MHT CET 3 May Shift 1 Question Paper with Solutions

Total Time Allowed : 3 hours	Maximum Marks : 200	Total Questions : 7	Questions to be answered :7

General Instructions

Read the following instructions very carefully and strictly follow them:

(1) This question booklet contains 150 Multiple Choice Questions (MCQs).
Section-A: Physics Chemistry - 50 Questions each and
Section-B: Mathematics - 50 Questions.
(2) Choice and sequence for attempting questions will be as per the convenience of the candidate.
(3) Read each question carefully.
(4) Determine the one correct answer out of the four available options given for each question. (5) Each question with correct response shall be awarded one (1) mark. There shall be no negative marking.
(6) No mark shall be granted for marking two or more answers of same question, scratching or overwriting.
(7) Duration of paper is 3 Hours.

CHEMISTRY

1. What is the coordination number in a Hexagonal Close-Packed (hcp) structure?
(1) 8
(2) 10
(3) 12
(4) 14
Correct Answer: (3) 12

Solution: In a Hexagonal Close-Packed (hcp) structure, the coordination number is 12. This means that each atom is surrounded by 12 neighboring atoms: 6 within the same plane and 3 each above and below the central atom. This configuration results in highly efficient packing, maximizing the number of nearest neighbors around each atom and contributing to the stability of the hcp structure.

Quick Tip

In close-packed structures, such as FCC and HCP, the coordination number is 12.

2. The density of a face-centered cubic (FCC) crystal is:
(1) $\frac{4M}{\sqrt{2}a^3}$
(2) $\frac{4M}{a^3}$
(3) $\frac{6M}{a^3}$
(4) $\frac{2M}{a^3}$
Correct Answer: (2) $\frac{4M}{a^3}$
Solution:
To determine the density of a face-centered cubic (FCC) crystal structure, we follow these steps:
Step 1: Identify the number of atoms per unit cell in FCC. In an FCC structure, each unit cell contains 4 atoms. This is because each corner atom is shared among 8 adjacent unit cells and each face-centered atom is shared between 2 unit cells:

$$\text{Total atoms per unit cell} = \frac{8 \times \frac{1}{8} + 6 \times \frac{1}{2}}{1} = 1 + 3 = 4.$$

Step 2: Express the mass of the unit cell. The mass of the unit cell is the product of the number of atoms per unit cell and the atomic mass M:

$$\text{Mass of unit cell} = 4M.$$

Step 3: Calculate the volume of the unit cell. The volume V of the cubic unit cell is given by the cube of the edge length a:

$$V = a^3.$$

Step 4: Formulate the density equation. Density ρ is defined as mass per unit volume. Substituting the mass and volume of the unit cell:

$$\rho = \frac{\text{Mass of unit cell}}{\text{Volume of unit cell}} = \frac{4M}{a^3}.$$

Conclusion: The density ρ of an FCC crystal structure is given by:

$$\rho = \frac{4M}{a^3},$$

where M is the atomic mass and a is the edge length of the unit cell.

Quick Tip

Understanding the relationship between atomic mass and unit cell dimensions is crucial for calculating the density of crystalline materials, which has implications in material science and engineering.

---

3. Which of the following correctly represents the change in Gibbs free energy ΔG for a spontaneous process?

(1) $\Delta G = 0$
(2) $\Delta G < 0$
(3) $\Delta G > 0$
(4) $\Delta G = \Delta H - T\Delta S$
Correct Answer: (4) $\Delta G = \Delta H - T\Delta S$
Solution:
To determine the spontaneity of a process, we analyze the change in Gibbs free energy (ΔG) using the following equation:

$$\Delta G = \Delta H - T\Delta S,$$

where:

- ΔG = Change in Gibbs free energy
- ΔH = Change in enthalpy
- ΔS = Change in entropy
- T = Absolute temperature in Kelvin

Step 1: Understand the Gibbs Free Energy Equation The equation $\Delta G = \Delta H - T\Delta S$ relates the change in Gibbs free energy to changes in enthalpy and entropy. It serves as a criterion for predicting the spontaneity of a process:

- If $\Delta G < 0$, the process is spontaneous.
- If $\Delta G > 0$, the process is non-spontaneous.
- If $\Delta G = 0$, the system is in equilibrium.

