MHT CET 4 May Shift 2 Question Paper with Solutions

Total Time Allowed : 3 hours	Maximum Marks : 200	Total Questions : 7	Questions to be answered :7

General Instructions

Read the following instructions very carefully and strictly follow them:

(1) This question booklet contains 150 Multiple Choice Questions (MCQs).
Section-A: Physics Chemistry - 50 Questions each and
Section-B: Mathematics - 50 Questions.
(2) Choice and sequence for attempting questions will be as per the convenience of the candidate.
(3) Read each question carefully.
(4) Determine the one correct answer out of the four available options given for each question. (5) Each question with correct response shall be awarded one (1) mark. There shall be no negative marking.
(6) No mark shall be granted for marking two or more answers of same question, scratching or overwriting.
(7) Duration of paper is 3 Hours.

MATHEMATICS

1. The variance of the first 50 even natural numbers is:
(1) 833
(2) $\frac{437}{4}$
(3) $\frac{833}{4}$
(4) 437

Correct Answer: (1) 833.

Solution: Step 1: Determine the first 50 even natural numbers. The sequence of the first 50 even natural numbers is:

$$2, 4, 6, \ldots, 100.$$

This sequence forms an arithmetic progression characterized by:

- First term $a = 2$
- Common difference $d = 2$
- Number of terms $n = 50$

Step 2: Calculate the sum and the mean of the sequence. The sum S of an arithmetic progression can be calculated using the formula:

$$S = \frac{n}{2}\left(2a + (n-1)d\right)$$

Substituting the known values:

$$S = \frac{50}{2}\left(2 \times 2 + (50-1) \times 2\right) = 25 \times (4 + 98) = 25 \times 102 = 2550$$

The mean μ is then:

$$\mu = \frac{S}{n} = \frac{2550}{50} = 51$$

Step 3: Compute the sum of squares of the sequence. The sum of squares of the first n even natural numbers is given by:

$$\sum (2i)^2 = 4 \sum i^2 = 4 \times \frac{n(n+1)(2n+1)}{6}$$

Plugging in $n = 50$:

$$4 \times \frac{50 \times 51 \times 101}{6} = 4 \times \frac{257550}{6} = 4 \times 42925 = 171700$$

Step 4: Determine the variance of the sequence. First, find the expected value of X^2:

$$E(X^2) = \frac{\sum X^2}{n} = \frac{171700}{50} = 3434$$

Then, calculate the variance σ^2:

$$\sigma^2 = E(X^2) - \mu^2 = 3434 - 51^2 = 3434 - 2601 = 833$$

Conclusion: The variance of the first 50 even natural numbers is 833.

Quick Tip

Variance measures the spread of a set of numbers. In this case, a variance of 833 indicates a relatively wide dispersion of the first 50 even natural numbers around the mean of 51.

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2. Integrate the function $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$**:**
Correct Answer: $e^x \cdot \tan\left(\frac{x}{2}\right) + C$.

Solution:
Step 1: Simplify the given trigonometric expression. We utilize the identity:

$$\frac{1+\sin x}{1+\cos x} = \tan\left(\frac{x}{2}\right).$$

Step 2: Substitute the identity into the integral. Applying this substitution, the integral becomes:

$$I = \int e^x \cdot \tan\left(\frac{x}{2}\right)\, dx.$$

Step 3: Apply substitution method. Let $u = \frac{x}{2}$, which implies $du = \frac{1}{2}\,dx$ and therefore $dx = 2\,du$. Substituting these into the integral:

$$I = 2\int e^{2u} \cdot \tan(u)\, du.$$

Step 4: Integrate the expression. This integral is standard, and its solution is:

$$I = e^{2u} \cdot \tan(u) + C.$$

Step 5: Revert the substitution to the original variable. Substituting back $u = \frac{x}{2}$ into the result:

$$I = e^x \cdot \tan\left(\frac{x}{2}\right) + C.$$

Conclusion: The integral simplifies to:

$$\int e^x \cdot \tan\left(\frac{x}{2}\right)\, dx = e^x \cdot \tan\left(\frac{x}{2}\right) + C.$$

Quick Tip

Using appropriate trigonometric identities and substitution techniques can greatly simplify the process of integrating complex trigonometric expressions.

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3. The solution of the differential equation $x\cos y\, dy = (xe^x \log x + e^x)\, dx$ **is:**
Correct Answer: $xe^x + C$.

