gas and V_{ideal} is the theoretical molar volume of an ideal gas.

Step 3: Detailed Explanation:

Given values from the problem: Compressibility factor (Z) = 1.05 The conditions specified are Standard Temperature and Pressure (STP). In traditional textbook problems, the historical standard value for the molar volume of an ideal gas at STP (0°C, 1 atm) is commonly used:

$$V_{\text{ideal}} = 22.4 \text{ L/mol} = 22.4 \text{ dm}^3/\text{mol}$$

Rearranging the compressibility factor formula to solve for V_{real} :

$$V_{\text{real}} = Z \times V_{\text{ideal}}$$

Substitute the given values:

$$V_{\text{real}} = 1.05 \times 22.4 \text{ dm}^3$$

$$V_{\text{real}} = 23.52 \text{ dm}^3$$

This calculated result matches option C exactly.

Step 4: Final Answer:

The molar volume of the real gas is 23.52dm³.

💡 Quick Tip

If $Z > 1$, the real gas is harder to compress and occupies a larger volume than an ideal gas because repulsive forces dominate. If $Z < 1$, it occupies a smaller volume because attractive forces dominate. Here $Z = 1.05$, so intuitively expect an answer slightly larger than 22.4.

26. Identify the type of defect from following in stainless steel.

- (A) Substitutional impurity defect
- (B) Interstitial impurity defect
- (C) Metal excess defect
- (D) Frenkel defect

Correct Answer: (A) Substitutional impurity defect

Solution:

Step 1: Understanding the Concept:

Point defects in crystal lattices can be intrinsic (like stoichiometric defects such as Schottky or Frenkel) or extrinsic (impurity defects). Solid solutions or alloys are typical, everyday examples where impurity defects deliberately occur.

Step 2: Key Formula or Approach:

Approach: Classify the defect based on the relative atomic sizes of the constituent elements blended in the stainless steel alloy.

Step 3: Detailed Explanation:

Let's analyze the physical nature of stainless steel: - Stainless steel is an alloy primarily composed of Iron (Fe) thoroughly mixed with significant amounts of Chromium (Cr) and often Nickel (Ni). - Iron, chromium, and nickel are all d-block transition metals located close to each other in the periodic table. Because of this, they possess very similar atomic radii. - Because their atomic sizes are comparable, chromium and nickel atoms can easily replace and take the place of iron atoms at regular lattice sites in the crystal structure of the metal without severely distorting the lattice. - When an impurity atom directly occupies a regular lattice site of the host atom, the defect is classified as a **substitutional impurity defect**. - In contrast, if the impurity atoms were much smaller (like Carbon in regular carbon steel), they would squeeze into the spaces between the host atoms, creating an **interstitial impurity defect**. While stainless steel does contain trace carbon, its primary classifying feature as a distinct alloy is the substitution by Cr and Ni. Thus, in the context of solid state chemistry classifications, it's widely cited as a classic example of a substitutional solid solution.

Step 4: Final Answer:

Stainless steel exhibits a substitutional impurity defect.

 Quick Tip

Substitutional alloys form successfully when the atomic radii of the constituent elements are within about 15% of each other. Common examples include brass (Cu/Zn), bronze (Cu/Sn), and stainless steel (Fe/Cr/Ni).

27. Which from following is a lanthanoid element?

- (A) Er
- (B) Am
- (C) Np
- (D) Lr

Correct Answer: (A) Er

Solution:

Step 1: Understanding the Concept:

The f-block elements in the periodic table are divided into two distinct series: the lanthanoids (or lanthanides) and the actinoids (or actinides). We need to identify which of the given chemical symbols correctly belongs to the lanthanoid series.

Step 2: Key Formula or Approach:

Approach: Locate the given elements by their atomic numbers to distinguish between the $4f$ series (lanthanoids) and the $5f$ series (actinoids).

Step 3: Detailed Explanation:

- **Lanthanoids:** These are the 14 elements immediately following lanthanum (La, atomic number 57), ranging from Cerium (Ce, $Z = 58$) to Lutetium (Lu, $Z = 71$). They involve the progressive filling of the $4f$ subshell. - **Actinoids:** These are the 14 elements immediately following actinium (Ac, atomic number 89), ranging from Thorium (Th, $Z = 90$) to Lawrencium (Lr, $Z = 103$). They involve the filling of the $5f$ subshell. Let's classify each option: - (A) **Er** (Erbium): Atomic number 68. It falls squarely in the range $58 - 71$, so it is a lanthanoid. - (B) **Am** (Americium): Atomic number 95. It falls in the range $90 - 103$, so it is an actinoid. - (C) **Np** (Neptunium): Atomic number 93. It falls in the range $90 - 103$, so it is an actinoid. - (D) **Lr** (Lawrencium): Atomic number 103. It is the very last element of the actinoid series.

Step 4: Final Answer:

Erbium (Er) is a lanthanoid element.

 Quick Tip

Familiarize yourself with the first few and last few elements of these series. Also, knowing that elements discovered synthetically (often named after places or scientists like Americium, Neptunium, Lawrencium) are almost exclusively actinides can help quickly narrow down choices.

28. Which from following formulae is used to obtain value of E_{cell}° for a reaction

taking place in Dry cell?

- (A) $\frac{-\Delta G^\circ}{F}$
- (B) $\frac{-\Delta G^\circ}{2F}$
- (C) $\frac{-\Delta G^\circ}{3F}$
- (D) $\frac{-\Delta G^\circ}{4F}$

Correct Answer: (B) $\frac{-\Delta G^\circ}{2F}$

Solution:

Step 1: Understanding the Concept:

The standard Gibbs free energy change (ΔG°) of an electrochemical cell reaction is fundamentally related to the standard cell potential (E°_{cell}) by the core thermodynamic equation $\Delta G^\circ = -nFE^\circ_{\text{cell}}$, where n represents the number of moles of electrons transferred in the balanced cell reaction, and F is the Faraday constant.

Step 2: Key Formula or Approach:

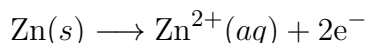
The rearranged formula to solve for the cell potential is:

$$E^\circ_{\text{cell}} = \frac{-\Delta G^\circ}{nF}$$

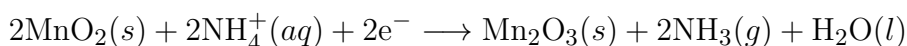
Approach: Determine the value of 'n' for the overall chemical reaction occurring specifically in a Leclanché cell (dry cell).

Step 3: Detailed Explanation:

To find the correct formula, we must identify the total electron transfer in a dry cell. The half-reactions operating in a standard dry cell are: At Anode (Oxidation of zinc container):



At Cathode (Reduction of manganese dioxide):



The balanced overall cell reaction involves the net transfer of exactly 2 electrons. Therefore, $n = 2$. Substitute $n = 2$ into the general thermodynamic relationship:

$$E^\circ_{\text{cell}} = \frac{-\Delta G^\circ}{nF}$$
$$E^\circ_{\text{cell}} = \frac{-\Delta G^\circ}{2F}$$

Step 4: Final Answer:

The correct formula is $\frac{-\Delta G^\circ}{2F}$.

 Quick Tip

Remember that zinc acts as the anode in most common primary batteries (like dry cells and button cells), and its simple oxidation from Zn to Zn^{2+} provides exactly 2 electrons, solidly establishing $n = 2$ for these systems.

29. Which of the following solutions exhibits highest freezing point depression?

- (A) 0.1 m NaCl
- (B) 0.05 m MgSO_4
- (C) 1 m AlPO_4
- (D) 0.05 m $\text{Al}_2(\text{SO}_4)_3$

Correct Answer: (D) 0.05 m $\text{Al}_2(\text{SO}_4)_3$

Solution:

Step 1: Understanding the Concept:

Freezing point depression (ΔT_f) is a colligative property, meaning its magnitude depends entirely on the total number of dissolved solute particles in the solution, not their identity.

Step 2: Key Formula or Approach:

For electrolytes, the depression is calculated using the formula:

$$\Delta T_f = i \cdot K_f \cdot m$$

where i is the van 't Hoff factor (number of ions produced per formula unit) and m is the molality. The solution with the highest mathematical product of ($i \times m$) will exhibit the highest freezing point depression.

Step 3: Detailed Explanation:

Let's systematically evaluate the ($i \times m$) value for each option, assuming 100% complete dissociation for strong electrolytes. - **(A) 0.1 m NaCl:** Dissociates into Na^+ and Cl^- . So, $i = 2$. Effective concentration = $i \times m = 2 \times 0.1 = 0.2$ - **(B) 0.05 m MgSO_4 :** Dissociates into Mg^{2+} and SO_4^{2-} . So, $i = 2$. Effective concentration = $i \times m = 2 \times 0.05 = 0.1$ - **(D) 0.05 m $\text{Al}_2(\text{SO}_4)_3$:** Dissociates into 2Al^{3+} and 3SO_4^{2-} . So, $i = 2 + 3 = 5$. Effective concentration = $i \times m = 5 \times 0.05 = 0.25$ - **(C) 1 m AlPO_4 :** While mathematically $1 \times 2 = 2$, aluminum phosphate (AlPO_4) is practically insoluble in water. Therefore, a true 1 m aqueous solution cannot exist, and its actual particle concentration in solution is virtually zero. In the context of standard multiple-choice questions comparing strong soluble electrolytes, highly insoluble salts are intended as distractors. Comparing the realistic soluble salts, (D) yields the largest effect. Comparing the practical values: 0.25 (from option D) is greater than 0.20 (from A) and 0.10 (from B).

Step 4: Final Answer:

The 0.05 m $\text{Al}_2(\text{SO}_4)_3$ solution has the highest effective concentration of particles and thus exhibits the highest freezing point depression.

 Quick Tip

Always calculate the product of concentration $\times$ number of ions (i). Compounds with high ionic charges like $\text{Al}_2(\text{SO}_4)_3$ produce many ions per unit ($i = 5$), often making them the correct answer even if their stated molarity/molality appears slightly lower.

30. Calculate the energy associated with third orbit of He^+ .

- (A) $-4.8 \times 10^{-19} \text{ J}$
- (B) $-1.45 \times 10^{-19} \text{ J}$
- (C) $-19.36 \times 10^{-19} \text{ J}$
- (D) $-9.69 \times 10^{-19} \text{ J}$

Correct Answer: (D) $-9.69 \times 10^{-19} \text{ J}$

Solution:

Step 1: Understanding the Concept:

According to the Bohr model of the atom, the energy of an electron residing in the n -th orbit of a hydrogen-like species (a one-electron species like H, He^+ , Li^{2+}) is strictly quantized and can be calculated using a specific formula involving the atomic number (Z) and the principal quantum number (n).

Step 2: Key Formula or Approach:

The energy E_n of the n -th orbit in Joules is given by:

$$E_n = -2.18 \times 10^{-18} \left(\frac{Z^2}{n^2} \right) \text{ J}$$

Step 3: Detailed Explanation:

For the helium ion (He^+): Atomic number, $Z = 2$ We need the energy for the third orbit, so $n = 3$ Substitute these values directly into the formula:

$$E_3 = -2.18 \times 10^{-18} \times \left(\frac{2^2}{3^2} \right)$$

$$E_3 = -2.18 \times 10^{-18} \times \left(\frac{4}{9} \right)$$

$$E_3 = -2.18 \times 10^{-18} \times 0.4444\dots$$

$$E_3 \approx -0.9688 \times 10^{-18} \text{ J}$$

To precisely match the scientific notation format presented in the options, adjust the decimal place:

$$E_3 \approx -9.688 \times 10^{-19} \text{ J}$$

Rounding to two decimal places, we get $-9.69 \times 10^{-19} \text{ J}$.

Step 4: Final Answer:

The energy associated with the third orbit is $-9.69 \times 10^{-19} \text{ J}$.

 Quick Tip

You can also memorize the energy formula in electron-volts: $E_n = -13.6 \cdot \frac{Z^2}{n^2} \text{ eV}$. Calculating this gives $-13.6 \cdot (4/9) = -6.04 \text{ eV}$. Multiply by $1.6 \times 10^{-19} \text{ J/eV}$ to quickly get $\approx -9.66 \times 10^{-19} \text{ J}$, confirming the correct order of magnitude.

31. What is the pH of buffer solution formed by mixing 0.01 M acetic acid and 0.05 M sodium acetate? ($\text{pK}_a = 4.7447$)

- (A) 2.80
- (B) 3.00
- (C) 5.44
- (D) 6.5

Correct Answer: (C) 5.44

Solution:

Step 1: Understanding the Concept:

An aqueous mixture of a weak acid (like acetic acid) and its conjugate base (like sodium acetate) forms an acidic buffer solution. The pH of such a buffer can be calculated directly using the Henderson-Hasselbalch equation.

Step 2: Key Formula or Approach:

The Henderson-Hasselbalch equation for an acidic buffer is:

$$\text{pH} = \text{pK}_a + \log_{10} \left(\frac{[\text{Salt}]}{[\text{Acid}]} \right)$$

Step 3: Detailed Explanation:

Given values from the problem: Concentration of acid, $[\text{Acid}] = 0.01 \text{ M}$ Concentration of salt (conjugate base), $[\text{Salt}] = 0.05 \text{ M}$ $\text{pK}_a = 4.7447$ Substitute these values into the equation:

$$\text{pH} = 4.7447 + \log_{10} \left(\frac{0.05}{0.01} \right)$$

$$\text{pH} = 4.7447 + \log_{10}(5)$$

The mathematical value of $\log_{10}(5)$ is approximately 0.699.

$$\text{pH} \approx 4.7447 + 0.699$$

$$\text{pH} \approx 5.4437$$

Rounding to two decimal places to match the options, the pH is 5.44.

Step 4: Final Answer:

The pH of the given buffer solution is 5.44.

 Quick Tip

Since the concentration of the basic component (salt) is strictly higher than the acidic component ($0.05 > 0.01$), the log term evaluates to a positive number, meaning the final pH will be greater than the pK_a . This logic helps instantly eliminate options (A) and (B).

32. Identify the geometry of TeF_4 molecule from the following.

- (A) Linear
- (B) Tetrahedral
- (C) Trigonal bipyramidal
- (D) Trigonal

Correct Answer: (C) Trigonal bipyramidal

Solution:

Step 1: Understanding the Concept:

The structural geometry of a molecule can be accurately predicted using Valence Shell Electron Pair Repulsion (VSEPR) theory by counting the number of bonding electron pairs and non-bonding lone pairs around the central atom.

Step 2: Key Formula or Approach:

Approach: Apply VSEPR theory by determining the number of valence electrons on the central atom, calculating the steric number (bonding domains + lone pairs), and identifying the corresponding electron geometry.

Step 3: Detailed Explanation:

1. **Determine valence electrons:** The central atom is Tellurium (Te), which belongs to Group 16 of the periodic table (same as Oxygen and Sulfur). Therefore, it possesses 6 valence electrons. 2. **Calculate bonding and lone pairs:** Te forms 4 single covalent bonds with 4 Fluorine (F) atoms. - Number of bonding pairs (bp) = 4. - Electrons utilized in bonding = 4. - Remaining valence electrons = $6 - 4 = 2$ electrons, which constitutes exactly 1 lone pair (lp). 3. **Determine steric number and hybridization:** Steric number (total electron domains) = $\text{bp} + \text{lp} = 4 + 1 = 5$. A steric number of 5 directly corresponds to sp^3d hybridization. 4. **Identify the geometry:** The base *electron-pair geometry* for 5 domains is **trigonal bipyramidal**. Due to the presence of one lone pair occupying an equatorial position to minimize repulsion, the actual physical *molecular shape* is "see-saw". Because "see-saw" is not among the provided options, the question is utilizing the term "geometry" to refer to the fundamental electron-pair geometry that dictates the overall spatial structure, which is trigonal bipyramidal.

Step 4: Final Answer:

The parent electron-pair geometry of TeF_4 is trigonal bipyramidal.

 Quick Tip

In multiple-choice questions, if the specific resultant molecular shape (like V-shape, see-saw, square planar) is not listed, look for the parent electron domain geometry (tetrahedral, trigonal bipyramidal, octahedral) associated with the steric number.

33. Which of the following set of elements is present in apatite?

- (A) Mg, P and S
- (B) Ca, P and O
- (C) Al, Mg and P
- (D) Ca, S and O

Correct Answer: (B) Ca, P and O

Solution:

Step 1: Understanding the Concept:

Apatite represents a group of very common phosphate minerals. Knowing the general chemical composition of this mineral family allows us to easily identify its core constituent elements.

Step 2: Key Formula or Approach:

Approach: Recall the general chemical formula for the apatite group of minerals to identify the primary elements that form its lattice structure.

Step 3: Detailed Explanation:

The apatite group consists of isomorphous minerals characterized by the general chemical formula $\text{Ca}_5(\text{PO}_4)_3(\text{F}, \text{Cl}, \text{OH})$. Common specific varieties include: - Fluorapatite: $\text{Ca}_5(\text{PO}_4)_3\text{F}$ (the most common geological form) - Chlorapatite: $\text{Ca}_5(\text{PO}_4)_3\text{Cl}$ - Hydroxylapatite: $\text{Ca}_5(\text{PO}_4)_3\text{OH}$ (the primary mineral constituent of bone and teeth) By examining the general chemical formula, the primary elements that constantly make up the structural framework of all apatite minerals are: - Calcium (Ca) - Phosphorus (P) - Oxygen (O) Looking at the given options, only set (B) contains this specific combination of elements.

Step 4: Final Answer:

Apatite fundamentally contains the elements Ca, P, and O.

💡 Quick Tip

Apatite is the principal industrial ore of phosphorus. The mineralogical suffix "-apatite" usually points specifically to a calcium phosphate mineral matrix.

34. Which of the following is not negatively charged sol?

- (A) As_2S_3
- (B) Haemoglobin
- (C) Clay
- (D) Congo Red

Correct Answer: (B) Haemoglobin

Solution:**Step 1: Understanding the Concept:**

Colloidal particles naturally carry an electrical charge, which can be positive or negative depending on the chemical nature of the dispersed phase and the specific method of preparation. We need to classify the given sols based on their standard accepted charge.

Step 2: Key Formula or Approach:

Approach: Classify the given colloidal sols based on standard charge conventions for basic/acidic components, metal sulfides, and biological macromolecules.

Step 3: Detailed Explanation:

Let's review the classification of common textbook sols based on their charge: - **Positively charged sols:** Metallic hydroxides [like $\text{Fe}(\text{OH})_3$, $\text{Al}(\text{OH})_3$], basic dyes (like methylene blue), and biologically important basic macromolecules like **haemoglobin** in blood. - **Negatively charged sols:** Metal sols (like Cu, Ag, Au), metallic sulfides (like As_2S_3 , Sb_2S_3), acid dyes (like eosin, **Congo red**), and naturally occurring sols like starch, gum, gelatin, and **clay**. Based strictly on this standard classification: - As_2S_3 is a well-known negatively charged sol. - Clay is a naturally occurring negatively charged sol. - Congo Red is an acidic dye, hence it forms a negatively charged sol. - Haemoglobin is a widely cited example of a positively charged macromolecular sol.

Step 4: Final Answer:

Haemoglobin is not a negatively charged sol; it is positively charged.

💡 Quick Tip

A simple rule of thumb for exam questions: Basic components (like basic dyes and metal hydroxides) form positive sols, while acidic components (like acid dyes and metal sulfides) form negative sols.

35. The rate constant for the reaction, $2\text{N}_2\text{O}_{5(\text{g})} \longrightarrow 2\text{N}_2\text{O}_{4(\text{g})} + \text{O}_{2(\text{g})}$ is $4.98 \times 10^{-4} \text{ s}^{-1}$. What is the order of reaction?

- (A) 2
- (B) 1
- (C) Zero
- (D) 3

Correct Answer: (B) 1

Solution:**Step 1: Understanding the Concept:**

The kinetic order of a chemical reaction can be unequivocally determined solely by inspecting the units of its rate constant (k).

Step 2: Key Formula or Approach:

The general unit for the rate constant of an n -th order reaction is mathematically given by:

$$\text{Unit of } k = (\text{mol L}^{-1})^{1-n} \text{ s}^{-1}$$

where n represents the overall order of the reaction.

Step 3: Detailed Explanation:

The problem explicitly states that the rate constant is $k = 4.98 \times 10^{-4} \text{ s}^{-1}$. Notice carefully that the unit is simply s^{-1} (inverse seconds), and it contains no concentration terms (neither mol nor L). Let's logically equate the general unit formula to the given unit:

$$(\text{mol L}^{-1})^{1-n} \text{ s}^{-1} = \text{s}^{-1}$$

For this equation to hold true algebraically, the exponent of the concentration term must be exactly zero, because any non-zero value raised to the power of 0 equals 1.

$$1 - n = 0$$

Solving for n :

$$n = 1$$

Therefore, the reaction is a first-order reaction. The stoichiometry of the balanced equation ($2\text{N}_2\text{O}_5$) does not dictate the order, which is a purely experimentally determined quantity reflected accurately in the units of k .

Step 4: Final Answer:

The order of the reaction is 1.

💡 Quick Tip

Always look exclusively at the units of the rate constant to find the order! Zero order: $\text{mol L}^{-1}\text{s}^{-1}$ First order: s^{-1} Second order: $\text{L mol}^{-1}\text{s}^{-1}$

36. Identify thermoplastic polymer from following.

- (A) Polyvinyl
- (B) Bakelite
- (C) Terylene
- (D) Neoprene

Correct Answer: (A) Polyvinyl

Solution:**Step 1: Understanding the Concept:**

Polymers are broadly classified based on their intermolecular forces and thermal behavior into four categories: elastomers, fibers, thermoplastics, and thermosetting polymers. Thermoplastics are linear or slightly branched long-chain molecules capable of repeatedly softening on heating and hardening on cooling without chemical change.

Step 2: Key Formula or Approach:

Approach: Classify the given polymers based on their thermal properties and structural cross-linking to identify the one that melts upon heating.

Step 3: Detailed Explanation:

Let's examine the classification of each option: - **(A) Polyvinyl:** Examples include Polyvinyl Chloride (PVC) and polyvinyl acetate. These are predominantly linear polymers with intermolecular forces intermediate between elastomers and fibers. They soften upon heating and can be easily remolded, classifying them strictly as **thermoplastics**. - **(B) Bakelite:** It is a phenol-formaldehyde resin. It undergoes extensive 3D cross-linking upon heating in molds to become an infusible, hard, rigid mass. It cannot be remelted or reshaped, making it a **thermosetting polymer**. - **(C) Terylene:** Also known as Dacron, it is a polyester. It possesses strong intermolecular forces like hydrogen bonding or dipole-dipole interactions, giving it high tensile strength. It is classified primarily as a **fiber**. - **(D) Neoprene:** It is a synthetic rubber. Its polymer chains are held together by the weakest intermolecular forces, allowing it to be easily stretched and return to its original shape. It is classified as an **elastomer**.

Step 4: Final Answer:

Polyvinyl is the thermoplastic polymer.

💡 Quick Tip

Common thermoplastics to memorize: Polythene, Polystyrene, Polyvinyls (PVC). Common thermosetting plastics: Bakelite, Melamine-formaldehyde, Urea-formaldehyde resins.

37. What is the number of unpaired electrons in Ti in +3 state?

- (A) 4
- (B) 3
- (C) 1
- (D) Zero

Correct Answer: (C) 1

Solution:

Step 1: Understanding the Concept:

To find the exact number of unpaired electrons in a transition metal ion, we first write the full electron configuration of the neutral atom and then systematically remove electrons to form the specified ion. Electrons are always removed from the outermost shell first (highest principal quantum number n).

Step 2: Key Formula or Approach:

Approach: Write the electron configuration for the neutral atom (Ti), remove 3 electrons to form the ion (Ti^{3+}), and apply Hund's rule to the remaining d-electrons.

Step 3: Detailed Explanation:

1. **Electron configuration of neutral Titanium (Ti):** The atomic number of Ti is 22. Its ground-state electron configuration is $[\text{Ar}]3d^24s^2$. 2. **Electron configuration of Ti^{+3} ion:** To form a +3 ion, the titanium atom must logically lose 3 electrons. Electrons are lost from the outermost shell (4s) first, and only then from the inner shell (3d). - Remove 2 electrons from the 4s orbital. - Remove 1 electron from the 3d orbital. The resulting configuration for Ti^{3+} is $[\text{Ar}]3d^14s^0$, or simply $[\text{Ar}]3d^1$. 3. **Count unpaired electrons:** The 3d subshell has 5 degenerate orbitals. According to Hund's rule, electrons fill degenerate orbitals singly first. The single electron will occupy one of these orbitals unpaired. Therefore, there is exactly 1 unpaired electron.

Step 4: Final Answer:

The Ti^{+3} ion has 1 unpaired electron.

 Quick Tip

Always remember to remove electrons from the highest principle quantum number (n) first. For transition metals, this means removing ns electrons **before** removing nd electrons when forming cations.

38. Calculate the total number of tetrahedral and octahedral voids formed in 0.6 mol of a compound if it forms hcp structure.

- (A) 3.613×10^{24}
- (B) 7.226×10^{24}
- (C) 1.084×10^{24}
- (D) 2.913×10^{24}

Correct Answer: (C) 1.084×10^{24}

Solution:

Step 1: Understanding the Concept:

In any close-packed lattice structure (like hcp or ccp/fcc), the number of voids generated depends directly and proportionally on the number of constituent particles forming the lattice.

Step 2: Key Formula or Approach:

If the total number of close-packed particles (atoms) is N , then by geometric derivation: Number of octahedral voids = N Number of tetrahedral voids = $2N$ Total number of voids = $N + 2N = 3N$

Step 3: Detailed Explanation:

1. **Calculate the number of atoms (N):** Amount of compound given = 0.6 mol Using Avogadro's number ($N_A \approx 6.022 \times 10^{23} \text{ mol}^{-1}$):

$$N = 0.6 \text{ mol} \times (6.022 \times 10^{23} \text{ atoms/mol})$$

$$N = 3.6132 \times 10^{23} \text{ atoms}$$

2. **Calculate total voids:** Total voids = $3 \times N$

$$\text{Total voids} = 3 \times (3.6132 \times 10^{23})$$

$$\text{Total voids} = 10.8396 \times 10^{23}$$

3. **Format the mathematical answer:** Adjusting the decimal to standard scientific notation format:

$$\text{Total voids} = 1.08396 \times 10^{24} \approx 1.084 \times 10^{24}$$

Step 4: Final Answer:

The total number of combined voids is 1.084×10^{24} .

 Quick Tip

Remember the simple 1 : 2 ratio rule: for every 1 atom forming the crystal lattice, there is exactly 1 octahedral void and 2 tetrahedral voids, making 3 total voids per lattice atom.

39. Calculate the concentration of dissolved gas in water at 25°C if partial pressure of gas at same temperature is 0.15 atm . [$K_H = 0.15 \text{ mol dm}^{-3} \text{ atm}^{-1}$]

- (A) 0.0261 M
- (B) 0.0182 M
- (C) 0.0293 M
- (D) 0.0225 M

Correct Answer: (D) 0.0225 M

Solution:

Step 1: Understanding the Concept:

Henry's Law states that the solubility (concentration) of a gas in a liquid is directly proportional to the partial pressure of that specific gas located above the surface of the liquid.

Step 2: Key Formula or Approach:

The mathematical form of Henry's Law utilized based on the given units is:

$$C = K_H \times P$$

where: C = concentration of the dissolved gas (solubility) K_H = Henry's law constant P = partial pressure of the gas

Step 3: Detailed Explanation:

Given values from the problem statement: Partial pressure (P) = 0.15 atm Henry's law constant (K_H) = 0.15 moldm⁻³atm⁻¹ Substitute the numerical values directly into the law's equation:

$$C = (0.15 \text{ moldm}^{-3}\text{atm}^{-1}) \times (0.15 \text{ atm})$$

$$C = 0.0225 \text{ moldm}^{-3}$$

Since standard volume conversions state that 1 dm³ = 1 L, the concentration unit moldm⁻³ is exactly equivalent to standard Molarity (M).

$$C = 0.0225 \text{ M}$$

Step 4: Final Answer:

The concentration of the dissolved gas is 0.0225 M.

 Quick Tip

Pay very close attention to the specific units of Henry's law constant (K_H). The formula form varies ($P = K_H \cdot X$ vs $C = K_H \cdot P$) depending entirely on the units provided. Here, moldm⁻³atm⁻¹ dictates multiplying K_H by pressure.

40. Which from following is an essential amino acid?

- (A) Tyrosine
- (B) Serine
- (C) Histidine
- (D) Glycine

Correct Answer: (C) Histidine

Solution:

Step 1: Understanding the Concept:

Biologically, amino acids are classified into essential and non-essential amino acids based strictly on whether the human body can synthesize them internally. Essential amino acids cannot be synthesized by the body and must be obtained through the diet.

Step 2: Key Formula or Approach:

Approach: Differentiate between amino acids synthesized by the human metabolic pathways (non-essential) and those that must be obtained from dietary sources (essential) by recalling

the standard lists.

Step 3: Detailed Explanation:

Let's review the biochemical classification of the amino acids given in the options: - **(A) Tyrosine:** This is a non-essential amino acid. The human body can synthesize it internally by hydroxylating another amino acid, phenylalanine. - **(B) Serine:** This is a non-essential amino acid. It can be readily synthesized in the body from metabolic intermediates like 3-phosphoglycerate. - **(C) Histidine:** This is classified as an **essential** amino acid. While adults can synthesize very small amounts, it is not sufficient to meet physiological needs, and it is strictly essential for infants and growing children. - **(D) Glycine:** This is the simplest amino acid structurally and is non-essential, being readily synthesized in the body from serine.

Step 4: Final Answer:

Histidine is an essential amino acid.

 Quick Tip

A highly useful mnemonic for remembering the 10 essential amino acids is **PVT TIM HALL**: Phenylalanine, Valine, Threonine, Tryptophan, Isoleucine, Methionine, Histidine, Arginine, Leucine, Lysine.

41. What is the mass in grams of 0.25 mol water?

- (A) 1.5 g
- (B) 2.5 g
- (C) 4.5 g
- (D) 3.5 g

Correct Answer: (C) 4.5 g

Solution:

Step 1: Understanding the Concept:

The mass of a given substance can be calculated directly from its number of moles if its molar mass is known. The molar mass is defined as the mass of exactly one mole of a substance expressed in grams per mole (g/mol).

Step 2: Key Formula or Approach:

The fundamental relationship connecting mass, moles, and molar mass is:

$$\text{Mass} = \text{Moles} \times \text{Molar Mass}$$

Step 3: Detailed Explanation:

First, we must determine the molar mass of water (H_2O). Using standard atomic masses: Atomic mass of Hydrogen (H) = 1 g/mol Atomic mass of Oxygen (O) = 16 g/mol The molar mass of H_2O is calculated as:

$$\text{Molar mass of } \text{H}_2\text{O} = (2 \times 1) + 16 = 2 + 16 = 18 \text{ g/mol}$$

The problem provides the number of moles: Given number of moles = 0.25 mol Substitute these known values into the mass formula:

$$\text{Mass} = 0.25 \text{ mol} \times 18 \text{ g/mol}$$

To simplify the calculation, recognize that 0.25 is equivalent to the fraction $\frac{1}{4}$:

$$\text{Mass} = \frac{1}{4} \times 18 \text{ g} = 4.5 \text{ g}$$

Step 4: Final Answer:

The mass of 0.25 mol water is 4.5 g.

 Quick Tip

Memorizing the molar mass of extremely common compounds like water (18 g/mol), carbon dioxide (44 g/mol), and ammonia (17 g/mol) can save you valuable time during exams.

42. Which from following statements is correct regarding a detergent sodium lauryl sulphate?

- (A) It is anionic detergent and is used in toothpaste.
- (B) It is cationic detergent and is used as household detergent.
- (C) It is cationic detergent and is used as hair conditioner.
- (D) It is an ether and used as liquid detergent.

Correct Answer: (A) It is anionic detergent and is used in toothpaste.

Solution:

Step 1: Understanding the Concept:

Synthetic detergents are broadly classified into three categories—anionic, cationic, and non-ionic—based on the electrical charge carried by their hydrophilic (water-loving) head group. Sodium lauryl sulphate is a ubiquitous synthetic detergent, and knowing its classification helps deduce its common uses.

Step 2: Key Formula or Approach:

Approach: Analyze the chemical structure and name of sodium lauryl sulphate to determine its ionic nature upon dissociation, and then relate this to standard commercial applications.

Step 3: Detailed Explanation:

The chemical formula for sodium lauryl sulphate (often abbreviated as SLS or SDS) is $\text{CH}_3(\text{CH}_2)_{11}\text{OSO}_3^- \text{Na}^+$.

When dissolved in an aqueous solution, it dissociates completely into a long-chain alkyl sulphate anion ($\text{CH}_3(\text{CH}_2)_{11}\text{OSO}_3^-$) and a sodium cation (Na^+). Because the actual cleansing action is performed by the large, negatively charged anionic part of the molecule, it is formally classified as an **anionic detergent**. Anionic detergents are highly effective and are widely utilized for general household cleaning. More specifically, sodium lauryl sulphate is a very common foaming agent and surfactant used in personal care products, particularly in **toothpastes** and shampoos, to create a rich lather. Let's evaluate the given options based on these facts: - (A) Correctly identifies it as an anionic detergent and notes its widespread use in toothpaste. - (B) Incorrectly identifies it as a cationic detergent. - (C) Incorrectly identifies it as cationic (cationic detergents, such as cetyltrimethylammonium bromide, are typically used in hair conditioners). - (D) Incorrectly identifies its functional group as an ether; it is chemically a sulfate ester salt.

Step 4: Final Answer:

Sodium lauryl sulphate is an anionic detergent and is used in toothpaste.

💡 Quick Tip

Most common, heavy-duty household detergents and personal care foaming agents are anionic (like SLS). Cationic detergents are more expensive, often possess germicidal properties, and are used in specialized products like hair conditioners and fabric softeners.

43. Identify the structural formula of phloroglucinol.

- (A) Benzene ring with -OH groups at 1, 3, 5 positions
- (B) Benzene ring with -OH groups at 1, 2, 3 positions
- (C) Benzene ring with -OH groups at 1, 2, 4 positions
- (D) Benzene ring with -OH groups at 1, 3 positions

Correct Answer: (A) Benzene ring with -OH groups at 1, 3, 5 positions

Solution:**Step 1: Understanding the Concept:**

Phloroglucinol is the widely accepted common or trivial name for a specific positional isomer of benzenetriol. Knowing the IUPAC names that correspond to these common historical names is essential for identifying their correct chemical structures.

Step 2: Key Formula or Approach:

Approach: Match the given common name "phloroglucinol" to its systematic IUPAC nomenclature to deduce its exact substitution pattern on the central benzene ring.

Step 3: Detailed Explanation:

Benzenetriol (a benzene ring with three hydroxyl groups) exists as three distinct positional isomers based on the relative placement of the -OH groups: 1. **1,2,3-benzenetriol:** Also commonly known as **pyrogallol**. In this isomer, all three -OH groups are situated adjacent to one another. 2. **1,2,4-benzenetriol:** Also commonly known as **hydroxyquinol**. Here, two -OH groups are adjacent, and the third is separated by one unsubstituted carbon. 3. **1,3,5-benzenetriol:** Also commonly known as **phloroglucinol**. In this highly symmetric isomer, the three -OH groups are distributed evenly around the ring, occupying alternating carbon positions. Based on the visual structures provided in the original options: - Option (A) clearly shows a benzene ring with hydroxyl groups attached at the 1, 3, and 5 positions. This represents 1,3,5-benzenetriol, which is definitively phloroglucinol. - Option (B) shows a benzene ring with hydroxyl groups at the 1, 2, and 3 positions, representing pyrogallol. - Option (C) shows a benzene ring with hydroxyl groups at the 1, 2, and 4 positions, representing hydroxyquinol. - Option (D) shows a benzene ring with only two hydroxyl groups located at the 1 and 3 positions, representing a diol known as resorcinol.

Step 4: Final Answer:

The correct structural formula for phloroglucinol is a benzene ring with -OH groups at the 1, 3, and 5 positions.

 Quick Tip

To easily remember the three trihydroxybenzenes: **P**yro gallol is packed tightly together (1,2,3), **H**ydroxyquinol has a single gap (1,2,4), and **P**hloroglucinol is perfectly symmetric (1,3,5).

44. Which of the following equations is correct regarding rate of disappearance of reactant and appearance of product for $\text{N}_{2(g)} + 3\text{H}_{2(g)} \longrightarrow 2\text{NH}_{3(g)}$

- (A) $3 \frac{d[\text{N}_2]}{dt} = \frac{1}{2} \frac{d[\text{N}_2]}{dt}$
(B) $\frac{1}{2} \frac{d[\text{N}_2]}{dt} = \frac{1}{3} \frac{d[\text{H}_2]}{dt}$
(C) $2 \frac{d[\text{NH}_3]}{dt} = 3 \frac{d[\text{H}_2]}{dt}$
(D) $3 \frac{d[\text{NH}_3]}{dt} = -2 \frac{d[\text{H}_2]}{dt}$

Correct Answer: (D) $3 \frac{d[\text{NH}_3]}{dt} = -2 \frac{d[\text{H}_2]}{dt}$

Solution:

Step 1: Understanding the Concept:

The overall rate of a chemical reaction can be unambiguously expressed in terms of the rate of change of concentration of any participating reactant or product. To equate these individual species rates to one another, they must be divided by their respective stoichiometric coefficients from the balanced chemical equation. Furthermore, reactants are assigned a negative sign (since their concentration decreases over time), while products are assigned a positive sign.

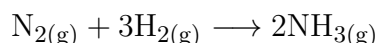
Step 2: Key Formula or Approach:

For any general balanced reaction $aA + bB \longrightarrow cC + dD$, the relationship between the rates is defined as:

$$\text{Rate} = -\frac{1}{a} \frac{d[A]}{dt} = -\frac{1}{b} \frac{d[B]}{dt} = +\frac{1}{c} \frac{d[C]}{dt} = +\frac{1}{d} \frac{d[D]}{dt}$$

Step 3: Detailed Explanation:

Given the balanced chemical equation for the synthesis of ammonia:



Let's apply the general formula to this specific reaction. The overall rate of reaction can be expressed as:

$$\text{Rate} = -\frac{d[\text{N}_2]}{dt} = -\frac{1}{3} \frac{d[\text{H}_2]}{dt} = +\frac{1}{2} \frac{d[\text{NH}_3]}{dt}$$

We need to evaluate the given options to find the mathematical equation that correctly represents a part of this fundamental relationship. Let's specifically look at the relationship between Hydrogen (H_2) and Ammonia (NH_3):

$$-\frac{1}{3} \frac{d[\text{H}_2]}{dt} = \frac{1}{2} \frac{d[\text{NH}_3]}{dt}$$

To clear the fractions and find a match among the options, we can multiply the entire equation by the lowest common multiple of the denominators (which is 6):

$$6 \times \left(-\frac{1}{3} \frac{d[\text{H}_2]}{dt} \right) = 6 \times \left(\frac{1}{2} \frac{d[\text{NH}_3]}{dt} \right)$$

$$-2 \frac{d[\text{H}_2]}{dt} = 3 \frac{d[\text{NH}_3]}{dt}$$

Rearranging this equation slightly gives:

$$3 \frac{d[\text{NH}_3]}{dt} = -2 \frac{d[\text{H}_2]}{dt}$$

This derived equation perfectly matches option (D). Let's briefly check why others fail: (A) relates a species to itself incorrectly, (B) mixes up the coefficients and signs, (C) lacks the necessary negative sign indicating disappearance.