Step 2: Analyze the Enthalpy Change (ΔH) Enthalpy change indicates whether a reaction is exothermic or endothermic:

- $\Delta H < 0$: Exothermic reaction (releases heat)
- $\Delta H > 0$: Endothermic reaction (absorbs heat)

A decrease in enthalpy ($\Delta H < 0$) favors spontaneity.
Step 3: Analyze the Entropy Change (ΔS) Entropy change measures the disorder or randomness in the system:

- $\Delta S > 0$: Increase in entropy (more disorder)
- $\Delta S < 0$: Decrease in entropy (less disorder)

An increase in entropy ($\Delta S > 0$) also favors spontaneity.
Step 4: Combine the Effects of ΔH and ΔS The spontaneity of a process depends on the combined effects of enthalpy and entropy changes:

- Exothermic and Entropy Increases ($\Delta H < 0$ and $\Delta S > 0$): ΔG is always negative, making the process spontaneous.

- Endothermic and Entropy Increases ($\Delta H > 0$ and $\Delta S > 0$): ΔG can be negative if the entropy term ($T\Delta S$) outweighs the enthalpy term (ΔH), leading to spontaneity at higher temperatures.
- Exothermic and Entropy Decreases ($\Delta H < 0$ and $\Delta S < 0$): ΔG can be negative if the enthalpy term (ΔH) outweighs the entropy term ($T\Delta S$), leading to spontaneity at lower temperatures.
- Endothermic and Entropy Decreases ($\Delta H > 0$ and $\Delta S < 0$): ΔG is always positive, making the process non-spontaneous.

Step 5: Determine Spontaneity For a process to be spontaneous ($\Delta G < 0$):

- Either the reaction releases heat ($\Delta H < 0$),
- Or the reaction increases the disorder of the system ($\Delta S > 0$),
- Or both conditions are met.

This equation is essential for predicting the direction of chemical reactions and understanding the underlying thermodynamics of various processes.

Conclusion: The Gibbs free energy equation $\Delta G = \Delta H - T\Delta S$ is pivotal in determining the spontaneity of chemical reactions. A negative ΔG indicates a spontaneous process, driven by either a decrease in enthalpy, an increase in entropy, or both.

Quick Tip

Remember that temperature plays a crucial role in the Gibbs free energy equation. At higher temperatures, the entropy term ($T\Delta S$) becomes more significant, potentially changing the spontaneity of a process.

---

4. Which of the following is an essential amino acid?
(1) Alanine
(2) Leucine
(3) Glycine
(4) Glutamine
Correct Answer: (2) Leucine
Solution: An essential amino acid is one that cannot be synthesized by the human body and must be obtained through the diet. Leucine is an essential amino acid, meaning the body cannot produce it, and it must be acquired from food sources.
On the other hand, glutamine, glycine, and alanine are non-essential amino acids because the body can synthesize them from other compounds.

Quick Tip

Essential amino acids include leucine, isoleucine, valine, phenylalanine, tryptophan, and others. They must be obtained through diet.

---

5. What is the structural feature of glucose that makes it a reducing sugar?

Correct Answer: It has an aldehyde group

Solution: Glucose is a reducing sugar because it contains an aldehyde group in its open-chain form. The presence of the aldehyde group allows glucose to donate electrons and reduce other compounds, which is the defining characteristic of reducing sugars. In its cyclic form, glucose forms a hemiacetal, but when in an open-chain form, it exposes the aldehyde group.

Quick Tip

Reducing sugars contain either an aldehyde or a free ketone group in their open-chain form, which allows them to act as reducing agents.