Solution:
We are given the differential equation:

$$x\cos y\, dy = (xe^x \log x + e^x)\, dx.$$

This equation can be rearranged to separate the variables:

$$\cos y\, dy = \left(e^x \log x + \frac{e^x}{x}\right)\, dx.$$

Step 1: Integrate both sides. Integrate the left side with respect to y and the right side with respect to x:

$$\int \cos y\, dy = \int \left(e^x \log x + \frac{e^x}{x}\right) dx.$$

$$\sin y = \int e^x \log x\, dx + \int \frac{e^x}{x}\, dx + C.$$

Step 2: Evaluate the integrals on the right-hand side. For the first integral, use integration by parts: Let $u = \log x$, so $du = \frac{1}{x}dx$, and $dv = e^x dx$, so $v = e^x$.
Applying the integration by parts formula $\int u\,dv = uv - \int v\,du$, we get:

$$\int e^x \log x\,dx = e^x \log x - \int \frac{e^x}{x}\,dx.$$

Therefore, the right side becomes:

$$\sin y = e^x \log x - \int \frac{e^x}{x}\,dx + \int \frac{e^x}{x}\,dx + C.$$

The integrals $\int \frac{e^x}{x}\,dx$ cancel out:

$$\sin y = e^x \log x + C.$$

Conclusion: The solution to the differential equation is:

$$\sin y = e^x \log x + C.$$

Quick Tip

Using integration by parts is effective for solving integrals involving products of functions, such as exponential and logarithmic terms in this example.

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4. Find the expected value and variance of X for the following p.m.f:

x	-2	-1	0	1	2
$P(X)$	0.2	0.3	0.1	0.15	0.25

Correct Answer: 2.2475.

Solution: The expected value $E(X)$ is given by:

$$E(X) = \sum x \cdot P(X = x) = (-2)(0.2) + (-1)(0.3) + (0)(0.1) + (1)(0.15) + (2)(0.25)$$

$$E(X) = -0.4 - 0.3 + 0 + 0.15 + 0.5 = -0.05.$$

The expected value of X^2 is:

$$E(X^2) = \sum x^2 \cdot P(X = x) = (-2)^2(0.2) + (-1)^2(0.3) + (0)^2(0.1) + (1)^2(0.15) + (2)^2(0.25)$$

$$E(X^2) = 0.8 + 0.3 + 0 + 0.15 + 1 = 2.25.$$

The variance $\text{Var}(X)$ is given by:

$$\text{Var}(X) = E(X^2) - (E(X))^2 = 2.25 - (-0.05)^2 = 2.25 - 0.0025 = 2.2475.$$

Thus, the variance is 2.2475.

> 💡 Quick Tip
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> To calculate the variance, use the formula Variance $= E(X^2) - (E(X))^2$, where $E(X^2)$ is the expected value of X^2.

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5. If the statement $p \leftrightarrow (q \rightarrow p)$ is false, then the true statement is:
(1) p
(2) $p \rightarrow (p \vee \sim q)$
(3) $p \wedge (\sim pq)$
(4) $(p \vee \sim q) \rightarrow p$

Correct Answer: (2) $p \rightarrow (p \vee \sim q)$.

Solution:
We are given the logical statement $p \leftrightarrow (q \rightarrow p)$. This biconditional expression is false only when one part is true and the other is false.

Case 1: Let $p =$ True and $q =$ False. - $q \rightarrow p$ evaluates to False $\rightarrow$ True $=$ True. - Therefore, $p \leftrightarrow (q \rightarrow p)$ becomes True $\leftrightarrow$ True $=$ True. - This case does not make the statement false.

Case 2: Let $p =$ False and $q =$ True. - $q \rightarrow p$ evaluates to True $\rightarrow$ False $=$ False. - Therefore, $p \leftrightarrow (q \rightarrow p)$ becomes False $\leftrightarrow$ False $=$ True. - This case also does not make the statement false.

Case 3: Let $p =$ False and $q =$ False. - $q \rightarrow p$ evaluates to False $\rightarrow$ False $=$ True. - Therefore, $p \leftrightarrow (q \rightarrow p)$ becomes False $\leftrightarrow$ True $=$ False. - This case satisfies the condition where the biconditional statement is false.
Next, we evaluate the provided options under the scenario $p =$ False and $q =$ False:

- **Option 1:** p is false.
- **Option 2:** $p \rightarrow (p \vee \sim q)$. Substituting the values:
$$\text{False} \rightarrow (\text{False} \vee \text{True}) = \text{False} \rightarrow \text{True} = \text{True}.$$
- **Option 3:** $p \wedge (\sim p \wedge q)$. Substituting the values:
$$\text{False} \wedge (\text{True} \wedge \text{False}) = \text{False} \wedge \text{False} = \text{False}.$$
- **Option 4:** $(p \vee \sim q) \rightarrow p$. Substituting the values:
$$(\text{False} \vee \text{True}) \rightarrow \text{False} = \text{True} \rightarrow \text{False} = \text{False}.$$

Among these options, **Option 2** is the only statement that evaluates to true when $p =$ False and $q =$ False.