Step 4: Final Answer:

The correct relationship is $3 \frac{d[\text{NH}_3]}{dt} = -2 \frac{d[\text{H}_2]}{dt}$.

💡 Quick Tip

Always remember the required negative sign for reactants (denoting disappearance) and the positive sign for products (denoting appearance). Dividing by the stoichiometric coefficient "normalizes" the individual rate so it accurately represents the single, overall "reaction rate".

45. If salicylic acid (138 u) reacts with acetic anhydride (102 u) to form aspirin (180 u) calculate % atom economy.

- (A) 25%
- (B) 50%
- (C) 65%
- (D) 75%

Correct Answer: (D) 75%

Solution:

Step 1: Understanding the Concept:

Atom economy is a vital metric in green chemistry that measures the proportion of reactant atoms that become part of the desired product in a chemical reaction. It evaluates the efficiency of a synthetic route in terms of minimizing waste.

Step 2: Key Formula or Approach:

The formula for calculating percentage atom economy is:

$$\% \text{ Atom Economy} = \left(\frac{\text{Molecular mass of desired product}}{\text{Sum of molecular masses of all reactants}} \right) \times 100$$

Step 3: Detailed Explanation:

From the problem description, the reaction is the synthesis of aspirin from salicylic acid and acetic anhydride.

The relevant molecular masses are provided directly in the question:

Mass of Salicylic acid (Reactant 1) = 138 u

Mass of Acetic anhydride (Reactant 2) = 102 u

Mass of Aspirin (Desired Product) = 180 u

First, calculate the sum of the masses of the reactants to find the total theoretical mass input:

$$\text{Total mass of reactants} = 138 \text{ u} + 102 \text{ u} = 240 \text{ u}$$

Next, substitute these values into the atom economy formula:

$$\% \text{ Atom Economy} = \left(\frac{180}{240} \right) \times 100$$

Simplify the fraction:

$$\% \text{ Atom Economy} = \left(\frac{3}{4} \right) \times 100$$

$$\% \text{ Atom Economy} = 0.75 \times 100 = 75\%$$

Step 4: Final Answer:

The percentage atom economy for this reaction is 75%.

Quick Tip

Always ensure you are dividing the mass of the **desired product** by the sum of the masses of **all reactants**. Do not erroneously include the mass of any byproducts (like acetic acid in this specific synthesis) in the denominator.

46. Identify a molecule having highest number of lone pair of electrons in valence shell of central atom.

- (A) NH_3
- (B) SF_4
- (C) ICl_3
- (D) PCl_3

Correct Answer: (C) ICl_3

Solution:

Step 1: Understanding the Concept:

To accurately determine the number of lone pairs residing on the central atom of a molecule, we must apply the principles of VSEPR theory. This involves finding the total number of valence electrons on the central atom and subtracting the electrons that are shared in covalent bonds to find the remaining non-bonding electrons (lone pairs).

Step 2: Key Formula or Approach:

The number of lone pairs (LP) can be quickly calculated using the logic:

$$\text{LP} = \frac{\text{Valence } e^- \text{ on central atom} - \text{Number of bonding } e^-}{2}$$

For molecules with single bonds, this simplifies to: $\text{LP} = \frac{\text{Group Number} - \text{Valency}}{2}$.

Step 3: Detailed Explanation:

Let's systematically analyze each molecule to count the lone pairs:

(A) **NH₃**: The central atom is Nitrogen (N), which is located in Group 15 and possesses 5 valence electrons. It forms 3 single bonds with 3 Hydrogen atoms, utilizing 3 electrons. Remaining non-bonding electrons = $5 - 3 = 2$. This pair constitutes exactly **1 lone pair**.

(B) **SF₄**: The central atom is Sulfur (S), located in Group 16, possessing 6 valence electrons. It forms 4 single bonds with 4 Fluorine atoms, utilizing 4 electrons. Remaining non-bonding electrons = $6 - 4 = 2$. This pair constitutes exactly **1 lone pair**.

(C) **ICl₃**: The central atom is Iodine (I), a halogen located in Group 17, possessing 7 valence electrons. It forms 3 single bonds with 3 Chlorine atoms, utilizing 3 electrons. Remaining non-bonding electrons = $7 - 3 = 4$. These 4 electrons constitute exactly **2 lone pairs**.

(D) **PCl₃**: The central atom is Phosphorus (P), located in Group 15, possessing 5 valence electrons. It forms 3 single bonds with 3 Chlorine atoms, utilizing 3 electrons. Remaining non-bonding electrons = $5 - 3 = 2$. This pair constitutes exactly **1 lone pair**.

Comparing the tabulated results, ICl₃ clearly has the highest number of lone pairs on its central atom among the given choices.

Step 4: Final Answer:

The molecule ICl₃ has the highest number of lone pairs (two) on its central atom.

 Quick Tip

Interhalogen compounds such as ClF₃ and ICl₃ are classic, high-yield exam examples of molecules with multiple lone pairs on the central atom, which leads to their characteristic T-shaped molecular geometry.

47. Which among the following compounds has lowest boiling point?

- (A) (C₂H₅)₂NH
- (B) C₂H₅N(CH₃)₂
- (C) *n* - C₄H₉OH
- (D) C₂H₅COOH

Correct Answer: (B) C₂H₅N(CH₃)₂

Solution:

Step 1: Understanding the Concept:

The boiling point of a molecular compound is primarily dictated by the strength of its intermolecular forces. Stronger forces, such as hydrogen bonding, require significantly more thermal energy to overcome during the transition from liquid to gas, resulting in a higher observed boiling point.

Step 2: Key Formula or Approach:

Approach: Identify the dominant types of intermolecular forces present in each compound. Specifically, look for the presence or absence of hydrogen bonding, a particularly strong dipole-dipole interaction that occurs only when a Hydrogen atom is directly covalently bonded to a highly electronegative atom (N, O, or F).

Step 3: Detailed Explanation:

Let's evaluate the intermolecular forces in each given compound:

(A) (C₂H₅)₂NH (**Diethylamine**): This is classified as a secondary (2°) amine. It contains

one polar N – H covalent bond. Therefore, discrete molecules of diethylamine can form intermolecular hydrogen bonds with each other, leading to a relatively high boiling point.

(B) $\text{C}_2\text{H}_5\text{N}(\text{CH}_3)_2$ (Ethyltrimethylamine): This is classified as a tertiary (3°) amine. The central nitrogen atom is bonded exclusively to three carbon groups and has zero N – H bonds. Without a hydrogen atom directly bonded to an electronegative atom, it cannot act as a hydrogen bond donor to itself. Its molecules are held together only by much weaker dipole-dipole interactions and London dispersion forces, leading to a significantly lower boiling point compared to its isomers.

(C) $n - \text{C}_4\text{H}_9\text{OH}$ (1-Butanol): This is a primary alcohol. It contains a highly polar -OH group, which engages in extensive and strong intermolecular hydrogen bonding. Alcohols generally exhibit higher boiling points than amines of comparable molar mass because oxygen is more electronegative than nitrogen, making the O – H hydrogen bond stronger than the N – H bond.

(D) $\text{C}_2\text{H}_5\text{COOH}$ (Propanoic acid): This is a carboxylic acid. It can form very strong hydrogen bonds, often pairing up to form stable, discrete dimers even in the liquid state. This extensive and robust hydrogen bonding gives carboxylic acids the highest boiling points among the common organic classes listed here.

Comparing all options, the tertiary amine stands out as it entirely lacks hydrogen bonding capability, giving it the weakest intermolecular forces and unequivocally the lowest boiling point.

Step 4: Final Answer:

The compound with the lowest boiling point is $\text{C}_2\text{H}_5\text{N}(\text{CH}_3)_2$.

💡 Quick Tip

When comparing isomers or molecules of very similar molar mass, the general boiling point order is typically: Carboxylic Acids > Alcohols > 1° Amines > 2° Amines > 3° Amines > Alkanes. Tertiary amines always stand out uniquely because they completely lack hydrogen bonding.

48. Calculate the molar mass of an element if it forms fcc unit cell structure [mass of unit cell = 1.8×10^{-22} g, $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$]

- (A) 27.0 g mol^{-1}
- (B) 24.4 g mol^{-1}
- (C) 21.0 g mol^{-1}
- (D) 30.2 g mol^{-1}

Correct Answer: (A) 27.0 g mol^{-1}

Solution:

Step 1: Understanding the Concept:

In solid-state chemistry, the macroscopic properties of a crystal can be deduced directly from its fundamental geometric building block, the unit cell. The total mass of a single unit cell is simply determined by multiplying the number of atoms it contains by the absolute mass of one individual atom.

Step 2: Key Formula or Approach:

The mass of a unit cell (m_{cell}) is mathematically related to the molar mass (M) of the element by the formula:

$$m_{\text{cell}} = \frac{Z \times M}{N_A}$$

where: Z = number of atoms per unit cell M = molar mass of the element (g mol^{-1}) N_A = Avogadro's number (mol^{-1}) Rearranging this equation algebraically to solve for molar mass gives:

$$M = \frac{m_{\text{cell}} \times N_A}{Z}$$

Step 3: Detailed Explanation:

First, we must correctly identify the value of Z based on the given crystal lattice type. The problem states the element forms a face-centered cubic (fcc) unit cell structure. For an fcc lattice, there are 8 atoms located at the corners (each contributing $1/8$ to the cell) and 6 atoms situated on the faces (each contributing $1/2$).

$$Z = \left(8 \times \frac{1}{8}\right) + \left(6 \times \frac{1}{2}\right) = 1 + 3 = 4 \text{ atoms per unit cell}$$

Now, list the known values provided explicitly in the problem: Mass of unit cell (m_{cell}) = 1.8×10^{-22} g Avogadro's number (N_A) = $6.022 \times 10^{23} \text{ mol}^{-1}$ Substitute these numerical values into the rearranged formula:

$$M = \frac{(1.8 \times 10^{-22}) \times (6.022 \times 10^{23})}{4}$$

Perform the multiplication in the numerator:

$$M = \frac{1.8 \times 6.022 \times 10^1}{4}$$

$$M = \frac{10.8396 \times 10}{4}$$

$$M = \frac{108.396}{4}$$

Divide to find the final molar mass:

$$M = 27.099 \text{ g mol}^{-1}$$

Rounding to one decimal place to match the format of the given options, we get 27.0 g mol^{-1} .

Step 4: Final Answer:

The calculated molar mass is 27.0 g mol^{-1} .

💡 Quick Tip

Always commit to memory the Z values for the common cubic crystal lattices: Simple Cubic (SC) $Z = 1$, Body-Centered Cubic (BCC) $Z = 2$, and Face-Centered Cubic (FCC or ccp) $Z = 4$.

49. Calculate the osmotic pressure of 0.03 mole of non electrolyte solute dissolved in 0.1dm³ of water at 300 K. [R = 0.082dm³ atm mol⁻¹ K⁻¹]

- (A) 7.4 atm
- (B) 6.4 atm
- (C) 8.0 atm
- (D) 5.6 atm

Correct Answer: (A) 7.4 atm

Solution:

Step 1: Understanding the Concept:

Osmotic pressure (π) is an important colligative property of solutions that depends strictly on the concentration of solute particles, not their chemical identity. It can be calculated using an equation mathematically identical to the ideal gas law.

Step 2: Key Formula or Approach:

The standard formula for calculating osmotic pressure is:

$$\pi = iCRT \quad \text{or} \quad \pi = i \frac{n}{V} RT$$

where: i = van 't Hoff factor (which is 1 for a non-electrolyte) n = number of moles of solute
 V = volume of solution in liters (dm³) R = ideal gas constant T = absolute temperature in Kelvin

Step 3: Detailed Explanation:

Let's clearly list the numerical values given in the problem statement: Number of moles (n) = 0.03 mol Volume of solution (V) = 0.1 dm³ (Note: Since it is a dilute solution, the volume of the solvent water is an acceptable approximation for the volume of the solution). Temperature (T) = 300 K Gas constant (R) = 0.082 dm³ atm mol⁻¹ K⁻¹ Because the solute is explicitly described as a "non-electrolyte", it does not dissociate into ions in solution, meaning $i = 1$. Substitute these values directly into the osmotic pressure formula:

$$\pi = (1) \times \left(\frac{0.03 \text{ mol}}{0.1 \text{ dm}^3} \right) \times (0.082 \text{ dm}^3 \text{ atm mol}^{-1} \text{ K}^{-1}) \times (300 \text{ K})$$

Simplify the concentration term first:

$$\pi = (0.3 \text{ mol dm}^{-3}) \times (0.082 \text{ dm}^3 \text{ atm mol}^{-1} \text{ K}^{-1}) \times (300 \text{ K})$$

Perform the multiplication:

$$\pi = 0.3 \times 0.082 \times 300 \text{ atm}$$

$$\pi = 0.3 \times 24.6 \text{ atm}$$

$$\pi = 7.38 \text{ atm}$$

Rounding this result to one decimal place to appropriately match the provided options gives 7.4 atm.

Step 4: Final Answer:

The calculated osmotic pressure is 7.4 atm.

💡 Quick Tip

Always verify that your units are perfectly consistent before calculating. If R is given with units involving dm^3 (which is equivalent to liters), ensure your volume is also expressed in dm^3 . The term "non-electrolyte" is a critical clue that tells you to set the van 't Hoff factor i exactly to 1.

50. Identify the correct decreasing order of stability of complexes formed by divalent metal ions with same ligand.

- (A) $\text{Cu}^{2+} > \text{Mn}^{2+} > \text{Cd}^{2+}$
- (B) $\text{Cd}^{2+} > \text{Mn}^{2+} > \text{Cu}^{2+}$
- (C) $\text{Mn}^{2+} > \text{Cd}^{2+} > \text{Cu}^{2+}$
- (D) $\text{Cu}^{2+} > \text{Cd}^{2+} > \text{Mn}^{2+}$

Correct Answer: (D) $\text{Cu}^{2+} > \text{Cd}^{2+} > \text{Mn}^{2+}$

Solution:

Step 1: Understanding the Concept:

The thermodynamic stability of coordination complexes formed by transition metal ions with a given common ligand depends on several factors, including the charge density of the metal ion, ligand field stabilization energy (CFSE), and the nature of the metal-ligand bond (covalency). For divalent ions of the 3d transition series, the Irving-Williams series provides a very reliable general trend.

Step 2: Key Formula or Approach:

Approach: Apply the Irving-Williams series to firmly establish the high stability of the Cu^{2+} complex, and then compare the remaining ions (Mn^{2+} vs Cd^{2+}) based on general principles of coordination chemistry, specifically size, charge, and degree of covalency.

Step 3: Detailed Explanation:

Let's critically analyze the chemical properties of each metal ion provided: - **Cu^{2+} ($3d^9$):** According to the well-established empirical Irving-Williams series ($\text{Ba}^{2+} < \text{Sr}^{2+} < \text{Ca}^{2+} < \text{Mg}^{2+} < \text{Mn}^{2+} < \text{Fe}^{2+} < \text{Co}^{2+} < \text{Ni}^{2+} < \text{Cu}^{2+} > \text{Zn}^{2+}$), Copper(II) routinely forms the most stable complexes among all the divalent 3d transition metals with most ligands. This exceptional stability is largely attributed to additional stabilization derived from the Jahn-Teller effect. - **Mn^{2+} ($3d^5$):** This is typically a high-spin d^5 system, meaning its Crystal Field Stabilization Energy (CFSE) is exactly zero in weak or moderate ligand fields. Because it completely lacks this extra stabilizing energy and possesses a relatively large ionic radius for its row, it generally forms the least stable complexes among the later 3d metals. - **Cd^{2+} ($4d^{10}$):** Cadmium is a group 12 element, situated immediately below Zinc. Like Mn^{2+} , it possesses a spherically symmetric electron cloud and zero CFSE. However, being a 4d transition metal, it is larger and significantly more polarizable (often termed a "softer" acid) than Mn^{2+} . Because of its higher effective nuclear charge (Z_{eff}) compared to early 3d metals, it tends to form much more covalent, and therefore generally stronger, bonds with many common ligands (especially nitrogen or softer donors) than Mn^{2+} . As a comparative example, the formation constant ($\log K_f$) for the standard EDTA complex of Cd^{2+} is ≈ 16.5 , which is substantially higher than the corresponding value for Mn^{2+} (≈ 13.9). Therefore, comparing the three ions, Cu^{2+} is

unequivocally the most stable due to its position in the Irving-Williams series and Jahn-Teller distortion. Between the two ions with zero CFSE, the heavier, more polarizable Cd^{2+} forms more stable complexes than the lighter, harder Mn^{2+} . The resulting sequence representing decreasing stability is: $\text{Cu}^{2+} > \text{Cd}^{2+} > \text{Mn}^{2+}$.

Step 4: Final Answer:

The correct decreasing order of stability is $\text{Cu}^{2+} > \text{Cd}^{2+} > \text{Mn}^{2+}$.

 Quick Tip

Remember that the Irving-Williams stability series sharply peaks at Cu^{2+} . When comparing 3d metals to 4d/5d metals possessing similar characteristics (like zero CFSE), the heavier metals typically form more stable complexes primarily due to increased covalency in the resulting metal-ligand bonds.

Maths

1. The general solution of $x(x-1)\frac{dy}{dx} = x^3(2x-1) + (x-2)y$ is

- (A) $y(x-1) = x^3 + c(x-1)$, where c is the constant of integration.
- (B) $y = x^3(x-1) + c$, where c is the constant of integration.
- (C) $y(x-1) = x^3(x-1) + cx^2$, where c is the constant of integration.
- (D) $y(x-1) = x^3(x-1) + c$, where c is the constant of integration.

Correct Answer: (C) $y(x-1) = x^3(x-1) + cx^2$, where c is the constant of integration.

Solution:

Step 1: Understanding the Concept:

The given equation is a first-order linear differential equation.

We need to rearrange it into the standard form $\frac{dy}{dx} + Py = Q$ to find the integrating factor.

Step 2: Key Formula or Approach:

The standard form is $\frac{dy}{dx} + Py = Q$.

The Integrating Factor (IF) is given by $e^{\int P dx}$.

The general solution is $y \cdot (\text{IF}) = \int Q \cdot (\text{IF}) dx + c$.

Step 3: Detailed Explanation:

Given differential equation is:

$$x(x-1)\frac{dy}{dx} = x^3(2x-1) + (x-2)y$$

Rearranging terms, we get:

$$x(x-1)\frac{dy}{dx} - (x-2)y = x^3(2x-1)$$

Dividing the entire equation by $x(x-1)$, we obtain the standard linear form:

$$\frac{dy}{dx} - \frac{x-2}{x(x-1)}y = \frac{x^3(2x-1)}{x(x-1)}$$

$$\frac{dy}{dx} + \left(\frac{2-x}{x(x-1)} \right) y = \frac{x^2(2x-1)}{x-1}$$

Here, $P = \frac{2-x}{x(x-1)}$ and $Q = \frac{x^2(2x-1)}{x-1}$.

To integrate P , we use partial fractions:

$$\frac{2-x}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$$2-x = A(x-1) + Bx$$

Putting $x = 0$, we get $2 = A(-1) \Rightarrow A = -2$.

Putting $x = 1$, we get $1 = B(1) \Rightarrow B = 1$.

So, $P = -\frac{2}{x} + \frac{1}{x-1}$.

Now, calculate the Integrating Factor (IF):

$$\text{IF} = e^{\int P dx} = e^{\int \left(-\frac{2}{x} + \frac{1}{x-1}\right) dx}$$

$$\text{IF} = e^{-2 \ln |x| + \ln |x-1|} = e^{\ln(x^{-2}) + \ln(x-1)} = e^{\ln\left(\frac{x-1}{x^2}\right)} = \frac{x-1}{x^2}$$

The general solution is given by:

$$y \cdot (\text{IF}) = \int Q \cdot (\text{IF}) dx + c$$

$$y \left(\frac{x-1}{x^2} \right) = \int \left(\frac{x^2(2x-1)}{x-1} \right) \left(\frac{x-1}{x^2} \right) dx + c$$

$$y \left(\frac{x-1}{x^2} \right) = \int (2x-1) dx + c$$

$$y \left(\frac{x-1}{x^2} \right) = x^2 - x + c = x(x-1) + c$$

Multiplying both sides by x^2 , we get:

$$y(x-1) = x^3(x-1) + cx^2$$

Step 4: Final Answer:

The general solution is $y(x-1) = x^3(x-1) + cx^2$.

💡 Quick Tip

Always simplify rational expressions using partial fractions before integrating to find the integrating factor.

Watch out for logarithmic properties like $e^{a \ln x} = x^a$.

2. The direction cosines of the line $x - y + 2z = 5$ and $3x + y + z = 6$ are

- (A) $\frac{-3}{5\sqrt{2}}, \frac{5}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}$
 (B) $\frac{3}{5\sqrt{2}}, \frac{-5}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}$
 (C) $\frac{3}{5\sqrt{2}}, \frac{5}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}$
 (D) $\frac{3}{5\sqrt{2}}, \frac{5}{5\sqrt{2}}, \frac{-4}{5\sqrt{2}}$

Correct Answer: (A) $\frac{-3}{5\sqrt{2}}, \frac{5}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}$

Solution:**Step 1: Understanding the Concept:**

The line of intersection of two planes is perpendicular to the normal vectors of both planes.

Therefore, the direction vector of the line is the cross product of the normal vectors of the two given planes.

Step 2: Key Formula or Approach:

Let the planes be P_1 and P_2 with normal vectors $\vec{n}_1$ and $\vec{n}_2$.

The direction vector of the line is $\vec{d} = \vec{n}_1 \times \vec{n}_2$.

The direction cosines (l, m, n) are given by $\frac{\vec{d}}{|\vec{d}|}$.

Step 3: Detailed Explanation:

The equations of the planes are:

Plane 1: $x - y + 2z = 5 \Rightarrow$ normal vector $\vec{n}_1 = \hat{i} - \hat{j} + 2\hat{k}$.

Plane 2: $3x + y + z = 6 \Rightarrow$ normal vector $\vec{n}_2 = 3\hat{i} + \hat{j} + \hat{k}$.

The direction vector $\vec{d}$ of the line is:

$$\vec{d} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix}$$

$$\vec{d} = \hat{i}(-1 \cdot 1 - 2 \cdot 1) - \hat{j}(1 \cdot 1 - 2 \cdot 3) + \hat{k}(1 \cdot 1 - (-1) \cdot 3)$$

$$\vec{d} = \hat{i}(-1 - 2) - \hat{j}(1 - 6) + \hat{k}(1 + 3)$$

$$\vec{d} = -3\hat{i} + 5\hat{j} + 4\hat{k}$$

The magnitude of the direction vector is:

$$|\vec{d}| = \sqrt{(-3)^2 + 5^2 + 4^2} = \sqrt{9 + 25 + 16} = \sqrt{50} = 5\sqrt{2}$$

The direction cosines are:

$$\left(\frac{-3}{5\sqrt{2}}, \frac{5}{5\sqrt{2}}, \frac{4}{5\sqrt{2}} \right)$$

Or its negative, but this exactly matches option (A).

Step 4: Final Answer:

The direction cosines are $\frac{-3}{5\sqrt{2}}, \frac{5}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}$.

💡 Quick Tip

The direction ratios of the line of intersection of two planes $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$ can be directly found using determinants: $(b_1c_2 - b_2c_1, c_1a_2 - c_2a_1, a_1b_2 - a_2b_1)$.

3. 20 is divided into two parts so that the product of the cube of one part and the square of the other part is maximum, then these two parts are

- (A) 15, 5
- (B) 16, 4
- (C) 12, 8
- (D) 14, 6

Correct Answer: (C) 12, 8

Solution:

Step 1: Understanding the Concept:

This is an optimization problem using derivatives.

We need to define a function for the product described and find its maximum value over a valid domain.

Step 2: Key Formula or Approach:

Let the two parts be x and $20 - x$.

The function to maximize is $P(x) = x^3(20 - x)^2$.

Find the critical points by setting $P'(x) = 0$ and verify the maximum using the first or second derivative test.

Step 3: Detailed Explanation:

Let the two parts be x and y , so $x + y = 20$, which gives $y = 20 - x$.

We want to maximize the product $P = x^3y^2 = x^3(20 - x)^2$.

Differentiating P with respect to x :

$$\frac{dP}{dx} = 3x^2(20 - x)^2 + x^3 \cdot 2(20 - x)(-1)$$

$$\frac{dP}{dx} = x^2(20 - x)[3(20 - x) - 2x]$$

$$\frac{dP}{dx} = x^2(20 - x)(60 - 3x - 2x)$$

$$\frac{dP}{dx} = x^2(20 - x)(60 - 5x)$$

To find critical points, set $\frac{dP}{dx} = 0$:

$$x^2(20 - x)(60 - 5x) = 0$$

This gives $x = 0, x = 20$, or $60 - 5x = 0 \Rightarrow 5x = 60 \Rightarrow x = 12$.

Since the parts must be positive for a meaningful division of 20 in this context, $x = 0$ and $x = 20$ yield a product of 0, which is a minimum.

So, the maximum occurs at $x = 12$.

We can verify it's a maximum using the first derivative test. For x slightly less than 12 (e.g., $x = 11$), $\frac{dP}{dx}$ is positive. For x slightly greater than 12 (e.g., $x = 13$), $\frac{dP}{dx}$ is negative. Thus, $P(x)$ has a local maximum at $x = 12$.

When $x = 12$, the other part is $y = 20 - 12 = 8$.

The two parts are 12 and 8.

Step 4: Final Answer:

The two parts are 12 and 8.

 Quick Tip

A useful shortcut for problems of the form “Divide a number N into two parts x, y to maximize $x^m y^n$ ” is that the parts are in the ratio of their powers, i.e., $x : y = m : n$. Here, $m = 3$ and $n = 2$, so the parts are divided in the ratio $3 : 2$. The parts are $\frac{3}{5} \times 20 = 12$ and $\frac{2}{5} \times 20 = 8$.

4. The acute angle between the diagonals of a parallelogram whose vertices are $A(2, -1)$, $B(0, 2)$, $C(2, 3)$ and $D(4, 0)$ is

- (A) $\cot^{-1} 2$
- (B) $\cot^{-1} \left(\frac{1}{3}\right)$
- (C) $\tan^{-1} 2$
- (D) $\tan^{-1} \left(\frac{2}{3}\right)$

Correct Answer: (C) $\tan^{-1} 2$

Solution:

Step 1: Understanding the Concept:

We are given the vertices of a parallelogram. We need to find the equations or vectors representing its diagonals and then find the angle between them.

The diagonals are the line segments connecting opposite vertices, AC and BD.

Step 2: Key Formula or Approach:

The angle θ between two vectors $\vec{u}$ and $\vec{v}$ is given by $\cos \theta = \frac{|\vec{u} \cdot \vec{v}|}{|\vec{u}||\vec{v}|}$.

Alternatively, if the lines have slopes m_1 and m_2 , the acute angle is given by $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$.

Step 3: Detailed Explanation:

Let's find the vectors representing the diagonals AC and BD.

Vector $\vec{AC} = (x_c - x_a)\hat{i} + (y_c - y_a)\hat{j} = (2 - 2)\hat{i} + (3 - (-1))\hat{j} = 0\hat{i} + 4\hat{j}$.

Vector $\vec{BD} = (x_d - x_b)\hat{i} + (y_d - y_b)\hat{j} = (4 - 0)\hat{i} + (0 - 2)\hat{j} = 4\hat{i} - 2\hat{j}$.

Now, let's find the acute angle θ between them using the dot product:

$$\cos \theta = \frac{|\vec{AC} \cdot \vec{BD}|}{|\vec{AC}||\vec{BD}|}$$

$$\vec{AC} \cdot \vec{BD} = (0)(4) + (4)(-2) = 0 - 8 = -8$$

$$|\vec{AC}| = \sqrt{0^2 + 4^2} = 4$$

$$|\vec{BD}| = \sqrt{4^2 + (-2)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$$

$$\cos \theta = \frac{|-8|}{4 \cdot 2\sqrt{5}} = \frac{8}{8\sqrt{5}} = \frac{1}{\sqrt{5}}$$

Since $\cos \theta = \frac{1}{\sqrt{5}}$, we can find $\tan \theta$ using a right-angled triangle where adjacent side is 1 and hypotenuse is $\sqrt{5}$.

Opposite side = $\sqrt{(\sqrt{5})^2 - 1^2} = \sqrt{5 - 1} = \sqrt{4} = 2$.

Therefore, $\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{2}{1} = 2$.

This implies $\theta = \tan^{-1} 2$.

Step 4: Final Answer:

The acute angle between the diagonals is $\tan^{-1} 2$.

💡 Quick Tip

When one line is vertical (undefined slope), as is the case for diagonal AC here, it's often easier and less error-prone to use the vector dot product method rather than the slope formula.

5. The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is

- (A) $\frac{3\sqrt{2}}{8}$
- (B) $\frac{2\sqrt{3}}{8}$
- (C) $\frac{3\sqrt{2}}{5}$
- (D) $\frac{\sqrt{3}}{4}$

Correct Answer: (A) $\frac{3\sqrt{2}}{8}$

Solution:

Step 1: Understanding the Concept:

The shortest distance between a line and a curve occurs along the common normal.

Alternatively, we can express the distance from a generic point on the curve to the line as a function of a single parameter and then minimize this function.

Step 2: Key Formula or Approach:

Let a point on the curve $x = y^2$ be parameterized as $P(t^2, t)$.

The perpendicular distance from a point (x_1, y_1) to the line $ax + by + c = 0$ is $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

We will express this distance d as a function of t and minimize it using calculus.

Step 3: Detailed Explanation:

The equation of the line is $x - y + 1 = 0$.

Let $P(t^2, t)$ be an arbitrary point on the parabola $x = y^2$.

The distance D from point P to the line is:

$$D(t) = \frac{|(1)(t^2) + (-1)(t) + 1|}{\sqrt{1^2 + (-1)^2}} = \frac{|t^2 - t + 1|}{\sqrt{2}}$$

To minimize the distance, we need to minimize the function inside the absolute value, let's call it $f(t) = t^2 - t + 1$.

Since $t^2 - t + 1 = \left(t - \frac{1}{2}\right)^2 + \frac{3}{4}$, it is always positive for all real t . Thus we can drop the absolute value for optimization.

Differentiating $f(t)$ with respect to t :

$$f'(t) = 2t - 1$$

Setting $f'(t) = 0$ for critical points:

$$2t - 1 = 0 \Rightarrow t = \frac{1}{2}$$

To confirm it's a minimum, find the second derivative: $f''(t) = 2 > 0$, so it is indeed a minimum.

Substitute $t = \frac{1}{2}$ back into the distance formula:

$$D_{min} = \frac{\left|\left(\frac{1}{2}\right)^2 - \frac{1}{2} + 1\right|}{\sqrt{2}}$$

$$D_{min} = \frac{\left|\frac{1}{4} - \frac{1}{2} + 1\right|}{\sqrt{2}} = \frac{\left|\frac{1-2+4}{4}\right|}{\sqrt{2}} = \frac{\frac{3}{4}}{\sqrt{2}}$$

Rationalizing the denominator:

$$D_{min} = \frac{3}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4 \times 2} = \frac{3\sqrt{2}}{8}$$

Step 4: Final Answer:

The shortest distance is $\frac{3\sqrt{2}}{8}$.

💡 Quick Tip

Another approach is to find a point on the curve where the tangent is parallel to the given line. The slope of the given line is 1. Differentiating $x = y^2$ with respect to y , we get $dx/dy = 2y$, so slope $dy/dx = 1/(2y)$. Setting $1/(2y) = 1$ gives $y = 1/2$, and then $x = 1/4$. The distance from $(1/4, 1/2)$ to $x - y + 1 = 0$ will yield the same result.

6. The equation of the circle passing through the point $(1, 1)$ and having two diameters along the pair of lines $x^2 - y^2 - 2x + 4y - 3 = 0$ is

- (A) $(x + 2)^2 + (y - 2)^2 = 4$
 (B) $(x - 3)^2 + (y - 1)^2 = 4$
 (C) $(x - 1)^2 + (y - 2)^2 = 1$
 (D) $(x + 1)^2 + (y + 2)^2 = 1$

Correct Answer: (C) $(x - 1)^2 + (y - 2)^2 = 1$

Solution:

Step 1: Understanding the Concept:

The diameters of a circle always intersect at its center.

The given equation represents a pair of straight lines. Since these lines are diameters, their point of intersection will be the center of the circle.

Once the center is found, the radius can be calculated as the distance from the center to the given point $(1, 1)$.

Step 2: Key Formula or Approach:

Factorize the combined equation of lines to find the individual lines and their intersection.

Alternatively, to find the intersection point (h, k) of a pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we can take partial derivatives: $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$.

Radius $r = \sqrt{(x - h)^2 + (y - k)^2}$.

Equation of circle is $(x - h)^2 + (y - k)^2 = r^2$.

Step 3: Detailed Explanation:

Let's find the center of the circle by finding the intersection of the pair of lines:

$$f(x, y) = x^2 - y^2 - 2x + 4y - 3 = 0$$

Using partial derivatives to find the intersection point:

$$\frac{\partial f}{\partial x} = 2x - 2 = 0 \Rightarrow x = 1$$

$$\frac{\partial f}{\partial y} = -2y + 4 = 0 \Rightarrow y = 2$$

So, the center of the circle is $(h, k) = (1, 2)$.

We can also verify this by completing squares to factorize the equation:

$$(x^2 - 2x) - (y^2 - 4y) - 3 = 0$$

$$(x^2 - 2x + 1) - (y^2 - 4y + 4) - 3 - 1 + 4 = 0$$

$$(x - 1)^2 - (y - 2)^2 = 0$$

$$((x - 1) - (y - 2))((x - 1) + (y - 2)) = 0$$

$$(x - y + 1)(x + y - 3) = 0$$

The lines are $x - y + 1 = 0$ and $x + y - 3 = 0$. Solving these gives $x = 1, y = 2$.

The circle passes through the point $(1, 1)$.

Radius $r = \text{distance from center to point}$

$$r = \sqrt{(1 - 1)^2 + (1 - 2)^2} = \sqrt{0^2 + (-1)^2} = \sqrt{1} = 1.$$

The equation of the circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 1)^2 + (y - 2)^2 = 1^2$$

$$(x - 1)^2 + (y - 2)^2 = 1$$

Step 4: Final Answer:

The equation of the circle is $(x - 1)^2 + (y - 2)^2 = 1$.

 Quick Tip

To quickly find the point of intersection of a pair of straight lines given by a second-degree general equation, set the partial derivatives with respect to x and y to zero and solve the resulting linear equations.

7. $\int \sin^5 x \, dx =$

- (A) $\cos x + \frac{2}{3} \cos^2 x - \frac{\cos^5 x}{5} + c$, where c is the constant of integration
 (B) $\cos x + \frac{2}{3} \cos^2 x + \frac{\cos^5 x}{5} + c$, where c is the constant of integration
 (C) $-\left(\cos x - \frac{2}{3} \cos^3 x + \frac{\cos^5 x}{5}\right) + c$, where c is the constant of integration
 (D) $\cos x - \frac{2}{3} \cos^2 x + \frac{\cos^5 x}{5} + c$, where c is the constant of integration

Correct Answer: (C) $-\left(\cos x - \frac{2}{3} \cos^3 x + \frac{\cos^5 x}{5}\right) + c$, where c is the constant of integration

Solution:

Step 1: Understanding the Concept:

To integrate an odd power of sine, we split off one $\sin x$ term and convert the remaining even power of sine to cosines using the identity $\sin^2 x = 1 - \cos^2 x$.

Then, we use the substitution method by letting $u = \cos x$.

Step 2: Key Formula or Approach:

$$\int \sin^{2k+1} x \, dx = \int (\sin^2 x)^k \sin x \, dx = \int (1 - \cos^2 x)^k \sin x \, dx$$

Substitute $u = \cos x$, then $du = -\sin x \, dx$.

Step 3: Detailed Explanation:

Let $I = \int \sin^5 x \, dx$.

Rewrite the integral as:

$$I = \int \sin^4 x \cdot \sin x \, dx$$

$$I = \int (\sin^2 x)^2 \cdot \sin x dx$$

Using the trigonometric identity $\sin^2 x = 1 - \cos^2 x$:

$$I = \int (1 - \cos^2 x)^2 \cdot \sin x dx$$

Let $u = \cos x$. Then $\frac{du}{dx} = -\sin x$, which implies $\sin x dx = -du$.
Substituting these into the integral:

$$I = \int (1 - u^2)^2 (-du)$$

$$I = - \int (1 - 2u^2 + u^4) du$$

Now, integrate term by term:

$$I = - \left(u - 2\frac{u^3}{3} + \frac{u^5}{5} \right) + c$$

$$I = -u + \frac{2}{3}u^3 - \frac{1}{5}u^5 + c$$

Factor out a negative sign to match the form of the options:

$$I = - \left(u - \frac{2}{3}u^3 + \frac{1}{5}u^5 \right) + c$$

Substitute back $u = \cos x$:

$$I = - \left(\cos x - \frac{2}{3} \cos^3 x + \frac{\cos^5 x}{5} \right) + c$$

Step 4: Final Answer:

The evaluated integral is $- \left(\cos x - \frac{2}{3} \cos^3 x + \frac{\cos^5 x}{5} \right) + c$.

💡 Quick Tip

For integrals of the form $\int \sin^m x \cos^n x dx$, if m is an odd integer, substitute $u = \cos x$. If n is an odd integer, substitute $u = \sin x$. If both are even, use double angle identities.

8. If the angle between the planes $x - 2y + 3z - 5 = 0$ and $x + \alpha y + 2z + 7 = 0$ is $\cos^{-1}\left(\frac{1}{14}\right)$, then the difference between the values of α is

- (A) $\frac{12}{11}$
- (B) $\frac{62}{55}$
- (C) $\frac{31}{11}$
- (D) $\frac{8}{5}$

Correct Answer: (B) $\frac{62}{55}$

Solution:

Step 1: Understanding the Concept:

The angle between two planes is the angle between their normal vectors.

Step 2: Key Formula or Approach:

Let the normal vectors of the planes be $\vec{n}_1$ and $\vec{n}_2$.

The angle θ between the planes is given by:

$$\cos \theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}$$

Step 3: Detailed Explanation:

The normal vectors of the given planes are:

$$\vec{n}_1 = \langle 1, -2, 3 \rangle$$

$$\vec{n}_2 = \langle 1, \alpha, 2 \rangle$$

We are given that $\theta = \cos^{-1}\left(\frac{1}{14}\right)$, so $\cos \theta = \frac{1}{14}$.

Using the formula:

$$\frac{1}{14} = \frac{|(1)(1) + (-2)(\alpha) + (3)(2)|}{\sqrt{1^2 + (-2)^2 + 3^2}\sqrt{1^2 + \alpha^2 + 2^2}}$$

$$\frac{1}{14} = \frac{|1 - 2\alpha + 6|}{\sqrt{1 + 4 + 9}\sqrt{1 + \alpha^2 + 4}}$$

$$\frac{1}{14} = \frac{|7 - 2\alpha|}{\sqrt{14}\sqrt{\alpha^2 + 5}}$$

Squaring both sides to remove the absolute value and square roots:

$$\left(\frac{1}{14}\right)^2 = \frac{(7 - 2\alpha)^2}{14(\alpha^2 + 5)}$$

$$\frac{1}{196} = \frac{49 - 28\alpha + 4\alpha^2}{14(\alpha^2 + 5)}$$

Cross-multiplying:

$$14(\alpha^2 + 5) = 196(4\alpha^2 - 28\alpha + 49)$$

Divide both sides by 14:

$$\alpha^2 + 5 = 14(4\alpha^2 - 28\alpha + 49)$$

$$\alpha^2 + 5 = 56\alpha^2 - 392\alpha + 686$$

Rearranging into a standard quadratic equation:

$$55\alpha^2 - 392\alpha + 681 = 0$$

This is a quadratic equation in α of the form $a\alpha^2 + b\alpha + c = 0$, where $a = 55, b = -392, c = 681$. Let the roots be α_1 and α_2 . The difference between the roots is $|\alpha_1 - \alpha_2| = \frac{\sqrt{b^2 - 4ac}}{|a|}$.

