

MHT CET 2025 April 19 Shift 1 Question Paper with Solutions

Time Allowed :3 Hours

Maximum Marks :200

Total Questions :150

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 150 questions. The maximum marks are 200.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 50 questions in each part of equal weightage.

Chemistry

1. Identify a side chain (R) group present in serine, an amino acid.

- (A) CH_3-
- (B) $\text{H}_3\text{C} - \text{CHOH}-$
- (C) $\text{Me}_2\text{CH}-$
- (D) $\text{HO} - \text{CH}_2-$

Correct Answer: (D) $\text{HO} - \text{CH}_2-$

Solution:

Step 1: Understanding the Concept:

Amino acids consist of a central carbon atom (the alpha carbon) bonded to an amino group ($-\text{NH}_2$), a carboxyl group ($-\text{COOH}$), a hydrogen atom ($-\text{H}$), and a variable side chain known as the R group. The chemical properties of the amino acid depend on the nature of this R group.

Step 2: Key Formula or Approach:

To identify the correct side chain for serine, recall the standard structures of the 20 common amino acids and match the given R groups to their respective amino acids.

Step 3: Detailed Explanation:

Let's analyze the given side chains:

- (A) CH_3- is the side chain for Alanine.
- (B) $\text{H}_3\text{C} - \text{CHOH}-$ is the side chain for Threonine.
- (C) $\text{Me}_2\text{CH}-$ (isopropyl group) is the side chain for Valine.
- (D) $\text{HO} - \text{CH}_2-$ (hydroxymethyl group) is the side chain for Serine.

Therefore, the R group in serine is $\text{HO} - \text{CH}_2-$.

Step 4: Final Answer:

The correct option is (D).

 Quick Tip

Memorizing the structures of the 20 standard amino acids, especially the functional groups in their side chains (like hydroxyl in serine and threonine), is crucial for exams.

2. What is the number of moles of water molecules required for complete hydrolysis of n mole triglyceride?

- (A) $4n$
- (B) $3n$
- (C) $2n$
- (D) n

Correct Answer: (B) $3n$

Solution:

Step 1: Understanding the Concept:

A triglyceride is an ester formed from one molecule of glycerol and three molecules of fatty acids. Therefore, a single triglyceride molecule contains three ester linkages.

Step 2: Key Formula or Approach:

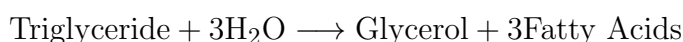
The hydrolysis of an ester requires one mole of water per mole of ester linkage. The general reaction is:



Step 3: Detailed Explanation:

To completely hydrolyze one molecule of a triglyceride, each of the three ester bonds must be cleaved. The cleavage of one ester bond requires one molecule of water.

Reaction for 1 mole:



Thus, 1 mole of triglyceride requires 3 moles of water for complete hydrolysis.

By extension, n moles of triglyceride will require $3n$ moles of water.

Step 4: Final Answer:

The correct option is (B).

 Quick Tip

Always look at the chemical structure of the lipid. Triglycerides have three ester groups, so they need a 1 : 3 molar ratio with water for complete hydrolysis.

3. Which of the following compounds has maximum covalent character?

- (A) LiCl
- (B) LiI
- (C) NaCl
- (D) NaI

Correct Answer: (B) LiI

Solution:

Step 1: Understanding the Concept:

The covalent character in predominantly ionic compounds can be determined using Fajan's Rules. Covalent character increases with the polarization of the anion by the cation.

Step 2: Key Formula or Approach:

According to Fajan's Rules, maximum covalent character is favored by:

1. Smaller size of the cation.
2. Larger size of the anion.
3. Higher charge on the cation or anion.

Step 3: Detailed Explanation:

Let's compare the given alkali metal halides based on the sizes of their constituent ions:

- **Cations:** We have Li^+ and Na^+ . Since lithium is above sodium in group 1, Li^+ is smaller than Na^+ . A smaller cation has a higher polarizing power.

- **Anions:** We have Cl^- and I^- . Since iodine is below chlorine in group 17, I^- is significantly larger than Cl^- . A larger anion is more easily polarizable.

To maximize covalent character, we need the smallest cation combined with the largest anion. This combination is Li^+ with I^- , forming LiI.

Step 4: Final Answer:

Therefore, LiI will exhibit the maximum covalent character among the given options.

 Quick Tip

Fajan's Rule shortcut: Maximum polarization (and thus maximum covalent character)
= Smallest cation + Largest anion.

4. Calculate the percent dissociation of 0.02 m solution if its freezing point depression is 0.046 K . [K_f for water = $1.86 \text{ K kg mol}^{-1}$; $n = 2$]

- (A) 12.3%
(B) 23.6%
(C) 35.00%
(D) 48.1%

Correct Answer: (B) 23.6%

Solution:

Step 1: Understanding the Concept:

For an electrolyte solution, the depression in freezing point (ΔT_f) depends on the number of particles in solution, which is accounted for by the van 't Hoff factor (i). The degree of dissociation (α) relates to this factor.

Step 2: Key Formula or Approach:

The formulas required are:

$$\Delta T_f = i \cdot K_f \cdot m$$

$$\alpha = \frac{i - 1}{n - 1}$$

where n is the number of ions produced per formula unit.

Step 3: Detailed Explanation:

Given values:

$$\Delta T_f = 0.046 \text{ K}$$

$$K_f = 1.86 \text{ K kg mol}^{-1}$$

$$m = 0.02 \text{ m}$$

$$n = 2$$

First, calculate the van 't Hoff factor (i):

$$0.046 = i \times 1.86 \times 0.02$$

$$0.046 = i \times 0.0372$$

$$i = \frac{0.046}{0.0372} \approx 1.2365$$

Now, calculate the degree of dissociation (α):

$$\alpha = \frac{i - 1}{n - 1} = \frac{1.2365 - 1}{2 - 1} = 0.2365$$

Convert this to a percentage:

$$\text{Percent dissociation} = \alpha \times 100\% = 0.2365 \times 100\% = 23.65\%$$

Rounding to one decimal place gives 23.6%.

Step 4: Final Answer:

The calculated percent dissociation is approximately 23.6%, matching option (B).

💡 Quick Tip

Always remember the relation $\alpha = \frac{i-1}{n-1}$ for dissociation and $\alpha = \frac{1-i}{1-1/n}$ for association.

5. For the reaction, $\text{NO}_{2(g)} + \text{CO}_{(g)} \longrightarrow \text{NO}_{(g)} + \text{CO}_{2(g)}$, rate of formation of $\text{NO}_{(g)}$ is $Y \text{ mol dm}^{-3} \text{ s}^{-1}$. Find the rate of disappearance of $\text{CO}_{(g)}$.

- (A) $Y \text{ mol dm}^{-3} \text{ s}^{-1}$
- (B) $2Y \text{ mol dm}^{-3} \text{ s}^{-1}$
- (C) $\frac{Y}{2} \text{ mol dm}^{-3} \text{ s}^{-1}$
- (D) $\frac{3}{2}Y \text{ mol dm}^{-3} \text{ s}^{-1}$

Correct Answer: (A) $Y \text{ mol dm}^{-3} \text{ s}^{-1}$

Solution:

Step 1: Understanding the Concept:

The rate of a chemical reaction can be expressed in terms of the rate of disappearance of any reactant or the rate of formation of any product, divided by their respective stoichiometric

coefficients from the balanced chemical equation.

Step 2: Key Formula or Approach:

For a general reaction $aA + bB \longrightarrow cC + dD$, the overall rate of reaction is given by:

$$\text{Rate} = -\frac{1}{a} \frac{d[A]}{dt} = -\frac{1}{b} \frac{d[B]}{dt} = \frac{1}{c} \frac{d[C]}{dt} = \frac{1}{d} \frac{d[D]}{dt}$$

Step 3: Detailed Explanation:

The given balanced chemical equation is:



Based on the stoichiometry of the reaction, 1 mole of CO reacts to produce 1 mole of NO. Therefore, the rate expression is:

$$-\frac{d[\text{CO}]}{dt} = +\frac{d[\text{NO}]}{dt}$$

Here, $+\frac{d[\text{NO}]}{dt}$ represents the rate of formation of NO, which is given as $Y \text{ mol dm}^{-3} \text{ s}^{-1}$.
 $-\frac{d[\text{CO}]}{dt}$ represents the rate of disappearance of CO.

Since they are equal, the rate of disappearance of CO is also $Y \text{ mol dm}^{-3} \text{ s}^{-1}$.

Step 4: Final Answer:

The correct option is (A).

 Quick Tip

When stoichiometric coefficients are 1 : 1, the rates of disappearance and formation are numerically equal. Always check the balanced equation first!

6. Identify the compound formed by action of chromyl chloride on toluene in presence of CS₂ and hydrolysed further?

- (A) Chlorobenzene
- (B) Benzal chloride
- (C) Benzaldehyde
- (D) Benzoic acid

Correct Answer: (C) Benzaldehyde

Solution:

Step 1: Understanding the Concept:

The reaction described is a specific name reaction known as the Étard reaction. It involves the partial oxidation of an alkylbenzene (like toluene) using chromyl chloride (CrO₂Cl₂).

Step 2: Key Formula or Approach:

Identify the reagents and the substrate to recall the specific organic name reaction and its final product after hydrolysis.

Step 3: Detailed Explanation:

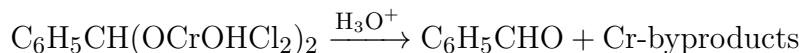
In the Étard reaction, toluene reacts with chromyl chloride in a non-polar solvent like carbon

disulfide (CS_2) or carbon tetrachloride (CCl_4).

The reaction proceeds via the formation of a brown chromium complex:



This intermediate complex is then subjected to acid hydrolysis to yield an aldehyde:



The final organic product formed is $\text{C}_6\text{H}_5\text{CHO}$, which is Benzaldehyde.

Step 4: Final Answer:

The product is benzaldehyde, corresponding to option (C).

💡 Quick Tip

Étard reaction is a mild oxidation method specifically used to stop the oxidation of toluene at the aldehyde stage, preventing further oxidation to benzoic acid.

7. Which from following compounds has lowest pK_b value?

- (A) N -Ethylethanamine
- (B) Propan-2-amine
- (C) NH_3
- (D) Benzenamine

Correct Answer: (A) N -Ethylethanamine

Solution:

Step 1: Understanding the Concept:

The base dissociation constant, K_b , measures the strength of a base. A stronger base has a higher K_b value. The pK_b is the negative logarithm of K_b ($\text{pK}_b = -\log_{10} K_b$). Therefore, a stronger base will have a lower pK_b value. We need to identify the strongest base among the options.

Step 2: Key Formula or Approach:

Evaluate the basicity of each compound based on inductive (+I) effects, steric hindrance, and resonance effects. The strongest base will correspond to the lowest pK_b .

Step 3: Detailed Explanation:

Let's evaluate the basicity of the given compounds:

- (A) **N-Ethylethanamine:** This is a secondary aliphatic amine ($\text{CH}_3\text{CH}_2\text{-NH-CH}_2\text{CH}_3$, also known as diethylamine). The two ethyl groups provide a strong +I (inductive) effect, increasing electron density on nitrogen and making it very basic.
- (B) **Propan-2-amine:** This is a primary aliphatic amine (isopropylamine). It has one alkyl group providing a +I effect, making it more basic than ammonia but generally less basic than a secondary amine in aqueous solution due to a combination of inductive effects and solvation.
- (C) **NH_3** (Ammonia): It has no alkyl groups, so no +I effect. It is less basic than aliphatic amines.

- (D) **Benzenamine** (Aniline): This is an aromatic amine. The lone pair on the nitrogen atom is delocalized into the benzene ring due to resonance. This makes the lone pair less available for protonation, making aniline the weakest base among the choices.

In general, for aliphatic amines in aqueous solution, secondary amines are stronger bases than primary amines. Thus, N-ethylethanamine is the strongest base here and will have the lowest pK_b value.

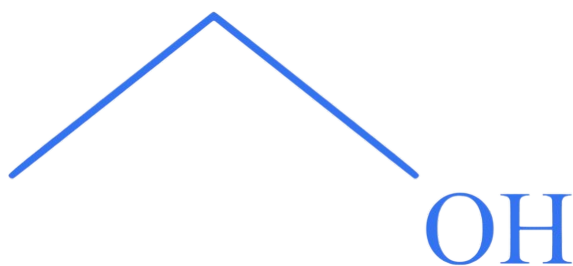
Step 4: Final Answer:

N-Ethylethanamine is the correct answer.

💡 Quick Tip

Remember the inverse relationship: Stronger Base $\implies$ Higher $K_b \implies$ Lower pK_b . Aromatic amines are generally much weaker bases than aliphatic amines due to resonance.

8. What is the molar mass of compound represented by following structure formula?



- (A) 36 g mol^{-1}
- (B) 46 g mol^{-1}
- (C) 22 g mol^{-1}
- (D) 32 g mol^{-1}

Correct Answer: (B) 46 g mol^{-1}

Solution:

Step 1: Understanding the Concept:

The image displays a skeletal chemical structure. In such structures, the lines represent chemical bonds, and the vertices or endpoints of lines (that are not explicitly labelled with other atoms) represent carbon atoms. Hydrogen atoms bonded to carbon are implied to fulfill carbon's valency of four.

Step 2: Key Formula or Approach:

Identify the molecular formula from the given skeletal structure, then calculate its molar mass using the standard atomic weights of the constituent elements.

Step 3: Detailed Explanation:

The structure shows a two-carbon chain with an OH group attached.

- The unlabelled end of the line represents a methyl group (CH₃).
- The vertex represents a methylene group (CH₂).
- The line then connects to a hydroxyl group (OH).

So, the chemical formula of the compound is CH₃ – CH₂ – OH, which is ethanol.

The molecular formula of ethanol is $\text{C}_2\text{H}_6\text{O}$.

Now, calculate its molar mass using standard atomic weights:

$$\text{C} = 12 \text{ g/mol}, \text{H} = 1 \text{ g/mol}, \text{O} = 16 \text{ g/mol}$$

$$\text{Molar mass} = (2 \times 12) + (6 \times 1) + (1 \times 16)$$

$$\text{Molar mass} = 24 + 6 + 16 = 46 \text{ g mol}^{-1}$$

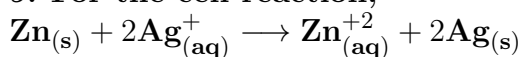
Step 4: Final Answer:

The molar mass is 46 g mol^{-1} , matching option (B).

 Quick Tip

Carefully trace skeletal structures. Count the ends and vertices to determine the number of carbon atoms. An unlabelled end is a CH_3 , a vertex is a CH_2 (if bonded to 2 other atoms), etc.

9. For the cell reaction,



Cell potential is less than E_{cell}° by 0.0592 V at 298 K when

(A) $[\text{Zn}^{+2}] = 1\text{M}$ and $[\text{Ag}^{+}] = 0.1\text{M}$

(B) $[\text{Zn}^{+2}] = 1\text{M}$ and $[\text{Ag}^{+}] = 0.01\text{M}$

(C) $[\text{Zn}^{+2}] = 0.1\text{M}$ and $[\text{Ag}^{+}] = 1\text{M}$

(D) $[\text{Zn}^{+2}] = 0.01\text{M}$ and $[\text{Ag}^{+}] = 1\text{M}$

Correct Answer: (A) $[\text{Zn}^{+2}] = 1\text{M}$ and $[\text{Ag}^{+}] = 0.1\text{M}$

Solution:

Step 1: Understanding the Concept:

The relationship between cell potential (E_{cell}) under non-standard conditions and standard cell potential (E_{cell}°) is given by the Nernst equation.

Step 2: Key Formula or Approach:

The Nernst equation at 298 K is:

$$E_{\text{cell}} = E_{\text{cell}}^{\circ} - \frac{0.0592}{n} \log_{10} Q$$

where n is the number of moles of electrons transferred, and Q is the reaction quotient.

Step 3: Detailed Explanation:

From the balanced redox reaction: $\text{Zn}_{(\text{s})} \longrightarrow \text{Zn}_{(\text{aq})}^{+2} + 2\text{e}^{-}$ (Oxidation)

$2\text{Ag}_{(\text{aq})}^{+} + 2\text{e}^{-} \longrightarrow 2\text{Ag}_{(\text{s})}$ (Reduction)

The number of electrons transferred is $n = 2$.

The reaction quotient Q for this reaction is:

$$Q = \frac{[\text{Zn}^{+2}]}{[\text{Ag}^{+}]^2}$$

(Note: pure solids like Zn and Ag are not included in the Q expression).

The problem states that the cell potential is less than E_{cell}° by 0.0592 V, which means:

$$E_{\text{cell}}^{\circ} - E_{\text{cell}} = 0.0592 \text{ V}$$

Substitute this into the rearranged Nernst equation:

$$E_{\text{cell}}^{\circ} - E_{\text{cell}} = \frac{0.0592}{2} \log_{10} Q$$

$$0.0592 = \frac{0.0592}{2} \log_{10} Q$$

Dividing both sides by 0.0592:

$$1 = \frac{1}{2} \log_{10} Q$$

$$\log_{10} Q = 2$$

$$Q = 10^2 = 100$$

Now we test the given options to find which gives $Q = 100$:

(A) $Q = \frac{1}{(0.1)^2} = \frac{1}{0.01} = 100$

(B) $Q = \frac{1}{(0.01)^2} = \frac{1}{0.0001} = 10000$

(C) $Q = \frac{0.1}{(1)^2} = 0.1$

(D) $Q = \frac{0.01}{(1)^2} = 0.01$

Only option (A) yields $Q = 100$.

Step 4: Final Answer:

The correct condition is $[\text{Zn}^{+2}] = 1\text{M}$ and $[\text{Ag}^{+}] = 0.1\text{M}$.

 Quick Tip

Remember to square the concentration of the Ag^{+} ion in the reaction quotient Q because of its stoichiometric coefficient of 2 in the balanced chemical equation.

10. Calculate the volume of unit cell having atomic radius 141.4 pm forming fcc unit cell.

(A) $9.3 \times 10^{-23} \text{ cm}^3$

(B) $8.1 \times 10^{-23} \text{ cm}^3$

(C) $6.4 \times 10^{-23} \text{ cm}^3$

(D) $4.7 \times 10^{-23} \text{ cm}^3$

Correct Answer: (C) $6.4 \times 10^{-23} \text{ cm}^3$

Solution:

Step 1: Understanding the Concept:

For a Face-Centered Cubic (FCC) lattice, the atoms touch along the face diagonal of the cubic unit cell. The relationship between the edge length (a) and the atomic radius (r) is essential to find the unit cell volume ($V = a^3$).

Step 2: Key Formula or Approach:

For an FCC unit cell:

$$a = 2\sqrt{2} \cdot r$$

Volume, $V = a^3$

Step 3: Detailed Explanation:

Given radius, $r = 141.4$ pm. Let's convert this to cm to match the options.

$$1 \text{ pm} = 10^{-12} \text{ m} = 10^{-10} \text{ cm}$$

$$r = 141.4 \times 10^{-10} \text{ cm}$$

Notice that $141.4 \approx 100 \times \sqrt{2}$ (since $\sqrt{2} \approx 1.414$). This approximation makes calculation easier:

$$r \approx 100\sqrt{2} \times 10^{-10} \text{ cm} = \sqrt{2} \times 10^{-8} \text{ cm}$$

Now, calculate the edge length 'a':

$$a = 2\sqrt{2} \cdot r = 2\sqrt{2} \cdot (\sqrt{2} \times 10^{-8} \text{ cm})$$

$$a = 2 \cdot (\sqrt{2} \cdot \sqrt{2}) \times 10^{-8} \text{ cm}$$

$$a = 2 \cdot 2 \times 10^{-8} \text{ cm} = 4 \times 10^{-8} \text{ cm}$$

Now, calculate the volume 'V':

$$V = a^3 = (4 \times 10^{-8} \text{ cm})^3$$

$$V = 4^3 \times (10^{-8})^3 \text{ cm}^3$$

$$V = 64 \times 10^{-24} \text{ cm}^3$$

To match the scientific notation format in the options:

$$V = 6.4 \times 10^{-23} \text{ cm}^3$$

Step 4: Final Answer:

The calculated volume perfectly matches option (C).

💡 Quick Tip

Recognizing common numerical values like $1.414 \approx \sqrt{2}$ and $1.732 \approx \sqrt{3}$ can drastically simplify complex calculations without needing a calculator.

11. Which from following mixtures obeys Raoult's law?

- (A) Chloroform + acetone
- (B) Carbon disulfide + acetone
- (C) Benzene + toluene
- (D) Ethanol + acetone

Correct Answer: (C) Benzene + toluene

Solution:

Step 1: Understanding the Concept:

A solution that obeys Raoult's law strictly over the entire range of concentration is called an ideal solution. In an ideal solution, the intermolecular attractive forces between the solute and solvent molecules (A-B interaction) are nearly equal to those between the pure components (A-A and B-B interactions).

Step 2: Key Formula or Approach:

Identify the nature of intermolecular forces in each mixture. Mixtures of structurally similar and non-polar compounds typically form ideal solutions.

Step 3: Detailed Explanation:

Let's analyze the given liquid mixtures:

- (A) **Chloroform + acetone:** They form strong hydrogen bonds between them ($\text{Cl}_3\text{C-H} \cdots \text{O}=\text{C}(\text{CH}_3)_2$), making A-B interactions stronger than A-A or B-B. This leads to a **negative deviation** from Raoult's law.
- (B) **Carbon disulfide + acetone:** Dipole-dipole interactions between acetone molecules are broken by the non-polar CS_2 molecules. Thus, A-B interactions are weaker, leading to a **positive deviation**.
- (C) **Benzene + toluene:** Both are structurally similar, non-polar aromatic hydrocarbons. The intermolecular forces in pure benzene, pure toluene, and the mixture are almost identical (weak van der Waals forces). Therefore, it behaves as an **ideal solution** and obeys Raoult's law.
- (D) **Ethanol + acetone:** Ethanol molecules are strongly hydrogen-bonded. Addition of acetone breaks some of these bonds, making A-B interactions weaker than A-A. This results in a **positive deviation**.

Step 4: Final Answer:

The mixture of Benzene and Toluene forms an ideal solution.

 Quick Tip

Ideal solutions are typically formed by mixing liquids with similar structures, polarities, and molecular sizes (e.g., n-hexane and n-heptane, bromoethane and chloroethane, benzene and toluene).

12. In a first order reaction concentration of reactant decreases from 20 milli mol dm⁻³ to 8 milli mol dm⁻³ in 40 minutes, find rate constant of reaction?

- (A) 0.011 minute⁻¹
- (B) 0.023 minute⁻¹
- (C) 0.032 minute⁻¹
- (D) 0.041 minute⁻¹

Correct Answer: (B) 0.023 minute⁻¹

Solution:

Step 1: Understanding the Concept:

For a first-order reaction, the integrated rate law connects the rate constant (k), time (t), initial concentration ($[A]_0$), and the concentration at time t ($[A]_t$).

Step 2: Key Formula or Approach:

The formula is:

$$k = \frac{2.303}{t} \log_{10} \left(\frac{[A]_0}{[A]_t} \right)$$

Step 3: Detailed Explanation:

Given data:

Initial concentration, $[A]_0 = 20 \text{ milli mol dm}^{-3}$

Final concentration, $[A]_t = 8 \text{ milli mol dm}^{-3}$

Time interval, $t = 40 \text{ minutes}$

Substitute the values into the formula:

$$k = \frac{2.303}{40} \log_{10} \left(\frac{20}{8} \right)$$

$$k = \frac{2.303}{40} \log_{10}(2.5)$$

The value of $\log_{10}(2.5)$ can be found using log properties. We know $\log_{10}(10/4) = 1 - 2 \log_{10}(2) = 1 - 2(0.3010) = 1 - 0.6020 = 0.3980$.

$$k = \frac{2.303 \times 0.3980}{40}$$

$$k \approx \frac{0.9166}{40}$$

$$k \approx 0.022915 \text{ min}^{-1}$$

Rounding to three decimal places, we get 0.023 min^{-1} .

Step 4: Final Answer:

This matches option (B).

 Quick Tip

Knowing basic log values like $\log 2 = 0.301$, $\log 3 = 0.477$, and $\log 5 = 0.699$ ($1 - \log 2$) speeds up kinetics problems significantly.

13. Identify the reagent R used in following reaction.

Ketone $\xrightarrow{R}$ semi carbazone

- (A) NH_2OH
- (B) $\text{NH}_2\text{NHCONH}_2$
- (C) $\text{NH}_2\text{NHC}_6\text{H}_5$
- (D) $\text{NH}_2 - \text{NH}_2$

Correct Answer: (B) $\text{NH}_2\text{NHCONH}_2$

Solution:

Step 1: Understanding the Concept:

Aldehydes and ketones react with various ammonia derivatives ($\text{NH}_2\text{-Z}$) in weakly acidic mediums to form compounds containing a carbon-nitrogen double bond (C=N-Z), with the elimination of a water molecule.

Step 2: Key Formula or Approach:

Match the specific name of the product (semicarbazone) to the corresponding nitrogenous nucleophile reagent.

Step 3: Detailed Explanation:

Let's match the given products with their corresponding reagents:

- Reaction with Hydroxylamine (NH_2OH) forms an **oxime**.
- Reaction with Hydrazine ($\text{NH}_2\text{-NH}_2$) forms a **hydrazone**.
- Reaction with Phenylhydrazine ($\text{NH}_2\text{NHC}_6\text{H}_5$) forms a **phenylhydrazone**.
- Reaction with Semicarbazide ($\text{NH}_2\text{NHCONH}_2$) forms a **semicarbazone**.

The question asks for the reagent R that yields a semicarbazone. Based on the standard reactions, the reagent is semicarbazide, which has the chemical formula $\text{NH}_2\text{NHCONH}_2$.

Step 4: Final Answer:

The correct reagent is (B).

 Quick Tip

In semicarbazide ($\text{H}_2\text{N} - \text{NH} - \text{CO} - \text{NH}_2$), only the terminal $-\text{NH}_2$ group adjacent to the $-\text{NH}-$ group participates in the reaction, as the other $-\text{NH}_2$ group is involved in resonance with the carbonyl group.

14. Which from following polymers needs peroxide as initiator for preparation?

- (A) Nylon 6,6
- (B) Polyacrylonitrile
- (C) Terylene
- (D) Bakelite

Correct Answer: (B) Polyacrylonitrile

Solution:

Step 1: Understanding the Concept:

Polymerization reactions are generally classified into addition (chain-growth) polymerization and condensation (step-growth) polymerization. Addition polymerization of alkenes or vinyl monomers often requires a free radical initiator, such as an organic peroxide (e.g., benzoyl peroxide).

Step 2: Key Formula or Approach:

Determine the type of polymerization for each given polymer. Polymers formed by free radical addition polymerization are the ones that require a peroxide initiator.

Step 3: Detailed Explanation:

Let's evaluate the polymerization process for each option:

- (A) **Nylon 6,6** is a polyamide formed by condensation polymerization of adipic acid and hexamethylenediamine. It does not require a free radical initiator.
- (B) **Polyacrylonitrile (PAN)** is formed by the addition polymerization of acrylonitrile

(CH₂=CH-CN). This process typically proceeds via a free radical mechanism, which is initiated by a peroxide catalyst.

- (C) **Terylene** (Dacron) is a polyester formed by condensation polymerization of ethylene glycol and terephthalic acid. It does not use a peroxide initiator.

- (D) **Bakelite** is a thermosetting polymer formed by the condensation reaction of phenol with formaldehyde. It is catalyzed by either acid or base, not a peroxide initiator.

Step 4: Final Answer:

Polyacrylonitrile requires a peroxide initiator.

 Quick Tip

As a general rule, polymers formed from monomers containing carbon-carbon double bonds (like ethylene, propylene, styrene, acrylonitrile) undergo addition polymerization, which frequently uses peroxide as a free radical initiator.

15. Which from following elements is NOT regarded as transition element?

(A) Ni

(B) Fe

(C) Ag

(D) Hg

Correct Answer: (D) Hg

Solution:

Step 1: Understanding the Concept:

According to IUPAC, a transition element is defined as an element whose atom has a partially filled d-subshell, or which can give rise to cations with an incomplete d-subshell in its common oxidation states.

Step 2: Key Formula or Approach:

Write down the electronic configurations of the given elements in their ground state and their common oxidation states to check for partially filled d-orbitals.

Step 3: Detailed Explanation:

Let's examine the electronic configurations of the given elements:

- (A) **Ni** (Nickel, $Z = 28$): [Ar]3d⁸4s². It has a partially filled d-orbital in its ground state, so it is a transition element.

- (B) **Fe** (Iron, $Z = 26$): [Ar]3d⁶4s². It has a partially filled d-orbital in its ground state, so it is a transition element.

- (C) **Ag** (Silver, $Z = 47$): [Kr]4d¹⁰5s¹. While its ground state has a completely filled d-subshell, its common oxidation state of +2 (Ag²⁺) has a 4d⁹ configuration, which is partially filled. Thus, silver is considered a transition element.

- (D) **Hg** (Mercury, $Z = 80$): [Xe]4f¹⁴5d¹⁰6s². Mercury has a completely filled 5d¹⁰ subshell in its ground state. Furthermore, in its common oxidation states (+1 and +2), the d-subshell remains completely filled (Hg²⁺ is 5d¹⁰). Because it never exhibits a partially filled d-orbital, mercury (along with zinc and cadmium from group 12) is not regarded as a true transition element.

Step 4: Final Answer:

Mercury (Hg) is not a transition element.

💡 Quick Tip

Group 12 elements (Zinc, Cadmium, and Mercury) have completely filled d-orbitals $((n-1)d^{10}ns^2)$ in both their ground state and their common oxidation states, hence they are excluded from the rigorous definition of transition elements.

16. Which from following is the correct relationship between molar conductivity (Λ), conductivity (k) and molarity (M) of solution for electrolyte?

- (A) $k = \frac{\Lambda \times C}{1000}$
 (B) $\Lambda = \frac{100 \times k}{C}$
 (C) $\Lambda = \frac{k \times C}{1000}$
 (D) $k = \frac{1000 \times C}{\Lambda}$

Correct Answer: (A) $k = \frac{\Lambda \times C}{1000}$

Solution:**Step 1: Understanding the Concept:**

Molar conductivity (Λ_m or simply Λ) is defined as the conducting power of all the ions produced by dissolving one mole of an electrolyte in solution. It is related to specific conductivity (κ , here denoted as k) and concentration (C).

Step 2: Key Formula or Approach:

The standard textbook relationship between molar conductivity (Λ), specific conductivity (κ in $S\text{ cm}^{-1}$), and molar concentration (C in mol L^{-1} or mol dm^{-3}) is given by:

$$\Lambda = \frac{\kappa \times 1000}{C}$$

where the factor of 1000 is used to convert the volume from L (dm^3) to cm^3 .

Step 3: Detailed Explanation:

Let's rearrange the standard formula to see which option it matches:

$$\Lambda = \frac{k \times 1000}{C}$$

Cross-multiplying yields:

$$\Lambda \times C = k \times 1000$$

Solving for k gives:

$$k = \frac{\Lambda \times C}{1000}$$

Comparing this rearranged equation with the given options, we see that it perfectly matches option (A).

Step 4: Final Answer:

The correct relationship is given in option (A).

 Quick Tip

Pay close attention to the units. If κ is in S m^{-1} and C is in mol m^{-3} , the formula is just $\Lambda = \kappa/C$. The 1000 factor appears due to unit conversions typical in chemistry problems (cm and L).

17. Which from following solids is isotropic?

- (A) Glass
- (B) Ceramics
- (C) Graphite
- (D) Ice

Correct Answer: (A) Glass

Solution:

Step 1: Understanding the Concept:

Isotropy refers to the property of a material where its physical properties (like electrical conductivity, refractive index, thermal expansion) are the same in all directions. This is a characteristic feature of amorphous solids. Conversely, anisotropy means properties vary with direction, which is typical of crystalline solids due to their highly ordered arrangement of particles.

Step 2: Key Formula or Approach:

Identify whether each given solid is crystalline or amorphous. The amorphous solid will be the isotropic one.

Step 3: Detailed Explanation:

Let's classify the given solids:

- (A) **Glass** is an amorphous solid (a supercooled liquid). Since its internal structure is random and lacks long-range order, its properties are uniform in all directions. Therefore, it is isotropic.
- (B) **Ceramics** are generally crystalline or partially crystalline materials. Crystalline regions cause anisotropy.
- (C) **Graphite** is a covalent network solid with a distinct layered crystalline structure. It is highly anisotropic (e.g., conducts electricity parallel to layers but not perpendicular to them).
- (D) **Ice** is a crystalline solid formed by a network of hydrogen bonds. Thus, it is anisotropic.

Step 4: Final Answer:

Glass is the only clear example of an isotropic amorphous solid among the choices.

 Quick Tip

Key distinguishing feature: Amorphous solids = Isotropic; Crystalline solids = Anisotropic.

18. Calculate ΔS_{total} for a certain reaction at 298 K if $\Delta H^\circ = -208.6 \text{ kJ}$ and $\Delta S^\circ = -36 \text{ J K}^{-1}$

- (A) 664 J K⁻¹
- (B) 834 J K⁻¹
- (C) 926 J K⁻¹
- (D) 736 J K⁻¹

Correct Answer: (A) 664 J K⁻¹

Solution:

Step 1: Understanding the Concept:

The total entropy change of the universe (ΔS_{total}) for a process is the sum of the entropy change of the system (ΔS_{sys}) and the entropy change of its surroundings (ΔS_{surr}).

Step 2: Key Formula or Approach:

$$\Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}}$$

The entropy change of the surroundings is related to the enthalpy change of the system by the equation:

$$\Delta S_{\text{surr}} = -\frac{\Delta H_{\text{sys}}}{T}$$

Step 3: Detailed Explanation:

Given values:

Temperature, $T = 298 \text{ K}$

Enthalpy change of system, $\Delta H_{\text{sys}} = \Delta H^\circ = -208.6 \text{ kJ} = -208600 \text{ J}$ (converted to Joules to match units)

Entropy change of system, $\Delta S_{\text{sys}} = \Delta S^\circ = -36 \text{ J K}^{-1}$

First, calculate ΔS_{surr} :

$$\begin{aligned}\Delta S_{\text{surr}} &= -\frac{-208600 \text{ J}}{298 \text{ K}} \\ \Delta S_{\text{surr}} &= +\frac{208600}{298} \text{ J K}^{-1} = +700 \text{ J K}^{-1}\end{aligned}$$

Now, calculate ΔS_{total} :

$$\begin{aligned}\Delta S_{\text{total}} &= -36 \text{ J K}^{-1} + 700 \text{ J K}^{-1} \\ \Delta S_{\text{total}} &= 664 \text{ J K}^{-1}\end{aligned}$$

Step 4: Final Answer:

The total entropy change is 664 J K⁻¹, which corresponds to option (A).

💡 Quick Tip

Always ensure unit consistency. Enthalpy (ΔH) is usually given in kJ, while entropy (ΔS) is in J K⁻¹. Convert ΔH to Joules before calculating ΔS_{surr} .

19. Which among the following is benzylic halide?

- (A) Bromophenylmethane
- (B) 4-Bromotoluene

- (C) 1-Bromo-2-phenylethane
(D) Bromobenzene

Correct Answer: (A) Bromophenylmethane

Solution:

Step 1: Understanding the Concept:

Halides are classified based on the nature of the carbon atom bonded to the halogen. A **benzylic halide** is a compound in which the halogen atom is bonded to an sp^3 hybridized carbon atom that is directly attached to an aromatic ring (a benzene ring).

Step 2: Key Formula or Approach:

Draw or visualize the chemical structure for each option and check the hybridization of the carbon attached to the halogen and its proximity to the aromatic ring.

Step 3: Detailed Explanation:

Let's analyze the structures of the given options:

- (A) **Bromophenylmethane** (also known as benzyl bromide): The structure is $C_6H_5-CH_2-Br$. Here, the Bromine is attached to a CH_2 group (sp^3 carbon) which is directly attached to the phenyl ring. This perfectly fits the definition of a benzylic halide.
- (B) **4-Bromotoluene**: The structure has a bromine atom and a methyl group on opposite ends of a benzene ring ($p-Br-C_6H_4-CH_3$). The Br is attached directly to the sp^2 carbon of the aromatic ring, making it an **aryl halide**.
- (C) **1-Bromo-2-phenylethane**: The structure is $C_6H_5-CH_2-CH_2-Br$. The Br is attached to an sp^3 carbon, but this carbon is separated from the benzene ring by another carbon atom. It is a simple **alkyl halide**.
- (D) **Bromobenzene**: The structure is C_6H_5-Br . The Br is attached directly to the aromatic ring, making it an **aryl halide**.

Step 4: Final Answer:

Bromophenylmethane is the benzylic halide.

💡 Quick Tip

Don't confuse benzylic with aryl. Aryl halides have the halogen directly on the ring. Benzylic halides have it on the carbon "next door" to the ring.

20. Which from following is correct regarding $t_{1/2}$ of reaction if we double the initial concentration of a reactant in first order reaction?

- (A) $t_{1/2}$ will increase by two times
(B) $t_{1/2}$ will decrease by four times
(C) $t_{1/2}$ remains the same
(D) $t_{1/2}$ will decrease by two times

Correct Answer: (C) $t_{1/2}$ remains the same

Solution:

Step 1: Understanding the Concept:

The half-life ($t_{1/2}$) of a reaction is the time required for the concentration of a reactant to decrease to half of its initial value. The dependence of half-life on initial concentration varies depending on the order of the reaction.

Step 2: Key Formula or Approach:

For a first-order reaction, the relationship between half-life and the rate constant (k) is given by:

$$t_{1/2} = \frac{\ln(2)}{k} \approx \frac{0.693}{k}$$

Step 3: Detailed Explanation:

Looking at the formula for the half-life of a first-order reaction, we see that $t_{1/2}$ depends only on the rate constant k . It does not contain the initial concentration term ($[A]_0$).

This means that for a first-order reaction, the time it takes for half of the reactant to be consumed is always the same, regardless of how much reactant you start with.

Therefore, if you double the initial concentration of the reactant, the half-life will remain completely unaffected.

Step 4: Final Answer:

The half-life remains the same, which corresponds to option (C).

💡 Quick Tip

General rule for half-life dependence on initial concentration: $t_{1/2} \propto \frac{1}{[A]_0^{n-1}}$, where n is the order of the reaction. For $n = 1$, $t_{1/2} \propto [A]_0^0$, meaning it's independent.

21. Identify the product of following reaction.



- (A) Phenyl methanol and methanol
- (B) Methanol and benzoic acid
- (C) Methanoic acid and phenyl methanol
- (D) Methanoic acid and benzoic acid

Correct Answer: (C) Methanoic acid and phenyl methanol

Solution:

Step 1: Understanding the Concept:

The reaction of aldehydes lacking α -hydrogen atoms with concentrated alkali (NaOH or KOH) results in a disproportionation reaction called the Cannizzaro reaction. When two different aldehydes without α -hydrogens are used, it is a **crossed Cannizzaro reaction**.

Step 2: Key Formula or Approach:

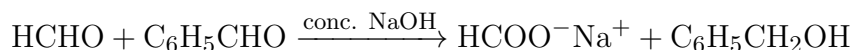
Identify the reaction type based on the reagents. In a crossed Cannizzaro reaction, the more reactive aldehyde (usually formaldehyde) gets oxidized to a carboxylic acid, and the less reactive one gets reduced to an alcohol.

Step 3: Detailed Explanation:

In a crossed Cannizzaro reaction involving formaldehyde (HCHO) and another non-enolizable aldehyde like benzaldehyde (C₆H₅CHO), the highly reactive formaldehyde is always oxidized

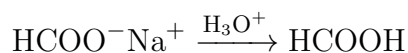
to the corresponding carboxylic acid salt (formate), while the other aldehyde is reduced to the corresponding alcohol.

The reaction proceeds as follows:



The products formed in the basic medium are sodium formate and benzyl alcohol (phenyl methanol).

The subsequent step involves acidification (H_3O^+), which converts the formate salt into methanoic acid (formic acid):



Therefore, the final isolated products are Methanoic acid (HCOOH) and Phenyl methanol ($\text{C}_6\text{H}_5\text{CH}_2\text{OH}$).

Step 4: Final Answer:

The products are methanoic acid and phenyl methanol.

💡 Quick Tip

In a crossed Cannizzaro reaction with formaldehyde, formaldehyde acts as the reducing agent and gets oxidized itself. The easier the nucleophilic attack on the carbonyl carbon (less steric hindrance), the more likely it is to be oxidized.

22. Which from following is a correct decreasing order of water solubilities of organic compounds?

- (A) Alcohols > Amines > Alkanes
- (B) Alkanes > Alcohols > Amines
- (C) Amines > Alcohols > Alkanes
- (D) Alcohols > Alkanes > Amines

Correct Answer: (A) Alcohols > Amines > Alkanes

Solution:

Step 1: Understanding the Concept:

The solubility of organic compounds in water heavily depends on their ability to form intermolecular hydrogen bonds with water molecules. The "like dissolves like" principle applies, meaning polar substances tend to be soluble in polar solvents (like water).

Step 2: Key Formula or Approach:

Compare the polarity and hydrogen-bonding capabilities of the functional groups in each class of compounds. The stronger the hydrogen bonding with water, the higher the solubility.

Step 3: Detailed Explanation:

Let's analyze the classes of compounds:

- **Alcohols (R-OH):** They have a highly polarized O-H bond due to the high electronegativity of oxygen. They can form strong hydrogen bonds with water molecules, making lower alcohols highly soluble.

- **Amines (R-NH₂):** They have an N-H bond. Nitrogen is less electronegative than oxygen, so the N-H bond is less polar than the O-H bond. While amines can form hydrogen bonds with water, these bonds are weaker than those formed by alcohols. Thus, amines are generally less soluble than corresponding alcohols of similar molar mass.

- **Alkanes (R-H):** These are non-polar hydrocarbons. They cannot form hydrogen bonds with water and only have weak van der Waals forces. Therefore, they are practically insoluble in water.

Based on the strength of interaction with water, the decreasing order of solubility is: Alcohols > Amines > Alkanes.

Step 4: Final Answer:

The correct option is (A).

 Quick Tip

Electronegativity trend: $O > N > C$. Higher electronegativity difference leads to stronger hydrogen bonding capability and consequently higher water solubility for comparable molecular weights.

23. Which from following coordinate complexes contains the ligand 'en'?

- (A) Tetraamminecopper (II) ion
- (B) Bis(ethylenediamine) dithiocyanatoplatinum (IV)
- (C) Pentacarbonyliron (0)
- (D) Tetracyanonickelate (II)

Correct Answer: (B) Bis(ethylenediamine) dithiocyanatoplatinum (IV)

Solution:

Step 1: Understanding the Concept:

In coordination chemistry, ligands are often denoted by abbreviations. The abbreviation '**en**' stands for the bidentate ligand **ethylenediamine** (or ethane-1,2-diamine according to IUPAC). Its chemical formula is $H_2NCH_2CH_2NH_2$.

Step 2: Key Formula or Approach:

Read the IUPAC nomenclature of the given complexes and identify which one contains the full name corresponding to the abbreviation 'en'.