Quick Tip

When analyzing logical biconditional statements, test various truth values for the components involved to identify when the statement becomes false. Then compare this to the given options.

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6. The statement $[(p \to q) \wedge \sim q] \to r$ is a tautology when r is equivalent to:
(1) $p \wedge \sim q$
(2) $q \vee p$
(3) $p \wedge q$
(4) $\sim q$

Correct Answer: (4) $\sim q$.

Solution:
A tautology is a logical statement that is always true, irrespective of the truth values of its individual components.
Consider the statement $[(p \to q) \wedge \sim q] \to r$. For this statement to be a tautology, it must hold true under all possible truth assignments of p, q, and r.

Analyzing the Antecedent: The antecedent of the implication is $(p \to q) \wedge \sim q$. This conjunction is true only when both $p \to q$ is true and $\sim q$ (not q) is true.
- $\sim q$ being true implies that q is false.
- For $p \to q$ to be true while q is false, p must also be false. (Recall that $p \to q$ is only false when p is true and q is false.)

Therefore, the antecedent $(p \to q) \wedge \sim q$ is true exclusively when both p and q are false.
Implications for r:
The entire statement $[(p \to q) \wedge \sim q] \to r$ is a conditional statement that evaluates to false only when the antecedent is true and the consequent r is false. To ensure that the entire statement is always true (i.e., a tautology), r must be true in every scenario where the antecedent is true.
Since the antecedent is true only when $\sim q$ is true, r must also be true whenever $\sim q$ is true. This relationship implies that r must logically follow $\sim q$, meaning r should be equivalent to $\sim q$.

Conclusion: For the statement $[(p \to q) \wedge \sim q] \to r$ to be a tautology, r must be equivalent to $\sim q$.

💡 Quick Tip

A tautology remains true under all possible truth values of its variables. Ensuring that consequents align with the conditions of the antecedents is key to forming tautological statements.

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7. A lot of 100 bulbs contains 10 defective bulbs. Five bulbs are selected at random from the lot and are sent to the retail store. Then the probability that the store will receive at most one defective bulb is:
(1) $\frac{7}{5}\left(\frac{9}{10}\right)^4$
(2) $\frac{7}{5}\left(\frac{9}{10}\right)^5$
(3) $\frac{6}{5}\left(\frac{9}{10}\right)^4$
(4) $\frac{6}{5}\left(\frac{9}{10}\right)^5$

Correct Answer: (1) $\frac{7}{5}\left(\frac{9}{10}\right)^4$.

Solution:
We are given:

- Total number of bulbs, $N = 100$
- Defective bulbs, $K = 10$
- Non-defective bulbs, $N - K = 90$
- Bulbs selected, $n = 5$

We need to find the probability of selecting at most one defective bulb, i.e., $P(X \leq 1)$.
Step 1: Probability of selecting 0 defective bulbs Selecting all 5 bulbs from the 90 non-defective ones:

$$P(X = 0) = \frac{\binom{90}{5}}{\binom{100}{5}} \approx 0.5837$$

Step 2: Probability of selecting 1 defective bulb Selecting 1 defective bulb from 10 and 4 non-defective bulbs from 90:

$$P(X = 1) = \frac{\binom{10}{1}\binom{90}{4}}{\binom{100}{5}} \approx 0.31$$

Step 3: Total probability Summing the probabilities for 0 and 1 defective bulbs:

$$P(X \leq 1) = P(X = 0) + P(X = 1) = 0.5837 + 0.31 = 0.8937$$

Using Hypergeometric Distribution The hypergeometric distribution formula is:

$$P(X = k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

Thus,

$$P(X \leq 1) = \frac{\binom{10}{0}\binom{90}{5}}{\binom{100}{5}} + \frac{\binom{10}{1}\binom{90}{4}}{\binom{100}{5}} \approx 0.8937$$

Conclusion: The probability of selecting at most one defective bulb out of five is approximately 0.8937.

Quick Tip

The hypergeometric distribution is ideal for scenarios without replacement, such as selecting defective items from a finite population.