First, calculate the discriminant $D = b^2 - 4ac$:

$$D = (-392)^2 - 4(55)(681)$$

$$D = 153664 - 220(681)$$

$$D = 153664 - 149820 = 3844$$

Now, calculate the difference between the values:

$$|\alpha_1 - \alpha_2| = \frac{\sqrt{3844}}{55}$$

We know that $60^2 = 3600$ and $62^2 = (60 + 2)^2 = 3600 + 240 + 4 = 3844$. So, $\sqrt{3844} = 62$.

$$|\alpha_1 - \alpha_2| = \frac{62}{55}$$

Step 4: Final Answer:

The difference between the values of α is $\frac{62}{55}$.

 Quick Tip

To find the difference between the roots of a quadratic equation $ax^2 + bx + c = 0$ directly without finding the individual roots, use the formula $|\alpha - \beta| = \frac{\sqrt{b^2 - 4ac}}{|a|}$. This saves significant calculation time.

9. If the shortest distance between the lines $\frac{x-k}{2} = \frac{y-4}{3} = \frac{z-3}{4}$ and $\frac{x-2}{4} = \frac{y-4}{6} = \frac{z-7}{8}$ is $\frac{13}{\sqrt{29}}$, then $k =$

- (A) 1
- (B) -1
- (C) 2
- (D) -2

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

First, observe the direction ratios of the two lines.

Line 1 has direction ratios (2, 3, 4).

Line 2 has direction ratios (4, 6, 8), which can be simplified to (2, 3, 4) by dividing by 2.

Since their direction ratios are proportional, the two lines are parallel.

We need to use the formula for the shortest distance between two parallel lines.

Step 2: Key Formula or Approach:

The distance d between two parallel lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}$ is given by:

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

Step 3: Detailed Explanation:

Let's extract the necessary vectors from the line equations.

Line 1 passes through $\vec{a}_1 = k\hat{i} + 4\hat{j} + 3\hat{k}$ and is parallel to $\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

Line 2 passes through $\vec{a}_2 = 2\hat{i} + 4\hat{j} + 7\hat{k}$. It is also parallel to $\vec{b}$ since (4, 6, 8) is parallel to (2, 3, 4).

Now, calculate the vector difference $\vec{a}_2 - \vec{a}_1$:

$$\vec{a}_2 - \vec{a}_1 = (2 - k)\hat{i} + (4 - 4)\hat{j} + (7 - 3)\hat{k} = (2 - k)\hat{i} + 0\hat{j} + 4\hat{k}$$

Next, find the cross product $(\vec{a}_2 - \vec{a}_1) \times \vec{b}$:

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 - k & 0 & 4 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= \hat{i}(0 \cdot 4 - 4 \cdot 3) - \hat{j}((2 - k) \cdot 4 - 4 \cdot 2) + \hat{k}((2 - k) \cdot 3 - 0 \cdot 2)$$

$$= \hat{i}(-12) - \hat{j}(8 - 4k - 8) + \hat{k}(6 - 3k)$$

$$= -12\hat{i} + 4k\hat{j} + (6 - 3k)\hat{k}$$

Now find the magnitude of this cross product:

$$|(\vec{a}_2 - \vec{a}_1) \times \vec{b}| = \sqrt{(-12)^2 + (4k)^2 + (6 - 3k)^2}$$

$$= \sqrt{144 + 16k^2 + (36 - 36k + 9k^2)}$$

$$= \sqrt{25k^2 - 36k + 180}$$

Find the magnitude of the direction vector $\vec{b}$:

$$|\vec{b}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$$

We are given that the shortest distance $d = \frac{13}{\sqrt{29}}$.

$$\frac{\sqrt{25k^2 - 36k + 180}}{\sqrt{29}} = \frac{13}{\sqrt{29}}$$

Equating the numerators and squaring:

$$25k^2 - 36k + 180 = 13^2 = 169$$

$$25k^2 - 36k + 11 = 0$$

Factorize the quadratic equation:

$$25k^2 - 25k - 11k + 11 = 0$$

$$25k(k - 1) - 11(k - 1) = 0$$

$$(25k - 11)(k - 1) = 0$$

This gives $k = 1$ or $k = \frac{11}{25}$.

Comparing with the given options, $k = 1$ is the correct choice.

Step 4: Final Answer:

The value of k is 1.

💡 Quick Tip

Always check if given lines are parallel by comparing their direction ratios before blindly applying the general skew lines shortest distance formula, which involves a determinant and is more complex.

10. The acute angle between the lines $x = -2 + 2t, y = 3 - 4t, z = -4 + t$ and $x = -2 - t, y = 3 + 2t, z = -4 + 3t$ is

- (A) $\cos^{-1} \left(\frac{1}{\sqrt{6}} \right)$
- (B) $\cos^{-1} \left(\frac{1}{\sqrt{5}} \right)$
- (C) $\sin^{-1} \left(\frac{2}{\sqrt{5}} \right)$
- (D) $\cos^{-1} \left(\frac{2}{\sqrt{6}} \right)$

Correct Answer: (A) $\cos^{-1} \left(\frac{1}{\sqrt{6}} \right)$

Solution:

Step 1: Understanding the Concept:

The equations of the lines are given in parametric form.

The coefficients of the parameter ' t ' represent the components of the direction vectors of the lines.

The angle between two lines is the angle between their direction vectors.

Step 2: Key Formula or Approach:

If a line is given by $\vec{r} = \vec{a} + t\vec{v}$, then $\vec{v}$ is its direction vector.

The angle θ between two vectors $\vec{v}_1$ and $\vec{v}_2$ is given by:

$$\cos \theta = \frac{|\vec{v}_1 \cdot \vec{v}_2|}{|\vec{v}_1||\vec{v}_2|}$$

Step 3: Detailed Explanation:

From the first line equations: $x = -2 + 2t, y = 3 - 4t, z = -4 + t$, the direction vector is:

$$\vec{v}_1 = 2\hat{i} - 4\hat{j} + 1\hat{k}$$

From the second line equations: $x = -2 - t, y = 3 + 2t, z = -4 + 3t$, the direction vector is:

$$\vec{v}_2 = -1\hat{i} + 2\hat{j} + 3\hat{k}$$

Now calculate the dot product of the two direction vectors:

$$\vec{v}_1 \cdot \vec{v}_2 = (2)(-1) + (-4)(2) + (1)(3) = -2 - 8 + 3 = -7$$

Calculate the magnitudes of the direction vectors:

$$|\vec{v}_1| = \sqrt{2^2 + (-4)^2 + 1^2} = \sqrt{4 + 16 + 1} = \sqrt{21}$$

$$|\vec{v}_2| = \sqrt{(-1)^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$$

Use the cosine formula to find the acute angle:

$$\cos \theta = \frac{|-7|}{\sqrt{21} \cdot \sqrt{14}} = \frac{7}{\sqrt{3 \cdot 7} \cdot \sqrt{2 \cdot 7}}$$

$$\cos \theta = \frac{7}{\sqrt{3 \cdot 2 \cdot 7 \cdot 7}} = \frac{7}{7\sqrt{6}} = \frac{1}{\sqrt{6}}$$

Therefore, the acute angle θ is:

$$\theta = \cos^{-1} \left(\frac{1}{\sqrt{6}} \right)$$

Step 4: Final Answer:

The acute angle is $\cos^{-1} \left(\frac{1}{\sqrt{6}} \right)$.

💡 Quick Tip

When asked for the acute angle between lines or planes, always take the absolute value of the dot product in the numerator.

11. If the curves $y^2 = 6x$ and $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is

- (A) 4
- (B) $\frac{7}{2}$

- (C) 6
(D) $\frac{9}{2}$

Correct Answer: (D) $\frac{9}{2}$

Solution:

Step 1: Understanding the Concept:

Two curves intersect at right angles (orthogonally) if their tangents at the point of intersection are perpendicular.

This means the product of their slopes at the point of intersection must be -1 .

Step 2: Key Formula or Approach:

Let (x_1, y_1) be the point of intersection.

Find the slope $m_1 = \left. \frac{dy}{dx} \right|_{(x_1, y_1)}$ for the first curve.

Find the slope $m_2 = \left. \frac{dy}{dx} \right|_{(x_1, y_1)}$ for the second curve.

Apply the condition $m_1 \cdot m_2 = -1$.

Step 3: Detailed Explanation:

Let the curves intersect at (x_1, y_1) .

Curve 1: $y^2 = 6x$

Differentiating with respect to x :

$$2y \frac{dy}{dx} = 6 \Rightarrow \frac{dy}{dx} = \frac{3}{y}$$

Slope at (x_1, y_1) is $m_1 = \frac{3}{y_1}$.

Curve 2: $9x^2 + by^2 = 16$

Differentiating with respect to x :

$$18x + 2by \frac{dy}{dx} = 0 \Rightarrow 2by \frac{dy}{dx} = -18x \Rightarrow \frac{dy}{dx} = -\frac{9x}{by}$$

Slope at (x_1, y_1) is $m_2 = -\frac{9x_1}{by_1}$.

Since they intersect orthogonally, $m_1 \cdot m_2 = -1$:

$$\left(\frac{3}{y_1} \right) \left(-\frac{9x_1}{by_1} \right) = -1$$

$$-\frac{27x_1}{by_1^2} = -1$$

$$by_1^2 = 27x_1$$

We also know that the intersection point (x_1, y_1) lies on both curves.

From Curve 1, we have $y_1^2 = 6x_1$.

Substitute $y_1^2 = 6x_1$ into the condition $by_1^2 = 27x_1$:

$$b(6x_1) = 27x_1$$

Assuming the point of intersection is not the origin (if $x_1 = 0$, then $y_1 = 0$, but $(0, 0)$ does not satisfy the second curve $9(0)^2 + b(0)^2 = 16 \Rightarrow 0 = 16$, which is false), we can divide both sides by $x_1 \neq 0$:

$$6b = 27$$

$$b = \frac{27}{6} = \frac{9}{2}$$

Step 4: Final Answer:

The value of b is $\frac{9}{2}$.

 Quick Tip

For orthogonal intersection problems, always remember to substitute the relation from the simpler curve's equation into the $m_1 m_2 = -1$ condition to eliminate coordinates.

12. The value of $\sqrt{3} \cot 20^\circ - 4 \cos 20^\circ$ is equal to

- (A) 1
- (B) -1
- (C) 0
- (D) $\frac{1}{2}$

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

We need to simplify a trigonometric expression involving standard but non-common angles. The general approach is to convert everything to sines and cosines, and use compound angle and double angle identities to simplify.

Step 2: Key Formula or Approach:

Use $\cot \theta = \frac{\cos \theta}{\sin \theta}$.

Use $\sin 2\theta = 2 \sin \theta \cos \theta$.

Use $\sqrt{3} = 2 \sin 60^\circ$ or $2 \cos 30^\circ$.

Use sum-to-product or product-to-sum formulas.

Step 3: Detailed Explanation:

Let the given expression be $E = \sqrt{3} \cot 20^\circ - 4 \cos 20^\circ$.

Convert $\cot$ to $\cos / \sin$:

$$E = \sqrt{3} \frac{\cos 20^\circ}{\sin 20^\circ} - 4 \cos 20^\circ$$

Taking common denominator:

$$E = \frac{\sqrt{3} \cos 20^\circ - 4 \sin 20^\circ \cos 20^\circ}{\sin 20^\circ}$$

Apply the double angle identity $\sin(2\theta) = 2 \sin \theta \cos \theta$ to the second term in numerator:

$$E = \frac{\sqrt{3} \cos 20^\circ - 2(2 \sin 20^\circ \cos 20^\circ)}{\sin 20^\circ}$$

$$E = \frac{\sqrt{3} \cos 20^\circ - 2 \sin 40^\circ}{\sin 20^\circ}$$

Multiply and divide the numerator by 2 to introduce a known trigonometric value:

$$E = \frac{2 \left(\frac{\sqrt{3}}{2} \cos 20^\circ - \sin 40^\circ \right)}{\sin 20^\circ}$$

Substitute $\frac{\sqrt{3}}{2} = \sin 60^\circ$:

$$E = \frac{2 (\sin 60^\circ \cos 20^\circ) - 2 \sin 40^\circ}{\sin 20^\circ}$$

Use the product-to-sum formula $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$ for the first term:

$$2 \sin 60^\circ \cos 20^\circ = \sin(60^\circ + 20^\circ) + \sin(60^\circ - 20^\circ) = \sin 80^\circ + \sin 40^\circ$$

Substitute this back into the expression:

$$E = \frac{(\sin 80^\circ + \sin 40^\circ) - 2 \sin 40^\circ}{\sin 20^\circ}$$

$$E = \frac{\sin 80^\circ - \sin 40^\circ}{\sin 20^\circ}$$

Now, use the sum-to-product formula $\sin C - \sin D = 2 \cos \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$:

$$\sin 80^\circ - \sin 40^\circ = 2 \cos \left(\frac{80^\circ + 40^\circ}{2} \right) \sin \left(\frac{80^\circ - 40^\circ}{2} \right)$$

$$= 2 \cos(60^\circ) \sin(20^\circ)$$

Since $\cos 60^\circ = \frac{1}{2}$:

$$\sin 80^\circ - \sin 40^\circ = 2 \left(\frac{1}{2} \right) \sin 20^\circ = \sin 20^\circ$$

Finally, substitute this back into E :

$$E = \frac{\sin 20^\circ}{\sin 20^\circ} = 1$$

Step 4: Final Answer:

The value of the expression is 1.

 Quick Tip

Recognizing patterns like $\sqrt{3}$ or 1 as $2 \sin 60^\circ$ or $2 \cos 60^\circ$ after factoring out a 2 is a very common trick in trigonometric simplification problems.

13. Let $\bar{a}$ and $\bar{b}$ be two vectors such that $|\bar{a}| = 1$, $|\bar{b}| = 4$, $\bar{a} \cdot \bar{b} = 2$. If $\bar{c} = (2\bar{a} \times \bar{b}) - 3\bar{b}$, then the angle between $\bar{b}$ and $\bar{c}$ is

- (A) $\frac{\pi}{3}$
- (B) $\frac{\pi}{6}$
- (C) $\frac{3\pi}{4}$
- (D) $\frac{5\pi}{6}$

Correct Answer: (D) $\frac{5\pi}{6}$

Solution:

Step 1: Understanding the Concept:

We need to find the angle θ between vectors $\bar{b}$ and $\bar{c}$.

The cosine of the angle is given by $\cos \theta = \frac{\bar{b} \cdot \bar{c}}{|\bar{b}| |\bar{c}|}$.

We need to compute the dot product $\bar{b} \cdot \bar{c}$ and the magnitude $|\bar{c}|$.

Step 2: Key Formula or Approach:

Dot product definition: $\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|}$.

Properties of cross product: $\vec{v} \cdot (\vec{u} \times \vec{v}) = 0$ because $\vec{u} \times \vec{v}$ is perpendicular to $\vec{v}$.

Lagrange's Identity: $|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$.

Step 3: Detailed Explanation:

Given: $|\vec{a}| = 1, |\vec{b}| = 4, \vec{a} \cdot \vec{b} = 2$, and $\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$.

First, let's find the dot product $\vec{b} \cdot \vec{c}$:

$$\vec{b} \cdot \vec{c} = \vec{b} \cdot (2(\vec{a} \times \vec{b}) - 3\vec{b})$$

$$\vec{b} \cdot \vec{c} = 2(\vec{b} \cdot (\vec{a} \times \vec{b})) - 3(\vec{b} \cdot \vec{b})$$

Since $\vec{a} \times \vec{b}$ is orthogonal to both $\vec{a}$ and $\vec{b}$, the dot product $\vec{b} \cdot (\vec{a} \times \vec{b}) = 0$.
Also, $\vec{b} \cdot \vec{b} = |\vec{b}|^2$.

$$\vec{b} \cdot \vec{c} = 2(0) - 3|\vec{b}|^2 = -3(4^2) = -3(16) = -48$$

Next, we need the magnitude of $\vec{c}$. Let's calculate $|\vec{c}|^2$:

$$|\vec{c}|^2 = \vec{c} \cdot \vec{c} = (2(\vec{a} \times \vec{b}) - 3\vec{b}) \cdot (2(\vec{a} \times \vec{b}) - 3\vec{b})$$

Using the property that $(\vec{u} \times \vec{v}) \cdot \vec{v} = 0$:

$$|\vec{c}|^2 = 4|\vec{a} \times \vec{b}|^2 - 12(\vec{a} \times \vec{b}) \cdot \vec{b} + 9|\vec{b}|^2$$

$$|\vec{c}|^2 = 4|\vec{a} \times \vec{b}|^2 - 0 + 9(16) = 4|\vec{a} \times \vec{b}|^2 + 144$$

We need to find $|\vec{a} \times \vec{b}|^2$. Using Lagrange's identity:

$$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$$

$$|\vec{a} \times \vec{b}|^2 = (1^2)(4^2) - (2)^2 = 16 - 4 = 12$$

Substitute this back into the equation for $|\vec{c}|^2$:

$$|\vec{c}|^2 = 4(12) + 144 = 48 + 144 = 192$$

$$|\vec{c}| = \sqrt{192} = \sqrt{64 \times 3} = 8\sqrt{3}$$

Now, calculate the angle θ :

$$\cos \theta = \frac{\vec{b} \cdot \vec{c}}{|\vec{b}||\vec{c}|} = \frac{-48}{4 \cdot 8\sqrt{3}} = \frac{-48}{32\sqrt{3}} = \frac{-3}{2\sqrt{3}}$$

Rationalizing the denominator:

$$\cos \theta = \frac{-3\sqrt{3}}{2 \cdot 3} = -\frac{\sqrt{3}}{2}$$

The principal value for which $\cos \theta = -\frac{\sqrt{3}}{2}$ is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

Step 4: Final Answer:

The angle is $\frac{5\pi}{6}$.

💡 Quick Tip

Remember the orthogonality property: the dot product of a vector with a cross product involving itself is always zero, i.e., $\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$. This greatly simplifies vector expansion expressions.

14. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are unit vectors such that $\vec{a} \cdot \vec{b} = \frac{1}{2}$, $\vec{c} \cdot \vec{d} = \frac{1}{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c} \times \vec{d}$ is $\frac{\pi}{6}$, then the value of $|\vec{a}\vec{b}\vec{d}\vec{c} - \vec{a}\vec{b}\vec{c}\vec{d}| =$

- (A) $\frac{3}{2}$
- (B) $\frac{3}{4}$
- (C) $\frac{3}{8}$
- (D) 2

Correct Answer: (C) $\frac{3}{8}$

Solution:

Step 1: Understanding the Concept:

We are dealing with a vector triple product identity in a disguised form.

The expression $[\vec{a}\vec{b}\vec{d}]\vec{c} - [\vec{a}\vec{b}\vec{c}]\vec{d}$ represents the expansion of a cross product of two cross products.

Step 2: Key Formula or Approach:

Scalar triple product notation: $[\vec{u}\vec{v}\vec{w}] = (\vec{u} \times \vec{v}) \cdot \vec{w}$.

Vector quadruple product identity: $(\vec{u} \times \vec{v}) \times (\vec{w} \times \vec{x}) = [\vec{u}\vec{v}\vec{x}]\vec{w} - [\vec{u}\vec{v}\vec{w}]\vec{x}$.

Magnitude of cross product: $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin \theta$.

Step 3: Detailed Explanation:

Let's analyze the given expression inside the magnitude:

$$E = [\vec{a}\vec{b}\vec{d}]\vec{c} - [\vec{a}\vec{b}\vec{c}]\vec{d}$$

We know the identity for $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$.

Let $\vec{u} = \vec{a} \times \vec{b}$. Then:

$$\vec{u} \times (\vec{c} \times \vec{d}) = (\vec{u} \cdot \vec{d})\vec{c} - (\vec{u} \cdot \vec{c})\vec{d}$$

$$\vec{u} \times (\vec{c} \times \vec{d}) = ((\vec{a} \times \vec{b}) \cdot \vec{d})\vec{c} - ((\vec{a} \times \vec{b}) \cdot \vec{c})\vec{d}$$

$$\vec{u} \times (\vec{c} \times \vec{d}) = [\vec{a}\vec{b}\vec{d}]\vec{c} - [\vec{a}\vec{b}\vec{c}]\vec{d}$$

So, the given expression is exactly $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d})$.

We need to find its magnitude: $|(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d})|$.

Let $\vec{v}_1 = \bar{a} \times \bar{b}$ and $\vec{v}_2 = \bar{c} \times \bar{d}$.

The magnitude is $|\vec{v}_1 \times \vec{v}_2| = |\vec{v}_1||\vec{v}_2| \sin(\text{angle between } \vec{v}_1 \text{ and } \vec{v}_2)$.

We are given the angle between $\bar{a} \times \bar{b}$ and $\bar{c} \times \bar{d}$ is $\frac{\pi}{6}$.

So, magnitude = $|\bar{a} \times \bar{b}||\bar{c} \times \bar{d}| \sin\left(\frac{\pi}{6}\right)$.

Now, let's find $|\bar{a} \times \bar{b}|$ and $|\bar{c} \times \bar{d}|$.

Given $\bar{a}, \bar{b}$ are unit vectors, and $\bar{a} \cdot \bar{b} = \frac{1}{2}$.

$|\bar{a}||\bar{b}| \cos \theta_{ab} = \frac{1}{2} \Rightarrow (1)(1) \cos \theta_{ab} = \frac{1}{2} \Rightarrow \cos \theta_{ab} = \frac{1}{2}$.

Thus, $\sin \theta_{ab} = \sqrt{1 - \cos^2 \theta_{ab}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$.

So, $|\bar{a} \times \bar{b}| = |\bar{a}||\bar{b}| \sin \theta_{ab} = (1)(1) \left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$.

Similarly, for unit vectors $\bar{c}, \bar{d}$ with $\bar{c} \cdot \bar{d} = \frac{1}{2}$, we have $|\bar{c} \times \bar{d}| = \frac{\sqrt{3}}{2}$.

Now, substitute these back into the magnitude expression:

Magnitude = $\left(\frac{\sqrt{3}}{2}\right) \left(\frac{\sqrt{3}}{2}\right) \sin\left(\frac{\pi}{6}\right)$

Magnitude = $\left(\frac{3}{4}\right) \left(\frac{1}{2}\right) = \frac{3}{8}$.

Step 4: Final Answer:

The value is $\frac{3}{8}$.

💡 Quick Tip

Recognizing standard vector identities is crucial. The form $[\vec{u}\vec{v}\vec{y}]\vec{x} - [\vec{u}\vec{v}\vec{x}]\vec{y}$ should immediately trigger the thought of $(\vec{u} \times \vec{v}) \times (\vec{x} \times \vec{y})$.

15. If $\bar{a} = 4\hat{i} + 3\hat{j} + \hat{k}$, $\bar{b} = \hat{i} - 2\hat{j} + 2\hat{k}$ then $\bar{a} \times (\bar{a} \times (\bar{a} \times (\bar{a} \times \bar{b}))) =$

(A) $676\bar{a}$

(B) $676\bar{b}$

(C) $625\bar{a}$

(D) $625\bar{b}$

Correct Answer: (B) $676\bar{b}$

Solution:

Step 1: Understanding the Concept:

We have a repeated cross product. We should use the vector triple product formula to simplify it step by step from the inside out.

The vector triple product formula is $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$.

Step 2: Key Formula or Approach:

Apply the formula to the innermost triple product: $\bar{a} \times (\bar{a} \times \vec{v})$, where $\vec{v} = \bar{a} \times \bar{b}$.

Notice that a dot product might simplify to zero, making subsequent steps very easy.

Step 3: Detailed Explanation:

First, let's calculate the dot product $\vec{a} \cdot \vec{b}$ and the magnitude squared $|\vec{a}|^2$.

$\vec{a} = \langle 4, 3, 1 \rangle$ and $\vec{b} = \langle 1, -2, 2 \rangle$.

$$\vec{a} \cdot \vec{b} = (4)(1) + (3)(-2) + (1)(2) = 4 - 6 + 2 = 0.$$

Since $\vec{a} \cdot \vec{b} = 0$, the vectors $\vec{a}$ and $\vec{b}$ are perpendicular.

$$|\vec{a}|^2 = \vec{a} \cdot \vec{a} = 4^2 + 3^2 + 1^2 = 16 + 9 + 1 = 26.$$

Let's evaluate the expression from the inside:

$$\text{Let } \vec{v}_1 = \vec{a} \times \vec{b}.$$

$$\text{Next term is } \vec{v}_2 = \vec{a} \times \vec{v}_1 = \vec{a} \times (\vec{a} \times \vec{b}).$$

Using the vector triple product formula:

$$\vec{a} \times (\vec{a} \times \vec{b}) = (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}.$$

Since $\vec{a} \cdot \vec{b} = 0$, this simplifies to:

$$\vec{v}_2 = 0\vec{a} - |\vec{a}|^2\vec{b} = -26\vec{b}.$$

$$\text{Now, the next term is } \vec{v}_3 = \vec{a} \times \vec{v}_2 = \vec{a} \times (-26\vec{b}) = -26(\vec{a} \times \vec{b}).$$

Finally, evaluate the entire expression:

$$\vec{a} \times \vec{v}_3 = \vec{a} \times (-26(\vec{a} \times \vec{b})) = -26[\vec{a} \times (\vec{a} \times \vec{b})].$$

$$\text{We already found that } \vec{a} \times (\vec{a} \times \vec{b}) = -26\vec{b}.$$

So, substituting this back:

$$\text{Result} = -26(-26\vec{b}) = (-26)^2\vec{b} = 676\vec{b}.$$

Step 4: Final Answer:

The result is $676\vec{b}$.

💡 Quick Tip

Always check the dot product of the base vectors in repeated cross product problems. If $\vec{a} \cdot \vec{b} = 0$, then $\vec{a} \times (\vec{a} \times \vec{b})$ simplifies immensely to $-|\vec{a}|^2\vec{b}$. This pattern repeats.

16. If X is a binomial variable with range $\{0, 1, 2, 3, 4\}$ and $P(X = 3) = 3P(X = 4)$ then the parameter ' p ' of the binomial distribution is

- (A) $\frac{1}{4}$
- (B) $\frac{3}{4}$
- (C) $\frac{1}{3}$
- (D) $\frac{2}{5}$

Correct Answer: None of the options (4/7). Assuming a typo in the question where it meant $P(X = 2) = 3P(X = 3)$, the answer would be C.

Solution:**Step 1: Understanding the Concept:**

A binomial random variable $X \sim B(n, p)$ represents the number of successes in n independent trials, each with probability of success p .

The range of X is $\{0, 1, 2, \dots, n\}$. Given the range is $\{0, 1, 2, 3, 4\}$, we identify $n = 4$.

Step 2: Key Formula or Approach:

The probability mass function of a binomial distribution is given by:

$$P(X = r) = \binom{n}{r} p^r q^{n-r}$$

where $q = 1 - p$.

Substitute the given values into the equation $P(X = 3) = 3P(X = 4)$ and solve for p .

Step 3: Detailed Explanation:

Given $n = 4$ and $P(X = 3) = 3P(X = 4)$.

Let's express both probabilities using the binomial formula:

$$P(X = 3) = \binom{4}{3} p^3 q^{4-3} = 4p^3 q$$

$$P(X = 4) = \binom{4}{4} p^4 q^{4-4} = 1 \cdot p^4 \cdot 1 = p^4$$

Substitute these into the given equation:

$$4p^3 q = 3p^4$$

Since p is a probability parameter for a distribution taking values up to 4, we assume $p > 0$. We can divide both sides by p^3 :

$$4q = 3p$$

We know that $q = 1 - p$. Substitute this:

$$4(1 - p) = 3p$$

$$4 - 4p = 3p$$

$$4 = 7p$$

$$p = \frac{4}{7}$$

The value derived mathematically is $\frac{4}{7}$, which is not among the given options (A: $1/4$, B: $3/4$, C: $1/3$, D: $2/5$).

Note: It is highly probable there is a typographical error in the original question. A common

variant is $P(X = 2) = 3P(X = 3)$, which yields $6p^2q^2 = 3(4p^3q) \Rightarrow 6q = 12p \Rightarrow q = 2p \Rightarrow 1 - p = 2p \Rightarrow p = 1/3$ (Option C). However, solving the text exactly as presented yields $4/7$.

Step 4: Final Answer:

Based on the exact text, the parameter p is $\frac{4}{7}$.

 Quick Tip

In exams, if your derived mathematically sound answer is missing from the options, check for common typographical errors in the question (like misread numbers or indices) that might lead to one of the given choices.

17. If a statement q has truth value False and $(p \wedge q) \leftrightarrow r$ has truth value True then which of the following has truth value true?

- (A) $p \wedge q$
- (B) $p \vee r$
- (C) $p \wedge r$
- (D) $(p \wedge r) \rightarrow (p \vee r)$

Correct Answer: (D) $(p \wedge r) \rightarrow (p \vee r)$

Solution:

Step 1: Understanding the Concept:

We are dealing with propositional logic. We need to evaluate the truth values of compound statements based on the given conditions.

The logical connectives are AND ($\wedge$), OR ($\vee$), implies ($\rightarrow$), and iff ($\leftrightarrow$).

Step 2: Key Formula or Approach:

Use truth tables or known properties:

$p \wedge F \equiv F$ (Anything AND False is False).

$F \leftrightarrow r \equiv T$ if and only if r is False (Both sides must have same truth value).

$F \rightarrow \text{Anything} \equiv T$ (A conditional with a false premise is always true).

Step 3: Detailed Explanation:

Given:

Truth value of q is False ($q = F$).

Truth value of $(p \wedge q) \leftrightarrow r$ is True.

First, evaluate the truth value of $(p \wedge q)$:

Since $q = F$, the conjunction $p \wedge F$ will always be False, regardless of the truth value of p .

So, $(p \wedge q) = F$.

Now substitute this into the biconditional statement:

$F \leftrightarrow r$ is True.

A biconditional statement $A \leftrightarrow B$ is true if and only if both A and B have the same truth

value.

Since the left side is F and the whole statement is True, the right side r must also be F .

So, $r = F$.

We don't know the truth value of p , it could be True or False. Let's analyze the options:

(A) $p \wedge q$: We already established this is F .

(B) $p \vee r$: Since $r = F$, this becomes $p \vee F \equiv p$. Its truth value depends on p , so it's not necessarily True.

(C) $p \wedge r$: Since $r = F$, this becomes $p \wedge F \equiv F$.

(D) $(p \wedge r) \rightarrow (p \vee r)$: From our evaluation, $(p \wedge r) = F$.

The statement becomes $F \rightarrow (p \vee r)$.

In logic, an implication $A \rightarrow B$ is always True if the antecedent A is False (vacuous truth).

Therefore, $F \rightarrow (p \vee r)$ is True, regardless of the value of $p \vee r$.

Step 4: Final Answer:

The statement $(p \wedge r) \rightarrow (p \vee r)$ is True.

💡 Quick Tip

Remember the 'vacuous truth' property of implications: If the premise (if-part) is false, the entire conditional statement is considered true, no matter what the conclusion is. This is often the key to solving such problems.

18. The logically equivalent statement of $(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)$ is

- (A) $(\sim p) \wedge q$
- (B) $(\sim p) \vee (\sim q)$
- (C) $(\sim p) \wedge (\sim q)$
- (D) $p \vee q$

Correct Answer: (B) $(\sim p) \vee (\sim q)$

Solution:

Step 1: Understanding the Concept:

We need to simplify a logical expression using Boolean algebra laws or logical equivalences such as Distributive Law, Complement Law, and Identity Law.

Step 2: Key Formula or Approach:

Distributive Law: $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$.

Distributive Law: $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$.

Complement Law: $A \vee \sim A \equiv T$.

Identity Law: $A \wedge T \equiv A$.

Step 3: Detailed Explanation:

Let the given expression be $S = (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)$.

We can group the first two terms together:

$$S = [(\sim p \wedge q) \vee (\sim p \wedge \sim q)] \vee (p \wedge \sim q)$$

Apply the Distributive Law in reverse (factoring out $\sim p$) to the grouped terms:

$$[(\sim p \wedge q) \vee (\sim p \wedge \sim q)] \equiv \sim p \wedge (q \vee \sim q)$$

By the Complement Law, $q \vee \sim q \equiv T$ (True).

So, the grouped term simplifies to $\sim p \wedge T \equiv \sim p$.

Now substitute this back into S :

$$S = \sim p \vee (p \wedge \sim q)$$

Apply the Distributive Law to this new expression:

$$S \equiv (\sim p \vee p) \wedge (\sim p \vee \sim q)$$

Again, using the Complement Law, $\sim p \vee p \equiv T$.

$$S \equiv T \wedge (\sim p \vee \sim q)$$

By Identity Law, this simplifies to:

$$S \equiv \sim p \vee \sim q$$

Step 4: Final Answer:

The logically equivalent statement is $(\sim p) \vee (\sim q)$.

💡 Quick Tip

Always look for common terms to factor out using the distributive law. Expressions like $(A \wedge B) \vee (A \wedge \sim B)$ quickly reduce to A .

19. Two cards are drawn simultaneously from a well shuffled pack of 52 cards. If X is the random variable of getting queens, then the value of $2E(X) + 3E(X^2)$ for the number of queens is

- (A) $\frac{132}{221}$
- (B) $\frac{108}{221}$
- (C) $\frac{176}{221}$
- (D) $\frac{68}{221}$

Correct Answer: (C) $\frac{176}{221}$

Solution:

Step 1: Understanding the Concept:

We are drawing 2 cards without replacement from a deck of 52. The random variable X denotes the number of queens drawn.

X can take values 0, 1, or 2.

We need to find the probability distribution of X to calculate the expected values $E(X)$ and $E(X^2)$.

Step 2: Key Formula or Approach:

The probability $P(X = x)$ is calculated using combinations: $\frac{\text{favorable outcomes}}{\text{total outcomes}}$.

Total number of queens = 4, non-queens = 48.

$$E(X) = \sum x \cdot P(X = x)$$

$$E(X^2) = \sum x^2 \cdot P(X = x)$$

Step 3: Detailed Explanation:

Let's find the probabilities for each value of X .

Total ways to draw 2 cards from 52 is $\binom{52}{2} = \frac{52 \times 51}{2 \times 1} = 26 \times 51 = 1326$.

- $P(X = 0)$: Drawing 0 queens means drawing 2 non-queens from 48.

$$P(X = 0) = \frac{\binom{48}{2}}{\binom{52}{2}} = \frac{\frac{48 \times 47}{2}}{1326} = \frac{24 \times 47}{1326} = \frac{1128}{1326} = \frac{188}{221} \text{ (dividing by 6)}$$

- $P(X = 1)$: Drawing 1 queen from 4 and 1 non-queen from 48.

$$P(X = 1) = \frac{\binom{4}{1} \times \binom{48}{1}}{\binom{52}{2}} = \frac{4 \times 48}{1326} = \frac{192}{1326} = \frac{32}{221} \text{ (dividing by 6)}$$

- $P(X = 2)$: Drawing 2 queens from 4.

$$P(X = 2) = \frac{\binom{4}{2}}{\binom{52}{2}} = \frac{\frac{4 \times 3}{2}}{1326} = \frac{6}{1326} = \frac{1}{221} \text{ (dividing by 6)}$$

Let's check if probabilities sum to 1: $\frac{188+32+1}{221} = \frac{221}{221} = 1$. Correct.

Now calculate $E(X)$ and $E(X^2)$:

$$E(X) = \sum_{x=0}^2 x \cdot P(x) = 0 \left(\frac{188}{221} \right) + 1 \left(\frac{32}{221} \right) + 2 \left(\frac{1}{221} \right) = \frac{32 + 2}{221} = \frac{34}{221}$$

$$E(X^2) = \sum_{x=0}^2 x^2 \cdot P(x) = 0^2 \left(\frac{188}{221} \right) + 1^2 \left(\frac{32}{221} \right) + 2^2 \left(\frac{1}{221} \right) = \frac{32 + 4}{221} = \frac{36}{221}$$

Finally, evaluate the required expression:

$$2E(X) + 3E(X^2) = 2 \left(\frac{34}{221} \right) + 3 \left(\frac{36}{221} \right)$$

$$= \frac{68}{221} + \frac{108}{221} = \frac{176}{221}$$

Step 4: Final Answer:

The value is $\frac{176}{221}$.

💡 Quick Tip

For calculating expected value $E(X)$ when drawing n items without replacement from N items containing K successes, it follows a hypergeometric distribution where $E(X) = n \frac{K}{N}$. Here, $E(X) = 2 \times \frac{4}{52} = \frac{8}{52} = \frac{2}{13} = \frac{34}{221}$. This provides a quick check.

20. A random variable X has the following probability distribution

X :	0	1	2	3	4
P(X) :	k	2k	4k	2k	k

then the value of $P(1 \leq X < 4 \mid X \leq 2) =$

- (A) $\frac{5}{6}$
- (B) $\frac{6}{7}$
- (C) $\frac{7}{8}$
- (D) $\frac{8}{9}$

Correct Answer: (B) $\frac{6}{7}$

Solution:

Step 1: Understanding the Concept:

First, we need to find the value of k by using the property that the sum of all probabilities in a probability distribution equals 1.

Then, we calculate the conditional probability using the formula $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$.

Step 2: Key Formula or Approach:

$$\sum P(X) = 1$$

$$\text{Conditional Probability: } P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Let A be the event $1 \leq X < 4$ and B be the event $X \leq 2$.

Step 3: Detailed Explanation:

Sum of probabilities:

$$P(0) + P(1) + P(2) + P(3) + P(4) = 1$$

$$k + 2k + 4k + 2k + k = 1$$

$$10k = 1 \Rightarrow k = 0.1$$

However, we might not even need to substitute k 's numerical value if it cancels out.

We want to find $P(1 \leq X < 4 \mid X \leq 2)$.

Using the conditional probability formula:

$$P(1 \leq X < 4 \mid X \leq 2) = \frac{P((1 \leq X < 4) \cap (X \leq 2))}{P(X \leq 2)}$$

The condition $1 \leq X < 4$ means X can be 1, 2, or 3.

The condition $X \leq 2$ means X can be 0, 1, or 2.

The intersection of these two sets of outcomes is $\{1, 2, 3\} \cap \{0, 1, 2\} = \{1, 2\}$.

So, the numerator is $P(X = 1 \text{ or } X = 2) = P(X = 1) + P(X = 2)$.

$$\text{Numerator} = 2k + 4k = 6k.$$

The denominator is $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$.

$$\text{Denominator} = k + 2k + 4k = 7k.$$

Now substitute these back into the fraction:

$$P(1 \leq X < 4 \mid X \leq 2) = \frac{6k}{7k}$$

Since $k \neq 0$, we can cancel it out:

$$P(1 \leq X < 4 \mid X \leq 2) = \frac{6}{7}$$

Step 4: Final Answer:

The value is $\frac{6}{7}$.

💡 Quick Tip

In ratio-based probability problems involving an unknown constant k , the constant often cancels out. Calculating the actual value of k is an unnecessary step that consumes time. Focus on formulating the ratio first.