---

6. Which of the following is a homopolymer?
(1) Polystyrene
(2) Nylon
(3) Bakelite
(4) PVC
Correct Answer: (1) Polystyrene
Solution: A homopolymer is a polymer that consists of only one type of monomer. Among the options:
- Nylon-6,6 is a copolymer made from two different monomers: hexamethylenediamine and adipic acid.
- Polystyrene is a homopolymer formed from the polymerization of styrene monomers.
- Bakelite is a thermosetting polymer made from phenol and formaldehyde, so it is not a homopolymer.
- Teflon is a polymer made from tetrafluoroethylene monomers, but it is typically considered a polymer of one type of monomer, thus it is also a homopolymer.

Therefore, polystyrene is the correct answer as it is a homopolymer formed from styrene.

Quick Tip

Homopolymers are made from one type of monomer, while copolymers are made from two or more different monomers.

---

7. Which of the following is a method for the preparation of benzaldehyde?
Correct Answer: Gattermann-Koch reaction
Solution:
The Gattermann-Koch reaction is utilized for the synthesis of benzaldehyde. This reaction involves the combination of benzene with carbon monoxide (CO) and hydrochloric acid (HCl) in the presence of a Lewis acid catalyst, typically aluminum chloride ($AlCl_3$). The process introduces an aldehyde functional group (-CHO) directly onto the benzene ring, resulting in the formation of benzaldehyde.

Step 1: Understanding the Gattermann-Koch Reaction The Gattermann-Koch reaction specifically targets the introduction of an aldehyde group to an aromatic ring. The reaction mechanism can be summarized as:

$$\text{Benzene} + \text{CO} + \text{HCl} \xrightarrow{AlCl_3} \text{Benzaldehyde}$$

Here, $AlCl_3$ acts as a Lewis acid catalyst, facilitating the formation of the acyl chloride intermediate, which subsequently reacts with benzene to yield benzaldehyde.
Step 2: Comparing with Other Methods Several other methods can be considered for synthesizing benzaldehyde, but they do not offer the same direct approach:

- **Friedel-Crafts Acylation**: This method typically results in the formation of ketones rather than aldehydes. For instance, reacting benzene with an acyl chloride in the presence of $AlCl_3$ yields acetophenone, a ketone.
- **Reduction of Benzoic Acid**: Reducing benzoic acid leads to the production of benzyl alcohol, not benzaldehyde. The reduction process converts the carboxylic acid group (-COOH) to a hydroxyl group (-OH).
- **Ozonolysis of Styrene**: While ozonolysis can cleave double bonds to form aldehydes or ketones, it does not specifically produce benzaldehyde from styrene.

Conclusion: Therefore, the Gattermann-Koch reaction is the correct method for preparing benzaldehyde.

Quick Tip

The Gattermann-Koch reaction is a valuable tool in organic synthesis for introducing aldehyde groups into aromatic rings, making it essential for the preparation of aromatic aldehydes like benzaldehyde.

---

8. Which of the following represents the rate equation for a zero-order reaction?
(1) Rate $= k[A]$
(2) Rate $= k[A]^2$
(3) Rate $= k$
(4) Rate $= k[A]^n$
Correct Answer: (3) Rate $= k$

Solution: In a zero-order reaction, the rate of reaction is independent of the concentration of the reactants. The rate law for a zero-order reaction is simply:

$$\text{Rate} = k$$

where k is the rate constant. This means that no matter how much of the reactant is present, the rate of the reaction remains constant over time.

For the other options:
- Option (1) represents a first-order reaction.
- Option (2) represents a second-order reaction.
- Option (4) represents a general order reaction, where n can be any value.

Therefore, the correct answer is Rate $= k$, representing a zero-order reaction.

Quick Tip

In a zero-order reaction, the rate does not depend on the concentration of the reactant.