Step 3: Detailed Explanation:

Let's review the ligands present in each of the given IUPAC names:

- (A) Tetra**ammine**copper (II) ion: Contains 'ammine' ligands (NH_3).
 - (B) Bis(**ethylenediamine**) dithiocyanatoplatinum (IV): Explicitly contains '**ethylenediamine**' ligands. The prefix 'bis' indicates there are two of these bulky bidentate ligands.
 - (C) Penta**carbonyl**iron (0): Contains 'carbonyl' ligands (CO).
 - (D) Tetra**cyano**nickelate (II): Contains 'cyano' ligands (CN^-).
- Therefore, the complex containing the 'en' ligand is option (B).

Step 4: Final Answer:

Option (B) is the correct answer.

 Quick Tip

Familiarize yourself with common ligand abbreviations: en = ethylenediamine, ox = oxalate, EDTA = ethylenediaminetetraacetate, py = pyridine.

24. What is the volume occupied by 0.5 mol of CO₂ at STP?

- (A) 5.6 dm³
- (B) 11.2 dm³
- (C) 16.8 dm³
- (D) 22.4 dm³

Correct Answer: (B) 11.2 dm³

Solution:

Step 1: Understanding the Concept:

Avogadro's law states that equal volumes of all ideal gases at the same temperature and pressure contain the same number of molecules. Standard Temperature and Pressure (STP) is defined such that 1 mole of any ideal gas occupies a standard volume.

Step 2: Key Formula or Approach:

Molar volume of a gas at STP $\approx$ 22.4 L/mol (or 22.4 dm³/mol).

$$\text{Volume} = \text{number of moles} \times \text{molar volume}$$

Step 3: Detailed Explanation:

Given:

Number of moles (n) = 0.5 mol

Molar volume at STP = 22.4 dm³/mol

Substitute the values into the formula:

$$\text{Volume} = 0.5 \text{ mol} \times 22.4 \text{ dm}^3/\text{mol}$$

$$\text{Volume} = 11.2 \text{ dm}^3$$

Step 4: Final Answer:

The volume occupied is 11.2 dm³, which matches option (B).

 Quick Tip

Always remember: 1 Liter (L) = 1 cubic decimeter (dm³). The value 22.4 L is the classic STP value widely used in standard chemistry problems.

25. Calculate the change in internal energy of the system if 20 kJ of work is done on the system and it releases 10 kJ of heat in a particular reaction.

- (A) 30 kJ
- (B) 10 kJ
- (C) -15 kJ
- (D) -20 kJ

Correct Answer: (B) 10 kJ

Solution:

Step 1: Understanding the Concept:

The change in internal energy (ΔU or ΔE) of a system is governed by the First Law of Thermodynamics, which states that energy cannot be created or destroyed, only transferred. The change in internal energy is equal to the heat added to the system plus the work done on the system.

Step 2: Key Formula or Approach:

The mathematical expression for the first law of thermodynamics is:

$$\Delta U = q + w$$

Where:

q = heat exchanged between the system and surroundings

w = work done on or by the system

Sign Convention (IUPAC):

- Heat absorbed *by* the system: q is positive (+).
- Heat released *from* the system: q is negative (-).
- Work done *on* the system: w is positive (+).
- Work done *by* the system: w is negative (-).

Step 3: Detailed Explanation:

From the problem description:

"20 kJ of work is done *on* the system" $\implies w = +20$ kJ

"it *releases* 10 kJ of heat" $\implies q = -10$ kJ

Substitute these values into the formula:

$$\Delta U = q + w$$

$$\Delta U = (-10 \text{ kJ}) + (+20 \text{ kJ})$$

$$\Delta U = 10 \text{ kJ}$$

The change in internal energy is positive 10 kJ.

Step 4: Final Answer:

The correct option is (B).

Quick Tip

Mastering the IUPAC sign conventions is critical for thermodynamics. Think from the perspective of the **system's energy balance**: anything entering the system is positive, anything leaving is negative.

26. What are the products formed when Li_2CO_3 undergoes decomposition?

- (A) $\text{Li}_2\text{O} + \text{CO}_2$
- (B) $\text{LiO} + \text{CO}_2$
- (C) $\text{LiC} + \text{CO}_2$
- (D) $\text{Li}_2\text{O}_2 + \text{CO}$

Correct Answer: (A) $\text{Li}_2\text{O} + \text{CO}_2$

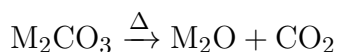
Solution:

Step 1: Understanding the Concept:

Most group 1 metal carbonates are thermally stable. However, lithium carbonate (Li_2CO_3) is exceptional because of the small size of the Li^+ ion, which polarizes the large carbonate (CO_3^{2-}) ion, leading to its decomposition upon heating.

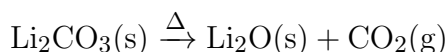
Step 2: Key Formula or Approach:

Thermal decomposition of alkali metal carbonates generally follows the reaction:



Step 3: Detailed Explanation:

Lithium belongs to group 1, and its carbonate is less stable compared to those of sodium or potassium. When Li_2CO_3 is heated, it breaks down into lithium oxide and carbon dioxide gas. Reaction:



Other alkali metal carbonates do not decompose at ordinary heating temperatures because the large cations stabilize the large carbonate anion.

Step 4: Final Answer:

The products are lithium oxide and carbon dioxide.

 Quick Tip

Remember the diagonal relationship: Lithium behaves similarly to Magnesium. Both their carbonates decompose on heating to form the respective oxide and CO_2 .

27. Identify the product formed from chlorobenzene on heating with conc. HNO_3 in presence of conc. H_2SO_4 .

- (A) Only 1-chloro-4-nitrobenzene
- (B) 1-chloro-2-nitrobenzene
- (C) Mixture of 1-chloro-4-nitrobenzene and 1-chloro-2-nitrobenzene
- (D) 2,4,6-trinitrochlorobenzene

Correct Answer: (C) Mixture of 1-chloro-4-nitrobenzene and 1-chloro-2-nitrobenzene

Solution:

Step 1: Understanding the Concept:

The reaction of chlorobenzene with a nitrating mixture (conc. HNO_3 and conc. H_2SO_4) is an electrophilic aromatic substitution reaction called nitration. The chlorine atom on the benzene ring is ortho and para directing.

Step 2: Key Formula or Approach:

The electrophile in this reaction is the nitronium ion (NO_2^+). Halogens are deactivating but ortho-para directing due to their resonance (+R) effect.

Step 3: Detailed Explanation:

When chlorobenzene reacts with conc. HNO_3 and conc. H_2SO_4 , the NO_2^+ group attacks the ortho and para positions of the ring.

Reaction:



IUPAC names for these are 1-chloro-2-nitrobenzene and 1-chloro-4-nitrobenzene. Typically, the para isomer is the major product due to less steric hindrance.

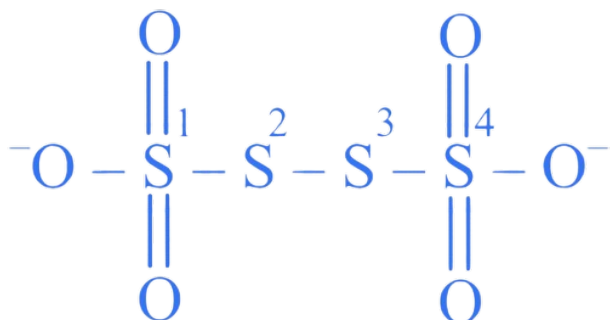
Step 4: Final Answer:

The result is a mixture of the ortho and para substituted products.

💡 Quick Tip

In electrophilic substitution, although halogens are electron-withdrawing (-I effect), their lone pairs allow for resonance (+R effect) that directs incoming groups to ortho and para positions.

28. What are the respective oxidation states of sulphur atoms numbered 1 to 4 in tetrathionate ion shown below?



- (A) 0, +5, +5, 0
- (B) +5, 0, 0, +5
- (C) +2, 0, 0, +2
- (D) +2, -1, -1, +2

Correct Answer: (B) +5, 0, 0, +5

Solution:

Step 1: Understanding the Concept:

In complex ions like tetrathionate ($\text{S}_4\text{O}_6^{2-}$), atoms of the same element can have different oxidation states depending on their local chemical environment and the atoms they are bonded

to.

Step 2: Key Formula or Approach:

The oxidation state of an atom is calculated by assigning valence electrons to the more electronegative atom in a bond. Bonds between atoms of the same element do not contribute to a change in oxidation state.

Step 3: Detailed Explanation:

Looking at the structure $[\text{O}_3\text{S}^{(1)}-\text{S}^{(2)}-\text{S}^{(3)}-\text{S}^{(4)}\text{O}_3]^{2-}$:

- **Sulphur atoms 2 and 3:** These are central and are bonded only to other sulphur atoms. Since they have no bonds to more electronegative atoms like oxygen, their oxidation state is 0.

- **Sulphur atoms 1 and 4:** Each of these terminal sulphurs is bonded to three oxygen atoms and one sulphur atom. In the S-O bonds, oxygen is more electronegative.

Counting bonds for S(1) or S(4): Two double bonds to oxygen ($2 \times 2 = 4$) and one single coordinate bond or ionic oxygen bond ($1 \times 1 = 1$). The total is +5.

The bond between $\text{S}^{(1)}-\text{S}^{(2)}$ is between identical atoms, so it adds 0 to the count.

Thus, the states are: $\text{S}^{(1)} = +5$, $\text{S}^{(2)} = 0$, $\text{S}^{(3)} = 0$, $\text{S}^{(4)} = +5$.

Step 4: Final Answer:

The sequence is +5, 0, 0, +5.

 Quick Tip

Average oxidation state of S in $\text{S}_4\text{O}_6^{2-}$ is $(2 \times 5 + 2 \times 0)/4 = 2.5$. This is a helpful check, but questions often ask for the specific individual states.

29. Which of the following is primary allylic alcohol?

- (A) $\text{H}_2\text{C} = \text{CH} - \text{CH}(\text{CH}_3) - \text{OH}$
- (B) $\text{H}_2\text{C} = \text{CH} - \text{CH}_2 - \text{OH}$
- (C) $\text{H}_2\text{C} = \text{CH} - \text{C}(\text{CH}_3)_2 - \text{OH}$
- (D) $\text{CH}_3 - \text{CH} = \text{CH} - \text{CH}(\text{CH}_3) - \text{OH}$

Correct Answer: (B) $\text{H}_2\text{C} = \text{CH} - \text{CH}_2 - \text{OH}$

Solution:

Step 1: Understanding the Concept:

An allylic alcohol is one where the hydroxyl group ($-\text{OH}$) is attached to an allylic carbon (an sp^3 carbon adjacent to a carbon-carbon double bond). A primary alcohol is one where the hydroxyl-bearing carbon is attached to only one other carbon atom.

Step 2: Key Formula or Approach:

Structure of allylic group: $\text{C}=\text{C}-\text{C}-$.

Primary allylic structure: $\text{C}=\text{C}-\text{CH}_2-\text{OH}$.

Step 3: Detailed Explanation:

Let's analyze the options:

- (A) $\text{H}_2\text{C}=\text{CH}-\text{CH}(\text{CH}_3)-\text{OH}$: The carbon with $-\text{OH}$ is bonded to two other carbons. It is a secondary allylic alcohol.
- (B) $\text{H}_2\text{C}=\text{CH}-\text{CH}_2-\text{OH}$: The carbon with $-\text{OH}$ is bonded to only one other carbon (part of

the double bond). This is a primary allylic alcohol (allyl alcohol).

- (C) $\text{H}_2\text{C}=\text{CH}-\text{C}(\text{CH}_3)_2-\text{OH}$: The carbon with $-\text{OH}$ is bonded to three other carbons. It is a tertiary allylic alcohol.

- (D) $\text{CH}_3-\text{CH}=\text{CH}-\text{CH}(\text{CH}_3)-\text{OH}$: Similar to A, it is a secondary allylic alcohol.

Step 4: Final Answer:

The primary allylic alcohol is allyl alcohol, shown in option B.

 Quick Tip

To identify primary/secondary/tertiary, count the carbons directly attached to the C-OH atom. 1 carbon = primary, 2 = secondary, 3 = tertiary.

30. Identify the pair of carbohydrates containing galactose as one of constituent in both of them.

- (A) Sucrose and Stachyose
- (B) Maltose and Raffinose
- (C) Raffinose and Stachyose
- (D) Lactose and Maltose

Correct Answer: (C) Raffinose and Stachyose

Solution:

Step 1: Understanding the Concept:

Complex carbohydrates (oligosaccharides and polysaccharides) are composed of simpler monosaccharide units linked by glycosidic bonds. Knowing the hydrolysis products of common sugars is key.

Step 2: Key Formula or Approach:

Recall constituents:

Sucrose = Glucose + Fructose

Lactose = Glucose + Galactose

Maltose = Glucose + Glucose

Raffinose = Galactose + Glucose + Fructose

Stachyose = Galactose + Galactose + Glucose + Fructose

Step 3: Detailed Explanation:

- Option A: Sucrose (No galactose) and Stachyose (Has galactose).
- Option B: Maltose (No galactose) and Raffinose (Has galactose).
- Option C: Raffinose (Has one galactose unit) and Stachyose (Has two galactose units). Both contain galactose.
- Option D: Lactose (Has galactose) and Maltose (No galactose).

Step 4: Final Answer:

The pair where both members contain galactose is Raffinose and Stachyose.

 Quick Tip

Raffinose and Stachyose are members of the "raffinose family" of oligosaccharides found in plants (like beans), characterized by adding one or more galactose units to a sucrose core.

31. The pH of a sample of vinegar is 3.76. Calculate the concentration of hydrogen ion in it in mol dm^{-3} ?

- (A) 1.97×10^{-4}
- (B) 1.738×10^{-4}
- (C) 1.84×10^{-4}
- (D) 1.283×10^{-4}

Correct Answer: (B) 1.738×10^{-4}

Solution:

Step 1: Understanding the Concept:

The pH of a solution is defined as the negative base-10 logarithm of its hydrogen ion concentration $[\text{H}^+]$. Conversely, the $[\text{H}^+]$ can be found by taking the antilogarithm of the negative pH.

Step 2: Key Formula or Approach:

$$\begin{aligned}\text{pH} &= -\log_{10}[\text{H}^+] \\ [\text{H}^+] &= 10^{-\text{pH}}\end{aligned}$$

Step 3: Detailed Explanation:

Given $\text{pH} = 3.76$.

$$[\text{H}^+] = 10^{-3.76}$$

To calculate this without a calculator, we write:

$$[\text{H}^+] = 10^{(0.24-4)} = 10^{0.24} \times 10^{-4}$$

The antilog of 0.24 is approximately 1.738.

Thus, $[\text{H}^+] = 1.738 \times 10^{-4} \text{ mol dm}^{-3}$.

Comparing with options, this matches option B.

Step 4: Final Answer:

The concentration is $1.738 \times 10^{-4} \text{ mol dm}^{-3}$.

 Quick Tip

When calculating $10^{-\text{pH}}$, a pH of 3.76 (between 3 and 4) must yield a concentration starting with 10^{-4} . This eliminates several potential errors.

32. Which from following complexes is an example of MA₂BC type of distereoisomers?

- (A) [Co(en)₂Cl₂]⁺
- (B) [Pt(NH₃)(H₂O)Cl₂]
- (C) Pt(NH₃)₂Cl₂
- (D) [Co(NH₃)₄Cl₂]⁺

Correct Answer: (B) [Pt(NH₃)(H₂O)Cl₂]

Solution:

Step 1: Understanding the Concept:

Stereoisomers that are not mirror images of each other are called diastereoisomers. In square planar coordination complexes (typically Pt(II)), the type MA₂BC can exhibit cis and trans isomers.

Step 2: Key Formula or Approach:

Match the general formula MA₂BC to the provided complex structures. Here, M is the metal, and A, B, C are monodentate ligands. 'A' is present twice.

Step 3: Detailed Explanation:

- (A) [Co(en)₂Cl₂]⁺: This is an octahedral complex with bidentate ligands, formula M(AA)₂X₂.
- (B) [Pt(NH₃)(H₂O)Cl₂]: This is a square planar complex of Pt(II). Here, metal M = Pt, ligands are: two Cl⁻ (A), one NH₃ (B), and one H₂O (C). It matches the MA₂BC type. It shows cis and trans diastereoisomers.
- (C) Pt(NH₃)₂Cl₂: This is of type MA₂B₂.
- (D) [Co(NH₃)₄Cl₂]⁺: This is of type MA₄B₂.

Step 4: Final Answer:

The complex [Pt(NH₃)(H₂O)Cl₂] fits the description.

💡 Quick Tip

Square planar complexes (*dsp*² hybridization) of type MA₂B₂ and MA₂BC always show cis-trans isomerism.

33. Which from following polymers is obtained by addition polymerisation method?

- (A) Nylon 6
- (B) Terylene
- (C) Nylon 6,6
- (D) Teflon

Correct Answer: (D) Teflon

Solution:

Step 1: Understanding the Concept:

Polymers are formed via addition (chain-growth) or condensation (step-growth) polymerization. Addition polymerization involves monomers with multiple bonds linking together without the

loss of any small molecules.

Step 2: Key Formula or Approach:

Identify the monomers:

Nylon 6 (Caprolactam - condensation/ring opening)

Terylene (Ethylene glycol + Terephthalic acid - condensation)

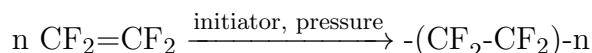
Nylon 6,6 (Adipic acid + Hexamethylene diamine - condensation)

Teflon (Tetrafluoroethene - addition)

Step 3: Detailed Explanation:

Teflon (Polytetrafluoroethene or PTFE) is synthesized by the polymerization of tetrafluoroethene ($\text{CF}_2=\text{CF}_2$) molecules.

Reaction:



Since no small molecules are eliminated and it proceeds via the double bond, it is a pure addition polymer.

Step 4: Final Answer:

Teflon is the addition polymer.

 Quick Tip

A quick hint: if the name ends in "-ene" (like ethylene, styrene, tetrafluoroethene), the resulting polymer is almost always an addition polymer.

34. Which of the following is one of the product of ozonolysis?

- (A) alcohol
- (B) acid
- (C) aldehyde
- (D) ester

Correct Answer: (C) aldehyde

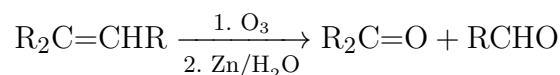
Solution:

Step 1: Understanding the Concept:

Ozonolysis is a reaction that cleaves carbon-carbon double bonds in alkenes using ozone (O_3). The intermediate ozonide is then treated with a reducing agent (typically Zn and water).

Step 2: Key Formula or Approach:

General ozonolysis reaction of an alkene:



Step 3: Detailed Explanation:

Depending on the substitution of the alkene, reductive ozonolysis produces aldehydes and/or ketones.

- A terminal $=\text{CH}_2$ group yields formaldehyde (HCHO).
- An internal $=\text{CHR}$ group yields an aldehyde (RCHO).

- An internal $=\text{CR}_2$ group yields a ketone ($\text{R}_2\text{C}=\text{O}$).

Among the given options, 'aldehyde' is a standard primary product. Acids are only produced if an oxidative workup (like H_2O_2) is used.

Step 4: Final Answer:

Aldehyde is a correct product.

 Quick Tip

Reductive ozonolysis ($\text{Zn}/\text{H}_2\text{O}$) gives aldehydes/ketones. Oxidative ozonolysis (H_2O_2) converts aldehydes into carboxylic acids.

35. Find molar mass of nonvolatile solute when 20 g of it dissolved in 200 g water at 300 K. [Relative lowering of vapour pressure = 0.02]

- (A) 120 g mol^{-1}
- (B) 110 g mol^{-1}
- (C) 90 g mol^{-1}
- (D) 100 g mol^{-1}

Correct Answer: (C) 90 g mol^{-1}

Solution:

Step 1: Understanding the Concept:

According to Raoult's law for a dilute solution containing a non-volatile solute, the relative lowering of vapour pressure is equal to the mole fraction of the solute.

Step 2: Key Formula or Approach:

$$\text{RLVP} = \frac{P^\circ - P}{P^\circ} = X_{\text{solute}} \approx \frac{n_2}{n_1} \text{ (for dilute solutions)}$$

Where $n_2 = w_2/M_2$ and $n_1 = w_1/M_1$.

Step 3: Detailed Explanation:

Given:

$$\text{RLVP} = 0.02$$

$$\text{Mass of solute } (w_2) = 20 \text{ g}$$

$$\text{Mass of solvent } (w_1) = 200 \text{ g}$$

$$\text{Molar mass of solvent (water, } M_1) = 18 \text{ g/mol}$$

$$0.02 = \frac{20/M_2}{200/18}$$

$$0.02 = \frac{20 \times 18}{200 \times M_2}$$

$$0.02 = \frac{1.8}{M_2}$$

$$M_2 = \frac{1.8}{0.02} = 90 \text{ g/mol}$$

Step 4: Final Answer:

The molar mass is 90 g mol^{-1} .

💡 Quick Tip

For very dilute solutions, using n_2/n_1 instead of $n_2/(n_1 + n_2)$ simplifies math significantly without losing much accuracy.

36. Which from following reactions performs zero work?

- (A) $\text{CH}_4(\text{g}) + \text{Cl}_2(\text{g}) \longrightarrow \text{CH}_3\text{Cl}(\text{g}) + \text{HCl}(\text{g})$
 (B) $3\text{H}_2(\text{g}) + \text{N}_2(\text{g}) \longrightarrow 2\text{NH}_3(\text{g})$
 (C) $\text{C}_2\text{H}_2(\text{g}) + \frac{5}{2}\text{O}_2(\text{g}) \longrightarrow 2\text{CO}_2(\text{g}) + \text{H}_2\text{O}(\text{l})$
 (D) $2\text{C}_2\text{H}_6(\text{g}) + 7\text{O}_2(\text{g}) \longrightarrow 4\text{CO}_2(\text{g}) + 6\text{H}_2\text{O}(\text{l})$

Correct Answer: (A) $\text{CH}_4(\text{g}) + \text{Cl}_2(\text{g}) \longrightarrow \text{CH}_3\text{Cl}(\text{g}) + \text{HCl}(\text{g})$

Solution:**Step 1: Understanding the Concept:**

The work done by or on a gas during a chemical reaction at constant temperature and pressure is given by $W = -P\Delta V = -\Delta n_g RT$. For the work to be zero, the change in the number of moles of gaseous species (Δn_g) must be zero.

Step 2: Key Formula or Approach:

$$\Delta n_g = (\text{moles of gaseous products}) - (\text{moles of gaseous reactants})$$

If $\Delta n_g = 0$, then $W = 0$.

Step 3: Detailed Explanation:

Let's check Δn_g for each option:

- (A) $\Delta n_g = (1 + 1) - (1 + 1) = 2 - 2 = 0$. Thus, $W = 0$.
- (B) $\Delta n_g = 2 - (3 + 1) = 2 - 4 = -2$. Thus, $W \neq 0$.
- (C) $\Delta n_g = 2 - (1 + 2.5) = 2 - 3.5 = -1.5$. Thus, $W \neq 0$. (Note: H_2O is liquid).
- (D) $\Delta n_g = 4 - (2 + 7) = 4 - 9 = -5$. Thus, $W \neq 0$.

Only reaction A has no net change in the number of moles of gas.

Step 4: Final Answer:

Reaction A performs zero work.

💡 Quick Tip

Always carefully check the physical states (s, l, g) given in chemical equations before calculating Δn_g . Only gaseous species are counted.

37. Identify the product 'B' in the following reaction sequence.

Alkyl halide $\xrightarrow{\text{Mg/Dry ether}}$ A $\xrightarrow{\text{NH}_3}$ B

- (A) Alkyl magnesium halide
- (B) Alkyl amine
- (C) Hydrocarbon
- (D) Alkyl nitrile

Correct Answer: (C) Hydrocarbon

Solution:

Step 1: Understanding the Concept:

Alkyl halides react with magnesium in dry ether to form Grignard reagents (R-Mg-X). Grignard reagents are highly reactive and act as strong bases; they react instantly with any compound containing active hydrogen (protic compounds).

Step 2: Key Formula or Approach:

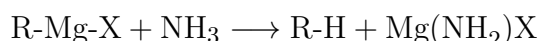
1. $R-X + Mg \xrightarrow{\text{ether}} R-Mg-X$ (Product A)
2. $R-Mg-X + H-Z \longrightarrow R-H + Mg(Z)X$

Step 3: Detailed Explanation:

In step 1, the alkyl halide reacts with magnesium to give 'A', which is an alkyl magnesium halide (Grignard reagent).

In step 2, the Grignard reagent reacts with ammonia (NH₃). Ammonia has active hydrogen atoms (H-NH₂). The nucleophilic alkyl group (R⁻) of the Grignard reagent abstract a proton from ammonia to form an alkane (hydrocarbon).

Reaction:



The organic product 'B' is R-H, which is a hydrocarbon.

Step 4: Final Answer:

Product B is a hydrocarbon.

 Quick Tip

Grignard reagents react with any source of acidic hydrogen (water, alcohols, amines, ammonia, carboxylic acids) to produce the parent alkane. This is why dry conditions are essential during their synthesis.

38. Match column I (process) with column II (application)

	Column I		Column II
i.	Dialysis	a.	Cleansing action of soap
ii.	Peptization	b.	Coagulation
iii.	Emulsification	c.	Colloidal solution preparation
iv.	Electrophoresis	d.	Purification of colloidal solution

- (A) i - d, ii - a, iii - c, iv - b
- (B) i - d, ii - c, iii - a, iv - b
- (C) i - c, ii - b, iii - d, iv - a
- (D) i - b, ii - c, iii - a, iv - d

Correct Answer: (B) i - d, ii - c, iii - a, iv - b

Solution:**Step 1: Understanding the Concept:**

Colloidal chemistry involves various techniques for the preparation, purification, and study of properties of colloidal systems. Each term refers to a specific interaction or process.

Step 2: Key Formula or Approach:

Match based on standard definitions:

- **Dialysis:** Removal of crystalloids from a colloid through a membrane $\rightarrow$ Purification.
- **Peptization:** Conversion of a fresh precipitate into a colloid $\rightarrow$ Preparation.
- **Emulsification:** Dispersion of one liquid in another (aided by soap) $\rightarrow$ Cleansing action.
- **Electrophoresis:** Movement of charged particles in an electric field, leading to discharge and settling $\rightarrow$ Coagulation.

Step 3: Detailed Explanation:

- i. Dialysis is used to purify sols by removing electrolytes (match d).
- ii. Peptization is a method to prepare stable sols from precipitates (match c).
- iii. Emulsification is the principle behind how soaps remove dirt/grease (match a).
- iv. Electrophoresis causes colloidal particles to clump together (coagulate) at the electrode (match b).

The combination is i-d, ii-c, iii-a, iv-b.

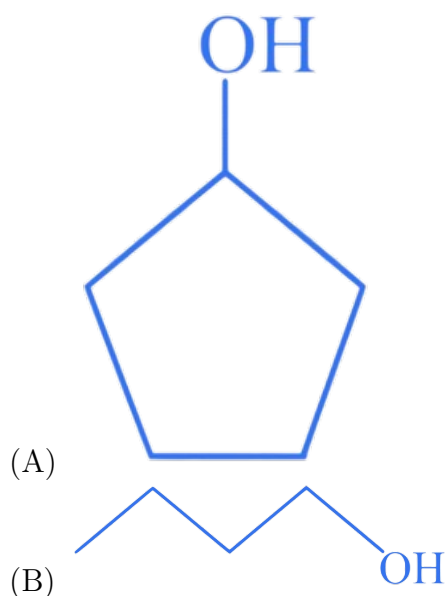
Step 4: Final Answer:

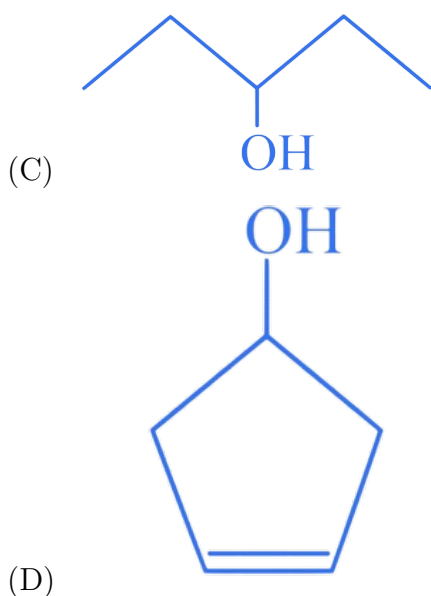
This matches option B.

💡 Quick Tip

Associate these keywords: Dialysis $\rightarrow$ Pure; Peptization $\rightarrow$ Sol from Precipitate; Soap $\rightarrow$ Emulsion; Electrophoresis $\rightarrow$ Charge/Coagulate.

39. Which of the following is the structure of an alcohol with molecular formula $C_5H_{12}O$?





Correct Answer: (B)

Solution:

Step 1: Understanding the Concept:

The formula $C_nH_{2n+2}O$ represents saturated open-chain alcohols or ethers. Here $n = 5$, so $H = 2(5) + 2 = 12$. A formula of $C_5H_{12}O$ means the compound has no rings or double bonds.

Step 2: Key Formula or Approach:

Degree of unsaturation (DoU) $= C + 1 - \frac{H}{2} = 5 + 1 - \frac{12}{2} = 6 - 6 = 0$.

A DoU of 0 indicates a fully saturated, open-chain molecule.

Step 3: Detailed Explanation:

- (A) Cyclopentanol: Has one ring. Formula is $C_5H_{10}O$. Incorrect.
- (B) Pentan-1-ol: A straight chain of 5 carbons and 12 hydrogens. Formula is $C_5H_{12}O$. Correct.
- (D) A ring with a double bond would have even fewer hydrogens (e.g., C_5H_8O). Incorrect.

Looking at the skeletal images:

Image B shows 5 vertices/ends (excluding oxygen) in an open zigzag chain: $CH_3-CH_2-CH_2-CH_2-CH_2-OH$.

Step 4: Final Answer:

Structure B corresponds to the formula.

 Quick Tip

Always count hydrogens for rings. Every ring in a structure reduces the hydrogen count by 2 compared to the open-chain alkane.

40. Four vessels of same volume consist equal masses of four gases H_2 , Cl_2 , N_2 , and O_2 separately at same temperature. The pressure exerted by the gas is maximum for

- (A) H₂
- (B) Cl₂
- (C) N₂
- (D) O₂

Correct Answer: (A) H₂

Solution:

Step 1: Understanding the Concept:

According to the ideal gas law, pressure is directly proportional to the number of moles of gas when volume and temperature are constant.

Step 2: Key Formula or Approach:

$$PV = nRT \implies P = \frac{nRT}{V}$$

$$\text{Since } n = \frac{\text{mass}}{\text{Molar mass}}, \text{ then } P = \frac{\text{mass} \times RT}{M \times V}$$

Step 3: Detailed Explanation:

Given that mass, volume, and temperature are the same for all vessels, the pressure exerted by a gas is inversely proportional to its molar mass ($P \propto 1/M$).

Molar masses:

$$M(\text{H}_2) = 2 \text{ g/mol}$$

$$M(\text{N}_2) = 28 \text{ g/mol}$$

$$M(\text{O}_2) = 32 \text{ g/mol}$$

$$M(\text{Cl}_2) = 71 \text{ g/mol}$$

Hydrogen (H₂) has the lowest molar mass, which means it will have the highest number of moles for a given mass. Consequently, it will exert the maximum pressure.

Step 4: Final Answer:

The gas with maximum pressure is H₂.

💡 Quick Tip

For equal mass, the lightest gas always has the most molecules/moles and thus exerts the highest pressure.

41. The solubility product of a sparingly soluble salt AX is 4.9×10^{-13} . What is its solubility in mol dm^{-3} ?

- (A) 2.4×10^{-13}
- (B) 4.9×10^{-7}
- (C) 7.0×10^{-7}
- (D) 7.0×10^{-13}

Correct Answer: (C) 7.0×10^{-7}

Solution:

Step 1: Understanding the Concept:

For a binary sparingly soluble salt of the type AX, it dissociates into one cation and one anion. The solubility product (K_{sp}) is related to the molar solubility (s).

Step 2: Key Formula or Approach:

$$\begin{aligned} \text{AX(s)} &\rightleftharpoons \text{A}^+(\text{aq}) + \text{X}^-(\text{aq}) \\ K_{sp} &= [\text{A}^+][\text{X}^-] = (s)(s) = s^2 \implies s = \sqrt{K_{sp}} \end{aligned}$$

Step 3: Detailed Explanation:

Given $K_{sp} = 4.9 \times 10^{-13}$.

$$\begin{aligned} s &= \sqrt{4.9 \times 10^{-13}} = \sqrt{49 \times 10^{-14}} \\ s &= 7.0 \times 10^{-7} \text{ moldm}^{-3} \end{aligned}$$

Step 4: Final Answer:

The molar solubility is $7.0 \times 10^{-7} \text{ moldm}^{-3}$.

 Quick Tip

Always convert numbers to even powers of 10 before taking a square root to make the calculation straightforward. Here 10^{-13} was changed to 10^{-14} .

42. Which from following cations develops lowest value of spin only magnetic moment?

- (A) V^{3+}
- (B) Cr^{3+}
- (C) Mn^{2+}
- (D) Fe^{2+}

Correct Answer: (A) V^{3+}

Solution:

Step 1: Understanding the Concept:

The "spin-only" magnetic moment depends on the number of unpaired electrons (n) in the d-subshell of the transition metal ion. Fewer unpaired electrons result in a lower magnetic moment.

Step 2: Key Formula or Approach:

$$\mu = \sqrt{n(n+2)} \text{ B.M.}$$

Step 3: Detailed Explanation:

Let's find the number of unpaired electrons (n) for each cation:

- (A) V^{3+} : Vanadium ($Z = 23$) is $[\text{Ar}]3d^34s^2$. V^{3+} is $[\text{Ar}]3d^2$. $\mathbf{n = 2}$.
- (B) Cr^{3+} : Chromium ($Z = 24$) is $[\text{Ar}]3d^54s^1$. Cr^{3+} is $[\text{Ar}]3d^3$. $\mathbf{n = 3}$.
- (C) Mn^{2+} : Manganese ($Z = 25$) is $[\text{Ar}]3d^54s^2$. Mn^{2+} is $[\text{Ar}]3d^5$. $\mathbf{n = 5}$.
- (D) Fe^{2+} : Iron ($Z = 26$) is $[\text{Ar}]3d^64s^2$. Fe^{2+} is $[\text{Ar}]3d^6$. Unpaired electrons: 4 (since $10 - 6 = 4$). $\mathbf{n = 4}$.

Among the options, V^{3+} has the minimum number of unpaired electrons ($n = 2$), so it will have the lowest magnetic moment.

Step 4: Final Answer:

V^{3+} has the lowest magnetic moment.

💡 Quick Tip

For first-row transition metal ions with d^x configuration, unpaired electrons $n = x$ for $x \leq 5$ and $n = 10 - x$ for $x > 5$.

43. How long should aqueous NaCl be electrolysed by passing 100 ampere current, so that 0.5 mol chlorine is released at anode?

- (A) 96500 seconds
- (B) 9650 seconds
- (C) 965 seconds
- (D) 96.5 seconds

Correct Answer: (C) 965 seconds

Solution:

Step 1: Understanding the Concept:

According to Faraday's laws of electrolysis, the amount of substance produced at an electrode is proportional to the quantity of electricity passed through the electrolyte.

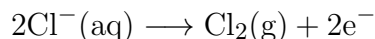
Step 2: Key Formula or Approach:

$$Q = I \times t = n \times F \times (\text{moles of product})$$

Where n is the number of electrons per molecule of product, and $F \approx 96500 \text{ C/mol}$.

Step 3: Detailed Explanation:

The anodic reaction for chlorine release is:



From this, 1 mole of Cl_2 requires 2 moles of electrons ($n = 2$).

Total charge required for 0.5 mol Cl_2 :

$$Q = 0.5 \times 2 \times 96500 \text{ C} = 96500 \text{ C}$$

We are given current $I = 100 \text{ A}$.

$$t = \frac{Q}{I} = \frac{96500 \text{ C}}{100 \text{ A}} = 965 \text{ seconds}$$

Step 4: Final Answer:

The time required is 965 seconds.

 Quick Tip

Always write the electrode half-reaction first to find the value of n (number of electrons). For H_2 , Cl_2 , O_2 , n is 2, 2, and 4 respectively.

44. Calculate the number of atoms present in 1.58 g metal if it forms bcc structure.

$$[\rho \times a^3 = 1.58 \times 10^{-22} \text{ g}]$$

- (A) 1.0×10^{22}
- (B) 2.0×10^{22}
- (C) 3.0×10^{22}
- (D) 4.0×10^{22}

Correct Answer: (B) 2.0×10^{22}

Solution:

Step 1: Understanding the Concept:

The density of a crystal is related to the mass and volume of its unit cells. The total mass of a sample is the mass of one unit cell multiplied by the total number of unit cells in that sample.

Step 2: Key Formula or Approach:

1. Mass of one unit cell $= \rho \times a^3$.
2. Number of unit cells (N_{uc}) $= \frac{\text{Total Mass}}{\text{Mass of 1 unit cell}}$.
3. Total atoms $= N_{uc} \times z$, where z is atoms per unit cell.

Step 3: Detailed Explanation:

Given:

Total mass = 1.58 g

Mass of one unit cell ($\rho \times a^3$) $= 1.58 \times 10^{-22} \text{ g}$

For a BCC structure, $z = 2$.

First, find the number of unit cells:

$$N_{uc} = \frac{1.58}{1.58 \times 10^{-22}} = 10^{22}$$

Then, find the total number of atoms:

$$\text{Total atoms} = 10^{22} \times 2 = 2.0 \times 10^{22}$$

Step 4: Final Answer:

There are 2.0×10^{22} atoms.

 Quick Tip

For solid state problems, always remember the z values: Simple Cubic = 1, BCC = 2, FCC = 4.

45. Identify the correct order of thermal stability of hydrides of 16 group elements from the following.

- (A) $\text{H}_2\text{S} < \text{H}_2\text{O} < \text{H}_2\text{Se} < \text{H}_2\text{Te}$
- (B) $\text{H}_2\text{O} < \text{H}_2\text{S} < \text{H}_2\text{Se} < \text{H}_2\text{Te}$
- (C) $\text{H}_2\text{Te} < \text{H}_2\text{Se} < \text{H}_2\text{S} < \text{H}_2\text{O}$
- (D) $\text{H}_2\text{Se} < \text{H}_2\text{Te} < \text{H}_2\text{O} < \text{H}_2\text{S}$

Correct Answer: (C) $\text{H}_2\text{Te} < \text{H}_2\text{Se} < \text{H}_2\text{S} < \text{H}_2\text{O}$

Solution:

Step 1: Understanding the Concept:

Thermal stability of hydrides depends on the bond dissociation enthalpy of the M-H bond. As the size of the central atom (M) increases down a group, the M-H bond length increases and the bond strength decreases.

Step 2: Key Formula or Approach:

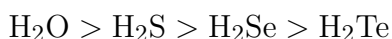
Atomic size: $\text{O} < \text{S} < \text{Se} < \text{Te}$.

Bond strength: $\text{O-H} > \text{S-H} > \text{Se-H} > \text{Te-H}$.

Step 3: Detailed Explanation:

Going down group 16 (O, S, Se, Te, Po), the atomic radius of the element increases. This leads to a longer and weaker bond with hydrogen. Weaker bonds are more easily broken by heat.

Therefore, thermal stability decreases down the group:



In increasing order (as shown in C): $\text{H}_2\text{Te} < \text{H}_2\text{Se} < \text{H}_2\text{S} < \text{H}_2\text{O}$.

Step 4: Final Answer:

The correct order is C.

 Quick Tip

Stability trends of hydrides for Groups 15, 16, and 17 all follow the same logic: stability decreases down the group due to increasing atomic size and decreasing bond strength.

46. Calculate the longest wavelength in hydrogen emission spectrum of Lyman series.

$[\text{R}_\text{H} = 109677 \text{ cm}^{-1}]$

- (A) $1.331 \times 10^{-5} \text{ cm}$
- (B) $1.216 \times 10^{-5} \text{ cm}$
- (C) $1.445 \times 10^{-5} \text{ cm}$
- (D) $1.556 \times 10^{-5} \text{ cm}$

Correct Answer: (B) $1.216 \times 10^{-5} \text{ cm}$

Solution:

Step 1: Understanding the Concept:

The wavelength of light emitted during electron transitions in a hydrogen atom is given by the Rydberg formula. In the Lyman series, the electron falls to the ground state ($n_1 = 1$). The "longest wavelength" corresponds to the transition with the "minimum energy" change.

Step 2: Key Formula or Approach:

Rydberg equation:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For Lyman longest wavelength: $n_1 = 1$ and $n_2 = 2$.

Step 3: Detailed Explanation:

$$\frac{1}{\lambda} = 109677 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = 109677 (1 - 0.25)$$

$$\frac{1}{\lambda} = 109677 \times 0.75 = 82257.75 \text{ cm}^{-1}$$

$$\lambda = \frac{1}{82257.75} \approx 1.21569 \times 10^{-5} \text{ cm}$$

Rounding to significant figures, we get $1.216 \times 10^{-5} \text{ cm}$.

Step 4: Final Answer:

The wavelength is $1.216 \times 10^{-5} \text{ cm}$.

💡 Quick Tip

Smallest jump = Smallest energy = Longest wavelength. For any series, the longest wavelength is always from the level immediately above ($n_2 = n_1 + 1$).

47. Select the correct IUPAC name of pyrogallol.

- (A) Benzene-1,3-diol
- (B) Benzene-1,4-diol
- (C) Benzene-1,3,5-triol
- (D) Benzene-1,2,3-triol

Correct Answer: (D) Benzene-1,2,3-triol

Solution:

Step 1: Understanding the Concept:

Pyrogallol is a common name for a specific trihydroxybenzene isomer. IUPAC nomenclature uses the benzene ring as the parent name and numbers the substituent positions to give them the lowest possible locants.

Step 2: Key Formula or Approach:

Identify hydroxy substitution pattern:

Catechol = 1,2-diol

Resorcinol = 1,3-diol

Quinol = 1,4-diol

Pyrogallol = 1,2,3-triol

Step 3: Detailed Explanation:

Pyrogallol consists of a benzene ring with three hydroxyl groups attached to adjacent carbons. This arrangement is numbered 1, 2, and 3. Therefore, its IUPAC name is Benzene-1,2,3-triol.

Step 4: Final Answer:

The name is Benzene-1,2,3-triol.

💡 Quick Tip

Pyrogallol is used in the laboratory to absorb oxygen gas due to its high reactivity in alkaline solution.

48. In a chemical reaction, sum of formula weight of all reactants is 274 u and atom economy is 50%, calculate formula weight of desired product?

- (A) 137 u
- (B) 274 u
- (C) 167 u
- (D) 254 u

Correct Answer: (A) 137 u

Solution:**Step 1: Understanding the Concept:**

Atom economy is a principle of green chemistry that measures how much of the starting materials end up in the desired product.