21. The area of the region bounded by $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$ is

- (A) $\frac{1}{2}(\pi - 2)$ sq. units
- (B) $\frac{3}{2}(\pi - 2)$ sq. units
- (C) $\frac{5}{4}(\pi - 2)$ sq. units
- (D) $\frac{3}{3}(\pi - 2)$ sq. units

Correct Answer: (B) $\frac{3}{2}(\pi - 2)$ sq. units

Solution:

Step 1: Understanding the Concept:

We need to find the area between an ellipse and a straight line.

The line connects the x-intercept and y-intercept of the ellipse in the first quadrant.

The bounded area is the difference between the area of the quarter-ellipse and the area of the right-angled triangle formed by the line.

Step 2: Key Formula or Approach:

The ellipse equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a = 3$ and $b = 2$.

The line equation is $\frac{x}{a} + \frac{y}{b} = 1$.

Area of ellipse = πab . Area of its quadrant in first quadrant = $\frac{1}{4}\pi ab$.

Area of triangle formed by intercepts a, b on axes = $\frac{1}{2} \cdot \text{base} \cdot \text{height} = \frac{1}{2}ab$.

Required bounded Area = (Area of quarter ellipse) – (Area of triangle).

Step 3: Detailed Explanation:

The given curve is an ellipse $\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$. Its semi-major axis is $a = 3$ and semi-minor axis is $b = 2$.

It intersects the axes at $(3, 0), (-3, 0), (0, 2), (0, -2)$.

The given line is $\frac{x}{3} + \frac{y}{2} = 1$. Its x-intercept is 3 and y-intercept is 2.

Thus, the line passes through the points $(3, 0)$ and $(0, 2)$.

The region bounded by the ellipse and the line lies entirely in the first quadrant.

The area under the ellipse in the first quadrant is:

$$A_{\text{ellipse_quadrant}} = \frac{1}{4} \cdot \pi \cdot a \cdot b = \frac{1}{4} \cdot \pi \cdot 3 \cdot 2 = \frac{6\pi}{4} = \frac{3\pi}{2} \text{ sq. units.}$$

The area of the right-angled triangle formed by the line and the coordinate axes is:

$$A_{\text{triangle}} = \frac{1}{2} \cdot \text{base} \cdot \text{height} = \frac{1}{2} \cdot 3 \cdot 2 = 3 \text{ sq. units.}$$

The required bounded area A is the difference between these two areas:

$$A = A_{\text{ellipse_quadrant}} - A_{\text{triangle}}$$

$$A = \frac{3\pi}{2} - 3$$

Taking $\frac{3}{2}$ common:

$$A = \frac{3}{2}(\pi - 2) \text{ sq. units.}$$

Step 4: Final Answer:

The area of the region is $\frac{3}{2}(\pi - 2)$ sq. units.

💡 Quick Tip

For standard bounded regions between $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x}{a} + \frac{y}{b} = 1$, the area is always $\frac{ab}{4}(\pi - 2)$. Here $a = 3, b = 2 \Rightarrow \frac{6}{4}(\pi - 2) = \frac{3}{2}(\pi - 2)$. Memorizing this saves integration time.

22. If $f(x) = \begin{cases} mx + 1, & x \leq \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}, (m, n \in \mathbb{Z})$ then

(A) $m = 1, n = 0$

(B) $m = \frac{n\pi}{2}$

(C) $m = n = \frac{\pi}{2}$

(D) $n = \frac{m\pi}{2}$

Correct Answer: (D) $n = \frac{m\pi}{2}$

Solution:

Step 1: Understanding the Concept:

A piecewise function is continuous at a boundary point if the left-hand limit, the right-hand limit, and the function value at that point are all equal.

We will set up these equations for $x = \frac{\pi}{2}$.

Step 2: Key Formula or Approach:

For continuity at $x = a$:

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$$

Step 3: Detailed Explanation:

The given function is:

$$f(x) = \begin{cases} mx + 1, & x \leq \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$$

We check continuity at $x = \frac{\pi}{2}$.

1. Function value at $x = \frac{\pi}{2}$:

Using the first piece of the function: $f\left(\frac{\pi}{2}\right) = m\left(\frac{\pi}{2}\right) + 1$.

2. Left-hand limit (LHL) as $x \rightarrow \left(\frac{\pi}{2}\right)^-$:

LHL = $\lim_{x \rightarrow (\pi/2)^-} (mx + 1) = m\left(\frac{\pi}{2}\right) + 1$.

3. Right-hand limit (RHL) as $x \rightarrow \left(\frac{\pi}{2}\right)^+$:

RHL = $\lim_{x \rightarrow (\pi/2)^+} (\sin x + n) = \sin\left(\frac{\pi}{2}\right) + n = 1 + n$.

For the function to be continuous, LHL = RHL:

$$m\left(\frac{\pi}{2}\right) + 1 = 1 + n$$

Subtracting 1 from both sides gives:

$$m\frac{\pi}{2} = n$$

This matches option (D).

Self-correction/Note regarding $m, n \in \mathbb{Z}$: The condition that m, n are integers would strictly imply $m = 0, n = 0$ because π is irrational. If $m \neq 0$, then $\pi = \frac{2n}{m}$, which would make π a rational number. However, the question simply asks for the derived relationship, which is $n = \frac{m\pi}{2}$. Even if $m = 0, n = 0$, this relation holds true ($0 = 0 \cdot \pi/2$). Option D is the generalized relation that must be satisfied.

Step 4: Final Answer:

The correct relation is $n = \frac{m\pi}{2}$.

Quick Tip

Don't get bogged down by extra conditions like $m, n \in \mathbb{Z}$ until you have derived the primary relationship. Often, the options just reflect the direct algebraic consequence of continuity.

23. $\int_{-2}^2 |x^2 - x - 2| dx =$

- (A) $\frac{17}{3}$
- (B) $\frac{19}{3}$
- (C) 19
- (D) 17

Correct Answer: (B) $\frac{19}{3}$

Solution:

Step 1: Understanding the Concept:

To integrate a function involving an absolute value, we first need to determine the intervals where the expression inside the absolute value is positive and negative.

We find the roots of the expression to split the integration limits.

Step 2: Key Formula or Approach:

Let $f(x) = x^2 - x - 2$. Find roots of $f(x) = 0$.

Split the integral $\int_a^b |f(x)|dx = \int_a^c |f(x)|dx + \int_c^b |f(x)|dx$ where c is a root.

Evaluate $|f(x)| = f(x)$ if $f(x) \geq 0$, and $|f(x)| = -f(x)$ if $f(x) < 0$.

Step 3: Detailed Explanation:

Consider the function inside the absolute value: $f(x) = x^2 - x - 2$.

Factorizing it: $f(x) = x^2 - 2x + x - 2 = x(x - 2) + 1(x - 2) = (x - 2)(x + 1)$.

The roots are $x = -1$ and $x = 2$.

We analyze the sign of $f(x)$ on the interval $[-2, 2]$:

- For $x \in [-2, -1]$, e.g., $x = -1.5$: $f(-1.5) = (-1.5 - 2)(-1.5 + 1) = (-)(-) = +ve$. So, $|x^2 - x - 2| = x^2 - x - 2$.

- For $x \in [-1, 2]$, e.g., $x = 0$: $f(0) = (0 - 2)(0 + 1) = (-)(+) = -ve$. So, $|x^2 - x - 2| = -(x^2 - x - 2) = -x^2 + x + 2$.

Now, split the definite integral:

$$I = \int_{-2}^2 |x^2 - x - 2|dx = \int_{-2}^{-1} (x^2 - x - 2)dx + \int_{-1}^2 (-x^2 + x + 2)dx$$

Let's evaluate the first integral I_1 :

$$I_1 = \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x \right]_{-2}^{-1}$$

$$I_1 = \left(\frac{(-1)^3}{3} - \frac{(-1)^2}{2} - 2(-1) \right) - \left(\frac{(-2)^3}{3} - \frac{(-2)^2}{2} - 2(-2) \right)$$

$$I_1 = \left(-\frac{1}{3} - \frac{1}{2} + 2 \right) - \left(-\frac{8}{3} - 2 + 4 \right)$$

$$I_1 = \left(\frac{-2 - 3 + 12}{6} \right) - \left(-\frac{8}{3} + 2 \right) = \frac{7}{6} - \left(\frac{-8 + 6}{3} \right)$$

$$I_1 = \frac{7}{6} - \left(-\frac{2}{3} \right) = \frac{7}{6} + \frac{4}{6} = \frac{11}{6}$$

Now, let's evaluate the second integral I_2 :

$$I_2 = \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x \right]_{-1}^2$$

$$I_2 = \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2(2) \right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2(-1) \right)$$

$$I_2 = \left(-\frac{8}{3} + 2 + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$$

$$I_2 = \left(6 - \frac{8}{3}\right) - \left(\frac{2 + 3 - 12}{6}\right) = \left(\frac{18 - 8}{3}\right) - \left(-\frac{7}{6}\right)$$

$$I_2 = \frac{10}{3} + \frac{7}{6} = \frac{20}{6} + \frac{7}{6} = \frac{27}{6} = \frac{9}{2}$$

Sum the two parts to get the total integral:

$$I = I_1 + I_2 = \frac{11}{6} + \frac{9}{2} = \frac{11}{6} + \frac{27}{6} = \frac{38}{6} = \frac{19}{3}$$

Step 4: Final Answer:

The value of the integral is $\frac{19}{3}$.

💡 Quick Tip

Always double-check arithmetic with fractions in piecewise integration problems. Errors frequently happen during the substitution of limits and combining fractions.

24. $\lim_{x \rightarrow 0} \frac{e^{\tan x} - e^x}{\tan x - x} =$

- (A) 1
- (B) 0
- (C) $\frac{1}{2}$
- (D) $\frac{1}{4}$

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

This is a limit problem of the form $\frac{0}{0}$. We can manipulate the expression algebraically to use standard limits.

Step 2: Key Formula or Approach:

Standard limit: $\lim_{u \rightarrow 0} \frac{e^u - 1}{u} = 1$.

We will factor out e^x from the numerator to create an expression that matches this standard form.

Step 3: Detailed Explanation:

We need to evaluate:

$$L = \lim_{x \rightarrow 0} \frac{e^{\tan x} - e^x}{\tan x - x}$$

Factor out e^x from the numerator:

$$L = \lim_{x \rightarrow 0} \frac{e^x(e^{\tan x - x} - 1)}{\tan x - x}$$

$$L = \lim_{x \rightarrow 0} \frac{e^x(e^{\tan x - x} - 1)}{\tan x - x}$$

We can use the properties of limits to separate this into a product of limits:

$$L = \left(\lim_{x \rightarrow 0} e^x \right) \cdot \left(\lim_{x \rightarrow 0} \frac{e^{\tan x - x} - 1}{\tan x - x} \right)$$

Let $u = \tan x - x$.

As $x \rightarrow 0$, we know $\tan x \rightarrow 0$, so $u \rightarrow 0 - 0 = 0$.

The second limit becomes the standard exponential limit:

$$\lim_{u \rightarrow 0} \frac{e^u - 1}{u} = 1$$

The first limit is simply evaluating the continuous function e^x at 0:

$$\lim_{x \rightarrow 0} e^x = e^0 = 1$$

Multiplying the two results:

$$L = 1 \cdot 1 = 1$$

Step 4: Final Answer:

The limit evaluates to 1.

💡 Quick Tip

When seeing $e^{f(x)} - e^{g(x)}$, factoring out $e^{g(x)}$ to form $e^{g(x)}(e^{f(x)-g(x)} - 1)$ often neatly sets up the standard limit form if the denominator is related to $f(x) - g(x)$.

25. The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 20y$ to the end of its latus rectum is

- (A) 100 sq. units
- (B) 40 sq. units
- (C) 20 sq. units
- (D) 50 sq. units

Correct Answer: (D) 50 sq. units

Solution:

Step 1: Understanding the Concept:

We need to find the coordinates of the vertex and the endpoints of the latus rectum for the given parabola, and then calculate the area of the triangle formed by these three points.

Step 2: Key Formula or Approach:

For a parabola $x^2 = 4ay$:

Vertex is $V(0, 0)$.

Focus is $S(0, a)$.

Length of latus rectum is $4a$.

Endpoints of latus rectum are $L(2a, a)$ and $L'(-2a, a)$.

Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$.

Step 3: Detailed Explanation:

The given equation of the parabola is $x^2 = 20y$.

Comparing this with the standard form $x^2 = 4ay$, we get:

$4a = 20 \Rightarrow a = 5$.

The vertex of the parabola is at the origin, $V(0, 0)$.

The focus is at $S(0, a) = (0, 5)$.

The latus rectum is a line segment passing through the focus and perpendicular to the axis of symmetry (y-axis). It lies on the line $y = 5$.

The length of the latus rectum is $4a = 20$.

The endpoints of the latus rectum are equidistant from the y-axis, at a distance of $2a = 10$.

So, the endpoints are $L(10, 5)$ and $L'(-10, 5)$.

We need the area of the triangle VLL' .

The base of this triangle is the length of the latus rectum, $LL' = 20$.

The height of the triangle corresponding to this base is the perpendicular distance from the vertex $V(0, 0)$ to the line LL' ($y = 5$), which is equal to $a = 5$.

Area of triangle $VLL' = \frac{1}{2} \times \text{base} \times \text{height}$

Area $= \frac{1}{2} \times 20 \times 5 = 10 \times 5 = 50$ sq. units.

Step 4: Final Answer:

The area of the triangle is 50 sq. units.

 Quick Tip

For any standard parabola $y^2 = 4ax$ or $x^2 = 4ay$, the area of the triangle formed by the vertex and the ends of the latus rectum is simply $2a^2$. Here $a = 5$, so area = $2(5^2) = 50$. This is a handy formula to remember.

26. The value of $\int_{-1}^1 (\sqrt{1+x+x^2} - \sqrt{1-x+x^2}) dx$ is

- (A) -1
- (B) 1
- (C) 0
- (D) 2

Correct Answer: (C) 0

Solution:

Step 1: Understanding the Concept:

We are evaluating a definite integral over a symmetric interval $[-a, a]$.

We should always check if the integrand is an even or odd function. If $f(x)$ is odd, $\int_{-a}^a f(x) dx = 0$.

Step 2: Key Formula or Approach:

Function property:

Odd function: $f(-x) = -f(x)$

Even function: $f(-x) = f(x)$

Integral property: $\int_{-a}^a f(x) dx = 0$ if $f(x)$ is an odd function.

Step 3: Detailed Explanation:

Let the integrand be $f(x) = \sqrt{1+x+x^2} - \sqrt{1-x+x^2}$.

We evaluate $f(-x)$ to check for symmetry:

$$f(-x) = \sqrt{1+(-x)+(-x)^2} - \sqrt{1-(-x)+(-x)^2}$$

$$f(-x) = \sqrt{1-x+x^2} - \sqrt{1+x+x^2}$$

Factoring out a negative sign:

$$f(-x) = -\left(\sqrt{1+x+x^2} - \sqrt{1-x+x^2}\right)$$

$$f(-x) = -f(x)$$

Since $f(-x) = -f(x)$ for all x in the domain, $f(x)$ is an odd function.

According to the property of definite integrals for odd functions over symmetric intervals:

$$\int_{-a}^a f(x)dx = 0$$

Therefore,

$$\int_{-1}^1 (\sqrt{1+x+x^2} - \sqrt{1-x+x^2})dx = 0$$

Step 4: Final Answer:

The value of the integral is 0.

💡 Quick Tip

Whenever you see integration limits of the form $[-a, a]$, immediately test the integrand for even/odd symmetry. It is the most common trick to save massive amounts of calculation time.

27. If two numbers p and q are chosen randomly from the set $\{1, 2, 3, 4\}$, one by one, with replacement, then the probability of getting $p^2 \geq 4q$ is

- (A) $\frac{1}{4}$
- (B) $\frac{7}{16}$
- (C) $\frac{1}{2}$
- (D) $\frac{9}{16}$

Correct Answer: (B) $\frac{7}{16}$

Solution:

Step 1: Understanding the Concept:

We are drawing two numbers p and q with replacement, so all pairs (p, q) are equally likely.

We need to find the total number of possible pairs and the number of pairs that satisfy the given condition $p^2 \geq 4q$.

Step 2: Key Formula or Approach:

Total outcomes = (number of choices for p) $\times$ (number of choices for q).

Probability = $\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$.

Step 3: Detailed Explanation:

The set of numbers is $S = \{1, 2, 3, 4\}$.

Since numbers are drawn with replacement, $p \in S$ and $q \in S$.

Total number of possible pairs $(p, q) = 4 \times 4 = 16$.

We want to find pairs satisfying the condition $p^2 \geq 4q$. Let's test each possible value of p :

- Case 1: If $p = 1$, then $p^2 = 1$. The condition is $1 \geq 4q$.

Since minimum value of q is 1, $4q \geq 4$. Thus, $1 \geq 4$ is false. No q satisfies this. (0 pairs)

- Case 2: If $p = 2$, then $p^2 = 4$. The condition is $4 \geq 4q \Rightarrow 1 \geq q$.

The only value from the set satisfying this is $q = 1$. So, the pair is $(2, 1)$. (1 pair)

- Case 3: If $p = 3$, then $p^2 = 9$. The condition is $9 \geq 4q \Rightarrow 2.25 \geq q$.

The possible values for q are 1, 2. So, the pairs are $(3, 1), (3, 2)$. (2 pairs)

- Case 4: If $p = 4$, then $p^2 = 16$. The condition is $16 \geq 4q \Rightarrow 4 \geq q$.

The possible values for q are 1, 2, 3, 4. So, the pairs are $(4, 1), (4, 2), (4, 3), (4, 4)$. (4 pairs)

Total number of favorable outcomes = $0 + 1 + 2 + 4 = 7$.

The required probability = $\frac{\text{favorable outcomes}}{\text{total outcomes}} = \frac{7}{16}$.

Step 4: Final Answer:

The probability is $\frac{7}{16}$.

💡 Quick Tip

For small sample spaces like this, systematic manual enumeration of cases based on the independent variable (like p here) is the safest and fastest method to avoid missing any pairs.

28. The function defined by $f(x) = \frac{2x+3}{3x+4}, x \neq -\frac{4}{3}$ is

(A) only one one

(B) only onto

(C) onto for $y \neq \frac{2}{3}$ and one-one

(D) neither one-one nor onto

Correct Answer: (C) onto for $y \neq \frac{2}{3}$ and one-one

Solution:

Step 1: Understanding the Concept:

We need to determine if the function is injective (one-one) and/or surjective (onto).

A function is one-one if $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

A function is onto if for every y in the codomain, there exists an x in the domain such that $f(x) = y$.

Step 2: Key Formula or Approach:

For one-one: Set $\frac{2x_1+3}{3x_1+4} = \frac{2x_2+3}{3x_2+4}$ and solve.

For onto: Express x in terms of y ($y = f(x)$) and find the range of possible y values. Compare it with the typical codomain (real numbers $\mathbb{R}$) or interpret the given options.

Step 3: Detailed Explanation:

Check for One-One (Injectivity):

Let $f(x_1) = f(x_2)$.

$$\frac{2x_1 + 3}{3x_1 + 4} = \frac{2x_2 + 3}{3x_2 + 4}$$

Cross-multiply:

$$(2x_1 + 3)(3x_2 + 4) = (2x_2 + 3)(3x_1 + 4)$$

$$6x_1x_2 + 8x_1 + 9x_2 + 12 = 6x_1x_2 + 8x_2 + 9x_1 + 12$$

Cancel common terms from both sides:

$$8x_1 + 9x_2 = 8x_2 + 9x_1$$

$$9x_2 - 8x_2 = 9x_1 - 8x_1$$

$$x_2 = x_1$$

Since $f(x_1) = f(x_2)$ implies $x_1 = x_2$, the function is one-one.

Check for Onto (Surjectivity):

Let $y = f(x) = \frac{2x+3}{3x+4}$.

We will solve for x in terms of y :

$$y(3x + 4) = 2x + 3$$

$$3xy + 4y = 2x + 3$$

$$3xy - 2x = 3 - 4y$$

$$x(3y - 2) = 3 - 4y$$

$$x = \frac{3 - 4y}{3y - 2}$$

For x to be a real number, the denominator cannot be zero.

So, $3y - 2 \neq 0 \Rightarrow y \neq \frac{2}{3}$.

Also, we must ensure $x \neq -\frac{4}{3}$. If $\frac{3-4y}{3y-2} = -\frac{4}{3}$, then $9 - 12y = -12y + 8 \Rightarrow 9 = 8$, which is absurd. So x will never be $-\frac{4}{3}$.

The range of the function is all real numbers except $\frac{2}{3}$.

If the codomain is assumed to be $\mathbb{R}$, the function is not onto. However, option (C) specifically states "onto for $y \neq \frac{2}{3}$ ", meaning it is a bijection to its range $\mathbb{R} \setminus \{\frac{2}{3}\}$. This matches our findings perfectly.

Step 4: Final Answer:

The function is onto for $y \neq \frac{2}{3}$ and one-one.

💡 Quick Tip

For a rational function of the form $f(x) = \frac{ax+b}{cx+d}$, it is always one-one (provided $ad - bc \neq 0$). Its range is $\mathbb{R} \setminus \{\frac{a}{c}\}$. Here $a/c = 2/3$. Thus it's onto if the codomain excludes $2/3$.

29. The equation $|z + 1 - i| = |z - 1 + i|$ represents a (where z is a complex number)

- (A) Straight line passing through the origin and the first and third quadrant.
- (B) Straight line passing through the origin and the second and fourth quadrant.
- (C) Straight line passing through the point $(1, -1)$ and having slope -1 .
- (D) Straight line passing through the point $(2, 1)$ and having slope $\frac{1}{2}$.

Correct Answer: (A) Straight line passing through the origin and the first and third quadrant.

Solution:

Step 1: Understanding the Concept:

The geometric interpretation of $|z - z_1| = |z - z_2|$ is the perpendicular bisector of the line segment joining the points z_1 and z_2 in the complex plane.

Alternatively, we can substitute $z = x + iy$ and solve algebraically to find the locus equation in Cartesian coordinates.

Step 2: Key Formula or Approach:

Substitute $z = x + iy$.

Magnitude of a complex number $a + ib$ is $\sqrt{a^2 + b^2}$.

Simplify the equation to find the relation between x and y .

Step 3: Detailed Explanation:

Let $z = x + iy$.

The given equation is $|x + iy + 1 - i| = |x + iy - 1 + i|$.

Rearranging real and imaginary parts:

$$|(x + 1) + i(y - 1)| = |(x - 1) + i(y + 1)|$$

Using the definition of modulus:

$$\sqrt{(x+1)^2 + (y-1)^2} = \sqrt{(x-1)^2 + (y+1)^2}$$

Squaring both sides to remove square roots:

$$(x+1)^2 + (y-1)^2 = (x-1)^2 + (y+1)^2$$

Expanding the squares:

$$(x^2 + 2x + 1) + (y^2 - 2y + 1) = (x^2 - 2x + 1) + (y^2 + 2y + 1)$$

Canceling $x^2, y^2, 1, 1$ from both sides:

$$2x - 2y = -2x + 2y$$

Bringing all terms to one side:

$$4x - 4y = 0$$

$$4x = 4y \Rightarrow y = x$$

The equation $y = x$ represents a straight line.

It passes through the origin $(0, 0)$.

Since the slope is 1 (positive), it passes through the first quadrant (where $x > 0, y > 0$) and the third quadrant (where $x < 0, y < 0$).

Step 4: Final Answer:

It is a straight line passing through the origin and the first and third quadrant.

💡 Quick Tip

Geometrically, $|z - (-1 + i)| = |z - (1 - i)|$ means the distance of z from $A(-1, 1)$ is equal to its distance from $B(1, -1)$. The locus is the perpendicular bisector of segment AB. The midpoint of AB is $(0, 0)$. The slope of AB is $\frac{-1-1}{1-(-1)} = -1$. The slope of the perpendicular bisector is 1. Equation: $y - 0 = 1(x - 0) \Rightarrow y = x$.

30. If $\int \frac{2x+3}{(x-1)(x^2+1)} dx = \log_e \left\{ (x-1)^{\frac{5}{2}} (x^2+1)^a \right\} - \frac{1}{2} \tan^{-1} x + A$ where **A is an arbitrary constant, then the value of a is**

- (A) $\frac{5}{4}$
- (B) $-\frac{5}{4}$
- (C) $-\frac{5}{6}$
- (D) $-\frac{5}{6}$

Correct Answer: (B) $-\frac{5}{4}$

Solution:

Step 1: Understanding the Concept:

This is an integration problem requiring partial fraction decomposition.

The denominator has a linear factor $(x - 1)$ and an irreducible quadratic factor $(x^2 + 1)$.

After integration, we need to compare the result with the given expression to find the value of the exponent 'a'.

Step 2: Key Formula or Approach:

Set up partial fractions: $\frac{2x+3}{(x-1)(x^2+1)} = \frac{B}{x-1} + \frac{Cx+D}{x^2+1}$.

Solve for constants B, C, D .

Integrate each term separately.

Use log properties like $n \ln x = \ln(x^n)$ and $\ln x + \ln y = \ln(xy)$ to combine terms.

Step 3: Detailed Explanation:

Let's decompose the integrand into partial fractions:

$$\frac{2x + 3}{(x - 1)(x^2 + 1)} = \frac{B}{x - 1} + \frac{Cx + D}{x^2 + 1}$$

Multiply through by the common denominator $(x - 1)(x^2 + 1)$:

$$2x + 3 = B(x^2 + 1) + (Cx + D)(x - 1)$$

To find B , let $x = 1$:

$$2(1) + 3 = B(1^2 + 1) + 0 \Rightarrow 5 = 2B \Rightarrow B = \frac{5}{2}$$

To find D , let $x = 0$:

$$3 = B(1) + (D)(-1) \Rightarrow 3 = \frac{5}{2} - D \Rightarrow D = \frac{5}{2} - 3 = -\frac{1}{2}$$

To find C , compare the coefficients of x^2 on both sides:

$$0 = B + C \Rightarrow C = -B = -\frac{5}{2}$$

Now, substitute the partial fractions back into the integral:

$$\int \frac{2x + 3}{(x - 1)(x^2 + 1)} dx = \int \left(\frac{\frac{5}{2}}{x - 1} + \frac{-\frac{5}{2}x - \frac{1}{2}}{x^2 + 1} \right) dx$$

Split the integral:

$$= \frac{5}{2} \int \frac{1}{x-1} dx - \frac{5}{2} \int \frac{x}{x^2+1} dx - \frac{1}{2} \int \frac{1}{x^2+1} dx$$

For the second integral, multiply and divide by 2:

$$\begin{aligned} &= \frac{5}{2} \ln|x-1| - \frac{5}{4} \int \frac{2x}{x^2+1} dx - \frac{1}{2} \tan^{-1} x \\ &= \frac{5}{2} \ln|x-1| - \frac{5}{4} \ln|x^2+1| - \frac{1}{2} \tan^{-1} x + C_1 \end{aligned}$$

Combine the log terms using properties of logarithms:

$$\begin{aligned} &= \ln(|x-1|^{5/2}) + \ln((x^2+1)^{-5/4}) - \frac{1}{2} \tan^{-1} x + C_1 \\ &= \ln\left(|x-1|^{5/2}(x^2+1)^{-5/4}\right) - \frac{1}{2} \tan^{-1} x + C_1 \end{aligned}$$

Comparing this result with the given expression $\log_e \left\{ (x-1)^{\frac{5}{2}}(x^2+1)^a \right\} - \frac{1}{2} \tan^{-1} x + A$, we can see that:

$$a = -\frac{5}{4}$$

Step 4: Final Answer:

The value of a is $-\frac{5}{4}$.

💡 Quick Tip

Instead of solving a system of equations for partial fractions, strategically choosing values like $x = 1$, $x = 0$, and comparing highest degree coefficients is a much faster way to find the constants.

31. The money invested in a company is compounded continuously. Rs. 400 invested today becomes Rs. 800 in 6 years, then at the end of 33 years, it will become .. ($\sqrt{2} = 1.4142$)

- (A) 9050.88
- (B) 18101.76

- (C) 6788.16
(D) 12067.84

Correct Answer: (B) 18101.76

Solution:

Step 1: Understanding the Concept:

When money is compounded continuously, the amount A after time t is given by the formula $A = Pe^{rt}$, where P is the principal amount and r is the continuous compounding rate.

Step 2: Key Formula or Approach:

Use the formula $A = Pe^{rt}$.

First, use the given condition ($P = 400$, $A = 800$, $t = 6$) to find the value of e^r .

Then, use this value to calculate the final amount for $t = 33$.

Step 3: Detailed Explanation:

Given principal $P = 400$.

After $t = 6$ years, the amount $A = 800$.

Substituting these values into the formula:

$$800 = 400e^{6r}$$

$$e^{6r} = \frac{800}{400} = 2$$

To find e^r , we take the 6th root of both sides:

$$(e^{6r})^{1/6} = 2^{1/6} \Rightarrow e^r = 2^{1/6}$$

Now, we need to find the amount at the end of $t = 33$ years.

$$A_{33} = 400e^{33r}$$

$$A_{33} = 400(e^r)^{33}$$

Substitute $e^r = 2^{1/6}$:

$$A_{33} = 400(2^{1/6})^{33}$$

$$A_{33} = 400 \cdot 2^{33/6}$$

Simplify the fraction $33/6$ by dividing numerator and denominator by 3:

$$A_{33} = 400 \cdot 2^{11/2}$$

We can write $2^{11/2}$ as $2^5 \cdot 2^{1/2}$:

$$A_{33} = 400 \cdot 2^5 \cdot \sqrt{2}$$

$$A_{33} = 400 \cdot 32 \cdot \sqrt{2}$$

$$A_{33} = 12800 \cdot \sqrt{2}$$

Given $\sqrt{2} = 1.4142$, substitute this value:

$$A_{33} = 12800 \cdot 1.4142$$

$$A_{33} = 128 \cdot 141.42$$

$$A_{33} = 18101.76$$

Step 4: Final Answer:

The amount will become Rs. 18101.76.

💡 Quick Tip

Avoid calculating the exact decimal value of the rate r . Keep it in exponential form like $e^r = 2^{1/6}$ to maintain precision and simplify subsequent calculations involving powers.

32. If $\bar{a}$ and $\bar{b}$ are unit vectors and θ is the angle between them, then $\bar{a} + \bar{b}$ is a unit vector when θ is

- (A) $\frac{\pi}{3}$
- (B) $\frac{2\pi}{3}$
- (C) $\frac{\pi}{2}$
- (D) $\frac{\pi}{4}$

Correct Answer: (B) $\frac{2\pi}{3}$

Solution:

Step 1: Understanding the Concept:

We are given that $\bar{a}$ and $\bar{b}$ are unit vectors, which means their magnitudes are 1. We need to find the angle θ such that their sum $\bar{a} + \bar{b}$ is also a unit vector.

Step 2: Key Formula or Approach:

The magnitude squared of the sum of two vectors is given by:

$$|\bar{a} + \bar{b}|^2 = |\bar{a}|^2 + |\bar{b}|^2 + 2\bar{a} \cdot \bar{b}$$

Recall that the dot product $\bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}| \cos \theta$.

Step 3: Detailed Explanation:

Given that $\bar{a}$ and $\bar{b}$ are unit vectors, so $|\bar{a}| = 1$ and $|\bar{b}| = 1$.

It is also given that $\bar{a} + \bar{b}$ is a unit vector, so $|\bar{a} + \bar{b}| = 1$.

Squaring both sides of this equation:

$$|\bar{a} + \bar{b}|^2 = 1^2 = 1$$

Expand the left side using vector properties:

$$|\bar{a}|^2 + |\bar{b}|^2 + 2(\bar{a} \cdot \bar{b}) = 1$$

Substitute the dot product formula:

$$|\bar{a}|^2 + |\bar{b}|^2 + 2|\bar{a}||\bar{b}| \cos \theta = 1$$

Substitute the known magnitudes ($|\bar{a}| = 1$, $|\bar{b}| = 1$):

$$1^2 + 1^2 + 2(1)(1) \cos \theta = 1$$

$$1 + 1 + 2 \cos \theta = 1$$

$$2 + 2 \cos \theta = 1$$

Subtract 2 from both sides:

$$2 \cos \theta = 1 - 2$$

$$2 \cos \theta = -1$$

$$\cos \theta = -\frac{1}{2}$$

The principal value of θ for which $\cos \theta = -1/2$ is in the second quadrant.

$$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Step 4: Final Answer:

The angle θ is $\frac{2\pi}{3}$.

💡 Quick Tip

A useful geometric fact to remember: if three vectors of equal magnitude form a closed triangle, it must be an equilateral triangle. Here, $\vec{a}$, $\vec{b}$, and $-(\vec{a} + \vec{b})$ form such a triangle. The interior angles are 60° . The angle between the vectors $\vec{a}$ and $\vec{b}$ is the exterior angle, which is $180^\circ - 60^\circ = 120^\circ$ or $2\pi/3$.

33. A regular polygon has 20 sides. The number of triangles that can be drawn by using the vertices but not using the sides are

- (A) 1140
- (B) 800
- (C) 340
- (D) 20

Correct Answer: (B) 800

Solution:

Step 1: Understanding the Concept:

We are forming triangles using the vertices of a 20-sided polygon.

We need to exclude any triangles that share one or two sides with the polygon itself.

We can solve this by finding the total number of triangles and subtracting those with shared sides, or by using a direct formula.

Step 2: Key Formula or Approach:

For a polygon with n sides (and n vertices):

1. Total number of triangles formed by vertices = $\binom{n}{3}$.
2. Number of triangles sharing exactly two sides with the polygon = n (these are formed by any three consecutive vertices).
3. Number of triangles sharing exactly one side with the polygon = $n(n-4)$ (choose one side in n ways, the third vertex must not be adjacent to this side's endpoints, leaving $n-4$ choices).
4. Number of triangles sharing NO sides = Total - (sharing 2 sides) - (sharing 1 side).

Alternative direct formula: $\frac{n(n-4)(n-5)}{6}$.

Step 3: Detailed Explanation:

Here, $n = 20$.

Let's use the subtraction method first.

Total number of triangles = $\binom{20}{3} = \frac{20 \times 19 \times 18}{3 \times 2 \times 1} = 20 \times 19 \times 3 = 1140$.

Triangles sharing 2 sides = $n = 20$.

Triangles sharing 1 side = $n(n-4) = 20(20-4) = 20 \times 16 = 320$.

Triangles sharing zero sides = Total - (sharing 2 sides) - (sharing 1 side)
 $= 1140 - 20 - 320 = 1140 - 340 = 800$.

Alternatively, using the direct formula:

Number of such triangles = $\frac{n(n-4)(n-5)}{6}$

$$= \frac{20 \times (20-4) \times (20-5)}{6}$$

$$= \frac{20 \times 16 \times 15}{6}$$

$$= \frac{20 \times 8 \times 30}{6} \text{ (simplifying first) or just cancel: } \frac{15}{6} = \frac{5}{2}, \text{ then } \frac{16}{2} = 8.$$

$$= 20 \times 8 \times 5 = 160 \times 5 = 800.$$

Step 4: Final Answer:

The number of such triangles is 800.

💡 Quick Tip

Memorizing the direct formula $\frac{n(n-4)(n-5)}{6}$ for "triangles with no sides common to the polygon" can save valuable time during competitive exams. Similarly, the number of diagonals is $\frac{n(n-3)}{2}$.

34. $\int \frac{dx}{2+\cos x} =$

(A) $2 \tan^{-1} \left(\frac{1}{\sqrt{3}} \tan \frac{x}{2} \right) + c$, where c is the constant of integration

(B) $\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{1}{\sqrt{3}} \tan \frac{x}{2} \right) + c$, where c is the constant of integration

(C) $\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{1}{\sqrt{3}} \tan \frac{x}{2} \right) + c$, where c is the constant of integration

(D) $\sqrt{3} \tan^{-1} \left(\frac{1}{\sqrt{3}} \tan \frac{x}{2} \right) + c$, where c is the constant of integration

Correct Answer: (B) $\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{1}{\sqrt{3}} \tan \frac{x}{2} \right) + c$, where c is the constant of integration

Solution:

Step 1: Understanding the Concept:

This is a standard integral of the form $\int \frac{dx}{a+b \cos x}$.

The universal substitution for such integrals is to express $\cos x$ in terms of $\tan(x/2)$.

Step 2: Key Formula or Approach:

Use the half-angle substitution:

Let $t = \tan\left(\frac{x}{2}\right)$.

Then, $\cos x = \frac{1-t^2}{1+t^2}$ and $dx = \frac{2dt}{1+t^2}$.

Also, use the standard integral $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$.

Step 3: Detailed Explanation:

Let $I = \int \frac{dx}{2+\cos x}$.

Substitute $t = \tan(x/2)$:

$$I = \int \frac{\frac{2dt}{1+t^2}}{2 + \frac{1-t^2}{1+t^2}}$$

Multiply numerator and denominator by $(1+t^2)$:

$$I = \int \frac{2dt}{2(1+t^2) + (1-t^2)}$$

$$I = \int \frac{2dt}{2 + 2t^2 + 1 - t^2}$$

Combine like terms in the denominator:

$$I = \int \frac{2dt}{t^2 + 3}$$

Factor out the 2 and rewrite the denominator in the form $x^2 + a^2$:

$$I = 2 \int \frac{dt}{t^2 + (\sqrt{3})^2}$$

Now apply the standard formula $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$:

Here, x is t and a is $\sqrt{3}$.

$$I = 2 \left[\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) \right] + c$$

Substitute back $t = \tan(x/2)$:

$$I = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\tan(x/2)}{\sqrt{3}}\right) + c$$

$$I = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{1}{\sqrt{3}} \tan \frac{x}{2}\right) + c$$

Step 4: Final Answer:

The evaluated integral is $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{1}{\sqrt{3}} \tan \frac{x}{2}\right) + c$.

 Quick Tip

Whenever you see an integral of the form $\frac{1}{a \pm b \sin x \pm c \cos x}$, the Weierstrass substitution ($t = \tan(x/2)$) is almost always the most reliable and systematic way to solve it, converting trigonometric expressions into rational algebraic expressions.

35. If $A + B = \frac{\pi}{2}$ then the maximum value of $\cos A \cdot \cos B$ is

- (A) $\frac{1}{\sqrt{2}}$
- (B) $\frac{1}{2}$
- (C) $-\frac{1}{2}$
- (D) $-\frac{1}{\sqrt{2}}$

Correct Answer: (B) $\frac{1}{2}$

Solution:

Step 1: Understanding the Concept:

We need to find the maximum value of a product of two cosines, given a relation between their angles.

We can convert this product into a single trigonometric function to easily find its maximum.

Step 2: Key Formula or Approach:

Use the given relation $A + B = \frac{\pi}{2}$ to express B in terms of A .

Use the complementary angle identity: $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$.

Use the double angle formula: $\sin(2\theta) = 2 \sin \theta \cos \theta$.

The maximum value of $\sin(x)$ is 1.

Step 3: Detailed Explanation:

Let the given expression be $y = \cos A \cdot \cos B$.

We are given that $A + B = \frac{\pi}{2}$, which means $B = \frac{\pi}{2} - A$.

Substitute B into the expression:

$$y = \cos A \cdot \cos\left(\frac{\pi}{2} - A\right)$$

Using the identity $\cos\left(\frac{\pi}{2} - A\right) = \sin A$, we get:

$$y = \cos A \cdot \sin A$$

Multiply and divide by 2 to use the double angle formula:

$$y = \frac{1}{2}(2 \sin A \cos A)$$

$$y = \frac{1}{2} \sin(2A)$$

We know that the sine function oscillates between -1 and 1 .