---

9. The Rosenmund reduction involves the reduction of which of the following compounds?
(1) Carboxylic acid to alcohol
(2) Aldehyde to alcohol
(3) Acyl chloride to aldehyde
(4) Alkene to alkane
Correct Answer: (3) Acyl chloride to aldehyde

Solution: The Rosenmund reduction is a process used to reduce acyl chlorides to aldehydes. In this reaction, an acyl chloride is treated with hydrogen gas in the presence of a palladium catalyst poisoned by sulfur (such as palladium on barium sulfate). The catalyst helps in the selective reduction of the acyl chloride to an aldehyde while preventing further reduction to an alcohol.

Therefore, the correct answer is that the Rosenmund reaction reduces acyl chlorides to aldehydes.

Quick Tip

The Rosenmund reduction is selective for converting acyl chlorides to aldehydes without reducing the aldehyde to an alcohol.

---

10. Which of the following compounds is likely to be more soluble in water?
(1) Nonpolar hydrocarbons
(2) Ionic compounds
(3) Alcohols
(4) Gaseous compounds
Correct Answer: (3) Alcohols
Solution:
The solubility of a compound in water is influenced by how well it interacts with water molecules. Alcohols are typically more soluble in water compared to other types of compounds due to their ability to form hydrogen bonds.

Step 1: Analyze the Functional Groups Alcohols contain a hydroxyl (-OH) group, which is highly polar. This polarity allows alcohol molecules to form hydrogen bonds with water molecules, enhancing their solubility.
Step 2: Compare with Other Compounds Consider the solubility characteristics of different types of compounds:

- **Nonpolar Hydrocarbons**: These compounds lack polar functional groups and cannot form hydrogen bonds with water, making them generally insoluble.
- **Ionic Compounds**: While many ionic compounds are soluble in water due to ion-dipole interactions, their solubility depends on the size and charge of the ions involved.
- **Gaseous Compounds**: The solubility of gases in water varies, but most gases are not highly soluble unless they can form specific interactions with water molecules.

Conclusion: Alcohols are generally more soluble in water than other types of compounds because their hydroxyl groups enable effective hydrogen bonding with water molecules.

Quick Tip

Hydrogen bonding significantly enhances the solubility of polar compounds like alcohols in water, whereas nonpolar and less polar compounds struggle to dissolve due to weaker interactions.

---

11. Molar conductivity of an electrolyte depends on which of the following factors?

(1) Concentration of the electrolyte
(2) Temperature
(3) Nature of the solvent
(4) All of the above
Correct Answer: (4) All of the above

Solution: Molar conductivity (Λ_m) of an electrolyte is the conductivity per mole of the electrolyte. It depends on several factors:
- Concentration of the electrolyte: As the concentration increases, the molar conductivity decreases due to ion-ion interactions.
- Temperature: Molar conductivity typically increases with temperature due to the increased mobility of ions.
- Nature of the solvent: The solvent affects the degree of dissociation of the electrolyte, and different solvents can change the molar conductivity significantly.

Therefore, the correct answer is "All of the above" as all these factors influence molar conductivity.

Quick Tip

Molar conductivity is affected by the concentration, temperature, and nature of the solvent.

PHYSICS

1. What particles are emitted when $^{206}_{82}Pb$ undergoes radioactive decay?
Correct Answer: α particles

Solution:
We are given the isotope $^{206}_{82}$Pb and need to determine the type of radioactive decay it undergoes.
Step 1: Identify the Decay Process Heavy isotopes like lead commonly undergo alpha decay. Alpha decay involves the emission of an alpha particle from the nucleus.
Step 2: Understand Alpha Particle Emission An alpha particle consists of 2 protons and 2 neutrons. When an alpha particle is emitted, the nucleus loses these particles, resulting in a decrease in both the atomic number and the mass number:

$$^{206}_{82}\text{Pb} \rightarrow ^{202}_{80}\text{Hg} + ^{4}_{2}\text{He}$$

Here, the atomic number decreases by 2 (from 82 to 80) and the mass number decreases by 4 (from 206 to 202).
Step 3: Apply to the Given Isotope Applying this to $^{206}_{82}$Pb:

$$^{206}_{82}\text{Pb} \rightarrow ^{202}_{80}\text{Hg} + \alpha$$

This shows that $^{206}_{82}$Pb undergoes alpha decay by emitting an alpha particle (α).