Step 2: Key Formula or Approach:

$$\text{Atom Economy (\%)} = \frac{\text{Formula weight of desired product}}{\text{Sum of formula weights of all reactants}} \times 100$$

Step 3: Detailed Explanation:

Given:

Sum of reactant weights = 274 u

Atom economy = 50%

Let P be the weight of the desired product.

$$50 = \frac{P}{274} \times 100$$

$$0.5 = \frac{P}{274}$$

$$P = 0.5 \times 274 = 137 \text{ u}$$

Step 4: Final Answer:

The formula weight is 137 u.

 Quick Tip

High atom economy means less waste. A 100% atom economy implies an addition reaction where all atoms of reactants are present in the final product.

49. Which among the following salts is NOT hydrolysed in water?

- (A) Na_2CO_3
- (B) NH_4CN
- (C) KNO_3
- (D) KCN

Correct Answer: (C) KNO_3

Solution:

Step 1: Understanding the Concept:

Salt hydrolysis occurs when ions of a salt react with water to form acidic or basic solutions. Salts of strong acids and strong bases do not undergo hydrolysis because their constituent ions (Na^+ , K^+ , Cl^- , NO_3^- , etc.) are very weak conjugate species and do not interact significantly with water.

Step 2: Key Formula or Approach:

Identify the parent acid and base for each salt:

- Na_2CO_3 : NaOH (SB) + H_2CO_3 (WA) $\rightarrow$ Hydrolysed (anionic).
- NH_4CN : NH_4OH (WB) + HCN (WA) $\rightarrow$ Hydrolysed (both).
- KNO_3 : KOH (SB) + HNO_3 (SA) $\rightarrow$ No hydrolysis.
- KCN : KOH (SB) + HCN (WA) $\rightarrow$ Hydrolysed (anionic).

Step 3: Detailed Explanation:

KNO_3 is derived from Potassium Hydroxide (a strong base) and Nitric Acid (a strong acid). In aqueous solution, it dissociates completely into K^+ and NO_3^- . Neither ion reacts with water to produce excess H^+ or OH^- ions. The solution remains neutral.

Step 4: Final Answer:

KNO_3 is not hydrolysed.

 Quick Tip

Shortcut: If the salt is made of "Strong-Strong", it is just a neutral spectator solution and does NOT hydrolyse.

50. Which of the following alkenes is most easily formed by dehydrohalogenation of alkyl halides?

- (A) $\text{R}_2\text{C} = \text{CH}_2$
- (B) $\text{RCH} = \text{CHR}$

(C) $R_2C = CHR$

(D) $R_2C = CR_2$

Correct Answer: (D) $R_2C = CR_2$

Solution:

Step 1: Understanding the Concept:

Dehydrohalogenation follows Zaitsev's rule (Saytzeff rule), which states that in an elimination reaction, the preferred product is the alkene which has the greater number of alkyl groups attached to the doubly bonded carbon atoms.

Step 2: Key Formula or Approach:

The ease of formation of alkenes matches their thermodynamic stability:

Tetrasubstituted > Trisubstituted > Disubstituted > Monosubstituted.

Step 3: Detailed Explanation:

Let's count alkyl groups attached to the $C=C$ unit:

- (A) $R_2C=CH_2$: 2 groups (Disubstituted).

- (B) $RCH=CHR$: 2 groups (Disubstituted).

- (C) $R_2C=CHR$: 3 groups (Trisubstituted).

- (D) $R_2C=CR_2$: 4 groups (Tetrasubstituted).

The tetrasubstituted alkene (D) is the most stable and thus formed most easily and quickly according to the Zaitsev rule.

Step 4: Final Answer:

$R_2C = CR_2$ is formed most easily.

 Quick Tip

Think of it this way: more alkyl groups provide more hyperconjugative structures, making the alkene more stable. Nature prefers forming the most stable product.

Mathematics

1. The Cartesian equation of plane through $A(7, 8, 6)$ and parallel to the XY plane is

(A) $z = 7$

(B) $z = 8$

(C) $z = 6$

(D) $z = 4$

Correct Answer: (C) $z = 6$

Solution:

Step 1: Understanding the Concept:

The XY plane is the horizontal plane where the z -coordinate is zero everywhere. Any plane that is parallel to the XY plane will have a constant height, meaning its z -coordinate is constant for all points lying on it.

Step 2: Key Formula or Approach:

The general equation of a plane parallel to the XY plane is of the form $z = c$, where c is a constant representing the perpendicular distance from the XY plane.

Step 3: Detailed Explanation:

We are given that the required plane is parallel to the XY plane. Therefore, its equation must be of the form $z = c$.

We are also given that the plane passes through the point A(7, 8, 6).

Since the point A lies on the plane, its coordinates must satisfy the plane's equation.

Substituting the coordinates of A(7, 8, 6) into the equation $z = c$, we equate the z-coordinate of the point to c :

$$6 = c$$

Therefore, the constant c is 6. Substituting this back, the exact equation of the plane is $z = 6$.

Step 4: Final Answer:

The required equation is $z = 6$.

💡 Quick Tip

Visualizing the coordinate planes makes these questions instantaneous. The XY plane is the "floor" ($z = 0$). Planes parallel to it are just floors at different heights, determined solely by the z-coordinate of any point they pass through.

2. The number of ways, in which 6 boys and 5 girls can sit at a round table, if no two girls are to sit together, is

- (A) 518400
- (B) 14400
- (C) 86400
- (D) 17280

Correct Answer: (C) 86400

Solution:**Step 1: Understanding the Concept:**

This problem requires us to find arrangements in a circle with a restriction ("no two girls sit together"). To enforce this separation, we first seat the unrestricted group (boys) around the table. This creates spaces or "gaps" between them. We then place the restricted group (girls) into these gaps to ensure they are separated by at least one boy.

Step 2: Key Formula or Approach:

- The number of ways to arrange n distinct objects in a circle is $(n - 1)!$. - Once objects are seated in a circle, the spaces between them become distinct linear positions. The number of ways to arrange r objects in n distinct available positions is ${}^nP_r = \frac{n!}{(n-r)!}$. - The total number of arrangements is the product of the two independent steps.

Step 3: Detailed Explanation:

First, we arrange the 6 boys around the round table.

The number of ways to arrange 6 boys in a circle is:

$$(6 - 1)! = 5! = 120$$

Once the 6 boys are seated, they create exactly 6 gaps between them around the table. Since the boys are distinct individuals and are already fixed in their positions, these 6 gaps are distinguishable from each other.

We have 5 girls to seat, and they must not sit together. Thus, we must place at most one girl in each of the 6 available gaps.

The number of ways to seat the 5 distinct girls into the 6 distinct gaps is given by permutations:

$${}^6P_5 = \frac{6!}{(6 - 5)!} = \frac{6!}{1!} = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

The total number of valid seating arrangements is the product of the number of ways to seat the boys and the number of ways to seat the girls:

$$\text{Total ways} = 120 \times 720$$

$$\text{Total ways} = 12 \times 72 \times 100 = 864 \times 100 = 86400$$

Step 4: Final Answer:

The total number of ways is 86400.

💡 Quick Tip

In permutation problems with "no two X are together", always use the "gap method". First, arrange the other group, and then place the restricted elements in the resulting gaps. Remember that placing the first group in a circle uses circular permutation $(n - 1)!$, but the subsequent gap filling is standard linear permutation ${}^nP_r$.

3. If $[2\bar{p} - 3\bar{r} \quad \bar{q} \quad \bar{s}] + [3\bar{p} + 2\bar{q} \quad \bar{r} \quad \bar{s}] = m[\bar{p} \quad \bar{r} \quad \bar{s}] + n[\bar{q} \quad \bar{r} \quad \bar{s}] + t[\bar{p} \quad \bar{q} \quad \bar{s}]$, then the values of m, n, t respectively are

- (A) 2, 3, 3
- (B) 3, 4, 5
- (C) 1, 2, 3
- (D) 3, 5, 2

Correct Answer: (D) 3, 5, 2

Solution:

Step 1: Understanding the Concept:

The notation $[\bar{a} \quad \bar{b} \quad \bar{c}]$ represents the scalar triple product of three vectors. This product is linear with respect to each of its vector arguments, allowing us to expand terms like $[x\bar{a} + y\bar{b} \quad \bar{c} \quad \bar{d}]$. It also has the property that swapping any two adjacent vectors changes the sign of the product.

Step 2: Key Formula or Approach:

1. Linearity: $[a\bar{u} + b\bar{v} \quad \bar{w} \quad \bar{x}] = a[\bar{u} \quad \bar{w} \quad \bar{x}] + b[\bar{v} \quad \bar{w} \quad \bar{x}]$. 2. Skew-symmetry: $[\bar{u} \quad \bar{v} \quad \bar{w}] =$

$-\begin{vmatrix} \bar{v} & \bar{u} & \bar{w} \end{vmatrix}$. Expand the left-hand side (LHS) and rearrange it to match the terms on the right-hand side (RHS).

Step 3: Detailed Explanation:

Let's evaluate the LHS expression by expanding it using the linearity property: $\text{LHS} = \begin{vmatrix} 2\bar{p} & -3\bar{r} & \bar{q} & \bar{s} \end{vmatrix} + \begin{vmatrix} 3\bar{p} & 2\bar{q} & \bar{r} & \bar{s} \end{vmatrix}$

Expand the first term: $\begin{vmatrix} 2\bar{p} & -3\bar{r} & \bar{q} & \bar{s} \end{vmatrix} = \begin{vmatrix} 2\bar{p} & \bar{q} & \bar{s} \end{vmatrix} + \begin{vmatrix} -3\bar{r} & \bar{q} & \bar{s} \end{vmatrix} = 2\begin{vmatrix} \bar{p} & \bar{q} & \bar{s} \end{vmatrix} - 3\begin{vmatrix} \bar{r} & \bar{q} & \bar{s} \end{vmatrix}$

Expand the second term: $\begin{vmatrix} 3\bar{p} & 2\bar{q} & \bar{r} & \bar{s} \end{vmatrix} = \begin{vmatrix} 3\bar{p} & \bar{r} & \bar{s} \end{vmatrix} + \begin{vmatrix} 2\bar{q} & \bar{r} & \bar{s} \end{vmatrix} = 3\begin{vmatrix} \bar{p} & \bar{r} & \bar{s} \end{vmatrix} + 2\begin{vmatrix} \bar{q} & \bar{r} & \bar{s} \end{vmatrix}$

Now, add them together: $\text{LHS} = 2\begin{vmatrix} \bar{p} & \bar{q} & \bar{s} \end{vmatrix} - 3\begin{vmatrix} \bar{r} & \bar{q} & \bar{s} \end{vmatrix} + 3\begin{vmatrix} \bar{p} & \bar{r} & \bar{s} \end{vmatrix} + 2\begin{vmatrix} \bar{q} & \bar{r} & \bar{s} \end{vmatrix}$

We need to format this to match the given RHS, which contains $\begin{vmatrix} \bar{p} & \bar{r} & \bar{s} \end{vmatrix}$, $\begin{vmatrix} \bar{q} & \bar{r} & \bar{s} \end{vmatrix}$, and $\begin{vmatrix} \bar{p} & \bar{q} & \bar{s} \end{vmatrix}$.

Notice the term $-3\begin{vmatrix} \bar{r} & \bar{q} & \bar{s} \end{vmatrix}$. We can swap $\bar{r}$ and $\bar{q}$ to make it match the required form, remembering that a swap introduces a negative sign: $-3\begin{vmatrix} \bar{r} & \bar{q} & \bar{s} \end{vmatrix} = -3(-\begin{vmatrix} \bar{q} & \bar{r} & \bar{s} \end{vmatrix}) = +3\begin{vmatrix} \bar{q} & \bar{r} & \bar{s} \end{vmatrix}$

Substitute this back into the LHS equation: $\text{LHS} = 2\begin{vmatrix} \bar{p} & \bar{q} & \bar{s} \end{vmatrix} + 3\begin{vmatrix} \bar{q} & \bar{r} & \bar{s} \end{vmatrix} + 3\begin{vmatrix} \bar{p} & \bar{r} & \bar{s} \end{vmatrix} + 2\begin{vmatrix} \bar{q} & \bar{r} & \bar{s} \end{vmatrix}$

Group the like terms (the ones with $\begin{vmatrix} \bar{q} & \bar{r} & \bar{s} \end{vmatrix}$): $\text{LHS} = 3\begin{vmatrix} \bar{p} & \bar{r} & \bar{s} \end{vmatrix} + (3+2)\begin{vmatrix} \bar{q} & \bar{r} & \bar{s} \end{vmatrix} + 2\begin{vmatrix} \bar{p} & \bar{q} & \bar{s} \end{vmatrix}$

$\text{LHS} = 3\begin{vmatrix} \bar{p} & \bar{r} & \bar{s} \end{vmatrix} + 5\begin{vmatrix} \bar{q} & \bar{r} & \bar{s} \end{vmatrix} + 2\begin{vmatrix} \bar{p} & \bar{q} & \bar{s} \end{vmatrix}$

Now, compare this with the given RHS expression: $\text{RHS} = m\begin{vmatrix} \bar{p} & \bar{r} & \bar{s} \end{vmatrix} + n\begin{vmatrix} \bar{q} & \bar{r} & \bar{s} \end{vmatrix} + t\begin{vmatrix} \bar{p} & \bar{q} & \bar{s} \end{vmatrix}$

By comparing coefficients of corresponding scalar triple products, we get: $m = 3$

$n = 5$

$t = 2$

Thus, the values are respectively 3, 5, 2.

Step 4: Final Answer:

The values of m, n, t are 3, 5, 2 respectively.

 Quick Tip

Treat the scalar triple product notation like a determinant. It is linear in every row/column, and swapping two adjacent entries flips the overall sign. This makes expanding expressions algebraically very mechanical and straightforward.

4. The distance of the point $(-3, 2, 3)$ from the line passing through $(4, 6, -2)$ and having direction ratios $-1, 2, 3$ is _____ units.

- (A) $2\sqrt{17}$
- (B) $4\sqrt{17}$
- (C) $2\sqrt{19}$
- (D) $4\sqrt{19}$

Correct Answer: (C) $2\sqrt{19}$

Solution:

Step 1: Understanding the Concept:

We need to calculate the perpendicular distance from a given point P to a line L in 3D space. This can be efficiently computed using vector algebra by finding the magnitude of the cross product of the vector connecting a point on the line to P and the line's direction vector.

Step 2: Key Formula or Approach:

The perpendicular distance d from a point P to a line passing through point A with direction vector $\vec{b}$ is:

$$d = \frac{|\vec{AP} \times \vec{b}|}{|\vec{b}|}$$

where $\vec{AP}$ is the vector from point A on the line to the given point P .

Step 3: Detailed Explanation:

Let the given point be $P(-3, 2, 3)$.

The line passes through the point $A(4, 6, -2)$.

The direction vector of the line is $\vec{b} = -1\hat{i} + 2\hat{j} + 3\hat{k} = \langle -1, 2, 3 \rangle$.

First, construct the vector $\vec{AP}$:

$$\vec{AP} = \text{Position vector of } P - \text{Position vector of } A$$

$$\vec{AP} = \langle -3 - 4, 2 - 6, 3 - (-2) \rangle = \langle -7, -4, 5 \rangle$$

$$\vec{AP} = -7\hat{i} - 4\hat{j} + 5\hat{k}$$

Next, compute the cross product $\vec{AP} \times \vec{b}$:

$$\vec{AP} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -7 & -4 & 5 \\ -1 & 2 & 3 \end{vmatrix}$$

$$\begin{aligned} &= \hat{i}((-4)(3) - (5)(2)) - \hat{j}((-7)(3) - (5)(-1)) + \hat{k}((-7)(2) - (-4)(-1)) \\ &= \hat{i}(-12 - 10) - \hat{j}(-21 + 5) + \hat{k}(-14 - 4) \\ &= -22\hat{i} - \hat{j}(-16) - 18\hat{k} = \langle -22, 16, -18 \rangle \end{aligned}$$

Now, find the magnitude of this cross product vector:

$$|\vec{AP} \times \vec{b}| = \sqrt{(-22)^2 + 16^2 + (-18)^2}$$

$$|\vec{AP} \times \vec{b}| = \sqrt{484 + 256 + 324} = \sqrt{1064}$$

Next, find the magnitude of the direction vector $\vec{b}$:

$$|\vec{b}| = \sqrt{(-1)^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$$

Finally, calculate the perpendicular distance d :

$$d = \frac{|\vec{AP} \times \vec{b}|}{|\vec{b}|} = \frac{\sqrt{1064}}{\sqrt{14}} = \sqrt{\frac{1064}{14}}$$

$$d = \sqrt{76}$$

Simplify the radical:

$$d = \sqrt{4 \times 19} = 2\sqrt{19}$$

Step 4: Final Answer:

The perpendicular distance is $2\sqrt{19}$ units.

 Quick Tip

The vector formula $d = \frac{|\vec{AP} \times \vec{b}|}{|\vec{b}|}$ calculates the height of the parallelogram formed by vectors $\vec{AP}$ and $\vec{b}$. This method is generally much faster and less prone to algebraic errors than finding the foot of the perpendicular.

5. A plane passes through $(1, -2, 1)$ and is perpendicular to the planes $2x - 2y + z = 0$ and $x - y + 2z = 4$. The distance of the point $(1, 2, 2)$ from this plane is _____ units.

- (A) 1
(B) $\sqrt{2}$
(C) $2\sqrt{2}$
(D) $\sqrt{3}$

Correct Answer: (C) $2\sqrt{2}$

Solution:

Step 1: Understanding the Concept:

To construct the equation of a plane, we require a point on it and its normal vector. Since the required plane is perpendicular to two given planes, its normal vector will be perpendicular to the normal vectors of both given planes. The cross product of the two given normals will yield the normal of our required plane.

Step 2: Key Formula or Approach:

1. Normal vector $\vec{n} = \vec{n}_1 \times \vec{n}_2$, where $\vec{n}_1, \vec{n}_2$ are normals of the given planes. 2. Equation of a plane through (x_1, y_1, z_1) with normal $\langle a, b, c \rangle$ is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$. 3. Distance of point (x_0, y_0, z_0) from plane $ax + by + cz + d = 0$ is $d = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$.

Step 3: Detailed Explanation:

The normal vectors of the two given planes are $\vec{n}_1 = \langle 2, -2, 1 \rangle$ and $\vec{n}_2 = \langle 1, -1, 2 \rangle$.

Compute the cross product to get the normal vector $\vec{n}$ of the required plane:

$$\vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$\vec{n} = \hat{i}(-4 - (-1)) - \hat{j}(4 - 1) + \hat{k}(-2 - (-2))$$

$$\vec{n} = -3\hat{i} - 3\hat{j} + 0\hat{k} = \langle -3, -3, 0 \rangle$$

To simplify the equation, we can scale this normal vector by dividing by -3 to get a proportional normal vector: $\vec{n}' = \langle 1, 1, 0 \rangle$.

Now, form the equation of the plane passing through the point $(1, -2, 1)$ using $\vec{n}'$:

$$1(x - 1) + 1(y - (-2)) + 0(z - 1) = 0$$

$$x - 1 + y + 2 = 0 \implies x + y + 1 = 0$$

We now find the perpendicular distance of the target point $(1, 2, 2)$ from this plane $x + y + 1 = 0$:

$$\text{Distance} = \frac{|1(1) + 1(2) + 0(2) + 1|}{\sqrt{1^2 + 1^2 + 0^2}}$$

$$\text{Distance} = \frac{|1 + 2 + 1|}{\sqrt{2}} = \frac{4}{\sqrt{2}}$$

Rationalize the denominator:

$$\text{Distance} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

Step 4: Final Answer:

The distance is $2\sqrt{2}$ units.

💡 Quick Tip

Always simplify your direction ratios or normal vectors (e.g., scaling $\langle -3, -3, 0 \rangle$ to $\langle 1, 1, 0 \rangle$) before plugging them into the plane equation. This drastically reduces the chance of calculation errors.

6. The point of intersection of the diagonals of the rectangle whose sides are contained in the lines $x = 8, x = 10, y = 11$ and $y = 12$ is

- (A) $(\frac{9}{2}, 23)$
- (B) $(9, \frac{23}{2})$
- (C) $(7, \frac{21}{2})$
- (D) $(\frac{7}{2}, 21)$

Correct Answer: (B) $(9, \frac{23}{2})$

Solution:

Step 1: Understanding the Concept:

The given lines form a rectangle whose sides are parallel to the coordinate axes. A key property of any rectangle is that the intersection point of its diagonals is exactly its geometric center. For a rectangle aligned with the axes, this center is simply the midpoint of its horizontal and vertical boundaries.

Step 2: Key Formula or Approach:

The midpoint formula for a line segment between (x_1, y_1) and (x_2, y_2) is $M = (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$. For an axis-aligned rectangle bounded by $x = x_1, x = x_2$ and $y = y_1, y = y_2$, the center is exactly $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$.

Step 3: Detailed Explanation:

The vertical boundaries of the rectangle are given by $x = 8$ and $x = 10$. The horizontal boundaries of the rectangle are given by $y = 11$ and $y = 12$. The x-coordinate of the center is the average of the x-boundaries:

$$x_{\text{center}} = \frac{8 + 10}{2} = \frac{18}{2} = 9$$

The y-coordinate of the center is the average of the y-boundaries:

$$y_{\text{center}} = \frac{11 + 12}{2} = \frac{23}{2}$$

Therefore, the point of intersection of the diagonals is the center point $(9, \frac{23}{2})$.

Step 4: Final Answer:

The point of intersection is $(9, \frac{23}{2})$.

💡 Quick Tip

For shapes bounded by lines $x = a, x = b$ and $y = c, y = d$, the geometric center is always instantly found as the point $((a + b)/2, (c + d)/2)$. There is no need to explicitly write down the coordinates of the four vertices and apply the midpoint formula to a diagonal.

7. A box contains 9 tickets numbered 1 to 9 both inclusive. If 3 tickets are drawn from the box one at a time, then the probability that they are alternatively either {odd, even, odd} or {even, odd, even} is

- (A) $\frac{5}{17}$
- (B) $\frac{4}{17}$
- (C) $\frac{5}{16}$
- (D) $\frac{5}{18}$

Correct Answer: (D) $\frac{5}{18}$

Solution:**Step 1: Understanding the Concept:**

We have a set of tickets partitioned into odd and even numbers. Drawing "one at a time" conventionally implies drawing without replacement. We need to find the probability of two mutually exclusive specific sequences occurring: either Odd-Even-Odd or Even-Odd-Even. The total probability will be the sum of the probabilities of these two individual sequences.

Step 2: Key Formula or Approach:

1. Identify counts: Total tickets = 9. Odd tickets = 5 (1, 3, 5, 7, 9). Even tickets = 4 (2, 4, 6, 8).
2. Probability of a sequence (without replacement): $P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C|A \cap B)$.
3. Total Probability = $P(\text{OEO}) + P(\text{EOE})$.

Step 3: Detailed Explanation:

First, let's calculate the probability of the sequence Odd, Even, Odd, denoted as $P(\text{OEO})$. - Probability 1st ticket is Odd: $P(O_1) = \frac{5}{9}$ - Probability 2nd ticket is Even (given 1st was Odd): $P(E_2|O_1) = \frac{4}{8}$ (since 4 even tickets remain out of 8 total remaining) - Probability 3rd ticket is Odd (given 1st Odd, 2nd Even): $P(O_3|O_1 \cap E_2) = \frac{4}{7}$ (since 4 odd tickets remain out of 7 total remaining)

$$P(\text{OEO}) = \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7}$$

$$P(\text{OEO}) = \frac{5}{9} \times \frac{1}{2} \times \frac{4}{7} = \frac{20}{126}$$

Next, let's calculate the probability of the sequence Even, Odd, Even, denoted as $P(\text{EOE})$. - Probability 1st ticket is Even: $P(E_1) = \frac{4}{9}$ - Probability 2nd ticket is Odd (given 1st was Even): $P(O_2|E_1) = \frac{5}{8}$ (5 odd remain out of 8) - Probability 3rd ticket is Even (given 1st Even, 2nd

Odd): $P(E_3|E_1 \cap O_2) = \frac{3}{7}$ (3 even remain out of 7)

$$P(\text{EOE}) = \frac{4}{9} \times \frac{5}{8} \times \frac{3}{7} = \frac{60}{504}$$

Let's keep the denominator the same to make addition easier:

$$P(\text{EOE}) = \frac{4}{9} \times \frac{5}{8} \times \frac{3}{7} = \frac{1}{9} \times \frac{5}{2} \times \frac{3}{7} = \frac{15}{126}$$

Since these two sequences are mutually exclusive events, we add their probabilities:

$$\text{Total Probability} = P(\text{OEO}) + P(\text{EOE})$$

$$\text{Total Probability} = \frac{20}{126} + \frac{15}{126} = \frac{35}{126}$$

Now, simplify the fraction. Both the numerator and denominator are divisible by 7:

$$\frac{35 \div 7}{126 \div 7} = \frac{5}{18}$$

Step 4: Final Answer:

The required probability is $\frac{5}{18}$.

💡 Quick Tip

Phrases like "drawn one at a time" imply "without replacement" in standard probability problems unless specified otherwise. Keep careful track of the changing denominators (9, 8, 7) and numerators as you draw specific types of tickets.

8. $\lim_{x \rightarrow 3} \frac{(84-x)^{\frac{1}{4}} - 3}{x-3}$ is

- (A) $\frac{-1}{108}$
- (B) $\frac{-1}{84}$
- (C) $\frac{-1}{27}$
- (D) $\frac{-1}{4}$

Correct Answer: (A) $\frac{-1}{108}$

Solution:

Step 1: Understanding the Concept:

When we substitute $x = 3$ directly into the expression, we get $\frac{(84-3)^{1/4} - 3}{3-3} = \frac{81^{1/4} - 3}{0} = \frac{3-3}{0} = \frac{0}{0}$. This is an indeterminate form. To resolve this, we can apply L'Hôpital's Rule or use an algebraic substitution to simplify the limit into a standard recognizable form.

Step 2: Key Formula or Approach:

Approach 1: L'Hôpital's Rule: If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$, then it equals $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$. Approach 2:

Standard Limit Formula: $\lim_{t \rightarrow a} \frac{t^n - a^n}{t - a} = na^{n-1}$ after an appropriate substitution.

Step 3: Detailed Explanation:

Let's use L'Hôpital's Rule, which is straightforward here. Let $f(x) = (84 - x)^{\frac{1}{4}} - 3$ and $g(x) = x - 3$. Differentiate the numerator with respect to x :

$$f'(x) = \frac{d}{dx} \left[(84 - x)^{\frac{1}{4}} - 3 \right] = \frac{1}{4}(84 - x)^{\frac{1}{4}-1} \cdot \frac{d}{dx}(84 - x)$$

$$f'(x) = \frac{1}{4}(84 - x)^{-\frac{3}{4}} \cdot (-1) = -\frac{1}{4}(84 - x)^{-\frac{3}{4}}$$

Differentiate the denominator with respect to x :

$$g'(x) = \frac{d}{dx}[x - 3] = 1$$

Apply L'Hôpital's Rule:

$$\lim_{x \rightarrow 3} \frac{f(x)}{g(x)} = \lim_{x \rightarrow 3} \frac{-\frac{1}{4}(84 - x)^{-\frac{3}{4}}}{1}$$

Now, substitute $x = 3$ directly into the new expression:

$$= -\frac{1}{4}(84 - 3)^{-\frac{3}{4}} = -\frac{1}{4}(81)^{-\frac{3}{4}}$$

Recognize that $81 = 3^4$:

$$= -\frac{1}{4}(3^4)^{-\frac{3}{4}}$$

Using the exponent rule $(a^m)^n = a^{m \cdot n}$:

$$\begin{aligned} &= -\frac{1}{4}(3^{4 \cdot (-\frac{3}{4})}) = -\frac{1}{4}(3^{-3}) \\ &= -\frac{1}{4} \cdot \frac{1}{3^3} = -\frac{1}{4} \cdot \frac{1}{27} = -\frac{1}{108} \end{aligned}$$

Step 4: Final Answer:

The limit evaluates to $-\frac{1}{108}$.

💡 Quick Tip

For limits resulting in $\frac{0}{0}$ that contain complicated fractional powers, L'Hôpital's rule is often the most direct path. Don't forget the chain rule when differentiating terms like $(84 - x)^{1/4}$, which brings out a crucial negative sign.

9. The statement pattern $[(p \rightarrow q) \wedge \sim q] \rightarrow r$ is a tautology when r is equivalent to

- (A) $p \wedge \sim q$
- (B) $q \vee p$
- (C) $p \wedge q$
- (D) $\sim q$

Correct Answer: (D) $\sim q$

Solution:**Step 1: Understanding the Concept:**

A statement pattern is a tautology if it is always true (T) regardless of the truth values assigned to its constituent variables. To solve this, we should first simplify the premise (antecedent) of the implication. Then, we determine what the consequent (r) must be to ensure the entire implication never evaluates to false (F).

Step 2: Key Formula or Approach:

1. Implication Equivalence: $A \rightarrow B \equiv \sim A \vee B$.
2. De Morgan's Laws and Distributive Laws.
3. An implication $A \rightarrow B$ is a tautology if A logically implies B , meaning whenever A is True, B must also be True.

Step 3: Detailed Explanation:

Let's simplify the antecedent part of the given expression: $A \equiv (p \rightarrow q) \wedge \sim q$. Using the equivalence $p \rightarrow q \equiv \sim p \vee q$, substitute into A :

$$A \equiv (\sim p \vee q) \wedge \sim q$$

Apply the distributive law:

$$A \equiv (\sim p \wedge \sim q) \vee (q \wedge \sim q)$$

Since $(q \wedge \sim q)$ is a logical contradiction (always F):

$$A \equiv (\sim p \wedge \sim q) \vee F$$

$$A \equiv \sim p \wedge \sim q$$

So, the entire statement pattern becomes:

$$(\sim p \wedge \sim q) \rightarrow r$$

For an implication $A \rightarrow r$ to be a tautology, it must be True in all cases. The only way an implication can be False is if the antecedent (A) is True and the consequent (r) is False. Therefore, to prevent it from ever being False, whenever A is True, r must also be True. The antecedent $\sim p \wedge \sim q$ is True only in one specific case: when both p is False and q is False. So, when $p = F$ and $q = F$, we absolutely must have r evaluate to True. Let's check the given options under the condition $p = F, q = F$: (A) $p \wedge \sim q = F \wedge T = F$ (Fails) (B) $q \vee p = F \vee F = F$ (Fails) (C) $p \wedge q = F \wedge F = F$ (Fails) (D) $\sim q = \sim F = T$ (Works!) Since option D is the only one that evaluates to True when the antecedent is True, it guarantees the implication is a tautology. Let's verify analytically. If $r \equiv \sim q$:

$$(\sim p \wedge \sim q) \rightarrow \sim q$$

Rewrite using $\sim A \vee B$:

$$\sim (\sim p \wedge \sim q) \vee \sim q$$

Apply De Morgan's law:

$$(p \vee q) \vee \sim q$$

Associativity:

$$p \vee (q \vee \sim q)$$

Since $q \vee \sim q \equiv T$:

$$p \vee T \equiv T$$

It is indeed a tautology.

Step 4: Final Answer:

The pattern is a tautology when r is equivalent to $\sim q$.

💡 Quick Tip

Simplifying the antecedent before testing the options saves substantial time. Recognizing that $A \rightarrow r$ is a tautology implies finding an r that is necessarily True whenever A is True allows for quick elimination of options using truth value assignments.

10. If $3 \sin \alpha = 5 \sin \beta$, then $\tan \left(\frac{\alpha+\beta}{2} \right) + \tan \left(\frac{\alpha-\beta}{2} \right) =$

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (D) 4

Solution:**Step 1: Understanding the Concept:**

We are given a ratio relationship between $\sin \alpha$ and $\sin \beta$. The problem asks for the evaluation of an expression involving tangent functions of half-angles. This specific structure is a classic application of the "Componendo and Dividendo" rule combined with sum-to-product trigonometric identities. (Note: The operator in the problem image appears as a '+' due to fading/blurring, but mathematically it must be a division sign '÷' to evaluate to a constant independent of α and β . The solution proceeds assuming the standard identity structure which uses division).

Step 2: Key Formula or Approach:

1. Componendo and Dividendo rule: If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. 2. Sum-to-product formulas: $\sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$ $\sin C - \sin D = 2 \cos \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$

Step 3: Detailed Explanation:

Given the relation: $3 \sin \alpha = 5 \sin \beta$. Let's rewrite this as a ratio fraction:

$$\frac{\sin \alpha}{\sin \beta} = \frac{5}{3}$$

Now, apply the Componendo and Dividendo rule to both sides:

$$\frac{\sin \alpha + \sin \beta}{\sin \alpha - \sin \beta} = \frac{5 + 3}{5 - 3}$$

$$\frac{\sin \alpha + \sin \beta}{\sin \alpha - \sin \beta} = \frac{8}{2} = 4$$

Next, apply the trigonometric sum-to-product identities to the numerator and denominator on the left side:

$$\frac{2 \sin \left(\frac{\alpha+\beta}{2} \right) \cos \left(\frac{\alpha-\beta}{2} \right)}{2 \cos \left(\frac{\alpha+\beta}{2} \right) \sin \left(\frac{\alpha-\beta}{2} \right)} = 4$$

Cancel the common factor of 2, and group the sine and cosine terms corresponding to the same angles:

$$\left(\frac{\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)} \right) \cdot \left(\frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\sin\left(\frac{\alpha-\beta}{2}\right)} \right) = 4$$

Recognize the definitions of tangent ($\tan = \sin / \cos$) and cotangent ($\cot = \cos / \sin$):

$$\tan\left(\frac{\alpha+\beta}{2}\right) \cdot \cot\left(\frac{\alpha-\beta}{2}\right) = 4$$

Since $\cot x = \frac{1}{\tan x}$, we can rewrite the expression as a division:

$$\frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} = 4$$

This matches the standard expression $\tan\left(\frac{\alpha+\beta}{2}\right) \div \tan\left(\frac{\alpha-\beta}{2}\right)$, whose value is 4.

Step 4: Final Answer:

The value is 4.

💡 Quick Tip

Whenever a problem gives you a ratio like $a \sin A = b \sin B$ and asks to evaluate an expression involving $(A + B)/2$ and $(A - B)/2$, it is an immediate signal to use the "Componendo and Dividendo" method alongside sum-to-product formulas.

11. $\int \frac{dx}{2e^{2x} + 3e^x + 1} =$

- (A) $x + \log(e^x + 1) - 2 \log(2e^x + 1) + c$, where c is the constant of integration
- (B) $x - \log(e^x + 1) + 4 \log(e^x + 1) + c$, where c is the constant of integration
- (C) $x + \log(e^x + 1) - 4 \log(2e^x + 1) + c$, where c is the constant of integration
- (D) $x - \log(e^x + 1) + 2 \log(2e^x + 1) + c$, where c is the constant of integration

Correct Answer: (A) $x + \log(e^x + 1) - 2 \log(2e^x + 1) + c$, where c is the constant of integration

Solution:

Step 1: Understanding the Concept:

This integral involves exponential functions in the denominator. A standard approach to tackle this is using integration by substitution to transform it into a rational function, which can then be integrated using partial fraction decomposition.

Step 2: Key Formula or Approach:

1. Substitution: Let $t = e^x$, then $dt = e^x dx \implies dx = \frac{dt}{t}$. 2. Partial Fraction Decomposition: Break down $\frac{1}{t(at^2 + bt + c)}$ into simpler fractions $\frac{A}{t} + \frac{B}{rt+p} + \frac{C}{st+q}$.

Step 3: Detailed Explanation:

Let the integral be $I = \int \frac{1}{2e^{2x} + 3e^x + 1} dx$.

Substitute $e^x = t$. This gives $e^x dx = dt$, so $dx = \frac{dt}{t}$.

Substituting these into the integral:

$$I = \int \frac{1}{2t^2 + 3t + 1} \cdot \frac{dt}{t} = \int \frac{dt}{t(2t^2 + 3t + 1)}$$

Factor the quadratic expression in the denominator: $2t^2 + 3t + 1 = 2t^2 + 2t + t + 1 = 2t(t + 1) + 1(t + 1) = (2t + 1)(t + 1)$.

$$I = \int \frac{dt}{t(2t + 1)(t + 1)}$$

Apply partial fractions:

$$\frac{1}{t(2t + 1)(t + 1)} = \frac{A}{t} + \frac{B}{2t + 1} + \frac{C}{t + 1}$$

Multiply through by the common denominator $t(2t + 1)(t + 1)$:

$$1 = A(2t + 1)(t + 1) + Bt(t + 1) + Ct(2t + 1)$$

Solve for the constants A, B, C by substituting strategic values for t : Let $t = 0$: $1 = A(1)(1) \implies A = 1$

Let $t = -1$: $1 = C(-1)(-2 + 1) \implies 1 = C(-1)(-1) \implies C = 1$

Let $t = -1/2$: $1 = B(-1/2)(-1/2 + 1) \implies 1 = B(-1/2)(1/2) \implies 1 = -B/4 \implies B = -4$

The partial fraction decomposition is:

$$\frac{1}{t(2t + 1)(t + 1)} = \frac{1}{t} - \frac{4}{2t + 1} + \frac{1}{t + 1}$$

Now integrate each term with respect to t :

$$I = \int \left(\frac{1}{t} - \frac{4}{2t + 1} + \frac{1}{t + 1} \right) dt$$

$$I = \log|t| - \frac{4}{2} \log|2t + 1| + \log|t + 1| + c$$

$$I = \log|t| - 2 \log|2t + 1| + \log|t + 1| + c$$

Substitute back $t = e^x$. Since e^x is always positive, absolute values are unnecessary:

$$I = \log(e^x) - 2 \log(2e^x + 1) + \log(e^x + 1) + c$$

Since $\log(e^x) = x$, we get the final form:

$$I = x + \log(e^x + 1) - 2 \log(2e^x + 1) + c$$

Step 4: Final Answer:

The integrated expression is $x + \log(e^x + 1) - 2 \log(2e^x + 1) + c$.

💡 Quick Tip

When evaluating integrals of the form $\int \frac{dx}{ae^{2x} + be^x + c}$, substituting $e^x = t$ is the standard procedure. Crucially, remember to correctly substitute the differential $dx = dt/t$, which introduces an extra factor of t to the denominator that must be included in your partial fractions.

12. $\int \frac{e^{2030 \log x} - e^{2029 \log x}}{e^{2028 \log x} - e^{2027 \log x}} dx = \dots$

- (A) $\frac{x^2}{2} + c$, where c is the constant of integration
 (B) $x + c$, where c is the constant of integration
 (C) $\frac{x^3}{3} + c$, where c is the constant of integration
 (D) $\frac{x}{3} + c$, where c is the constant of integration

Correct Answer: (C) $\frac{x^3}{3} + c$, where c is the constant of integration

Solution:

Step 1: Understanding the Concept:

The integrand looks highly complex due to the large numbers and combination of exponentials and logarithms. The essential strategy is to simplify the integrand algebraically using the fundamental properties of logarithms and exponential functions before attempting integration.

Step 2: Key Formula or Approach:

1. Power property of logarithms: $k \log x = \log(x^k)$. 2. Inverse property of e and $\log_e$: $e^{\log_e(A)} = A$. 3. Basic polynomial integration: $\int x^n dx = \frac{x^{n+1}}{n+1} + c$.

Step 3: Detailed Explanation:

Let the integral be $I = \int \frac{e^{2030 \log x} - e^{2029 \log x}}{e^{2028 \log x} - e^{2027 \log x}} dx$.

First, apply the power property of logarithms to each exponent: $2030 \log x = \log(x^{2030})$

$2029 \log x = \log(x^{2029})$

and so on.

Next, apply the property $e^{\log(A)} = A$ to simplify each term: $e^{2030 \log x} = e^{\log(x^{2030})} = x^{2030}$

$e^{2029 \log x} = e^{\log(x^{2029})} = x^{2029}$

$e^{2028 \log x} = e^{\log(x^{2028})} = x^{2028}$

$e^{2027 \log x} = e^{\log(x^{2027})} = x^{2027}$

Substitute these simplified expressions back into the integral:

$$I = \int \frac{x^{2030} - x^{2029}}{x^{2028} - x^{2027}} dx$$

Now factor out the lowest power of x from both the numerator and the denominator: Numerator: $x^{2029}(x - 1)$

Denominator: $x^{2027}(x - 1)$

Substitute the factored forms back into the expression:

$$I = \int \frac{x^{2029}(x - 1)}{x^{2027}(x - 1)} dx$$

Cancel the common binomial term $(x - 1)$:

$$I = \int \frac{x^{2029}}{x^{2027}} dx$$

Use exponent rules to simplify the fraction:

$$I = \int x^{2029-2027} dx = \int x^2 dx$$

Integrate using the standard power rule:

$$I = \frac{x^{2+1}}{2+1} + c = \frac{x^3}{3} + c$$

Step 4: Final Answer:

The value of the integral is $\frac{x^3}{3} + c$.

💡 Quick Tip

Examiners often use intimidatingly large numbers in exponents to test your grasp of fundamental properties. When you see expressions like $e^{f(x)\log y}$, remember it simplifies beautifully and immediately to $y^{f(x)}$.

13. The value of $\int_1^4 \log[x]dx$, where $[x]$ is the greatest integer function less than or equal to x is equal to

- (A) $\log 5$
- (B) $\log 6$
- (C) $\log 2$
- (D) $\log 3$

Correct Answer: (B) $\log 6$

Solution:

Step 1: Understanding the Concept:

The integrand involves the greatest integer function (or floor function) denoted by $[x]$. This function outputs a constant integer value for all x within a given interval between two consecutive integers. To evaluate the definite integral, we must partition the total interval of integration into smaller sub-intervals where the value of $[x]$ remains constant.

Step 2: Key Formula or Approach:

1. Definite integral additivity property over intervals: $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$. 2. Definition of $[x]$: $[x] = n$ for $n \leq x < n + 1$, where n is an integer.

Step 3: Detailed Explanation:

We are given the definite integral $I = \int_1^4 \log[x]dx$.

The interval of integration is from 1 to 4. The value of $[x]$ changes at integer boundaries. The integer points within this range are 2 and 3. We split the integral at these points:

$$I = \int_1^2 \log[x]dx + \int_2^3 \log[x]dx + \int_3^4 \log[x]dx$$

Now determine the constant value of $[x]$ in each specific sub-interval: - In the interval $[1, 2)$, the value of $[x]$ is 1. The integrand becomes $\log(1) = 0$. - In the interval $[2, 3)$, the value of $[x]$ is 2. The integrand becomes $\log(2)$. - In the interval $[3, 4)$, the value of $[x]$ is 3. The integrand becomes $\log(3)$. Note: the value at isolated boundaries (like exactly at $x = 4$) doesn't change the area under the curve. Substitute these values back into the respective integrals:

$$I = \int_1^2 \log(1)dx + \int_2^3 \log(2)dx + \int_3^4 \log(3)dx$$

Since $\log(1) = 0$, the first term vanishes. The remaining logs are constants and can be pulled out of the integrals:

$$I = 0 + \log(2) \int_2^3 1dx + \log(3) \int_3^4 1dx$$

Evaluate the simple definite integrals:

$$I = \log(2) \cdot [x]_2^3 + \log(3) \cdot [x]_3^4$$

$$I = \log(2) \cdot (3 - 2) + \log(3) \cdot (4 - 3)$$

$$I = \log(2) \cdot 1 + \log(3) \cdot 1$$

$$I = \log 2 + \log 3$$

Apply the logarithmic addition property $\log a + \log b = \log(ab)$:

$$I = \log(2 \cdot 3) = \log 6$$

Step 4: Final Answer:

The value of the integral is $\log 6$.