Therefore, the maximum value of $\sin(2A)$ is 1 (which occurs when $2A = \frac{\pi}{2} \Rightarrow A = \frac{\pi}{4}$).

So, the maximum value of y is:

$$y_{\max} = \frac{1}{2} \times 1 = \frac{1}{2}$$

Step 4: Final Answer:

The maximum value is $\frac{1}{2}$.

💡 Quick Tip

Another way is to use AM-GM inequality if A, B are acute. Since $A + B = 90^\circ$, let's maximize $\cos A \sin A$. By AM-GM on $\cos^2 A$ and $\sin^2 A$: $\frac{\cos^2 A + \sin^2 A}{2} \geq \sqrt{\cos^2 A \sin^2 A} \Rightarrow \frac{1}{2} \geq |\cos A \sin A|$. So max value is $1/2$.

36. The magnitude of a vector which is orthogonal to the vector $\hat{i} + \hat{j} + \hat{k}$ and is coplanar with the vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ is

- (A) $\sqrt{2}$
- (B) $4\sqrt{2}$
- (C) 4
- (D) $2\sqrt{3}$

Correct Answer: (B) $4\sqrt{2}$

Solution:

Step 1: Understanding the Concept:

A vector coplanar with vectors $\vec{b}$ and $\vec{c}$ can be expressed as a linear combination of them, or it lies in the plane defined by them.

A vector orthogonal to vector $\vec{a}$ has a dot product of zero with $\vec{a}$.

The vector that satisfies both conditions (coplanar with $\vec{b}, \vec{c}$ and orthogonal to $\vec{a}$) is parallel to the vector triple product $\vec{a} \times (\vec{b} \times \vec{c})$. Given the specific options, the question implicitly asks for the magnitude of this exact vector construction without any arbitrary scaling factor.

Step 2: Key Formula or Approach:

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$.

Let $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$.

Let $\vec{c} = \hat{i} + 2\hat{j} + \hat{k}$.

The required vector is $\vec{v} = \vec{a} \times (\vec{b} \times \vec{c})$.

Vector Triple Product expansion: $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$.

Step 3: Detailed Explanation:

Let's calculate the dot products first:

$$\vec{a} \cdot \vec{c} = (\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} + \hat{k}) = (1)(1) + (1)(2) + (1)(1) = 1 + 2 + 1 = 4$$

$$\vec{a} \cdot \vec{b} = (\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} + \hat{j} + 2\hat{k}) = (1)(1) + (1)(1) + (1)(2) = 1 + 1 + 2 = 4$$

Now substitute these into the expansion formula to find $\vec{v}$:

$$\vec{v} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

$$\vec{v} = 4\vec{b} - 4\vec{c} = 4(\vec{b} - \vec{c})$$

Calculate $\vec{b} - \vec{c}$:

$$\vec{b} - \vec{c} = (\hat{i} + \hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} + \hat{k})$$

$$\vec{b} - \vec{c} = (1 - 1)\hat{i} + (1 - 2)\hat{j} + (2 - 1)\hat{k} = 0\hat{i} - \hat{j} + \hat{k} = -\hat{j} + \hat{k}$$

So, the required vector is:

$$\vec{v} = 4(-\hat{j} + \hat{k}) = -4\hat{j} + 4\hat{k}$$

Now, find its magnitude:

$$|\vec{v}| = \sqrt{0^2 + (-4)^2 + 4^2} = \sqrt{16 + 16} = \sqrt{32}$$

$$|\vec{v}| = \sqrt{16 \times 2} = 4\sqrt{2}$$

Step 4: Final Answer:

The magnitude of the vector is $4\sqrt{2}$.

💡 Quick Tip

The phrasing "a vector which is orthogonal to $\vec{a}$ and coplanar with $\vec{b}$ and $\vec{c}$ " is standard terminology referring specifically to the vector triple product $\vec{a} \times (\vec{b} \times \vec{c})$. Expanding it as $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$ is much faster than calculating two cross products.

37. The distance between the lines represented by the equation $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ is

- (A) $\frac{1}{\sqrt{5}}$ units
- (B) $\frac{1}{5}$ units
- (C) $\sqrt{5}$ units
- (D) 5 units

Correct Answer: (C) $\sqrt{5}$ units

Solution:

Step 1: Understanding the Concept:

The given equation is a second-degree general equation representing a pair of straight lines. Since the second-degree terms form a perfect square ($4x^2 + 4xy + y^2 = (2x + y)^2$), the lines are parallel.

We need to factorize the equation to find the individual equations of the lines and then use the formula for distance between parallel lines.

Step 2: Key Formula or Approach:

Factorize the equation into the form $(ax + by + c_1)(ax + by + c_2) = 0$.

The distance d between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by:

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

Step 3: Detailed Explanation:

Given equation: $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$

Observe the second-degree part: $4x^2 + 4xy + y^2 = (2x + y)^2$.

We can rewrite the given equation as:

$$(2x + y)^2 - 3(2x + y) - 4 = 0$$

This is a quadratic equation in terms of $(2x + y)$. Let $u = 2x + y$.

$$u^2 - 3u - 4 = 0$$

Factorizing the quadratic equation:

$$u^2 - 4u + u - 4 = 0$$

$$u(u - 4) + 1(u - 4) = 0$$

$$(u - 4)(u + 1) = 0$$

So, $u = 4$ or $u = -1$.

Substituting back $u = 2x + y$:

Line 1: $2x + y - 4 = 0$ (here $c_1 = -4$)

Line 2: $2x + y + 1 = 0$ (here $c_2 = 1$)

Both lines have $a = 2$ and $b = 1$.

The distance between them is:

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

$$d = \frac{|-4 - 1|}{\sqrt{2^2 + 1^2}}$$

$$d = \frac{|-5|}{\sqrt{4 + 1}} = \frac{5}{\sqrt{5}}$$

Rationalizing the denominator:

$$d = \frac{5}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$$

Step 4: Final Answer:

The distance is $\sqrt{5}$ units.

💡 Quick Tip

If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents parallel lines, then $h^2 = ab$ and $bg^2 = af^2$. The distance between them can also be directly calculated using the formula $2\sqrt{\frac{g^2 - ac}{a(a+b)}}$ or $2\sqrt{\frac{f^2 - bc}{b(a+b)}}$.

Here $a = 4, b = 1, c = -4, g = -3$. $d = 2\sqrt{\frac{(-3)^2 - 4(-4)}{4(4+1)}} = 2\sqrt{\frac{9+16}{20}} = 2\sqrt{\frac{25}{20}} = 2 \cdot \frac{5}{2\sqrt{5}} = \sqrt{5}$.

38. If $y = x^x + x^{\frac{1}{x}}$, then $\frac{dy}{dx}$ is equal to

- (A) $x^x(1 + \log x) + x^{\frac{1}{x}} \frac{1}{x^2}(1 - \log x)$
 (B) $(x^x + x^{\frac{1}{x}}) \left[1 + \log x + \frac{1}{x^2}(1 - \log x) \right]$

(C) $(x^x + x^{\frac{1}{x}}) \left[(1 + \log x) - \frac{1}{x^2}(1 - \log x) \right]$

(D) $x^x(1 + \log x) - x^{\frac{1}{x}} \frac{1}{x^2}(1 - \log x)$

Correct Answer: (A) $x^x(1 + \log x) + x^{\frac{1}{x}} \frac{1}{x^2}(1 - \log x)$

Solution:

Step 1: Understanding the Concept:

The function is a sum of two terms where both the base and exponent are variables ($f(x)^{g(x)}$ form).

We cannot directly use standard power or exponential rules. We must use logarithmic differentiation for each term separately.

Step 2: Key Formula or Approach:

Let $y = u + v$, then $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$.

For $u = f(x)^{g(x)}$, take natural logarithm: $\ln u = g(x) \ln f(x)$.

Differentiate implicitly: $\frac{1}{u} \frac{du}{dx} = g'(x) \ln f(x) + g(x) \frac{f'(x)}{f(x)}$.

Step 3: Detailed Explanation:

Let $y = u + v$ where $u = x^x$ and $v = x^{\frac{1}{x}}$.

Part 1: Differentiate $u = x^x$

Take natural logarithm on both sides:

$$\log u = \log(x^x) = x \log x$$

Differentiate with respect to x using the product rule:

$$\frac{1}{u} \frac{du}{dx} = (1) \cdot \log x + x \cdot \left(\frac{1}{x} \right)$$

$$\frac{1}{u} \frac{du}{dx} = \log x + 1$$

$$\frac{du}{dx} = u(1 + \log x) = x^x(1 + \log x)$$

Part 2: Differentiate $v = x^{\frac{1}{x}}$

Take natural logarithm on both sides:

$$\log v = \log \left(x^{\frac{1}{x}} \right) = \frac{1}{x} \log x$$

Differentiate with respect to x using the quotient rule (or product rule with x^{-1}):

Let's use quotient rule on $\frac{\log x}{x}$:

$$\frac{1}{v} \frac{dv}{dx} = \frac{x \cdot \frac{d}{dx}(\log x) - \log x \cdot \frac{d}{dx}(x)}{x^2}$$

$$\frac{1}{v} \frac{dv}{dx} = \frac{x \cdot \left(\frac{1}{x}\right) - \log x \cdot (1)}{x^2}$$

$$\frac{1}{v} \frac{dv}{dx} = \frac{1 - \log x}{x^2}$$

$$\frac{dv}{dx} = v \left(\frac{1 - \log x}{x^2} \right) = x^{\frac{1}{x}} \frac{1}{x^2} (1 - \log x)$$

Now, add the derivatives together:

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\frac{dy}{dx} = x^x(1 + \log x) + x^{\frac{1}{x}} \frac{1}{x^2} (1 - \log x)$$

Step 4: Final Answer:

The derivative is $x^x(1 + \log x) + x^{\frac{1}{x}} \frac{1}{x^2} (1 - \log x)$.

💡 Quick Tip

Remembering the derivative of x^x as $x^x(1 + \ln x)$ is very useful as it appears frequently. For functions of the form $u + v$, never take log of the whole sum directly (i.e., $\log(u + v) \neq \log u + \log v$). Always differentiate parts separately.

39. If the plane $\frac{x}{3} + \frac{y}{2} - \frac{z}{4} = 1$ cuts the co-ordinate axes at points A, B and C, then the area of the triangle ABC is

- (A) $\frac{\sqrt{61}}{2}$ sq. units
- (B) $2\sqrt{61}$ sq. units
- (C) $\sqrt{61}$ sq. units
- (D) $3\sqrt{61}$ sq. units

Correct Answer: (C) $\sqrt{61}$ sq. units

Solution:

Step 1: Understanding the Concept:

The given equation is of a plane in intercept form: $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

The intercepts on the x, y, and z axes are a , b , and c respectively.

The vertices of the triangle are the points where the plane intersects the axes.

The area of this triangle can be found using vectors or by relating it to the areas of its projections on the coordinate planes.

Step 2: Key Formula or Approach:

If a plane has intercepts a, b, c on the coordinate axes, the area Δ of the triangle formed by these intercept points is given by:

$$\Delta = \frac{1}{2} \sqrt{(ab)^2 + (bc)^2 + (ca)^2}$$

This comes from the relation $\Delta^2 = \Delta_{xy}^2 + \Delta_{yz}^2 + \Delta_{zx}^2$, where $\Delta_{xy} = \frac{1}{2}|ab|$ etc. are areas of projections.

Step 3: Detailed Explanation:

The equation of the plane is $\frac{x}{3} + \frac{y}{2} + \frac{z}{-4} = 1$.

Comparing with intercept form, we have:

x-intercept $a = 3 \Rightarrow$ point A is $(3, 0, 0)$

y-intercept $b = 2 \Rightarrow$ point B is $(0, 2, 0)$

z-intercept $c = -4 \Rightarrow$ point C is $(0, 0, -4)$

Using the formula for the area of the triangle:

$$\Delta = \frac{1}{2} \sqrt{(ab)^2 + (bc)^2 + (ca)^2}$$

Calculate the pairwise products:

$$ab = 3 \cdot 2 = 6$$

$$bc = 2 \cdot (-4) = -8$$

$$ca = (-4) \cdot 3 = -12$$

Now substitute these into the formula:

$$\Delta = \frac{1}{2} \sqrt{(6)^2 + (-8)^2 + (-12)^2}$$

$$\Delta = \frac{1}{2} \sqrt{36 + 64 + 144}$$

$$\Delta = \frac{1}{2} \sqrt{244}$$

Simplify the radical:

$$\sqrt{244} = \sqrt{4 \times 61} = 2\sqrt{61}$$

So, the area is:

$$\Delta = \frac{1}{2} (2\sqrt{61}) = \sqrt{61}$$

Step 4: Final Answer:

The area of the triangle is $\sqrt{61}$ sq. units.

💡 Quick Tip

The formula $\text{Area} = \frac{1}{2}\sqrt{(ab)^2 + (bc)^2 + (ca)^2}$ is a very fast shortcut for finding the area of a triangle formed by the intercepts of a plane. It's essentially the magnitude of the cross product $\frac{1}{2}|\vec{AB} \times \vec{AC}|$.

40. If $\tan^{-1}\left(\frac{x}{2}\right) + \tan^{-1}\left(\frac{y}{2}\right) + \tan^{-1}\left(\frac{z}{2}\right) = \frac{\pi}{2}$ then $xy + yz + zx =$

- (A) 0
- (B) 2
- (C) -1
- (D) 4

Correct Answer: (D) 4

Solution:**Step 1: Understanding the Concept:**

We are given an equation involving the sum of three inverse tangent functions.

We can use the standard trigonometric identity for the tangent of a sum of three angles.

Step 2: Key Formula or Approach:

If $A + B + C = \frac{\pi}{2}$, then $\tan(A + B + C)$ is undefined (approaches infinity).

The formula for $\tan(A + B + C)$ is:

$$\tan(A + B + C) = \frac{\sum \tan A - \prod \tan A}{1 - \sum \tan A \tan B}$$

For the fraction to approach infinity, the denominator must be zero:

$$1 - (\tan A \tan B + \tan B \tan C + \tan C \tan A) = 0$$

$$\Rightarrow \tan A \tan B + \tan B \tan C + \tan C \tan A = 1.$$

Step 3: Detailed Explanation:

$$\text{Let } A = \tan^{-1}\left(\frac{x}{2}\right) \Rightarrow \tan A = \frac{x}{2}$$

$$\text{Let } B = \tan^{-1}\left(\frac{y}{2}\right) \Rightarrow \tan B = \frac{y}{2}$$

$$\text{Let } C = \tan^{-1}\left(\frac{z}{2}\right) \Rightarrow \tan C = \frac{z}{2}$$

Given that $A + B + C = \frac{\pi}{2}$.

As derived above, this implies:

$$\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$$

Substitute the values of $\tan A$, $\tan B$, $\tan C$:

$$\left(\frac{x}{2}\right)\left(\frac{y}{2}\right) + \left(\frac{y}{2}\right)\left(\frac{z}{2}\right) + \left(\frac{z}{2}\right)\left(\frac{x}{2}\right) = 1$$

$$\frac{xy}{4} + \frac{yz}{4} + \frac{zx}{4} = 1$$

Multiply the entire equation by 4:

$$xy + yz + zx = 4$$

Step 4: Final Answer:

The value of $xy + yz + zx$ is 4.

 Quick Tip

Standard conditionally valid identities are great time-savers:

If $\sum \tan^{-1} x_i = \pi/2$, then $\sum x_i x_j = 1$.

If $\sum \tan^{-1} x_i = \pi$, then $\sum x_i = \prod x_i$.

41. In a triangle ABC, with usual notations if $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$ then $\angle A =$

- (A) $\frac{\pi}{2}$
- (B) $\frac{\pi}{4}$
- (C) $\frac{\pi}{3}$
- (D) $\frac{\pi}{6}$

Correct Answer: (A) $\frac{\pi}{2}$

Solution:

Step 1: Understanding the Concept:

The problem provides a relation involving sides and cosines of angles of a triangle.

The standard approach is to convert all trigonometric ratios into expressions involving sides using the Cosine Rule.

Step 2: Key Formula or Approach:

Cosine Rule formulas:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Substitute these into the given equation and simplify to find a relationship between the sides.

Step 3: Detailed Explanation:

Given equation:

$$\frac{2 \cos A}{a} + \frac{\cos B}{b} + \frac{2 \cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$$

Let's simplify the Right Hand Side (RHS) first:

$$\text{RHS} = \frac{a \cdot a + b \cdot b}{abc} = \frac{a^2 + b^2}{abc}$$

Now, substitute the Cosine Rule into the Left Hand Side (LHS):

$$\text{LHS} = \frac{2 \left(\frac{b^2 + c^2 - a^2}{2bc} \right)}{a} + \frac{\left(\frac{a^2 + c^2 - b^2}{2ac} \right)}{b} + \frac{2 \left(\frac{a^2 + b^2 - c^2}{2ab} \right)}{c}$$

$$\text{LHS} = \frac{b^2 + c^2 - a^2}{abc} + \frac{a^2 + c^2 - b^2}{2abc} + \frac{a^2 + b^2 - c^2}{abc}$$

To add these fractions, take a common denominator of $2abc$:

$$\text{LHS} = \frac{2(b^2 + c^2 - a^2) + (a^2 + c^2 - b^2) + 2(a^2 + b^2 - c^2)}{2abc}$$

Expand the numerator:

$$\text{Numerator} = 2b^2 + 2c^2 - 2a^2 + a^2 + c^2 - b^2 + 2a^2 + 2b^2 - 2c^2$$

Group like terms:

$$\text{For } a^2: -2a^2 + a^2 + 2a^2 = a^2$$

$$\text{For } b^2: 2b^2 - b^2 + 2b^2 = 3b^2$$

$$\text{For } c^2: 2c^2 + c^2 - 2c^2 = c^2$$

So, the numerator simplifies to $a^2 + 3b^2 + c^2$.

$$\text{LHS} = \frac{a^2 + 3b^2 + c^2}{2abc}$$

Now, equate the simplified LHS and RHS:

$$\frac{a^2 + 3b^2 + c^2}{2abc} = \frac{a^2 + b^2}{abc}$$

Multiply both sides by $2abc$ (since $a, b, c \neq 0$ for a valid triangle):

$$a^2 + 3b^2 + c^2 = 2(a^2 + b^2)$$

$$a^2 + 3b^2 + c^2 = 2a^2 + 2b^2$$

Rearrange to solve for a relationship:

$$3b^2 - 2b^2 + c^2 = 2a^2 - a^2$$

$$b^2 + c^2 = a^2$$

This equation $a^2 = b^2 + c^2$ represents the Pythagorean theorem for a triangle.

This means the triangle is right-angled, and the hypotenuse is side a .

Therefore, the angle opposite to side a , which is $\angle A$, must be 90° or $\frac{\pi}{2}$ radians.

Step 4: Final Answer:

The angle A is $\frac{\pi}{2}$.

💡 Quick Tip

When an equation mixes sides and angles of a triangle linearly, applying the Cosine Rule immediately translates the whole equation into lengths, which is purely algebraic and usually simplifies down to recognizable forms like Pythagorean theorem or isosceles conditions.

42. If $f(1) = 1$, $f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is

- (A) 9
- (B) 12
- (C) 15
- (D) 33

Correct Answer: (D) 33

Solution:

Step 1: Understanding the Concept:

We need to find the derivative of a composite function evaluated at a specific point.

This requires the repeated application of the chain rule.

Step 2: Key Formula or Approach:

Chain rule for $h(x) = f(g(x))$ is $h'(x) = f'(g(x)) \cdot g'(x)$.

For a triple composition $f(f(f(x)))$, the derivative is $f'(f(f(x))) \cdot f'(f(x)) \cdot f'(x)$.

Power rule with chain rule for $(f(x))^n$ is $n(f(x))^{n-1} \cdot f'(x)$.

Step 3: Detailed Explanation:

Let the function be $g(x) = f(f(f(x))) + (f(x))^2$.

We need to find $g'(1)$.

First, find the general derivative $g'(x)$ using the chain rule:

$$g'(x) = \frac{d}{dx}[f(f(f(x)))] + \frac{d}{dx}[(f(x))^2]$$

$$g'(x) = f'(f(f(x))) \cdot \frac{d}{dx}[f(f(x))] + 2(f(x))^1 \cdot f'(x)$$

$$g'(x) = f'(f(f(x))) \cdot f'(f(x)) \cdot f'(x) + 2f(x)f'(x)$$

Now, evaluate this derivative at $x = 1$:

$$g'(1) = f'(f(f(1))) \cdot f'(f(1)) \cdot f'(1) + 2f(1)f'(1)$$

We are given the values: $f(1) = 1$ and $f'(1) = 3$.

Let's evaluate the nested function values step-by-step:

Inner-most: $f(1) = 1$

Middle: $f(f(1)) = f(1) = 1$

Outer: $f(f(f(1))) = f(1) = 1$

Now substitute these back into the expression for $g'(1)$:

$$g'(1) = f'(1) \cdot f'(1) \cdot f'(1) + 2 \cdot f(1) \cdot f'(1)$$

Substitute the numerical values $f(1) = 1$ and $f'(1) = 3$:

$$g'(1) = (3) \cdot (3) \cdot (3) + 2 \cdot (1) \cdot (3)$$

$$g'(1) = 27 + 6$$

$$g'(1) = 33$$

Step 4: Final Answer:

The value of the derivative is 33.

 Quick Tip

When dealing with multiple nested functions like $f(f(f(1)))$, evaluate them from the inside out before substituting into the derivative formula to avoid confusion. Notice how $f(1) = 1$ makes all the nested calls evaluate to 1, simplifying the problem tremendously.

43. If $y = \log_e x^3 + 3 \sin^{-1} x + kx^2$ and $y' \left(\frac{1}{2} \right) = 2\sqrt{3}$, then $k =$

- (A) 6
- (B) -6
- (C) $2\sqrt{3}$
- (D) 1

Correct Answer: (B) -6

Solution:

Step 1: Understanding the Concept:

We need to find the derivative of the given function y with respect to x , evaluate it at $x = 1/2$, and set it equal to the given value $2\sqrt{3}$ to solve for the unknown constant k .

Step 2: Key Formula or Approach:

Use logarithm property: $\log_e(x^n) = n \log_e x$.

Derivatives:

$$\begin{aligned}\frac{d}{dx}(\ln x) &= \frac{1}{x} \\ \frac{d}{dx}(\sin^{-1} x) &= \frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx}(kx^2) &= 2kx\end{aligned}$$

Step 3: Detailed Explanation:

The given function is:

$$y = \log_e(x^3) + 3 \sin^{-1} x + kx^2$$

First, simplify the logarithmic term using exponent properties:

$$y = 3 \ln x + 3 \sin^{-1} x + kx^2$$

Now, differentiate y with respect to x :

$$y' = \frac{dy}{dx} = 3 \left(\frac{1}{x} \right) + 3 \left(\frac{1}{\sqrt{1-x^2}} \right) + k(2x)$$

$$y' = \frac{3}{x} + \frac{3}{\sqrt{1-x^2}} + 2kx$$

We are given that $y' \left(\frac{1}{2} \right) = 2\sqrt{3}$. Substitute $x = \frac{1}{2}$ into the derivative:

$$y' \left(\frac{1}{2} \right) = \frac{3}{(1/2)} + \frac{3}{\sqrt{1 - (1/2)^2}} + 2k \left(\frac{1}{2} \right)$$

$$2\sqrt{3} = 6 + \frac{3}{\sqrt{1 - 1/4}} + k$$

$$2\sqrt{3} = 6 + \frac{3}{\sqrt{3/4}} + k$$

$$2\sqrt{3} = 6 + \frac{3}{\sqrt{3}/2} + k$$

$$2\sqrt{3} = 6 + \frac{6}{\sqrt{3}} + k$$

Rationalize the term $\frac{6}{\sqrt{3}}$:

$$\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

Substitute this back:

$$2\sqrt{3} = 6 + 2\sqrt{3} + k$$

Subtract $2\sqrt{3}$ from both sides:

$$0 = 6 + k$$

$$k = -6$$

Step 4: Final Answer:

The value of k is -6 .

💡 Quick Tip

Always simplify terms like $\log(x^3)$ to $3 \log x$ before differentiating. It reduces the chance of making a chain rule error.

44. In a triangle ABC with usual notations, if a, b, c are in arithmetic progression, then, $\tan \frac{A}{2} \cdot \tan \frac{C}{2} =$

- (A) 3
- (B) $\frac{1}{13}$
- (C) -3
- (D) $\frac{1}{3}$

Correct Answer: (D) $\frac{1}{3}$

Solution:

Step 1: Understanding the Concept:

We are given that the sides a, b, c of a triangle are in Arithmetic Progression (A.P.). This gives us a specific relation between them.

We need to evaluate a product of half-angle tangents. We should use the formulas expressing half-angle tangents in terms of the sides of the triangle.

Step 2: Key Formula or Approach:

If a, b, c are in A.P., then $2b = a + c$.

Half-angle formulas for tangent:

$$\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\tan \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

where s is the semi-perimeter, $s = \frac{a+b+c}{2}$.

Step 3: Detailed Explanation:

Let's express the product $\tan \frac{A}{2} \cdot \tan \frac{C}{2}$ in terms of sides:

$$\tan \frac{A}{2} \cdot \tan \frac{C}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \cdot \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

Combine the square roots:

$$= \sqrt{\frac{(s-b)(s-c)(s-a)(s-b)}{s(s-a)s(s-c)}}$$

Cancel common terms $(s-a)$ and $(s-c)$ from numerator and denominator:

$$= \sqrt{\frac{(s-b)^2}{s^2}}$$

Since $s = \frac{a+b+c}{2}$, we have $s > b$, so $s-b > 0$. Thus, the square root simplifies to:

$$= \frac{s-b}{s}$$

Now, use the given condition that a, b, c are in A.P.:

$$2b = a + c$$

We know the semi-perimeter is:

$$s = \frac{a + b + c}{2}$$

Substitute $a + c = 2b$ into the semi-perimeter formula:

$$s = \frac{(a + c) + b}{2} = \frac{2b + b}{2} = \frac{3b}{2}$$

Now substitute $s = \frac{3b}{2}$ into our simplified expression for the product:

$$\frac{s - b}{s} = \frac{\frac{3b}{2} - b}{\frac{3b}{2}}$$

$$= \frac{\frac{3b - 2b}{2}}{\frac{3b}{2}}$$

$$= \frac{\frac{b}{2}}{\frac{3b}{2}}$$

The $b/2$ terms cancel out:

$$= \frac{1}{3}$$

Step 4: Final Answer:

The value is $\frac{1}{3}$.

💡 Quick Tip

This is a standard property of triangles that is very useful to remember: If sides a, b, c are in A.P., then $\tan(A/2) \tan(C/2) = 1/3$. Conversely, if $\tan(A/2) \tan(C/2) = 1/3$, then a, b, c are in A.P.

45. If $\tan 3\theta = \cot \theta$, then $\theta =$

- (A) $\frac{(2n+1)\pi}{8}, n \in \mathbb{Z}$
 (B) $\frac{(2n+1)\pi}{4}, n \in \mathbb{Z}$

- (C) $\frac{(n+2)\pi}{3}, n \in \mathbb{Z}$
 (D) $n\pi, n \in \mathbb{Z}$

Correct Answer: (A) $\frac{(2n+1)\pi}{8}, n \in \mathbb{Z}$

Solution:

Step 1: Understanding the Concept:

We need to solve a general trigonometric equation.

The best approach is to convert both sides to the same trigonometric function, either both tangent or both cotangent.

Step 2: Key Formula or Approach:

Complementary angle identity: $\cot \theta = \tan \left(\frac{\pi}{2} - \theta \right)$.

General solution formula for $\tan x = \tan y$:

If $\tan x = \tan y$, then $x = n\pi + y$, where n is any integer ($n \in \mathbb{Z}$).

Step 3: Detailed Explanation:

Given equation:

$$\tan 3\theta = \cot \theta$$

Convert $\cot \theta$ to $\tan$ using the identity:

$$\tan 3\theta = \tan \left(\frac{\pi}{2} - \theta \right)$$

Now we have an equation of the form $\tan A = \tan B$.

Applying the general solution formula:

$$3\theta = n\pi + \left(\frac{\pi}{2} - \theta \right)$$

where $n \in \mathbb{Z}$.

Rearrange to solve for θ . Bring $-\theta$ to the left side:

$$3\theta + \theta = n\pi + \frac{\pi}{2}$$

$$4\theta = \frac{2n\pi + \pi}{2}$$

Factor out π in the numerator:

$$4\theta = \frac{(2n + 1)\pi}{2}$$

Divide by 4 to isolate θ :

$$\theta = \frac{(2n + 1)\pi}{2 \times 4}$$

$$\theta = \frac{(2n+1)\pi}{8}$$

where n is an integer.

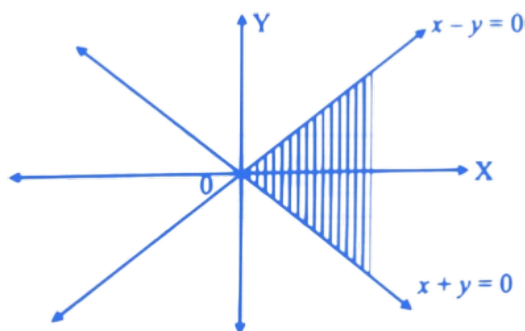
Step 4: Final Answer:

The general solution is $\theta = \frac{(2n+1)\pi}{8}, n \in \mathbb{Z}$.

💡 Quick Tip

Whenever you have an equation like $\sin x = \cos y$, $\tan x = \cot y$, or $\sec x = \csc y$, always use the complementary angle identities $(\pi/2 - \theta)$ to make the functions match before applying general solution formulas.

46. The shaded region in the following figure represents a solution set of



- (A) $x - y \geq 0, x + y \geq 0$
- (B) $x - y \leq 0, x + y \geq 0$
- (C) $x - y \geq 0, x + y \leq 0$
- (D) $x - y \leq 0, x + y \leq 0$

Correct Answer: (A) $x - y \geq 0, x + y \geq 0$

Solution:

Step 1: Understanding the Concept:

The problem asks to identify the system of linear inequalities that corresponds to a given shaded region on a graph.

The boundaries of the region are defined by equations. The shaded area represents the inequalities.

We can determine the correct inequality signs by picking a test point clearly inside the shaded region and plugging it into the options.

Step 2: Key Formula or Approach:

1. Identify the boundary lines from the graph.

2. Choose a test point (x_0, y_0) inside the shaded region.
3. Substitute the coordinates of the test point into the given inequalities. The set of inequalities that are all true for this point is the correct answer.

Step 3: Detailed Explanation:

From the figure, the boundary lines are given as:

Line 1: $x - y = 0$ (which is $y = x$)

Line 2: $x + y = 0$ (which is $y = -x$)

The shaded region lies to the right of the y-axis, between these two lines.

Let's select a simple test point clearly inside the shaded region, for example, a point on the positive x-axis.

Let's choose the test point $(1, 0)$.

Now, test this point $(x = 1, y = 0)$ against each option:

(A) $x - y \geq 0 \Rightarrow 1 - 0 \geq 0 \Rightarrow 1 \geq 0$ (True)

$x + y \geq 0 \Rightarrow 1 + 0 \geq 0 \Rightarrow 1 \geq 0$ (True)

Both conditions are satisfied. This is the likely answer.

(B) $x - y \leq 0 \Rightarrow 1 - 0 \leq 0 \Rightarrow 1 \leq 0$ (False)

No need to check the second inequality.

(C) $x - y \geq 0 \Rightarrow 1 - 0 \geq 0 \Rightarrow 1 \geq 0$ (True)

$x + y \leq 0 \Rightarrow 1 + 0 \leq 0 \Rightarrow 1 \leq 0$ (False)

(D) $x - y \leq 0 \Rightarrow 1 - 0 \leq 0 \Rightarrow 1 \leq 0$ (False)

Only option (A) holds true for our test point.

Alternatively, we can analyze geometrically:

The region is below the line $y = x$, which translates to $y \leq x$ or $x - y \geq 0$.

The region is above the line $y = -x$, which translates to $y \geq -x$ or $x + y \geq 0$.

Combining these confirms the result.

Step 4: Final Answer:

The solution set is $x - y \geq 0, x + y \geq 0$.

💡 Quick Tip

The "Test Point Method" is the fastest and most reliable way to solve linear inequality graph problems. Always pick a point that is obviously inside the shaded region, like $(1, 0)$ or $(0, 1)$, making calculation trivial.

47. With usual notations, in a triangle ABC , if θ is any real number, then $a \cos(B - \theta) + b \cos(A + \theta)$ is

- (A) $a \cos \theta$
- (B) $b \cos \theta$
- (C) $\cos \theta$
- (D) $c \cos \theta$

Correct Answer: (D) $c \cos \theta$

Solution:

Step 1: Understanding the Concept:

We are given an expression involving sides and angles of a triangle with an arbitrary angle θ . The approach is to expand the compound trigonometric terms using addition/subtraction formulas and then simplify using standard triangle properties like the Sine Rule and Projection Formula.

Step 2: Key Formula or Approach:

Compound angle formulas:

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\text{Sine Rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow a \sin B = b \sin A.$$

$$\text{Projection Formula: } c = a \cos B + b \cos A.$$

Step 3: Detailed Explanation:

Let the given expression be $E = a \cos(B - \theta) + b \cos(A + \theta)$.

Expand the cosine terms using the compound angle formulas:

$$E = a(\cos B \cos \theta + \sin B \sin \theta) + b(\cos A \cos \theta - \sin A \sin \theta)$$

Distribute the sides a and b :

$$E = a \cos B \cos \theta + a \sin B \sin \theta + b \cos A \cos \theta - b \sin A \sin \theta$$

Group the terms by $\cos \theta$ and $\sin \theta$:

$$E = (a \cos B + b \cos A) \cos \theta + (a \sin B - b \sin A) \sin \theta$$

Now apply properties of triangles.

From the Sine Rule, we know that $\frac{a}{\sin A} = \frac{b}{\sin B}$, which cross-multiplies to $a \sin B = b \sin A$. Therefore, the coefficient of $\sin \theta$ becomes zero:

$$a \sin B - b \sin A = 0.$$

From the Projection Formula, we know that $a \cos B + b \cos A$ is equal to side c .

Therefore, the coefficient of $\cos \theta$ becomes c :

$$a \cos B + b \cos A = c.$$

Substitute these findings back into the expression for E :

$$E = (c) \cos \theta + (0) \sin \theta$$

$$E = c \cos \theta$$

Step 4: Final Answer:

The expression is equal to $c \cos \theta$.

💡 Quick Tip

When an expression involves terms like $a \cos(\dots)$ and $b \cos(\dots)$, expanding them will almost always lead to combinations where the Sine Rule ($a \sin B - b \sin A = 0$) and Projection Formula ($a \cos B + b \cos A = c$) perfectly simplify the result.

48. If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, then $A^T A^{-1} =$

- (A) $\begin{bmatrix} \cos 2x & -\sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$
 (B) $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$
 (C) $\begin{bmatrix} -\cos 2x & \sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$
 (D) $\begin{bmatrix} -\cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$

Correct Answer: (B) $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

Solution:**Step 1: Understanding the Concept:**

We need to calculate the matrix product of the transpose of A and the inverse of A.

First, find A^T . Then find the determinant $|A|$ and adjugate matrix $\text{adj}(A)$ to compute A^{-1} . Finally, multiply the two matrices.

Step 2: Key Formula or Approach:

For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$:

Transpose $A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$.

Determinant $|A| = ad - bc$.

Inverse $A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Trigonometric identities:

$$\frac{1 - \tan^2 x}{1 + \tan^2 x} = \cos 2x$$

$$\frac{2 \tan x}{1 + \tan^2 x} = \sin 2x$$

Step 3: Detailed Explanation:

Let $t = \tan x$ for simplicity.

$$A = \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}$$

The transpose is:

$$A^T = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

The determinant is:

$$|A| = (1)(1) - (t)(-t) = 1 + t^2$$

The inverse is:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} 1 & -t \\ -(-t) & 1 \end{bmatrix} = \frac{1}{1+t^2} \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

Notice that $A^{-1} = \frac{1}{1+t^2} A^T$.

Now, calculate the product $A^T A^{-1}$:

$$A^T A^{-1} = A^T \left(\frac{1}{1+t^2} A^T \right) = \frac{1}{1+t^2} (A^T A^T)$$

Let's multiply A^T by itself:

$$\begin{aligned} A^T A^T &= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} \\ &= \begin{bmatrix} (1)(1) + (-t)(t) & (1)(-t) + (-t)(1) \\ (t)(1) + (1)(t) & (t)(-t) + (1)(1) \end{bmatrix} \\ &= \begin{bmatrix} 1-t^2 & -2t \\ 2t & 1-t^2 \end{bmatrix} \end{aligned}$$

Now, multiply by the scalar $\frac{1}{1+t^2}$:

$$A^T A^{-1} = \frac{1}{1+t^2} \begin{bmatrix} 1-t^2 & -2t \\ 2t & 1-t^2 \end{bmatrix} = \begin{bmatrix} \frac{1-t^2}{1+t^2} & \frac{-2t}{1+t^2} \\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{bmatrix}$$

Substitute $t = \tan x$ back into the matrix:

$$A^T A^{-1} = \begin{bmatrix} \frac{1-\tan^2 x}{1+\tan^2 x} & -\frac{2 \tan x}{1+\tan^2 x} \\ \frac{2 \tan x}{1+\tan^2 x} & \frac{1-\tan^2 x}{1+\tan^2 x} \end{bmatrix}$$

Using the double angle trigonometric identities:

$$A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$$

Step 4: Final Answer:

The resulting matrix is $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

 Quick Tip

Recognizing the matrix $\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ is related to a scaled rotation matrix. Factoring out $1/\cos x$ gives a matrix with entries like $\cos x, \sin x$. This connection often hints that double angle formulas will emerge upon squaring or multiplying by transpose.

49. The sum of the degree and order of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \sqrt[5]{\frac{dy}{dx}} - 5$ is

- (A) 1
- (B) 3
- (C) 5
- (D) 7

Correct Answer: (D) 7

Solution:

Step 1: Understanding the Concept:

The order of a differential equation is the order of the highest derivative present.

The degree is the power of the highest order derivative when the equation is expressed as a polynomial in all its derivatives. This means we must eliminate all fractional powers (radicals) involving derivatives.

Step 2: Key Formula or Approach:

Given equation: $\left(\frac{d^2y}{dx^2}\right)^{1/2} = \left(\frac{dy}{dx}\right)^{1/5} - 5$.

Identify highest derivative for order.

Algebraically manipulate the equation to remove radicals. If the equation is $A^{1/p} = B^{1/q} + C$, isolating terms and raising to LCM of powers isn't always straight forward. Let's substitute variables to simplify algebra.

Step 3: Detailed Explanation:

Let $y'' = \frac{d^2y}{dx^2}$ and $y' = \frac{dy}{dx}$.

The equation is $\sqrt{y''} = (y')^{1/5} - 5$.

The highest order derivative is y'' , which is a second-order derivative.

Therefore, the Order = 2.

To find the degree, we need to make it a polynomial in y'' and y' .