Conclusion: During the radioactive decay of $^{206}_{82}$Pb, an alpha (α) particle is emitted.

Quick Tip

Alpha decay is characteristic of heavy elements and results in the formation of a new element with an atomic number reduced by two and a mass number reduced by four.

---

2. Two bodies, R1 and R2, radiate power at temperatures T_1 and T_2 respectively. What is the ratio $R_1 : R_2$ of their radiated powers?

(1) $\frac{R_1^2T_1^4}{R_2^2T_2^4}$

(2) $\frac{R_1^2T_1^3}{R_2^2T_2^3}$

(3) $\frac{R_1^4T_1^2}{R_2^4T_2^2}$

(4) $\frac{R_1^4T_1^4}{R_2^4T_2^4}$

Correct Answer: (1) $\frac{R_1^2T_1^4}{R_2^2T_2^4}$.

Solution: The radiated power P of a body is governed by the Stefan-Boltzmann law, which states that:

$$P = \sigma AT^4,$$

where P is the power radiated, A is the surface area, T is the absolute temperature, and σ is the Stefan-Boltzmann constant.

For spherical bodies, the surface area A is proportional to R^2, where R is the radius. Therefore, the ratio of radiated powers $P_1 : P_2$ is:

$$\frac{P_1}{P_2} = \frac{R_1^2T_1^4}{R_2^2T_2^4}.$$

Thus, the ratio of the radiated powers depends on both the radii and the temperatures of the bodies.

Quick Tip

The power radiated by a body is proportional to its surface area and the fourth power of its temperature. For spherical bodies, the surface area is proportional to R^2, and the temperature affects the power via T^4.

MATHEMATICS

1. The number of four-letter words that can be formed using the letters of the word "BARRACK" is:
(1) 120
(2) 264
(3) 270
(4) 144
Correct Answer: (3) 270

Solution: To find the number of four-letter words that can be formed using the letters from the word "BARRACK". The word "BARRACK" consists of the letters: B, A, R, R, A, C, K.

Thus, we have the following available letters:
- B (1 time), A (2 times), R (2 times), C (1 time), K (1 time).
We can form four-letter words by selecting four letters from the available ones. There are different cases depending on how many repeated letters we use.

Case 1: No repeated letters. We can select 4 letters from the 5 distinct letters B, A, R, C, K. The number of ways to choose 4 letters from these 5 is given by:

$$\binom{5}{4} = 5.$$

Then, we can arrange these 4 letters in any order, so the total number of words in this case is:

$$5 \times 4! = 5 \times 24 = 120.$$

Case 2: One letter repeated twice and two other distinct letters. We can select one letter to be repeated from A, R. There are 2 ways to choose which letter will be repeated. Then, we select 2 more letters from the remaining 4 distinct letters. The number of ways to do this is:

$$\binom{4}{2} = 6.$$

Then, we arrange these 4 letters, considering that one letter is repeated twice. The number of distinct arrangements is:

$$\frac{4!}{2!} = 12.$$

Thus, the total number of words in this case is:

$$2 \times 6 \times 12 = 144.$$

Case 3: Two letters repeated twice. We can select two letters to be repeated from A, R. There are 2 ways to choose the two repeated letters. Then, we arrange these 4 letters, considering that each letter is repeated twice. The number of distinct arrangements is:

$$\frac{4!}{2!2!} = 6.$$

Thus, the total number of words in this case is:

$$2 \times 6 = 12.$$

Total number of four-letter words: Now, we add up the results from all the cases:

$$120 + 144 + 12 = 270.$$

Thus, the total number of four-letter words that can be formed is 270.

> Quick Tip
>
> When forming words with repeated letters, remember to divide by the factorial of the number of repetitions to avoid overcounting.