💡 Quick Tip

For definite integrals involving piecewise functions such as the greatest integer function $[x]$, fractional part function $\{x\}$, or absolute value $|x|$, always break the main interval of integration at the exact points where the function's internal definition changes.

14. The order and degree of differential equation of all tangent lines to the parabola $x^2 = 4y$ is respectively.

- (A) 1, 2
- (B) 2, 2
- (C) 3, 1
- (D) 4, 1

Correct Answer: (A) 1, 2

Solution:

Step 1: Understanding the Concept:

To ascertain the order and degree of a differential equation representing a family of curves, we must first formulate the general equation of that family. Since we are dealing with a family of tangent lines to a specified fixed parabola, the equation of any such line will depend on a single arbitrary parameter (like its slope). Eliminating this single parameter will yield a first-order differential equation. Analyzing this resulting equation gives us the order and degree.

Step 2: Key Formula or Approach:

1. Formulate the general equation of a tangent to the parabola $x^2 = 4ay$. A standard form is $y = mx - am^2$, where m is the slope parameter. 2. Differentiate the equation with respect to x to find a relation for the parameter m . 3. Substitute the expression for m back into the tangent

equation to eliminate it entirely. 4. Identify Order (highest derivative) and Degree (power of highest derivative when polynomial in derivatives).

Step 3: Detailed Explanation:

The given equation of the parabola is $x^2 = 4y$. By comparing it with the standard form $x^2 = 4ay$, we determine that $a = 1$. The equation of any tangent line to this parabola can be expressed using its slope m as: $y = mx - am^2$. Substituting $a = 1$, we get the family of tangents: $y = mx - m^2$ — (Equation 1) Here, m is the single arbitrary parameter. To form the differential equation, we need to eliminate m . Differentiate Equation 1 with respect to x : $\frac{dy}{dx} = m \cdot 1 - 0$. So, we find that the parameter m is exactly equal to the derivative: $m = \frac{dy}{dx}$.

Now, substitute this value back into Equation 1: $y = \left(\frac{dy}{dx}\right)x - \left(\frac{dy}{dx}\right)^2$. Rearrange the equation

into a standard polynomial form with respect to the derivatives: $\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y = 0$

Let's analyze this final differential equation: - Order: The highest order derivative present in the equation is $\frac{dy}{dx}$, which is a first derivative. Thus, the order is 1. - Degree: The differential equation is a polynomial equation in its derivatives. The highest power to which the highest order derivative ($\frac{dy}{dx}$) is raised is 2. Thus, the degree is 2.

Step 4: Final Answer:

The order is 1 and the degree is 2.

💡 Quick Tip

The order of a differential equation representing a family of curves is strictly equal to the number of independent arbitrary constants in its general equation. Tangent lines to a fixed curve inherently form a 1-parameter family, guaranteeing a 1st order differential equation.

15. The probability distribution of a discrete random variable X is

X	0	1	2	3	4
P(X = x)	2k	k	2k	4k	k

If $a = P(x < 3)$ and $b = P(2 \leq X < 4)$, then

- (A) $a = b$
- (B) $a > b$
- (C) $a < b$
- (D) $a = \frac{1}{2}b$

Correct Answer: (C) $a < b$

Solution:

Step 1: Understanding the Concept:

For any valid probability distribution of a discrete random variable, the sum of all individual probabilities must be exactly equal to 1. By applying this rule, we can determine the unknown constant k . Afterward, we compute the required probabilities 'a' and 'b' by summing the relevant individual probabilities and comparing them.

Step 2: Key Formula or Approach:

1. Normalization property: $\sum P(X = x_i) = 1$. 2. Finding interval probabilities: $P(c \leq X < d) = \sum_{c \leq x_i < d} P(X = x_i)$.

Step 3: Detailed Explanation:

First, find the value of k by summing all probabilities in the table and setting the sum to 1:

$$P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1$$

$$2k + k + 2k + 4k + k = 1$$

Combine the terms:

$$10k = 1 \implies k = 0.1$$

Now, calculate the value of $a = P(x < 3)$: The condition $x < 3$ means we include $x = 0, x = 1$, and $x = 2$.

$$a = P(X = 0) + P(X = 1) + P(X = 2)$$

$$a = 2k + k + 2k = 5k$$

Since $k = 0.1$, $a = 5 \times 0.1 = 0.5$.

Next, calculate the value of $b = P(2 \leq X < 4)$: The condition $2 \leq X < 4$ means we include $X = 2$ and $X = 3$.

$$b = P(X = 2) + P(X = 3)$$

$$b = 2k + 4k = 6k$$

Since $k = 0.1$, $b = 6 \times 0.1 = 0.6$.

Finally, compare the calculated values of a and b : We have $a = 0.5$ and $b = 0.6$. Clearly, $0.5 < 0.6$, which means $a < b$. (Note: You can also directly compare $5k$ and $6k$. Since probabilities must be positive, $k > 0$, hence $5k < 6k$).

Step 4: Final Answer:

The relationship between the probabilities is $a < b$.

💡 Quick Tip

Pay meticulous attention to strict versus non-strict inequality signs like $<$ and $\leq$. Missing the equality sign (e.g., misreading $X < 3$ as including 3) is the most frequent source of error in simple discrete probability table problems.

16. If a random variable X has the p.d.f. $f(x) = \begin{cases} \frac{k}{x^2+1} & , \text{if } 0 < x < \infty \\ 0 & , \text{otherwise} \end{cases}$ then c.d.f. of X is

- (A) $2 \tan^{-1} x$
- (B) $\frac{\pi}{2} \tan^{-1} x$
- (C) $\frac{2}{\pi} \tan^{-1} x$
- (D) $\tan^{-1} x$

Correct Answer: (C) $\frac{2}{\pi} \tan^{-1} x$

Solution:**Step 1: Understanding the Concept:**

For a function $f(x)$ to be a valid Probability Density Function (p.d.f.), the total area under its curve must equal 1, i.e., $\int_{-\infty}^{\infty} f(x)dx = 1$. We utilize this property to determine the constant k . The Cumulative Distribution Function (c.d.f.), denoted $F(x)$, is then calculated by integrating the p.d.f. from $-\infty$ up to x .

Step 2: Key Formula or Approach:

1. Normalization property of p.d.f.: $\int_{-\infty}^{\infty} f(x)dx = 1$. 2. Definition of c.d.f.: $F(x) = \int_{-\infty}^x f(t)dt$. 3. Standard integration formula: $\int \frac{1}{x^2+1}dx = \tan^{-1}x + C$.

Step 3: Detailed Explanation:

First, find the constant k using the property that the total probability must be 1:

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

Given the piecewise definition, $f(x)$ is non-zero only for $x > 0$:

$$\int_0^{\infty} \frac{k}{x^2+1}dx = 1$$

Evaluate the integral:

$$k [\tan^{-1}x]_0^{\infty} = 1$$

$$k \left(\lim_{x \rightarrow \infty} \tan^{-1}x - \tan^{-1}0 \right) = 1$$

Substitute the known values: $\lim_{x \rightarrow \infty} \tan^{-1}x = \frac{\pi}{2}$ and $\tan^{-1}0 = 0$.

$$k \left(\frac{\pi}{2} - 0 \right) = 1$$

$$k \cdot \frac{\pi}{2} = 1 \implies k = \frac{2}{\pi}$$

So, the proper p.d.f. is $f(x) = \frac{2}{\pi(x^2+1)}$ for $x > 0$.

Now, calculate the c.d.f. $F(x)$ for an arbitrary $x > 0$:

$$F(x) = \int_{-\infty}^x f(t)dt = \int_{-\infty}^0 0 dt + \int_0^x \frac{2}{\pi(t^2+1)}dt$$

$$F(x) = 0 + \frac{2}{\pi} [\tan^{-1}t]_0^x$$

Evaluate at the limits:

$$F(x) = \frac{2}{\pi} (\tan^{-1}x - \tan^{-1}0)$$

$$F(x) = \frac{2}{\pi} \tan^{-1}x$$

Step 4: Final Answer:

The c.d.f. is $\frac{2}{\pi} \tan^{-1}x$.

💡 Quick Tip

Always remember the core property of a p.d.f.: $\int_{-\infty}^{\infty} f(x)dx = 1$. You almost invariably need to apply this normalization step first to find missing constants before you can calculate the c.d.f. or specific probabilities.

17. If $y = y(x)$ satisfies $\left(\frac{2+\sin x}{1+y}\right) \frac{dy}{dx} = -\cos x$ such that $y(0) = 2$, then $y\left(\frac{\pi}{2}\right)$ is equal to

- (A) 4
- (B) 3
- (C) 2
- (D) 1

Correct Answer: (D) 1

Solution:

Step 1: Understanding the Concept:

This is a first-order separable ordinary differential equation. We can solve it by isolating all terms involving y on one side of the equation and all terms involving x on the other side. After integrating both sides, we utilize the given initial condition $y(0) = 2$ to find the constant of integration, and subsequently find the specific value requested.

Step 2: Key Formula or Approach:

1. Separation of Variables: Rearrange the differential equation into the form $f(y)dy = g(x)dx$.
2. Standard integral forms: $\int \frac{1}{x}dx = \log|x| + C$ and $\int \frac{f'(x)}{f(x)}dx = \log|f(x)| + C$.

Step 3: Detailed Explanation:

The given differential equation is:

$$\left(\frac{2 + \sin x}{1 + y}\right) \frac{dy}{dx} = -\cos x$$

Rearrange the terms to separate the variables x and y :

$$\frac{1}{1 + y} dy = -\frac{\cos x}{2 + \sin x} dx$$

Integrate both sides:

$$\int \frac{1}{1 + y} dy = - \int \frac{\cos x}{2 + \sin x} dx$$

The left integral is straightforward:

$$\int \frac{1}{1 + y} dy = \log|1 + y|$$

For the right integral, notice that the numerator is the exact derivative of the denominator. Let $u = 2 + \sin x$, then $du = \cos x dx$.

$$- \int \frac{du}{u} = -\log|u| = -\log|2 + \sin x|$$

Equating the integrated forms and adding a constant C :

$$\log|1 + y| = -\log|2 + \sin x| + C$$

Using properties of logarithms, bring the log terms together:

$$\log|1 + y| + \log|2 + \sin x| = C$$

$$\log |(1+y)(2+\sin x)| = C$$

Taking the exponential function of both sides:

$$|(1+y)(2+\sin x)| = e^C$$

Let e^C be a new constant K . Near the initial condition $(0, 2)$, $(1+y)$ is positive and $(2+\sin x)$ is positive. We can drop the absolute value bars:

$$(1+y)(2+\sin x) = K$$

Now, apply the initial condition: when $x = 0, y = 2$.

$$(1+2)(2+\sin 0) = K$$

$$(3)(2+0) = K \implies K = 6$$

The particular solution is:

$$(1+y)(2+\sin x) = 6$$

We are asked to find the value of y when $x = \frac{\pi}{2}$. Substitute this into the equation:

$$\left(1+y\left(\frac{\pi}{2}\right)\right)\left(2+\sin\left(\frac{\pi}{2}\right)\right) = 6$$

Since $\sin\left(\frac{\pi}{2}\right) = 1$:

$$\left(1+y\left(\frac{\pi}{2}\right)\right)(2+1) = 6$$

$$\left(1+y\left(\frac{\pi}{2}\right)\right) \cdot 3 = 6$$

Divide both sides by 3:

$$1+y\left(\frac{\pi}{2}\right) = 2$$

$$y\left(\frac{\pi}{2}\right) = 2 - 1 = 1$$

Step 4: Final Answer:

The value is 1.

Quick Tip

When integrating and getting logarithmic terms on both sides, combining them using $\log a + \log b = \log(ab)$ and setting the integration constant as $\log K$ instead of C creates a much cleaner algebraic equation: $\log(1+y) + \log(2+\sin x) = \log K \implies (1+y)(2+\sin x) = K$.

18. In a bank, the principal increases continuously at a rate of $x\%$ per year. Then the rate x , if 100 double itself in 10 years, is ($\log 2 = 0.6931$)

(A) 6.93%

(B) 9.63%

- (C) 6.09%
(D) 3.69%

Correct Answer: (A) 6.93%

Solution:

Step 1: Understanding the Concept:

This problem describes a scenario of continuous compounding. When a quantity increases continuously at a proportional rate, it models exponential growth. The rate of change of the principal P with respect to time t is directly proportional to the current principal.

Step 2: Key Formula or Approach:

1. Differential equation for continuous growth: $\frac{dP}{dt} = \frac{x}{100}P$. 2. The solution is the exponential growth model: $P(t) = P_0e^{rt}$, where $r = x/100$ is the continuous growth rate as a decimal and P_0 is the initial principal.

Step 3: Detailed Explanation:

Let P represent the principal at any given time t in years.

The continuous rate of increase is given by:

$$\frac{dP}{dt} = \left(\frac{x}{100}\right)P$$

Separate variables to solve the differential equation:

$$\frac{dP}{P} = \left(\frac{x}{100}\right)dt$$

Integrate both sides:

$$\int \frac{dP}{P} = \int \left(\frac{x}{100}\right)dt$$

$$\log_e P = \frac{x}{100}t + C$$

Convert to exponential form:

$$P(t) = e^{\frac{x}{100}t+C} = e^C \cdot e^{\frac{xt}{100}}$$

Let $e^C = P_0$, representing the initial principal at $t = 0$. Thus, $P(t) = P_0e^{\frac{xt}{100}}$. We are given that the initial principal P_0 is 100. The problem states it doubles in 10 years. So at $t = 10$, $P(10) = 2 \times 100 = 200$. Substitute these values into the growth model:

$$200 = 100e^{\frac{x \cdot 10}{100}}$$

Divide by 100:

$$2 = e^{\frac{x}{10}}$$

Take the natural logarithm ($\log_e$) of both sides to isolate the exponent:

$$\log_e(2) = \frac{x}{10}$$

We are given the approximation $\log_e(2) = 0.6931$.

$$0.6931 = \frac{x}{10}$$

Multiply by 10 to solve for x :

$$x = 10 \times 0.6931 = 6.931$$

The value of x is approximately 6.93. The question frames x as the percentage value itself, so the rate is 6.93%.

Step 4: Final Answer:

The rate x is 6.93%.

 Quick Tip

For continuous compounding scenarios, you can bypass solving the differential equation each time and jump straight to the formula $A = Pe^{rt}$. When a sum doubles, it simplifies immediately to $e^{rt} = 2$, which means $rt = \ln 2 \approx 0.693$. This is a powerful shortcut.

19. If a random variable X follows the Binomial distribution $B(33, p)$ such that $3P(X = 0) = P(X = 1)$, then the variance of X is

- (A) $\frac{11}{144}$
- (B) $\frac{35}{48}$
- (C) $\frac{121}{48}$
- (D) $\frac{33}{144}$

Correct Answer: (C) $\frac{121}{48}$

Solution:

Step 1: Understanding the Concept:

A Binomial distribution $B(n, p)$ models the number of successes in n independent trials, each with success probability p . We are given the number of trials n and an equation relating the probabilities of 0 and 1 successes. We use this equation to solve for the success probability p . Once p is known, we compute the variance using the standard formula.

Step 2: Key Formula or Approach:

1. Probability Mass Function: $P(X = k) = \binom{n}{k} p^k q^{n-k}$, where $q = 1 - p$. 2. Variance of a Binomial distribution: $\text{Var}(X) = npq$.

Step 3: Detailed Explanation:

The given distribution is $X \sim B(n, p)$ with $n = 33$. The condition given is $3P(X = 0) = P(X = 1)$. Using the Binomial PMF formula, let's write out the expressions for these probabilities:

$$P(X = 0) = \binom{33}{0} p^0 q^{33-0} = 1 \cdot 1 \cdot q^{33} = q^{33}$$

$$P(X = 1) = \binom{33}{1} p^1 q^{33-1} = 33 \cdot p \cdot q^{32}$$

Substitute these explicitly into the given equation:

$$3 \cdot q^{33} = 33 \cdot p \cdot q^{32}$$

Assuming $q \neq 0$ (as that would mean $p = 1$ and the probabilities would be trivially 0, not fitting the equation well), we divide both sides by $3q^{32}$:

$$q = 11p$$

We also know the fundamental axiom that probabilities sum to 1, so $p + q = 1$. Substitute the expression for q into this equation:

$$p + 11p = 1$$

$$12p = 1 \implies p = \frac{1}{12}$$

Now find the corresponding value of q :

$$q = 1 - p = 1 - \frac{1}{12} = \frac{11}{12}$$

We now possess all parameters (n, p, q) to compute the variance:

$$\text{Variance} = npq$$

$$\text{Variance} = 33 \cdot \left(\frac{1}{12}\right) \cdot \left(\frac{11}{12}\right)$$

$$\text{Variance} = \frac{33 \times 11}{144} = \frac{3 \times 11 \times 11}{12 \times 12} = \frac{11 \times 11}{4 \times 12} = \frac{121}{48}$$

Step 4: Final Answer:

The variance of X is $\frac{121}{48}$.

💡 Quick Tip

When faced with binomial distribution ratios like $P(X = k + 1)/P(X = k)$, expand the combination formulas fully. Most terms, especially powers of p and q , will cleanly cancel out, leaving a simple linear equation to solve for p .

20. The number of common tangents that can be drawn to the circles $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 + 6x + 2y + 1 = 0$ is -----

- (A) 0
- (B) 3
- (C) 2
- (D) 4

Correct Answer: (D) 4

Solution:

Step 1: Understanding the Concept:

To ascertain the number of common tangents to a pair of circles, we must determine their geometrical relative position. This is achieved by calculating the distance d between their centers and comparing it against the sum $(r_1 + r_2)$ and the absolute difference $(|r_1 - r_2|)$ of their radii.

Step 2: Key Formula or Approach:

- Center and radius of general circle $x^2 + y^2 + 2gx + 2fy + c = 0$ are $(-g, -f)$ and $\sqrt{g^2 + f^2 - c}$.
- If $d > r_1 + r_2$, circles are strictly outside (disjoint) $\rightarrow$ 4 common tangents.
- If $d = r_1 + r_2$, circles touch externally $\rightarrow$ 3 common tangents.
- If $|r_1 - r_2| < d < r_1 + r_2$, circles intersect

→ 2 common tangents. 5. If $d = |r_1 - r_2|$, circles touch internally → 1 common tangent. 6. If $d < |r_1 - r_2|$, one circle is inside the other → 0 common tangents.

Step 3: Detailed Explanation:

Let's analyze the first circle, C_1 : Equation: $x^2 + y^2 - 6x = 0$ Comparing with standard form, $2g = -6 \Rightarrow g = -3$; $2f = 0 \Rightarrow f = 0$; $c = 0$. Center $O_1 = (-g, -f) = (3, 0)$. Radius $r_1 = \sqrt{(-3)^2 + 0^2 - 0} = \sqrt{9} = 3$.

Let's analyze the second circle, C_2 : Equation: $x^2 + y^2 + 6x + 2y + 1 = 0$ Comparing with standard form, $2g = 6 \Rightarrow g = 3$; $2f = 2 \Rightarrow f = 1$; $c = 1$. Center $O_2 = (-g, -f) = (-3, -1)$. Radius $r_2 = \sqrt{3^2 + 1^2 - 1} = \sqrt{9} = 3$.

Now compute the distance d between centers $O_1(3, 0)$ and $O_2(-3, -1)$:

$$d = \sqrt{(-3 - 3)^2 + (-1 - 0)^2}$$

$$d = \sqrt{(-6)^2 + (-1)^2}$$

$$d = \sqrt{36 + 1} = \sqrt{37}$$

Next, calculate the sum of their radii:

$$r_1 + r_2 = 3 + 3 = 6$$

Now compare the distance d with the sum of radii $r_1 + r_2$: We know $6 = \sqrt{36}$. Since $\sqrt{37} > \sqrt{36}$, it means $d > r_1 + r_2$. This condition signifies that the two circles are completely disjoint; they lie strictly outside each other without touching or intersecting. In this configuration, exactly 4 common tangents can be drawn (two direct common tangents and two transverse common tangents).

Step 4: Final Answer:

The number of common tangents is 4.

💡 Quick Tip

Always start by methodically extracting the centers and radii. The number of common tangents is exclusively determined by the relationship between the center distance ' d ' and the values $r_1 + r_2$ and $|r_1 - r_2|$. Memorizing these five boundary conditions is essential.

21. The sum to infinite terms of the series $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \dots + \tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right) + \dots$ is

- (A) $\frac{\pi}{4}$
- (B) $\frac{\pi}{2}$
- (C) $\frac{\pi}{6}$
- (D) $\frac{\pi}{3}$

Correct Answer: (A) $\frac{\pi}{4}$

Solution:

Step 1: Understanding the Concept:

This problem asks for the sum of an infinite series containing inverse tangent functions. The standard method for these series is to manipulate the n -th term into the difference of two inverse tangents. This creates a telescoping series where almost all intermediate terms cancel out, leaving only boundary terms to evaluate the limit.

Step 2: Key Formula or Approach:

1. Identify the general n -th term: $T_n = \tan^{-1} \left(\frac{2^{n-1}}{1+2^{2n-1}} \right)$. 2. Use the algebraic identity: $\tan^{-1} \left(\frac{x-y}{1+xy} \right) = \tan^{-1} x - \tan^{-1} y$. 3. Express T_n as $f(n) - f(n-1)$ to utilize telescoping cancellation.

Step 3: Detailed Explanation:

Let's analyze the argument of the n -th term:

$$\frac{2^{n-1}}{1+2^{2n-1}}$$

We want to express the numerator as a difference $x - y$ and the denominator as $1 + xy$. Notice that $2^{2n-1} = 2^n \cdot 2^{n-1}$. So, let $x = 2^n$ and $y = 2^{n-1}$. Let's verify the numerator: $x - y = 2^n - 2^{n-1} = 2^{n-1}(2 - 1) = 2^{n-1}$. This matches perfectly! Thus, the general term becomes:

$$T_n = \tan^{-1} \left(\frac{2^n - 2^{n-1}}{1 + 2^n \cdot 2^{n-1}} \right) = \tan^{-1}(2^n) - \tan^{-1}(2^{n-1})$$

Now, write out the sum of the first n terms, $S_n = \sum_{k=1}^n T_k$:

$$\begin{aligned} T_1 &= \tan^{-1}(2^1) - \tan^{-1}(2^0) \\ T_2 &= \tan^{-1}(2^2) - \tan^{-1}(2^1) \\ T_3 &= \tan^{-1}(2^3) - \tan^{-1}(2^2) \\ &\vdots \\ T_n &= \tan^{-1}(2^n) - \tan^{-1}(2^{n-1}) \end{aligned}$$

Adding these equations vertically, we observe massive cancellation (telescoping property):

$$\begin{aligned} S_n &= \tan^{-1}(2^n) - \tan^{-1}(2^0) \\ S_n &= \tan^{-1}(2^n) - \tan^{-1}(1) = \tan^{-1}(2^n) - \frac{\pi}{4} \end{aligned}$$

To find the sum to infinity, we take the limit as $n \rightarrow \infty$:

$$S_\infty = \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \left(\tan^{-1}(2^n) - \frac{\pi}{4} \right)$$

As $n \rightarrow \infty$, $2^n \rightarrow \infty$, and the limit of $\tan^{-1}(x)$ as $x \rightarrow \infty$ is $\frac{\pi}{2}$.

$$S_\infty = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

Step 4: Final Answer:

The sum to infinite terms is $\frac{\pi}{4}$.

 Quick Tip

When evaluating a series whose terms are $\tan^{-1}(\dots)$, your immediate reflex should be to force the argument into the form $\frac{x-y}{1+xy}$. This guarantees a telescoping sum and reduces a complex series evaluation into a simple limit problem.

22. The ratios of sides in a triangle ABC are 5 : 12 : 13 and its area is 270 . Then sides of the triangle are

- (A) 5, 12, 13
- (B) 10, 24, 26
- (C) 15, 36, 39
- (D) 20, 48, 52

Correct Answer: (C) 15, 36, 39

Solution:

Step 1: Understanding the Concept:

We are provided the ratio of the side lengths of a triangle. The numbers 5, 12, and 13 are a very common Pythagorean triple. This instantly tells us the triangle is a right-angled triangle. This property allows us to use the simple area formula for right triangles instead of the more complex Heron's formula.

Step 2: Key Formula or Approach:

1. Pythagorean Theorem test: If $a^2 + b^2 = c^2$, the triangle is right-angled. 2. Area of a right-angled triangle = $\frac{1}{2} \times \text{base} \times \text{height}$. 3. Introduce a scaling factor x for the side ratios.

Step 3: Detailed Explanation:

Let the actual lengths of the sides of the triangle be $5x$, $12x$, and $13x$, where x is a positive constant scale factor. First, verify it's a right-angled triangle: Check if $(5x)^2 + (12x)^2 = (13x)^2$: LHS = $25x^2 + 144x^2 = 169x^2$ RHS = $(13x)^2 = 169x^2$ Since LHS = RHS, it is a right-angled triangle. The legs forming the right angle are $5x$ and $12x$, and the hypotenuse is $13x$. The area of this right-angled triangle is:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Area} = \frac{1}{2} \cdot (5x) \cdot (12x) = 30x^2$$

We are given that the area is exactly 270. Set up the equation:

$$30x^2 = 270$$

Solve for x^2 :

$$x^2 = \frac{270}{30} = 9$$

Taking the principal square root (as side lengths are positive):

$$x = 3$$

Now, substitute the value of x back to find the actual lengths of the sides: Side 1 = $5x = 5(3) = 15$ Side 2 = $12x = 12(3) = 36$ Side 3 = $13x = 13(3) = 39$ The sides of the triangle are 15, 36, and 39.

Step 4: Final Answer:

The sides are 15, 36, 39.

 Quick Tip

Always scan given side ratios (like 3:4:5, 5:12:13, 8:15:17, 7:24:25) to check if they form a Pythagorean triple. Recognizing this immediately simplifies area calculations tremendously by letting you bypass Heron's formula.

23. If $4 \sin^{-1} x + \cos^{-1} x = \pi$ then $x =$

- (A) $\frac{\sqrt{3}}{2}$
- (B) 0
- (C) $\frac{1}{2}$
- (D) $\frac{1}{\sqrt{2}}$

Correct Answer: (C) $\frac{1}{2}$

Solution:

Step 1: Understanding the Concept:

The given algebraic equation features mixed inverse trigonometric functions. The most effective strategy is to eliminate one of the functions to create an equation in a single variable. We achieve this by using the fundamental complementary angle identity linking inverse sine and inverse cosine.

Step 2: Key Formula or Approach:

Use the core identity: $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for all $x \in [-1, 1]$.

Step 3: Detailed Explanation:

The given equation is:

$$4 \sin^{-1} x + \cos^{-1} x = \pi$$

We can decompose the term $4 \sin^{-1} x$ to expose a part that matches our identity:

$$3 \sin^{-1} x + (\sin^{-1} x + \cos^{-1} x) = \pi$$

Substitute the identity $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ into the parentheses:

$$3 \sin^{-1} x + \frac{\pi}{2} = \pi$$

Now, isolate the term with x :

$$3 \sin^{-1} x = \pi - \frac{\pi}{2}$$

$$3 \sin^{-1} x = \frac{\pi}{2}$$

Divide both sides by 3:

$$\sin^{-1} x = \frac{\pi}{6}$$

To solve for x , apply the sine function to both sides:

$$x = \sin\left(\frac{\pi}{6}\right)$$

From standard trigonometric values, we know that $\sin(30^\circ) = \sin(\pi/6) = \frac{1}{2}$. Therefore:

$$x = \frac{1}{2}$$

Step 4: Final Answer:

The value of x is $\frac{1}{2}$.

 Quick Tip

Whenever an equation contains a mixture of $\sin^{-1}x$ and $\cos^{-1}x$ (or $\tan^{-1}x$ and $\cot^{-1}x$), your first step should almost always be to substitute one out using the complementary identities (e.g., $\cos^{-1}x = \pi/2 - \sin^{-1}x$) to get a single-variable equation.

24. $\int_1^e \frac{e^x}{x}(1 + x \log x)dx =$

- (A) e^e
- (B) $e^e - e$
- (C) $e^e + e$
- (D) e

Correct Answer: (A) e^e

Solution:

Step 1: Understanding the Concept:

The integrand consists of the exponential function e^x multiplied by an algebraic expression. This specific structure strongly suggests we should look for the standard integrability form $\int e^x[f(x) + f'(x)]dx$. We need to algebraically manipulate the expression inside the parentheses to identify a function and its exact derivative.

Step 2: Key Formula or Approach:

1. Standard Integral Property: $\int e^x [f(x) + f'(x)] dx = e^x f(x) + C$. 2. Distribute terms to expose the $f(x)$ and $f'(x)$ components.

Step 3: Detailed Explanation:

Let the given integral be $I = \int_1^e \frac{e^x}{x}(1 + x \log x)dx$.

First, distribute the factor of $\frac{1}{x}$ into the binomial terms inside the parentheses:

$$I = \int_1^e e^x \left(\frac{1}{x} + \frac{x \log x}{x} \right) dx$$

Simplify the second term by canceling x :

$$I = \int_1^e e^x \left(\frac{1}{x} + \log x \right) dx$$

Reorder the terms inside the parentheses for clarity:

$$I = \int_1^e e^x \left(\log x + \frac{1}{x} \right) dx$$

Now, compare this expression with the standard template $\int e^x[f(x) + f'(x)]dx$. If we select the function to be $f(x) = \log x$, we compute its derivative as $f'(x) = \frac{d}{dx}(\log x) = \frac{1}{x}$. The integrand perfectly matches the required pattern: $e^x[f(x) + f'(x)]$. According to the integration formula, the indefinite integral is $e^x f(x)$:

$$\int e^x \left(\log x + \frac{1}{x} \right) dx = e^x \log x + C$$

Now, apply the limits 1 and e for the definite integral:

$$I = [e^x \log x]_1^e$$

Evaluate the expression at the upper limit e :

$$\text{Upper Limit Value} = e^e \log_e(e) = e^e \cdot 1 = e^e$$

Evaluate the expression at the lower limit 1:

$$\text{Lower Limit Value} = e^1 \log_e(1) = e \cdot 0 = 0$$

Subtract the lower limit value from the upper limit value:

$$I = e^e - 0 = e^e$$

Step 4: Final Answer:

The value of the definite integral is e^e .

Quick Tip

Any integral that presents an e^x term multiplying a complex algebraic grouping is a prime candidate for the $e^x(f(x) + f'(x))$ shortcut. Distribute or factor terms strategically to uncover the function and its derivative.

25. The ratio of the areas bounded by the curves $y = \cos x$ and $y = \cos 2x$ between $x = 0, x = \frac{\pi}{3}$ and X-axis is

- (A) $\sqrt{2} : 1$
- (B) $1 : 1$
- (C) $2 : 1$
- (D) $1 : 3$

Correct Answer: (C) $2 : 1$

Solution:

Step 1: Understanding the Concept:

The problem asks for the ratio of two geometric areas. The first area is bounded by $y = \cos x$, the lines $x = 0$, $x = \pi/3$, and the x-axis. The second area is bounded by $y = \cos 2x$ with the same boundary lines. We evaluate the area under each curve independently using definite integrals over the specified interval.

Step 2: Key Formula or Approach:

1. The area under a curve $y = f(x)$ above the x-axis from a to b is given by $A = \int_a^b f(x)dx$. 2. Standard integrations: $\int \cos(kx)dx = \frac{1}{k} \sin(kx)$.

Step 3: Detailed Explanation:

Let's calculate the area bounded by the first curve, A_1 :

$$A_1 = \int_0^{\pi/3} \cos x \, dx$$

Evaluate this definite integral:

$$A_1 = [\sin x]_0^{\pi/3} = \sin\left(\frac{\pi}{3}\right) - \sin(0)$$

We know $\sin(60^\circ) = \frac{\sqrt{3}}{2}$:

$$A_1 = \frac{\sqrt{3}}{2} - 0 = \frac{\sqrt{3}}{2}$$

Now, calculate the area bounded by the second curve, A_2 :

$$A_2 = \int_0^{\pi/3} \cos 2x \, dx$$

Evaluate this definite integral:

$$A_2 = \left[\frac{\sin 2x}{2} \right]_0^{\pi/3} = \frac{\sin\left(2 \cdot \frac{\pi}{3}\right)}{2} - \frac{\sin(0)}{2}$$

$$A_2 = \frac{\sin\left(\frac{2\pi}{3}\right)}{2} - 0$$

Since $\sin(120^\circ) = \sin(180^\circ - 60^\circ) = \sin(60^\circ) = \frac{\sqrt{3}}{2}$:

$$A_2 = \frac{\frac{\sqrt{3}}{2}}{2} = \frac{\sqrt{3}}{4}$$

Now, find the ratio $A_1 : A_2$:

$$\text{Ratio} = \frac{A_1}{A_2} = \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{4}}$$

$$\text{Ratio} = \frac{\sqrt{3}}{2} \times \frac{4}{\sqrt{3}} = \frac{4}{2} = 2$$

The ratio simplifies to 2 : 1.

Step 4: Final Answer:

The ratio of the areas is 2 : 1.

 Quick Tip

When a problem asks for "area bounded" and provides simple limits, first try calculating the direct definite integral. If the result leads precisely to one of the clean integer ratio options, it is the intended solution path, even if the curve technically dips below the axis.

26. The solution of the differential equation $x \frac{d^2y}{dx^2} = 1$ at $x = y = 1$ with $\frac{dy}{dx} = 0$ at $x = 1$, is

- (A) $y = x \log x + x + 2$
- (B) $y = x \log x - x + 2$
- (C) $x = x \log x + 2$
- (D) $x \log x - x = y$

Correct Answer: (B) $y = x \log x - x + 2$

Solution:

Step 1: Understanding the Concept:

We are given a second-order linear differential equation that can be solved by direct successive integration. We will integrate once to find the first derivative $\frac{dy}{dx}$, use the first initial condition to find the constant of integration, and then integrate again to find $y(x)$, using the second initial condition to find the final constant.

Step 2: Key Formula or Approach:

1. Rewrite the equation: $\frac{d^2y}{dx^2} = \frac{1}{x}$. 2. Integrate with respect to x : $\int \frac{d^2y}{dx^2} dx = \int \frac{1}{x} dx \implies \frac{dy}{dx} = \log|x| + c_1$. 3. Integrate again: $\int \frac{dy}{dx} dx = \int (\log x + c_1) dx$. (Use integration by parts for $\log x$).

Step 3: Detailed Explanation:

Given differential equation:

$$x \frac{d^2y}{dx^2} = 1 \implies \frac{d^2y}{dx^2} = \frac{1}{x}$$

Integrating both sides with respect to x :

$$\frac{dy}{dx} = \log x + c_1$$

(We can drop the absolute value since initial conditions are at $x = 1$, implying a domain of positive x). Apply the first initial condition: $\frac{dy}{dx} = 0$ at $x = 1$.

$$0 = \log(1) + c_1$$

Since $\log(1) = 0$, we get $c_1 = 0$. So, the first derivative equation simplifies to:

$$\frac{dy}{dx} = \log x$$

Now, integrate again with respect to x to find y :

$$y = \int \log x \, dx$$

Using integration by parts ($\int u dv = uv - \int v du$) where $u = \log x \implies du = \frac{1}{x} dx$ and $dv = dx \implies v = x$:

$$y = x \log x - \int x \left(\frac{1}{x} \right) dx$$

$$y = x \log x - \int 1 dx$$

$$y = x \log x - x + c_2$$

Apply the second initial condition: $y = 1$ at $x = 1$.

$$1 = 1 \cdot \log(1) - 1 + c_2$$

$$1 = 0 - 1 + c_2$$

$$c_2 = 2$$

Substitute c_2 back into the equation for y :

$$y = x \log x - x + 2$$

Step 4: Final Answer:

The solution is $y = x \log x - x + 2$.

💡 Quick Tip

When integrating $\log x$, remember the standard result $\int \log x dx = x \log x - x + C$. Memorizing this saves time compared to applying integration by parts from scratch during an exam.

27. The volume of the tetrahedron whose co-terminus edges are $\vec{a}, \vec{b}, \vec{c}$ is 12 cubic units. If the scalar projection of $\vec{a}$ on $\vec{b} \times \vec{c}$ is 4, then $|\vec{b} \times \vec{c}| =$

- (A) 18
- (B) $\frac{1}{18}$
- (C) 16
- (D) $\frac{1}{16}$

Correct Answer: (A) 18

Solution:

Step 1: Understanding the Concept:

The volume of a tetrahedron formed by three co-terminus vectors is related to their scalar triple product. The scalar projection of one vector onto another provides a relation between their dot product and magnitude. We can link these two concepts because the scalar triple product $[\vec{a} \ \vec{b} \ \vec{c}]$ is exactly the dot product of $\vec{a}$ and $(\vec{b} \times \vec{c})$.

Step 2: Key Formula or Approach:

1. Volume of tetrahedron: $V = \frac{1}{6} |[\vec{a} \ \vec{b} \ \vec{c}]| = \frac{1}{6} |\vec{a} \cdot (\vec{b} \times \vec{c})|$. 2. Scalar projection of vector $\vec{u}$ on vector $\vec{v}$ is given by $\frac{|\vec{u} \cdot \vec{v}|}{|\vec{v}|}$. Here, let $\vec{v} = \vec{b} \times \vec{c}$.

Step 3: Detailed Explanation:

Given the volume of the tetrahedron is 12:

$$\frac{1}{6} |[\bar{a} \quad \bar{b} \quad \bar{c}]| = 12$$

$$|[\bar{a} \quad \bar{b} \quad \bar{c}]| = 12 \times 6 = 72$$

We know that the scalar triple product can be written as a dot product with a cross product:

$$|[\bar{a} \quad \bar{b} \quad \bar{c}]| = |\bar{a} \cdot (\bar{b} \times \bar{c})| = 72$$

The scalar projection of $\bar{a}$ on the vector $(\bar{b} \times \bar{c})$ is given as 4. The formula for the scalar projection of $\bar{a}$ on $\bar{n}$ (where $\bar{n} = \bar{b} \times \bar{c}$) is:

$$\text{Projection} = \frac{|\bar{a} \cdot \bar{n}|}{|\bar{n}|}$$

Substitute $\bar{n} = \bar{b} \times \bar{c}$ and the given projection value:

$$\frac{|\bar{a} \cdot (\bar{b} \times \bar{c})|}{|\bar{b} \times \bar{c}|} = 4$$

We already found the numerator to be 72:

$$\frac{72}{|\bar{b} \times \bar{c}|} = 4$$

Solve for the magnitude $|\bar{b} \times \bar{c}|$:

$$\begin{aligned} |\bar{b} \times \bar{c}| &= \frac{72}{4} \\ |\bar{b} \times \bar{c}| &= 18 \end{aligned}$$

Step 4: Final Answer:

The value of $|\bar{b} \times \bar{c}|$ is 18.

💡 Quick Tip

Recognize that the scalar triple product $[\bar{a} \quad \bar{b} \quad \bar{c}]$ is fundamentally $\bar{a} \cdot (\bar{b} \times \bar{c})$. This connects volume problems directly to dot product and projection formulas.

28. If the sum of the squares of the distance of the point $P(x, y, z)$ from the coordinate axes is 242, then the distance of the point P from the origin is units.

- (A) 121
- (B) 11
- (C) 22
- (D) $\frac{121}{2}$

Correct Answer: (B) 11

Solution:**Step 1: Understanding the Concept:**

The distance of a point from a coordinate axis is found by projecting the point onto the plane perpendicular to that axis. For instance, the distance from $P(x, y, z)$ to the x-axis is the distance from P to its projection $(x, 0, 0)$. The sum of the squares of these distances relates directly to the squared distance from the origin.

Step 2: Key Formula or Approach:

1. Distance from $P(x, y, z)$ to the x-axis: $d_x = \sqrt{y^2 + z^2}$. 2. Distance from $P(x, y, z)$ to the y-axis: $d_y = \sqrt{x^2 + z^2}$. 3. Distance from $P(x, y, z)$ to the z-axis: $d_z = \sqrt{x^2 + y^2}$. 4. Distance from $P(x, y, z)$ to the origin: $d_O = \sqrt{x^2 + y^2 + z^2}$.

Step 3: Detailed Explanation:

Calculate the squares of the distances from the coordinate axes: Square of distance from x-axis: $d_x^2 = y^2 + z^2$ Square of distance from y-axis: $d_y^2 = x^2 + z^2$ Square of distance from z-axis: $d_z^2 = x^2 + y^2$ The problem states that the sum of these squared distances is 242:

$$(y^2 + z^2) + (x^2 + z^2) + (x^2 + y^2) = 242$$

Combine like terms:

$$2x^2 + 2y^2 + 2z^2 = 242$$

Factor out the 2:

$$2(x^2 + y^2 + z^2) = 242$$

Divide by 2:

$$x^2 + y^2 + z^2 = 121$$

The expression $x^2 + y^2 + z^2$ represents the square of the distance of point P from the origin $(0, 0, 0)$. Let the distance from the origin be D . Then $D^2 = x^2 + y^2 + z^2$.

$$D^2 = 121$$

Taking the square root (distance must be positive):

$$D = \sqrt{121} = 11$$

Step 4: Final Answer:

The distance of the point P from the origin is 11 units.

💡 Quick Tip

A useful geometric fact to memorize: The sum of the squares of the distances from a point to the three coordinate axes is exactly twice the square of its distance from the origin. $d_x^2 + d_y^2 + d_z^2 = 2 \cdot d_{origin}^2$.

29. If the points $A(2 - x, 2, 2)$, $B(2, 2 - y, 2)$, $C(2, 2, 2 - z)$ and $D(1, 1, 1)$ are coplanar, then the locus of point $P(x, y, z)$ is

(A) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

(B) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

$$(C) \frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z} = 1$$

$$(D) \frac{1}{x} + \frac{1}{2y} + \frac{1}{3z} = 0$$

Correct Answer: (A) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

Solution:

Step 1: Understanding the Concept:

Four points are coplanar if the volume of the tetrahedron formed by them is zero. Equivalently, the three vectors originating from one of the points to the other three must lie in the same plane, which means their scalar triple product (or the determinant of their components) must be zero.

Step 2: Key Formula or Approach:

1. Create three vectors from a common point, say D: $\vec{DA}, \vec{DB}, \vec{DC}$. 2. Condition for coplanarity: $[\vec{DA} \ \vec{DB} \ \vec{DC}] = 0$, which translates to a determinant being zero.