Let's isolate y' first, as it has a higher root.

$$\sqrt{y''} + 5 = (y')^{1/5}$$

Raise both sides to the power of 5:

$$(\sqrt{y''} + 5)^5 = y'$$

Now, expand the left side using the Binomial Theorem:

$$(\sqrt{y''})^5 + \binom{5}{1}(\sqrt{y''})^4(5) + \binom{5}{2}(\sqrt{y''})^3(5^2) + \binom{5}{3}(\sqrt{y''})^2(5^3) + \binom{5}{4}(\sqrt{y''})^1(5^4) + 5^5 = y'$$

$$(y'')^{5/2} + 25(y'')^2 + 250(y'')^{3/2} + 1250y'' + 3125(y'')^{1/2} + 3125 = y'$$

We still have fractional powers of y'' : $1/2, 3/2, 5/2$. Let's gather all terms with fractional powers on one side and integer powers on the other.

$$(y'')^{5/2} + 250(y'')^{3/2} + 3125(y'')^{1/2} = y' - 25(y'')^2 - 1250y'' - 3125$$

Factor out $(y'')^{1/2}$ on the left side:

$$(y'')^{1/2}[(y'')^2 + 250y'' + 3125] = y' - 25(y'')^2 - 1250y'' - 3125$$

Now, to eliminate the fractional power $(y'')^{1/2}$, we square both sides:

$$\left((y'')^{1/2}[(y'')^2 + 250y'' + 3125]\right)^2 = (y' - 25(y'')^2 - 1250y'' - 3125)^2$$

$$y''[(y'')^2 + 250y'' + 3125]^2 = (y' - 25(y'')^2 - 1250y'' - 3125)^2$$

This equation is now a polynomial in y'' and y' .

The degree is the highest exponent of the highest order derivative, y'' .

Let's find the term with the highest power of y'' on both sides.

On the left side: The highest power term comes from multiplying y'' with the square of the highest power term in the bracket.

LHS highest power term $\approx y'' \cdot ((y'')^2)^2 = y'' \cdot (y'')^4 = (y'')^5$.

On the right side: The highest power term comes from squaring the highest power term of y'' .

RHS highest power term $\approx (-25(y'')^2)^2 = 625(y'')^4$.

Comparing both sides, the overall highest power of y'' in the entire expanded equation is 5.

Therefore, the Degree = 5.

The sum of degree and order is $5 + 2 = 7$.

Step 4: Final Answer:

The sum of the degree and order is 7.

Quick Tip

To quickly find the degree of an equation like $f(y'')^{1/m} = g(y')^{1/n} + C$, it's not simply the LCM of m and n . You must isolate the highest order derivative's radical and repeatedly raise to powers to clear it. In this case, $(\sqrt{y''} + 5)^5 = y'$ yields a $(y'')^{5/2}$ term. Squaring again makes it $(y'')^5$, thus degree is 5.

50. The differential equation whose solution represents the family $x^2y = 4e^x + c$, where c is an arbitrary constant, is

- (A) $x \frac{dy}{dx} + xy = 0$
(B) $x^2 \frac{dy}{dx} + (2x - xy) = 0$
(C) $x \frac{dy}{dx} + (x - 2)y = 0$
(D) $x^2 \frac{dy}{dx} + 2xy - 4e^x = 0$

Correct Answer: (D) $x^2 \frac{dy}{dx} + 2xy - 4e^x = 0$

Solution:

Step 1: Understanding the Concept:

To form a differential equation from a given family of curves with one arbitrary constant, we need to differentiate the given equation with respect to the independent variable (x) once. The goal is to obtain an equation that does not contain the arbitrary constant ' c '.

Step 2: Key Formula or Approach:

Given equation: $x^2y = 4e^x + c$.

Differentiate both sides with respect to x using the product rule on the left side.

The constant ' c ' will disappear upon differentiation since $\frac{d}{dx}(c) = 0$.

Step 3: Detailed Explanation:

The given family of curves is:

$$x^2y = 4e^x + c$$

Differentiate both sides with respect to x :

$$\frac{d}{dx}(x^2 \cdot y) = \frac{d}{dx}(4e^x + c)$$

Apply the product rule ($u'v + uv'$) to the left side:

$$\left(\frac{d}{dx}(x^2) \right) \cdot y + x^2 \cdot \left(\frac{d}{dx}(y) \right) = \frac{d}{dx}(4e^x) + \frac{d}{dx}(c)$$

$$(2x) \cdot y + x^2 \cdot \frac{dy}{dx} = 4e^x + 0$$

Rearrange the terms to match the format of the given options:

$$x^2 \frac{dy}{dx} + 2xy = 4e^x$$

Bring $4e^x$ to the left side:

$$x^2 \frac{dy}{dx} + 2xy - 4e^x = 0$$

This equation matches option (D).

Step 4: Final Answer:

The differential equation is $x^2 \frac{dy}{dx} + 2xy - 4e^x = 0$.

 Quick Tip

If an equation is explicitly of the form $f(x, y) = g(x) + c$, forming the differential equation is trivial because differentiating once immediately annihilates the constant without needing to substitute it back from the original equation.

Physics

1. In hydrogen spectrum, the ratio of wavelengths of the last line of Lyman series and that of the last line of Balmer series is

- (A) 1
- (B) 0.5
- (C) 0.25
- (D) 0.2

Correct Answer: (C) 0.25

Solution:

Step 1: Understanding the Concept:

The wavelengths of spectral lines in the hydrogen emission spectrum are calculated using the Rydberg formula.

The 'last line' of any spectral series, also known as the series limit, corresponds to the shortest possible wavelength in that series.

This shortest wavelength occurs when an electron transitions from the highest possible energy level ($n_2 = \infty$) down to the base principal quantum number (n_1) for that series.

Step 2: Key Formula or Approach:

The Rydberg formula is given by:

$$\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where R is the Rydberg constant, n_1 is the lower energy level, and n_2 is the higher energy level

from which the electron transitions.

Step 3: Detailed Explanation:

For the Lyman series, the base energy level is $n_1 = 1$.

For the last line of the Lyman series, the transition is from $n_2 = \infty$.

Let its wavelength be λ_L . Substituting these values into the formula:

$$\frac{1}{\lambda_L} = R \left(\frac{1}{1^2} - \frac{1}{\infty^2} \right) = R(1 - 0) = R$$

$$\Rightarrow \lambda_L = \frac{1}{R}$$

For the Balmer series, the base energy level is $n_1 = 2$.

For the last line of the Balmer series, the transition is from $n_2 = \infty$.

Let its wavelength be λ_B . Substituting these values into the formula:

$$\frac{1}{\lambda_B} = R \left(\frac{1}{2^2} - \frac{1}{\infty^2} \right) = R \left(\frac{1}{4} - 0 \right) = \frac{R}{4}$$

$$\Rightarrow \lambda_B = \frac{4}{R}$$

Now, we must find the ratio of these two wavelengths:

$$\text{Ratio} = \frac{\lambda_L}{\lambda_B} = \frac{\frac{1}{R}}{\frac{4}{R}} = \frac{1}{4} = 0.25$$

Step 4: Final Answer:

The ratio of the wavelengths is 0.25.

💡 Quick Tip

For the series limit (last line) of any hydrogen spectral series, $n_2 = \infty$, which makes the term $1/n_2^2 = 0$. This implies that the wavelength λ is directly proportional to n_1^2 .

2. For a perfectly black body, coefficient of emission is

- (A) zero.
- (B) unity.
- (C) less than one (non-zero).
- (D) infinity.

Correct Answer: (B) unity.

Solution:**Step 1: Understanding the Concept:**

A perfectly black body is defined as an ideal physical body that absorbs all incident electromagnetic radiation, regardless of its frequency or angle of incidence.

The coefficient of emission (or emissivity), denoted by e , is a measure of a material's ability to emit infrared energy compared to a black body.

Step 2: Key Formula or Approach:

According to Kirchhoff's law of thermal radiation, for an arbitrary body in thermal equilibrium with its surroundings, its emissivity (e) is equal to its absorptivity (a).

Mathematically, this is expressed as:

$$e = a$$

Step 3: Detailed Explanation:

By definition, a perfectly black body is a perfect absorber of radiation.

This means it absorbs 100% of the radiation that falls on it.

Therefore, its coefficient of absorption (a) is exactly 1.

Using Kirchhoff's law ($e = a$), it directly follows that the coefficient of emission (e) for a perfectly black body must also be exactly 1.

A value of 1 is referred to as "unity".

Step 4: Final Answer:

The coefficient of emission for a perfectly black body is unity.

💡 Quick Tip

Remember the theoretical limits: for a perfectly black body $a = 1$ and $e = 1$. For a perfectly reflecting body $a = 0$ and $e = 0$. Real objects have values between 0 and 1.

3. The potential difference ($V_A - V_B$) between the points A and B in the given figure is



- (A) 6 V
- (B) -3 V
- (C) 9 V
- (D) 3 V

Correct Answer: (C) 9 V

Solution:**Step 1: Understanding the Concept:**

To find the potential difference between two arbitrary points in a circuit branch, we can apply Kirchhoff's Voltage Law (KVL) by traversing the path from the starting point to the end point. We sum up all the potential drops and gains along the path.

Step 2: Key Formula or Approach:

When traversing the circuit in the direction of the current I :

1. The potential change across a resistor R is $-IR$ (a drop).
2. The potential change across a battery from the positive terminal to the negative terminal is $-V$ (a drop).

The governing equation from point A to point B is:

$$V_A + \sum (\text{Potential Changes}) = V_B$$

Step 3: Detailed Explanation:

Let's analyze the given circuit branch traversing from point A to point B.

The current I is given as 2 A, flowing from A towards B.

We start at point A with an initial potential V_A .

Moving across the first resistor ($R_1 = 2 \Omega$) in the direction of the current results in a potential drop:

$$\Delta V_{R1} = -I \times R_1 = -(2 \text{ A}) \times (2 \Omega) = -4 \text{ V}$$

Next, we cross the battery. We are moving from the longer parallel line (+) to the shorter thicker line (-).

This indicates a potential drop equal to the electromotive force of the battery:

$$\Delta V_{\text{battery}} = -3 \text{ V}$$

Finally, moving across the second resistor ($R_2 = 1 \Omega$) in the direction of the current results in another potential drop:

$$\Delta V_{R2} = -I \times R_2 = -(2 \text{ A}) \times (1 \Omega) = -2 \text{ V}$$

Equating this to the potential at point B (V_B), we write the full equation:

$$V_A - 4 \text{ V} - 3 \text{ V} - 2 \text{ V} = V_B$$

$$V_A - 9 \text{ V} = V_B$$

Rearranging the terms to find the required difference ($V_A - V_B$):

$$V_A - V_B = 9 \text{ V}$$

Step 4: Final Answer:

The potential difference is 9 V.

💡 Quick Tip

Always strictly follow a sign convention. If you travel with the current, resistors cause a drop (negative). If you hit the positive terminal of a battery first, it's a drop (negative). Consistently applying this prevents sign errors.

4. Which one of the following person is in an inertial frame of reference?

- (A) A person (man) in a train which is slowing down to stop.
- (B) A person (child) revolving in a merry-go-round.
- (C) A person (driver) in a bus which is moving with constant velocity.
- (D) A person (pilot) in an aeroplane which is taking off.

Correct Answer: (C) A person (driver) in a bus which is moving with constant velocity.

Solution:

Step 1: Understanding the Concept:

An inertial frame of reference is defined as a frame of reference that is not undergoing any acceleration.

In such a frame, Newton's first law of motion (the law of inertia) holds true without the need for fictitious forces.

A non-inertial frame is one that is accelerating or rotating.

Step 2: Key Formula or Approach:

The defining condition for an inertial frame is that the acceleration vector is zero:

$$\vec{a} = \frac{d\vec{v}}{dt} = 0$$

This means the velocity vector $\vec{v}$ must be constant in both magnitude (speed) and direction.

Step 3: Detailed Explanation:

Let's systematically evaluate each given option based on the condition $\vec{a} = 0$:

(A) A train slowing down to stop is experiencing deceleration. Because its speed is changing, $\vec{a} \neq 0$. This is a non-inertial frame.

(B) A merry-go-round revolves in a circular path. Even if the speed is constant, the direction of motion is continuously changing, resulting in a centripetal acceleration. Because $\vec{a} \neq 0$, this

is a non-inertial frame.

(C) A bus moving with a constant velocity has both constant speed and a constant direction of motion in a straight line. Here, $\vec{a} = 0$. This perfectly describes an inertial frame.

(D) An aeroplane taking off is increasing its speed to gain lift. Because its speed is changing rapidly, $\vec{a} \neq 0$. This is a non-inertial frame.

Step 4: Final Answer:

The driver in a bus moving with constant velocity is in an inertial frame of reference.

💡 Quick Tip

Any change in velocity—whether it's speeding up, slowing down, or changing direction (like turning or revolving)—means the frame is accelerating and therefore non-inertial.

5. Two discs A and B of same material and thickness have radii R and $3R$ respectively. Their moments of inertia about their axis will be in the ratio

- (A) 3 : 1
- (B) 1 : 9
- (C) 1 : 81
- (D) 1 : 27

Correct Answer: (C) 1 : 81

Solution:

Step 1: Understanding the Concept:

The moment of inertia (I) of a uniform solid disc rotating about its central perpendicular axis depends on its mass (M) and its radius (r).

If discs are made of the same material and have the same thickness, their masses are not constant; rather, mass depends on the volume, which in turn depends on the radius.

Step 2: Key Formula or Approach:

The standard formula for the moment of inertia of a disc is:

$$I = \frac{1}{2}Mr^2$$

The mass M can be expressed in terms of density (ρ) and volume (V):

$$M = \rho \times V$$

For a disc with radius r and uniform thickness t , the volume is $V = \pi r^2 t$. Therefore, mass becomes:

$$M = \rho(\pi r^2 t)$$

Step 3: Detailed Explanation:

Substitute the expression for mass into the moment of inertia formula:

$$I = \frac{1}{2}(\rho\pi r^2 t)r^2 = \frac{1}{2}\rho\pi tr^4$$

Since both discs A and B are made of the same material, they have the same density (ρ).

They are also given to have the same thickness (t).

Therefore, the entire term ($\frac{1}{2}\rho\pi t$) is a constant for both discs.

This establishes a direct proportionality between the moment of inertia and the fourth power of the radius:

$$I \propto r^4$$

We are given the radii of the two discs: $r_A = R$ and $r_B = 3R$.

Now, we set up the ratio of their moments of inertia:

$$\frac{I_A}{I_B} = \frac{r_A^4}{r_B^4} = \left(\frac{r_A}{r_B}\right)^4$$

Substituting the given radii:

$$\frac{I_A}{I_B} = \left(\frac{R}{3R}\right)^4 = \left(\frac{1}{3}\right)^4 = \frac{1}{81}$$

Step 4: Final Answer:

The ratio of their moments of inertia is 1 : 81.

💡 Quick Tip

When comparing properties of objects made of the "same material and thickness", always expand mass into density $\times$ area $\times$ thickness. This quickly reveals how mass scales with the linear dimensions.

6. When an observer moves towards a stationary source with velocity ' V_1 ', the apparent frequency of emitted note is ' F_1 '. When observer moves away from stationary source with velocity ' V_1 ' the apparent frequency is ' F_2 '. If ' v ' is velocity of sound in air and $\frac{F_1}{F_2} = 2$, then $\frac{v}{V_1}$ is equal to

- (A) 6
(B) 5

- (C) 3
(D) 4

Correct Answer: (C) 3

Solution:

Step 1: Understanding the Concept:

This problem applies the Doppler effect for sound waves.

The apparent frequency heard by an observer changes depending on the relative motion between the observer and the source of the sound.

Step 2: Key Formula or Approach:

The general formula for apparent frequency f' due to the Doppler effect is:

$$f' = f_0 \left(\frac{v \pm v_o}{v \mp v_s} \right)$$

where f_0 is the true frequency emitted, v is the speed of sound, v_o is the observer's velocity, and v_s is the source's velocity.

Step 3: Detailed Explanation:

In the given problem, the source is stationary, which means $v_s = 0$.

Case 1: The observer moves towards the source with velocity V_1 .

Approaching the source increases the apparent frequency, so we use the plus sign (+) in the numerator.

$$F_1 = f_0 \left(\frac{v + V_1}{v} \right)$$

Case 2: The observer moves away from the source with velocity V_1 .

Receding from the source decreases the apparent frequency, so we use the minus sign (−) in the numerator.

$$F_2 = f_0 \left(\frac{v - V_1}{v} \right)$$

We are given the condition that the ratio of these frequencies is:

$$\frac{F_1}{F_2} = 2$$

Substituting our expressions for F_1 and F_2 into this ratio:

$$\frac{f_0 \left(\frac{v+V_1}{v} \right)}{f_0 \left(\frac{v-V_1}{v} \right)} = 2$$

The terms f_0 and the denominator v cancel out:

$$\frac{v + V_1}{v - V_1} = 2$$

Now, solve for the relationship between v and V_1 . Cross-multiply:

$$v + V_1 = 2(v - V_1)$$

$$v + V_1 = 2v - 2V_1$$

Rearrange the terms to group v and V_1 :

$$V_1 + 2V_1 = 2v - v$$

$$3V_1 = v$$

Finally, we want the ratio $\frac{v}{V_1}$:

$$\frac{v}{V_1} = 3$$

Step 4: Final Answer:

The value of the ratio is 3.

💡 Quick Tip

To avoid confusion with signs in the Doppler equation, remember this simple rule: any motion that brings the source and observer closer together increases the frequency. Any motion that moves them apart decreases the frequency.

7. For the following reaction, the particle ' x ' is ${}_6\text{C}^{11} \longrightarrow {}_5\text{B}^{11} + \beta + \text{X}$

- (A) proton
- (B) neutrino
- (C) anti neutrino
- (D) neutron

Correct Answer: (B) neutrino

Solution:

Step 1: Understanding the Concept:

In any balanced nuclear reaction, both the total mass number (represented by A , the superscript) and the total atomic number (represented by Z , the subscript) must be conserved.

By applying these conservation laws, we can deduce the properties of unknown particles emitted during radioactive decay.

Step 2: Key Formula or Approach:

The nuclear reaction is:



We enforce two rules:

1. Conservation of mass number: $A_1 = A_2 + A_\beta + A_X$
2. Conservation of atomic number: $Z_1 = Z_2 + Z_\beta + Z_X$

Step 3: Detailed Explanation:

Let's analyze the given equation: ${}_6\text{C}^{11} \longrightarrow {}_5\text{B}^{11} + \beta + \text{X}$.

First, apply the conservation of mass number (A):

$$11 = 11 + A_\beta + A_X$$

A beta particle (whether an electron or a positron) has a mass number of zero ($A_\beta = 0$).

$$11 = 11 + 0 + A_X \implies A_X = 0$$

The unknown particle X has no mass number.

Next, apply the conservation of atomic number (Z):

$$6 = 5 + Z_\beta + Z_X$$

Here, we must identify the type of beta decay. Since the atomic number decreases from 6 to 5 (a proton turns into a neutron), the emitted β particle must carry a positive charge to balance the equation.

Thus, the β particle is a positron (β^+ or ${}_{+1}e^0$), which means $Z_\beta = +1$.

$$6 = 5 + 1 + Z_X$$

$$6 = 6 + Z_X \implies Z_X = 0$$

The unknown particle X has a mass number of 0 and an atomic charge of 0.

In nuclear physics, β^+ (positron) emission is always accompanied by the emission of an electron neutrino (ν_e) to conserve lepton number.

Conversely, β^- (electron) emission is accompanied by an antineutrino ($\bar{\nu}_e$).

Since this is a β^+ decay, particle X must be a neutrino.

Step 4: Final Answer:

The particle 'x' is a neutrino.

💡 Quick Tip

A useful mnemonic: Beta-minus decay emits an Anti-neutrino (both have negative connotations). Beta-plus decay emits a regular neutrino.

8. In fundamental mode, the time required for the sound wave to reach up to closed end of a pipe filled with air is 't' second. The frequency of vibration of air column is (Neglect end correction)

- (A) $(4t)^{-1}$
- (B) $(2t)^{-1}$
- (C) $4t$
- (D) $2t$

Correct Answer: (A) $(4t)^{-1}$

Solution:**Step 1: Understanding the Concept:**

When an air column inside a pipe closed at one end vibrates in its fundamental mode, it forms a stationary wave.

The simplest standing wave pattern has an antinode at the open end and a node at the closed end.

This pattern encompasses exactly one-quarter of a full wavelength.

Step 2: Key Formula or Approach:

For a pipe of length L closed at one end, the fundamental wavelength λ is given by:

$$L = \frac{\lambda}{4} \implies \lambda = 4L$$

The frequency (f) of the wave is related to wave speed (v) and wavelength (λ) by the equation:

$$f = \frac{v}{\lambda}$$

Additionally, the speed of the wave is the distance traveled divided by the time taken. If it takes time t to travel the length L of the pipe:

$$v = \frac{L}{t}$$

Step 3: Detailed Explanation:

The problem states that the sound wave travels from the open end to the closed end in time 't'.

The distance it covers is the length of the pipe, L .

Thus, we can express the speed of sound as $v = \frac{L}{t}$.

We also established that for the fundamental mode, the wavelength is $\lambda = 4L$.

Substitute these expressions for v and λ into the fundamental frequency formula:

$$f = \frac{v}{\lambda} = \frac{\left(\frac{L}{t}\right)}{4L}$$

Simplifying the resulting fraction:

$$f = \frac{L}{t \times 4L}$$

The L terms cancel out from the numerator and the denominator:

$$f = \frac{1}{4t}$$

This can be expressed using a negative exponent format as found in the options:

$$f = (4t)^{-1}$$

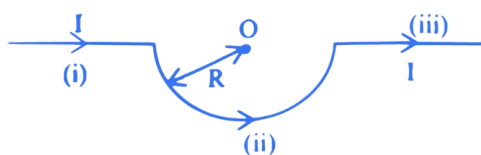
Step 4: Final Answer:

The frequency of vibration is $(4t)^{-1}$.

💡 Quick Tip

You can also think about this conceptually using periods. A full wave cycle is 4 times the length of the closed pipe. If it takes time t to travel one length, it takes $4t$ to complete one full cycle. Time for one cycle is the period $T = 4t$. Frequency is $1/T = 1/(4t)$.

9. A wire has three different sections as shown in figure. The magnitude of the magnetic field produced at the centre 'O' of the semicircle by three sections together is (μ_0 = permeability of free space)



- (A) $\frac{\mu_0 I}{4R}$
- (B) $\frac{\mu_0 I}{2R}$
- (C) $\frac{\mu_0 I}{4\pi R}$
- (D) $\frac{\mu_0 I}{2\pi R}$

Correct Answer: (A) $\frac{\mu_0 I}{4R}$

Solution:

Step 1: Understanding the Concept:

To find the total magnetic field at point O caused by a complex wire shape, we use the principle of superposition.

We will evaluate the magnetic field contribution from each distinct section of the wire independently and sum them up.

Step 2: Key Formula or Approach:

The Biot-Savart Law determines the magnetic field.

1. For a straight current-carrying wire, if the point of interest lies precisely on the line extending the wire, the angle between the current element $d\vec{l}$ and the position vector $\vec{r}$ is 0° or 180° . Since $\sin(0^\circ) = \sin(180^\circ) = 0$, the magnetic field contribution is zero.
2. For a circular arc of radius R subtending an angle θ at the center, the magnetic field at the center is $B = \frac{\mu_0 I}{4\pi R} \theta$.

Step 3: Detailed Explanation:

Let's systematically calculate the field from each of the three sections at center O .

Section (i): This is a straight horizontal wire segment. As indicated by the dashed line in the figure, its extended axis passes directly through the point O .

Consequently, its contribution to the magnetic field at O is zero: $B_1 = 0$.

Section (iii): Similar to the first section, this is a straight wire segment whose extended axis also passes straight through O .

Therefore, its contribution to the magnetic field at O is also zero: $B_3 = 0$.

Section (ii): This section is a semicircular arc with radius R .

A complete circle subtends an angle of 2π radians, so a semicircle subtends an angle of π radians ($\theta = \pi$).

Applying the formula for a circular arc:

$$B_2 = \frac{\mu_0 I}{4\pi R} \times \pi = \frac{\mu_0 I}{4R}$$

Alternatively, recalling that the field at the center of a full circular loop is $\frac{\mu_0 I}{2R}$, a semicircle generates exactly half of this field:

$$B_2 = \frac{1}{2} \left(\frac{\mu_0 I}{2R} \right) = \frac{\mu_0 I}{4R}$$

The total magnetic field is the vector sum of these contributions. Since all segments lie in the same plane, their fields (if non-zero) would be collinear.

$$B_{\text{total}} = B_1 + B_2 + B_3 = 0 + \frac{\mu_0 I}{4R} + 0 = \frac{\mu_0 I}{4R}$$

Step 4: Final Answer:

The magnitude of the magnetic field at O is $\frac{\mu_0 I}{4R}$.

💡 Quick Tip

Whenever a problem asks for the magnetic field at a point, immediately check if any straight wire segments point directly at or away from that point. You can instantly assign a zero contribution to those parts, saving calculation time.

10. A student measures time for 20 oscillations of a simple pendulum as 30 s, 32 s, 35 s and 35 s . If the minimum division in the measuring clock is 1 s , then correct mean time (in second) is

- (A) (33 ± 2)
- (B) (32 ± 3)
- (C) (33 ± 3)
- (D) (32 ± 2)

Correct Answer: (A) (33 ± 2)

Solution:**Step 1: Understanding the Concept:**

When an experiment yields multiple readings for a single physical quantity, the most probable value is represented by the arithmetic mean.

To quantify the uncertainty or precision of the measurements, we calculate the mean absolute error.

The final result is standardized in the format: Mean Value $\pm$ Mean Absolute Error.

Step 2: Key Formula or Approach:

1. The arithmetic mean (t_m) is calculated as:

$$t_m = \frac{t_1 + t_2 + \cdots + t_n}{n}$$

2. The absolute error for each reading (Δt_i) is the absolute difference from the mean:

$$\Delta t_i = |t_m - t_i|$$

3. The mean absolute error (Δt_m) is the average of these absolute errors:

$$\Delta t_m = \frac{\Delta t_1 + \Delta t_2 + \cdots + \Delta t_n}{n}$$

Step 3: Detailed Explanation:

The recorded time measurements are: $t_1 = 30$ s, $t_2 = 32$ s, $t_3 = 35$ s, $t_4 = 35$ s.

First, we find the mean time (t_m):

$$t_m = \frac{30 + 32 + 35 + 35}{4} = \frac{132}{4} = 33 \text{ s}$$

Next, we evaluate the absolute error for each individual measurement:

$$\Delta t_1 = |33 - 30| = 3 \text{ s}$$

$$\Delta t_2 = |33 - 32| = 1 \text{ s}$$

$$\Delta t_3 = |33 - 35| = 2 \text{ s}$$

$$\Delta t_4 = |33 - 35| = 2 \text{ s}$$

Now, calculate the mean absolute error (Δt_m):

$$\Delta t_m = \frac{3 + 1 + 2 + 2}{4} = \frac{8}{4} = 2 \text{ s}$$

The minimum division (least count) of the clock is 1 s. Since our calculated mean absolute error (2 s) is larger than the least count, the error in measurement dominates the precision limit.

The final correct mean time is expressed by combining the mean value and the mean absolute error.

Final expressed value = $t_m \pm \Delta t_m = (33 \pm 2)$ s.

Step 4: Final Answer:

The correct mean time is (33 ± 2) .

💡 Quick Tip

When calculating absolute errors, remember that the magnitude is always positive. Taking the absolute value ensures that positive and negative deviations don't cancel each other out when you average them.

11. Light of wavelength ' λ ' falls on a metal having work function $\frac{hc}{\lambda_0}$. Photoelectric effect will take place only if (λ_0 is the threshold wavelength)

- (A) $\lambda \geq \lambda_0$
- (B) $\lambda \geq 2\lambda_0$
- (C) $\lambda \leq \lambda_0$
- (D) $\lambda = 4\lambda_0$

Correct Answer: (C) $\lambda \leq \lambda_0$

Solution:

Step 1: Understanding the Concept:

The photoelectric effect occurs only when the energy of the incident light is greater than or equal to the work function of the metal. The work function is the minimum energy required to eject an electron from the metal surface.

Step 2: Key Formula or Approach:

The energy E of an incident photon is given by $E = \frac{hc}{\lambda}$, where h is Planck's constant, c is the speed of light, and λ is the wavelength.

The work function Φ is given as $\frac{hc}{\lambda_0}$, where λ_0 is the threshold wavelength.

The condition for the photoelectric effect is: $E \geq \Phi$.

Step 3: Detailed Explanation:

Substitute the expressions for E and Φ into the condition:

$$\frac{hc}{\lambda} \geq \frac{hc}{\lambda_0}$$

Since h and c are positive constants, we can cancel them from both sides:

$$\frac{1}{\lambda} \geq \frac{1}{\lambda_0}$$

Taking the reciprocal of both sides reverses the inequality sign (since both wavelengths are positive quantities):

$$\lambda \leq \lambda_0$$

Step 4: Final Answer:

The photoelectric effect will take place only if $\lambda \leq \lambda_0$.

 Quick Tip

Remember the inverse relationship between energy and wavelength. Higher energy means lower wavelength. So, to have enough energy (above the threshold), you need a wavelength shorter than or equal to the threshold wavelength.

12. For a particle moving in a circle with constant angular speed, which of the following statements is 'false'?

- (A) The velocity vector is tangent to the circle.
- (B) The acceleration vector is tangent to the circle.
- (C) The velocity and acceleration vectors are perpendicular to each other.
- (D) The acceleration vector points to the centre of the circle.

Correct Answer: (B) The acceleration vector is tangent to the circle.

Solution:

Step 1: Understanding the Concept:

A particle moving in a circle with a constant angular speed is undergoing uniform circular motion. In this type of motion, the magnitude of velocity (speed) is constant, but the direction of velocity changes continuously.

Step 2: Key Formula or Approach:

In uniform circular motion:

1. The velocity vector $\vec{v}$ is always directed along the tangent to the circular path at any point.
2. The acceleration is purely centripetal, meaning it arises solely from the change in direction of velocity. The formula for centripetal acceleration magnitude is $a_c = \frac{v^2}{r}$, and its direction is always radially inward towards the center of the circle.

Step 3: Detailed Explanation:

Let's evaluate each statement:

- (A) "The velocity vector is tangent to the circle." - This is true for any circular motion.
- (C) "The velocity and acceleration vectors are perpendicular to each other." - Since velocity is tangential and acceleration is radial (pointing to the center), they are indeed perpendicular (90° to each other). This is true.
- (D) "The acceleration vector points to the centre of the circle." - This describes centripetal acceleration, which is correct for uniform circular motion. This is true.
- (B) "The acceleration vector is tangent to the circle." - Tangential acceleration only exists if the particle's speed (or angular speed) is changing. Since the angular speed is constant, the tangential acceleration is zero. The net acceleration is purely radial. Thus, this statement is false.

Step 4: Final Answer:

The false statement is that the acceleration vector is tangent to the circle.

💡 Quick Tip

Keywords "constant angular speed" imply uniform circular motion. In uniform circular motion, tangential acceleration $a_t = 0$ and only centripetal acceleration a_c exists.

13. In Young's double slit experiment, the distance between screen and aperture is 1 m . The slit width is 2 mm . Light of 6000 Å is used. If a thin glass plate ($\mu = 1.5$) of thickness 0.04 mm is placed over one of the slits, then there will be a lateral displacement of the fringes by

- (A) 0.5 cm
- (B) 1 cm
- (C) 1.5 cm
- (D) 2 cm

Correct Answer: (B) 1 cm

Solution:

Step 1: Understanding the Concept:

When a transparent medium (like a glass plate) of thickness t and refractive index μ is introduced in the path of one of the interfering beams in Young's Double Slit Experiment (YDSE), the entire fringe pattern shifts laterally.

Step 2: Key Formula or Approach:

The introduction of the slab creates an additional optical path difference of $(\mu - 1)t$.

The formula for the lateral shift (displacement) y_0 of the central fringe (and thus the whole pattern) is:

$$y_0 = \frac{D}{d}(\mu - 1)t$$

where D is the distance to the screen, d is the slit separation (often referred to as slit width in this context), μ is the refractive index, and t is the thickness of the plate.

Step 3: Detailed Explanation:

Given values:

Distance to screen, $D = 1$ m

Slit separation, $d = 2$ mm $= 2 \times 10^{-3}$ m

Refractive index, $\mu = 1.5$

Thickness of plate, $t = 0.04$ mm $= 0.04 \times 10^{-3}$ m

Now, substitute these values into the displacement formula:

$$y_0 = \frac{1}{2 \times 10^{-3}}(1.5 - 1) \times (0.04 \times 10^{-3})$$

$$y_0 = \frac{1}{2 \times 10^{-3}} \times 0.5 \times 0.04 \times 10^{-3}$$

The 10^{-3} terms cancel out:

$$y_0 = \frac{0.5 \times 0.04}{2}$$

$$y_0 = \frac{0.02}{2} = 0.01 \text{ m}$$

Convert the result to centimeters to match the options:

$$y_0 = 0.01 \text{ m} = 1 \text{ cm}$$

Step 4: Final Answer:

The lateral displacement of the fringes is 1 cm.

💡 Quick Tip

Notice that the wavelength of light (6000 Å) given in the problem is actually not needed to calculate the total shift of the pattern. The shift depends only on the geometry (D , d) and the properties of the slab (μ , t).

14. For an ideal diode, in forward and reverse biased condition the resistance is respectively

- (A) infinite, zero.
- (B) infinite, infinite.
- (C) zero, zero.
- (D) zero, infinite.

Correct Answer: (D) zero, infinite.

Solution:

Step 1: Understanding the Concept:

A p-n junction diode acts as a one-way valve for electric current. It allows current to flow easily in one direction (forward bias) and severely restricts it in the opposite direction (reverse bias). An "ideal" diode is a theoretical model that perfectly exhibits this behavior without any non-ideal characteristics like voltage drops or leakage currents.

Step 2: Key Formula or Approach:

The defining characteristics of an ideal diode are:

1. In the forward-biased condition, it acts as a perfect short circuit, meaning it offers zero opposition to current flow.
2. In the reverse-biased condition, it acts as a perfect open circuit, meaning it completely blocks current flow.

Step 3: Detailed Explanation:

Based on the ideal model:

- Forward Bias Resistance: Since it acts as a short circuit, the resistance is exactly zero (0Ω).
 - Reverse Bias Resistance: Since it acts as an open circuit (no current flows regardless of the applied voltage), the resistance is considered to be infinite ($\infty \Omega$).
- Therefore, the resistance values are zero and infinite, respectively.

Step 4: Final Answer:

The resistance is zero, infinite respectively.

💡 Quick Tip

Always distinguish between an "ideal" diode and a "practical" diode. A practical Si diode has a forward resistance of a few ohms and a forward voltage drop of $0.7V$, and a very high (but finite) reverse resistance. An ideal diode simplifies this to 0Ω and $\infty \Omega$.

15. In Young's double slit experiment, when light of wavelength 600 nm is used, 18 fringes are observed on the screen. If the wavelength of light is changed to 400 nm , the number of fringes observed on the screen is

- (A) 12
- (B) 27
- (C) 22
- (D) 24

Correct Answer: (B) 27

Solution:

Step 1: Understanding the Concept:

In Young's Double Slit Experiment, the fringe width (the distance between two consecutive bright or dark fringes) depends on the wavelength of the light used.

If the experimental setup (screen size, slit distance, etc.) remains fixed, the total width of the interference pattern observed on the screen is constant.

Step 2: Key Formula or Approach:

The fringe width β is given by $\beta = \frac{\lambda D}{d}$, where λ is the wavelength, D is the distance to the screen, and d is the slit separation.

Let L be the fixed width of the observation field on the screen. The number of fringes n observed in this field is $n = \frac{L}{\beta}$.

Substituting β : $n = \frac{Ld}{\lambda D}$.

Since L , d , and D are constant for a given setup, the product of the number of fringes and the wavelength is a constant:

$$n_1 \lambda_1 = n_2 \lambda_2 = \text{constant}$$

Step 3: Detailed Explanation:

Given values:

Initial wavelength, $\lambda_1 = 600 \text{ nm}$

Initial number of fringes, $n_1 = 18$

New wavelength, $\lambda_2 = 400 \text{ nm}$

We need to find the new number of fringes, n_2 .

Using the relation established above:

$$n_1 \lambda_1 = n_2 \lambda_2$$

$$18 \times 600 \text{ nm} = n_2 \times 400 \text{ nm}$$

Solving for n_2 :

$$n_2 = \frac{18 \times 600}{400}$$

$$n_2 = 18 \times \frac{6}{4} = 18 \times 1.5$$

$$n_2 = 27$$

Step 4: Final Answer:

The number of fringes observed will be 27.

💡 Quick Tip

Fringe width is directly proportional to wavelength. If you decrease the wavelength, the fringes become narrower, so more of them can fit into the same visible area on the screen. Hence, an inverse relationship $n \propto 1/\lambda$.

16. If $\vec{A} = \hat{i} + \hat{j} + 3\hat{k}$, $\vec{B} = -\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{C} = 2\hat{i} - 2\hat{j} - 8\hat{k}$, then the angle between the vectors $\vec{P} = \vec{A} + \vec{B} + \vec{C}$ and $\vec{Q} = (\vec{A} \times \vec{B})$ is (in degree)

- (A) 0°
- (B) 45°
- (C) 90°
- (D) 60°

Correct Answer: (C) 90°

Solution:**Step 1: Understanding the Concept:**

To find the angle between two vectors $\vec{P}$ and $\vec{Q}$, we use their dot product. If the dot product of two non-zero vectors is zero, they are orthogonal (perpendicular), meaning the angle between them is 90° .

Step 2: Key Formula or Approach:

1. Vector addition: $\vec{P} = \vec{A} + \vec{B} + \vec{C}$ is found by adding the corresponding $\hat{i}, \hat{j}, \hat{k}$ components.
2. Cross product: $\vec{Q} = \vec{A} \times \vec{B}$ is calculated using the determinant method.
3. Dot product: $\vec{P} \cdot \vec{Q} = |\vec{P}||\vec{Q}| \cos \theta$. If $\vec{P} \cdot \vec{Q} = 0$, then $\cos \theta = 0 \implies \theta = 90^\circ$.

Step 3: Detailed Explanation:

First, calculate vector $\vec{P}$:

$$\vec{P} = (\hat{i} + \hat{j} + 3\hat{k}) + (-\hat{i} + \hat{j} + 4\hat{k}) + (2\hat{i} - 2\hat{j} - 8\hat{k})$$

$$\vec{P} = (1 - 1 + 2)\hat{i} + (1 + 1 - 2)\hat{j} + (3 + 4 - 8)\hat{k}$$

$$\vec{P} = 2\hat{i} + 0\hat{j} - 1\hat{k} = 2\hat{i} - \hat{k}$$

Next, calculate vector $\vec{Q} = \vec{A} \times \vec{B}$:

$$\vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 3 \\ -1 & 1 & 4 \end{vmatrix}$$

$$\vec{Q} = \hat{i}[(1)(4) - (3)(1)] - \hat{j}[(1)(4) - (3)(-1)] + \hat{k}[(1)(1) - (1)(-1)]$$

$$\vec{Q} = \hat{i}[4 - 3] - \hat{j}[4 + 3] + \hat{k}[1 + 1]$$

$$\vec{Q} = 1\hat{i} - 7\hat{j} + 2\hat{k}$$

Now, find the dot product $\vec{P} \cdot \vec{Q}$:

$$\vec{P} \cdot \vec{Q} = (2\hat{i} + 0\hat{j} - 1\hat{k}) \cdot (1\hat{i} - 7\hat{j} + 2\hat{k})$$

$$\vec{P} \cdot \vec{Q} = (2)(1) + (0)(-7) + (-1)(2)$$

$$\vec{P} \cdot \vec{Q} = 2 + 0 - 2 = 0$$

Since the dot product is zero and neither $\vec{P}$ nor $\vec{Q}$ is a null vector, the vectors are perpendicular. Therefore, the angle θ is 90° .