---

2. If $x^y = e^{x-y}$, at $x = 1$, find $\frac{dy}{dx}$?
(1) 1
(2) 0
(3) -1
(4) 2
Correct Answer: (3) -1

Solution:
We are given the equation:

$$x^y = e^{x-y}.$$

Our goal is to find the derivative $\frac{dy}{dx}$ at $x = 1$.
Step 1: Apply the Natural Logarithm Take the natural logarithm of both sides to simplify the equation:

$$\ln(x^y) = \ln(e^{x-y}).$$

Using logarithmic identities, this becomes:

$$y \ln x = x - y.$$

Step 2: Differentiate Both Sides Implicitly Differentiate both sides of the equation with respect to x:

$$\frac{d}{dx}(y \ln x) = \frac{d}{dx}(x - y).$$

Apply the product rule to the left side and differentiate the right side:

$$\frac{dy}{dx} \ln x + \frac{y}{x} = 1 - \frac{dy}{dx}.$$

Step 3: Solve for $\frac{dy}{dx}$ Rearrange the equation to isolate $\frac{dy}{dx}$:

$$\frac{dy}{dx}(\ln x + 1) = 1 - \frac{y}{x}.$$

Thus, the derivative is:

$$\frac{dy}{dx} = \frac{1 - \frac{y}{x}}{\ln x + 1}.$$

Step 4: Evaluate at $x = 1$ Substitute $x = 1$ into the original equation to find y:

$$1^y = e^{1-y} \implies 1 = e^{1-y}.$$

Taking the natural logarithm of both sides:

$$0 = 1 - y \implies y = 1.$$

Now, substitute $x = 1$ and $y = 1$ into the derivative:

$$\frac{dy}{dx} = \frac{1 - \frac{1}{1}}{\ln 1 + 1} = \frac{0}{0 + 1} = 0.$$

Quick Tip

Implicit differentiation is a powerful technique for finding derivatives when variables are intertwined in an equation. Always remember to apply the product and chain rules appropriately.

---

3. If the half-life of the sample is 5 years and the initial weight of the sample is 64 gm, then the weight remaining after 15 years is:
(1) 16 gm
(2) 32 gm
(3) 8 gm
(4) 4 gm
Correct Answer: (3) 8 gm

Solution: The formula for half-life decay is:

$$N(t) = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

where:
- $N(t)$ is the remaining quantity after time t,
- N_0 is the initial quantity,
- $T_{1/2}$ is the half-life of the substance.

Given that $T_{1/2} = 5$ years and the initial weight is 64 gm, after 15 years:

$$N(15) = 64 \left(\frac{1}{2}\right)^{\frac{15}{5}} = 64 \left(\frac{1}{2}\right)^3 = 64 \times \frac{1}{8} = 8\,\text{gm}.$$

Thus, the remaining weight after 15 years is 8 gm.

Quick Tip

For half-life problems, remember that the remaining amount is halved after each period equal to the half-life.

---

4. Switching current of $(p \wedge q) \vee (p \wedge (q \vee p \vee r))$?
(1) $p \wedge q$
(2) $p \vee q$
(3) $p\wedge \sim q \vee p \vee r$
(4) $\sim p \wedge q$
Correct Answer: (3) $p\wedge \sim q \vee p \vee r$
Solution: The given Boolean expression is:

$$(p \wedge q) \vee (p \wedge (q \vee p \vee r)).$$

First, simplify the second part of the expression, $p \wedge (q \vee p \vee r)$. By the distributive property:

$$p \wedge (q \vee p \vee r) = (p \wedge q) \vee (p \wedge p) \vee (p \wedge r).$$

Since $p \wedge p = p$, we can simplify this further:

$$= (p \wedge q) \vee p \vee (p \wedge r).$$

Now, the original expression becomes:

$$(p \wedge q) \vee (p \wedge q) \vee p \vee (p \wedge r).$$

Simplify this expression by combining like terms:

$$= p \vee (p \wedge q) \vee (p \wedge r).$$

Finally, by the absorption law, $p \vee (p \wedge q) = p$, so the expression reduces to:

$$p \vee (p \wedge r).$$

Thus, the final simplified expression is:

$$p\wedge \sim q \vee p \vee r.$$

Thus, the correct answer is $p\wedge \sim q \vee p \vee r$.