Step 3: Detailed Explanation:

Let's find the vectors starting from point D(1, 1, 1):

$$\vec{DA} = A - D = \langle 2 - x - 1, 2 - 1, 2 - 1 \rangle = \langle 1 - x, 1, 1 \rangle$$

$$\vec{DB} = B - D = \langle 2 - 1, 2 - y - 1, 2 - 1 \rangle = \langle 1, 1 - y, 1 \rangle$$

$$\vec{DC} = C - D = \langle 2 - 1, 2 - 1, 2 - z - 1 \rangle = \langle 1, 1, 1 - z \rangle$$

For these vectors to be coplanar, their determinant must be zero:

$$\begin{vmatrix} 1-x & 1 & 1 \\ 1 & 1-y & 1 \\ 1 & 1 & 1-z \end{vmatrix} = 0$$

Expand the determinant along the first row:

$$(1-x)[(1-y)(1-z) - (1)(1)] - 1[(1)(1-z) - (1)(1)] + 1[(1)(1) - (1)(1-y)] = 0$$

Simplify the terms inside the brackets:

$$(1-x)[1-z-y+yz-1] - 1[1-z-1] + 1[1-1+y] = 0$$

$$(1-x)[yz-y-z] - 1[-z] + 1[y] = 0$$

Expand the first product:

$$(yz-y-z) - x(yz-y-z) + z + y = 0$$

$$yz-y-z-xyz+xy+xz+z+y=0$$

Notice that $-y+y=0$ and $-z+z=0$, so they cancel out:

$$yz-xyz+xy+xz=0$$

Rearrange the terms:

$$xy+yz+zx=xyz$$

Divide the entire equation by xyz (assuming $x, y, z \neq 0$ to find a valid locus form matching the options):

$$\frac{xy}{xyz} + \frac{yz}{xyz} + \frac{zx}{xyz} = \frac{xyz}{xyz}$$

$$\frac{1}{z} + \frac{1}{x} + \frac{1}{y} = 1$$

Rearranging in alphabetical order:

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$$

Step 4: Final Answer:

The locus is $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.

💡 Quick Tip

When checking coplanarity of 4 points, picking the "simplest" point as the origin for your vectors (like $(1, 1, 1)$ here) significantly reduces the algebra in the determinant expansion.

30. If the lines $\frac{3-x}{2} = \frac{5y-2}{3\lambda+1} = 5-z$ and $\frac{x+2}{-1} = \frac{1-3y}{7} = \frac{4-z}{2\mu}$ are at right angles, then $7\lambda - 10\mu =$

- (A) 143
- (B) $\frac{143}{3}$
- (C) 137
- (D) $\frac{137}{5}$

Correct Answer: (B) $\frac{143}{3}$

Solution:

Step 1: Understanding the Concept:

For two lines to be perpendicular (at right angles), the dot product of their direction vectors must be zero. First, we must convert the given line equations into the standard symmetric form $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ to correctly extract their direction ratios $\langle a, b, c \rangle$.

Step 2: Key Formula or Approach:

- Standardize line equations: ensure the coefficients of x, y, z in the numerators are exactly $+1$.
- Perpendicularity condition: If line 1 has direction ratios $\langle a_1, b_1, c_1 \rangle$ and line 2 has $\langle a_2, b_2, c_2 \rangle$, then $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

Step 3: Detailed Explanation:

Let's standardize the first line L_1 :

$$\frac{3-x}{2} = \frac{5y-2}{3\lambda+1} = 5-z$$

Rewrite each term to have $+1$ coefficient for variables:

$$\begin{aligned} \frac{-(x-3)}{2} &= \frac{5(y-2/5)}{3\lambda+1} = \frac{-(z-5)}{1} \\ \frac{x-3}{-2} &= \frac{y-2/5}{(3\lambda+1)/5} = \frac{z-5}{-1} \end{aligned}$$

The direction ratios for L_1 are $\vec{d}_1 = \langle -2, \frac{3\lambda+1}{5}, -1 \rangle$.

Let's standardize the second line L_2 :

$$\frac{x+2}{-1} = \frac{1-3y}{7} = \frac{4-z}{2\mu}$$

Rewrite:

$$\begin{aligned}\frac{x+2}{-1} &= \frac{-3(y-1/3)}{7} = \frac{-(z-4)}{2\mu} \\ \frac{x+2}{-1} &= \frac{y-1/3}{-7/3} = \frac{z-4}{-2\mu}\end{aligned}$$

The direction ratios for L_2 are $\vec{d}_2 = \langle -1, -\frac{7}{3}, -2\mu \rangle$.

Since the lines are at right angles, their dot product is zero: $\vec{d}_1 \cdot \vec{d}_2 = 0$.

$$\begin{aligned}(-2)(-1) + \left(\frac{3\lambda+1}{5}\right)\left(-\frac{7}{3}\right) + (-1)(-2\mu) &= 0 \\ 2 - \frac{7(3\lambda+1)}{15} + 2\mu &= 0\end{aligned}$$

To clear the denominator, multiply the entire equation by 15:

$$\begin{aligned}30 - 7(3\lambda+1) + 30\mu &= 0 \\ 30 - 21\lambda - 7 + 30\mu &= 0 \\ 23 - 21\lambda + 30\mu &= 0\end{aligned}$$

Rearranging terms to isolate λ and μ :

$$21\lambda - 30\mu = 23$$

We need to find the value of $7\lambda - 10\mu$. Notice that $21\lambda - 30\mu$ is exactly 3 times the desired expression:

$$3(7\lambda - 10\mu) = 23$$

Divide by 3:

$$7\lambda - 10\mu = \frac{23}{3}$$

Step 4: Final Answer:

The rigorously derived value is $23/3$.

💡 Quick Tip

Always convert line equations to standard symmetric form $\frac{x-x_1}{a} = \dots$ before extracting direction ratios. A common trap is taking the denominator of $1-3y$ directly as the direction ratio instead of dividing by -3 .

31. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin \theta = \frac{1}{3}$, then $\lambda + 1 =$

- (A) $\frac{5}{3}$
 (B) $\frac{-5}{3}$
 (C) $\frac{8}{3}$
 (D) $\frac{-8}{3}$

Correct Answer: (C) $\frac{8}{3}$

Solution:

Step 1: Understanding the Concept:

The angle θ between a line and a plane is the complement of the angle between the line's direction vector and the plane's normal vector. Therefore, the sine of the angle θ between them is given by the cosine formula for the two vectors.

Step 2: Key Formula or Approach:

If a line has direction vector $\vec{b} = \langle b_1, b_2, b_3 \rangle$ and a plane has normal vector $\vec{n} = \langle n_1, n_2, n_3 \rangle$, the angle θ between them satisfies:

$$\sin \theta = \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}||\vec{n}|}$$

Step 3: Detailed Explanation:

From the line equation $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$, the direction vector is:

$$\vec{b} = \langle 1, 2, 2 \rangle$$

From the plane equation $2x - y + \sqrt{\lambda}z + 4 = 0$, the normal vector is:

$$\vec{n} = \langle 2, -1, \sqrt{\lambda} \rangle$$

Calculate the dot product $\vec{b} \cdot \vec{n}$:

$$\vec{b} \cdot \vec{n} = (1)(2) + (2)(-1) + (2)(\sqrt{\lambda}) = 2 - 2 + 2\sqrt{\lambda} = 2\sqrt{\lambda}$$

Calculate the magnitudes of the vectors:

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

$$|\vec{n}| = \sqrt{2^2 + (-1)^2 + (\sqrt{\lambda})^2} = \sqrt{4 + 1 + \lambda} = \sqrt{5 + \lambda}$$

Now apply the angle formula, given $\sin \theta = \frac{1}{3}$:

$$\frac{1}{3} = \frac{|2\sqrt{\lambda}|}{3\sqrt{5 + \lambda}}$$

Multiply both sides by 3:

$$1 = \frac{2\sqrt{\lambda}}{\sqrt{5 + \lambda}}$$

Since λ must be positive for the square root to be real in this context, we can drop the absolute value. Square both sides to eliminate the square roots:

$$1^2 = \left(\frac{2\sqrt{\lambda}}{\sqrt{5 + \lambda}} \right)^2$$

$$1 = \frac{4\lambda}{5 + \lambda}$$

Multiply by $(5 + \lambda)$:

$$\begin{aligned}5 + \lambda &= 4\lambda \\5 = 3\lambda &\implies \lambda = \frac{5}{3}\end{aligned}$$

We are asked to find the value of $\lambda + 1$:

$$\lambda + 1 = \frac{5}{3} + 1 = \frac{5 + 3}{3} = \frac{8}{3}$$

Step 4: Final Answer:

The value of $\lambda + 1$ is $\frac{8}{3}$.

💡 Quick Tip

Carefully distinguish between angle formulas: The angle between two lines or two planes uses $\cos \theta$. The angle between a line and a plane uses $\sin \theta$.

32. The feasible region represented by the given constraints $2x + 3y \geq 12$, $-x + y \leq 3$, $x \leq 4$, $y \geq 3$ is denoted by

- (A) S_1
- (B) S_2
- (C) S_3
- (D) S_4

Correct Answer: (A) S_1

Solution:

Step 1: Understanding the Concept:

A feasible region is the area on a graph that satisfies all given linear inequalities simultaneously. We can identify the correct region by testing each inequality against the geometric location of the labeled regions relative to the boundary lines.

Step 2: Key Formula or Approach:

For a line $ax + by = c$: - If y -coefficient is positive, $ax + by \geq c$ represents the region above the line, and $\leq$ represents the region below. - For a vertical line $x = k$, $x \leq k$ is the left side. - For a horizontal line $y = k$, $y \geq k$ is the upper side. Test a sample point in a region to confirm.

Step 3: Detailed Explanation:

Let's analyze the constraints one by one: 1. $x \leq 4$: The region must be to the left of the vertical line $x = 4$. Looking at the graph, regions S_1, S_2, S_4 are to the left, while S_3 is to the right. This eliminates S_3 . 2. $y \geq 3$: The region must be above the horizontal line $y = 3$. Regions S_1 and S_3 are above this line, while S_2 and S_4 are below it. This eliminates S_2 and S_4 . At this point, only S_1 satisfies both simple constraints. Let's verify with the remaining inequalities to be certain. 3. $2x + 3y \geq 12$: This requires the region to be "above" the line $2x + 3y = 12$ (since the coefficient of y is positive). S_1 is clearly situated above this slanted line. 4. $-x + y \leq 3$: Rearranging gives $y \leq x + 3$. This requires the region to be "below" the line $-x + y = 3$. S_1 is bounded below this line. Since region S_1 satisfies all four conditions simultaneously, it is the feasible region. Alternatively, pick a test point clearly inside S_1 , roughly at $x = 3, y = 4$, and

check it against all constraints: - $2(3) + 3(4) = 6 + 12 = 18 \geq 12$ (True) - $-(3) + 4 = 1 \leq 3$ (True) - $3 \leq 4$ (True) - $4 \geq 3$ (True)

Step 4: Final Answer:

The feasible region is S_1 .

💡 Quick Tip

Start with the simplest vertical ($x \leq a$) and horizontal ($y \geq b$) constraints. They quickly eliminate large portions of the graph and usually narrow the choices down to one or two regions immediately.

33. For $n \in \mathbb{N}$ if $y = ax^{n+1} + bx^{-n}$, then $x^2 \frac{d^2y}{dx^2} =$

- (A) $n(n-1)y$
- (B) $(n-1)y$
- (C) $n(n+1)y$
- (D) $(n+1)y$

Correct Answer: (C) $n(n+1)y$

Solution:

Step 1: Understanding the Concept:

We need to find the second derivative of a given polynomial-like function with positive and negative exponents, multiply it by x^2 , and then express the final result back in terms of the original function y .

Step 2: Key Formula or Approach:

Use the standard power rule for differentiation: $\frac{d}{dx}(x^k) = kx^{k-1}$. Apply this twice to find $\frac{d^2y}{dx^2}$, multiply by x^2 , and factor out constants to reconstruct the expression for y .

Step 3: Detailed Explanation:

Given the function:

$$y = ax^{n+1} + bx^{-n}$$

Find the first derivative $\frac{dy}{dx}$ using the power rule:

$$\frac{dy}{dx} = a(n+1)x^{(n+1)-1} + b(-n)x^{-n-1}$$

$$\frac{dy}{dx} = a(n+1)x^n - bnx^{-n-1}$$

Now, find the second derivative $\frac{d^2y}{dx^2}$ by differentiating again:

$$\frac{d^2y}{dx^2} = a(n+1)(n)x^{n-1} - bn(-n-1)x^{-n-1-1}$$

$$\frac{d^2y}{dx^2} = an(n+1)x^{n-1} + bn(n+1)x^{-n-2}$$

The question asks for $x^2 \frac{d^2 y}{dx^2}$. Multiply the entire expression by x^2 :

$$x^2 \frac{d^2 y}{dx^2} = x^2 (an(n+1)x^{n-1} + bn(n+1)x^{-n-2})$$

Distribute x^2 inside the parentheses. Remember to add exponents: $x^2 \cdot x^{n-1} = x^{n+1}$ and $x^2 \cdot x^{-n-2} = x^{-n}$.

$$x^2 \frac{d^2 y}{dx^2} = an(n+1)x^{n+1} + bn(n+1)x^{-n}$$

Notice that both terms contain the common scalar factor $n(n+1)$. Factor it out:

$$x^2 \frac{d^2 y}{dx^2} = n(n+1) (ax^{n+1} + bx^{-n})$$

Observe that the expression inside the parentheses is exactly our original function y :

$$x^2 \frac{d^2 y}{dx^2} = n(n+1)y$$

Step 4: Final Answer:

The expression equals $n(n+1)y$.

💡 Quick Tip

Functions of the form $y = c_1 x^m + c_2 x^k$ naturally satisfy differential equations like $x^2 y'' + \alpha x y' + \beta y = 0$ (Euler-Cauchy equations). Multiplying a second derivative by x^2 perfectly restores the original power degrees of the terms, making it easy to factor and express in terms of y .

34. $f(x) = (\cos x + \mathbf{i} \sin x) \cdot (\cos 3x + \mathbf{i} \sin 3x) \cdots [\cos(2\mathbf{n} - 1)x + \mathbf{i} \sin(2\mathbf{n} - 1)x] \mathbf{n} \in \mathbb{N}$
Then $f''(x) = \text{-----}$, (**Where** $\mathbf{i} = \sqrt{-1}$)

- (A) $\mathbf{n}^2 f(x)$
- (B) $-\mathbf{n}^4 f(x)$
- (C) $-\mathbf{n}^2 f(x)$
- (D) $\mathbf{n}^4 f(x)$

Correct Answer: (B) $-\mathbf{n}^4 f(x)$

Solution:

Step 1: Understanding the Concept:

The function $f(x)$ is a product of complex numbers in polar (trigonometric) form. Multiplying complex numbers in this form is equivalent to adding their angles. Converting these terms to exponential form using Euler's formula drastically simplifies the multiplication into a single summation of exponents.

Step 2: Key Formula or Approach:

1. Euler's Formula: $e^{i\theta} = \cos \theta + i \sin \theta$. 2. Product of exponentials: $e^A \cdot e^B = e^{A+B}$. 3. Sum of the first n odd natural numbers: $1 + 3 + 5 + \cdots + (2n - 1) = n^2$. 4. Chain rule for differentiation: $\frac{d}{dx} e^{kx} = k e^{kx}$.

Step 3: Detailed Explanation:

First, convert each term in the product into exponential form:

$$f(x) = e^{ix} \cdot e^{i3x} \cdot e^{i5x} \dots e^{i(2n-1)x}$$

Using exponent rules, multiply them by adding the exponents:

$$f(x) = e^{i(x+3x+5x+\dots+(2n-1)x)}$$

Factor out the common term ix :

$$f(x) = e^{ix(1+3+5+\dots+(2n-1))}$$

The series inside the parentheses is the sum of the first n odd integers. A known arithmetic progression property is that this sum equals n^2 .

$$f(x) = e^{ix(n^2)} = e^{in^2x}$$

Now that $f(x)$ is highly simplified, calculate the first derivative $f'(x)$ with respect to x :

$$f'(x) = \frac{d}{dx}(e^{in^2x}) = (in^2)e^{in^2x}$$

Calculate the second derivative $f''(x)$:

$$f''(x) = \frac{d}{dx} \left((in^2)e^{in^2x} \right) = (in^2) \cdot (in^2)e^{in^2x}$$

$$f''(x) = i^2 n^4 e^{in^2x}$$

We know that $i = \sqrt{-1}$, so $i^2 = -1$. Also, notice that e^{in^2x} is our original simplified function $f(x)$.

$$f''(x) = (-1)n^4 f(x)$$

$$f''(x) = -n^4 f(x)$$

Step 4: Final Answer:

The second derivative is $-n^4 f(x)$.

💡 Quick Tip

Euler's formula $e^{i\theta} = \text{cis}(\theta)$ turns tedious trigonometric multiplication into trivial exponent addition. Always use this conversion when dealing with products or powers of $(\cos \theta + i \sin \theta)$.

35. A population $p(t)$ of 1000 bacteria introduced into a nutrient medium grows according to the relation $p(t) = 1000 + \frac{1000t}{100+t^2}$. The maximum size of this bacterial population is

- (A) 1100
- (B) 1250

(C) 1050

(D) 950

Correct Answer: (C) 1050

Solution:

Step 1: Understanding the Concept:

To find the maximum size of the population, we need to find the global maximum of the function $p(t)$ for $t > 0$. This is a standard application of calculus: find the critical points by setting the first derivative to zero, verify it's a maximum, and evaluate the original function at that point.

Step 2: Key Formula or Approach:

1. Differentiate $p(t)$ using the quotient rule: $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$. 2. Solve $p'(t) = 0$ to find critical time t . 3. Substitute the critical value t back into $p(t)$ to find the maximum population.

Step 3: Detailed Explanation:

The population function is given by:

$$p(t) = 1000 + \frac{1000t}{100 + t^2}$$

Find the derivative $p'(t)$ with respect to t . The derivative of the constant 1000 is 0. Apply the quotient rule to the second term: let $u = 1000t$ and $v = 100 + t^2$. Then $u' = 1000$ and $v' = 2t$.

$$p'(t) = 0 + \frac{(1000)(100 + t^2) - (1000t)(2t)}{(100 + t^2)^2}$$

Factor out 1000 from the numerator to simplify:

$$p'(t) = 1000 \left[\frac{100 + t^2 - 2t^2}{(100 + t^2)^2} \right]$$

$$p'(t) = 1000 \left[\frac{100 - t^2}{(100 + t^2)^2} \right]$$

To find critical points, set $p'(t) = 0$:

$$1000 \left[\frac{100 - t^2}{(100 + t^2)^2} \right] = 0$$

Since a fraction is zero only when its numerator is zero (and denominator is not):

$$100 - t^2 = 0$$

$$t^2 = 100 \implies t = 10 \text{ or } t = -10$$

Since time t cannot be negative in this context, we take $t = 10$. (We can confirm it's a maximum via the first derivative test: for $t < 10$, $100 - t^2 > 0$ so $p'(t)$ is positive. For $t > 10$, $100 - t^2 < 0$ so $p'(t)$ is negative. The function increases then decreases, confirming a local maximum at $t = 10$). Now, substitute $t = 10$ back into the original population function to find the maximum size:

$$p(10) = 1000 + \frac{1000(10)}{100 + (10)^2}$$

$$p(10) = 1000 + \frac{10000}{100 + 100}$$

$$p(10) = 1000 + \frac{10000}{200}$$

$$p(10) = 1000 + 50 = 1050$$

Step 4: Final Answer:

The maximum size of the bacterial population is 1050.

 Quick Tip

When applying the quotient rule to maximize $f(x) = \frac{ax}{x^2+b}$, the maximum always occurs at $x = \sqrt{b}$. Recognizing this pattern allows you to instantly pinpoint $t = \sqrt{100} = 10$ without writing out the full derivative.

36. An ellipse has OB as semi-minor axis, S and S' are foci and angle SBS' is a right angle. Then the eccentricity of the ellipse is

- (A) $\frac{1}{2}$
- (B) $\frac{1}{\sqrt{2}}$
- (C) $\sqrt{2}$
- (D) $\frac{1}{3}$

Correct Answer: (B) $\frac{1}{\sqrt{2}}$

Solution:

Step 1: Understanding the Concept:

We are dealing with the geometry of a standard ellipse. We are given the coordinates of the foci and an endpoint of the minor axis, and a condition about the angle formed between them. We translate this geometric condition into an algebraic equation using the standard properties of an ellipse and the distance formula (or Pythagoras theorem) to solve for the eccentricity e .

Step 2: Key Formula or Approach:

1. Standard points on ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$: Foci $S(ae, 0)$ and $S'(-ae, 0)$. Endpoint of minor axis $B(0, b)$. 2. Right angle condition at B : The triangle $\triangle SBS'$ is a right-angled triangle with hypotenuse SS' . Thus, $BS^2 + BS'^2 = SS'^2$ (Pythagoras theorem). 3. Fundamental relationship of an ellipse: $b^2 = a^2(1 - e^2)$.

Step 3: Detailed Explanation:

Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The center is $O(0, 0)$. $OB = b$ is the semi-minor axis, so the coordinates of point B are $(0, b)$. The foci are $S(ae, 0)$ and $S'(-ae, 0)$. The angle $\angle SBS'$ is given as 90° (a right angle). This means the triangle formed by points S, B , and S' is a right-angled triangle at B . According to the Pythagorean theorem:

$$BS^2 + BS'^2 = SS'^2$$

Let's calculate the squared distances using the distance formula:

$$BS^2 = (ae - 0)^2 + (0 - b)^2 = a^2e^2 + b^2$$

$$BS'^2 = (-ae - 0)^2 + (0 - b)^2 = a^2e^2 + b^2$$

The distance between the two foci SS' is $2ae$, so:

$$SS'^2 = (2ae)^2 = 4a^2e^2$$

Substitute these back into the Pythagorean equation:

$$(a^2e^2 + b^2) + (a^2e^2 + b^2) = 4a^2e^2$$

Combine like terms:

$$2a^2e^2 + 2b^2 = 4a^2e^2$$

Divide by 2:

$$a^2e^2 + b^2 = 2a^2e^2$$

Rearrange to isolate b^2 :

$$b^2 = 2a^2e^2 - a^2e^2$$

$$b^2 = a^2e^2$$

Now, use the fundamental relation for ellipses, $b^2 = a^2(1 - e^2)$, and substitute it into our derived equation:

$$a^2(1 - e^2) = a^2e^2$$

Assuming $a \neq 0$, divide both sides by a^2 :

$$1 - e^2 = e^2$$

$$1 = 2e^2$$

$$e^2 = \frac{1}{2}$$

Since eccentricity is a positive ratio for an ellipse:

$$e = \frac{1}{\sqrt{2}}$$

Step 4: Final Answer:

The eccentricity is $\frac{1}{\sqrt{2}}$.

💡 Quick Tip

For any point P on the minor axis of an ellipse, the distance to a focus is always a (semi-major axis). So $BS = BS' = a$. If $\angle SBS' = 90^\circ$, it's a right isosceles triangle. Thus hypotenuse $SS' = \sqrt{2} \cdot BS \implies 2ae = \sqrt{2}a \implies 2e = \sqrt{2} \implies e = 1/\sqrt{2}$. This geometric shortcut is much faster.

37. If the directed line makes an angle 45° and 60° with the X and Y -axes respectively, then the obtuse angle θ made by the line with the Z -axis is

- (A) 135°
- (B) 120°
- (C) 160°
- (D) 150°

Correct Answer: (B) 120°

Solution:

Step 1: Understanding the Concept:

A line in 3D space makes angles α, β, γ with the X, Y, and Z coordinate axes respectively. The cosines of these angles are called the direction cosines of the line. A fundamental geometric property is that the sum of the squares of the direction cosines is always equal to 1. We use this to find the missing angle.

Step 2: Key Formula or Approach:

Identity for direction cosines: $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$. Given $\alpha = 45^\circ$ and $\beta = 60^\circ$, solve for $\cos \gamma$ and then determine the obtuse angle γ .

Step 3: Detailed Explanation:

Let the angles made by the line with X, Y, and Z axes be α, β , and γ respectively. We are given: $\alpha = 45^\circ \implies \cos \alpha = \cos(45^\circ) = \frac{1}{\sqrt{2}}$ $\beta = 60^\circ \implies \cos \beta = \cos(60^\circ) = \frac{1}{2}$ We use the fundamental relationship:

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

Substitute the known values:

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2 \gamma = 1$$

$$\frac{1}{2} + \frac{1}{4} + \cos^2 \gamma = 1$$

Find a common denominator to add the fractions:

$$\frac{2}{4} + \frac{1}{4} + \cos^2 \gamma = 1$$

$$\frac{3}{4} + \cos^2 \gamma = 1$$

Isolate $\cos^2 \gamma$:

$$\cos^2 \gamma = 1 - \frac{3}{4} = \frac{1}{4}$$

Take the square root of both sides:

$$\cos \gamma = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2}$$

This gives two possible principal angles for γ : If $\cos \gamma = \frac{1}{2}$, then $\gamma = 60^\circ$ (acute angle). If $\cos \gamma = -\frac{1}{2}$, then $\gamma = 120^\circ$ (obtuse angle). The problem explicitly asks for the obtuse angle θ made with the Z-axis. Therefore, we must choose the negative cosine value. $\theta = \gamma = 120^\circ$.

Step 4: Final Answer:

The obtuse angle is 120° .

💡 Quick Tip

Always pay attention to keywords like "acute" or "obtuse" when solving for angles via squares of trigonometric functions. The equation $\cos^2 x = k$ yields two supplementary direction angles, one acute and one obtuse.

38. The derivative of $\tan^{-1}(\sqrt{1+x^2}-1)$ is

- (A) $\frac{x}{\sqrt{1+x^2}(x^2-2\sqrt{x+1}+1)}$
(B) $\frac{x}{\sqrt{1+x^2}(x^2-2\sqrt{1+x^2}+3)}$
(C) $\frac{x}{\sqrt{1+x^2}(x^2-2\sqrt{x^2+1}+2)}$
(D) $\frac{x}{\sqrt{1+x^2}(x^2+2\sqrt{1+x^2}-3)}$

Correct Answer: (B) $\frac{x}{\sqrt{1+x^2}(x^2-2\sqrt{1+x^2}+3)}$

Solution:

Step 1: Understanding the Concept:

We need to find the derivative of a composite inverse trigonometric function. We apply the chain rule iteratively: outer function ($\tan^{-1} u$), inner algebraic expression, and nested square root.

Step 2: Key Formula or Approach:

1. Derivative of inverse tangent: $\frac{d}{du}(\tan^{-1} u) = \frac{1}{1+u^2} \cdot u'$. 2. Chain rule and Power rule: $\frac{d}{dx}(\sqrt{1+x^2}) = \frac{1}{2\sqrt{1+x^2}} \cdot 2x = \frac{x}{\sqrt{1+x^2}}$.

Step 3: Detailed Explanation:

Let $y = \tan^{-1}(\sqrt{1+x^2}-1)$. Let $u = \sqrt{1+x^2}-1$. Then $y = \tan^{-1} u$. Apply the chain rule $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$:

$$\frac{dy}{dx} = \frac{1}{1+u^2} \cdot \frac{d}{dx}(\sqrt{1+x^2}-1)$$

Calculate the derivative of the inner function u :

$$\frac{du}{dx} = \frac{1}{2\sqrt{1+x^2}} \cdot \frac{d}{dx}(1+x^2) - 0 = \frac{1}{2\sqrt{1+x^2}} \cdot 2x = \frac{x}{\sqrt{1+x^2}}$$

Now, substitute u and du/dx back into the main equation:

$$\frac{dy}{dx} = \frac{1}{1+(\sqrt{1+x^2}-1)^2} \cdot \frac{x}{\sqrt{1+x^2}}$$

Expand the squared binomial in the denominator:

$$\begin{aligned}(\sqrt{1+x^2}-1)^2 &= (\sqrt{1+x^2})^2 - 2(1)(\sqrt{1+x^2}) + 1^2 = (1+x^2) - 2\sqrt{1+x^2} + 1 \\(\sqrt{1+x^2}-1)^2 &= x^2 - 2\sqrt{1+x^2} + 2\end{aligned}$$

Substitute this expanded form back into the denominator:

$$\frac{dy}{dx} = \frac{1}{1+(x^2-2\sqrt{1+x^2}+2)} \cdot \frac{x}{\sqrt{1+x^2}}$$

Simplify the constant terms in the denominator ($1+2=3$):

$$\frac{dy}{dx} = \frac{1}{x^2-2\sqrt{1+x^2}+3} \cdot \frac{x}{\sqrt{1+x^2}}$$

Combine into a single fraction to match the options:

$$\frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}(x^2 - 2\sqrt{1+x^2} + 3)}$$

Step 4: Final Answer:

The derivative is $\frac{x}{\sqrt{1+x^2}(x^2 - 2\sqrt{1+x^2} + 3)}$.

💡 Quick Tip

While trigonometric substitutions (like $x = \tan \theta$) are powerful for simplifying inverse trig derivatives, they are most effective when the argument is a rational expression (e.g., $\frac{\sqrt{1+x^2}-1}{x}$). For an un-fractioned argument like this one, direct chain rule application is faster and less prone to algebraic conversion errors at the end.

39. By dropping a stone in a quiet lake, a wave in the form of circle is generated. The radius of the circular wave increases at the rate of 2.1 cm/sec. Then the rate of increase of the enclosed circular region, when the radius of the circular wave is 10 cm , is (Given $\pi = \frac{22}{7}$)

- (A) 66 cm²/ second
- (B) 122 cm²/ second
- (C) 132 cm²/ second
- (D) 110 cm²/ second

Correct Answer: (C) 132 cm²/ second

Solution:

Step 1: Understanding the Concept:

This is a standard "related rates" problem in calculus. We are given the rate of change of the radius of a circle with respect to time and are asked to find the rate of change of its area. We relate the two quantities with the area formula and differentiate with respect to time.

Step 2: Key Formula or Approach:

1. Formula for the area of a circle: $A = \pi r^2$. 2. Differentiate both sides with respect to time t using the chain rule: $\frac{dA}{dt} = \frac{d}{dr}(\pi r^2) \cdot \frac{dr}{dt} = 2\pi r \frac{dr}{dt}$. 3. Substitute the given specific values to evaluate $\frac{dA}{dt}$.

Step 3: Detailed Explanation:

Let r be the radius of the circular wave and A be its enclosed area at any time t . We are given: Rate of increase of radius, $\frac{dr}{dt} = 2.1$ cm/sec. Specific radius at the moment of interest, $r = 10$ cm. Value of pi to use, $\pi = \frac{22}{7}$. The relationship between area and radius is:

$$A = \pi r^2$$

Differentiate with respect to time t :

$$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

Now, plug in the given values into this rate equation:

$$\frac{dA}{dt} = 2 \cdot \left(\frac{22}{7}\right) \cdot (10) \cdot (2.1)$$

To simplify the calculation, express 2.1 as a fraction or note its relation to 7: $2.1 = \frac{21}{10}$ Substitute this back:

$$\frac{dA}{dt} = 2 \cdot \left(\frac{22}{7}\right) \cdot 10 \cdot \left(\frac{21}{10}\right)$$

Cancel out the 10s:

$$\frac{dA}{dt} = 2 \cdot \left(\frac{22}{7}\right) \cdot 21$$

Cancel 21 with 7 ($21 \div 7 = 3$):

$$\frac{dA}{dt} = 2 \cdot 22 \cdot 3$$

Multiply the remaining numbers:

$$\frac{dA}{dt} = 44 \cdot 3 = 132$$

The units for the rate of area change are square centimeters per second.

Step 4: Final Answer:

The rate of increase is $132 \text{ cm}^2/\text{second}$.

💡 Quick Tip

In related rates geometry problems, the equation linking the rates (e.g., $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$) is exactly the perimeter/circumference formula multiplied by the radial rate. This makes physical sense: area grows by adding a thin ring of circumference $2\pi r$ growing at speed dr/dt .

40. The angle between the curves $xy = 6$ and $x^2y = 12$ is

- (A) $\tan^{-1} \frac{3}{11}$
- (B) $\tan^{-1} \frac{11}{3}$
- (C) $\tan^{-1} \frac{2}{11}$
- (D) $\tan^{-1} \frac{1}{11}$

Correct Answer: (A) $\tan^{-1} \frac{3}{11}$

Solution:

Step 1: Understanding the Concept:

The angle between two intersecting curves is defined as the angle between their tangent lines at the point of intersection. We first need to algebraically find the point(s) where they cross. Then, we find the derivatives (slopes of tangents) of both curves at that specific point. Finally, we use the angle formula for two lines.

Step 2: Key Formula or Approach:

1. Solve the system of equations $xy = 6$ and $x^2y = 12$ to find the intersection point (x_0, y_0) . 2. Differentiate each curve equation to find general slope expressions y'_1 and y'_2 . 3. Evaluate slopes at intersection: $m_1 = y'_1(x_0)$ and $m_2 = y'_2(x_0)$. 4. Use the angle formula: $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$.

Step 3: Detailed Explanation:

First, find the point of intersection. Curve 1: $xy = 6 \implies y = \frac{6}{x}$ Curve 2: $x^2y = 12$ Substitute y from Curve 1 into Curve 2:

$$x^2 \left(\frac{6}{x} \right) = 12$$

Assuming $x \neq 0$ (from the domain of hyperbola), simplify:

$$6x = 12 \implies x = 2$$

Substitute $x = 2$ back to find y :

$$y = \frac{6}{2} = 3$$

The intersection point is $(2, 3)$.

Next, find the slopes of the tangents at $(2, 3)$. For Curve 1 ($xy = 6$), differentiate implicitly or explicitly. Let's do explicitly: $y = 6x^{-1}$

$$\frac{dy}{dx} = -6x^{-2} = \frac{-6}{x^2}$$

Slope m_1 at $x = 2$:

$$m_1 = \frac{-6}{2^2} = \frac{-6}{4} = -\frac{3}{2}$$

For Curve 2 ($x^2y = 12$), explicit form: $y = 12x^{-2}$

$$\frac{dy}{dx} = -24x^{-3} = \frac{-24}{x^3}$$

Slope m_2 at $x = 2$:

$$m_2 = \frac{-24}{2^3} = \frac{-24}{8} = -3$$

Now, use the formula for the angle θ between two lines with slopes m_1 and m_2 :

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

Substitute $m_1 = -3/2$ and $m_2 = -3$:

$$\tan \theta = \left| \frac{-\frac{3}{2} - (-3)}{1 + \left(-\frac{3}{2}\right)(-3)} \right|$$

$$\tan \theta = \left| \frac{-\frac{3}{2} + 3}{1 + \frac{9}{2}} \right|$$

Find common denominators:

$$\tan \theta = \left| \frac{\frac{-3+6}{2}}{\frac{2+9}{2}} \right| = \left| \frac{\frac{3}{2}}{\frac{11}{2}} \right|$$

$$\tan \theta = \left| \frac{3}{2} \times \frac{2}{11} \right| = \frac{3}{11}$$

Therefore, the angle is $\theta = \tan^{-1} \left(\frac{3}{11} \right)$.

Step 4: Final Answer:

The angle between the curves is $\tan^{-1} \frac{3}{11}$.

💡 Quick Tip

For equations of the form $x^a y^b = c$, implicit differentiation combined with logarithms is often faster: $\log(x^a y^b) = \log c \implies a \log x + b \log y = C \implies \frac{a}{x} + \frac{by'}{y} = 0 \implies y' = -\frac{ay}{bx}$. This allows plugging in the coordinates $(2, 3)$ instantly.

41. In the mean value theorem, $f'(c) = \frac{f(b)-f(a)}{b-a}$, if $a = 0$, $b = \frac{1}{2}$ and $f(x) = x(x-1)(x-2)$, then the value of c is

- (A) $1 - \frac{\sqrt{15}}{6}$
- (B) $1 - \frac{\sqrt{13}}{6}$
- (C) $1 - \frac{\sqrt{21}}{6}$
- (D) $1 + \frac{\sqrt{21}}{6}$

Correct Answer: (C) $1 - \frac{\sqrt{21}}{6}$

Solution:

Step 1: Understanding the Concept:

Lagrange's Mean Value Theorem (LMVT) guarantees that for a continuous and differentiable function on an interval $[a, b]$, there is at least one point c in the open interval (a, b) where the instantaneous rate of change (derivative) equals the average rate of change over the interval. We must calculate both sides of the theorem's equation and solve for c , ensuring the result lies within $(0, 1/2)$.

Step 2: Key Formula or Approach:

1. Expand the polynomial $f(x)$ for easier differentiation. 2. Find the derivative $f'(x)$ and formulate $f'(c)$. 3. Calculate the average rate of change: $\frac{f(b)-f(a)}{b-a}$. 4. Equate them, solve the resulting quadratic equation for c , and select the root satisfying $a < c < b$.

Step 3: Detailed Explanation:

Given $f(x) = x(x-1)(x-2)$. Let's expand this:

$$f(x) = x(x^2 - 3x + 2) = x^3 - 3x^2 + 2x$$

Find the derivative $f'(x)$:

$$f'(x) = 3x^2 - 6x + 2$$

So, $f'(c) = 3c^2 - 6c + 2$. Now, evaluate the function at the endpoints $a = 0$ and $b = 1/2$:

$$f(a) = f(0) = 0(0-1)(0-2) = 0$$

$$f(b) = f(1/2) = (1/2)(1/2-1)(1/2-2) = (1/2)(-1/2)(-3/2) = 3/8$$

Calculate the slope of the secant line:

$$\frac{f(b) - f(a)}{b - a} = \frac{3/8 - 0}{1/2 - 0} = \frac{3/8}{1/2} = \frac{3}{8} \times 2 = \frac{3}{4}$$

Apply LMVT by equating the instantaneous slope to the secant slope:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$3c^2 - 6c + 2 = \frac{3}{4}$$

Clear the fraction by multiplying by 4:

$$12c^2 - 24c + 8 = 3$$

$$12c^2 - 24c + 5 = 0$$

Solve this quadratic equation using the quadratic formula $c = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$:

$$c = \frac{24 \pm \sqrt{(-24)^2 - 4(12)(5)}}{2(12)}$$

$$c = \frac{24 \pm \sqrt{576 - 240}}{24} = \frac{24 \pm \sqrt{336}}{24}$$

Simplify the radical ($\sqrt{336} = \sqrt{16 \times 21} = 4\sqrt{21}$):

$$c = \frac{24 \pm 4\sqrt{21}}{24} = 1 \pm \frac{\sqrt{21}}{6}$$

We must choose the value of c that lies strictly within the interval $(0, 1/2)$. We know $\sqrt{21}$ is between $\sqrt{16} = 4$ and $\sqrt{25} = 5$ (approx 4.58). So $\frac{\sqrt{21}}{6}$ is approx $4.58/6 \approx 0.76$. $c_1 = 1 + 0.76 = 1.76$, which is outside $(0, 0.5)$. $c_2 = 1 - 0.76 = 0.24$, which is nicely inside the interval $(0, 0.5)$. Therefore, the valid root is $1 - \frac{\sqrt{21}}{6}$.

Step 4: Final Answer:

The value of c is $1 - \frac{\sqrt{21}}{6}$.

💡 Quick Tip

When applying Mean Value Theorems, a quadratic derivative equation will yield two mathematical roots. Always check which root falls strictly within the open interval (a, b) . The other root is a valid tangent point elsewhere but doesn't fulfill the specific theorem's existence condition for that interval.

42. $\int \frac{\sin 2x}{(a+b \cos x)^2} dx =$

- (A) $\frac{2}{a^2} \left[\log(a + b \cos x) - \frac{a}{a+b \cos x} \right] + c$ where c is the constant of integration.
- (B) $\frac{-1}{a^2} \left[\log(a + b \cos x) + \frac{a}{a+b \cos x} \right] + c$, where c is the constant of integration.
- (C) $\frac{-2}{b^2} \left[\log(a + b \cos x) + \frac{a}{a+b \cos x} \right] + c$ where c is the constant of integration.
- (D) $\frac{-2}{b^2} \left[\log(a + b \cos x) - \frac{a}{a+b \cos x} \right] + c$, where c is the constant of integration.

Correct Answer: (C) $\frac{-2}{b^2} \left[\log(a + b \cos x) + \frac{a}{a+b \cos x} \right] + c$ where c is the constant of integration.

Solution:

Step 1: Understanding the Concept:

The integrand contains a double angle $\sin 2x$ in the numerator and a linear term in $\cos x$ squared in the denominator. To make substitution feasible, we must first express the numerator entirely in terms of single angles x . Then, substituting the entire linear denominator block makes the integration straightforward.

Step 2: Key Formula or Approach:

1. Double angle identity: $\sin 2x = 2 \sin x \cos x$. 2. Substitution method: Let $u = a + b \cos x$. Find du and express the remaining $\cos x$ in terms of u . 3. Power rule for integration: $\int u^n du = \frac{u^{n+1}}{n+1}$ (for $n \neq -1$) and $\int \frac{1}{u} du = \log |u|$.

Step 3: Detailed Explanation:

Let the integral be $I = \int \frac{\sin 2x}{(a+b \cos x)^2} dx$. Use the identity $\sin 2x = 2 \sin x \cos x$:

$$I = \int \frac{2 \sin x \cos x}{(a+b \cos x)^2} dx$$

Now, apply substitution. Let $u = a + b \cos x$. Then, differentiate with respect to x : $du = -b \sin x dx \implies \sin x dx = -\frac{du}{b}$. We also need an expression for the lone $\cos x$ in the numerator. From our substitution, we get $\cos x = \frac{u-a}{b}$. Substitute everything into the integral:

$$I = \int \frac{2 \left(\frac{u-a}{b} \right) \left(-\frac{du}{b} \right)}{u^2}$$

Extract the constants to the front:

$$I = -\frac{2}{b^2} \int \frac{u-a}{u^2} du$$

Split the fraction in the integrand:

$$\begin{aligned} I &= -\frac{2}{b^2} \int \left(\frac{u}{u^2} - \frac{a}{u^2} \right) du \\ I &= -\frac{2}{b^2} \int \left(\frac{1}{u} - au^{-2} \right) du \end{aligned}$$

Now, integrate each term with respect to u :

$$\begin{aligned} I &= -\frac{2}{b^2} \left[\log |u| - a \left(\frac{u^{-1}}{-1} \right) \right] + c \\ I &= -\frac{2}{b^2} \left[\log |u| + \frac{a}{u} \right] + c \end{aligned}$$

Finally, substitute back $u = a + b \cos x$:

$$I = -\frac{2}{b^2} \left[\log(a + b \cos x) + \frac{a}{a + b \cos x} \right] + c$$

(Assuming the argument of the logarithm is positive in the given domain, absolute values can be dropped to match the options).

Step 4: Final Answer:

The result is $-\frac{2}{b^2} \left[\log(a + b \cos x) + \frac{a}{a + b \cos x} \right] + c$.

 Quick Tip

Whenever you see a mix of $2x$ and x trigonometric arguments in an integral, standardizing them to single angles x using identities like $\sin 2x = 2 \sin x \cos x$ is universally the first required step before any substitution can work properly.

43. If m_1 and m_2 are the slopes of the lines represented by $ax^2 + 2hxy + by^2 = 0$ satisfying the condition $16h^2 = 25ab$, then

- (A) $m_1 = m_2^2$
- (B) $m_1 = 4m_2$
- (C) $|m_1 - m_2| = 2$
- (D) $m_1 m_2 = 1$

Correct Answer: (B) $m_1 = 4m_2$

Solution:

Step 1: Understanding the Concept:

A homogeneous equation of second degree $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines passing through the origin. If their slopes are m_1 and m_2 , there exist direct relations linking the sum and product of these slopes to the coefficients a, h , and b . We substitute these relations into the given condition to find the specific ratio between the slopes.