Step 4: Final Answer:

The angle between the vectors is 90° .

💡 Quick Tip

Always check the dot product first before trying to use the full magnitude formula $\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$. Finding a zero dot product immediately gives you 90° and saves you the time of calculating magnitudes.

17. A coil of self-inductance L is connected in series with a bulb and an a. c. source. Brightness of the bulb decreases when

- (A) an iron rod is inserted in the coil.
- (B) frequency of a.c. source is decreased.
- (C) number of turns in the coil is reduced.
- (D) a capacitance of reactance ($X_C = X_L$) is included in the same circuit.

Correct Answer: (A) an iron rod is inserted in the coil.

Solution:

Step 1: Understanding the Concept:

The brightness of the bulb depends on the power dissipated by it, which in turn depends on the root-mean-square (RMS) current flowing through the circuit ($P \propto I_{\text{rms}}^2$).

To decrease the brightness, we need to decrease the current in the circuit.

In an AC circuit with an inductor and a resistor (bulb), the current is inversely proportional to the total impedance Z .

Step 2: Key Formula or Approach:

The impedance Z of an LR series circuit is $Z = \sqrt{R^2 + X_L^2}$, where $X_L = \omega L = 2\pi fL$ is the inductive reactance.

The RMS current is $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$.

To decrease current (and thus brightness), we must increase the impedance Z , which can be done by increasing the inductive reactance X_L .

Step 3: Detailed Explanation:

Let's analyze each option to see its effect on X_L or Z :

(A) Inserting an iron rod: Iron is a ferromagnetic material. Inserting it increases the magnetic permeability (μ) of the core. Since inductance $L \propto \mu$, L increases. An increase in L causes an increase in X_L (ωL), which increases the total impedance Z . A higher Z leads to a lower current, decreasing the bulb's brightness. This is the correct option.

(B) Decreasing frequency (f): Since $X_L = 2\pi fL$, decreasing f decreases X_L . This decreases Z , increases current, and increases brightness.

(C) Reducing number of turns (N): Since $L \propto N^2$, reducing N decreases L . This decreases X_L and Z , increasing current and brightness.

(D) Including capacitance with $X_C = X_L$: This creates a series resonant circuit. At resonance, the reactive components cancel out ($X_C - X_L = 0$), and the impedance is minimum ($Z = R$). This results in maximum current and maximum brightness.

Step 4: Final Answer:

Brightness decreases when an iron rod is inserted into the coil.

 Quick Tip

An inductor opposes changes in current. Adding a ferromagnetic core (like iron) significantly boosts its inductance, making it oppose AC current even more strongly, thus acting like a larger "AC resistor".

18. The period of S. H.M. of a particle is 16 second. The phase difference between the positions at $t = 2$ s and $t = 4$ s will be

- (A) π
- (B) $\frac{\pi}{2}$
- (C) $\frac{\pi}{4}$
- (D) $\frac{\pi}{8}$

Correct Answer: (C) $\frac{\pi}{4}$

Solution:

Step 1: Understanding the Concept:

In Simple Harmonic Motion (SHM), the phase angle ϕ determines the state (position and direction of motion) of the particle at any given time.

The phase changes uniformly with time.

Step 2: Key Formula or Approach:

The equation for displacement in SHM can be written as $x(t) = A \sin(\omega t + \phi_0)$.

The total phase at any time t is $\phi(t) = \omega t + \phi_0$.

The phase difference $\Delta\phi$ between two times t_1 and t_2 is the difference in their phases:

$$\Delta\phi = \phi(t_2) - \phi(t_1) = (\omega t_2 + \phi_0) - (\omega t_1 + \phi_0) = \omega(t_2 - t_1)$$

The angular frequency ω is related to the time period T by $\omega = \frac{2\pi}{T}$.

Step 3: Detailed Explanation:

Given values:

Time period, $T = 16$ s

Time interval, $\Delta t = t_2 - t_1 = 4$ s $-$ 2 s $= 2$ s

First, find the angular frequency ω :

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{16} = \frac{\pi}{8} \text{ rad/s}$$

Now, calculate the phase difference $\Delta\phi$:

$$\Delta\phi = \omega \times \Delta t$$

$$\Delta\phi = \frac{\pi}{8} \times 2$$

$$\Delta\phi = \frac{2\pi}{8} = \frac{\pi}{4}$$

Step 4: Final Answer:

The phase difference will be $\frac{\pi}{4}$.

💡 Quick Tip

You can also use a simple proportion: a full time period T corresponds to a phase change of 2π . Therefore, a time interval Δt corresponds to a phase change of $\frac{\Delta t}{T} \times 2\pi$. Here, $\frac{2}{16} \times 2\pi = \frac{1}{8} \times 2\pi = \frac{\pi}{4}$.

19. A body cools from 60°C to 40°C in 6 minutes. After next 6 minutes its temperature will be (Temperature of the surroundings is 10°C)

- (A) 24°C
- (B) 28°C
- (C) 18°C
- (D) 32°C

Correct Answer: (B) 28°C

Solution:

Step 1: Understanding the Concept:

According to Newton's Law of Cooling, the rate of change of temperature of an object is proportional to the difference between its own average temperature and the ambient temperature of its surroundings.

Step 2: Key Formula or Approach:

The approximate form of Newton's law of cooling is:

$$\frac{T_1 - T_2}{t} = K \left(\frac{T_1 + T_2}{2} - T_s \right)$$

where T_1 is the initial temperature, T_2 is the final temperature, t is the time taken, T_s is the surrounding temperature, and K is a cooling constant.

Step 3: Detailed Explanation:

Given $T_s = 10^\circ\text{C}$.

Case 1: First 6 minutes

$T_1 = 60^\circ\text{C}$, $T_2 = 40^\circ\text{C}$, $t = 6$ mins.

Substitute these into the formula to find K :

$$\frac{60 - 40}{6} = K \left(\frac{60 + 40}{2} - 10 \right)$$

$$\frac{20}{6} = K(50 - 10)$$

$$\frac{10}{3} = 40K \implies K = \frac{10}{3 \times 40} = \frac{1}{12} \text{ min}^{-1}$$

Case 2: Next 6 minutes

Now the initial temperature is $T_1 = 40^\circ\text{C}$. Let the new final temperature be T . $t = 6$ mins. Substitute the known values and the calculated K :

$$\frac{40 - T}{6} = \frac{1}{12} \left(\frac{40 + T}{2} - 10 \right)$$

Simplify the right side:

$$\frac{40 - T}{6} = \frac{1}{12} \left(\frac{40 + T - 20}{2} \right)$$

$$\frac{40 - T}{6} = \frac{1}{12} \left(\frac{20 + T}{2} \right) = \frac{20 + T}{24}$$

Cross-multiply to solve for T :

$$24(40 - T) = 6(20 + T)$$

Divide both sides by 6:

$$4(40 - T) = 20 + T$$

$$160 - 4T = 20 + T$$

$$160 - 20 = T + 4T$$

$$140 = 5T$$

$$T = \frac{140}{5} = 28^{\circ}\text{C}$$

Step 4: Final Answer:

The temperature after the next 6 minutes will be 28°C .

💡 Quick Tip

As an object gets closer to room temperature, its cooling rate decreases. So in the first 6 mins it dropped 20°C . In the next 6 mins, it must drop LESS than 20°C . Since $40 - 20 = 20^{\circ}\text{C}$, the final answer must be definitely greater than 20°C . This eliminates option C immediately.

20. A parallel beam of light is incident normally on a plane surface absorbing 50% of the light and reflecting the rest. If the incident beam carries 90 W of power, the force exerted by it on the surface is ($C = \text{velocity of light in air} = 3 \times 10^8 \text{ m/s}$)

- (A) $4.5 \times 10^{-7} \text{ N}$
- (B) $1.5 \times 10^{-7} \text{ N}$
- (C) $3 \times 10^{-7} \text{ N}$
- (D) $9 \times 10^{-7} \text{ N}$

Correct Answer: (A) $4.5 \times 10^{-7} \text{ N}$

Solution:

Step 1: Understanding the Concept:

Light carries momentum. When it strikes a surface, it transfers some or all of its momentum to the surface, exerting a radiation pressure and, consequently, a force.

The momentum of a photon is $p = E/c$. The total force exerted depends on how much light is absorbed vs. reflected.

Absorbed photons transfer momentum p . Reflected photons transfer momentum $2p$ (because their direction is reversed).

Step 2: Key Formula or Approach:

For a surface that absorbs a fraction a and reflects a fraction r (such that $a + r = 1$), the force exerted by a beam of power P is:

$$F = F_{\text{absorbed}} + F_{\text{reflected}}$$

$$F = a \left(\frac{P}{c} \right) + r \left(\frac{2P}{c} \right)$$

Alternatively, since $a = 1 - r$, this simplifies to:

$$F = \frac{P}{c}(1 - r) + \frac{2Pr}{c} = \frac{P}{c}(1 + r)$$

Step 3: Detailed Explanation:

Given values:

Power, $P = 90 \text{ W}$

Speed of light, $c = 3 \times 10^8 \text{ m/s}$

Reflection fraction, $r = 50\% = 0.5$ (since it absorbs 50%, it reflects the remaining 50%).

Substitute these values into the formula:

$$F = \frac{P}{c}(1 + r)$$

$$F = \frac{90}{3 \times 10^8}(1 + 0.5)$$

$$F = (30 \times 10^{-8}) \times 1.5$$

$$F = 45 \times 10^{-8} \text{ N}$$

Adjusting into standard scientific notation:

$$F = 4.5 \times 10^{-7} \text{ N}$$

Step 4: Final Answer:

The force exerted on the surface is $4.5 \times 10^{-7} \text{ N}$.

💡 Quick Tip

To remember the force formula $F = \frac{P}{c}(1 + r)$: For a perfectly absorbing surface ($r = 0$), $F = P/c$. For a perfectly reflecting surface ($r = 1$), $F = 2P/c$ (double the momentum transfer). For a mixed surface, it's a weighted average.

21. A tyre of a vehicle is filled with air having pressure 270 kPa at 27°C . The air pressure in the tyre when the temperature increases to 37°C is

- (A) 282 kPa
- (B) 270 kPa
- (C) 265 kPa
- (D) 279 kPa

Correct Answer: (D) 279 kPa

Solution:

Step 1: Understanding the Concept:

For a gas enclosed in a rigid container like a vehicle tyre (assuming the volume expansion of the tyre is negligible), the process is isochoric (constant volume).

According to Gay-Lussac's Law, for a fixed mass and constant volume of an ideal gas, the pressure is directly proportional to its absolute temperature (in Kelvin).

Step 2: Key Formula or Approach:

Gay-Lussac's Law equation:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

where P_1, T_1 are the initial pressure and absolute temperature, and P_2, T_2 are the final pressure and absolute temperature.

Note: Temperatures MUST be converted to Kelvin. $T(K) = T(^{\circ}\text{C}) + 273$.

Step 3: Detailed Explanation:

Given values:

Initial pressure, $P_1 = 270 \text{ kPa}$

Initial temperature, $t_1 = 27^{\circ}\text{C} \implies T_1 = 27 + 273 = 300 \text{ K}$

Final temperature, $t_2 = 37^{\circ}\text{C} \implies T_2 = 37 + 273 = 310 \text{ K}$

We need to find final pressure P_2 .

Rearranging the formula to solve for P_2 :

$$P_2 = P_1 \times \left(\frac{T_2}{T_1} \right)$$

Substitute the values:

$$P_2 = 270 \times \left(\frac{310}{300} \right)$$

$$P_2 = 270 \times \left(\frac{31}{30} \right)$$

$$P_2 = \left(\frac{270}{30} \right) \times 31$$

$$P_2 = 9 \times 31$$

$$P_2 = 279 \text{ kPa}$$

Step 4: Final Answer:

The new air pressure in the tyre is 279 kPa.

 Quick Tip

Never use Celsius in gas law equations! Always convert to Kelvin first. The ratio 37/27 would give a completely wrong (and much higher) answer than the correct ratio 310/300.

22. A 20Ω resistance, 10 mH inductance coil and $15\mu\text{F}$ capacitor are joined in series. When a suitable frequency alternating current source is joined to this combination, the circuit resonates. If the resistance is made $\frac{1}{3}$ rd, the resonant frequency

- (A) remains unchanged.
- (B) is doubled.
- (C) is quadrupled.
- (D) is halved.

Correct Answer: (A) remains unchanged.

Solution:

Step 1: Understanding the Concept:

Electrical resonance occurs in an AC series RLC circuit when the inductive reactance (X_L) exactly cancels out the capacitive reactance (X_C), making the circuit purely resistive. The frequency at which this happens is the resonant frequency.

Step 2: Key Formula or Approach:

The condition for resonance is $X_L = X_C$.

Since $X_L = \omega L = 2\pi f L$ and $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$, we equate them:

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

Solving for the resonant frequency f_r :

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Step 3: Detailed Explanation:

Looking at the formula for resonant frequency, $f_r = \frac{1}{2\pi\sqrt{LC}}$, it is evident that the resonant frequency depends solely on the values of the inductance (L) and the capacitance (C).

The formula contains no term for the resistance (R).

Therefore, changing the resistance to $\frac{1}{3}$ rd of its original value (or any other value) will have absolutely no effect on the point of resonance. It will change the sharpness (Q-factor) of the resonance and the maximum current, but not the frequency at which it occurs.

Step 4: Final Answer:

The resonant frequency remains unchanged.

💡 Quick Tip

In a series RLC circuit, Resistance (R) only affects the amplitude of the current at resonance (damping) and the bandwidth (quality factor), but it never shifts the central resonant frequency.

23. If the period of a oscillation of mass ' m ' suspended from a spring is 2 s , then the period of suspended mass ' $4m$ ' with the same spring will be

- (A) 1 s
- (B) 3 s
- (C) 2 s
- (D) 4 s

Correct Answer: (D) 4 s

Solution:**Step 1: Understanding the Concept:**

A mass attached to an ideal spring undergoes Simple Harmonic Motion (SHM). The time period of this oscillation depends on the mass attached and the stiffness (spring constant) of the spring.

Step 2: Key Formula or Approach:

The formula for the time period T of a spring-mass system is:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where m is the mass and k is the spring constant.

Step 3: Detailed Explanation:

For the first case, mass is m and time period is $T_1 = 2$ s.

$$T_1 = 2\pi\sqrt{\frac{m}{k}} = 2 \text{ s}$$

For the second case, the mass is changed to $m_2 = 4m$ while using the same spring (so k remains constant). Let the new time period be T_2 .

$$T_2 = 2\pi\sqrt{\frac{m_2}{k}} = 2\pi\sqrt{\frac{4m}{k}}$$

We can pull the 4 out of the square root (since $\sqrt{4} = 2$):

$$T_2 = 2\pi \left(2\sqrt{\frac{m}{k}} \right) = 2 \times \left(2\pi\sqrt{\frac{m}{k}} \right)$$

Notice that the term in the parenthesis is exactly T_1 :

$$T_2 = 2 \times T_1$$

Substitute the given value of $T_1 = 2$ s:

$$T_2 = 2 \times 2 \text{ s} = 4 \text{ s}$$

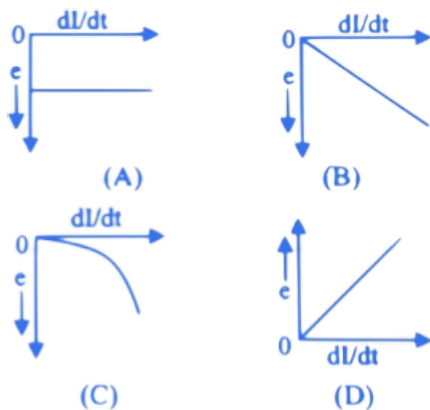
Step 4: Final Answer:

The new period of the suspended mass will be 4 s.

💡 Quick Tip

Since $T \propto \sqrt{m}$, if you multiply the mass by a factor of X , the period is multiplied by a factor of $\sqrt{X}$. Here, mass increased by $4\times$, so period increases by $\sqrt{4} = 2\times$.

24. The current flowing through an inductor of selfinductance L is continuously increasing at constant rate. The variation of induced e.m.f. (e) versus dI/dt is shown graphically by figure



- (A) B
- (B) A
- (C) D
- (D) C

Correct Answer: (B) A

Solution:

Step 1: Understanding the Concept:

According to Faraday's Law of electromagnetic induction and Lenz's Law, when the current through an inductor changes, an electromotive force (e.m.f.) is induced across it that opposes the change in current.

Step 2: Key Formula or Approach:

The formula for the induced e.m.f. (e) in a self-inductor is:

$$e = -L \frac{dI}{dt}$$

where L is the self-inductance (a positive constant characteristic of the coil), and $\frac{dI}{dt}$ is the rate of change of current.

Step 3: Detailed Explanation:

We need to find the graphical relationship between $y = e$ and $x = \frac{dI}{dt}$.

The equation $e = -L \left(\frac{dI}{dt} \right)$ takes the mathematical form $y = -mx$, where the slope m is L .

This equation describes a straight line passing through the origin.

Because the slope is negative (due to Lenz's law), as $\frac{dI}{dt}$ (the x -axis) is positive (current is increasing), the induced e.m.f. e (the y -axis) will be negative.

Looking at the provided graphs:

- (A) shows a straight line through the origin entering the 4th quadrant (positive x , negative y). This perfectly matches our equation.
- (B) shows a straight line through the origin in the 1st quadrant, implying $e = +L(dI/dt)$, which ignores Lenz's Law.
- (C) shows a non-linear curve, which is incorrect.
- (D) shows a vertical shift or different dependency.

Therefore, graph (A) correctly represents the variation.

Checking the options, Option (B) points to graph "A".

Step 4: Final Answer:

The variation is shown graphically by figure A, corresponding to option (B).

💡 Quick Tip

Always pay attention to signs when plotting graphs in physics. The negative sign in $e = -L(di/dt)$ is crucial. It physically means the induced emf creates a potential that opposes the rise in current (back emf), hence it falls in the negative y region for positive x .

25. Three point charges $+Q$, $+2Q$ and q are placed at the vertices of an equilateral triangle. The value of charge q in terms of Q , so that electrical potential energy of the system is zero, is given by

- (A) $q = \frac{1}{3}Q$
- (B) $q = \frac{2}{3}Q$
- (C) $q = -\frac{2}{3}Q$
- (D) $q = -\frac{1}{3}Q$

Correct Answer: (C) $q = -\frac{2}{3}Q$

Solution:

Step 1: Understanding the Concept:

The electrical potential energy of a system of point charges is the sum of the potential energies of all unique pairs of charges in the system.

Step 2: Key Formula or Approach:

For a system of three charges q_1, q_2, q_3 separated by distances r_{12}, r_{23}, r_{13} , the total electrostatic potential energy U is:

$$U = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}} \right)$$

For an equilateral triangle, all side lengths are equal. Let the side length be ' a '.

Step 3: Detailed Explanation:

Let the three charges be $q_1 = +Q$, $q_2 = +2Q$, and $q_3 = q$.

The distances are $r_{12} = r_{23} = r_{13} = a$.

Substitute the charges into the potential energy formula:

$$U = \frac{1}{4\pi\epsilon_0} \left(\frac{(Q)(2Q)}{a} + \frac{(2Q)(q)}{a} + \frac{(Q)(q)}{a} \right)$$

We are given that the total potential energy of the system is zero ($U = 0$):

$$0 = \frac{1}{4\pi\epsilon_0 a} (2Q^2 + 2Qq + Qq)$$

Since $\frac{1}{4\pi\epsilon_0 a}$ is a non-zero constant, the sum inside the parenthesis must be zero:

$$2Q^2 + 3Qq = 0$$

Assuming $Q \neq 0$, we can divide the entire equation by Q :

$$2Q + 3q = 0$$

Now, solve for q in terms of Q :

$$3q = -2Q$$

$$q = -\frac{2}{3}Q$$

Step 4: Final Answer:

The value of charge q is $-\frac{2}{3}Q$.

💡 Quick Tip

When asked to make total potential energy zero, ensure that at least one charge is negative to balance the positive energy of the like-charge pairs. If all charges were the same sign, the potential energy would be strictly positive.

26. The surface energy of a liquid drop is ' V '. It is sprayed into 1000 equal droplets. The surface energy of all the droplets is

- (A) V
- (B) $10 V$
- (C) $100 V$
- (D) $1000 V$

Correct Answer: (B) $10 V$

Solution:

Step 1: Understanding the Concept:

Surface energy is the energy required to create a certain area of a liquid surface. It is directly proportional to the surface area.

When a large drop is sprayed into many smaller droplets, the total volume remains constant, but the total surface area increases. Because the surface area increases, the total surface energy also increases.

Step 2: Key Formula or Approach:

1. Conservation of Volume: $V_{\text{initial}} = n \times V_{\text{final_droplet}}$, where n is the number of droplets.

$$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3 \implies R = n^{1/3}r \implies r = \frac{R}{n^{1/3}}.$$

2. Surface Energy Formula: $E = T \times A$, where T is surface tension and $A = 4\pi R^2$ is the surface area.

Step 3: Detailed Explanation:

Let the initial radius of the large drop be R .

Its initial surface energy is given as V :

$$V = T \times 4\pi R^2$$

The drop is broken into $n = 1000$ equal droplets of radius r .

Using volume conservation to find the relationship between R and r :

$$R^3 = 1000r^3$$

Taking the cube root of both sides:

$$R = 10r \implies r = \frac{R}{10}$$

Now calculate the total surface energy of the new 1000 droplets, let's call it V_{new} :

$$V_{\text{new}} = 1000 \times (\text{Surface energy of one small droplet})$$

$$V_{\text{new}} = 1000 \times (T \times 4\pi r^2)$$

Substitute $r = R/10$:

$$V_{\text{new}} = 1000 \times T \times 4\pi \left(\frac{R}{10}\right)^2$$

$$V_{\text{new}} = 1000 \times T \times 4\pi \frac{R^2}{100}$$

$$V_{\text{new}} = \left(\frac{1000}{100}\right) \times (T \times 4\pi R^2)$$

$$V_{\text{new}} = 10 \times (V)$$

Step 4: Final Answer:

The total surface energy of all the droplets is 10 V .

💡 Quick Tip

A useful shortcut for breaking 1 drop into n identical droplets: The total surface energy increases by a factor of $n^{1/3}$. Here, $n = 1000$, so the energy multiplies by $1000^{1/3} = 10$.

27. When an n-p-n junction transistor is used as an amplifier in common emitter mode,

- (A) the base emitter junction is reverse biased.
- (B) the input impedance is high and the output impedance is low.
- (C) input signal is applied between the base and emitter and the output is obtained between collector and emitter.
- (D) the input voltage and the output voltage are in phase.

Correct Answer: (C) input signal is applied between the base and emitter and the output is obtained between collector and emitter.

Solution:

Step 1: Understanding the Concept:

A Bipolar Junction Transistor (BJT) can be configured in three modes: Common Base (CB), Common Emitter (CE), and Common Collector (CC).

The CE configuration is the most widely used for amplification because it provides both good voltage gain and current gain.

Step 2: Detailed Explanation:

Let's analyze the properties of a CE amplifier to evaluate the options:

- **Biasing for Active Region:** For a transistor to work as an amplifier, it must operate in the active region. This requires the Base-Emitter (BE) junction to be *forward biased* and the Base-Collector (BC) junction to be *reverse biased*. Thus, option (A) is incorrect.
- **Impedances:** A CE amplifier typically has a moderate input impedance (due to forward-biased BE junction) and a moderately high output impedance (due to reverse-biased BC junction). The property of "high input and low output impedance" is characteristic of the Common Collector (emitter follower) configuration. Thus, option (B) is incorrect.
- **Connections:** "Common Emitter" means the emitter terminal is common to both the input and output circuits. Therefore, the input signal is applied across the Base and Emitter, and the amplified output is taken across the Collector and Emitter. This precisely matches option (C).
- **Phase Relationship:** In a CE amplifier, the output voltage is 180° out of phase with the input voltage (it acts as an inverter). Thus, option (D) is incorrect.

Step 4: Final Answer:

The correct statement is that the input signal is applied between the base and emitter and the output is obtained between collector and emitter.

 Quick Tip

"Common Emitter" literally dictates the wiring: the Emitter is the shared common ground for both the input loop (Base-Emitter) and the output loop (Collector-Emitter).

28. The average force applied on the walls of a closed container depends as T^x where T is the temperature of an ideal gas. The value of x is

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

The force exerted by a gas on the walls of its container is the result of the constant macroscopic pressure of the gas acting over the area of the walls.

We need to relate this force to the absolute temperature of the ideal gas.

Step 2: Key Formula or Approach:

1. Pressure and Force: Pressure P is defined as force F per unit area A , so $F = P \times A$.
2. Ideal Gas Law: The state of an ideal gas is governed by the equation $PV = nRT$, where P is pressure, V is volume, n is number of moles, R is the universal gas constant, and T is absolute temperature.

Step 3: Detailed Explanation:

From the Ideal Gas Law, we can express pressure as:

$$P = \frac{nRT}{V}$$

Substitute this expression for pressure into the force equation:

$$F = \left(\frac{nRT}{V} \right) \times A$$

Since the gas is in a "closed container", its volume V , the surface area A of the container walls, and the amount of gas n are all constants. The gas constant R is inherently constant.

Let $k = \frac{nRA}{V}$, which represents a combined constant value.

Then the equation becomes:

$$F = k \cdot T$$

This shows that the average force F is directly proportional to the temperature T to the first power.

$$F \propto T^1$$

Comparing this with the given relationship $F \propto T^x$, we find:

$$x = 1$$

Step 4: Final Answer:

The value of x is 1.

💡 Quick Tip

At a microscopic level, pressure is proportional to the average kinetic energy of gas molecules, which is directly proportional to absolute Temperature T . Since Volume is fixed, collision rate increases, making total force directly proportional to T .

29. A hollow cylinder has a charge of ' q ' C within it. If ϕ is the electric flux associated with the curved surface B, the flux linked with the plane surface A will be



- (A) $\frac{\phi}{3}$
- (B) $\frac{q}{\epsilon_0} - \phi$
- (C) $\frac{q}{3\epsilon_0}$
- (D) $\frac{1}{2} \left(\frac{q}{\epsilon_0} - \phi \right)$

Correct Answer: (D) $\frac{1}{2} \left(\frac{q}{\epsilon_0} - \phi \right)$

Solution:

Step 1: Understanding the Concept:

Gauss's Law states that the total net electric flux (Φ_{total}) passing through a closed surface is equal to the net enclosed charge divided by the permittivity of free space (ϵ_0).

Step 2: Key Formula or Approach:

Gauss's Law equation:

$$\Phi_{\text{total}} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

A cylinder consists of three surfaces: two flat circular plane surfaces (faces A and C) and one curved lateral surface (B).

Therefore, $\Phi_{\text{total}} = \Phi_A + \Phi_B + \Phi_C$.

Step 3: Detailed Explanation:

According to Gauss's Law, for the whole cylinder enclosing charge q :

$$\Phi_A + \Phi_B + \Phi_C = \frac{q}{\epsilon_0}$$

We are given that the flux associated with the curved surface B is ϕ , so $\Phi_B = \phi$.

Assuming the charge q is symmetrically placed inside the cylinder (which is standard for such problems unless stated otherwise), the electric field distribution is symmetric.

Due to this symmetry, the flux passing through the left flat face (C) is identical to the flux passing through the right flat face (A).

So, $\Phi_C = \Phi_A$.

Substitute these into the total flux equation:

$$\Phi_A + \phi + \Phi_A = \frac{q}{\varepsilon_0}$$

$$2\Phi_A + \phi = \frac{q}{\varepsilon_0}$$

Now, solve for Φ_A (the flux linked with plane surface A):

$$2\Phi_A = \frac{q}{\varepsilon_0} - \phi$$

$$\Phi_A = \frac{1}{2} \left(\frac{q}{\varepsilon_0} - \phi \right)$$

Step 4: Final Answer:

The flux linked with the plane surface A is $\frac{1}{2} \left(\frac{q}{\varepsilon_0} - \phi \right)$.

💡 Quick Tip

Always break down a complex 3D shape into its constituent simple surfaces when applying Gauss's Law. Look for geometric symmetries (like the two identical end caps of a cylinder) to simplify the number of unknown flux variables.

30. A diatomic gas ($\gamma = \frac{7}{5}$) is compressed adiabatically to volume $\frac{V_0}{32}$, where V_0 is its initial volume. The initial temperature of the gas is T_i in kelvin and the final temperature is xT_i in kelvin. The value of x is

- (A) 5
- (B) 4
- (C) 3
- (D) 2

Correct Answer: (B) 4

Solution:**Step 1: Understanding the Concept:**

An adiabatic process is a thermodynamic process in which no heat is exchanged with the surroundings.

For an ideal gas undergoing a reversible adiabatic process, the temperature and volume are related.

Step 2: Key Formula or Approach:

The relationship between temperature (T) and volume (V) for an adiabatic process is given by:

$$TV^{\gamma-1} = \text{constant}$$

Therefore, for two states 1 (initial) and 2 (final):

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

Step 3: Detailed Explanation:

Given values:

Adiabatic constant, $\gamma = \frac{7}{5}$

Initial Volume, $V_1 = V_0$

Final Volume, $V_2 = \frac{V_0}{32}$

Initial Temperature, $T_1 = T_i$

Final Temperature, $T_2 = xT_i$

Substitute these values into the adiabatic equation:

$$T_i \cdot (V_0)^{\frac{7}{5}-1} = (xT_i) \cdot \left(\frac{V_0}{32}\right)^{\frac{7}{5}-1}$$

First, calculate the exponent: $\frac{7}{5} - 1 = \frac{7-5}{5} = \frac{2}{5}$.

The equation becomes:

$$T_i \cdot V_0^{\frac{2}{5}} = xT_i \cdot \left(\frac{V_0}{32}\right)^{\frac{2}{5}}$$

We can cancel out the initial temperature T_i from both sides:

$$V_0^{\frac{2}{5}} = x \cdot \frac{V_0^{\frac{2}{5}}}{32^{\frac{2}{5}}}$$

Now, cancel out $V_0^{\frac{2}{5}}$ from both sides:

$$1 = x \cdot \frac{1}{32^{\frac{2}{5}}}$$

Solve for x :

$$x = 32^{\frac{2}{5}}$$

To compute $32^{\frac{2}{5}}$, we can write it as $(32^{\frac{1}{5}})^2$.
Since $2^5 = 32$, the fifth root of 32 is 2 ($32^{\frac{1}{5}} = 2$).

$$x = (2)^2$$

$$x = 4$$

Step 4: Final Answer:

The value of x is 4.

💡 Quick Tip

Familiarity with powers of 2 is very helpful for these adiabatic compression problems. Knowing that $32 = 2^5$ allows you to quickly evaluate fractional exponents like $(32)^{2/5} = (2^5)^{2/5} = 2^2 = 4$.

31. The coefficient of mutual induction is 2 H and induced e.m.f. across secondary is 2 kV . Current in the primary is reduced from 6 A to 3 A . The time required for the change of current is

- (A) 4×10^{-3} s
- (B) 6×10^{-3} s
- (C) 2×10^{-3} s
- (D) 3×10^{-3} s

Correct Answer: (D) 3×10^{-3} s

Solution:

Step 1: Understanding the Concept:

Mutual induction is the phenomenon where a changing current in one coil (primary) induces an electromotive force (e.m.f.) in a nearby second coil (secondary).

The magnitude of the induced e.m.f. is proportional to the rate of change of current in the primary coil.

Step 2: Key Formula or Approach:

The formula relating these quantities is Faraday's Law adapted for mutual inductance:

$$|e| = M \left| \frac{\Delta I}{\Delta t} \right|$$

where e is the induced e.m.f. in the secondary, M is the coefficient of mutual induction, ΔI is the change in primary current, and Δt is the time interval over which this change occurs.

Step 3: Detailed Explanation:

Given values:

Mutual inductance, $M = 2 \text{ H}$

Induced e.m.f., $|e| = 2 \text{ kV} = 2000 \text{ V}$

Initial current, $I_1 = 6 \text{ A}$

Final current, $I_2 = 3 \text{ A}$

Change in current magnitude, $|\Delta I| = |I_2 - I_1| = |3 - 6| = |-3| = 3 \text{ A}$

We need to find the time Δt . Rearrange the formula:

$$\Delta t = \frac{M \times |\Delta I|}{|e|}$$

Substitute the given values into the equation:

$$\Delta t = \frac{2 \times 3}{2000}$$

$$\Delta t = \frac{6}{2000}$$

$$\Delta t = \frac{3}{1000}$$

Converting to scientific notation:

$$\Delta t = 3 \times 10^{-3} \text{ s}$$

Step 4: Final Answer:

The time required for the change is $3 \times 10^{-3} \text{ s}$.

💡 Quick Tip

Always ensure your units are consistent before calculating. Convert kiloVolts (kV) to Volts (V) to match standard SI units like Henrys (H) and Amperes (A) so your resulting time is correctly in seconds (s).

32. A liquid drop having surface energy E is spread into 729 droplets of same size. The final surface energy of the droplets is

- (A) 6 E
- (B) 9 E
- (C) E
- (D) 3 E

Correct Answer: (B) 9 E

Solution:

Step 1: Understanding the Concept:

When a large liquid drop breaks into multiple smaller droplets, the total volume is conserved, but the total exposed surface area increases significantly.

Because surface energy is directly proportional to surface area, the total surface energy of the system increases after fragmentation.

Step 2: Key Formula or Approach:

1. Volume Conservation: $V_{\text{large_drop}} = n \times V_{\text{small_droplet}}$

$$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3 \implies R = n^{1/3}r$$

2. Surface Energy: $E = T \times A$, where T is surface tension and $A = 4\pi R^2$ is the spherical surface area.

Step 3: Detailed Explanation:

Let the initial radius of the large drop be R and its surface tension be T .

Initial surface energy, $E = T \times 4\pi R^2$.

It is divided into $n = 729$ equal droplets of radius r .

Using volume conservation to relate R and r :

$$R^3 = 729r^3$$

Taking the cube root of both sides (since $9^3 = 729$):

$$R = 9r \implies r = \frac{R}{9}$$

Now, calculate the final total surface energy E_{final} of the 729 droplets:

$$E_{\text{final}} = 729 \times (\text{Surface energy of one small droplet})$$

$$E_{\text{final}} = 729 \times (T \times 4\pi r^2)$$

Substitute $r = R/9$ into the equation:

$$E_{\text{final}} = 729 \times T \times 4\pi \left(\frac{R}{9}\right)^2$$

$$E_{\text{final}} = 729 \times T \times 4\pi \frac{R^2}{81}$$

$$E_{\text{final}} = \left(\frac{729}{81}\right) \times (T \times 4\pi R^2)$$

Since $729/81 = 9$:

$$E_{\text{final}} = 9 \times E$$

Step 4: Final Answer:

The final surface energy of the droplets is 9 E.

💡 Quick Tip

Remember the short-cut rule: If 1 drop splits into n equal droplets, the new total surface area (and thus surface energy) is $n^{1/3}$ times the original. Here $n = 729$, so $729^{1/3} = 9$. The new energy is $9E$.

33. A body is projected vertically from earth's surface with $\left(\frac{1}{3}\right)^{\text{rd}}$ of escape velocity. The maximum height reached by the body is (R = radius of earth)

- (A) $\frac{R}{4}$
- (B) $\frac{R}{8}$
- (C) $\frac{R}{9}$
- (D) $\frac{R}{6}$

Correct Answer: (B) $\frac{R}{8}$

Solution:

Step 1: Understanding the Concept:

When an object is thrown upwards, it exchanges its initial kinetic energy for gravitational potential energy as it rises. At its maximum height, its velocity becomes zero.

We use the principle of conservation of mechanical energy to solve this.

Step 2: Key Formula or Approach:

1. Escape Velocity (v_e) from Earth's surface: $v_e = \sqrt{\frac{2GM}{R}}$

2. Conservation of Energy: $(K.E. + P.E.)_{\text{surface}} = (K.E. + P.E.)_{\text{max_height}}$

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = 0 - \frac{GMm}{r}$$

where v is projection velocity, R is Earth's radius, and $r = R + h$ is the distance from Earth's center at maximum height h .

Step 3: Detailed Explanation:

Given projection velocity $v = \frac{1}{3}v_e = \frac{1}{3}\sqrt{\frac{2GM}{R}}$.

Substitute this velocity into the energy conservation equation:

$$\frac{1}{2}m \left(\frac{1}{3}\sqrt{\frac{2GM}{R}} \right)^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

Square the velocity term:

$$\frac{1}{2}m \left(\frac{1}{9} \frac{2GM}{R} \right) - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

Simplify the kinetic energy term:

$$\frac{GMm}{9R} - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

Divide the entire equation by GMm to simplify:

$$\frac{1}{9R} - \frac{1}{R} = -\frac{1}{R+h}$$

Find a common denominator for the left side:

$$\frac{1-9}{9R} = -\frac{1}{R+h}$$

$$\frac{-8}{9R} = \frac{-1}{R+h}$$

Multiply by -1 and take the reciprocal of both sides:

$$\frac{9R}{8} = R+h$$

Solve for the height h :

$$h = \frac{9R}{8} - R$$

$$h = \frac{9R - 8R}{8} = \frac{R}{8}$$

Step 4: Final Answer:

The maximum height reached by the body is $\frac{R}{8}$.

💡 Quick Tip

A useful derived formula for maximum height h when projected with velocity $v = n \cdot v_e$ (where $n < 1$) is: $h = \frac{n^2}{1-n^2}R$. Here, $n = 1/3$, so $h = \frac{(1/9)}{1-(1/9)}R = \frac{1/9}{8/9}R = \frac{R}{8}$. This is much faster for exams!

34. Two planar concentric rings of metal wire having radii r_1 and r_2 ($r_1 > r_2$) are placed in air. The current I is flowing through the coil of larger radius. The mutual inductance between the coils is given by (μ_0 = permeability of free space)

- (A) $\frac{\mu_0 \pi r_1^2}{2r_2}$
- (B) $\frac{\mu_0 \pi r_2^2}{2r_1}$
- (C) $\frac{\mu_0 \pi (r_1 + r_2)^2}{2r_1}$
- (D) $\frac{\mu_0 \pi (r_1 - r_2)^2}{2r_2}$

Correct Answer: (B) $\frac{\mu_0 \pi r_2^2}{2r_1}$

Solution:

Step 1: Understanding the Concept:

Mutual inductance (M) relates the magnetic flux linked with one coil due to a current flowing in a neighboring coil.

Because $r_1 \gg r_2$ typically applies for such textbook approximations (though not explicitly stated, it's the standard derivation path), we assume the magnetic field produced by the large coil is uniform across the small area of the inner coil.

Step 2: Key Formula or Approach:

- The magnetic field B_1 at the center of a circular loop of radius r_1 carrying current I is: $B_1 = \frac{\mu_0 I}{2r_1}$.
- The magnetic flux Φ_2 passing through the smaller inner loop (radius r_2 , Area $A_2 = \pi r_2^2$) is approximately: $\Phi_2 \approx B_1 \times A_2$.
- Mutual inductance M is defined by: $\Phi_2 = M \times I$.

Step 3: Detailed Explanation:

Calculate the magnetic field created by the larger coil at its center:

$$B_1 = \frac{\mu_0 I}{2r_1}$$

Since r_2 is usually considered much smaller than r_1 , we assume this field B_1 is uniform over the entire area of the smaller inner coil.

