MHT CET 3 May Shift 2 Question Paper with Solutions

Total Time Allowed : 3 hours	**Maximum Marks :** 200	**Total Questions :** 7	**Questions to be answered :7**

General Instructions

Read the following instructions very carefully and strictly follow them:

(1) This question booklet contains 150 Multiple Choice Questions (MCQs).
Section-A: Physics Chemistry - 50 Questions each and
Section-B: Mathematics - 50 Questions.
(2) Choice and sequence for attempting questions will be as per the convenience of the candidate.
(3) Read each question carefully.
(4) Determine the one correct answer out of the four available options given for each question. (5) Each question with correct response shall be awarded one (1) mark. There shall be no negative marking.
(6) No mark shall be granted for marking two or more answers of same question, scratching or overwriting.
(7) Duration of paper is 3 Hours.

1. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is:
(1) $\frac{1}{2}$
(2) 1
(3) $\frac{1}{\sqrt{2}}$
(4) 2

Correct Answer: (3) $\frac{1}{\sqrt{2}}$.

Solution:
Step 1: Given $y = \sec(\tan^{-1} x)$.
Let $\theta = \tan^{-1}(x)$. After this, by the definition of the inverse tangent function, we have:

$$\tan(\theta) = x.$$

Step 2: Differentiate the expression for y. It comes:

$$\sec^2(\theta) = 1 + \tan^2(\theta) = 1 + x^2,$$

so:

$$\sec(\theta) = \sqrt{1 + x^2}.$$

Thus, $y = \sec(\theta) = \sqrt{1 + x^2}$.

Step 3: Differentiate y with respect to x. Now, differentiate $y = \sqrt{1+x^2}$ using the chain rule:

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1+x^2}} \cdot 2x = \frac{x}{\sqrt{1+x^2}}.$$

Step 4: Calculate $\frac{dy}{dx}$ at $x = 1$. Substitute $x = 1$ into the derivative:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+1^2}} = \frac{1}{\sqrt{2}}.$$

The correct answer is $\frac{1}{\sqrt{2}}$.

Quick Tip

When differentiating $\sec(\tan^{-1} x)$, use the fact that $\sec(\theta) = \sqrt{1+\tan^2(\theta)}$ and apply the chain rule to differentiate the function.

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2. If $y = \log_e \left[e^{3x} \left(\frac{x-4}{x+3}\right)^{3/2}\right]$, then find $\frac{dy}{dx}$:

(1) $3 + \frac{21}{2(x-4)(x+3)}$

(2) $3 + \frac{21}{(x-4)(x+3)}$

(3) $3 + \frac{21}{2(x+3)(x-4)}$

(4) $3 + \frac{7}{(x-4)(x+3)}$

Correct Answer: (1) $3 + \frac{21}{2(x-4)(x+3)}$.

Solution: We have the function:

$$y = \log_e \left[e^{3x} \left(\frac{x-4}{x+3}\right)^{3/2}\right].$$

Step 1: Simplify the logarithmic expression using properties of logarithms.

$$y = \log_e \left(e^{3x}\right) + \log_e \left(\left(\frac{x-4}{x+3}\right)^{3/2}\right).$$

Using the property of logarithms $\log_e(a^b) = b \log_e(a)$, we have:

$$y = 3x + \frac{3}{2} \log_e \left(\frac{x-4}{x+3}\right).$$

Step 2: Differentiate y with respect to x. The derivative of $3x$ is 3, and for the second term, apply the chain rule to differentiate the logarithmic expression:

$$\frac{dy}{dx} = 3 + \frac{3}{2} \cdot \frac{d}{dx} \left[\log_e \left(\frac{x-4}{x+3}\right)\right].$$

Now, use the derivative of a logarithmic function $\frac{d}{dx}\left[\log_e\left(\frac{u}{v}\right)\right] = \frac{1}{u/v} \cdot \frac{d}{dx}\left(\frac{u}{v}\right)$:

$$\frac{d}{dx}\left[\log_e\left(\frac{x-4}{x+3}\right)\right] = \frac{1}{\frac{x-4}{x+3}} \cdot \frac{d}{dx}\left[\frac{x-4}{x+3}\right].$$

Then differentiate $\frac{x-4}{x+3}$ using quotient rule:

$$\frac{d}{dx}\left[\frac{x-4}{x+3}\right] = \frac{(x+3)(1)-(x-4)(1)}{(x+3)^2} = \frac{7}{(x+3)^2}.$$

Substitute this into derivative expression:

$$\frac{dy}{dx} = 3 + \frac{3}{2} \cdot \frac{7}{(x-4)(x+3)} = 3 + \frac{21}{2(x-4)(x+3)}.$$

The correct answer is $3 + \frac{21}{2(x-4)(x+3)}$.

Quick Tip

When differentiating logarithmic functions, first simplify the expression using logarithmic properties, then apply the chain rule to differentiate.

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3. Find the differential equation of the family of all circles, whose center lies on the x-axis and touches the y-axis at the origin.

(1) $2xy\frac{dy}{dx} = y^2 - x^2$
(2) $2xy\frac{dy}{dx} = x^2 - y^2$
(3) $x^2 + y^2 = 2xy\frac{dy}{dx}$
(4) $x^2 + y^2 = 2y\frac{dy}{dx}$

Correct Answer: (1) $2xy\frac{dy}{dx} = y^2 - x^2$.

Solution: The equation of a circle with center at $(h, 0)$ and radius h is given by:

$$(x-h)^2 + y^2 = h^2.$$

$$x^2 + y^2 + h^2 - 2hx = h^2$$

$$x^2 + y^2 - 2hx = 0$$

$$2x + 2y\frac{dy}{dx} - 2h = 0$$

$$h = x + y\frac{dy}{dx}$$

$$x^2 + y^2 - 2x\left(x + y\frac{dy}{dx}\right) = 0$$

$$x^2 + y^2 - 2x^2 - 2xy\frac{dy}{dx} = 0$$

$$y^2 - x^2 - 2xy\frac{dy}{dx} = 0$$

Then, equation of the family of circles is:

$$2xy\frac{dy}{dx} = y^2 - x^2.$$

Quick Tip

When differentiating implicit equations, apply the chain rule and remember to differentiate each term with respect to x. For a family of circles, center and radius conditions are key.

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4. If $f(x) = 3x + 6$, $g(x) = 4x + k$, and $f \circ g(x) = g \circ f(x)$, then find k:
(1) 3
(2) 6
(3) 9
(4) 12

Correct Answer: (3) 9.

Solution: Given the functions $f(x) = 3x+6$ and $g(x) = 4x+k$ and that $f\circ g(x) = g\circ f(x)$, meaning:

$$f(g(x)) = g(f(x)).$$

First compute both compositions:

$$f(g(x)) = f(4x + k) = 3(4x + k) + 6 = 12x + 3k + 6.$$

$$g(f(x)) = g(3x + 6) = 4(3x + 6) + k = 12x + 24 + k.$$

Then, equate the two expressions:

$$12x + 3k + 6 = 12x + 24 + k.$$

Canceling out the $12x$ terms:

$$3k + 6 = 24 + k.$$

Solve for k:

$$3k - k = 24 - 6,$$

$$2k = 18,$$

$$k = 9.$$

The value of k is 9.

Quick Tip

When solving problems with function compositions, equate the two expressions and solve for the unknowns. Pay attention to the structure of the functions to avoid mistakes.