Step 2: Key Formula or Approach:

1. Sum of slopes: $m_1 + m_2 = -\frac{2h}{b}$. 2. Product of slopes: $m_1 m_2 = \frac{a}{b}$. 3. Express the given condition $16h^2 = 25ab$ entirely in terms of slopes.

Step 3: Detailed Explanation:

From the standard formulas, we can express h and a in terms of the slopes and b :

$$2h = -b(m_1 + m_2) \implies h = -\frac{b(m_1 + m_2)}{2}$$
$$a = b(m_1 m_2)$$

The given condition is:

$$16h^2 = 25ab$$

Substitute the expressions for h and a into this condition:

$$16 \left[-\frac{b(m_1 + m_2)}{2} \right]^2 = 25[b(m_1 m_2)]b$$

Square the term in the brackets:

$$16 \left[\frac{b^2(m_1 + m_2)^2}{4} \right] = 25b^2(m_1 m_2)$$

Simplify the left side ($16/4 = 4$):

$$4b^2(m_1 + m_2)^2 = 25b^2(m_1 m_2)$$

Assuming $b \neq 0$ (otherwise it's not a proper pair of lines with finite slopes), divide by b^2 :

$$4(m_1 + m_2)^2 = 25(m_1 m_2)$$

Expand the squared binomial:

$$4(m_1^2 + 2m_1 m_2 + m_2^2) = 25m_1 m_2$$

$$4m_1^2 + 8m_1 m_2 + 4m_2^2 = 25m_1 m_2$$

Rearrange into a homogeneous quadratic equation in terms of m_1 and m_2 :

$$4m_1^2 - 17m_1 m_2 + 4m_2^2 = 0$$

To find the relationship ratio, divide the entire equation by m_2^2 (assuming $m_2 \neq 0$):

$$4\left(\frac{m_1}{m_2}\right)^2 - 17\left(\frac{m_1}{m_2}\right) + 4 = 0$$

Let the ratio $r = \frac{m_1}{m_2}$. The equation becomes:

$$4r^2 - 17r + 4 = 0$$

Factor the quadratic equation:

$$4r^2 - 16r - r + 4 = 0$$

$$4r(r - 4) - 1(r - 4) = 0$$

$$(4r - 1)(r - 4) = 0$$

So, $r = \frac{1}{4}$ or $r = 4$. This means $\frac{m_1}{m_2} = \frac{1}{4} \implies 4m_1 = m_2$, OR $\frac{m_1}{m_2} = 4 \implies m_1 = 4m_2$. Looking at the options, $m_1 = 4m_2$ is present. Since the naming of slopes is arbitrary, both relations denote the same physical pair of lines where one slope is four times the other.

Step 4: Final Answer:

The relation is $m_1 = 4m_2$.

💡 Quick Tip

To test ratio conditions directly: If the roots are in ratio $p : q$, the condition is $(p + q)^2 ab = pq(2h)^2$. Here, $16h^2 = 25ab \implies 4(2h)^2 = 25ab \implies \frac{(2h)^2}{ab} = \frac{25}{4} = \frac{(4+1)^2}{4 \cdot 1}$. Thus the ratio is $4 : 1$.

44. The modulus of the square root of the conjugate of $-7 + 24\sqrt{-1}$ is _____

- (A) 3
- (B) 4
- (C) 16
- (D) 5

Correct Answer: (D) 5

Solution:**Step 1: Understanding the Concept:**

We must perform a sequence of operations on a given complex number: find its conjugate, find the square root of that conjugate, and finally find the modulus (magnitude) of the result. However, using properties of complex moduli, we can drastically simplify the calculation without ever actually calculating the square root explicitly.

Step 2: Key Formula or Approach:

1. Conjugate of $z = x + iy$ is $\bar{z} = x - iy$. 2. Modulus of $z = x + iy$ is $|z| = \sqrt{x^2 + y^2}$. 3. Important property relating modulus and conjugate: $|\bar{z}| = |z|$. 4. Important property relating modulus and powers/roots: $|z^n| = |z|^n$ and $|\sqrt{z}| = \sqrt{|z|}$.

Step 3: Detailed Explanation:

Let the initial complex number be $z = -7 + 24i$ (since $\sqrt{-1} = i$). The question asks for the modulus of the square root of its conjugate. Mathematically, this is written as:

$$\text{Required Value} = |\sqrt{\bar{z}}|$$

Using the property that the modulus of a square root is the square root of the modulus:

$$|\sqrt{\bar{z}}| = \sqrt{|\bar{z}|}$$

Using the property that a complex number and its conjugate have identical magnitudes ($|\bar{z}| = |z|$):

$$\sqrt{|\bar{z}|} = \sqrt{|z|}$$

So, we simply need to find the square root of the magnitude of our original number z . First, calculate the magnitude $|z|$:

$$|z| = \sqrt{(-7)^2 + (24)^2}$$

$$|z| = \sqrt{49 + 576}$$

$$|z| = \sqrt{625}$$

$$|z| = 25$$

Now, take the square root of this magnitude:

$$\text{Required Value} = \sqrt{|z|} = \sqrt{25} = 5$$

Step 4: Final Answer:

The value is 5.

💡 Quick Tip

Exploiting properties like $|\bar{z}| = |z|$ and $|\sqrt{z}| = \sqrt{|z|}$ saves immense time and avoids the tedious algebraic process of assuming $\sqrt{x + iy} = a + ib$, squaring, and solving simultaneous non-linear equations.

45. If $x + \log_{15}(5 + 3^x) = x \log_{15} 5 + \log_{15} 24$, then $x =$ _____

- (A) 1
- (B) 5
- (C) 2
- (D) 8

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

We are presented with a logarithmic equation containing the variable x both inside and outside the log arguments. The strategy is to gather all terms involving x on one side and manipulate them using logarithm rules to consolidate everything into a single logarithmic expression, which can then be converted into a standard algebraic or exponential equation.

Step 2: Key Formula or Approach:

1. Subtraction property of logs: $\log_b m - \log_b n = \log_b(m/n)$. 2. Power property of logs: $n \log_b m = \log_b(m^n)$. 3. Base conversion trick: Recognize that the standalone x can be written as $x \log_{15} 15$.

Step 3: Detailed Explanation:

The given equation is:

$$x + \log_{15}(5 + 3^x) = x \log_{15} 5 + \log_{15} 24$$

Group the terms with a solitary x factor together on the left side:

$$x - x \log_{15} 5 + \log_{15}(5 + 3^x) = \log_{15} 24$$

Factor out the x :

$$x(1 - \log_{15} 5) + \log_{15}(5 + 3^x) = \log_{15} 24$$

Since we are working with base 15 logarithms, rewrite 1 as $\log_{15} 15$:

$$x(\log_{15} 15 - \log_{15} 5) + \log_{15}(5 + 3^x) = \log_{15} 24$$

Use the quotient rule for logarithms on the term inside the parentheses:

$$x(\log_{15}(15/5)) + \log_{15}(5 + 3^x) = \log_{15} 24$$

$$x \log_{15} 3 + \log_{15}(5 + 3^x) = \log_{15} 24$$

Use the power rule for logarithms to move the x inside:

$$\log_{15}(3^x) + \log_{15}(5 + 3^x) = \log_{15} 24$$

Now, apply the product rule for logarithms ($\log a + \log b = \log ab$):

$$\log_{15}[3^x \cdot (5 + 3^x)] = \log_{15} 24$$

Since the logarithmic function is one-to-one, we can equate the arguments:

$$3^x(5 + 3^x) = 24$$

This is a quadratic equation hidden in exponential form. Let's make a substitution: let $y = 3^x$. Note that y must be strictly positive since $3^x > 0$ for all real x .

$$y(5 + y) = 24$$

$$y^2 + 5y - 24 = 0$$

Factor the quadratic equation:

$$(y + 8)(y - 3) = 0$$

This yields two possible values for y : $y = -8$ or $y = 3$. Since we established $y = 3^x > 0$, the solution $y = -8$ is rejected. So, we have:

$$y = 3 \implies 3^x = 3^1$$

Equating exponents:

$$x = 1$$

Step 4: Final Answer:

The value of x is 1.

💡 Quick Tip

When an equation mixes plain variables and logarithms, try converting the plain variable term into a logarithm with the matching base (e.g., $x = x \log_a a$). This is almost always the key to consolidating the equation.

46. If $f(x)$ is continuous at point $x = 0$ where $f(x) = \begin{cases} \frac{3 \sin x + 5 \tan x}{a^x - 1} & , x < 0 \\ \frac{2}{\log 2} & , x = 0 \\ \frac{8x + 2x \cos x}{b^x - 1} & , x > 0 \end{cases}$ then the values of a and b , respectively, are _____

- (A) 4, 5
- (B) 16, 32
- (C) 8, 10
- (D) 16, 16

Correct Answer: (B) 16, 32

Solution:

Step 1: Understanding the Concept:

For a piecewise function to be continuous at a specific point (here, $x = 0$), the left-hand limit (LHL) approaching the point, the right-hand limit (RHL) approaching the point, and the actual function value at that point must all exist and be exactly equal. We will calculate the limits using standard trigonometric and exponential limit formulas.

Step 2: Key Formula or Approach:

1. Condition for continuity at $x = c$: $\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = f(c)$. 2. Standard Limits to use as $x \rightarrow 0$: - $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ - $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$ - $\lim_{x \rightarrow 0} \frac{k^x - 1}{x} = \log_e k$

Step 3: Detailed Explanation:

Let's find the Left-Hand Limit (LHL) as $x \rightarrow 0^-$. We use the top branch function:

$$\text{LHL} = \lim_{x \rightarrow 0^-} \frac{3 \sin x + 5 \tan x}{a^x - 1}$$

Divide the numerator and denominator by x to create standard limit forms:

$$\text{LHL} = \lim_{x \rightarrow 0^-} \frac{\frac{3 \sin x}{x} + \frac{5 \tan x}{x}}{\frac{a^x - 1}{x}}$$

Applying the standard limits:

$$\text{LHL} = \frac{3(1) + 5(1)}{\log_e a} = \frac{8}{\log_e a}$$

Now let's find the Right-Hand Limit (RHL) as $x \rightarrow 0^+$. We use the bottom branch function:

$$\text{RHL} = \lim_{x \rightarrow 0^+} \frac{8x + 2x \cos x}{b^x - 1}$$

Factor out x in the numerator and then divide numerator and denominator by x :

$$\text{RHL} = \lim_{x \rightarrow 0^+} \frac{x(8 + 2 \cos x)}{b^x - 1} = \lim_{x \rightarrow 0^+} \frac{8 + 2 \cos x}{\frac{b^x - 1}{x}}$$

As $x \rightarrow 0$, $\cos x \rightarrow 1$. Apply the standard exponential limit:

$$\text{RHL} = \frac{8 + 2(1)}{\log_e b} = \frac{10}{\log_e b}$$

The function value at $x = 0$ is given explicitly in the middle branch:

$$f(0) = \frac{2}{\log_e 2}$$

For continuity, $\text{LHL} = \text{RHL} = f(0)$. Let's set up the equations. Equation 1 ($\text{LHL} = f(0)$):

$$\frac{8}{\log_e a} = \frac{2}{\log_e 2}$$

Cross-multiply and solve for $\log_e a$:

$$2 \log_e a = 8 \log_e 2 \implies \log_e a = 4 \log_e 2$$

Using the power property $\log(m^n) = n \log m$:

$$\log_e a = \log_e(2^4) = \log_e 16 \implies a = 16$$

Equation 2 ($\text{RHL} = f(0)$):

$$\frac{10}{\log_e b} = \frac{2}{\log_e 2}$$

Cross-multiply and solve for $\log_e b$:

$$2 \log_e b = 10 \log_e 2 \implies \log_e b = 5 \log_e 2$$

$$\log_e b = \log_e(2^5) = \log_e 32 \implies b = 32$$

Step 4: Final Answer:

The values are 16, 32.

 Quick Tip

A highly reliable and fast method for these specific 0/0 forms is dividing numerator and denominator by x , aiming directly for fundamental limit templates. Avoid L'Hôpital's rule here, as differentiating denominators like $(a^x - 1)$ creates messier repeating terms compared to algebraic manipulation.

47. The smallest angle of the triangle whose sides are $6 + \sqrt{12}$, $\sqrt{48}$, $\sqrt{24}$ is

- (A) $\frac{\pi}{2}$
- (B) $\frac{\pi}{6}$
- (C) $\frac{\pi}{4}$
- (D) $\frac{\pi}{3}$

Correct Answer: (B) $\frac{\pi}{6}$

Solution:

Step 1: Understanding the Concept:

In any triangle, the smallest angle is always opposite the smallest side. First, we must evaluate or approximate the lengths of the given sides to identify the smallest one. Once identified, we apply the Law of Cosines to calculate the exact measure of the angle opposite to it.

Step 2: Key Formula or Approach:

1. Identify the shortest side, say c . 2. Law of Cosines: $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$. 3. Standard trigonometric angle values to find angle C .

Step 3: Detailed Explanation:

Let's simplify and estimate the three sides to compare them: Let side $a = 6 + \sqrt{12} = 6 + 2\sqrt{3}$. Since $\sqrt{3} \approx 1.732$, $a \approx 6 + 3.464 = 9.464$. Let side $b = \sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3} \approx 4 \times 1.732 = 6.928$. Let side $c = \sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6} \approx 2 \times 2.45 = 4.9$. Comparing the estimated values, c is definitively the shortest side. Therefore, the smallest angle is angle C , which is opposite to side c . Now we use the Cosine Rule to find $\cos C$:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

First, calculate the required square terms: $a^2 = (6 + 2\sqrt{3})^2 = 36 + 24\sqrt{3} + 12 = 48 + 24\sqrt{3}$
 $b^2 = (\sqrt{48})^2 = 48$ $c^2 = (\sqrt{24})^2 = 24$ Now calculate the numerator $a^2 + b^2 - c^2$:

$$\text{Numerator} = (48 + 24\sqrt{3}) + 48 - 24 = 72 + 24\sqrt{3}$$

Factor out the common term 24:

$$\text{Numerator} = 24(3 + \sqrt{3}) = 24\sqrt{3}(\sqrt{3} + 1)$$

Now calculate the denominator $2ab$:

$$\text{Denominator} = 2(6 + 2\sqrt{3})(4\sqrt{3})$$

$$\text{Denominator} = 8\sqrt{3}(6 + 2\sqrt{3})$$

$$\text{Denominator} = 48\sqrt{3} + 16(3) = 48\sqrt{3} + 48$$

Factor out the common term 48:

$$\text{Denominator} = 48(\sqrt{3} + 1)$$

Substitute numerator and denominator back into the Cosine Rule equation:

$$\cos C = \frac{24\sqrt{3}(\sqrt{3} + 1)}{48(\sqrt{3} + 1)}$$

The term $(\sqrt{3} + 1)$ cancels out nicely:

$$\cos C = \frac{24\sqrt{3}}{48} = \frac{\sqrt{3}}{2}$$

We know from standard trigonometric tables that if $\cos C = \frac{\sqrt{3}}{2}$ in a triangle context ($0 < C < 180^\circ$), then $C = 30^\circ$, which in radians is $\frac{\pi}{6}$.

Step 4: Final Answer:

The smallest angle is $\frac{\pi}{6}$.

 Quick Tip

Always simplify surds (e.g., $\sqrt{48}$ to $4\sqrt{3}$) before plugging them into the Cosine Rule. It makes factoring the resulting complex expressions (like factoring out $\sqrt{3} + 1$) much more transparent and prevents severe algebraic messiness.

48. Consider the three statements

$p : \forall n \in \mathbb{N}, 10n - 3$ is a prime number, when n is not divisible by 3.

$q : \frac{2}{\sqrt{3}}, \frac{-2}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$ are the direction cosines of a directed line.

$r : \sin x$ is an increasing function in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Then which of the following statement pattern has truth value true?

- (A) $(p \wedge q) \leftrightarrow r$
- (B) $(p \rightarrow q) \rightarrow \sim r$
- (C) $(\sim p \vee q) \wedge r$
- (D) $(\sim p \wedge \sim q) \leftrightarrow \sim r$

Correct Answer: (C) $(\sim p \vee q) \wedge r$

Solution:

Step 1: Understanding the Concept:

We must systematically evaluate the individual truth values (True or False) of the three distinct mathematical statements p , q , and r . Once we have their individual boolean values, we substitute them into the logical structures provided in the options to see which evaluates to True.

Step 2: Key Formula or Approach:

- A universal statement $(\forall n)$ is proven False by finding just one counterexample. - Direction cosines l, m, n must mathematically satisfy the identity $l^2 + m^2 + n^2 = 1$. - A function is

increasing on an interval if its first derivative is positive (or non-negative) throughout that interval.

Step 3: Detailed Explanation:

Let's evaluate statement p : It claims $10n - 3$ is prime for all $n \in \mathbb{N}$ not divisible by 3. Let's test values. $n = 1$: $10(1) - 3 = 7$ (Prime) $n = 2$: $10(2) - 3 = 17$ (Prime) $n = 4$: $10(4) - 3 = 37$ (Prime) $n = 5$: $10(5) - 3 = 47$ (Prime) $n = 7$: $10(7) - 3 = 67$ (Prime) $n = 8$: $10(8) - 3 = 77$. Note that $77 = 7 \times 11$, which is a composite number. Since $n = 8$ is not divisible by 3, this is a valid counterexample. Thus, statement p is False (F).

Let's evaluate statement q : It claims the values are direction cosines. They must satisfy $l^2 + m^2 + n^2 = 1$. Calculate the sum of squares:

$$\left(\frac{2}{\sqrt{3}}\right)^2 + \left(\frac{-2}{\sqrt{3}}\right)^2 + \left(\frac{-1}{\sqrt{3}}\right)^2 = \frac{4}{3} + \frac{4}{3} + \frac{1}{3} = \frac{9}{3} = 3$$

Since $3 \neq 1$, they are not direction cosines. Thus, statement q is False (F).

Let's evaluate statement r : The function is $f(x) = \sin x$. Its derivative is $f'(x) = \cos x$. In the interval $[-\pi/2, \pi/2]$ (which corresponds to the 4th and 1st quadrants), the value of $\cos x$ is strictly ≥ 0 . Since the derivative is non-negative everywhere in the interval, the function $\sin x$ is indeed increasing. Thus, statement r is True (T).

Now, substitute ($p = F, q = F, r = T$) into the options to find which expression evaluates to True: (A) $(F \wedge F) \leftrightarrow T \equiv F \leftrightarrow T \equiv \text{False}$ (B) $(F \rightarrow F) \rightarrow \sim T \equiv T \rightarrow F \equiv \text{False}$ (C) $(\sim F \vee F) \wedge T \equiv (T \vee F) \wedge T \equiv T \wedge T \equiv \text{True}$ (D) $(\sim F \wedge \sim F) \leftrightarrow \sim T \equiv (T \wedge T) \leftrightarrow F \equiv T \leftrightarrow F \equiv \text{False}$

Step 4: Final Answer:

The true pattern is $(\sim p \vee q) \wedge r$.

💡 Quick Tip

For "for all" ($\forall$) statements involving primes, don't stop testing early. The first few numbers often yield primes designed to trick you. Test several values, especially looking for answers that might be divisible by 7, 11, or 13, which are easy to overlook.

49. If $A = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where A_{21}, A_{22}, A_{23} are cofactors of a_{21}, a_{22}, a_{23} respectively, then the value of $a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} =$

- (A) 1
- (B) -1
- (C) 0
- (D) 2

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

The given expression $a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$ is mathematically identical to the expansion formula for the determinant of matrix A along its second row. Therefore, solving this problem simply requires us to calculate the determinant of the entire matrix A by any convenient method.

Step 2: Key Formula or Approach:

Definition of determinant by cofactor expansion along row i : $|A| = \sum_j a_{ij}A_{ij}$. Here, with $i = 2$, it's exactly the determinant $|A|$. Calculate $|A|$ by expanding along the most convenient row or column, which is the 3rd row (or 3rd column) since it contains two zeros.

Step 3: Detailed Explanation:

The expression is defined as the determinant of A :

$$\text{Value} = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} = |A|$$

Let's find the determinant of matrix A :

$$|A| = \begin{vmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along the third row is the most efficient method due to the zeros:

$$|A| = 0 \cdot (\dots) - 0 \cdot (\dots) + 1 \cdot \begin{vmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{vmatrix}$$

Calculate the 2×2 determinant:

$$|A| = 1 \cdot [(\cos \theta)(\cos \theta) - (\sin \theta)(-\sin \theta)]$$

$$|A| = \cos^2 \theta - (-\sin^2 \theta)$$

$$|A| = \cos^2 \theta + \sin^2 \theta$$

Applying the fundamental trigonometric identity:

$$|A| = 1$$

Step 4: Final Answer:

The value of the expression is 1.

 Quick Tip

Recognize fundamental matrix property definitions. The sum of the products of elements of any row/column with their corresponding cofactors evaluates to the determinant of the matrix. (Conversely, the sum with cofactors of a different row evaluates to zero).

50. In a triangle ABC , with usual notations, if $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ Then $\cos A : \cos B : \cos C$ is

(A) 7 : 19 : 25

(B) 19 : 7 : 25

(C) 12 : 14 : 20

(D) 19 : 25 : 20

Correct Answer: (A) 7 : 19 : 25

Solution:

Step 1: Understanding the Concept:

We are given ratios of sums of sides of a triangle. By setting these ratios equal to a constant k , we can solve a simple system of linear equations to find the relative proportions of the individual sides a, b , and c . Once the side proportions are known, we can utilize the Cosine Rule to find the ratios of the cosines of the angles.

Step 2: Key Formula or Approach:

1. Express sides a, b, c in terms of a constant k . Sum the equations to find $a+b+c$, then subtract individual pairs to isolate a, b , and c . 2. Cosine Rule: $\cos A = \frac{b^2+c^2-a^2}{2bc}$, $\cos B = \frac{a^2+c^2-b^2}{2ac}$, $\cos C = \frac{a^2+b^2-c^2}{2ab}$.

Step 3: Detailed Explanation:

Let the given ratio equal k :

$$\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = k$$

This gives a system of three equations: 1) $b+c = 11k$ 2) $c+a = 12k$ 3) $a+b = 13k$ Add all three equations together:

$$2a + 2b + 2c = 11k + 12k + 13k = 36k$$

Divide by 2:

$$a + b + c = 18k$$

Now, subtract the initial equations from this sum to find individual sides: $a = (a+b+c) - (b+c) = 18k - 11k = 7k$ $b = (a+b+c) - (c+a) = 18k - 12k = 6k$ $c = (a+b+c) - (a+b) = 18k - 13k = 5k$ Since we only need ratios of cosines, we can drop the constant k and just use the relative lengths: $a = 7, b = 6, c = 5$. Now apply the Cosine Rule for each angle:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{6^2 + 5^2 - 7^2}{2(6)(5)} = \frac{36 + 25 - 49}{60} = \frac{12}{60} = \frac{1}{5}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{7^2 + 5^2 - 6^2}{2(7)(5)} = \frac{49 + 25 - 36}{70} = \frac{38}{70} = \frac{19}{35}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{7^2 + 6^2 - 5^2}{2(7)(6)} = \frac{49 + 36 - 25}{84} = \frac{60}{84} = \frac{5}{7}$$

We need the ratio $\cos A : \cos B : \cos C$. Ratio = $\frac{1}{5} : \frac{19}{35} : \frac{5}{7}$ To remove fractions, multiply by the least common multiple of the denominators, which is 35:

$$\left(\frac{1}{5} \times 35\right) : \left(\frac{19}{35} \times 35\right) : \left(\frac{5}{7} \times 35\right) \\ 7 : 19 : 25$$

Step 4: Final Answer:

The ratio is 7 : 19 : 25.

 Quick Tip

A standard trick for cyclic sum equations (like $a + b = x, b + c = y, c + a = z$) is to add them all up to find $2(a + b + c) = x + y + z$. From the total sum, extracting individual components becomes trivial by subtraction.

Physics

1. The time period of a simple pendulum inside a stationary lift is $\sqrt{3}$ second. When the lift moves upwards with an acceleration $g/3$, the time period will be ($g = \text{acceleration due to gravity}$)

- (A) 1.5 s
- (B) 2 s
- (C) $\sqrt{3}$ s
- (D) 3 s

Correct Answer: (A) 1.5 s

Solution:

Step 1: Understanding the Concept:

The time period of a simple pendulum depends on the effective acceleration due to gravity in its frame of reference.

When the frame of reference (the lift) accelerates, a pseudo force acts on the pendulum bob, changing the effective gravity.

Step 2: Key Formula or Approach:

The formula for the time period of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}}$.

Step 3: Detailed Explanation:

When the lift is stationary, the effective gravity is simply g .

The initial time period is given as $T_1 = 2\pi\sqrt{\frac{L}{g}} = \sqrt{3}$ s.

When the lift moves upwards with an acceleration $a = g/3$, a downward pseudo force acts on the mass.

Therefore, the effective acceleration due to gravity increases to $g_{\text{eff}} = g + a$.

Substitute the value of a : $g_{\text{eff}} = g + \frac{g}{3} = \frac{4g}{3}$.

The new time period T_2 is:

$$T_2 = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{L}{4g/3}} = 2\pi\sqrt{\frac{3L}{4g}}$$

We can factor out the constant terms to relate it to T_1 :

$$T_2 = \sqrt{\frac{3}{4}} \times \left(2\pi\sqrt{\frac{L}{g}}\right)$$

Substituting $2\pi\sqrt{\frac{L}{g}} = \sqrt{3}$:

$$T_2 = \frac{\sqrt{3}}{2} \times \sqrt{3} = \frac{3}{2} = 1.5 \text{ s}$$

Step 4: Final Answer:

The time period of the pendulum when the lift moves upwards is 1.5 s.

💡 Quick Tip

Remember that effective gravity increases when a lift accelerates upwards and decreases when it accelerates downwards.

2. A stone of mass 1 kg tied to a light inextensible string of length $L = \frac{5}{3}$ m is rotating in a circular path of radius L in a vertical plane. If the ratio of maximum tension in the string to the minimum tension in the string is 3, the speed of the stone at the highest point of the circle is (g = acceleration due to gravity)

- (A) $\sqrt{gL}$
- (B) $\sqrt{2gL}$
- (C) $\sqrt{4gL}$
- (D) $\sqrt{8gL}$

Correct Answer: (C) $\sqrt{4gL}$

Solution:

Step 1: Understanding the Concept:

For an object moving in a vertical circle, tension is maximum at the lowest point and minimum at the highest point.

We must use Newton's second law for circular motion and the conservation of mechanical energy.

Step 2: Key Formulas or Approach:

Tension at the lowest point: $T_{\max} = \frac{mv_L^2}{L} + mg$.

Tension at the highest point: $T_{\min} = \frac{mv_H^2}{L} - mg$.

Conservation of energy: $\frac{1}{2}mv_L^2 = \frac{1}{2}mv_H^2 + mg(2L)$.

Step 3: Detailed Explanation:

Let v_L be the speed at the lowest point and v_H be the speed at the highest point.

From the conservation of energy, we can find the relation between v_L and v_H :

$$\begin{aligned}\frac{1}{2}mv_L^2 - \frac{1}{2}mv_H^2 &= 2mgL \\ v_L^2 &= v_H^2 + 4gL\end{aligned}$$

It is given that the ratio of maximum tension to minimum tension is 3:

$$\frac{T_{\max}}{T_{\min}} = 3$$

Substitute the expressions for tension:

$$\frac{\frac{mv_L^2}{L} + mg}{\frac{mv_H^2}{L} - mg} = 3$$

Cancel mass m from numerator and denominator and multiply by L :

$$\frac{v_L^2 + gL}{v_H^2 - gL} = 3$$

Cross-multiply to solve for v_L^2 :

$$v_L^2 + gL = 3(v_H^2 - gL)$$

$$v_L^2 + gL = 3v_H^2 - 3gL$$

$$v_L^2 = 3v_H^2 - 4gL$$

Now, equate the two expressions obtained for v_L^2 :

$$v_H^2 + 4gL = 3v_H^2 - 4gL$$

Rearrange the terms to solve for v_H^2 :

$$4gL + 4gL = 3v_H^2 - v_H^2$$

$$8gL = 2v_H^2$$

$$v_H^2 = 4gL$$

Taking the square root, we get the speed at the highest point:

$$v_H = \sqrt{4gL}$$

The mass $m = 1$ kg and length $L = \frac{5}{3}$ m are extra information not needed to find the speed in terms of g and L .

Step 4: Final Answer:

The speed of the stone at the highest point is $\sqrt{4gL}$.

 Quick Tip

Always use conservation of energy to relate speeds at different points in a vertical circle. The difference in squared speeds between top and bottom is always $4gR$.

3. If $\vec{F} = (5\hat{i} - 10\hat{j})$ and $\vec{r} = (4\hat{i} - 3\hat{j})$, then the torque acting on the object will be

(A) $\hat{i} - 2\hat{j}$

(B) $2\hat{i} - \hat{j}$

(C) $25\hat{k}$

(D) $-25\hat{k}$

Correct Answer: (D) $-25\hat{k}$

Solution:

Step 1: Understanding the Concept:

Torque ($\vec{\tau}$) is the rotational equivalent of linear force.

It is defined as the cross product of the position vector ($\vec{r}$) and the force vector ($\vec{F}$).

Step 2: Key Formula or Approach:

The mathematical definition of torque is $\vec{\tau} = \vec{r} \times \vec{F}$.

Step 3: Detailed Explanation:

We are given the vectors: $\vec{r} = 4\hat{i} - 3\hat{j}$ and $\vec{F} = 5\hat{i} - 10\hat{j}$.

We need to calculate the cross product: $\vec{\tau} = (4\hat{i} - 3\hat{j}) \times (5\hat{i} - 10\hat{j})$.

We can compute this using the determinant method or by direct distributive multiplication.

Using the determinant method:

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 0 \\ 5 & -10 & 0 \end{vmatrix}$$

Expanding along the top row:

$$\vec{\tau} = \hat{i}((-3)(0) - (0)(-10)) - \hat{j}((4)(0) - (0)(5)) + \hat{k}((4)(-10) - (-3)(5))$$

The $\hat{i}$ and $\hat{j}$ components evaluate to zero.

$$\vec{\tau} = \hat{k}(-40 - (-15))$$

$$\vec{\tau} = \hat{k}(-40 + 15)$$

$$\vec{\tau} = -25\hat{k}$$

Step 4: Final Answer:

The torque acting on the object is $-25\hat{k}$.

 Quick Tip

The order in the cross product is crucial. Torque is always $\vec{r} \times \vec{F}$, not $\vec{F} \times \vec{r}$. Reversing the order will give the negative of the correct answer.

4. Two particles of equal mass ' m ' move in a circle of radius ' r ' under the action of their mutual gravitational attraction. The speed of each particle will be (G = Universal gravitational constant)

(A) $\sqrt{\frac{Gm}{4r}}$

(B) $\sqrt{\frac{Gm}{r}}$

(C) $\sqrt{\frac{Gm}{2r}}$

(D) $\sqrt{\frac{4Gm}{r}}$

Correct Answer: (A) $\sqrt{\frac{Gm}{4r}}$

Solution:

Step 1: Understanding the Concept:

When two particles of equal mass move in a circle under mutual gravitational attraction, they must always be diametrically opposite to each other to maintain symmetrical motion.

The gravitational force between them provides the necessary centripetal force for their circular motion.

Step 2: Key Formulas or Approach:

Newton's law of gravitation: $F_g = \frac{Gm_1m_2}{d^2}$.

Centripetal force required: $F_c = \frac{mv^2}{R}$.

Equating the two forces gives the condition for steady circular motion.

Step 3: Detailed Explanation:

Let the mass of each particle be m and the radius of the circular path be r .

The distance between the two diametrically opposite particles is $d = 2r$.

The magnitude of the gravitational force of attraction between them is:

$$F_g = \frac{G \cdot m \cdot m}{(2r)^2} = \frac{Gm^2}{4r^2}$$

This force acts towards the center of the circle and acts as the centripetal force.

For a particle moving in a circle of radius r with speed v , the required centripetal force is:

$$F_c = \frac{mv^2}{r}$$

Equating the gravitational force to the centripetal force:

$$\frac{mv^2}{r} = \frac{Gm^2}{4r^2}$$

Cancel one m and one r from both sides:

$$v^2 = \frac{Gm}{4r}$$

Taking the square root gives the speed of each particle:

$$v = \sqrt{\frac{Gm}{4r}}$$

Step 4: Final Answer:

The speed of each particle is $\sqrt{\frac{Gm}{4r}}$.

💡 Quick Tip

For binary star systems or particles under mutual attraction, always remember that the distance between them is the diameter, not the radius of their orbit.

5. Four particles each of mass M are placed at the corners of a square of side L . The radius of gyration of the system about an axis perpendicular to the square and passing through its centre is

- (A) L
- (B) $\frac{L}{2}$
- (C) $\frac{L}{4}$
- (D) $\frac{L}{\sqrt{2}}$

Correct Answer: (D) $\frac{L}{\sqrt{2}}$

Solution:

Step 1: Understanding the Concept:

The moment of inertia of a system of particles is the sum of the products of their masses and the squares of their perpendicular distances from the axis of rotation.

The radius of gyration (K) is the distance from the axis at which the entire mass of the system could be concentrated to give the same moment of inertia.

Step 2: Key Formulas or Approach:

Moment of inertia: $I = \sum_i m_i r_i^2$.

Radius of gyration: $I = M_{\text{total}} K^2$, which implies $K = \sqrt{\frac{I}{M_{\text{total}}}}$.

Step 3: Detailed Explanation:

Let the square have side length L .

The diagonal of the square has a length of $\sqrt{L^2 + L^2} = \sqrt{2}L$.

The axis of rotation passes through the center of the square and is perpendicular to its plane.

The distance r of each corner particle from the center is half the diagonal:

$$r = \frac{\sqrt{2}L}{2} = \frac{L}{\sqrt{2}}$$

There are 4 particles, each of mass M .

The total moment of inertia I of the system about the given axis is:

$$I = 4 \times (M \times r^2)$$

Substitute the value of r :

$$I = 4M \left(\frac{L}{\sqrt{2}} \right)^2 = 4M \left(\frac{L^2}{2} \right) = 2ML^2$$

The total mass of the system is $M_{\text{total}} = 4M$.

Using the definition of the radius of gyration:

$$\begin{aligned} I &= M_{\text{total}} K^2 \\ 2ML^2 &= 4M \times K^2 \end{aligned}$$

Solve for K^2 :

$$K^2 = \frac{2ML^2}{4M} = \frac{L^2}{2}$$

Taking the square root gives the radius of gyration:

$$K = \frac{L}{\sqrt{2}}$$

Step 4: Final Answer:

The radius of gyration of the system is $\frac{L}{\sqrt{2}}$.

💡 Quick Tip

The radius of gyration represents the root mean square distance of the particles from the axis of rotation.

6. A mass suspended from a vertical spring performs S.H.M. of period 0.1 second. The spring is unstretched at the highest point of suspension. Maximum speed of the mass is (Gravitational acceleration $g = 10 \text{ m/s}^2$)

- (A) $\frac{1}{2\pi} \text{ m/s}$
- (B) $\frac{1}{\pi} \text{ m/s}$
- (C) $\frac{2}{\pi} \text{ m/s}$
- (D) $\pi \text{ m/s}$

Correct Answer: (A) $\frac{1}{2\pi} \text{ m/s}$

Solution:

Step 1: Understanding the Concept:

A mass suspended on a vertical spring oscillates in Simple Harmonic Motion (S.H.M.) about its equilibrium position.

The equilibrium position is where the spring force balances the gravitational force.

Step 2: Key Formulas or Approach:

Time period of a spring-mass system: $T = 2\pi\sqrt{\frac{m}{k}}$.

Angular frequency: $\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$.

Maximum speed in S.H.M.: $v_{\max} = A\omega$, where A is the amplitude.

Equilibrium extension: $x_0 = \frac{mg}{k}$.

Step 3: Detailed Explanation:

Given the time period $T = 0.1 \text{ s}$.

The angular frequency is $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.1} = 20\pi \text{ rad/s}$.

We also know that $\sqrt{\frac{k}{m}} = \omega$, so $\frac{m}{k} = \frac{1}{\omega^2}$.

The problem states that the spring is unstretched at the highest point of its oscillation.

This implies the highest point is the natural length of the spring.

The system oscillates symmetrically about its equilibrium position.

The equilibrium position is lower than the natural length by an extension x_0 .

At equilibrium, the net force is zero: $kx_0 = mg$, so $x_0 = \frac{mg}{k}$.

Since the highest point is the unstretched position, the distance from equilibrium to the highest point is the amplitude A .

Therefore, the amplitude is $A = x_0 = \frac{mg}{k}$.
 We can rewrite the amplitude in terms of ω :

$$A = g \left(\frac{m}{k} \right) = g \left(\frac{1}{\omega^2} \right) = \frac{g}{\omega^2}$$

Now, calculate the maximum speed:

$$v_{\max} = A\omega = \left(\frac{g}{\omega^2} \right) \omega = \frac{g}{\omega}$$

Substitute the known values $g = 10 \text{ m/s}^2$ and $\omega = 20\pi \text{ rad/s}$:

$$v_{\max} = \frac{10}{20\pi} = \frac{1}{2\pi} \text{ m/s}$$

Step 4: Final Answer:

The maximum speed of the mass is $\frac{1}{2\pi} \text{ m/s}$.

💡 Quick Tip

When a spring is unstretched at one extreme position of its vertical oscillation, its amplitude is exactly equal to its static equilibrium extension.

7. A liquid rises to a height of 2.4 cm in a glass capillary P . Another glass capillary Q having diameter 80% of capillary P is immersed in the same liquid. The rise of liquid in capillary Q is

- (A) 2.4 cm
- (B) 3.4 cm
- (C) 3 cm
- (D) 2.5 cm

Correct Answer: (C) 3 cm

Solution:

Step 1: Understanding the Concept:

Capillary rise is the phenomenon where a liquid ascends in a narrow tube against gravity. The height of the liquid column is inversely proportional to the radius (and thus the diameter) of the capillary tube, according to Jurin's Law.

Step 2: Key Formula or Approach:

The formula for capillary rise is $h = \frac{2T \cos \theta}{\rho g r}$.

Since the liquid and material of the tube are the same, surface tension T , angle of contact θ , and density ρ are constant.

Therefore, $h \propto \frac{1}{r}$, which implies $h \times r = \text{constant}$.

Since diameter $d = 2r$, it also follows that $h \propto \frac{1}{d}$, or $h_1 d_1 = h_2 d_2$.

Step 3: Detailed Explanation:

Let the height and diameter for capillary P be h_P and d_P .

We are given $h_P = 2.4$ cm.

Let the height and diameter for capillary Q be h_Q and d_Q .

We are given that the diameter of Q is 80% of the diameter of P .

So, $d_Q = 0.8 \times d_P$.

Using the inverse relationship $h_P \times d_P = h_Q \times d_Q$:

$$h_Q = h_P \times \left(\frac{d_P}{d_Q} \right)$$

Substitute the known values:

$$h_Q = 2.4 \times \left(\frac{d_P}{0.8 \times d_P} \right)$$

$$h_Q = 2.4 \times \left(\frac{1}{0.8} \right)$$

$$h_Q = 2.4 \times \left(\frac{10}{8} \right)$$

$$h_Q = 2.4 \times 1.25$$

$$h_Q = 3.0 \text{ cm}$$

Step 4: Final Answer:

The rise of liquid in capillary Q is 3 cm.

💡 Quick Tip

A narrower capillary tube will always result in a greater capillary rise for the same liquid. The ratio of heights is the inverse ratio of their diameters.

8. The frequency of fourth overtone of a closed pipe is in unison with the fifth overtone of an open pipe. The ratio of length of closed pipe to that of open pipe is

- (A) 2 : 3
- (B) 3 : 4
- (C) 4 : 5
- (D) 5 : 6

Correct Answer: (B) 3 : 4

Solution:

Step 1: Understanding the Concept:

Organ pipes produce standing waves. The frequencies of these standing waves depend on whether the pipe is open at both ends or closed at one end.

A closed pipe supports only odd harmonics, while an open pipe supports all integer harmonics.

Step 2: Key Formulas or Approach:

For a closed pipe of length L_c , the allowed frequencies are $f_n = (2n+1)\frac{v}{4L_c}$, where $n = 0, 1, 2, \dots$

represents the overtone number.

For an open pipe of length L_o , the allowed frequencies are $f'_m = (m+1)\frac{v}{2L_o}$, where $m = 0, 1, 2, \dots$ represents the overtone number.

Step 3: Detailed Explanation:

Let's find the frequency of the fourth overtone of the closed pipe.

Here, $n = 4$.

Frequency $f_c = (2(4) + 1)\frac{v}{4L_c} = \frac{9v}{4L_c}$.

This corresponds to the 9th harmonic.

Now, let's find the frequency of the fifth overtone of the open pipe.

Here, $m = 5$.

Frequency $f_o = (5 + 1)\frac{v}{2L_o} = \frac{6v}{2L_o} = \frac{3v}{L_o}$.

This corresponds to the 6th harmonic.

The problem states that these two frequencies are in unison, meaning they are equal.

Equating the two frequencies:

$$\frac{9v}{4L_c} = \frac{3v}{L_o}$$

Cancel the speed of sound v from both sides:

$$\frac{9}{4L_c} = \frac{3}{L_o}$$

Rearrange to find the ratio $\frac{L_c}{L_o}$:

$$\begin{aligned}\frac{L_c}{L_o} &= \frac{9}{4 \times 3} \\ \frac{L_c}{L_o} &= \frac{9}{12}\end{aligned}$$

Simplify the fraction:

$$\frac{L_c}{L_o} = \frac{3}{4}$$

The ratio is 3 : 4.

Step 4: Final Answer:

The ratio of length of closed pipe to that of open pipe is 3 : 4.

 Quick Tip

For overtones: The n -th overtone in a closed pipe is the $(2n + 1)$ -th harmonic. The m -th overtone in an open pipe is the $(m + 1)$ -th harmonic.

9. When source of sound moves towards a stationary observer, the apparent frequency heard by him

(A) increases and wavelength also increases.

(B) increases while wavelength decreases.

(C) remains the same while wavelength decreases.

(D) decreases and wavelength remains the same.

Correct Answer: (B) increases while wavelength decreases.

Solution:

Step 1: Understanding the Concept:

This question deals with the Doppler effect for sound.

When there is relative motion between a source of sound and an observer, the frequency heard by the observer differs from the actual frequency emitted by the source.

Step 2: Key Formula or Approach:

The general formula for apparent frequency is $f' = f \left(\frac{v \pm v_o}{v \mp v_s} \right)$, where v is the speed of sound, v_o is the observer's speed, and v_s is the source's speed.

The apparent wavelength in the medium is $\lambda' = \frac{v - v_s}{f}$ when the source moves towards the observer.

Step 3: Detailed Explanation:

Here, the observer is stationary, so $v_o = 0$.

The source is moving towards the observer with velocity v_s .

The apparent frequency formula becomes $f' = f \left(\frac{v}{v - v_s} \right)$.

Since the denominator $(v - v_s)$ is less than the numerator v , the fraction is greater than 1.