The area of the smaller coil is:

$$A_2 = \pi r_2^2$$

Calculate the magnetic flux linked with the smaller coil:

$$\Phi_2 = B_1 \times A_2$$

$$\Phi_2 = \left(\frac{\mu_0 I}{2r_1} \right) \times (\pi r_2^2)$$

$$\Phi_2 = \frac{\mu_0 \pi r_2^2}{2r_1} I$$

By the definition of mutual inductance ($\Phi_2 = MI$), we equate the terms:

$$MI = \frac{\mu_0 \pi r_2^2}{2r_1} I$$

Canceling the current I :

$$M = \frac{\mu_0 \pi r_2^2}{2r_1}$$

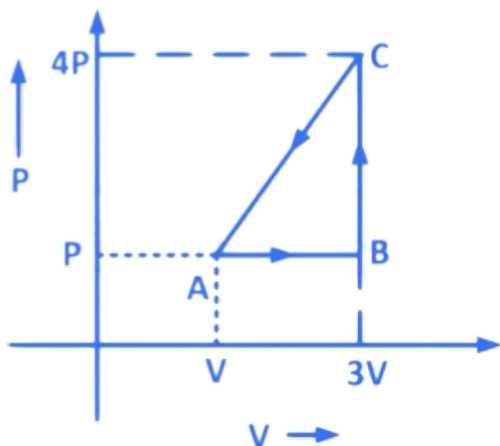
Step 4: Final Answer:

The mutual inductance between the coils is $\frac{\mu_0 \pi r_2^2}{2r_1}$.

💡 Quick Tip

To remember the formula, note that mutual inductance must have units of Henrys. The dimension works out. It is always directly proportional to the area of the smaller loop (πr_{small}^2) and inversely proportional to the radius of the larger loop (r_{large}).

35. The work done by a gas as it is taken in a cyclic process (shown in graph) is



- (A) 2 pv
- (B) -2 pv
- (C) 3 pv
- (D) -3 pv

Correct Answer: (D) -3 pv

Solution:

Step 1: Understanding the Concept:

In a Pressure-Volume (PV) diagram, the net work done by a gas during a cyclic process is equal to the area enclosed by the cycle on the graph.

The sign of the work depends on the direction of the cycle:

- Clockwise cycle: Net work done by the gas is positive ($W > 0$).
- Anti-clockwise cycle: Net work done by the gas is negative ($W < 0$).

Step 2: Key Formula or Approach:

1. Identify the shape and calculate its area: Area of Triangle = $\frac{1}{2} \times \text{base} \times \text{height}$.
2. Determine the direction (clockwise vs. anti-clockwise) to assign the correct sign to the work.

Step 3: Detailed Explanation:

First, let's analyze the shape of the cycle in the graph. It is a right-angled triangle with vertices at points roughly corresponding to (V, P) , $(3V, P)$, and $(3V, 4P)$.

Let's find the lengths of the base and height of this triangle:

- Base (along the V-axis): The process goes from V to $3V$. So, base = $3V - V = 2V$.
- Height (along the P-axis): The process goes from P to $4P$. So, height = $4P - P = 3P$.

Now, calculate the magnitude of the area enclosed:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Area} = \frac{1}{2} \times (2V) \times (3P) = 3PV$$

Next, determine the sign. Observe the arrows on the path in the graph:

- The bottom path goes from (V, P) to $(3V, P)$ (Rightwards).

- The rightmost path goes from $(3V, P)$ to $(3V, 4P)$ (Upwards).
- The diagonal path goes from $(3V, 4P)$ back to (V, P) (Down-Left).

Tracing this path reveals that the cycle is moving in an **anti-clockwise** direction.

An anti-clockwise cycle means work is done ON the gas, so the net work done BY the gas is negative.

$$W = -\text{Area} = -3PV$$

Step 4: Final Answer:

The work done by the gas is -3 pv .

💡 Quick Tip

Always double-check the arrows. A common trap is calculating the area correctly but assigning the wrong sign. Clockwise = Positive Work (Expansion dominates). Anti-clockwise = Negative Work (Compression dominates).

36. A conducting sphere of radius ' R ' is given a charge ' Q ' uniformly. The electric field and the electric potential at the centre of the sphere are respectively [ϵ_0 = permittivity of free space]

- (A) zero and $\frac{Q}{4\pi\epsilon_0 R}$
- (B) $\frac{Q}{4\pi\epsilon_0 R^2}$ and zero
- (C) $\frac{Q}{4\pi\epsilon_0 R}$ and $\frac{Q}{4\pi\epsilon_0 R^2}$
- (D) zero and zero

Correct Answer: (A) zero and $\frac{Q}{4\pi\epsilon_0 R}$

Solution:

Step 1: Understanding the Concept:

For a solid conducting sphere (or a hollow spherical shell), any excess charge placed on it will instantly repel itself and reside entirely on its outer surface.

Due to this charge distribution, the electrical properties inside the conductor have specific boundary conditions.

Step 2: Key Formula or Approach:

1. Electric Field (E): Inside a conductor in electrostatic equilibrium, the electric field is always zero everywhere. $E_{\text{inside}} = 0$.
2. Electric Potential (V): Since $E = -\frac{dV}{dr} = 0$, the potential V must be a constant throughout the entire volume of the sphere. This constant value is equal to the potential at the surface of the sphere, which is calculated as if all charge were concentrated at the center: $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$.

Step 3: Detailed Explanation:

Applying the principles to the center of the sphere (which is inside the conductor):

- The Electric Field at the center is strictly zero. The symmetric distribution of positive charge on the surface pulls equally in all outward directions, canceling the field out perfectly at the center.

- The Electric Potential at the center is not zero. It requires work to bring a test charge from infinity to the surface of the sphere against the repulsion of Q . Once on the surface, moving it anywhere inside (including the center) requires no additional work because $E = 0$. Thus, the potential at the center is the same as on the surface:

$$V_{\text{center}} = V_{\text{surface}} = \frac{Q}{4\pi\epsilon_0 R}$$

Step 4: Final Answer:

The electric field is zero and the electric potential is $\frac{Q}{4\pi\epsilon_0 R}$.

💡 Quick Tip

"Field inside is flat (zero), Potential inside is plateau (constant)." A common mistake is assuming that because the field is zero, the potential must also be zero. Potential is the integral of the field, so a zero field just means the potential doesn't change from its surface value.

37. An inclined plane makes an angle 30° with the horizontal. A solid sphere rolling down an inclined plane from rest without slipping has linear acceleration ($g = \text{acceleration due gravity}$) ($\sin 30^\circ = 0.5$)

- (A) $\frac{5}{7}g$
- (B) $\frac{5}{14}g$
- (C) $\frac{2}{3}g$
- (D) $\frac{g}{3}$

Correct Answer: (B) $\frac{5}{14}g$

Solution:**Step 1: Understanding the Concept:**

When a rigid body rolls down an incline without slipping, its potential energy is converted into both translational kinetic energy and rotational kinetic energy.

Because some energy goes into rotation, its linear acceleration will be less than an object simply sliding down a frictionless incline ($g \sin \theta$).

Step 2: Key Formula or Approach:

The linear acceleration a of a body rolling without slipping down an incline of angle θ is given by:

$$a = \frac{g \sin \theta}{1 + \frac{K^2}{R^2}}$$

where g is gravity, θ is the angle of incline, and K^2/R^2 is a shape factor derived from the moment of inertia $I = MK^2$.

For a solid sphere, the moment of inertia is $I = \frac{2}{5}MR^2$, so its shape factor is $\frac{K^2}{R^2} = \frac{2}{5}$.

Step 3: Detailed Explanation:

Given values:

Incline angle, $\theta = 30^\circ$

$\sin 30^\circ = 0.5 = \frac{1}{2}$

Shape factor for solid sphere, $\frac{K^2}{R^2} = \frac{2}{5}$

Substitute these values into the acceleration formula:

$$a = \frac{g \sin(30^\circ)}{1 + \frac{2}{5}}$$

$$a = \frac{g \left(\frac{1}{2}\right)}{\frac{5}{5} + \frac{2}{5}}$$

$$a = \frac{\frac{g}{2}}{\frac{7}{5}}$$

$$a = \frac{g}{2} \times \frac{5}{7}$$

$$a = \frac{5g}{14}$$

Step 4: Final Answer:

The linear acceleration is $\frac{5g}{14}$.

💡 Quick Tip

Memorize the K^2/R^2 values for standard shapes: Solid sphere (2/5), Hollow sphere (2/3), Solid cylinder/disc (1/2), Hollow cylinder/ring (1). The larger this fraction, the slower the object accelerates because more energy goes into spinning it.

38. Two pipes of lengths L_1 and L_2 , open at both ends are joined in series. If ' f_1 ' and ' f_2 ' are the fundamental frequencies of two pipes, then the fundamental

frequency of series combination will be (neglect end correction)

- (A) $\frac{f_1 f_2}{f_1 - f_2}$
- (B) $f_1 + f_2$
- (C) $\frac{f_1 f_2}{f_1 + f_2}$
- (D) $\sqrt{f_1^2 + f_2^2}$

Correct Answer: (C) $\frac{f_1 f_2}{f_1 + f_2}$

Solution:

Step 1: Understanding the Concept:

The fundamental frequency of an open organ pipe depends on the speed of sound in the air inside it and the length of the pipe.

When two pipes are joined in series, they form a single, longer open pipe whose length is the sum of the individual lengths.

Step 2: Key Formula or Approach:

The fundamental frequency f of an open pipe of length L is given by:

$$f = \frac{v}{2L}$$

From this, we can express length in terms of frequency:

$$L = \frac{v}{2f}$$

When joined in series, the new total length is $L_{\text{series}} = L_1 + L_2$.

The new fundamental frequency will be $f_{\text{series}} = \frac{v}{2L_{\text{series}}}$.

Step 3: Detailed Explanation:

Let the lengths of the two pipes be L_1 and L_2 .

Their fundamental frequencies are:

$$f_1 = \frac{v}{2L_1} \implies L_1 = \frac{v}{2f_1}$$

$$f_2 = \frac{v}{2L_2} \implies L_2 = \frac{v}{2f_2}$$

When connected in series, the combined length L' is:

$$L' = L_1 + L_2$$

Substitute the expressions for L_1 and L_2 :

$$L' = \frac{v}{2f_1} + \frac{v}{2f_2}$$

Factor out $\frac{v}{2}$:

$$L' = \frac{v}{2} \left(\frac{1}{f_1} + \frac{1}{f_2} \right)$$

Find a common denominator:

$$L' = \frac{v}{2} \left(\frac{f_2 + f_1}{f_1 f_2} \right)$$

The fundamental frequency of this combined series pipe f' is:

$$f' = \frac{v}{2L'}$$

Substitute the expression we found for L' :

$$f' = \frac{v}{2 \left[\frac{v}{2} \left(\frac{f_1 + f_2}{f_1 f_2} \right) \right]}$$

The $\frac{v}{2}$ terms cancel out:

$$f' = \frac{1}{\frac{f_1 + f_2}{f_1 f_2}}$$

$$f' = \frac{f_1 f_2}{f_1 + f_2}$$

Step 4: Final Answer:

The fundamental frequency of the series combination is $\frac{f_1 f_2}{f_1 + f_2}$.

💡 Quick Tip

Notice the mathematical similarity to parallel resistors or series capacitors. When lengths add linearly ($L = L_1 + L_2$), and frequency is inversely proportional to length ($f \propto 1/L$), the frequencies combine using the product-over-sum rule.

39. A long wire carrying a steady current is bent into a circle of single turn. The magnetic field at the centre of the coil is ' B '. If it is bent into a circular loop of radius ' r_1 ' having ' n ' turns, the magnetic field at the centre of the coil for same current is

- (A) $\frac{B}{n^2}$
- (B) $\frac{B}{n}$
- (C) $n^2 B$
- (D) nB

Correct Answer: (C) $n^2 B$

Solution:**Step 1: Understanding the Concept:**

The magnetic field at the center of a circular coil depends on the current, the radius of the coil, and the number of turns.

When a single piece of wire is rewound from 1 turn to n turns, its length remains constant, but the radius of the new loops must be smaller to accommodate the extra turns.

Step 2: Key Formula or Approach:

1. Length constraint: The total length of the wire L is constant. For 1 turn of radius R : $L = 2\pi R$. For n turns of radius r_1 : $L = n \times (2\pi r_1)$.

2. Magnetic field formula: The magnetic field at the center of a coil with N turns and radius r is $B = \frac{\mu_0 N I}{2r}$.

Step 3: Detailed Explanation:

Let the initial single-turn loop have a radius R .

Its magnetic field at the center is given as B :

$$B = \frac{\mu_0(1)I}{2R} = \frac{\mu_0 I}{2R}$$

The total length of the wire is $L = 2\pi R$.

Now, the same wire is bent into n turns of radius r_1 .

Conserving the length of the wire:

$$L = n \times 2\pi r_1$$

$$2\pi R = n \times 2\pi r_1$$

$$R = nr_1 \implies r_1 = \frac{R}{n}$$

Now, calculate the new magnetic field B' at the center of this new n -turn coil:

$$B' = \frac{\mu_0 n I}{2r_1}$$

Substitute $r_1 = R/n$ into the equation:

$$B' = \frac{\mu_0 n I}{2(R/n)}$$

The $1/n$ in the denominator moves to the numerator:

$$B' = \frac{\mu_0 n^2 I}{2R}$$

Separate the n^2 term from the rest:

$$B' = n^2 \left(\frac{\mu_0 I}{2R} \right)$$

Since the term in parentheses is the original magnetic field B :

$$B' = n^2 B$$

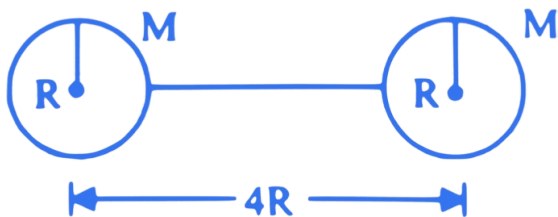
Step 4: Final Answer:

The new magnetic field at the center is $n^2 B$.

💡 Quick Tip

When a wire is rewound into n turns, it has a double-compounding effect: the field increases by a factor of n because there are n loops adding up, and it increases by another factor of n because the radius of each loop is now $1/n$ th the size (bringing the wire closer to the center). Hence, $n \times n = n^2$.

40. Two spheres each of mass M and radius R are connected with a massless rod of length $4R$. The moment of inertia of the system about an axis passing through the centre of one of the spheres and perpendicular to the rod will be



- (A) $\frac{21}{5}MR^2$
- (B) $\frac{84}{5}MR^2$
- (C) $\frac{42}{5}MR^2$
- (D) $\frac{5}{21}MR^2$

Correct Answer: (B) $\frac{84}{5}MR^2$

Solution:

Step 1: Understanding the Concept:

The total moment of inertia of a system is the scalar sum of the moments of inertia of its individual components about the specified axis.

For objects not rotating about their own center of mass, we must use the Parallel Axis Theorem.

Step 2: Key Formula or Approach:

1. Moment of inertia of a solid sphere about its central diameter: $I_{cm} = \frac{2}{5}MR^2$. (Assuming standard solid spheres as is convention when not specified otherwise).
2. Parallel Axis Theorem: $I = I_{cm} + Md^2$, where d is the perpendicular distance from the center of mass to the axis of rotation.
3. Total system inertia: $I_{\text{total}} = I_1 + I_2$.

Step 3: Detailed Explanation:

The axis of rotation passes through the center of the first sphere (Sphere 1) and is perpendicular to the connecting rod.

Sphere 1:

Since the axis passes right through its center of mass, its moment of inertia is simply its standard formula:

$$I_1 = \frac{2}{5}MR^2$$

Sphere 2:

The axis is parallel to its central diameter but shifted by a distance d .

Looking at the diagram, the distance d between the center of Sphere 1 and the center of Sphere 2 is explicitly given by the dimension line as $4R$.

Using the Parallel Axis Theorem for Sphere 2:

$$I_2 = I_{cm,2} + Md^2$$

$$I_2 = \frac{2}{5}MR^2 + M(4R)^2$$

$$I_2 = \frac{2}{5}MR^2 + 16MR^2$$

Find a common denominator to add:

$$I_2 = \frac{2}{5}MR^2 + \frac{80}{5}MR^2 = \frac{82}{5}MR^2$$

Total System:

Add the inertias of both spheres (the rod is massless, so $I_{\text{rod}} = 0$):

$$I_{\text{total}} = I_1 + I_2$$

$$I_{\text{total}} = \frac{2}{5}MR^2 + \frac{82}{5}MR^2$$

$$I_{\text{total}} = \frac{84}{5}MR^2$$

Step 4: Final Answer:

The moment of inertia of the system is $\frac{84}{5}MR^2$.

💡 Quick Tip

Always scrutinize diagrams carefully to determine if distances are given surface-to-surface or center-to-center. The dimension lines here clearly mark the center points of the spheres.

41. In Young's double slit experiment, for the n th dark fringe ($n = 1, 2, 3, \dots$) the phase difference of the interfering waves in radian will be

- (A) $n\frac{\pi}{2}$
- (B) $(2n + 1)\pi$
- (C) $(2n - 1)\pi$
- (D) $(2n - 1)\frac{\pi}{2}$

Correct Answer: (C) $(2n - 1)\pi$

Solution:**Step 1: Understanding the Concept:**

In Young's double-slit experiment, an interference pattern consisting of alternate bright and dark fringes is formed on the screen.

Destructive interference occurs when the waves from the two slits arrive out of phase, producing a dark fringe.

Step 2: Key Formula or Approach:

For destructive interference (dark fringes), the phase difference $\Delta\phi$ between the two interfering waves must be an odd multiple of π .

Mathematically, this is expressed as:

$$\Delta\phi = (2n - 1)\pi \quad \text{where } n = 1, 2, 3, \dots$$

Alternatively, it can be written as $(2n + 1)\pi$ if $n = 0, 1, 2, \dots$

Step 3: Detailed Explanation:

The problem specifies the index sequence as $n = 1, 2, 3, \dots$

- For the 1st dark fringe ($n = 1$), the phase difference should be π . Substituting $n = 1$ into $(2n - 1)\pi$ gives $(2(1) - 1)\pi = \pi$.

- For the 2nd dark fringe ($n = 2$), the phase difference should be 3π . Substituting $n = 2$ into $(2n - 1)\pi$ gives $(2(2) - 1)\pi = 3\pi$.

This perfectly matches the standard sequence for odd multiples of π starting from π .

Let's check option (B) which is $(2n + 1)\pi$: If $n = 1$, it gives 3π , which is the 2nd dark fringe, not the 1st. So it is incorrect for the given index $n = 1, 2, 3, \dots$.

Step 4: Final Answer:

The phase difference for the n th dark fringe is $(2n - 1)\pi$.

💡 Quick Tip

Always check the starting value of n . If n starts from 1, use $(2n - 1)$ for odd numbers. If n starts from 0, use $(2n + 1)$ for odd numbers.

42. A water drop of 0.01 cm^3 is squeezed between two glass plates and spreads in to area of 10 cm^2 . If surface tension of water is 70 dyne/cm then the normal force required to separate glass plates from each other will be

- (A) 12 N
- (B) 14 N
- (C) 16 N
- (D) 28 N

Correct Answer: (B) 14 N

Solution:

Step 1: Understanding the Concept:

When a liquid drop is squeezed between two flat plates, it forms a thin film. The surface tension of the liquid creates a pressure difference across the curved free surface of the film (meniscus) at the edges, leading to an attractive force between the plates.

Step 2: Key Formula or Approach:

The normal force F required to separate the two plates is given by:

$$F = \frac{2TA}{t}$$

where T is the surface tension, A is the wetted area, and t is the thickness of the film. Since the volume V of the drop remains constant, $V = A \times t$, which means $t = \frac{V}{A}$. Substituting this into the force equation:

$$F = \frac{2TA}{\frac{V}{A}} = \frac{2TA^2}{V}$$

Step 3: Detailed Explanation:

Given values (let's use CGS units first for calculation, then convert to SI):

Volume, $V = 0.01 \text{ cm}^3 = 10^{-2} \text{ cm}^3$

Area, $A = 10 \text{ cm}^2$

Surface tension, $T = 70 \text{ dyne/cm}$

Substitute these into the derived formula:

$$F = \frac{2 \times 70 \times (10)^2}{10^{-2}}$$

$$F = \frac{140 \times 100}{10^{-2}}$$

$$F = 14000 \times 10^2$$

$$F = 1,400,000 \text{ dynes} = 14 \times 10^5 \text{ dynes}$$

To convert dynes to Newtons (SI unit), we use the relation $1 \text{ N} = 10^5 \text{ dynes}$.

$$F = \frac{14 \times 10^5}{10^5} \text{ N}$$

$$F = 14 \text{ N}$$

Step 4: Final Answer:

The normal force required is 14 N.

💡 Quick Tip

You can also solve this entirely in SI units from the start. $V = 10^{-8} \text{ m}^3$, $A = 10^{-3} \text{ m}^2$, $T = 0.07 \text{ N/m}$. Then $F = \frac{2(0.07)(10^{-3})^2}{10^{-8}} = \frac{0.14 \times 10^{-6}}{10^{-8}} = 14 \text{ N}$. This often avoids conversion mistakes at the end.

43. A null point is obtained at 200 cm on potentiometer wire when cell in secondary circuit is shunted by 5Ω . When a resistance of 15Ω is used for shunting, null point moves to 300 cm . The internal resistance of the cell is

- (A) 3Ω
- (B) 4Ω
- (C) 5Ω
- (D) 6Ω

Correct Answer: (C) 5Ω

Solution:

Step 1: Understanding the Concept:

A potentiometer can be used to determine the internal resistance of a primary cell.

When the cell is shunted by a resistance R , the balancing length l corresponds to the terminal potential difference V across the shunt.

The e.m.f. E of the cell corresponds to the balancing length l_0 when it is not shunted (open circuit).

Step 2: Key Formula or Approach:

The formula for the internal resistance r is:

$$r = R \left(\frac{l_0 - l}{l} \right)$$

Since we have two different shunting cases, we can set up two equations with the same unknown l_0 .

For case 1: $r = R_1 \left(\frac{l_0 - l_1}{l_1} \right)$

For case 2: $r = R_2 \left(\frac{l_0 - l_2}{l_2} \right)$

Equating them gives: $R_1 \left(\frac{l_0 - l_1}{l_1} \right) = R_2 \left(\frac{l_0 - l_2}{l_2} \right)$.

Step 3: Detailed Explanation:

Given values:

$R_1 = 5 \Omega$, $l_1 = 200 \text{ cm}$

$R_2 = 15 \Omega$, $l_2 = 300 \text{ cm}$

Substitute these into the equated formula to find l_0 :

$$5 \left(\frac{l_0 - 200}{200} \right) = 15 \left(\frac{l_0 - 300}{300} \right)$$

Simplify the fractions:

$$\frac{l_0 - 200}{40} = \frac{l_0 - 300}{20}$$

Multiply both sides by 40:

$$l_0 - 200 = 2(l_0 - 300)$$

$$l_0 - 200 = 2l_0 - 600$$

Rearrange to solve for l_0 :

$$600 - 200 = 2l_0 - l_0$$

$$l_0 = 400 \text{ cm}$$

Now, substitute l_0 back into either of the internal resistance equations (let's use case 1):

$$r = 5 \left(\frac{400 - 200}{200} \right)$$

$$r = 5 \left(\frac{200}{200} \right)$$

$$r = 5 \times 1 = 5 \Omega$$

Step 4: Final Answer:

The internal resistance of the cell is 5Ω .

💡 Quick Tip

A higher shunt resistance draws less current, making the terminal voltage closer to the actual EMF. Thus, the balancing length increases ($300 \text{ cm} > 200 \text{ cm}$), which confirms the logical consistency of the problem's numbers.

44. A resistance of 200Ω and an inductor of $\frac{1}{2\pi}$ H are connected in series to a.c. voltage of 40 V and 100 Hz frequency. The phase angle between the voltage and current is

- (A) $\tan^{-1}(1/5)$
- (B) $\tan^{-1}(1/4)$
- (C) $\tan^{-1}(1/3)$
- (D) $\tan^{-1}(0.5)$

Correct Answer: (D) $\tan^{-1}(0.5)$

Solution:

Step 1: Understanding the Concept:

In a series LR alternating current circuit, the voltage leads the current by a certain phase angle ϕ .

This phase angle is determined by the ratio of the inductive reactance (X_L) to the resistance (R).

Step 2: Key Formula or Approach:

The formula for the phase angle ϕ is:

$$\tan \phi = \frac{X_L}{R}$$

where $X_L = 2\pi fL$ is the inductive reactance.

f is the frequency of the AC source, and L is the inductance.

Step 3: Detailed Explanation:

Given values:

Resistance, $R = 200 \, \Omega$

Inductance, $L = \frac{1}{2\pi} \, \text{H}$

Frequency, $f = 100 \, \text{Hz}$

First, calculate the inductive reactance X_L :

$$X_L = 2\pi fL$$

$$X_L = 2\pi \times 100 \times \left(\frac{1}{2\pi}\right)$$

The 2π terms cancel out:

$$X_L = 100 \, \Omega$$

Now, calculate the tangent of the phase angle:

$$\tan \phi = \frac{X_L}{R}$$

$$\tan \phi = \frac{100}{200}$$

$$\tan \phi = \frac{1}{2} = 0.5$$

Therefore, the phase angle is:

$$\phi = \tan^{-1}(0.5)$$

Step 4: Final Answer:

The phase angle is $\tan^{-1}(0.5)$.

 Quick Tip

Notice that the applied voltage magnitude (40 V) is extra information not needed to find the phase angle. Phase angle depends entirely on the component values (R, L) and the driving frequency (f).

45. A wire of length L , diameter ' d ', density of material ' e ' is under tension ' T ', having fundamental frequency of vibration n_A . Another wire of length $2L$, tension $2T$, density $2e$ and diameter $3d$ has fundamental frequency of vibration n_B . The ratio $n_B : n_A$ is

- (A) 1 : 2
- (B) 1 : 4
- (C) 1 : 6
- (D) 1 : 8

Correct Answer: (C) 1 : 6

Solution:

Step 1: Understanding the Concept:

The fundamental frequency of a stretched string (wire) depends on its length, the tension applied, and its linear mass density (mass per unit length).

Linear mass density can be further broken down into volume density and cross-sectional area.

Step 2: Key Formula or Approach:

The fundamental frequency n is given by:

$$n = \frac{1}{2L} \sqrt{\frac{T}{\mu}}$$

where μ is the linear mass density.

$$\mu = \frac{\text{Mass}}{\text{Length}} = \frac{\text{Volume} \times \text{density}}{L} = \frac{(\pi r^2 L) \times e}{L} = \pi r^2 e$$

Substituting radius $r = d/2$:

$$\mu = \pi \left(\frac{d}{2}\right)^2 e = \frac{\pi d^2 e}{4}$$

Substitute μ back into the frequency formula:

$$n = \frac{1}{2L} \sqrt{\frac{T}{\frac{\pi d^2 e}{4}}} = \frac{1}{2L} \frac{2}{d} \sqrt{\frac{T}{\pi e}} = \frac{1}{Ld} \sqrt{\frac{T}{\pi e}}$$

This gives the proportionality: $n \propto \frac{1}{L \cdot d} \sqrt{\frac{T}{e}}$.

Step 3: Detailed Explanation:

Let's set up the ratio for the two wires A and B:

$$\frac{n_B}{n_A} = \left(\frac{L_A}{L_B}\right) \left(\frac{d_A}{d_B}\right) \sqrt{\left(\frac{T_B}{T_A}\right) \left(\frac{e_A}{e_B}\right)}$$

Given parameters for wire A: $L_A = L, d_A = d, e_A = e, T_A = T$.

Given parameters for wire B: $L_B = 2L, d_B = 3d, e_B = 2e, T_B = 2T$.

Substitute these into the ratio equation:

$$\frac{n_B}{n_A} = \left(\frac{L}{2L}\right) \left(\frac{d}{3d}\right) \sqrt{\left(\frac{2T}{T}\right) \left(\frac{e}{2e}\right)}$$

Simplify the terms:

$$\frac{n_B}{n_A} = \left(\frac{1}{2}\right) \left(\frac{1}{3}\right) \sqrt{2 \times \frac{1}{2}}$$

$$\frac{n_B}{n_A} = \left(\frac{1}{6}\right) \sqrt{1}$$

$$\frac{n_B}{n_A} = \frac{1}{6}$$

Step 4: Final Answer:

The ratio $n_B : n_A$ is 1 : 6.

💡 Quick Tip

Always derive the full proportionality $n \propto \frac{1}{L \cdot d} \sqrt{\frac{T}{e}}$ before plugging in numbers. A common mistake is forgetting that d (diameter) comes out of the square root, while density e stays inside.

46. ' n ' small spherical drops of same size which are charged to ' V ' volt each coalesce to form a single big drop. The potential of the big drop is

- (A) $\frac{V}{n}$
- (B) $n \cdot V$
- (C) $n^{1/3} \cdot V$
- (D) $n^{2/3} \cdot V$

Correct Answer: (D) $n^{2/3} \cdot V$

Solution:**Step 1: Understanding the Concept:**

When multiple small charged drops merge into one large drop, two physical quantities are conserved: total volume and total charge.

The electric potential of a spherical drop depends on both its charge and its radius.

Step 2: Key Formula or Approach:

1. Electric Potential of a sphere: $V = \frac{kq}{r}$.
2. Conservation of Charge: $Q_{\text{big}} = n \times q_{\text{small}}$.
3. Conservation of Volume: $V_{\text{big_drop}} = n \times V_{\text{small_drop}} \implies \frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3 \implies R = n^{1/3}r$.

Step 3: Detailed Explanation:

Let a small drop have radius r and charge q . Its potential is $V = \frac{kq}{r}$.

When n such drops coalesce, the big drop has a total charge:

$$Q = nq$$

And its new radius R is found from volume conservation:

$$R = n^{1/3}r$$

The potential V' of the new big drop is:

$$V' = \frac{kQ}{R}$$

Substitute the expressions for Q and R :

$$V' = \frac{k(nq)}{n^{1/3}r}$$

Separate the terms relating to n and the original small drop's potential:

$$V' = \left(\frac{n}{n^{1/3}}\right) \left(\frac{kq}{r}\right)$$

Using exponent rules ($n^1/n^{1/3} = n^{1-1/3} = n^{2/3}$):

$$V' = n^{2/3} \times V$$

Step 4: Final Answer:

The potential of the big drop is $n^{2/3} \cdot V$.

 Quick Tip

For n coalescing drops: Radius $R = n^{1/3}r$. Charge $Q = nq$. Capacitance $C = n^{1/3}c$. Potential $V = n^{2/3}v$. Energy $U = n^{5/3}u$. Surface Charge Density $\sigma = n^{1/3}\sigma_0$. Memorizing these scaling factors saves a lot of time.

47. In a transistor (common emitter configuration) the ratio of power gain to voltage gain is (α and β are current ratios)

- (A) $\frac{\beta}{\alpha}$
- (B) $\alpha\beta$
- (C) α
- (D) β

Correct Answer: (D) β

Solution:

Step 1: Understanding the Concept:

In an electronic amplifier, power gain is defined as the ratio of output power to input power. Power can be expressed as the product of voltage and current ($P = V \times I$). Therefore, power gain is inherently linked to voltage gain and current gain.

Step 2: Key Formula or Approach:

$$\text{Power Gain } (A_p) = \frac{\text{Output Power}}{\text{Input Power}}$$

$$A_p = \frac{V_{\text{out}} \times I_{\text{out}}}{V_{\text{in}} \times I_{\text{in}}} = \left(\frac{V_{\text{out}}}{V_{\text{in}}} \right) \times \left(\frac{I_{\text{out}}}{I_{\text{in}}} \right)$$

$$A_p = \text{Voltage Gain } (A_v) \times \text{Current Gain } (A_i)$$

Step 3: Detailed Explanation:

The question asks for the ratio of power gain to voltage gain:

$$\text{Ratio} = \frac{A_p}{A_v}$$

From the relationship derived above:

$$\frac{A_p}{A_v} = A_i$$

So, the ratio is simply equal to the current gain of the configuration.

The problem specifies a "common emitter configuration".

In a common emitter (CE) configuration, the input current is the base current (I_b) and the output current is the collector current (I_c).

The current gain in CE mode is denoted by β (beta):

$$A_i = \beta = \frac{I_c}{I_b}$$

(Note: α is the current gain for common base configuration, $\alpha = I_c/I_e$).
Therefore, the ratio $\frac{A_p}{A_v} = \beta$.

Step 4: Final Answer:

The ratio is β .

💡 Quick Tip

Power = Voltage $\times$ Current is a universal rule. Hence, Power Gain = Voltage Gain $\times$ Current Gain. Just identify the configuration (CE $\rightarrow \beta$, CB $\rightarrow \alpha$) to know the specific symbol for current gain.

48. A particle oscillates in straight line simple harmonically with period 8 second and amplitude $4\sqrt{2}$ m. Particle starts from mean position. The ratio of the distance travelled by it in 1st second of its motion to that in 2nd second is ($\sin 45^\circ = 1/\sqrt{2}$, $\sin \frac{\pi}{2} = 1$)

- (A) 1 : 8
- (B) 1 : 4
- (C) 1 : 2
- (D) 1 : ($\sqrt{2} - 1$)

Correct Answer: (D) 1 : ($\sqrt{2} - 1$)

Solution:

Step 1: Understanding the Concept:

For a particle executing Simple Harmonic Motion (SHM) starting from the mean position, its displacement is given by a sine function.

We need to calculate its position at $t = 1$ s and $t = 2$ s to determine the distances covered in the specific time intervals.

Step 2: Key Formula or Approach:

Equation of motion: $y(t) = A \sin(\omega t)$

Angular frequency: $\omega = \frac{2\pi}{T}$

Distance in 1st second (d_1): $|y(1) - y(0)|$

Distance in 2nd second (d_2): $|y(2) - y(1)|$ (assuming it hasn't reversed direction, which it doesn't since $t = 2\text{s} = T/4$).

Step 3: Detailed Explanation:

Given values:

Time period, $T = 8$ s

Amplitude, $A = 4\sqrt{2}$ m

Calculate angular frequency: $\omega = \frac{2\pi}{8} = \frac{\pi}{4}$ rad/s.

The equation for displacement is: $y(t) = 4\sqrt{2} \sin\left(\frac{\pi}{4}t\right)$.

- At $t = 0$ s, $y(0) = 0$.

- At $t = 1$ s (end of 1st second):

$$y(1) = 4\sqrt{2} \sin\left(\frac{\pi}{4} \times 1\right) = 4\sqrt{2} \sin(45^\circ)$$

$$y(1) = 4\sqrt{2} \times \left(\frac{1}{\sqrt{2}}\right) = 4 \text{ m.}$$

- At $t = 2$ s (end of 2nd second):

$$y(2) = 4\sqrt{2} \sin\left(\frac{\pi}{4} \times 2\right) = 4\sqrt{2} \sin\left(\frac{\pi}{2}\right)$$

$$y(2) = 4\sqrt{2} \times 1 = 4\sqrt{2} \text{ m.}$$

Now, calculate the distances:

Distance travelled in 1st second (d_1) is the change in position from $t = 0$ to $t = 1$:

$$d_1 = y(1) - y(0) = 4 - 0 = 4 \text{ m}$$

Distance travelled in 2nd second (d_2) is the change in position from $t = 1$ to $t = 2$:

$$d_2 = y(2) - y(1) = 4\sqrt{2} - 4 = 4(\sqrt{2} - 1) \text{ m}$$

Find the ratio $d_1 : d_2$:

$$\text{Ratio} = \frac{4}{4(\sqrt{2} - 1)} = \frac{1}{\sqrt{2} - 1}$$

Step 4: Final Answer:

The ratio is $1 : (\sqrt{2} - 1)$.

 Quick Tip

In SHM, velocity is maximum at the mean position and decreases towards the extremes. Hence, it covers more distance in the 1st second (4m) than in the 2nd second ($4(\sqrt{2} - 1) \approx 4(0.414) = 1.65\text{m}$). The ratio confirms this.

49. The length of the compound microscope is 15 cm . The magnifying power for relaxed eye is 25 . If the focal length of eye lens is 6 cm then the object distance for objective lens will be

- (A) 1.3 cm
- (B) 1.5 cm
- (C) 1.7 cm
- (D) 1.9 cm

Correct Answer: (B) 1.5 cm

Solution:**Step 1: Understanding the Concept:**

A compound microscope consists of an objective lens and an eyepiece.

For a "relaxed eye" (normal adjustment), the final image is formed at infinity. This happens when the intermediate image formed by the objective lens falls exactly at the focal point of the eyepiece.

Step 2: Key Formula or Approach:

1. Tube length (L): For a relaxed eye, the distance between the lenses is $L = v_o + f_e$, where v_o is the image distance for the objective and f_e is the focal length of the eyepiece.

2. Magnifying Power (M): The total magnification for normal adjustment is $M = m_o \times m_e = \left(\frac{v_o}{u_o}\right) \times \left(\frac{D}{f_e}\right)$, where u_o is the object distance and D is the least distance of distinct vision (standard value $D = 25$ cm).

Step 3: Detailed Explanation:

Given values:

Length of microscope, $L = 15$ cm

Magnifying power, $M = 25$

Focal length of eyepiece, $f_e = 6$ cm

Least distance of distinct vision, $D = 25$ cm (standard assumption)

First, find the image distance v_o produced by the objective lens using the tube length formula:

$$L = v_o + f_e$$

$$15 = v_o + 6$$

$$v_o = 15 - 6 = 9 \text{ cm}$$

Next, use the magnifying power formula to find u_o :

$$M = \left(\frac{v_o}{u_o}\right) \times \left(\frac{D}{f_e}\right)$$

Substitute the known values:

$$25 = \left(\frac{9}{u_o}\right) \times \left(\frac{25}{6}\right)$$

Divide both sides by 25:

$$1 = \left(\frac{9}{u_o}\right) \times \left(\frac{1}{6}\right)$$

$$1 = \frac{9}{6 \cdot u_o}$$

Simplify the fraction:

$$1 = \frac{3}{2 \cdot u_o}$$

Solve for u_o :

$$u_o = \frac{3}{2} = 1.5 \text{ cm}$$

Step 4: Final Answer:

The object distance for the objective lens is 1.5 cm.

💡 Quick Tip

”Relaxed eye” always means the final image is at infinity. This immediately tells you two things: $m_e = D/f_e$ (instead of $1 + D/f_e$), and the separation between lenses $L = v_o + f_e$ (the objective’s image falls on the eyepiece’s focal point).

50. The magnetic susceptibility of iron is 5499 . The relative permeability of iron will be

- (A) 5500×10^7
- (B) 5500×10^{-7}
- (C) 5500
- (D) 5501

Correct Answer: (C) 5500

Solution:

Step 1: Understanding the Concept:

Magnetic susceptibility (χ_m) is a measure of how much a material will become magnetized in an applied magnetic field.

Relative permeability (μ_r) is the ratio of the permeability of a specific medium to the permeability of free space.

These two dimensionless quantities are directly related to each other.

Step 2: Key Formula or Approach:

The fundamental relationship between relative permeability (μ_r) and magnetic susceptibility (χ_m) is given by:

$$\mu_r = 1 + \chi_m$$

Step 3: Detailed Explanation:

Given value:

Magnetic susceptibility, $\chi_m = 5499$

Substitute this value into the relationship formula:

$$\mu_r = 1 + 5499$$

$$\mu_r = 5500$$

Step 4: Final Answer:

The relative permeability of iron is 5500.

💡 Quick Tip

For ferromagnetic materials like iron, χ_m is very large and positive (in the thousands). Consequently, μ_r is also very large. The +1 difference is mathematically required, but practically, $\mu_r \approx \chi_m$ for such highly magnetic materials.