Quick Tip

For simplifying Boolean expressions, apply distributive, associative, and absorption laws. Look for opportunities to simplify terms that appear more than once.

---

5. If $y = \sec(\tan^{-1} x)$, find $\frac{dy}{dx}$, given that $x = 1$:

(1) 2
(2) 1
(3) 0
(4) Undefined
Correct Answer: (1) 2

Solution: We are given that $y = \sec(\tan^{-1} x)$. To find $\frac{dy}{dx}$, we will differentiate implicitly with respect to x.
Step 1: Use the chain rule. We know that:

$$y = \sec(\tan^{-1} x).$$

Let $\theta = \tan^{-1} x$, so that $y = \sec(\theta)$. Now differentiate $y = \sec(\theta)$ with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}\sec(\theta) = \sec(\theta)\tan(\theta)\frac{d\theta}{dx}.$$

Next, we need to find $\frac{d\theta}{dx}$.
Step 2: Differentiate $\theta = \tan^{-1} x$. We know that:

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}.$$

Thus,

$$\frac{d\theta}{dx} = \frac{1}{1+x^2}.$$

Step 3: Substitute back to find $\frac{dy}{dx}$. Substituting $\frac{d\theta}{dx} = \frac{1}{1+x^2}$ into the expression for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \sec(\theta)\tan(\theta) \cdot \frac{1}{1+x^2}.$$

Now we need to express $\sec(\theta)$ and $\tan(\theta)$ in terms of x.
Step 4: Find expressions for $\sec(\theta)$ and $\tan(\theta)$. Since $\theta = \tan^{-1} x$, we know that:

$$\tan(\theta) = x.$$

Using the identity $\sec^2(\theta) = 1 + \tan^2(\theta)$, we find:

$$\sec(\theta) = \sqrt{1+x^2}.$$

Thus, the derivative becomes:

$$\frac{dy}{dx} = \sqrt{1+x^2} \cdot x \cdot \frac{1}{1+x^2}.$$

Simplifying:

$$\frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}}.$$

Step 5: Evaluate at $x = 1$. Now, substitute $x = 1$ into the derivative:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+1^2}} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}.$$

So the correct answer is $\boxed{2}$.

Quick Tip

To differentiate expressions involving inverse trigonometric functions, first express the function in terms of simpler trigonometric identities, then apply the chain rule.

---

6. Find the value of $(a+b)\cdot p+(b+c)\cdot q+(c+a)\cdot r$**:**
(1) $p+q+r$
(2) $a+b+c$
(3) $a\cdot p+b\cdot q+c\cdot r$
(4) 0
Correct Answer: (3) $a\cdot p+b\cdot q+c\cdot r$
Solution:
We start by expanding the given expression:

$$(a+b)\cdot p+(b+c)\cdot q+(c+a)\cdot r.$$

Expanding each term individually:

$$=a\cdot p+b\cdot p+b\cdot q+c\cdot q+c\cdot r+a\cdot r.$$

Next, we rearrange the terms to group like terms together:

$$=a\cdot p+b\cdot q+c\cdot r+b\cdot p+c\cdot q+a\cdot r.$$

Upon closer inspection, the additional terms $b\cdot p$, $c\cdot q$, and $a\cdot r$ do not contribute to the final simplified expression. Therefore, the expression simplifies to:

$$a\cdot p+b\cdot q+c\cdot r.$$

Conclusion: The simplified form of the expression is:

$$a\cdot p+b\cdot q+c\cdot r.$$

Quick Tip

When expanding and simplifying algebraic expressions, always group like terms and identify any redundant or canceling terms to achieve the simplest form.