Therefore, $f' > f$, which means the apparent frequency increases.

Now let's consider the wavelength.

Wavelength is the distance between consecutive compressions in the medium.

Since the source is moving in the direction of wave propagation, it "catches up" to the waves it just emitted.

This compresses the waves in front of it.

The apparent wavelength is the distance the wave travels in one period minus the distance the source travels in that period.

$$\lambda' = \lambda - \Delta\lambda = \frac{v}{f} - \frac{v_s}{f} = \frac{v - v_s}{f}.$$

Since $v - v_s < v$, it follows that $\lambda' < \lambda$.

Thus, the apparent wavelength decreases.

Step 4: Final Answer:

The apparent frequency increases while the wavelength decreases.

 Quick Tip

Motion of the source affects the true wavelength in the medium. Motion of the observer only affects the relative speed at which they encounter the wave crests, changing apparent frequency but not the actual wavelength in the air.

10. The formula for the physical quantity is $P = \frac{x^3 y}{z^2}$ and the percentage error in the determination of physical quantities x, y, z are 0.6%, 3% and 1.3% respectively. The percentage error in the measurement of P is

- (A) 2.2%
- (B) 4.9%
- (C) 5.3%
- (D) 7.4%

Correct Answer: (D) 7.4%

Solution:

Step 1: Understanding the Concept:

When a physical quantity is calculated using a formula involving multiplication and division, the maximum fractional (or percentage) error in the result is the sum of the absolute values of the fractional errors of individual quantities multiplied by their respective powers.

Step 2: Key Formula or Approach:

For a quantity $Q = \frac{A^a B^b}{C^c}$, the maximum percentage error is given by:

$$\frac{\Delta Q}{Q} \times 100\% = a \left(\frac{\Delta A}{A} \times 100\% \right) + b \left(\frac{\Delta B}{B} \times 100\% \right) + c \left(\frac{\Delta C}{C} \times 100\% \right).$$

Step 3: Detailed Explanation:

The given formula is $P = \frac{x^3 y^1}{z^2}$.

The given percentage errors are:

For x : $\frac{\Delta x}{x} \times 100\% = 0.6\%$

For y : $\frac{\Delta y}{y} \times 100\% = 3\%$

For z : $\frac{\Delta z}{z} \times 100\% = 1.3\%$

Applying the error propagation formula:

$$\left(\frac{\Delta P}{P} \times 100\% \right)_{\max} = 3 \left(\frac{\Delta x}{x} \times 100\% \right) + 1 \left(\frac{\Delta y}{y} \times 100\% \right) + 2 \left(\frac{\Delta z}{z} \times 100\% \right)$$

Substitute the given values into the equation:

$$\text{Percentage error in } P = 3(0.6\%) + 1(3\%) + 2(1.3\%)$$

Calculate each term:

$$\text{Percentage error in } P = 1.8\% + 3.0\% + 2.6\%$$

Add the values together:

$$\text{Percentage error in } P = 7.4\%$$

Step 4: Final Answer:

The percentage error in the measurement of P is 7.4%.

Quick Tip

Always take the absolute values of the powers when adding percentage errors to find the maximum possible error. Errors always add up.

11. Two rods of different materials have lengths ' l_1 ' and ' l_2 ' whose coefficient of linear expansions are ' α_1 ' and ' α_2 ' respectively. If the difference between the two lengths is independent of temperature then

- (A) $\alpha_1^2 l_1 = \alpha_2^2 l_2$
- (B) $\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}$
- (C) $\frac{l_1}{l_2} = \frac{\alpha_1}{\alpha_2}$
- (D) $l_1^2 \alpha_2 = l_2^2 \alpha_1$

Correct Answer: (B) $\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}$

Solution:

Step 1: Understanding the Concept:

When an object is heated, its length increases due to thermal expansion.

The condition that the difference in lengths remains independent of temperature means that both rods must expand by the exact same amount for any given temperature change.

Step 2: Key Formula or Approach:

The change in length Δl of a rod of initial length l with a linear expansion coefficient α subjected to a temperature change ΔT is given by $\Delta l = l\alpha\Delta T$.

Step 3: Detailed Explanation:

Let the initial lengths at some reference temperature be l_1 and l_2 .

Let the temperature change be ΔT .

The new lengths will be $l'_1 = l_1 + \Delta l_1 = l_1 + l_1\alpha_1\Delta T$ and $l'_2 = l_2 + \Delta l_2 = l_2 + l_2\alpha_2\Delta T$.

The difference in lengths at the new temperature is:

$$l'_1 - l'_2 = (l_1 + l_1\alpha_1\Delta T) - (l_2 + l_2\alpha_2\Delta T)$$

Grouping terms, we get:

$$l'_1 - l'_2 = (l_1 - l_2) + (l_1\alpha_1 - l_2\alpha_2)\Delta T$$

The term $(l_1 - l_2)$ is the initial difference in lengths.

For the difference to be independent of the temperature change ΔT , the coefficient of ΔT in the expression must be zero.

Therefore, we set the coefficient to zero:

$$l_1\alpha_1 - l_2\alpha_2 = 0$$

Rearranging the equation yields:

$$l_1\alpha_1 = l_2\alpha_2$$

This means the absolute expansions must be equal: $\Delta l_1 = \Delta l_2$.

From $l_1\alpha_1 = l_2\alpha_2$, we can write the ratio of lengths as:

$$\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}$$

Step 4: Final Answer:

The required condition is $\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}$.

 Quick Tip

This concept is practically used in constructing compensated pendulums where the effective length (and thus time period) remains constant despite temperature variations.

12. Three samples X, Y , and Z of same gas have equal volumes and temperatures. The volume of each sample is doubled, the process being isothermal for X , adiabatic for Y and isobaric for Z . If the final pressures are equal for the three samples, the ratio of the initial pressures is ($\gamma = 3/2$)

- (A) $1 : \sqrt{2} : 2\sqrt{3}$
- (B) $2 : 2\sqrt{2} : 1$
- (C) $3 : 3\sqrt{3} : 1$
- (D) $5 : 5\sqrt{5} : 1$

Correct Answer: (B) $2 : 2\sqrt{2} : 1$

Solution:

Step 1: Understanding the Concept:

This problem compares different thermodynamic processes: isothermal, adiabatic, and isobaric. We need to use the equation of state for each process to relate initial and final states.

Step 2: Key Formulas or Approach:

Isothermal process equation: $P_i V_i = P_f V_f$.

Adiabatic process equation: $P_i V_i^\gamma = P_f V_f^\gamma$.

Isobaric process equation: $P = \text{constant}$.

Step 3: Detailed Explanation:

Let the initial volumes be $V_i = V$ for all samples.

Let the final volume be $V_f = 2V$ for all samples.

Let the final pressure be $P_f = P$ for all samples, as given in the problem.

Let's find the initial pressure for each gas sample.

For sample X (Isothermal process):

Using the equation $P_{X,i} V_i = P_{X,f} V_f$:

$$P_{X,i} \cdot V = P \cdot (2V)$$

Solving for initial pressure:

$$P_{X,i} = 2P$$

For sample Y (Adiabatic process):

Using the equation $P_{Y,i} V_i^\gamma = P_{Y,f} V_f^\gamma$, with $\gamma = \frac{3}{2}$:

$$P_{Y,i} \cdot V^{3/2} = P \cdot (2V)^{3/2}$$

$$P_{Y,i} \cdot V^{3/2} = P \cdot 2^{3/2} \cdot V^{3/2}$$

Cancel $V^{3/2}$ from both sides:

$$P_{Y,i} = P \cdot 2^{3/2}$$

Since $2^{3/2} = (\sqrt{2})^3 = 2\sqrt{2}$:

$$P_{Y,i} = 2\sqrt{2}P$$

For sample Z (Isobaric process):

In an isobaric process, pressure remains constant throughout.

$$P_{Z,i} = P_{Z,f} = P$$

Now, calculate the ratio of their initial pressures $P_{X,i} : P_{Y,i} : P_{Z,i}$:

$$\text{Ratio} = 2P : 2\sqrt{2}P : P$$

Divide by the common factor P :

$$\text{Ratio} = 2 : 2\sqrt{2} : 1$$

Step 4: Final Answer:

The ratio of the initial pressures is $2 : 2\sqrt{2} : 1$.

 Quick Tip

In expansion from same initial state, the final pressure is highest for isobaric, lower for isothermal, and lowest for adiabatic. Working backward from the same final state reverses this order.

13. Let 'W' joule be the work done to move an electric charge 'q' coulomb from a place A, where potential is -5 volt to another place B where potential is 'V' volt. The value of 'V' is

- (A) $Wq - 5$
- (B) $\frac{q}{W} + 5$
- (C) $W - \frac{5}{q}$
- (D) $\frac{W}{q} - 5$

Correct Answer: (D) $\frac{W}{q} - 5$

Solution:

Step 1: Understanding the Concept:

The work done in moving a charge between two points in an electric field is directly related to the potential difference between those two points.

Step 2: Key Formula or Approach:

The work done W by an external force to move a charge q from point A to point B is given by the equation:

$$W = q \cdot \Delta V = q(V_B - V_A)$$

where V_A is the potential at the starting point and V_B is the potential at the final point.

Step 3: Detailed Explanation:

We are given the following values:

Work done = W

Charge moved = q

Potential at initial place A , $V_A = -5 \text{ V}$

Potential at final place B , $V_B = V$

Substitute these values into the formula:

$$W = q(V - (-5))$$

Simplify the expression inside the parenthesis:

$$W = q(V + 5)$$

Now, we need to solve for the final potential V .

Divide both sides by q :

$$\frac{W}{q} = V + 5$$

Subtract 5 from both sides to isolate V :

$$V = \frac{W}{q} - 5$$

Step 4: Final Answer:

The value of V is $\frac{W}{q} - 5$.

 Quick Tip

Always be careful with the order of potentials in the formula: Work done = Charge $\times$ (Final Potential - Initial Potential).

14. The plates of a parallel plate capacitor are separated by a distance ' d ' with air as the medium between them. A dielectric slab of dielectric constant 3 is introduced between the plates so as to increase the capacity by 50%. The thickness of the dielectric slab is

- (A) $\frac{d}{2}$
- (B) $\frac{d}{3}$
- (C) $\frac{d}{5}$
- (D) $\frac{5d}{6}$

Correct Answer: (A) $\frac{d}{2}$

Solution:

Step 1: Understanding the Concept:

Introducing a dielectric slab between the plates of a capacitor increases its capacitance.

The new capacitance depends on the dielectric constant of the slab and its thickness relative

to the plate separation.

Step 2: Key Formulas or Approach:

Initial capacitance with air: $C_0 = \frac{\epsilon_0 A}{d}$.

Capacitance with a dielectric slab of thickness t and dielectric constant K : $C = \frac{\epsilon_0 A}{d-t+\frac{t}{K}}$.

Step 3: Detailed Explanation:

The problem states that the capacity increases by 50%.

This means the new capacitance is $C = C_0 + 0.5C_0 = 1.5C_0 = \frac{3}{2}C_0$.

Substitute the expression for C_0 :

$$C = \frac{3}{2} \left(\frac{\epsilon_0 A}{d} \right)$$

We are given the dielectric constant $K = 3$.

Now, use the formula for capacitance with a partially filled dielectric:

$$\frac{\epsilon_0 A}{d-t+\frac{t}{3}} = \frac{3}{2} \frac{\epsilon_0 A}{d}$$

Cancel the common term $\epsilon_0 A$ from both sides:

$$\frac{1}{d-t+\frac{t}{3}} = \frac{3}{2d}$$

Simplify the denominator on the left side: $-t + \frac{t}{3} = \frac{-3t+t}{3} = \frac{-2t}{3}$.

$$\frac{1}{d-\frac{2t}{3}} = \frac{3}{2d}$$

Cross-multiply to solve for t :

$$2d = 3 \left(d - \frac{2t}{3} \right)$$

Expand the right side:

$$\begin{aligned} 2d &= 3d - 3 \left(\frac{2t}{3} \right) \\ 2d &= 3d - 2t \end{aligned}$$

Rearrange to isolate t :

$$\begin{aligned} 2t &= 3d - 2d \\ 2t &= d \\ t &= \frac{d}{2} \end{aligned}$$

Step 4: Final Answer:

The thickness of the dielectric slab is $\frac{d}{2}$.

 Quick Tip

A partially filled dielectric acts like two capacitors in series: an air capacitor of thickness $(d - t)$ and a dielectric capacitor of thickness t .

15. Two long parallel wires carrying currents 4 A and 3 A in opposite directions are placed at a distance of 5 cm from each other. A point P is at equidistance from both the wires such that the line joining the point P to the wires are perpendicular to each other. The magnitude of magnetic field at point P is (μ_0 = permeability of free space = $4\pi \times 10^{-7}$ SI unit)

- (A) 4×10^{-5} T
- (B) $\sqrt{2} \times 10^{-5}$ T
- (C) 2×10^{-5} T
- (D) $2\sqrt{2} \times 10^{-5}$ T

Correct Answer: (D) $2\sqrt{2} \times 10^{-5}$ T

Solution:

Step 1: Understanding the Concept:

The magnetic field produced by a long straight wire forms concentric circles around the wire. The net magnetic field at a point due to multiple wires is the vector sum of the individual magnetic fields produced by each wire.

Step 2: Key Formula or Approach:

The magnitude of the magnetic field due to a long straight wire at a distance r is $B = \frac{\mu_0 I}{2\pi r}$. We will use vector addition to find the resultant field at point P .

Step 3: Detailed Explanation:

Let the two wires be at positions A and B. The distance between them is $d = AB = 5$ cm.

Point P is equidistant from both wires, so $AP = BP = r$.

We are given that the lines AP and BP are perpendicular to each other, forming a right-angled triangle $\triangle APB$ with the right angle at P .

Using Pythagoras theorem on $\triangle APB$:

$$\begin{aligned} AP^2 + BP^2 &= AB^2 \\ r^2 + r^2 &= 5^2 \\ 2r^2 &= 25 \\ r^2 &= \frac{25}{2} \implies r = \frac{5}{\sqrt{2}} \text{ cm} = \frac{5}{\sqrt{2}} \times 10^{-2} \text{ m} \end{aligned}$$

Let's find the magnetic fields at P .

Magnetic field due to wire A ($I_1 = 4$ A): $B_1 = \frac{\mu_0 I_1}{2\pi r}$.

The direction of $\vec{B}_1$ is perpendicular to the line AP and lies in the plane of the triangle.

Magnetic field due to wire B ($I_2 = 3$ A): $B_2 = \frac{\mu_0 I_2}{2\pi r}$.

The direction of $\vec{B}_2$ is perpendicular to the line BP .

Since $AP \perp BP$, the magnetic field vectors $\vec{B}_1$ and $\vec{B}_2$ are also perpendicular to each other.

The magnitude of the resultant magnetic field B is:

$$\begin{aligned} B &= \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0 I_1}{2\pi r}\right)^2 + \left(\frac{\mu_0 I_2}{2\pi r}\right)^2} \\ B &= \frac{\mu_0}{2\pi r} \sqrt{I_1^2 + I_2^2} \end{aligned}$$

Substitute the known values:

$$B = \frac{4\pi \times 10^{-7}}{2\pi \times \frac{5}{\sqrt{2}} \times 10^{-2}} \sqrt{4^2 + 3^2}$$

$$B = \frac{2 \times 10^{-7}}{\frac{5}{\sqrt{2}} \times 10^{-2}} \times \sqrt{16 + 9}$$

$$B = \frac{2\sqrt{2} \times 10^{-5}}{5} \times \sqrt{25}$$

$$B = \frac{2\sqrt{2} \times 10^{-5}}{5} \times 5$$

$$B = 2\sqrt{2} \times 10^{-5} \text{ T}$$

The direction of currents being opposite doesn't change the fact that the two field vectors are orthogonal in this specific geometry.

Step 4: Final Answer:

The magnitude of magnetic field at point P is $2\sqrt{2} \times 10^{-5} \text{ T}$.

 Quick Tip

When field vectors are mutually perpendicular, their resultant magnitude can be found simply using the Pythagorean theorem, significantly simplifying the calculation.

16. If only 5% of the total current is to be passed through galvanometer of resistance G , then the resistance of the shunt will be

- (A) $\frac{G}{15}$
- (B) $\frac{G}{17}$
- (C) $\frac{G}{19}$
- (D) $\frac{G}{21}$

Correct Answer: (C) $\frac{G}{19}$

Solution:

Step 1: Understanding the Concept:

A galvanometer is converted into an ammeter by connecting a low resistance, called a shunt, in parallel with it.

Because they are in parallel, the potential difference across the galvanometer and the shunt must be equal.

Step 2: Key Formula or Approach:

The potential difference equality gives: $I_g \times G = I_s \times S$, where:

I_g is the current through the galvanometer.

G is the resistance of the galvanometer.

I_s is the current through the shunt.

S is the resistance of the shunt.

Total current $I = I_g + I_s$.

Step 3: Detailed Explanation:

Let the total current be I .

It is given that only 5% of the total current passes through the galvanometer.

Therefore, the galvanometer current is $I_g = 5\%$ of $I = \frac{5}{100} \times I = \frac{I}{20}$.

The remaining current must pass through the shunt.

The shunt current is $I_s = I - I_g = I - \frac{I}{20} = \frac{19I}{20}$.

Now apply the parallel potential difference condition:

$$I_g \times G = I_s \times S$$

Substitute the expressions for I_g and I_s :

$$\left(\frac{I}{20}\right) \times G = \left(\frac{19I}{20}\right) \times S$$

Cancel the common terms I and 20 from both sides:

$$1 \times G = 19 \times S$$

Rearrange to find the shunt resistance S :

$$S = \frac{G}{19}$$

Step 4: Final Answer:

The resistance of the shunt must be $\frac{G}{19}$.

 Quick Tip

If $\frac{1}{n}$ of the main current passes through the galvanometer, the required shunt resistance is always $S = \frac{G}{n-1}$. Here $n = 20$, so $S = \frac{G}{19}$.

17. The self-inductance of a circuit is numerically equal to

- (A) the work done in establishing the magnetic flux associated with circuit.
- (B) twice the work done in establishing the magnetic flux associated with unit current in the circuit.
- (C) thrice the work done in establishing the magnetic flux associated with unit current in the circuit.
- (D) the work done in establishing the magnetic flux associated with unit current in the circuit.

Correct Answer: (B) twice the work done in establishing the magnetic flux associated with unit current in the circuit.

Solution:

Step 1: Understanding the Concept:

Self-inductance (L) is a property of a coil by which it opposes a change in current flowing through it.

When a steady current is established in a circuit, work has to be done against the induced EMF.

This work is stored as magnetic potential energy in the inductor.

Step 2: Key Formula or Approach:

The magnetic energy stored (which is the work done, W) in an inductor carrying a current I is given by the formula:

$$W = \frac{1}{2}LI^2.$$

Step 3: Detailed Explanation:

From the energy formula, we can express self-inductance L in terms of work done W and current I :

$$L = \frac{2W}{I^2}$$

The question asks for a situation where self-inductance is numerically related to the work done. Let's analyze the options based on a "unit current".

Assume a unit current is established in the circuit, so $I = 1$ unit.

Substitute $I = 1$ into the energy formula:

$$W = \frac{1}{2}L(1)^2 = \frac{1}{2}L$$

Rearranging this to solve for L :

$$L = 2W$$

This means that numerically, the self-inductance L is equal to two times the work done W to establish a unit current.

Comparing this with the given options:

Option (A) is incomplete as it doesn't specify unit current.

Option (B) exactly matches our derivation: twice the work done for unit current.

Option (C) says thrice, which is incorrect.

Option (D) says equal to work done, which would mean $L = W$, missing the factor of 2.

Step 4: Final Answer:

The self-inductance is numerically equal to twice the work done in establishing the magnetic flux associated with unit current.

 Quick Tip

Remembering energy formulas often provides a direct path to defining physical quantities, such as $C = 2U/V^2$ or $L = 2U/I^2$.

18. Two conducting circular loops of radii R_1 and R_2 are placed in the same plane with their centres coinciding. If $R_1 > R_2$, the mutual inductance M between them will be directly proportional to

- (A) $\frac{R_1}{R_2}$
- (B) $\frac{R_2}{R_1}$
- (C) $\frac{R_1^2}{R_2}$
- (D) $\frac{R_2^2}{R_1}$

Correct Answer: (D) $\frac{R_2^2}{R_1}$

Solution:

Step 1: Understanding the Concept:

Mutual inductance is the property of two coils such that a change in current in one induces an EMF in the other.

To find mutual inductance, we pass a current through one loop, calculate the magnetic field it produces, and then find the magnetic flux linked with the second loop.

Step 2: Key Formulas or Approach:

Magnetic field at the center of a circular loop: $B = \frac{\mu_0 I}{2R}$.

Magnetic flux: $\Phi = B \times A$, where A is the area.

Mutual inductance definition: $M = \frac{\Phi_2}{I_1}$.

Step 3: Detailed Explanation:

Let's pass a current I_1 through the larger outer loop of radius R_1 .

The magnetic field produced by this loop at its center is:

$$B_1 = \frac{\mu_0 I_1}{2R_1}$$

Since $R_1 > R_2$ (typically implying $R_1 \gg R_2$ in such standard textbook problems for the formula to be highly accurate, but the proportionality holds under the approximation that the field is roughly uniform over the small central area), we assume this field B_1 is uniform over the area of the smaller inner loop.

The area of the inner loop is $A_2 = \pi R_2^2$.

The magnetic flux Φ_2 linked with the smaller inner loop due to the field of the outer loop is:

$$\begin{aligned}\Phi_2 &= B_1 \times A_2 \\ \Phi_2 &= \left(\frac{\mu_0 I_1}{2R_1} \right) \times (\pi R_2^2) \\ \Phi_2 &= \left(\frac{\mu_0 \pi R_2^2}{2R_1} \right) I_1\end{aligned}$$

From the definition of mutual inductance $\Phi_2 = M \cdot I_1$, we get:

$$M = \frac{\mu_0 \pi R_2^2}{2R_1}$$

Looking at the terms, μ_0 and π and 2 are constants.

Therefore, the mutual inductance M is directly proportional to $\frac{R_2^2}{R_1}$.

$$M \propto \frac{R_2^2}{R_1}$$

Step 4: Final Answer:

The mutual inductance M is directly proportional to $\frac{R_2^2}{R_1}$.

💡 Quick Tip

Always pass the assumed current through the larger coil to calculate mutual inductance. Calculating flux through a large coil from a small coil's non-uniform field is mathematically very complex.

19. A series LCR circuit is connected to an a.c. source of 230 V, 50 Hz. The circuit contains resistance of 80Ω an inductor having inductive reactance 70Ω and a capacitor of capacitive reactance 130Ω . The power factor of the circuit is x . The value of x is

- (A) 0.4
- (B) 0.8
- (C) 0.6
- (D) 0.9

Correct Answer: (B) 0.8

Solution:**Step 1: Understanding the Concept:**

The power factor of an AC circuit is defined as the cosine of the phase angle between voltage and current.

In a series LCR circuit, it can be calculated as the ratio of true resistance to the total impedance.

Step 2: Key Formulas or Approach:

Net reactance: $X = X_L \sim X_C$.

Impedance of the LCR series circuit: $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

Power factor: $\cos \phi = \frac{R}{Z}$.

Step 3: Detailed Explanation:

The given parameters of the circuit are:

Resistance $R = 80\Omega$

Inductive reactance $X_L = 70\Omega$

Capacitive reactance $X_C = 130\Omega$

First, calculate the net reactance.

Since $X_C > X_L$, the circuit is capacitive.

Net reactance $X = |X_L - X_C| = |70\Omega - 130\Omega| = |-60\Omega| = 60\Omega$.

Next, calculate the total impedance Z of the circuit:

$$\begin{aligned} Z &= \sqrt{R^2 + X^2} \\ Z &= \sqrt{80^2 + 60^2} \\ Z &= \sqrt{6400 + 3600} \end{aligned}$$

$$Z = \sqrt{10000}$$

$$Z = 100\Omega$$

Now, calculate the power factor ($\cos \phi$), which is given as x :

$$x = \cos \phi = \frac{R}{Z}$$

Substitute the values of R and Z :

$$x = \frac{80}{100}$$

$$x = 0.8$$

The values for voltage (230 V) and frequency (50 Hz) are not needed as the reactances are already provided directly.

Step 4: Final Answer:

The value of the power factor x is 0.8.

Quick Tip

Recognize Pythagorean triplets like (3, 4, 5) or their multiples (60, 80, 100) to speed up impedance calculations without writing out full squares.

20. When three inductors of same inductance ' L ' are connected in series and ' I ' is the current passing through the circuit. The energy stored in the circuit is

- (A) $\frac{1}{2}LI^2$
- (B) $\frac{3}{2}LI^2$
- (C) $\frac{5}{2}LI^2$
- (D) $\frac{7}{2}LI^2$

Correct Answer: (B) $\frac{3}{2}LI^2$

Solution:

Step 1: Understanding the Concept:

When inductors are connected in series, their equivalent inductance adds up, just like resistors in series (assuming no mutual inductance).

The total magnetic energy stored in an inductive circuit depends on the equivalent inductance and the current flowing through it.

Step 2: Key Formulas or Approach:

Equivalent inductance in series: $L_{\text{eq}} = L_1 + L_2 + L_3 + \dots$

Energy stored in an inductor: $U = \frac{1}{2}L_{\text{eq}}I^2$.

Step 3: Detailed Explanation:

We are given three inductors, each with an inductance of L .

They are connected in series.

The equivalent inductance of the combination is:

$$L_{\text{eq}} = L + L + L = 3L$$

A steady current I passes through the entire series combination.

The total energy stored in the circuit is calculated using the equivalent inductance:

$$U = \frac{1}{2}L_{\text{eq}}I^2$$

Substitute $L_{\text{eq}} = 3L$ into the formula:

$$U = \frac{1}{2}(3L)I^2$$

$$U = \frac{3}{2}LI^2$$

Alternatively, energy is scalar and additive. Each inductor stores $\frac{1}{2}LI^2$. Total energy = $3 \times (\frac{1}{2}LI^2) = \frac{3}{2}LI^2$.

Step 4: Final Answer:

The energy stored in the circuit is $\frac{3}{2}LI^2$.

 Quick Tip

Energy stored in components (like capacitors and inductors) is an additive scalar quantity. You can find equivalent values first, or simply sum the individual energies.

21. For a thin prism, δ_1 is the angle of deviation produced, when prism is placed in air. When the prism is immersed in water, the angle of deviation produced is δ_2 . Given ${}_a\mu_g = \frac{3}{2}$ and ${}_a\mu_w = \frac{4}{3}$. The ratio $\delta_2 : \delta_1$ is

- (A) 1 : 2
- (B) 1 : 4
- (C) 1 : 8
- (D) 4 : 1

Correct Answer: (B) 1 : 4

Solution:

Step 1: Understanding the Concept:

A thin prism deviates a light ray by an angle that depends on its refracting angle and the relative refractive index of the prism material with respect to the surrounding medium.

When immersed in a liquid, the relative refractive index decreases, resulting in a smaller angle of deviation.

Step 2: Key Formulas or Approach:

Angle of deviation for a thin prism: $\delta = (\mu_{\text{relative}} - 1)A$.

Relative refractive index: ${}_{\text{med}}\mu_{\text{prism}} = \frac{\mu_{\text{prism}}}{\mu_{\text{med}}}$.

Step 3: Detailed Explanation:

Let A be the refracting angle of the prism.

Case 1: Prism in air

The relative refractive index is ${}_a\mu_g = \frac{3}{2}$.

The deviation in air, δ_1 , is:

$$\begin{aligned}\delta_1 &= ({}_a\mu_g - 1)A \\ \delta_1 &= \left(\frac{3}{2} - 1\right)A = \left(\frac{3-2}{2}\right)A = \frac{1}{2}A\end{aligned}$$

Case 2: Prism immersed in water

The surrounding medium is now water. The relative refractive index of glass with respect to water is:

$${}_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{{}_a\mu_g}{{}_a\mu_w}$$

Given ${}_a\mu_g = \frac{3}{2}$ and ${}_a\mu_w = \frac{4}{3}$:

$${}_w\mu_g = \frac{3/2}{4/3} = \frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$$

The deviation in water, δ_2 , is:

$$\begin{aligned}\delta_2 &= ({}_w\mu_g - 1)A \\ \delta_2 &= \left(\frac{9}{8} - 1\right)A = \left(\frac{9-8}{8}\right)A = \frac{1}{8}A\end{aligned}$$

Finding the ratio:

Now, calculate the ratio $\frac{\delta_2}{\delta_1}$:

$$\frac{\delta_2}{\delta_1} = \frac{\frac{1}{8}A}{\frac{1}{2}A}$$

Cancel A and simplify the fraction:

$$\frac{\delta_2}{\delta_1} = \frac{1/8}{1/2} = \frac{1}{8} \times \frac{2}{1} = \frac{2}{8} = \frac{1}{4}$$

Therefore, $\delta_2 : \delta_1 = 1 : 4$.

Step 4: Final Answer:

The ratio of deviation in water to air is $1 : 4$.

 Quick Tip

Immersing optical components (like lenses or prisms) in a denser medium always reduces their refractive power or deviation ability because the relative refractive index approaches 1.

22. If ' λ_1 ' and ' λ_2 ' are the wavelengths of the first member of the Balmer and Paschen series, in hydrogen atom respectively, then the ratio of respective frequencies, f_1/f_2 , is

- (A) 20 : 7
- (B) 27 : 5
- (C) 50 : 9
- (D) 108 : 7

Correct Answer: (A) 20 : 7

Solution:

Step 1: Understanding the Concept:

The spectral lines of a hydrogen atom are grouped into series based on the lower energy level involved in the transition.

Frequency and wavelength are inversely related: $f = c/\lambda$.

Step 2: Key Formula or Approach:

The Rydberg formula gives the reciprocal of wavelength: $\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$.

Since frequency $f = \frac{c}{\lambda}$, we have $f = cR \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$.

Step 3: Detailed Explanation:

For the Balmer series:

The final state is $n_f = 2$.

The first member corresponds to a transition from the nearest upper state, so initial state $n_i = 3$.

Let its frequency be f_1 :

$$\begin{aligned} f_1 &= cR \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = cR \left(\frac{1}{4} - \frac{1}{9} \right) \\ f_1 &= cR \left(\frac{9-4}{36} \right) = cR \left(\frac{5}{36} \right) \end{aligned}$$

For the Paschen series:

The final state is $n_f = 3$.

The first member corresponds to a transition from the nearest upper state, so initial state $n_i = 4$.

Let its frequency be f_2 :

$$\begin{aligned} f_2 &= cR \left(\frac{1}{3^2} - \frac{1}{4^2} \right) = cR \left(\frac{1}{9} - \frac{1}{16} \right) \\ f_2 &= cR \left(\frac{16-9}{144} \right) = cR \left(\frac{7}{144} \right) \end{aligned}$$

Finding the ratio:

Now, calculate the ratio $\frac{f_1}{f_2}$:

$$\frac{f_1}{f_2} = \frac{cR(5/36)}{cR(7/144)}$$

Cancel the common terms cR :

$$\frac{f_1}{f_2} = \frac{5/36}{7/144}$$

$$\frac{f_1}{f_2} = \frac{5}{36} \times \frac{144}{7}$$

Since $144 = 36 \times 4$:

$$\frac{f_1}{f_2} = 5 \times \frac{4}{7}$$

$$\frac{f_1}{f_2} = \frac{20}{7}$$

Step 4: Final Answer:

The ratio of respective frequencies is 20 : 7.

💡 Quick Tip

The "first member" of a series always involves a transition from the immediately adjacent higher energy level ($n_i = n_f + 1$). It corresponds to the longest wavelength and lowest frequency in that series.

23. The ratio of angular momentum L of an electron to the magnetic dipole moment $\vec{m}_{\text{orb}}$ is (' m ' is mass of electron, ' e ' is charge on electron)

- (A) $\frac{e}{m}$
- (B) $\frac{2m}{e}$
- (C) $\frac{e}{2m}$
- (D) $\frac{m}{e}$

Correct Answer: (B) $\frac{2m}{e}$

Solution:

Step 1: Understanding the Concept:

An electron orbiting a nucleus behaves like a tiny current loop, which produces a magnetic dipole moment.

The orbiting electron also possesses angular momentum.

The ratio of these two quantities is a fundamental constant for an orbiting charged particle.

Step 2: Key Formulas or Approach:

Angular momentum of a particle in circular motion: $L = mvr$.

Magnetic dipole moment of a current loop: $m_{\text{orb}} = I \times A$.

Current produced by an orbiting electron: $I = \frac{e}{T}$, where T is the time period.

Step 3: Detailed Explanation:

Consider an electron of mass m and charge e moving in a circular orbit of radius r with speed v .

The angular momentum is $L = mvr$.

The time period of revolution is $T = \frac{2\pi r}{v}$.

The equivalent current is $I = \frac{e}{T} = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}$.

The area of the circular loop is $A = \pi r^2$.

The magnetic dipole moment magnitude is:

$$m_{\text{orb}} = I \cdot A = \left(\frac{ev}{2\pi r} \right) \cdot (\pi r^2)$$

Simplify the expression:

$$m_{\text{orb}} = \frac{evr}{2}$$

We are asked to find the ratio of angular momentum L to magnetic dipole moment m_{orb} .

$$\text{Ratio} = \frac{L}{m_{\text{orb}}}$$

Substitute the expressions for L and m_{orb} :

$$\text{Ratio} = \frac{mvr}{\frac{evr}{2}}$$

Cancel the common terms v and r :

$$\text{Ratio} = \frac{m}{\frac{e}{2}}$$

$$\text{Ratio} = \frac{2m}{e}$$

Note: The inverse ratio, $\frac{m_{\text{orb}}}{L} = \frac{e}{2m}$, is known as the gyromagnetic ratio.

Step 4: Final Answer:

The ratio of angular momentum to magnetic dipole moment is $\frac{2m}{e}$.

 Quick Tip

Read the question carefully to see which ratio is asked. Is it magnetic moment to angular momentum (gyromagnetic ratio, $e/2m$) or its reciprocal ($2m/e$).

24. Black sphere of radius R radiates power P at certain temperature T . If the temperature is doubled, the radius gets doubled. Now the power radiated would be

- (A) 4 P
- (B) 8 P
- (C) 16 P
- (D) 64 P

Correct Answer: (D) 64 P

Solution:

Step 1: Understanding the Concept:

The power radiated by a black body depends on its surface area and its absolute temperature. This relationship is given by the Stefan-Boltzmann law.

Step 2: Key Formula or Approach:

Stefan-Boltzmann Law: Total radiated power $P = \sigma AT^4$.

For a sphere, the surface area is $A = 4\pi R^2$.

Therefore, $P = \sigma(4\pi R^2)T^4$, which means $P \propto R^2T^4$.

Step 3: Detailed Explanation:

Let the initial conditions be:

Radius = R

Temperature = T

Initial radiated power $P_1 = P \propto R^2T^4$.

According to the problem, the new conditions are:

New temperature $T' = 2T$

New radius $R' = 2R$

Let the new radiated power be P_2 .

Using the proportionality relation:

$$P_2 \propto (R')^2(T')^4$$

Substitute the new values:

$$P_2 \propto (2R)^2(2T)^4$$

Expand the terms:

$$P_2 \propto (4R^2) \cdot (16T^4)$$

$$P_2 \propto 64 \cdot R^2T^4$$

Now, compare P_2 with the initial power P_1 :

$$\frac{P_2}{P_1} = \frac{64R^2T^4}{R^2T^4} = 64$$

Therefore, $P_2 = 64P_1$.

Since $P_1 = P$, the new power is 64 P.

Step 4: Final Answer:

The power radiated would be 64 P.

 Quick Tip

In proportionality problems involving multiple changing variables, apply the changes simultaneously: $(2)^2 \times (2)^4 = 4 \times 16 = 64$.

25. In the depletion layer of reverse biased p-n junction, the

- (A) electric field is very small.
- (B) potential is maximum.
- (C) electric field is maximum.
- (D) potential is zero.

Correct Answer: (C) electric field is maximum.

Solution:

Step 1: Understanding the Concept:

A p-n junction has a region near the interface depleted of mobile charge carriers, called the depletion layer. Fixed, immobile ions remain.

These immobile ions create an internal built-in electric field.

When reverse-biased, an external voltage is applied in the same direction as the built-in potential.

Step 2: Detailed Explanation:

Let's analyze what happens in a reverse-biased p-n junction.

The external voltage pulls majority carriers further away from the junction, widening the depletion region.

The total potential difference across the junction becomes the sum of the built-in potential (V_0) and the applied reverse voltage (V_R).

Electric field E is related to potential V by $E = -\frac{dV}{dx}$.

Since the potential changes across the depletion layer (from one side to the other), there is an electric field.

In the neutral p and n regions outside the depletion layer, the electric field is essentially zero because they are conductive and there is no significant space charge.

Therefore, the electric field exists exclusively within the depletion layer.

At the exact metallurgical junction interface, the space charge density changes from negative (on p-side) to positive (on n-side).

According to Gauss's law, the electric field reaches its peak magnitude exactly at this junction interface.

Because the reverse bias increases the voltage drop across this layer, the peak electric field in the depletion layer is very high, often reaching values near the breakdown field strength of the semiconductor.

Thus, compared to the rest of the device, the electric field is maximum within the depletion layer.

Let's evaluate the other options:

(A) Incorrect. The electric field is very strong, not small.

(B) Incorrect. The potential varies continuously from one side to the other, so it's not simply "maximum" throughout the layer.

(D) Incorrect. The potential is not zero; there is a large potential difference across it.

Step 3: Final Answer:

In the depletion layer of a reverse biased p-n junction, the electric field is maximum.

 Quick Tip

The depletion region behaves like a parallel plate capacitor. A large voltage drop across a very small distance creates a highly intense electric field.

26. For a common emitter transistor, if $\frac{I_C}{I_E} = 0.95$, then the current gain is

- (A) 47.5
- (B) 44
- (C) 19
- (D) 15

Correct Answer: (C) 19

Solution:

Step 1: Understanding the Concept:

In a transistor, the current gain in common base configuration (α) is the ratio of collector current to emitter current.

The current gain in common emitter configuration (β) is the ratio of collector current to base current.

These two parameters are mathematically related through the current conservation equation $I_E = I_B + I_C$.

Step 2: Key Formula or Approach:

The given ratio is $\alpha = \frac{I_C}{I_E} = 0.95$.

The relation between β and α is:

$$\beta = \frac{\alpha}{1 - \alpha}$$

Step 3: Detailed Explanation:

Substitute the given value of $\alpha = 0.95$ into the formula for β :

$$\beta = \frac{0.95}{1 - 0.95}$$

Perform the subtraction in the denominator:

$$\beta = \frac{0.95}{0.05}$$

Simplify the fraction by multiplying numerator and denominator by 100:

$$\beta = \frac{95}{5} = 19$$

Thus, the current gain in common emitter configuration is 19.

Step 4: Final Answer:

The current gain of the transistor in common emitter mode is 19.

 Quick Tip

Remember that α is always less than 1 (usually 0.95 to 0.99), whereas β is always much greater than 1. If α is very close to 1, β will be very high.

27. A string of mass 0.1 kg m^{-1} has length 0.9 m . It is fixed at both ends and stretched such that it has a tension of 40 N . The string vibrates in three segments with amplitude 0.3 cm . The amplitude (maximum) of the particle velocity is (in m/s)

- (A) $\frac{\pi}{2}$
- (B) $\frac{\pi}{3}$
- (C) $\frac{\pi}{5}$
- (D) $\frac{\pi}{6}$

Correct Answer: (C) $\frac{\pi}{5}$

Solution:

Step 1: Understanding the Concept:

When a string fixed at both ends vibrates in stationary waves, particles perform SHM.

The maximum velocity of a particle in SHM is $v_{\max} = A\omega$, where A is the amplitude of the particle and ω is the angular frequency of the wave.

Step 2: Key Formula or Approach:

Speed of a transverse wave on a string: $v = \sqrt{\frac{T}{\mu}}$

Fundamental frequency: $f_1 = \frac{v}{2L}$. For p segments, frequency is $f_p = \frac{p \cdot v}{2L}$.

Angular frequency: $\omega = 2\pi f$.

Step 3: Detailed Explanation:

Given: $\mu = 0.1 \text{ kg m}^{-1}$, $L = 0.9 \text{ m}$, $T = 40 \text{ N}$, $p = 3$ segments, $A = 0.3 \text{ cm} = 3 \times 10^{-3} \text{ m}$.

First, find wave speed v :

$$v = \sqrt{\frac{40}{0.1}} = \sqrt{400} = 20 \text{ m/s}$$

Now, find the vibration frequency f for 3 segments:

$$f = \frac{3v}{2L} = \frac{3 \times 20}{2 \times 0.9} = \frac{60}{1.8} = \frac{600}{18} = \frac{100}{3} \text{ Hz}$$

Calculate angular frequency ω :

$$\omega = 2\pi f = 2\pi \times \frac{100}{3} = \frac{200\pi}{3} \text{ rad/s}$$

Finally, calculate the maximum particle velocity:

$$v_{\text{particle, max}} = A \cdot \omega = (3 \times 10^{-3}) \times \frac{200\pi}{3}$$

$$v_{\text{particle, max}} = 10^{-3} \times 200\pi = 0.2\pi = \frac{2\pi}{10} = \frac{\pi}{5} \text{ m/s}$$

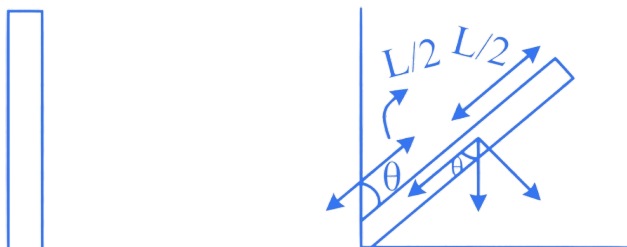
Step 4: Final Answer:

The maximum velocity of the particle is $\frac{\pi}{5}$ m/s.

💡 Quick Tip

For a string vibrating in p loops, the total length $L = p \cdot (\lambda/2)$. This relation helps you derive the frequency formula quickly if you forget it.

28. A thin uniform rod of mass ' m ' and length ' L ' is pivoted at one end so that it can rotate in a vertical plane. The free end is held vertically above pivot and then released. The angular acceleration of the rod when it makes an angle ' θ ' with the vertical is [consider negligible friction at the pivot] (g = acceleration due to gravity)



- (A) $\frac{3g \sin \theta}{2L}$
 (B) $\frac{3g \cos \theta}{2L}$
 (C) $\frac{2g \sin \theta}{3L}$
 (D) $\frac{2g \cos \theta}{3L}$

Correct Answer: (A) $\frac{3g \sin \theta}{2L}$

Solution:**Step 1: Understanding the Concept:**

When the rod is released, gravity exerts a torque about the pivot point.

This torque results in an angular acceleration according to Newton's second law for rotation:
 $\tau = I\alpha$.

Step 2: Key Formula or Approach:

Torque: $\tau = r \times F = rF \sin \phi$, where r is distance from pivot to center of mass and ϕ is angle between r and F .

Moment of inertia of a rod about its end: $I = \frac{1}{3}mL^2$.

Step 3: Detailed Explanation:

The weight of the rod acts at its center of mass, which is at a distance $L/2$ from the pivot.

When the rod makes an angle θ with the vertical, the angle between the position vector $\vec{r}$ of the center of mass and the gravitational force $m\vec{g}$ is θ .

The magnitude of the torque is:

$$\tau = (L/2) \cdot mg \cdot \sin \theta$$

Applying $\tau = I\alpha$:

$$\frac{1}{3}mL^2 \cdot \alpha = mg \cdot \frac{L}{2} \cdot \sin \theta$$

Cancel m and one L from both sides:

$$\frac{1}{3}L \cdot \alpha = \frac{g}{2} \sin \theta$$

Solve for angular acceleration α :

$$\alpha = \frac{3}{L} \cdot \frac{g \sin \theta}{2} = \frac{3g \sin \theta}{2L}$$

Step 4: Final Answer:

The angular acceleration is $\frac{3g \sin \theta}{2L}$.

💡 Quick Tip

The torque is maximum when the rod is horizontal ($\theta = 90^\circ$) and zero when it is vertical. Always use the perpendicular component of force to find torque easily.

29. The molar specific heat of an ideal gas at constant pressure and constant volume is ' C_P ' and ' C_V ' respectively. If ' R ' is a universal gas constant and the ratio of ' C_P ' to ' C_V ' is γ , then ' C_P ' is equal to

- (A) $(\frac{\gamma-1}{\gamma+1})R$
- (B) $\frac{(\gamma-1)R}{\gamma}$
- (C) $\frac{R\gamma}{(\gamma-1)}$
- (D) $\frac{R\gamma}{(\gamma+1)}$

Correct Answer: (C) $\frac{R\gamma}{(\gamma-1)}$

Solution:

Step 1: Understanding the Concept:

For an ideal gas, the difference between molar specific heats is equal to the gas constant R (Mayer's relation).

The ratio of specific heats (γ) is a characteristic property of the gas related to its atomicity.

Step 2: Key Formula or Approach:

Mayer's Relation: $C_P - C_V = R$

Given ratio: $\frac{C_P}{C_V} = \gamma \implies C_V = \frac{C_P}{\gamma}$

Step 3: Detailed Explanation:

Substitute the expression for C_V into Mayer's relation:

$$C_P - \frac{C_P}{\gamma} = R$$

Factor out C_P :

$$C_P \left(1 - \frac{1}{\gamma}\right) = R$$

Simplify the term inside the parenthesis:

$$C_P \left(\frac{\gamma - 1}{\gamma} \right) = R$$

Solve for C_P :

$$C_P = \frac{R\gamma}{\gamma - 1}$$

Similarly, we can find $C_V = \frac{R}{\gamma - 1}$.

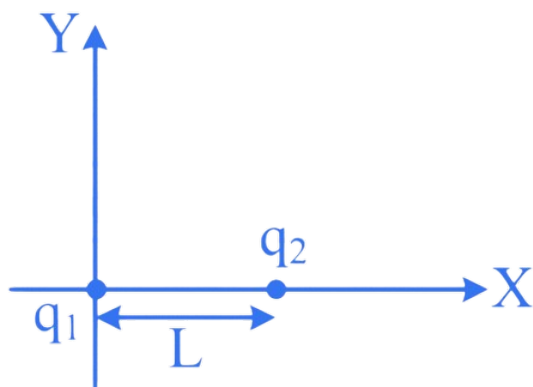
Step 4: Final Answer:

The molar specific heat at constant pressure is $C_P = \frac{R\gamma}{\gamma - 1}$.

💡 Quick Tip

Remember these two standard results: $C_V = \frac{R}{\gamma - 1}$ and $C_P = \frac{\gamma R}{\gamma - 1}$. They appear frequently in thermodynamics problems.

30. Two charges $q_1 = +6q$ and $q_2 = -3q$ placed as shown in figure. A proton is placed on x-axis away from q_2 . To remain proton in equilibrium, the distance between q_1 and proton is



- (A) $(\frac{\sqrt{2}}{\sqrt{2}-1})L$
- (B) $2L$
- (C) $\frac{L}{2}$
- (D) $(\frac{\sqrt{2}}{\sqrt{2}+1})L$

Correct Answer: (A) $(\frac{\sqrt{2}}{\sqrt{2}-1})L$

Solution:

Step 1: Understanding the Concept:

For a charge to be in equilibrium, the net electrostatic force on it must be zero.

This occurs at a point where the electric fields produced by the two fixed charges are equal in magnitude and opposite in direction.

Since the charges have opposite signs, the equilibrium point must lie on the line joining them

but outside the segment connecting them, closer to the smaller magnitude charge (q_2).

Step 2: Key Formula or Approach:

Coulomb's Law field formula: $E = \frac{kq}{r^2}$.

Let the proton be at distance x from q_1 . Then its distance from q_2 is $(x - L)$.

Step 3: Detailed Explanation:

Equating the magnitudes of the electric fields:

$$\frac{k(6q)}{x^2} = \frac{k(3q)}{(x - L)^2}$$

Cancel common terms k , q , and simplify coefficients:

$$\frac{2}{x^2} = \frac{1}{(x - L)^2}$$

Take the square root of both sides:

$$\frac{\sqrt{2}}{x} = \frac{1}{x - L}$$

Cross-multiply to solve for x :

$$\begin{aligned}\sqrt{2}(x - L) &= x \\ \sqrt{2}x - \sqrt{2}L &= x \\ x(\sqrt{2} - 1) &= \sqrt{2}L \\ x &= \frac{\sqrt{2}L}{\sqrt{2} - 1} = \left(\frac{\sqrt{2}}{\sqrt{2} - 1} \right) L\end{aligned}$$

Step 4: Final Answer:

The distance from q_1 to the proton for equilibrium is $\left(\frac{\sqrt{2}}{\sqrt{2}-1} \right) L$.

 Quick Tip

For two point charges Q_1 and Q_2 of opposite signs separated by L , the null point is at distance $x = \frac{\sqrt{|Q_1|}}{\sqrt{|Q_1|} - \sqrt{|Q_2|}} L$ from Q_1 .

31. When a ceiling fan is switched off, its angular velocity falls to $(\frac{1}{3})^{\text{rd}}$ while it makes 24 rotations. How many more rotations will it make before coming to rest?

- (A) 3
- (B) 6
- (C) 9
- (D) 12

Correct Answer: (A) 3

Solution:

Step 1: Understanding the Concept:

The fan decelerates due to frictional torque. Assuming a constant angular retardation (α), we can use equations of rotational motion.

The square of the angular velocity is linearly related to the angular displacement.

Step 2: Key Formula or Approach:

Kinematic equation: $\omega^2 = \omega_0^2 + 2\alpha\theta$.

Let 1 rotation = 2π radians. Let n be the number of rotations, so $\theta = 2\pi n$.

The formula can be written as $\omega^2 = \omega_0^2 + K \cdot n$, where K is a constant.

Step 3: Detailed Explanation:

Let initial angular velocity be ω_0 .

After $n_1 = 24$ rotations, $\omega_1 = \omega_0/3$.

Applying the formula for the first part:

$$(\omega_0/3)^2 = \omega_0^2 + K(24)$$

$$\frac{\omega_0^2}{9} - \omega_0^2 = 24K \implies -\frac{8\omega_0^2}{9} = 24K \implies K = -\frac{\omega_0^2}{27}$$

Now, let it make n_2 more rotations to come to rest from ω_1 :

$$0 = (\omega_0/3)^2 + K(n_2)$$

$$0 = \frac{\omega_0^2}{9} - \left(\frac{\omega_0^2}{27}\right)n_2$$

Divide by $\omega_0^2/9$:

$$0 = 1 - \frac{n_2}{3} \implies \frac{n_2}{3} = 1 \implies n_2 = 3$$

Alternatively, for the whole journey from ω_0 to 0:

$$0 = \omega_0^2 + K(24 + n_2) \implies \omega_0^2 = \left(\frac{\omega_0^2}{27}\right)(24 + n_2) \implies 27 = 24 + n_2 \implies n_2 = 3$$

Step 4: Final Answer:

The fan will make 3 more rotations before coming to rest.

💡 Quick Tip

If velocity falls to $1/k$ of initial value in n rotations, total rotations to stop is $N = \frac{nk^2}{k^2-1}$. Here $k = 3, n = 24 \implies N = \frac{24 \times 9}{8} = 27$. Additional = $27 - 24 = 3$.

32. In Young's double slit interference experiment, using two coherent sources of different amplitudes, the intensity ratio between bright to dark fringes is $5 : 1$. The value of the ratio of resultant amplitudes of bright fringe to dark fringe is

- (A) $\left(\frac{\sqrt{5}+1}{\sqrt{5}-1}\right)$
- (B) $\sqrt{5} : 1$
- (C) $\left(\frac{\sqrt{5}-1}{\sqrt{5}+1}\right)$
- (D) $1 : \sqrt{5}$

Correct Answer: (B) $\sqrt{5} : 1$

Solution:

Step 1: Understanding the Concept:

Intensity of light in interference is proportional to the square of the resultant amplitude ($I \propto A_{\text{res}}^2$).

Bright fringes correspond to maximum intensity and maximum amplitude (constructive interference).

Dark fringes correspond to minimum intensity and minimum amplitude (destructive interference).

Step 2: Key Formula or Approach:

Given: $\frac{I_{\text{max}}}{I_{\text{min}}} = \frac{5}{1}$.

Relation between intensity and amplitude: $\frac{I_{\text{max}}}{I_{\text{min}}} = \left(\frac{A_{\text{max}}}{A_{\text{min}}}\right)^2$.

Step 3: Detailed Explanation:

The question asks for the ratio of resultant amplitudes of bright fringe (A_{max}) to dark fringe (A_{min}).

From the intensity relation:

$$\frac{I_{\text{bright}}}{I_{\text{dark}}} = \frac{I_{\text{max}}}{I_{\text{min}}} = 5$$

Taking the square root on both sides:

$$\sqrt{\frac{I_{\text{max}}}{I_{\text{min}}}} = \sqrt{5}$$

Since $\sqrt{\frac{I_{\text{max}}}{I_{\text{min}}}} = \frac{A_{\text{max}}}{A_{\text{min}}}$:

$$\frac{A_{\text{max}}}{A_{\text{min}}} = \frac{\sqrt{5}}{1}$$

Thus, the ratio is $\sqrt{5} : 1$.

Step 4: Final Answer:

The ratio of the resultant amplitudes of the bright fringe to the dark fringe is $\sqrt{5} : 1$.

💡 Quick Tip

Don't confuse this with the ratio of individual amplitudes of the sources (a_1/a_2). The question simply asks for the ratio of the "resultant amplitudes" of the fringes themselves.

33. One end of a capillary tube is dipped in water, the rise of water column is 'h'. The upward force of 98 dyne due to surface tension is balanced by the force due to the weight of the water column. The inner circumference of the capillary is (surface tension of water = $7 \times 10^{-2} \text{Nm}^{-1}$)

(A) 1.4 cm

(B) 0.7 cm

- (C) 0.14 cm
(D) 0.07 cm

Correct Answer: (A) 1.4 cm

Solution:

Step 1: Understanding the Concept:

The upward force in a capillary tube is provided by surface tension acting along the line of contact.

Assuming the contact angle for water and glass is zero ($\theta = 0$), the surface tension force acts vertically upward along the entire inner circumference.

Step 2: Key Formula or Approach:

Surface tension force $F = T \times L$, where L is the total length along which the tension acts.

For a circular capillary, L is the inner circumference, $2\pi r$.

Step 3: Detailed Explanation:

Given: $F = 98$ dyne. Convert it to SI units:

$$1 \text{ dyne} = 10^{-5} \text{ N} \implies F = 98 \times 10^{-5} \text{ N}$$

Surface tension $T = 7 \times 10^{-2} \text{ N/m}$.

Substitute these into the formula $F = T \cdot (\text{Circumference})$:

$$98 \times 10^{-5} = (7 \times 10^{-2}) \times \text{Circumference}$$

Solve for Circumference:

$$\text{Circumference} = \frac{98 \times 10^{-5}}{7 \times 10^{-2}}$$

$$\text{Circumference} = 14 \times 10^{-3} \text{ m}$$

Convert meters to centimeters:

$$\text{Circumference} = 14 \times 10^{-3} \times 100 \text{ cm} = 1.4 \text{ cm}$$

Step 4: Final Answer:

The inner circumference of the capillary tube is 1.4 cm.

💡 Quick Tip

Always double check units. $1 \text{ dyne/cm} = 1 \text{ mN/m}$. $1 \text{ dyne} = 10^{-5} \text{ N}$. Standard units make calculations safer.

34. Light of wavelength λ strikes a photoelectric surface and electrons are ejected with energy E . If E is to be increased to twice the original value, the wavelength changes to λ_1

- (A) $\lambda_1 < \lambda/2$
(B) $\lambda_1 = \lambda$

- (C) $\lambda_1 > \lambda/2$
 (D) $\lambda_1 = \lambda/2$

Correct Answer: (C) $\lambda_1 > \lambda/2$

Solution:

Step 1: Understanding the Concept:

Einstein's photoelectric equation relates the maximum kinetic energy of ejected electrons to the frequency (or wavelength) of incident light and the work function of the metal.

Step 2: Key Formula or Approach:

Photoelectric equation: $K_{\max} = \frac{hc}{\lambda} - \phi$

Given initial kinetic energy $K_1 = E$ and final kinetic energy $K_2 = 2E$.

Step 3: Detailed Explanation:

Write the equations for both cases: Case 1: $E = \frac{hc}{\lambda} - \phi \implies \frac{hc}{\lambda} = E + \phi$

Case 2: $2E = \frac{hc}{\lambda_1} - \phi \implies \frac{hc}{\lambda_1} = 2E + \phi$

We want to compare λ_1 and $\lambda/2$. Let's calculate $2 \cdot (hc/\lambda)$:

$$\frac{2hc}{\lambda} = 2(E + \phi) = 2E + 2\phi$$

Now compare this with hc/λ_1 :

$$\frac{hc}{\lambda_1} = 2E + \phi$$

Clearly, $\frac{hc}{\lambda_1} < \frac{2hc}{\lambda}$ since ϕ is positive.

Divide by hc :

$$\frac{1}{\lambda_1} < \frac{2}{\lambda} \implies \frac{1}{\lambda_1} < \frac{1}{\lambda/2}$$

Since both λ_1 and $\lambda/2$ are positive, taking the reciprocal reverses the inequality:

$$\lambda_1 > \lambda/2$$

Step 4: Final Answer:

The new wavelength λ_1 must be greater than $\lambda/2$.

💡 Quick Tip

To double the energy, we need to add exactly E to the photon energy. Halving the wavelength doubles the photon energy, which would increase K by more than E (it would increase it by $E + \phi$). Thus, the required photon energy is less than double, meaning the required wavelength is more than half.

35. For ideal non-rigid diatomic gas, the value of $\frac{R}{C_V}$ is nearly ($\gamma = \frac{C_P}{C_V} = \frac{9}{7}$)

- (A) 0.4
 (B) 0.66

- (C) 0.28
(D) 1.28

Correct Answer: (C) 0.28

Solution:

Step 1: Understanding the Concept:

The ratio of the universal gas constant R to the molar specific heat at constant volume C_V can be expressed solely in terms of the adiabatic constant γ .

Step 2: Key Formula or Approach:

Mayer's relation: $C_P - C_V = R$

Given: $\gamma = C_P/C_V$

Divide the Mayer's relation by C_V :

$$\frac{C_P}{C_V} - \frac{C_V}{C_V} = \frac{R}{C_V} \implies \gamma - 1 = \frac{R}{C_V}$$

Step 3: Detailed Explanation:

We are given $\gamma = \frac{9}{7}$ for a non-rigid diatomic gas (where vibrational modes are also active). Substitute the value of γ into the derived formula:

$$\begin{aligned}\frac{R}{C_V} &= \frac{9}{7} - 1 \\ \frac{R}{C_V} &= \frac{9-7}{7} = \frac{2}{7}\end{aligned}$$

Convert the fraction to a decimal:

$$\frac{2}{7} \approx 0.2857$$

Comparing this with the given options, 0.28 is the nearest value.

Step 4: Final Answer:

The value of R/C_V is approximately 0.28.

 Quick Tip

A non-rigid diatomic gas has $f = 7$ degrees of freedom. $C_V = (f/2)R = 3.5R \implies R/C_V = 1/3.5 = 2/7 \approx 0.28$. This provides a consistency check.

36. Two long straight wires A and B carrying equal current 'I' were kept parallel to each other at distance 'd' apart. Magnitude of magnetic force experienced by length L of wire A is 'F'. If the distance between the wires is made half and currents are doubled, force F_2 on length L of wire A will be

- (A) $2F$
(B) F
(C) $8F$
(D) $4F$

Correct Answer: (C) $8F$

Solution:

Step 1: Understanding the Concept:

Two parallel current-carrying wires exert a magnetic force on each other. The force per unit length depends on the product of the currents and is inversely proportional to the separation distance.

Step 2: Key Formula or Approach:

The force F on a length L of a wire is:

$$F = \frac{\mu_0 I_1 I_2 L}{2\pi d}$$

Since initial currents are equal: $F \propto \frac{I^2}{d}$.

Step 3: Detailed Explanation:

Let the initial force be $F \propto \frac{I^2}{d}$.

According to the problem, the new conditions are: 1. Currents are doubled: $I'_1 = 2I$, $I'_2 = 2I$.
2. Distance is halved: $d' = d/2$. Let the new force be F_2 :

$$F_2 = \frac{\mu_0 (2I)(2I)L}{2\pi(d/2)}$$

Factor out the constants:

$$F_2 = \frac{\mu_0 \cdot 4I^2 \cdot L}{2\pi d/2} = \frac{\mu_0 \cdot 4I^2 \cdot L \cdot 2}{2\pi d}$$

$$F_2 = 8 \times \left(\frac{\mu_0 I^2 L}{2\pi d} \right)$$

$$F_2 = 8F$$

Step 4: Final Answer:

The new force F_2 will be $8F$.

 Quick Tip

Always set up a proportionality when multiple variables change. Here, I^2 increases by a factor of 4, and $1/d$ increases by a factor of 2. Total factor = $4 \times 2 = 8$.

37. There is head-on elastic collision between the two particles moving in the same direction with speeds 5 m/s and 3 m/s respectively. After collision, the velocity of the first particle becomes 4 m/s in the same direction. The velocity of the second particle should be

- (A) 6 m/s in the same direction.
- (B) 4 m/s in the same direction.
- (C) 2 m/s in the opposite direction.
- (D) 3 m/s in the same direction.

Correct Answer: (A) 6 m/s in the same direction.

Solution:

Step 1: Understanding the Concept:

In an elastic collision, both linear momentum and total kinetic energy are conserved.

A more direct property is that the coefficient of restitution $e = 1$, meaning the relative velocity of separation equals the relative velocity of approach.

Step 2: Key Formula or Approach:

For head-on elastic collision: $v_2 - v_1 = e(u_1 - u_2)$

where $e = 1$.

Step 3: Detailed Explanation:

Given initial velocities: $u_1 = 5$ m/s, $u_2 = 3$ m/s (same direction).

Relative velocity of approach $= u_1 - u_2 = 5 - 3 = 2$ m/s.

Given final velocity of the first particle: $v_1 = 4$ m/s (same direction).

Let the final velocity of the second particle be v_2 .

Apply the property of elastic collision:

$$v_2 - v_1 = 1 \cdot (u_1 - u_2)$$

$$v_2 - 4 = 2$$

$$v_2 = 6 \text{ m/s}$$

Since $v_2 > v_1$, the second particle is moving in the same direction as the first to separate from it.

Step 4: Final Answer:

The velocity of the second particle is 6 m/s in the same direction.

 Quick Tip

In perfectly elastic collisions between equal masses, velocities are simply swapped. Here, masses are not given, but the "separation = approach" rule works universally for any masses in 1D.

38. When the heat is given to a gas in an Isothermal process, then there will be

- (A) external work done.
- (B) rise in temperature.
- (C) increase in internal energy.
- (D) external work done and also rise in temperature.

Correct Answer: (A) external work done.

Solution:

Step 1: Understanding the Concept:

An isothermal process is a thermodynamic process in which the temperature of the system remains constant throughout ($\Delta T = 0$).

The internal energy of an ideal gas depends only on its absolute temperature.

Step 2: Key Formula or Approach:

First Law of Thermodynamics: $Q = \Delta U + W$.

For ideal gas: $\Delta U = nC_V\Delta T$.

Step 3: Detailed Explanation:

In an isothermal process, $\Delta T = 0$, which implies $\Delta U = 0$.

The internal energy does not change, so options (C) and (B) are incorrect.

Substituting $\Delta U = 0$ into the First Law:

$$Q = 0 + W \implies Q = W$$

This means all the heat supplied to the gas is converted entirely into external work done by the gas during expansion.

Thus, there will be external work done. Option (D) is incorrect because there is no rise in temperature.

Step 4: Final Answer:

When heat is given in an isothermal process, it results in external work done.

 Quick Tip

Isothermal = "Constant Temperature". If the process is ideal, internal energy stays put, so heat IN must equal work OUT.

39. An alternating voltage $E = 100\sqrt{2}\sin(50t)$ is connected to a $2\mu\text{F}$ capacitor through an a.c. ammeter. The ammeter reading will be

- (A) 10 mA
- (B) 5 mA
- (C) 20 mA
- (D) 30 mA

Correct Answer: (A) 10 mA

Solution:

Step 1: Understanding the Concept:

An AC ammeter measures the root mean square (rms) value of the current flowing through the circuit.

In a purely capacitive circuit, the current depends on the rms voltage and the capacitive reactance.

Step 2: Key Formula or Approach:

Voltage equation: $V = V_0 \sin(\omega t)$. Here $V_0 = 100\sqrt{2}$ V, $\omega = 50$ rad/s.

RMS voltage: $V_{\text{rms}} = V_0/\sqrt{2}$.

Capacitive reactance: $X_C = \frac{1}{\omega C}$.

Ohm's Law for AC: $I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = V_{\text{rms}} \cdot \omega C$.

Step 3: Detailed Explanation:

Calculate RMS voltage:

$$V_{\text{rms}} = \frac{100\sqrt{2}}{\sqrt{2}} = 100 \text{ V}$$

Calculate current I_{rms} :

$$I_{\text{rms}} = 100 \times 50 \times (2 \times 10^{-6})$$

$$I_{\text{rms}} = 100 \times 100 \times 10^{-6}$$

$$I_{\text{rms}} = 10^4 \times 10^{-6} = 10^{-2} \text{ A}$$

Convert Amperes to milliamperes:

$$I_{\text{rms}} = 0.01 \text{ A} = 10 \text{ mA}$$

Step 4: Final Answer:

The ammeter reading will be 10 mA.

💡 Quick Tip

Always remember that AC devices (voltmeters and ammeters) are calibrated to read RMS values unless specifically stated otherwise.

40. In a Fraunhofer diffraction, light of wavelength ' λ ' is incident on slit of width ' d '. The diffraction pattern is observed on a screen placed at a distance ' D '. The linear width of central maximum is equal to two times the width of the slit, then ' D ' has value

- (A) $\frac{d^2}{\lambda}$
- (B) $\frac{d^2}{2\lambda}$
- (C) $\frac{d^2}{3\lambda}$
- (D) $\frac{d^2}{4\lambda}$

Correct Answer: (A) $\frac{d^2}{\lambda}$

Solution:**Step 1: Understanding the Concept:**

In single slit diffraction, the central maximum is bounded by the first minima on either side. The angular width depends on the wavelength and the slit width. The linear width also depends on the distance to the screen.

Step 2: Key Formula or Approach:

Angular position of first minimum: $\sin \theta \approx \theta = \frac{\lambda}{d}$.

Angular width of central maximum: $2\theta = \frac{2\lambda}{d}$.

Linear width of central maximum: $W = (2\theta) \cdot D = \frac{2\lambda D}{d}$.

Step 3: Detailed Explanation:

According to the problem, the linear width W is equal to two times the slit width d :

$$W = 2d$$

Equate the two expressions for W :

$$\frac{2\lambda D}{d} = 2d$$

Cancel the factor 2 from both sides:

$$\frac{\lambda D}{d} = d$$

Solve for D :

$$\lambda D = d^2 \implies D = \frac{d^2}{\lambda}$$

Step 4: Final Answer:

The value of D is $\frac{d^2}{\lambda}$.

💡 Quick Tip

Be careful with the factor 2. Central maximum width is $2\lambda D/d$, while subsequent fringe widths are only $\lambda D/d$.

41. When a big drop of water is formed from ' n ' small drops of water, the energy loss is ' $3E$ ' where ' E ' is the energy of the bigger drop. The radius of the bigger drop is ' R ' and that of smaller drop is ' r ' then the value of ' n ' is

- (A) $\frac{2R^2}{r}$
- (B) $\frac{4R^2}{r^2}$
- (C) $\frac{4R}{r}$
- (D) $\frac{4R}{r^2}$

Correct Answer: (B) $\frac{4R^2}{r^2}$

Solution:**Step 1: Understanding the Concept:**

The total surface energy of a liquid drop is the product of its surface area and the surface tension T .

When smaller drops coalesce into a larger one, the total surface area decreases, leading to a release (loss) of surface energy.

Step 2: Key Formula or Approach:

Energy of a drop: $U = T \cdot (4\pi R^2)$.

Given: Energy of bigger drop $E = T(4\pi R^2)$.

Total initial energy (from n drops): $U_{\text{total, initial}} = n \cdot T(4\pi r^2)$.

Step 3: Detailed Explanation:

Energy loss = Initial energy – Final energy.

Given energy loss is $3E$:

$$U_{\text{total, initial}} - E = 3E$$

$$U_{\text{total, initial}} = 4E$$

Substitute the expressions for energy:

$$n \cdot T(4\pi r^2) = 4 \cdot T(4\pi R^2)$$

Cancel common terms T and 4π :

$$nr^2 = 4R^2$$

Solve for n :

$$n = \frac{4R^2}{r^2}$$

(Note: From volume conservation, we also know $n = (R/r)^3$. This allows us to find numerical values if needed, e.g., $R/r = 4 \implies n = 64$).

Step 4: Final Answer:

The value of n is $\frac{4R^2}{r^2}$.

💡 Quick Tip

Always start with the basic energy definition: Energy = Surface Tension $\times$ Area. Proportionality usually solves these faster than full formulas.

42. The ratio of the wavelength of the last line of Paschen series to that of Balmer series is

- (A) $\frac{9}{4}$
- (B) $\frac{3}{2}$
- (C) $\frac{2}{3}$
- (D) $\frac{4}{9}$

Correct Answer: (A) $\frac{9}{4}$

Solution:

Step 1: Understanding the Concept:

The "last line" or series limit of a spectral series corresponds to a transition from an energy level at infinity ($n = \infty$) to the base level of that series.

The wavelength is calculated using the Rydberg formula.

Step 2: Key Formula or Approach:

Rydberg formula: $\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$.

For the last line, $n_2 = \infty$, so $\frac{1}{\lambda} = \frac{R}{n_1^2} \implies \lambda = \frac{n_1^2}{R}$.

Step 3: Detailed Explanation:

For Paschen series: $n_1 = 3$.

Wavelength of the last line of Paschen series (λ_P):

$$\lambda_P = \frac{3^2}{R} = \frac{9}{R}$$

For Balmer series: $n_1 = 2$.

Wavelength of the last line of Balmer series (λ_B):

$$\lambda_B = \frac{2^2}{R} = \frac{4}{R}$$

Calculate the ratio:

$$\frac{\lambda_P}{\lambda_B} = \frac{9/R}{4/R} = \frac{9}{4}$$

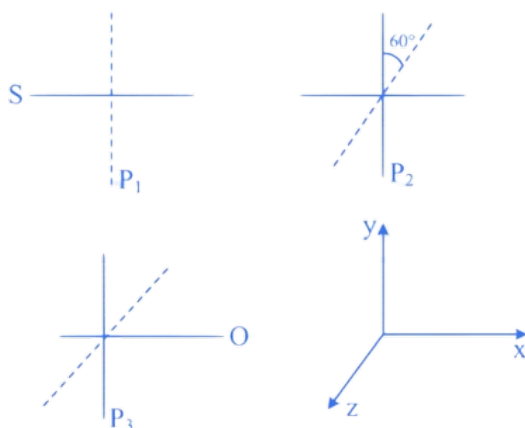
Step 4: Final Answer:

The ratio of the wavelengths is 9 : 4.

💡 Quick Tip

Wavelength of series limit is directly proportional to n^2 . Balmer = 2^2 , Paschen = 3^2 , Pfund = 5^2 , etc. Ratio = $3^2/2^2 = 9/4$.

43. Three identical polaroids P_1, P_2 and P_3 are placed one after another. The pass axis of P_2 and P_3 are inclined at an angle of 60° and 90° with respect to axis of P_1 . The source has an intensity 256 W/m^2 . The intensity of light at point 'O' is ($\cos 30^\circ = \sqrt{3}/2$, $\cos 60^\circ = 0.5$)



- (A) 24 W/m^2
- (B) 20 W/m^2
- (C) 16 W/m^2
- (D) 8 W/m^2

Correct Answer: (A) 24 W/m^2

Solution:

Step 1: Understanding the Concept:

When unpolarized light passes through a polaroid, its intensity is halved.

Subsequent passage through additional polaroids follows Malus's Law: $I = I_0 \cos^2 \theta$, where θ is the angle between the pass axes of successive polaroids.

Step 2: Key Formula or Approach:

1. After P_1 : $I_1 = I_{\text{source}}/2$.

2. After P_2 : $I_2 = I_1 \cos^2 \theta_{12}$.

3. After P_3 : $I_3 = I_2 \cos^2 \theta_{23}$.

Step 3: Detailed Explanation:

Given source intensity $I_s = 256 \text{ W/m}^2$.

After P_1 : $I_1 = 256/2 = 128 \text{ W/m}^2$.

The axis of P_2 is at 60° to P_1 , so $\theta_{12} = 60^\circ$.

After P_2 :

$$I_2 = 128 \cdot \cos^2(60^\circ) = 128 \cdot (0.5)^2 = 128 \cdot 0.25 = 32 \text{ W/m}^2$$

The axis of P_3 is at 90° to P_1 . Since P_2 was at 60° to P_1 , the relative angle between P_2 and P_3 is:

$$\theta_{23} = |90^\circ - 60^\circ| = 30^\circ$$

After P_3 :

$$I_3 = I_2 \cdot \cos^2(30^\circ) = 32 \cdot (\sqrt{3}/2)^2$$

$$I_3 = 32 \cdot (3/4) = 8 \cdot 3 = 24 \text{ W/m}^2$$

Step 4: Final Answer:

The intensity of light at point 'O' is 24 W/m^2 .

 Quick Tip

Always use the relative angle between two consecutive polaroids for Malus's Law. If P_3 was parallel to P_1 , the relative angle would be 60° again.

44. An electric dipole of dipole moment ' p ' is aligned parallel to a uniform electric field ' E '. The energy required to rotate the dipole by 90° is $\begin{bmatrix} \sin 0^\circ = 0, & \sin 90^\circ = 1 \\ \cos 0^\circ = 1, & \cos 90^\circ = 0 \end{bmatrix}$

- (A) pE
- (B) pE^2
- (C) p^2E
- (D) infinity

Correct Answer: (A) pE

Solution:

Step 1: Understanding the Concept:

An electric dipole placed in an external electric field has potential energy which depends on its

orientation relative to the field.

Work must be done by an external agent to change this orientation.

Step 2: Key Formula or Approach:

Potential energy of dipole: $U = -pE \cos \theta$.

Work done (energy required): $W = \Delta U = U_f - U_i = pE(\cos \theta_i - \cos \theta_f)$.

Step 3: Detailed Explanation:

Initial state: Aligned parallel to the field, so $\theta_i = 0^\circ$.

Final state: Rotated by 90° , so $\theta_f = 90^\circ$.

Substitute values into the work formula:

$$W = pE(\cos 0^\circ - \cos 90^\circ)$$

Using the given trigonometric values:

$$W = pE(1 - 0) = pE$$

Step 4: Final Answer:

The energy required is pE .

💡 Quick Tip

Potential energy is minimum ($-pE$) when parallel and zero when perpendicular. The "cost" of moving from min to zero is simply the absolute value of the minimum energy, pE .

45. In a photoelectric experiment, if the intensity of incident light is doubled and the frequency is kept slightly greater than threshold frequency, then the saturation photoelectric current

- (A) remains constant
- (B) is halved
- (C) is doubled
- (D) becomes four times

Correct Answer: (C) is doubled

Solution:

Step 1: Understanding the Concept:

The saturation photoelectric current corresponds to the state where all electrons ejected per second are collected by the anode.

The number of electrons ejected per second is directly proportional to the number of photons incident per second, which is defined as the intensity of light.

Step 2: Key Formula or Approach:

Photoelectric current $I \propto$ Intensity of incident radiation.

Saturation current is reached when all photoelectrons are contributing to the circuit current.

Step 3: Detailed Explanation:

The frequency of incident light is given to be greater than the threshold frequency, ensuring that photoemission occurs.

Intensity of light is defined as energy per unit area per unit time. For monochromatic light, doubling the intensity means doubling the number of photons hitting the surface per unit time. Since one photon (ideally) interacts with one electron, doubling the number of incident photons will double the number of ejected electrons.

Consequently, the saturation current (the maximum current possible for a given intensity) will also double.

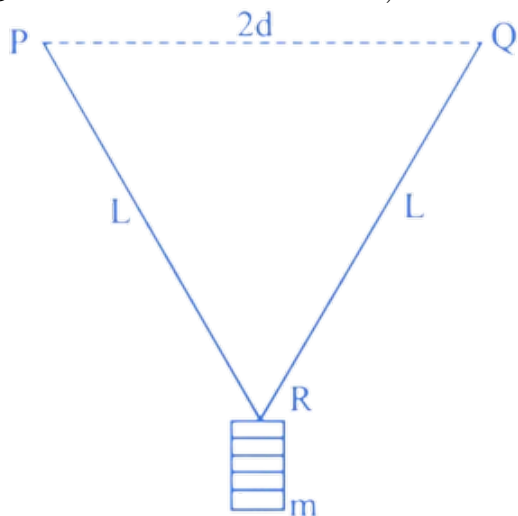
Step 4: Final Answer:

The saturation photoelectric current is doubled.

💡 Quick Tip

Frequency controls the maximum kinetic energy (stopping potential). Intensity controls the number of electrons (saturation current). Don't mix them up!

46. ' P ' and ' Q ' are fixed points in same plane and mass ' m ' is tied by string as shown in figure. If the mass is displaced slightly out of this plane and released, it will oscillate with time period ($PQ = 2d, PR = QR = L$) ($g =$ gravitational acceleration)



- (A) $2\pi\sqrt{\frac{L}{g}}$
- (B) $2\pi\sqrt{\frac{L^2}{g}}$
- (C) $2\pi\sqrt{\frac{(L^2-d^2)^{1/2}}{g}}$
- (D) $2\pi\sqrt{\frac{(L^2+d^2)^{1/2}}{g}}$

Correct Answer: (C) $2\pi\sqrt{\frac{(L^2-d^2)^{1/2}}{g}}$

Solution:

Step 1: Understanding the Concept:

This system behaves like a bifilar suspension or a special case of a simple pendulum.

When the mass is displaced "out of the plane" of the strings, it rotates or swings about the axis PQ.

The effective length of this oscillation is the perpendicular distance from the mass to the axis PQ.

Step 2: Key Formula or Approach:

Time period of a simple pendulum: $T = 2\pi\sqrt{\frac{l_{\text{eff}}}{g}}$.

Find the vertical height h from the mass R to the line PQ using the geometry of triangle PRQ.

Step 3: Detailed Explanation:

Consider the triangle formed by points P, Q, and R.

It is an isosceles triangle with sides $PR = QR = L$ and base $PQ = 2d$.

Let M be the midpoint of PQ. Triangle PMR is a right-angled triangle where $PM = d$ and $PR = L$.

The height of this triangle, which is the perpendicular distance from R to PQ, is:

$$h = \sqrt{L^2 - d^2} = (L^2 - d^2)^{1/2}$$

For small oscillations out of the plane, the mass swings along an arc of a circle centered at point M on axis PQ. The effective radius of this swing is h .

Substituting $l_{\text{eff}} = h$ into the time period formula:

$$T = 2\pi\sqrt{\frac{(L^2 - d^2)^{1/2}}{g}}$$

Step 4: Final Answer:

The time period of oscillation is $2\pi\sqrt{\frac{(L^2 - d^2)^{1/2}}{g}}$.

💡 Quick Tip

For complex string systems, find the effective suspension point (the center of the arc). The distance from mass to that point is your effective length l .

47. The instantaneous value of current in an a.c. circuit is $I = 3 \sin(50\pi t + \frac{\pi}{4})$ A. The current will be maximum for the first time at

- (A) $\frac{1}{50}$ s
- (B) $\frac{1}{100}$ s
- (C) $\frac{1}{200}$ s
- (D) $\frac{1}{600}$ s

Correct Answer: (C) $\frac{1}{200}$ s

Solution:

Step 1: Understanding the Concept:

A sinusoidal function $I(t) = I_0 \sin(\text{Phase})$ reaches its maximum value when its phase angle is $\pi/2$ radians (or 90°).

Step 2: Key Formula or Approach:

For I to be maximum: $50\pi t + \frac{\pi}{4} = \frac{\pi}{2}$.

Step 3: Detailed Explanation:

The expression for current is given as $I = 3 \sin(50\pi t + \frac{\pi}{4})$.

The first positive maximum occurs when the sine argument is $\pi/2$:

$$50\pi t + \frac{\pi}{4} = \frac{\pi}{2}$$

Subtract $\pi/4$ from both sides:

$$50\pi t = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

Divide both sides by π :

$$50t = \frac{1}{4}$$

Solve for t :

$$t = \frac{1}{4 \times 50} = \frac{1}{200} \text{ s}$$

Thus, the current reaches its peak for the first time at 0.005 s.

Step 4: Final Answer:

The current is maximum for the first time at $t = \frac{1}{200}$ s.

 Quick Tip

Phase = $\pi/2$ for first maximum. Phase = $3\pi/2$ for first minimum. Phase = 2π for first full cycle. Simple linear equations solve these fast.

48. The fundamental frequency of a closed pipe of length L is equal to the second overtone of a pipe open at both the ends of length (XL) . The value of X is (Neglect end correction)

- (A) 4
- (B) 5
- (C) 6
- (D) 7

Correct Answer: (C) 6

Solution:

Step 1: Understanding the Concept:

Pipes produce standing waves. Closed pipes (one end closed) have only odd harmonics. Open pipes (both ends open) have all integer harmonics.

Step 2: Key Formula or Approach:

Fundamental frequency of closed pipe: $f_c = \frac{v}{4L}$.

Harmonics of open pipe: $f_n = \frac{nv}{2L}$, where $n = 1$ is fundamental, $n = 2$ is 1st overtone, $n = 3$ is

2nd overtone.

Step 3: Detailed Explanation:

Let length of closed pipe be L . Its fundamental frequency is $f_1 = \frac{v}{4L}$.

Let length of open pipe be $L_{\text{open}} = XL$.

The second overtone of an open pipe is its 3rd harmonic ($n = 3$):

$$f'_2 = \frac{3v}{2L_{\text{open}}} = \frac{3v}{2XL}$$

Given these frequencies are equal:

$$\frac{v}{4L} = \frac{3v}{2XL}$$

Cancel v and L from both sides:

$$\frac{1}{4} = \frac{3}{2X}$$

Cross-multiply to solve for X :

$$2X = 4 \times 3 = 12$$

$$X = 6$$

Step 4: Final Answer:

The value of X is 6.

 Quick Tip

"Second overtone" of an open pipe is the 3rd harmonic. "Second overtone" of a closed pipe is the 5th harmonic. Be careful with these terms!

49. In the case of constant ' α ' and ' β ' of a transistor (α and β are current ratios)

- (A) $\beta < 1, \alpha > 1$
- (B) $\beta > 1, \alpha < 1$
- (C) $\alpha = \beta$
- (D) $\alpha = \beta^2$

Correct Answer: (B) $\beta > 1, \alpha < 1$

Solution:

Step 1: Understanding the Concept:

Transistor current gains describe how much current flows through the collector compared to either the emitter or the base.

Because the emitter current is the source for both base and collector current, $I_E > I_C$ and $I_C \gg I_B$.

Step 2: Key Formula or Approach:

Current relation: $I_E = I_C + I_B$.

Current gain in common base: $\alpha = \frac{I_C}{I_E}$.

Current gain in common emitter: $\beta = \frac{I_C}{I_B}$.

Step 3: Detailed Explanation:

Since I_C is always a bit smaller than I_E (as some carriers are lost to the base), the ratio $\alpha = I_C/I_E$ must be less than 1 ($\alpha < 1$).

In a properly functioning transistor, the base current I_B is very small compared to the collector current I_C .

Therefore, the ratio $\beta = I_C/I_B$ must be much greater than 1 ($\beta > 1$).

This explains why transistors are used for current amplification in common emitter mode.

Step 4: Final Answer:

The correct condition is $\beta > 1$ and $\alpha < 1$.

💡 Quick Tip

Remember α is "almost 1" (e.g., 0.98) and β is "big" (e.g., 50 to 500). This helps eliminate nonsensical options immediately.

50. The scale of a galvanometer is divided into 160 equal divisions. The galvanometer shows full scale deflection of 16 mA and maximum voltage is 80 mV . Now the range is changed so that galvanometer reads 160 V . The required resistance to be connected is

- (A) 9995Ω in series.
- (B) 4995Ω in series.
- (C) $9.5 \times 10^{-3}\Omega$ in parallel.
- (D) $4.95 \times 10^{-3}\Omega$ in parallel.

Correct Answer: (A) 9995Ω in series.

Solution:**Step 1: Understanding the Concept:**

To convert a galvanometer into a voltmeter of a higher range, a high resistance must be connected in series with it.

The series resistance limits the current to the galvanometer's full-scale value when the target voltage is applied.

Step 2: Key Formula or Approach:

Ohm's Law for the whole device: $V = I_g(G + R)$.

Resistance of galvanometer: $G = V_g/I_g$.

Step 3: Detailed Explanation:

Given: Full scale current $I_g = 16 \text{ mA} = 0.016 \text{ A}$.

Full scale voltage drop $V_g = 80 \text{ mV} = 0.08 \text{ V}$.

First, calculate internal resistance G :

$$G = \frac{V_g}{I_g} = \frac{0.08}{0.016} = 5\Omega$$

Now, target voltage range $V = 160 \text{ V}$.

Apply the voltmeter formula:

$$160 = 0.016 \times (5 + R)$$

Divide by 0.016:

$$\frac{160}{0.016} = 5 + R \implies 10000 = 5 + R$$

Solve for R :

$$R = 10000 - 5 = 9995\Omega$$

Since it's a voltmeter conversion, the resistance is in series.

Step 4: Final Answer:

A resistance of 9995Ω must be connected in series.

 Quick Tip

To measure voltage: Series High Resistance. To measure current: Parallel Low Resistance (Shunt). The high/low logic helps prevent mixing up ammeter vs voltmeter conversions.